MACHINE DESIGN

Maleev and Hartman

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Machine Design

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Preface to the Third Edition

The objective in preparing this Third Edition of Machine Design has been threefold: (1) to bring the book up to date; (2) to improve it as a teaching device; and (3) to increase ease of comprehension by the student.

To bring the book up to date the authors have now included, in addition to discussion of many lesser but nevertheless significant developments, topics of such major importance as those on the unified screw-thread system; new fastening devices; involute splines; new approaches to design of gears; and new materials, such as ductile iron and the various aluminum alloys.

To improve the book as a teaching device several sections have been elaborated; the topics in some of the chapters have been rearranged in order to bring out more clearly the importance of certain approaches to design problems; new sections have been introduced to reflect the increasing use of more thorough stress analysis, including statically indeterminate structures, curved beams, and shear in beams; the topics on dynamic stresses and stress concentration have been grouped together in a separate chapter to give them greater emphasis; and more emphasis has been assigned to the importance of understanding manufacturing processes, by the addition of two chapters and by grouping all of the chapters on manufacturing in a new Part II. In order to maintain flexibility of the text for classroom use, which was one of the aims of the previous edition, the new topics have been so presented that those instructors who do not consider them necessary for an undergraduate course can omit them without impairing the usefulness of the book.

To increase ease of comprehension by the student those portions of the previous edition which otherwise remain unchanged have been carefully re-edited by minor but important rephrasing whenever the student might benefit from increased clarity and definition.

With this edition Professor James B. Hartman enters as coauthor, but responsibility for the scope, organization, emphasis, and content of the book remains with the senior author.

The authors are indebted to all their colleagues who offered suggestions for improvement, and to Professors Sydney F. Duncan (University of Southern California), Boynton M. Green (Stanford University), Earl D. Hay

(Iowa State College), and Carl W. Nelson (University of California), for their constructive criticism of the manuscript. For many of the drawings and much of the engineering data, the authors also express their gratitude to the many industrial concerns most of which are mentioned by name throughout the body of the text.

V. L. M.
J. B. H.

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Abbreviations and Symbols

ABBREVIATIONS

Abbreviation	Meaning	Abbreviation	Meaning
abs	absolute	hp	horsepower
AGMA	American Gear Manufacturers Association	in.	inch(es)
		in.-lb	inch-pound(s)
AISI	American Iron and Steel Institute	kpsi	1000 pounds per square inch
ASA	American Standards Association	lb	pound(s)
		lb-ft	pound-foot (feet)
ASME	American Society of Mechanical Engineers	lb-in.	pound-inch(es)
		min	minute(s)
bhp	brake horsepower	psi	pound(s) per square inch
Bhn	Brinell hardness number	psig	pound(s) per square inch gauge
Btu	British thermal unit(s)		
cu	cubic (inches or feet)	rms	root mean square value
DFA	Drop Forging Association	rpm	revolutions per minute
F	Degree(s) Fahrenheit	SAE	Society of Automotive Engineers
fpm	feet per minute		
fps	feet per second	sec	second(s)
ft	foot, feet	sq	square (inches or feet)
ft-lb	foot-pound(s)	vibr	vibrations

SYMBOLS

[With Dimensions in the Technical System (F, L, T)]

Symbol	Units	Dimensions	Name of Quantity
a	fpsps	LT^{-2}	acceleration, linear
A	sq in.	L^2	area, total or of a cross section
b	in.	L	breadth, width
c	in.	L	distance from neutral axis to extreme fiber
C	....	....	constant (may have various subscripts)
C	lb	F	centrifugal force
d, D	in. or ft	L	diameter
e	in.	L	deformation, total
e	in.	L	eccentricity, as of force application
e	1	0	efficiency (mostly with a subscript)
E	psi	F/L^{-2}*	modulus of elasticity, direct (tension or compression)
f	1	0	coefficient of sliding friction
f	vibr per sec	T^{-1}	frequency of vibration
F	lb	F	force, concentrated load

* The notation / indicates that the two terms are at right angles.

Symbol	Units	Dimensions	Name of Quantity
g	fpsps	LT^{-2}	acceleration due to gravity, 32.2 fpsps
G	psi	$FL^{-2}(\theta)^{-1}$	modulus of elasticity, transverse (shear or torsion)
h	in.	L	height, depth, or thickness
i	1	0	number of elements if n is used for rpm
I	$in.^4$	L^4	moment of inertia, rectangular (for areas)
I	$lb\text{-}in.\text{-}sec^2$	FLT^2	moment of inertia, rotating mass
J	$in.^4$	L^4	moment of inertia, polar (for areas)
k, K	....	....	coefficients in empirical formulas, mostly with subscripts
K	1	0	stress-concentration factor due to discontinuities (also with subscripts)
k	in.	L	radius of gyration, rectangular
k_o	in.	L	radius of gyration, polar
l, L	in. or ft	L	length, distance
m	lb per (ft per sec^2)	$FL^{-1}T^2$	mass
M	lb-in. or lb-ft	FiL	moment of a force couple
n	1	0	factor of safety, design
n'	1	0	factor of safety, actual
n	1	0	number of elements or parts
n	rpm	T^{-1}	number of revolutions per minute
N	lb	F	force, normal
p	in.	L	pitch (also a ratio without denomination)
p	psi	FiL^{-2}*	pressure
P	hp	FLT^{-1}	horsepower, 33,000 ft-lb per min
Q	lb	F	load, total
r, R	in.	L	radius, radius of curvature
R	lb	F	force of reaction
s	psi	FiL^{-2}*	stress, direct or normal, tensile or compressive
s_c	psi	FiL^{-2}*	stress, compressive only when necessary to differentiate from tensile
s_n, s_o	psi	FiL^{-2}*	stress, nominal
s_s	psi	FL^{-2}	stress, shear or tangential
S_d	psi	FiL^{-2}* or FL^{-2}	design stress
S_e	psi		elastic limit
S_{en}	psi		endurance limit
t	deg F	0	temperature
t	sec, min, hr	T	time
T	lb-in. or lb-ft	FiL*	torque, torsional moment
T	sec	T	period (harmonic motion)
u	in.-lb per cu in.	FiL^{-2}*	modulus of resilience
U	in.-lb	FL	resilience
v	fps or fpm	LT^{-1}	velocity, linear
V	cu in.	L^3	volume
w	lb per cu in.	FL^{-3}	weight, specific
W	lb	F	weight, total, load
W	in.-lb or ft-lb	FL	work
y	in.	L	deflection
Y	1	0	Lewis factor in gear computations

* The notation i indicates that the two terms are at right angles.

Symbol	Units	Dimensions	Name of Quantity
Z	$in.^3$	L^3	section modulus, rectangular (for areas)
Z_o	$in.^3$	L^3	section modulus, polar (for areas)
Z	centipoise	$FL^{-2}T$	viscosity, absolute
Z_k	centistoke, sq ft per sec	L^2T^{-1}	viscosity, kinematic
α	1	0	coefficient of thermal expansion, linear
α β, γ	deg	0	angle between two lines
γ	1	0	specific gravity as compared with water
ϵ	1	0	unit deformation
θ	radian	LiL^{-1}*	angular distortion
θ	radian	LiL^{-1}*	angle
λ	deg	0	lead angle of worm or screw threads
μ	1	0	Poisson's ratio
μ_o	lb-sec per sq in.	$FL^{-2}T$	viscosity, absolute, Reynolds
π	1	0	ratio of circumference to diameter, 3.1416
ρ	lb per (ft per sec^2) per cu ft	$FL^{-4}T^2$	density, mass per unit of volume
ϕ	deg	0	angle of friction
ω	radian per sec	$(\theta)T^{-1}$	angular velocity

* The notation i indicates that the two terms are at right angles.

PART I: STRESSES AND MATERIALS

CHAPTER 1

Introduction

1–1. Machine design. Machine design is the art of developing new ideas for the construction of machines and expressing those ideas in the form of plans and drawings. The idea may be almost entirely new, as in the case of an invention or an improvement upon existing machinery; or it may be only partially new, as when a machine or a machine part is to differ in size, load, or materials from those already existing.

For a machine to be well-designed the parts must be strong enough for the duty required of them and must be adequate for the functions they must perform, but they must not involve unnecessary expenditure of material or prohibitive cost of construction.

To design well any machine or part, the designer must have a working knowledge of the elements of machine construction; must know how to analyze the applied forces and their reactions and how to determine the resulting stresses; must possess sufficient information about materials; and must understand the influence of shape, method of assembling, and working conditions of parts upon the operation and maintenance of the machine. Thus modern machine design involves the application of the principles of three fundamental engineering subjects: mechanisms, mechanics, and strength of materials, including elements of the theory of elasticity. In addition, possession of or access to experimental data on the performance of similar machines already existing is of great value.

1–2. Design procedure. The procedure for designing a machine usually involves the following main steps: (a) stating the problem; (b) analyzing the problem; (c) selecting the mechanism, materials, and stresses; (d) preparing the preliminary design; (e) revising the design; and (f) making final drawings.

In certain cases one or more of these steps may not be required or may be so simple as to disappear or to merge with another step. Whichever steps are included, however, the same logical sequence is used.

1–3. Stating the problem. To state a problem means to write down all data and also what is required. This information will indicate the nature of the problem and the purpose of the design and will help greatly in taking the necessary steps in proper sequence. Because putting down the data is a simple step in the solution of a problem, it is often overlooked or neglected, with resulting confusion and loss of time.

1–4. Analyzing the problem. A thorough analysis of all external and internal forces acting upon a machine part is essential. These forces may be classified as follows:

a. Useful loads due to the energy to be transmitted by the part
b. Dead-weight forces
c. Forces due to frictional resistance
d. Inertia forces due to changes in velocity
e. Centrifugal forces due to changes in the direction of motion
f. Forces due to changes of temperature
g. Forces due to procedures in manufacturing
h. Forces due to the shape of the part

Determination of forces. Forces must be determined with an accuracy consistent with the importance of the part. Complex loads should be resolved into combinations of simple forces or forces and couples. When forces and their relations change, as during the motion of the piston of an engine, the procedure must be repeated for two or more phases of the change in order to find the maximum stresses in the dangerous sections of the parts to be designed.

Assumptions. If the designer does not have sufficient information to evaluate accurately all the factors in an analysis, he should make certain assumptions based on similar, simpler, or more fully investigated conditions. No definite rules can be set for making such assumptions. However, the fewer the assumptions, the quicker and better will be the results. An assumption should be made only when there is no means of acquiring the necessary data. When an assumption is made it should always be on the safe side. If possible it should be checked in the course of preparing the design.

Experience is of great help in making necessary assumptions. The beginning designer should not become discouraged when he sees that he does not have the desired experience. He will acquire it by working systematically and by keeping a record of good and poor results.

Sketches. The result of an analysis of loads should be presented in the form of clear sketches showing all forces, with the direction, magnitude, and point of application of each, and the amount and position of each couple.

1–5. Selecting the mechanism, materials, and stresses. The three most important selections to be made in starting to design a machine are the mechanism, the materials to be used, and the allowable stresses.

Mechanism. The selection of the proper mechanism, or kinematic arrangement, for a machine part is usually indicated by the purpose for which the part is to be designed. Consideration is given to strength, wear, accuracy of motion, efficiency, and cost.

Materials. In the design of a certain machine part the selection of the most suitable material, from the large variety of modern materials available, is determined first by the shape of the part and secondly by the condition of loading to which the part is subjected. Resistance to corrosion is often an important consideration also, since corrosion greatly affects the life of various parts, particularly those subjected to repeated loads.

A designer must also keep costs in mind. If two materials are equally good for a specific application, the one that involves the least cost for the part should be selected. It must also be remembered that the cost of a manufactured object is the sum of the cost of the material and the cost of the labor necessary to make the object. Sometimes it is so much easier to machine a more expensive material that the saving in machining more than offsets the extra cost of the material. Since ordinarily a more expensive material is also a better one, the part so produced may be improved without an increase in cost.

Naturally, experience helps greatly in selecting the best materials. In discussing various machine details throughout this book, the proper material to be used will be mentioned.

Stresses. The selection of proper allowable stresses to be used in designing a part requires a working knowledge of materials. At the same time this selection must be based on an analysis of load variations, the stress distributon due to abrupt changes of sections, and other working conditions. Thus, in order to obtain uniform allowable stresses, it may be necessary in the calculations to use different nominal stresses for the same material in different parts of a machine or even in different sections of the same part.

1–6. Preparing the preliminary design. The purpose of the preliminary design is to determine the probable dimensions of all the parts that will form the machine. The chief consideration in establishing these dimensions is to provide sufficient strength, but rigidity and resistance to wear may also be important for certain parts.

Frequently the shape of a machine part depends on the operating conditions and the shape of adjoining parts to such an extent that the usual design procedure should be reversed. First the shape of the part is determined in accordance with service requirements; then the stresses in various sections are checked and necessary alterations are made in the dimensions.

If strength alone is the aim, the proper procedure is to design all sections of a part, and all parts of a machine, to be equally strong. An increase in a dimension of a section beyond the necessary size may mean not only a useless expenditure of material but also the weakening of an adjoining section and of the whole piece.

The designer should remember that the actual stresses in a part cannot always be computed by merely inserting numerical values for factors in

stress formulas. The proper selection of values for these factors and coefficients is possible only if the load application and the working conditions in general are clearly understood.

Parts that may develop resonance, whether through lateral or torsional vibration, must be checked for critical speeds.

Excessive wear must be avoided by using special materials with sufficiently large and rigid bearing areas; by making proper provision for lubrication; and by eliminating eccentric application of loads, which always tends to cause uneven wear.

If rigidity (that is, absence of excessive distortion) is essential for the operation of a machine or a part, the deflection of each part under the applied forces must be determined, and materials that have a high modulus of rigidity must be selected. Thus a construction of welded steel will often be preferable to one of cast iron. However, with our present knowledge of the strength and elasticity of materials and our knowledge of stress distribution, the determination of deflections by analysis can be carried out with sufficient accuracy only for simple sections. In determining deflections in parts with more-involved sections, either former experience with similar cases must be relied upon or special studies and experiments must be carried out to obtain relations between acting forces, dimensions, and deflections.

Sketches. The shapes decided upon in the preliminary design must be put on paper in the form of freehand sketches. A sufficient number of views and sections should be shown, and all the dimensions that can be computed should be included. Occasionally, oblique projections may help to present the design more clearly.

Calculations. All calculations must be as complete as possible and be noted in a neat and legible form. This is just as essential in a classroom as in an engineering office. Legibility facilitates the checking of results, whether it is done by the designer himself or by another person. Neatness in writing calculations should become a habit of the designer.

All assumptions and the reasoning behind them must be stated clearly so that the person who will check the calculations will know on what basis they were made.

1-7. Revising the design. Before working drawings are made from the sketches of the preliminary design, these sketches must be revised to take into consideration every practical requirement and contingency.

Manufacturing requirements. The first step in revision is to consider the problems of manufacturing, such as those involved in the making of patterns, in methods of forging, and in machining. The way a pattern can be simplified or made more cheaply, for instance, may suggest a change in the shape of a cast part. Sometimes the division of a complicated casting into two or more simpler ones may be advisable in order to decrease the

danger of having to scrap an entire defective piece. Due consideration might be given to the question of whether an improvement can be attained by changing a part ordinarily made of cast iron to a welded-steel construction. Forged pieces might be made less expensively by some changes in forging methods which would alter the shape. Sometimes it may be desirable to change the shape of a part so that the machining operation may be less expensive. The availability of existing patterns, fixtures, or tools may suggest other changes. In a manufacturing plant an experienced designer often consults the various foremen of the pattern shop, foundry, forge shop, and machine shop. The student designer can consult his instructor. He should also try to visualize the various manufacturing processes and the changes they suggest in the shape of his design.

The desirability of using standard commercial parts or of complying with the recommendations of the various committees on standardization may require changes of still another character.

Operation requirements. The second step in revision is to consider the requirements of the operation of the machine, such as provisions for lubrication and adjustment for wear. It may be found advisable to divide a part that is subject to severe wear into two pieces, one of which provides the strength while the other takes all the wear. The part which takes the wear should be made inexpensive and easily replaceable. The designer must also consider the safety problem, as by avoiding protruding parts in rotating members and by providing guards for gears and reciprocating parts.

Assembly requirements. The third and final step in revision begins when the designer starts to make assembly drawings. It may be found necessary to modify the shape of certain parts in order to avoid interference with adjoining parts or to connect them properly. Ease of assembly, maintenance, and dismantling should be considered carefully.

It may be advisable to change the shape of a part to give it a more pleasing appearance. Fanciful curves used in machinery fifty years ago are no longer in vogue. Simple lines that are appealing to the eye, and a general impression of well-balanced proportions, represent good design and help sell the finished machinery.

The assembly drawing should give all the dimensions and information needed for assembling and installing the machine, but it should show no unessential detail dimensions.

1-8. Making final drawings. After the assembly drawing has been made and all possible phases of revision have been taken into consideration, working drawings, or shop drawings, are made. The modern practice is to prepare them directly on vellum paper ready for blueprinting, and to show each part of the machine on a separate sheet to facilitate eventual altera-

tions. Despite the careful revision of the preliminary design, as just outlined, more changes may be found desirable after a piece has been made and used.

A working drawing must be clear, concise, and complete. It must have enough views and cross sections to show all details. The main view of a part should show it in the position it is designed to occupy. Every dimension must be given, so that there will be no occasion for guesswork and no necessity for scaling the drawing.

Tolerances. All dimensions that are important for correct assembly and interchangeability must be given in thousandths of an inch; and the tolerance, or limit of permissible deviation, should also be indicated. The use and magnitude of tolerances is discussed in Chapter 6.

Information and notes. A working drawing for a piece should contain, in the form of sections, dimensions, and notes, all the information that may assist the men in the shops in making the pattern for molding and casting the piece, in welding parts of it together, in machining it, or in heat-treating it. The sequence of machining operations may be given if it affects the accuracy of the final dimensions. The kind of finish should be indicated in order to obtain a piece with a finish required by its duty. This is explained in Chapter 6.

Bill of material. When the design of a mechanism or a subassembly of a machine has been completed, a bill of material, or list of all parts, is made. This list must show the number of every drawing, the name of the part on the drawing, the material of which the part is made, and the number of parts in the assembly. The bill of material must also show a list of all standard parts, such as bolts, nuts, washers, and cotter pins, their sizes, and the quantity of each needed.

CHAPTER 2

Static Stresses in Machine Parts

2–1. Deformations and stresses. When an external force, or *load*, is applied to a body, the body will move in the direction of the applied force unless that force is balanced by an opposite force called a *reaction*. Such forces always act in pairs (although mention of the reaction is often omitted). The action of this pair of forces produces a *deformation* of the body, also called a *strain*. The deformation may be only longitudinal, symbolized by e and by a plus sign (+) to indicate lengthening of the body or a minus sign (−) to indicate shortening. Sometimes the deformation is also angular, symbolized by α, denoting a change in the angle between adjacent elements of a body.

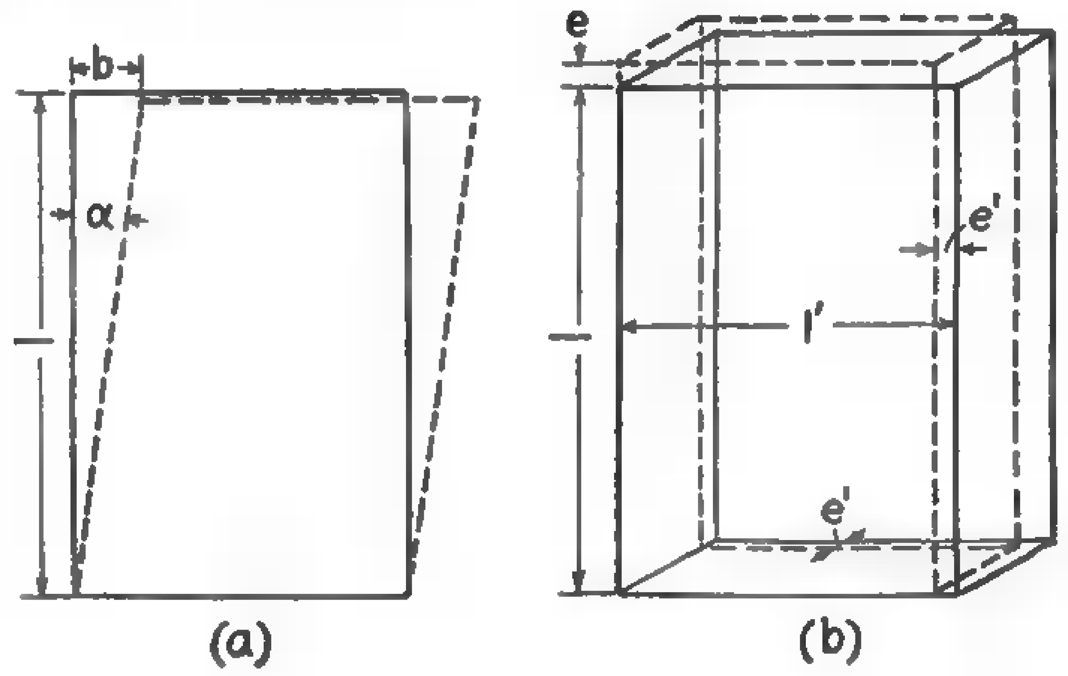


FIG. 2–1. Deformations.

Unit deformation. The deformation per unit length in the direction of the load is called *unit deformation*, or *strain*. It is symbolized by ϵ, and the units are in inches per inch. Thus,

$$\epsilon = \frac{e}{l} \tag{2–1}$$

Unit deformation α produced by angular distortion is called *shear strain* and is always measured in radians. Angular distortion is represented in Fig. 2–1a. Because the angle is very small, $\alpha = \sin\alpha = \tan\alpha$ and the shear strain may be expressed by the relation

$$\alpha = \frac{b}{l} \tag{2–2}$$

Poisson's ratio. When a bar is subjected to tension or compression and its length changes from l to $(l \pm e)$, as indicated in Fig. 2–1b, its transverse

TABLE 2–1

POISSON'S RATIO (μ)

Material	μ	Material	μ
Malleable cast iron	0.230	Zinc	0.331
Nickel	0.239	Copper	0.340
Gray cast iron	0.210–0.27	Brass	0.340
Cast steel	0.265	Ductile iron	0.34–0.37
Wrought iron	0.278	Aluminum, drawn	0.348
Steel, high-carbon	0.295	Inconel x	0.41
Steel, mild	0.303	Lead	0.431
Aluminum, cast	0.330	Rubber	0.45–0.49
Monel metal	0.32–0.37		

dimension l' changes to $(l' \mp e')$, the deformations e and e' having opposite signs. The ratio of the transverse strain $\epsilon' = e'/l'$ to the corresponding longitudinal strain $\epsilon = e/l$ is called *Poisson's ratio* and is designated μ. Thus

$$\mu = \frac{\epsilon'}{\epsilon} \tag{2-3}$$

Values of μ for various materials, found from tests, are given in Table 2–1.

Stresses. Any deformation of a body produced by external forces causes internal forces within its material. These internal forces are called *stresses.* A stress may be normal, tangential, or oblique to the plane on which it acts. A normal stress or the normal component of an oblique stress is referred to as a *direct stress.* Such a stress may be a tensile stress (+) or a compressive stress (−). A tangential stress is called *shear.* It may be produced either by a transverse load or by torsion.

Normal and tangential stresses seldom exist alone; any load usually produces a combination of both. The relative magnitudes of the stresses are determined by the type of load and by the location of the reference plane with respect to the direction of the load.

Unit stress, used in computations and usually called simply *stress,* is the amount of the internal force per unit area of the section. In the English system of units a stress is expressed in pounds per square inch. Direct unit stresses are designated by s, although s_c is sometimes used for compression. Shear is conveniently designated by s_s.

When several stresses act at a point, normal stresses that act on a plane through the point along which there is no shear are called *principal stresses.*

To bring out more clearly the meaning of stresses in various formulas, stresses *created* in a member will be designated by small letters, such as s, s_c, and s_s, while stresses that *characterize* a certain material will be designated by capital letters, such as S_e, S_u, and S_{en}.

Stress-strain diagram. The relation between loads on a specimen of the material and its deformations is determined by tests, and the results may be presented in a diagram like that in Fig. 2–2, where stresses are ordinates and strains are abscissas. The general form of the diagram is the same for either tensile or compressive stresses. Up to point e the strain is directly proportional to the stress (Hooke's law), and when the load is removed the body assumes its original shape and size. This tendency of the material to resume its original shape and size is called *elasticity,* and the stress S_e is called the *elastic limit.* The maximum stress up to which the strain of a material remains proportional to the stress is called the *proportional limit* S_p (not shown in Fig. 2–2). For the majority of metals S_p is very close to S_e. Therefore they are often considered equal and termed the *proportional elastic limit.* An increase of the stress above S_e is accompanied by a faster increase of deformation until point y is reached. At this point the resistance of the molecules begins to break down rapidly, and a sudden and large increase of deformation occurs without an increase in the load. The corresponding stress S_y is called the *yield point.* After the removal of a load causing a stress greater than S_e the body does not return to its original shape and a permanent set Oc remains. The maximum stress S_u, which results in the failure of the piece, is called the *ultimate strength.*

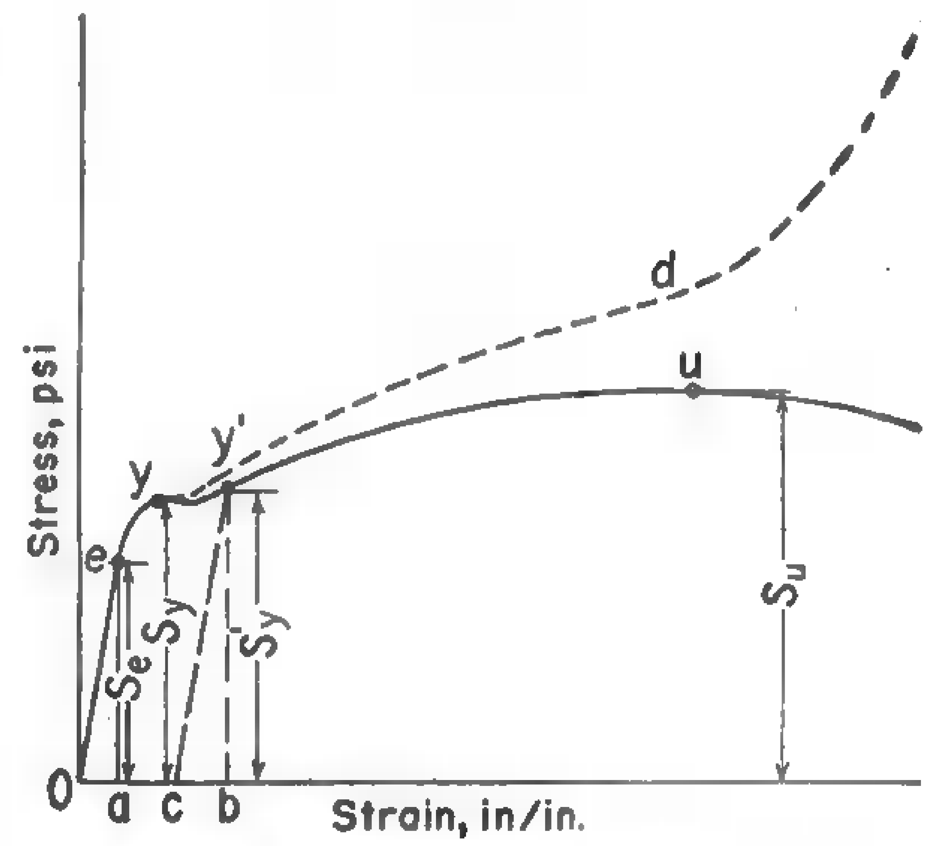


FIG. 2–2. Stress-strain diagram.

For the purpose of drawing a stress-strain diagram the stresses are referred to the original area of the test piece. If the actual areas, which gradually become smaller as the piece stretches, are used, the corrected stress curve would rise more rapidly, as shown by curve d in Fig. 2–2.

In order that a piece may retain its shape after the load has been removed, its stress must not exceed the elastic limit, a knowledge of which is therefore very important. In many instances it is easier to determine the proportional limit S_p. But some materials, such as cast iron or copper, do not have a proportional limit and do not follow Hooke's law. The deformation of such a material increases faster than the stress increases, even within the limits of elastic deformations. Accurate determination of the proportional elastic limit of the material is then difficult, and the yield point is often used instead. The yield point is therefore known also as the *commercial elastic limit.* However, the difference between the proportional elastic limit and the yield point may be considerable—up to 20 or 30 per cent; and so whenever possible the design should be based on the elastic limit and not on the yield point. In machine design the elastic limit is one of the most important properties of

a material. For a material that does not have a proportional elastic limit or for one for which a figure has not yet been established, the *apparent elastic limit* may be used. This means the limit stress which, when the load is removed, leaves such a small permanent deformation (0.02 per cent or less) that for all practical purposes it may be neglected.

2-2. The moduli of elasticity. A modulus of elasticity is defined as the ratio of the stress to the strain below the elastic limit.

The modulus of elasticity in tension, which is symbolized by E, is also called the *direct modulus*, or *Young's modulus*. It may be expressed by the relation

$$E = \frac{s}{\epsilon} \tag{2-4}$$

The value of E, which is expressed in pounds per square inch, is represented in Fig. 2-2 by the tangent of the angle eOa.

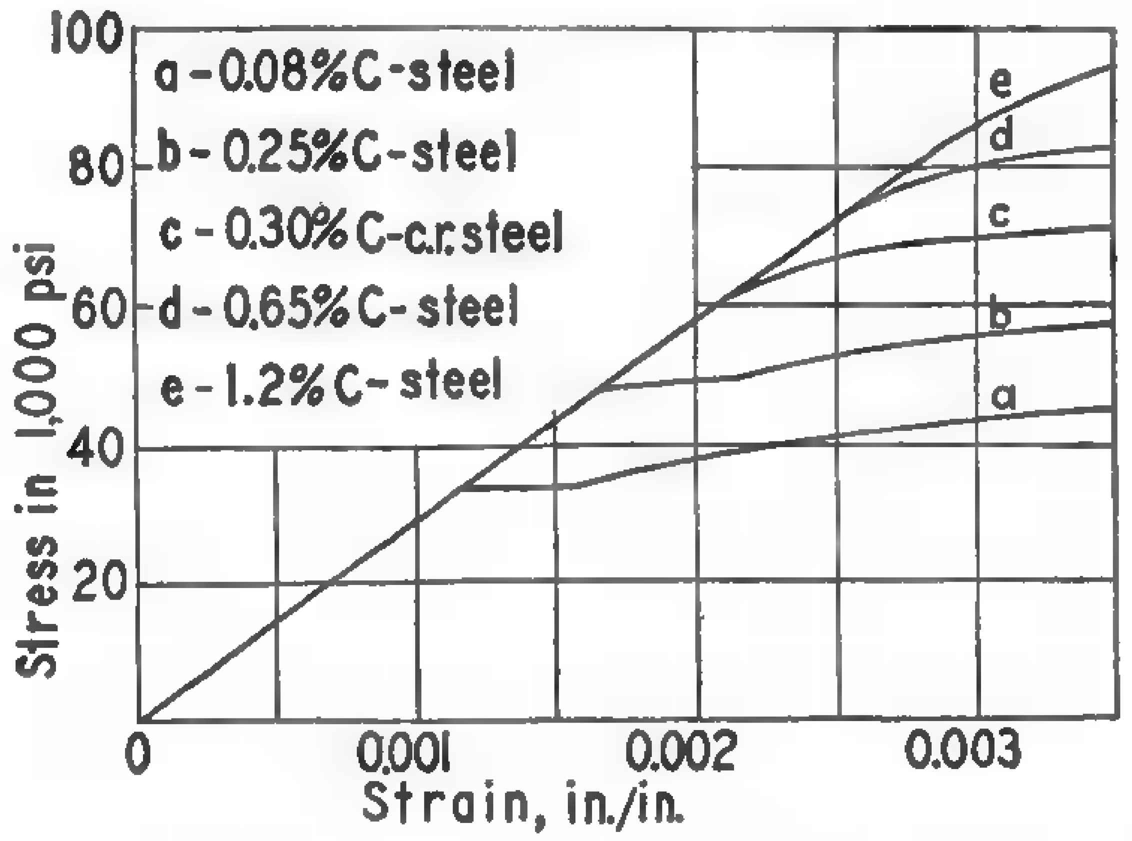


FIG. 2-3. Stress-strain diagrams for several steels.

The modulus of elasticity in compression is symbolized by E_c. It is determined in the same way as E. For most ductile materials the values of E_c and E are practically equal.

The modulus of elasticity does not depend upon the ultimate strength or elastic limit of a material, but it is a function of the stiffness of the material. Therefore it is often called the *modulus of rigidity* or the *coefficient of rigidity*. In Fig. 2-3 stress-strain diagrams are given for several grades of steel. These diagrams show that, regardless of the greatly varying values of ultimate strengths and elastic limits, the values of moduli of elasticity represented by the slopes of the lines of elastic deformation are almost the same for all these steels. The modulus decreases only slightly with an increase of the carbon content. A machine part made of soft steel therefore is practically as rigid as one made of high-carbon steel, although the one made of high-carbon steel may be two or more times as strong.

The modulus of elasticity in shear or torsion, which is symbolized by G, is also called the *transverse modulus*. It is equal to the ratio of the stress s_s and the deformation per unit length $c\theta/l$. Thus

$$G = \frac{s_s l}{c\theta} \tag{2-5}$$

where l is the length of the twisted bar, in inches; c is the distance from the surface to the center line, in inches; and θ is the angle of distortion, in radians.

In accordance with equation 2-2, $c\theta/l = \alpha$. If $c = 1$,

$$G = \frac{s_s}{\alpha} \tag{2-6}$$

Theoretically, G and E are related as follows:

$$G = \frac{E}{2(1+\mu)} \tag{2-7}$$

where μ is Poisson's ratio. Values of G computed by equation 2-7 are slightly smaller than those found from actual tests, but they may be used in the absence of information about G. Conversely, if E and G are known, μ may be computed with sufficient accuracy from the relation

$$\mu = \frac{E}{2G} - 1 \tag{2-8}$$

2-3. Simple stresses. Tension, compression, and shear are called *simple stresses* when they can be considered singly. A simple stress is considered to be distributed uniformly over the cross section of the part to which the force is applied.

Tensile stress. The stress s in simple tension in a machine member is equal to the external force F, in pounds, divided by the cross-sectional area A of the member, in square inches. Thus

$$s = \frac{F}{A} \tag{2-9}$$

From equation 2-1, the total elongation of a member l inches long is $e = \epsilon l$. Using for ϵ its value from equation 2-4 gives

$$e = \frac{sl}{E} \tag{2-10}$$

Then, using for s the value given by equation 2-9 results in

$$e = \frac{Fl}{AE} \tag{2-11}$$

Compressive stress. For a straight short compression member in which the resultant load F acts through the center of gravity of every cross section, the stress s is also found from equation 2-9.

If the length of a member is more than twenty times the least radius of gyration of its normal cross section, the member must be treated as a *column*, and the stresses are determined by one of the column formulas.

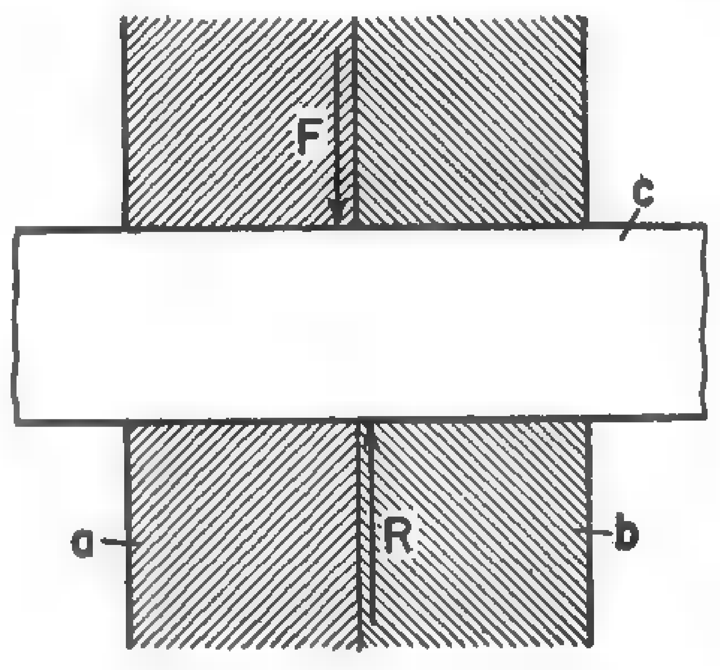


FIG. 2-4. Part in shear.

Shear stress. A shear stress is induced in a plane between two adjacent parts of a body by two equal external forces acting on opposite sides of the plane and in opposite directions along lines of action parallel to the plane considered. In Fig. 2-4 the load F acting downward upon plate a is transmitted to plate b through the dowel pin c and creates a reaction $R=F$ in plate b. The simultaneous action of the forces F and R induces shear stress in the dowel pin c. Ordinarily it is assumed that the shear stress is distributed uniformly over the cross section of the pin normal to its axis, in which case

$$s_s = \frac{F}{A} \tag{2-12}$$

The actual conditions are probably different. The stresses are greatest near the points of action of forces F and R and gradually decrease toward the center of the pin c. The assumption of uniform distribution of the shear stress over the whole area of the cross section is used only to simplify the computations. Such an assumption should never be made where shear is due to bending.

2-4. Torsion. Stresses produced by torsion and bending are termed *compound stresses,* because torsion and bending produce normal and tangential stresses in the same plane simultaneously.

Torsional stress. The stress produced in a member twisted by a couple is pure shear, but its intensity on any fiber depends on the distance to the fiber from the center line of the twisted member. For a circular section, the shear stress is proportional to the distance from this center line.

Equating the external torsional moment T to the internal resisting moment gives

$$T = s_s Z_o \tag{2-13}$$

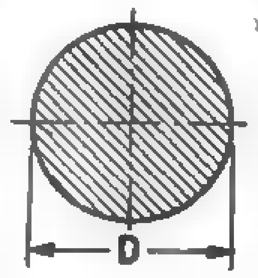
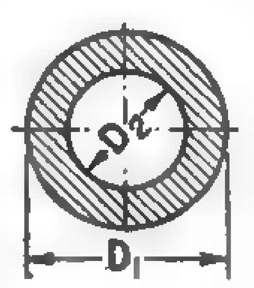
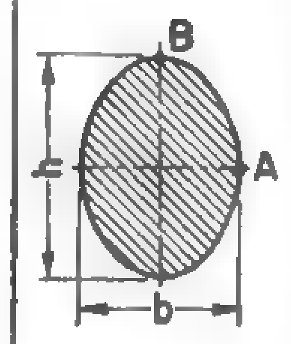
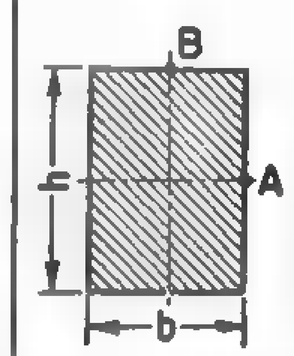
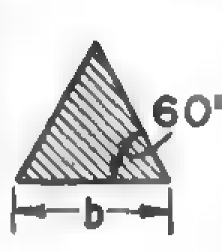
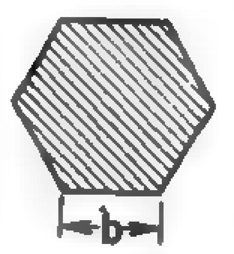
where Z_o is the polar section modulus and s_s is the maximum shear stress. For a solid or hollow circular section, $Z_o = J/c$, in which J is the polar moment of inertia and c is the distance from the axis to the most remote fiber where the stress is s_s.

For a solid round bar, $Z_o = \frac{1}{16}\pi D^3$ (see Table 2-2) and the maximum shear stress is expressed as follows:

$$s_s = \frac{16T}{\pi D^3} \tag{2-14}$$

TABLE 2-2

TORSION OF SHAFTS OF VARIOUS CROSS SECTIONS

Type	Cross Section	Polar Section Modulus $Z_o = \frac{J}{c}$	Polar Radius of Gyration k_o	Angular Deflection θ: In Terms of Torsional Moment T	Angular Deflection θ: In Terms of Maximum Stress s_s
a		$\frac{\pi D^3}{16}$	$\frac{D}{\sqrt{8}} = 0.354D$	$\frac{32l}{\pi D^4}\frac{T}{G}$	$\frac{2l}{D}\frac{s_s}{G}$, s_s at circumference
b		$\frac{\pi(D_1^4 - D_2^4)}{16D_1}$	$\sqrt{\frac{(D_1^2+D_2^2)}{8}} = 0.354\sqrt{D_1^2+D_2^2}$	$\frac{32l}{\pi(D_1^4-D_2^4)}\frac{T}{G}$	$\frac{2l}{D_1}\frac{s_s}{G}$, s_s at outer circumference
c		$\frac{\pi b^2 h}{16}$ * $h>b$	$\frac{1}{4}\sqrt{b^2+h^2}$	$\frac{16(b^2+h^2)l}{\pi b^3 h^3}\frac{T}{G}$	$\frac{(b^2+h^2)l}{bh^2}\frac{s_s}{G}$, s_s at A †
d		$\frac{2b^2h}{9}$ * $h>b$	$\sqrt{\frac{(b^2+h^2)}{12}} = 0.289\sqrt{b^2+h^2}$	$\frac{m(b^2+h^2)l}{b^3h^3}\frac{T}{G}$; $\frac{h}{b}$ = 1, 2, 4, 8; m = 3.56, 3.50, 3.35, 3.21; n = 0.79, 0.78, 0.74, 0.71	$\frac{n(b^2+h^2)l}{bh^2}\frac{s_s}{G}$, s_s at A ‡
e		$\frac{b^3}{20}$ *	$0.289b$	$\frac{46.2l}{b^4}\frac{T}{G}$	$\frac{2.31l}{b}\frac{s_s}{G}$, s_s at center of side
f		$0.92b^3$ *	$0.645b$	$\frac{0.967l}{b^4}\frac{T}{G}$	$\frac{0.9l}{b}\frac{s_s}{G}$, s_s at center of side

* This value is not a true value of Z_o but is the value of Z_o for a circular section of equal strength and may be used for determining the maximum stress by the formula $s_s = T/Z_o$.

† At B, shear stress $= 16T/\pi bh^2$.

‡ At B, shear stress $= 9T/2bh^2$.

Equation 2-13 does not apply for a noncircular section. Because of the uneven distortion of such a section, the greatest intensity of shear stress occurs in the outer fibers *closest* to the center line. The value of the maximum shear stress in torsion for one of the sections shown in Table 2-2 can be computed from the data in that table.

The relation between the angular deflection in radians and the torsional moment T for a section can be obtained by substituting for s_s in the equation for torque for that section (such as equation 2–14) the value of s_s from equation 2–5, and solving the resulting equation for θ. Thus, for a round shaft,

$$\theta=\frac{32lT}{\pi D^4G} \qquad (2\text{–}15)$$

Values of θ for other sections are given in the next-to-last column of Table 2–2.

The angular deflection θ of a bar may be expressed also as a function of the stress by applying equation 2–5. Thus for a round bar for which $c=D/2$,

$$\theta=\frac{2ls_s}{DG} \qquad (2\text{–}16)$$

Values of θ for other sections are given in the last column of Table 2–2.

Torque. It may be well to recall here the relation between the torque T, the horsepower P, and the speed n in revolutions per minute. If we designate by D the diameter at the end of which the tangential force F is applied and we express values in inches and pounds, the definition of horsepower is

$$P=\pi\left(\frac{D}{12}\right)\frac{nF}{60\times 550}$$

Since $FD/2=T$, solving for T gives

$$T=\frac{63{,}030P}{n} \qquad (2\text{–}17)$$

where T is expressed in pound-inches.

EXAMPLE 2–1. A shaft made of a seamless steel pipe with an outside diameter of 4.0 in. and a wall thickness of 0.226 in. transmits 250 hp at 375 rpm. Assume that the material is SAE 1010 steel. Determine (a) the maximum shear stress, (b) the shear stress at the inner surface of the shaft, and (c) the angular deflection of the shaft in a length of 10 ft.

The torque is, by equation 2–17,

$$T=\frac{63{,}030\times 250}{375}=42{,}000 \text{ lb-in.}$$

a) The polar section modulus of the shaft is determined from Table 2–2, for case b, when $D_1=4.0$ in. and $D_2=4-0.226\times 2=3.548$ in.:

$$Z_o=\frac{\pi(4.0^4-3.548^4)}{16\times 4.0}=4.85 \text{ in.}^3$$

Then, by equation 2–13,

$$s_s=\frac{42{,}000}{4.85}=8{,}660 \text{ psi}$$

b) In a round shaft the stress at any point is proportional to the distance from the center line. Since $D_2/2=1.774$ in., the shear stress at the inner surface is

$$\frac{8{,}660\times 1.774}{2.0}=7{,}680 \text{ psi}$$

c) From Table 4–2 the transverse modulus of elasticity for SAE 1010 steel is $G=11{,}700{,}000$ psi. By substituting known values in the expression in the fifth column of Table 2–2 for case b, and using $l=10\times 12=120$ in., we get

$$\theta=\frac{32\times 120\times 42{,}000}{\pi(4.0^4-3.548^4)\times 11{,}700{,}000}=0.045 \text{ radian}$$

This is equivalent to

$$\alpha=0.045\times\frac{180}{\pi}=2.58°$$

2–5. Bending. When a load is applied to a machine member transversely, it produces stresses of several kinds.

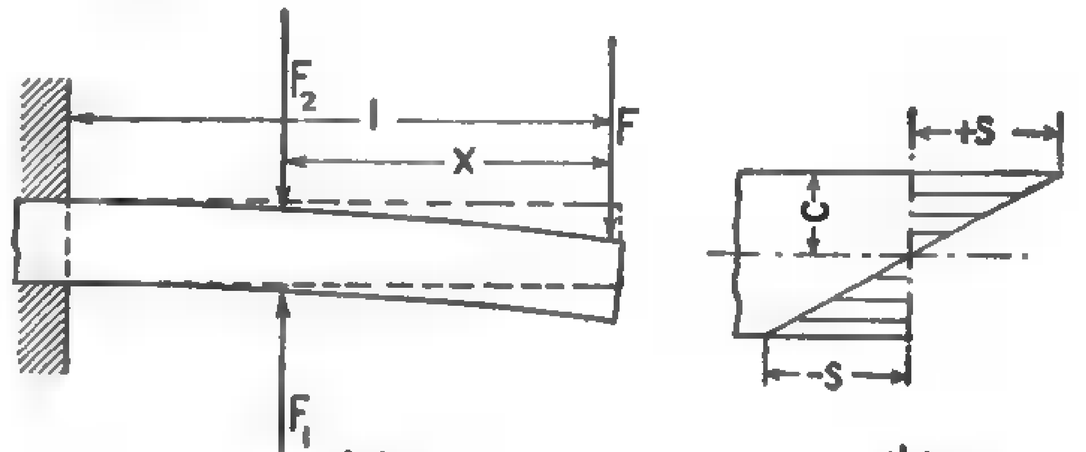


FIG. 2–5. Bending.

Tensile and compressive stresses. A cantilever beam carrying a concentrated load F at the end, Fig. 2–5a, may serve as an example of tensile and compressive stresses due to bending. The nature of the stresses at a section located at any distance x from the load F may be made evident by imagining that two opposite forces F_1 and F_2, each equal to F, are applied at this section. The forces F_1 and F form a couple whose moment is $M=Fx$. This couple bends the beam, producing a tensile stress on its upper, or convex, side and a compressive stress on the lower, or concave, side, as shown to a larger scale in Fig. 2–5b.

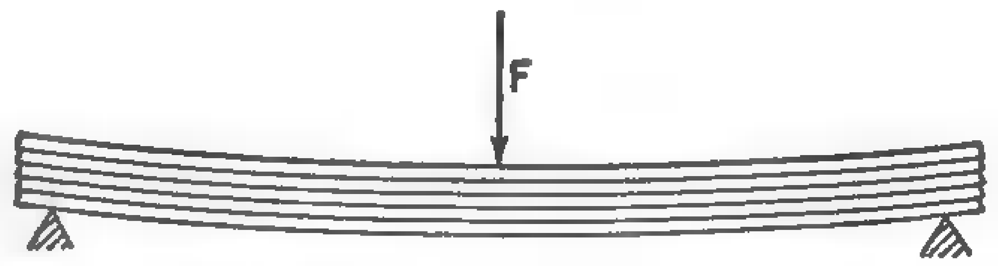


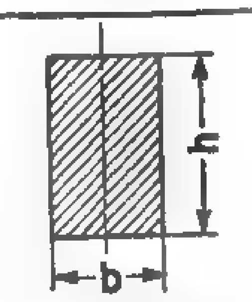
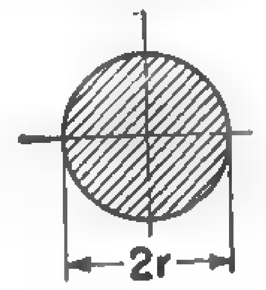
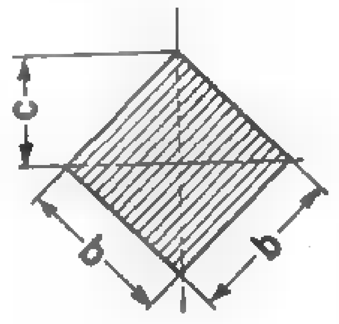
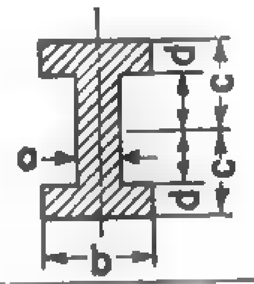
FIG. 2–6. Beam made up of several layers.

Shear stress. The remaining force F_2, Fig. 2–5a, produces a shear stress at right angles to the tensile and compressive stresses. This shear stress due to a transverse force, unlike a simple shear stress, is not distributed uniformly over the cross section of the beam. Its intensity is zero at the outer fibers and increases toward the neutral plane, where it is a maximum.

The distribution of shear stress in a beam can be best explained as follows: (1) If a shear stress exists on a plane at a certain point in a beam, there always exists a shear stress of equal intensity at that point on a plane at right angles to the first plane. (2) When a beam in bending assumes a

TABLE 2–3

SHEAR STRESS IN BEAMS, CAUSED BY BENDING

Type	Section	Shear Stress at a Distance y from Neutral Axis (psi)	Maximum Shear Stress (psi)
a		$\frac{3F}{2bh}\left[1-\left(\frac{2y}{h}\right)^2\right]$	$\frac{3F}{2bh}=1.5\frac{F}{A}$ (for $y=0$)
b		$\frac{4F}{3\pi r^2}\left[1-\left(\frac{y}{r}\right)^2\right]$	$\frac{4F}{3\pi r^2}=1.33\frac{F}{A}$ (for $y=0$)
c		$\frac{F\sqrt{2}}{b^2}\left[1+\frac{y\sqrt{2}}{b}-4\left(\frac{y}{b}\right)^2\right]$	$1.591\frac{F}{A}\left(\text{for } y=\frac{c}{4}\right)$
d		....	$\frac{3F}{4a}\left[\frac{bc^2-(b-a)d^2}{bc^3-(b-a)d^3}\right]$ (for $y=0$)

curved shape, the fibers in any two adjoining planes have a tendency to slide over one another, and shear stresses are created in planes parallel to the neutral plane. This is clearly shown in a simple beam composed of several layers, as indicated in Fig. 2–6. In this case the axial shear stress is zero at the outer fibers and its value gradually increases until it becomes maximum in the neutral plane.

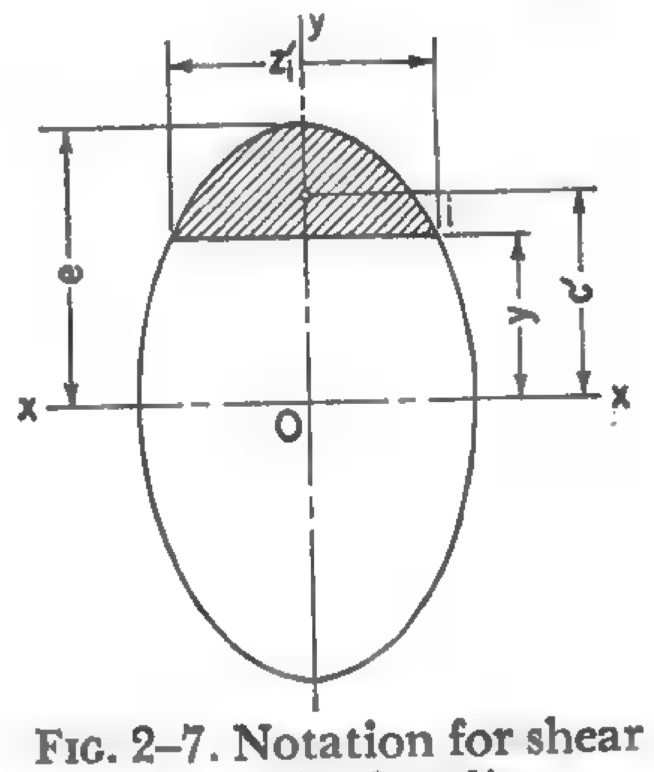


FIG. 2–7. Notation for shear caused by bending.

In accordance with statement 1 in the preceding paragraph, the transverse shear stress at any section follows the same pattern, and has the same value at each point, as the axial shear stress. A general expression for the shear stress at any point i in a beam, both in the axial plane and at right angles to it, may be written by using the designations of Fig. 2–7. This expression is

$$S_s=\frac{F_s}{Iz'}\int_{y}^{e} zy\,dy \qquad (2\text{–}18)$$

where F_s is the shear force, in pounds;

I is the moment of inertia of the full cross section with respect to the neutral axis $x-x$, in in.^{4}

Table 2–3 gives the values of the shear stresses found by equation 2–18 for several of the more important beam sections.

For an irregular cross section it may be convenient to use a modified equation. With the designations of Fig. 2–7,

$$S_s=\frac{F_sA'c'}{Iz} \qquad (2\text{–}19)$$

where A' is the cross-sectional area between the plane of the computed stress and the outer fiber of the beam, as the area shown cross-hatched in Fig. 2–7;

c' is the distance from the center of gravity, or *centroid*, of the cross-hatched area to the neutral axis or neutral plane.

In relatively long metal beams the actual shear stress is always small compared with the allowable value when the beam can safely withstand the tensile and compressive stresses, and shear is often neglected. In short beams, however, the shear stress may be large enough to be taken into account. Values of the maximum shear force for various types of beams and kinds of loads can be computed by using data from Table 2–4.

A valuable feature of shear diagrams, such as those shown in Table 2–4, in Fig. 2–15c, and elsewhere in the text, is that the bending moments have their highest values where the shear changes its sign. The change may be abrupt, as in cases c and d in Table 2–4, or it may be gradual, as in cases f and j.

The relation between the bending moment M, the normal tensile or compressive stress s, and the dimensions of the cross section of a beam may be obtained by equating the external moment to the internal-stress moment. Thus,

$$M=\frac{sI}{c} \qquad (2\text{–}20)$$

where I is the moment of inertia of the cross section with respect to the neutral axis normal to the direction of the load F, and where c is the distance from its center of gravity to the outermost fiber. The quantity I/c is called the *section modulus*. This definition is expressed by the equation

$$\frac{I}{c}=Z \qquad (2\text{–}21)$$

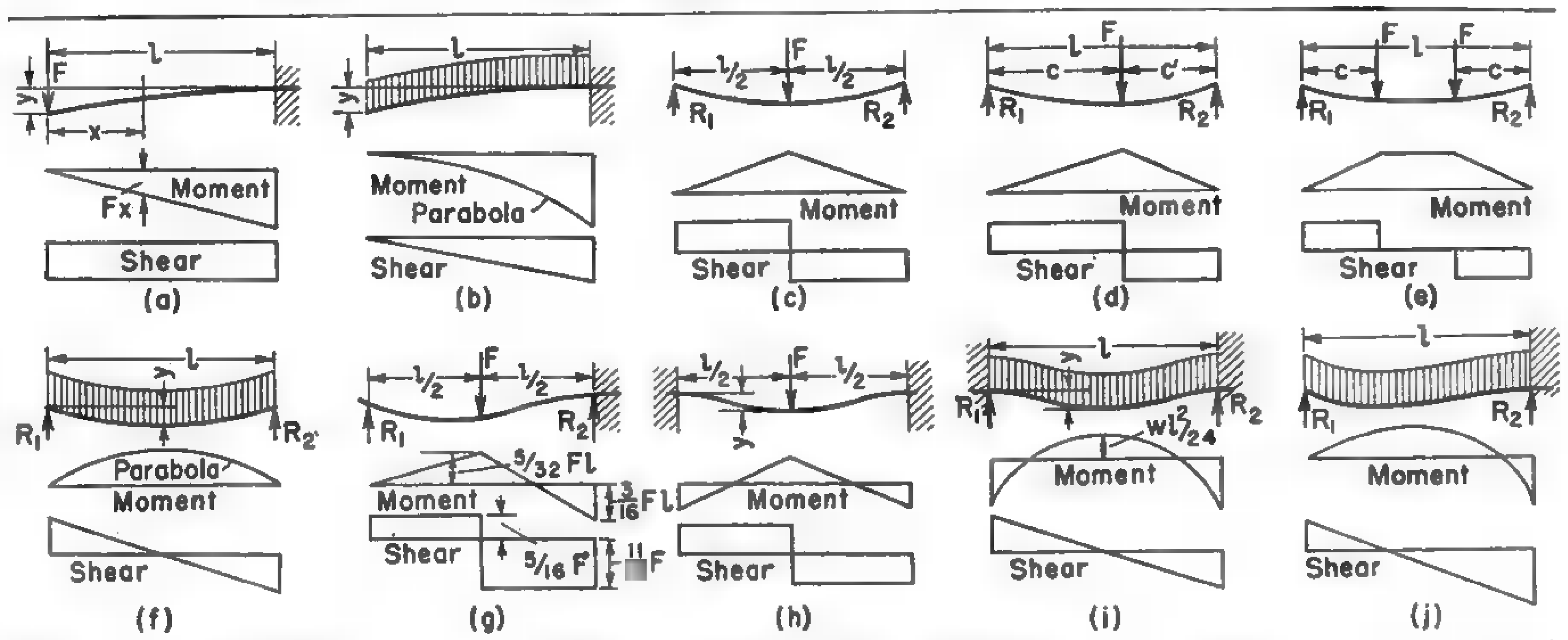
By substituting this value in equation 2–20 and solving for the stress, we get

$$s=\frac{M}{Z} \qquad (2\text{–}22)$$

Rigidity. When a load is placed on a beam, the beam is bent and every portion of the beam is moved in a direction parallel to the direction of the load. The distance that a point on the beam moves is called the *deflection*

TABLE 2-4

BEAMS OF UNIFORM CROSS SECTION, LOADED TRANSVERSELY



BEAM CASE	MAXIMUM BENDING MOMENT M	MAXIMUM SHEAR FORCE F_s	DANGEROUS SECTION	MAXIMUM DEFLECTION y	REACTIONS R_1	REACTIONS R_2
a	$-Fl$	$-F$	At support	$\frac{Fl^3}{3EI}$	F	..
b	$\frac{-wl^2}{2}$	$-wl = -F$	At support	$\frac{Fl^3}{8EI}$	F	..
c	$\frac{Fl}{4}$	$\pm\frac{F}{2}$	At center	$\frac{Fl^3}{48EI}$	$\frac{F}{2}$	$\frac{F}{2}$
d	$\frac{Fcc'}{l}$	$-\frac{Fc}{l}$	At load	$\frac{Fc'}{3EIl}\left[\frac{c(l+c')}{2}\right]^{\frac{3}{2}}$	$\frac{Fc'}{l}$	$\frac{Fc}{l}$
e	Fc	$\pm F$	Between the loads	$\frac{Fc(3l^2-4c^2)}{24EI}$	F	F
f	$\frac{wl^2}{8}$	$\pm\frac{wl}{2} = \pm\frac{F}{2}$	At center	$\frac{5Fl^3}{384EI}$	$\frac{F}{2}$	$\frac{F}{2}$
g	$-\frac{3}{16}Fl$	$-\frac{11}{16}F$	At fixed support	$\frac{7Fl^3}{768EI}$	$\frac{5}{16}F$	$\frac{11}{16}F$
h	$\pm\frac{Fl}{8}$	$\frac{F}{2}$	At support and at center	$\frac{Fl^3}{192EI}$	$\frac{F}{2}$	$\frac{F}{2}$
i	$-\frac{wl^2}{12}$	$\pm\frac{wl}{2} = \pm\frac{F}{2}$	At support	$\frac{Fl^3}{384EI}$	$\frac{F}{2}$	$\frac{F}{2}$
j	$\frac{wl^2}{8}$	$\frac{5}{8}wl = \frac{5}{8}F$	At fixed support	$\frac{Fl^3}{185EI}$	$\frac{3}{8}F$	$\frac{5}{8}F$

of the beam at that point. The deflection increases as the distance to the point from the support becomes greater, until a maximum value is reached. The rigidity of a beam is measured by its deflection, which can be determined by means of the equation

$$\frac{d^2y}{dx^2} = \frac{M}{EI} \tag{2-23}$$

where y is the deflection of the beam and x is the distance from the support. In order to use equation 2-23, an expression for the bending moment M in terms of x for the case considered must first be established and substituted in that equation; the resulting equation is then integrated twice to find y.

The maximum bending moments and deflections for the more frequently encountered types of bending loads are given in Table 2-4.

Table 2-5 gives properties of cross sections commonly adopted for beams.

EXAMPLE 2-2. Using the pipe of example 2-1 as a simple beam with a span of 10 ft between the supports, determine: (a) the load which, applied at the middle of the span, will produce a maximum bending stress of 13,400 psi; (b) the corresponding maximum deflection.

a) According to Table 2-5 the section modulus in bending is

$$Z = \frac{\pi(4.0^4 - 3.548^4)}{32 \times 4} = 2.425 \text{ in.}^3$$

Also, from Table 2-4, case c, the maximum moment is

$$M = F \times \frac{120}{4} = 30F$$

Substituting the values of s, Z, and M in equation 2-22 and solving the resulting equation for F, we get

$$F = 13{,}400 \times \frac{2.425}{30} = 1.083 \text{ lb}$$

b) The maximum deflection can be computed by substituting proper values in the expression in column 5 of Table 2-4 for case c; namely, $F = 1{,}083$ lb, $l = 120$ in., $E = 30{,}300{,}000$ psi (Table 4-2), $I = Zc = 2.425 \times 4/2 = 4.85$ in.^{3}

Hence,

$$y = \frac{1{,}083 \times 120^3}{48 \times 30{,}300{,}000 \times 4.85} = 0.265 \text{ in.}$$

2-6. Deflection of beams. When a beam is loaded, the neutral plane becomes a curved surface. Its intersection with a vertical plane drawn lengthwise through the center of the beam is called the *elastic curve*. The distance to any point on the elastic curve from the initial straight neutral axis of the unloaded beam is a measure of the *deflection* of the beam at that point.

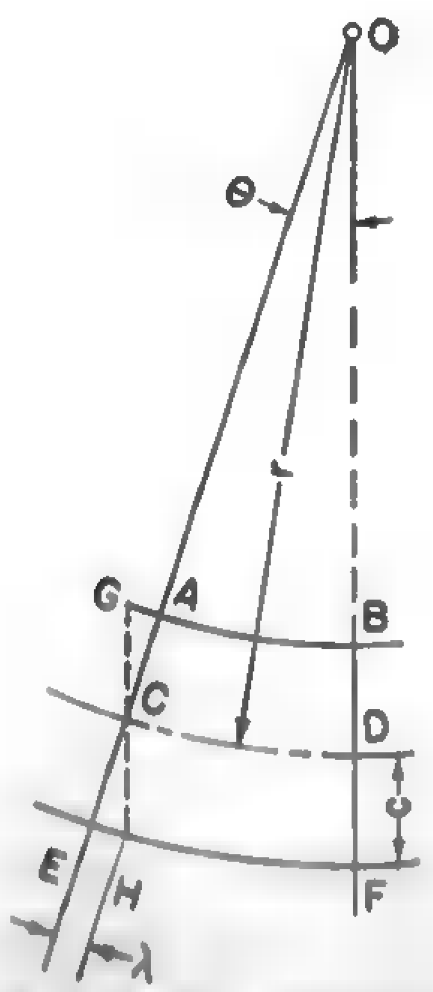
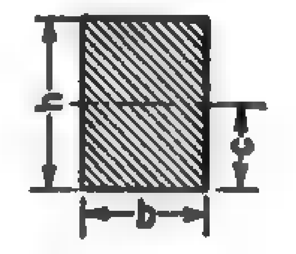
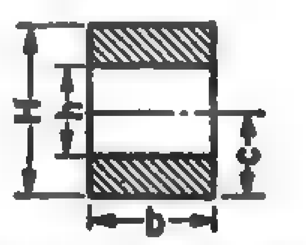
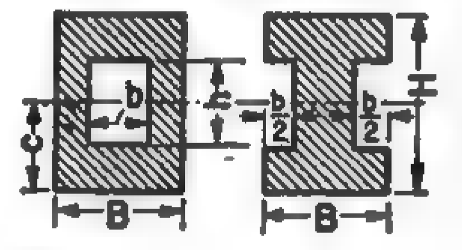
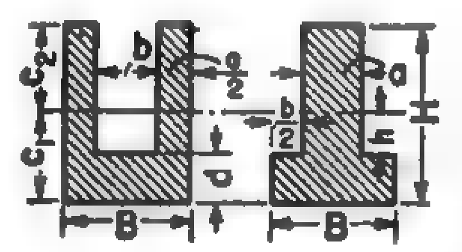
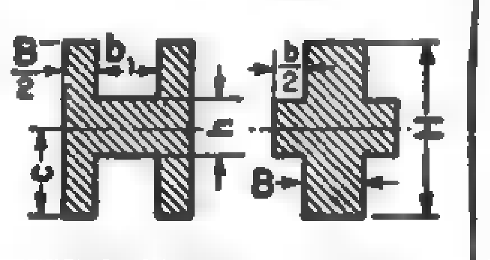
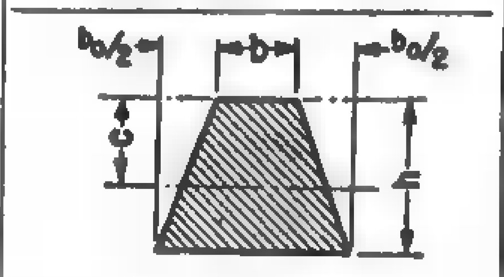
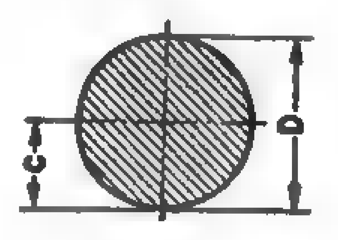
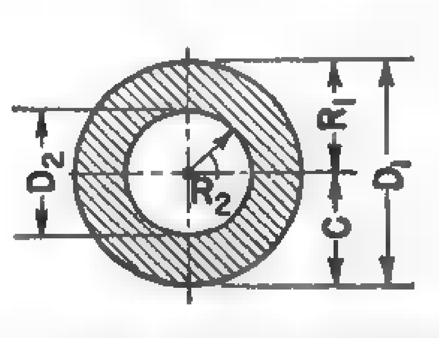
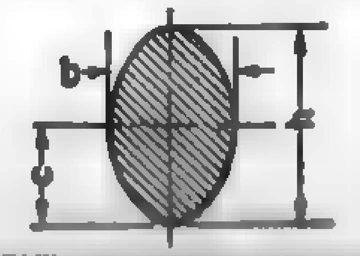


FIG. 2-8. Element of a bent beam.

Radius of curvature of elastic curve. The radius of curvature at any point on a curve is the radius of the circle that is tangent to the curve at the point. This radius is normal to the straight line that is tangent to the curve at the point. In general the elastic curve of a beam is not a circle. However, a very short length of the curve, such as $CD = \Delta l$ in Fig. 2-8, may be considered to be the arc of a circle. Before the load was applied to the beam, the section AE was in the position GH and was parallel to the section BF. After the load has been applied, the sections AE and BF make an angle θ with each other but remain perpendicular to the elastic curve CD. They are parts of

TABLE 2–5

PROPERTIES OF VARIOUS CROSS SECTIONS

Type	Section	Moment of Inertia I	Distance to Farthest Point c	Section Modulus $Z=\frac{I}{c}$	Radius of Gyration $k=\sqrt{\frac{I}{A}}$
a		$\frac{bh^3}{12}$	$h/$[illegible]	$\frac{bh^2}{6}$	$0.289h$
b		$\frac{b}{12}(H^3-h^3)$	$\frac{H}{2}$	$\frac{b(H^3-h^3)}{6H}$	$\sqrt{\frac{H^3-h^3}{12(H-h)}}$
c		$\frac{BH^3-bh^3}{12}$	$\frac{H}{2}$	$\frac{BH^3-bh^3}{6H}$	$\sqrt{\frac{BH^3-bh^3}{12(BH-bh)}}$
d		$\frac{Bc_1^3-bh^3+ac_2^3}{3}$ $h=c_1-d$	$c_1=\frac{aH^2+bd^2}{2(aH+bd)}$ $c_2=H-c_1$	$\frac{I}{c_1}$ and $\frac{I}{c_2}$	$\sqrt{\frac{I}{Bd+a(H-d)}}$
e		$\frac{BH^3+bh^3}{12}$	$\frac{H}{2}$	$\frac{BH^3+bh^3}{6H}$	$\sqrt{\frac{BH^3+bh^3}{12(BH+bh)}}$
f		$\frac{(6b^2+6bb_o+b_o^2)h^3}{36(2b+b_o)}$	$\frac{(3b+2b_o)h}{3(2b+b_o)}$	$\frac{(6b^2+6bb_o+b_o^2)h^2}{12(3b+b_o)}$	$\sqrt{\frac{I}{A}}$
g		$\frac{\pi D^4}{64}=\frac{\pi R^4}{4}$	$\frac{D}{2}=R$	$\frac{\pi D^3}{32}=0.0982D^3$	$\frac{D}{4}=\frac{R}{2}$
h		$\frac{\pi}{64}(D_1^4-D_2^4)$ $=\frac{\pi}{4}(R_1^4-R_2^4)$	$\frac{D_1}{2}=R_1$	$\frac{\pi(D_1^4-D_2^4)}{32D_1}$	$\frac{\sqrt{D_1^2+D_2^2}}{4}$ $=\frac{\sqrt{R_1^2+R_2^2}}{2}$
i		$\frac{\pi bh^3}{64}$	$\frac{h}{2}$	$\frac{\pi bh^2}{32}$	$\frac{h}{4}$

radii of the same circle, and the intersection of these radii at O is the *center of curvature*. Because of the bending of the beam, the fiber HF lengthens to EF. The increase in length is $EH=\lambda$, and the unit elongation, or strain, is

$$\epsilon=\frac{\lambda}{\Delta l} \tag{2–24}$$

Since the triangles COD and ECH are similar,

$$\frac{OD}{CD}=\frac{CH}{EH}$$

or

$$\frac{r}{\Delta l}=\frac{c}{\lambda} \tag{2–25}$$

Equating the values of ϵ from equations 2–4 and 2–24, and replacing s by the expression obtained by solving equation 2–20, gives

$$\frac{\lambda}{\Delta l}=\frac{s}{E}=\frac{Mc}{EI}$$

Also, from equation 2–25,

$$\frac{\lambda}{\Delta l}=\frac{c}{r}$$

Finally, by equating the two values of $\lambda/\Delta l$ and solving for r, there results

$$r=\frac{EI}{M} \tag{2–26}$$

From Fig. 2–8 it is evident that an increase in the deflection of a beam makes the radius of curvature smaller. Thus, the deflection is inversely proportional to the radius of curvature r. Therefore the deflection is directly proportional to M and inversely proportional to E and I.

This analysis also permits the establishment of a rule for determining the sign of the bending moment by noting the location of the center of curvature of the bent beam: If the center of curvature is *below* the beam, as in the case of a cantilever beam, the bending moment is considered *negative;* if the center of curvature is *above* the beam, as in the case of a simple beam, the bending moment is considered *positive.*

Deflection formula. In Fig. 2–9 let OU be the neutral axis of an unloaded cantilever beam supported at O and having its end U free; and let $OKRS$ represent to an exaggerated scale the elastic curve of the beam carrying a uniform load from O to P and having no load from P to U. For practical purposes it may be assumed that the point U moves downward vertically and its deflection is the distance US. The problem is to find the deflection QR of any point Q.

Suppose that $BCDB$ is the moment diagram. Divide the part OK of the elastic curve into a number of small segments, one of which is LN. Since the part PU of the beam does not carry any load, the part KS of the elastic

curve is a straight line. At L and N are drawn radii of curvature having a length r_i and forming an angle θ; the tangents LT and NV also form the angle θ with each other. If the length a_i of the arc LN is very small, it may be assumed that $LT = NV = l_i = x_i$. The bending of the portion of the beam from L to N causes point Q to deflect through the distance $TV = y_i$. But $y_i = l_i\theta$; $\theta = a_i/r_i$; and $r_i = EI/M_i$. Therefore,

$$y_i = \frac{l_i a_i}{r_i} = \frac{l_i a_i M_i}{EI} \tag{2-27}$$

Now the bending moment M_i at L is equal to the ordinate EG of the moment diagram. Also a_i is practically equal to the distance GH. Therefore a_iM_i is equal to the area A_i of the portion $EFHG$ of the moment diagram, and

$$y_i = \frac{x_i A_i}{EI} \tag{2-28}$$

The product x_iA_i is the moment of the moment area A_i with respect to point Q of the beam.

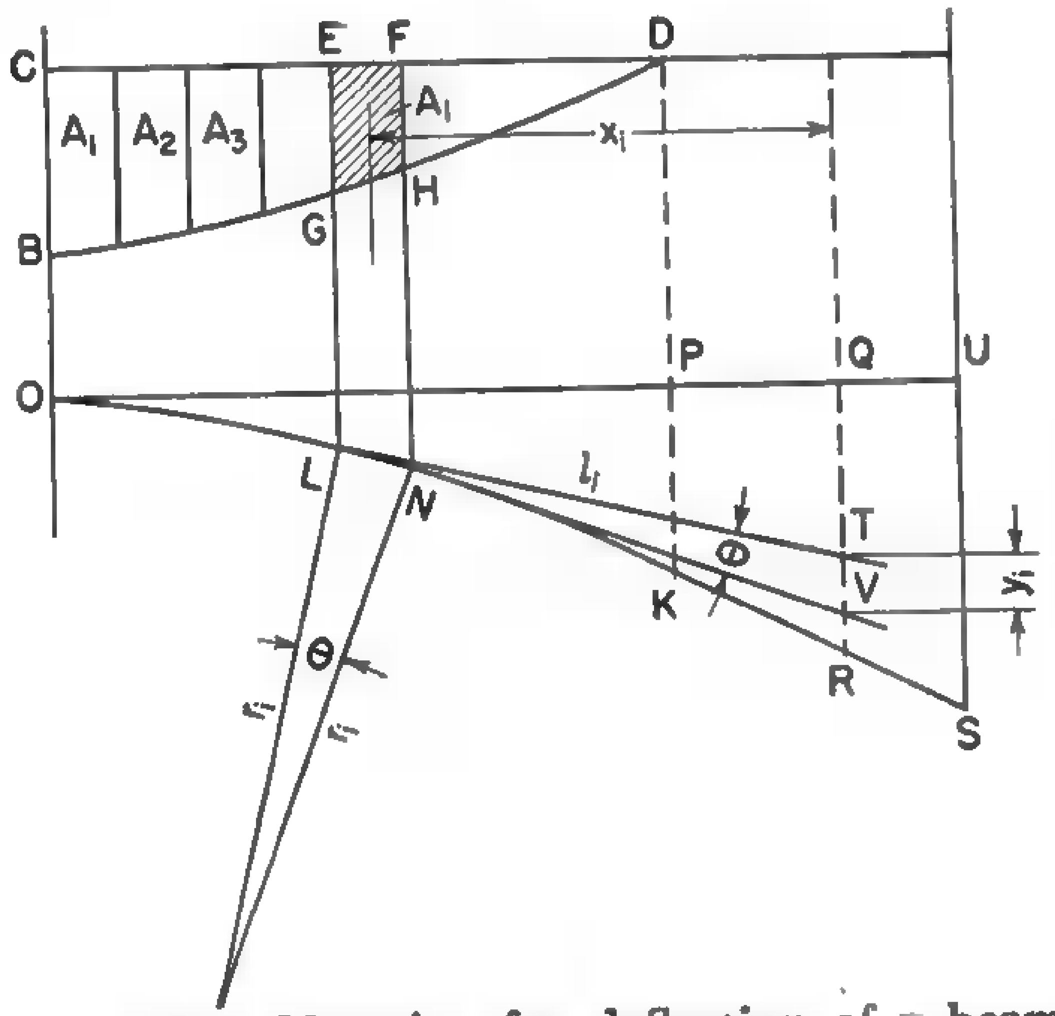


FIG. 2-9. Notation for deflection of a beam.

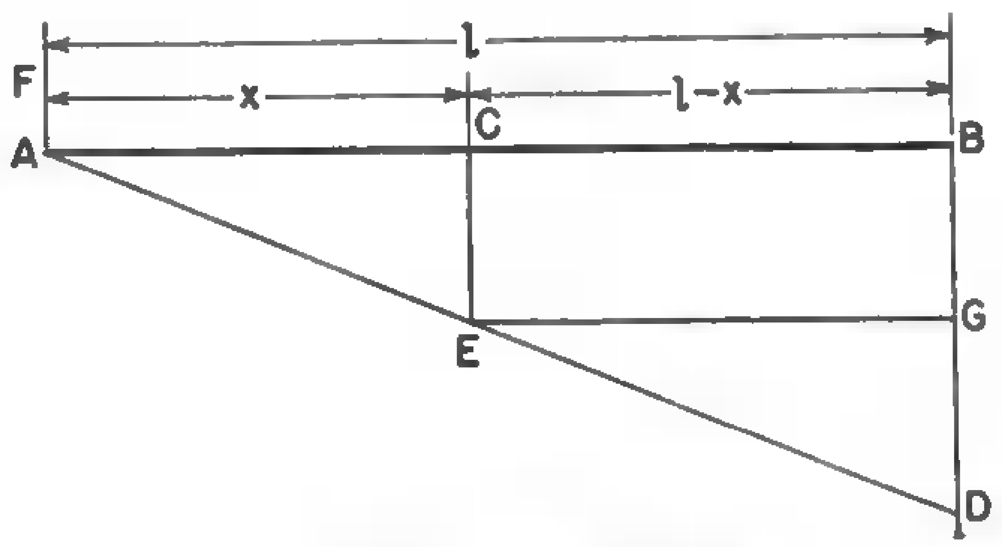


FIG. 2-10. Moment diagram.

The total deflection QR is the sum of the deflections y_i caused by the bending of the small parts into which the beam was divided. Thus the deflection of point Q is

$$y = y_1 + y_2 + \cdots + y_n = \frac{x_1A_1 + x_2A_2 + \cdots + x_nA_n}{EI} = \frac{\Sigma x_iA_i}{EI} \tag{2-29}$$

From a study of theoretical mechanics it is known that

$$\Sigma x_iA_i = \bar{x}A \tag{2-30}$$

where A is the entire area of the portion of the moment diagram between the support and the point at which the deflection is to be found and $\bar{x}$ is the distance from the center of gravity of that area to that point for a cantilever beam, and to the nearest support for a simple beam with symmetrical

loading. Thus

$$y = \frac{\bar{x}A}{EI} \tag{2-31}$$

EXAMPLE 2-3. Determine the expression for the deflection of a cantilever beam with a concentrated load on its free end.

The moment diagram for this beam is shown in Fig. 2-10, where the free end is at A and the support is at B. To find the deflection of a point C at a distance x from the free end, the moment area $CBDE$ between the point C and the support is divided into the rectangle $CBGE$ and the triangle EGD. Since the ordinate CE is equal to $-Fx$, the area of the rectangle and its moment arm are, respectively,

$$A = -Fx(l-x) \qquad \bar{x} = \frac{l-x}{2}$$

For the triangle EGD, the height $GD = BD - BG = -Fl - (-Fx) = -F(l-x)$. Hence,

$$A = \frac{-F(l-x)(l-x)}{2} = \frac{-F(l-x)^2}{2} \quad \text{and} \quad \bar{x} = \tfrac{2}{3}(l-x)$$

The sum of the moments of the rectangle and the triangle is

$$\Sigma x_iA_i = -\frac{Fx(l-x)^2}{2} - \frac{F(l-x)^3}{3} = -\frac{F(2l^3 - 3l^2x + x^3)}{6}$$

Hence, the deflection at C, by equation 2-31, is

$$y = -\frac{F(2l^3 - 3l^2x + x^3)}{6EI}$$

The maximum deflection is at the free end and is found by making x equal to zero. Thus

$$y_{max} = -\frac{F(2l^3)}{6EI} = -\frac{Fl^3}{3EI}$$

2-7. Statically indeterminate structures. Stresses in a machine part can be determinate only if all external forces, both loads and reactions, acting upon the part are known. The external loads must be given or found from data that are furnished. The reactions are determined, either completely or partially, from the following three conditions of equilibrium of a free body: (1) The summation of all horizontal forces must be zero. (2) The summation of all vertical forces must be zero. (3) The summation of the moments of all forces with respect to any point must be zero.

If the laws of equilibrium are sufficient for the determination of the reactions, the structure is said to be *statically determinate*.

Redundant elements. If a structure has more supports or members than are necessary for stability, the three equilibrium conditions are not sufficient for determining the reactions. Additional equations must then be set up by taking deformations of the structure into account. Such a structure is said to be *statically indeterminate*; and the supports or members that can be removed without destroying the stability of the structure are called *redundant elements*.

In Fig. 2-11a is shown a beam that is statically indeterminate *externally* because if the support at A were removed, the beam would remain a stable

cantilever beam. Therefore the support is redundant. The structures shown in Fig. 2–11b and c are statically indeterminate *internally* because if one member, as BD, were removed from either structure, the remaining members could carry the loads. Each structure therefore has one redundant member.

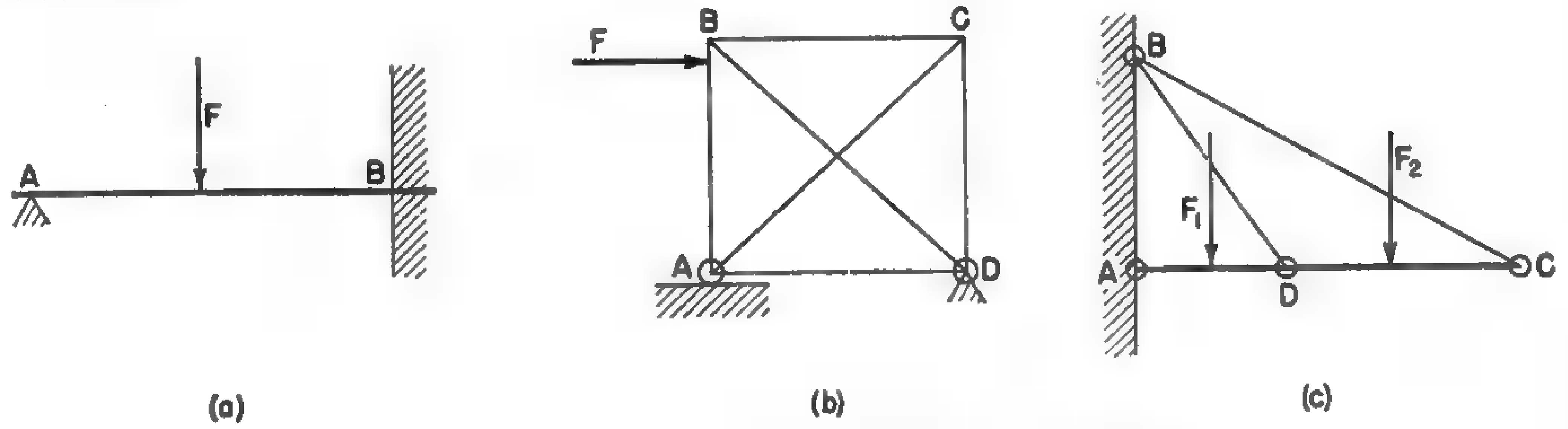


FIG. 2–11. Structures with redundant elements.

A general rule that helps in the design of a statically indeterminate structure may be stated as follows: In transmitting a load to its final support the stresses in the members take the *most rigid path.*

There are many ways of analyzing statically indeterminate structures. In the simplest method, deformations are used explicitly in setting up equations that give relations between the forces, moments, and stresses.

A typical method of analysis will be illustrated by the following examples.

EXAMPLE 2–4. A machine weighing 100 tons is supported on three very short columns. The columns have equal lengths and cross-sectional areas and are spaced as shown in Fig. 2–12a. Assuming that the machine frame and the supporting foundation are so rigid that their deformations can be neglected, find the load on each column.

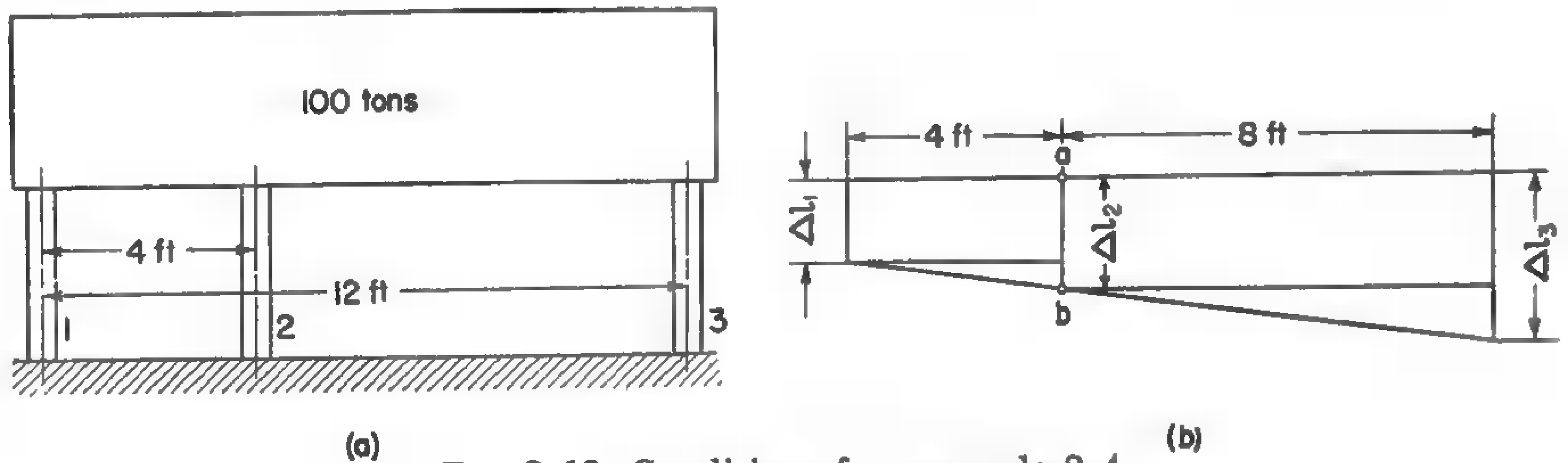


FIG. 2–12. Conditions for example 2–4.

If the loads on the columns are designated as F_1, F_2, and F_3, the conditions for equilibrium of vertical forces and equilibrium of moments may be written as follows:

$$F_1 + F_2 + F_3 = 100 \qquad \text{(a)}$$

and

$$6F_1 + 2F_2 = 6F_3 \qquad \text{(b)}$$

Because of the rigidity of the foundation and of the supported body, the tops of the columns, after deformations have taken place, will be on a straight line, as shown in Fig. 2–12b. From the similar triangles, it follows that

$$\frac{\Delta l_2 - \Delta l_1}{4} = \frac{\Delta l_3 - \Delta l_2}{8} \qquad \text{(c)}$$

The general expression for the total vertical deformation of any column is

$$\Delta l = \frac{Fl}{AE} \qquad \text{(d)}$$

where A is the cross-sectional area of the column, in square inches.

Since all three columns have the same length l, the same cross-sectional area A, and the same modulus of elasticity E, we will introduce the designation $l/AE = C$. Then

$$\Delta l_1 = F_1C \qquad \Delta l_2 = F_2C \qquad \Delta l_3 = F_3C \qquad \text{(e)}$$

Substituting values of Δl from equation e in equation c gives the equation

$$2F_1 - 3F_2 + F_3 = 0 \qquad \text{(f)}$$

Solving equations a, b, and f simultaneously, there results

$$F_1 = 28.6 \text{ tons} \qquad F_2 = 32.1 \text{ tons} \qquad F_3 = 39.3 \text{ tons}$$

EXAMPLE 2–5. Determine the loads on the columns described in example 2–4 if column 2 has been machined 0.0025 in. shorter than the others. The normal length of all three columns is 12 in., the area A of each cross section is 5 sq in., and the steel has a modulus of elasticity E of 30,200,000 psi.

As in example 2–4,

$$F_1 + F_2 + F_3 = 100 \qquad \text{(a)}$$

and

$$6F_1 + 2F_2 = 6F_3 \qquad \text{(b)}$$

Again the tops of the columns, after the deformations have taken place, will be on a straight line, as in Fig. 2–12b. Since column 2 was originally 0.0025 in. short, however, the distance corresponding to *ab* in Fig. 2–12b will be $\Delta l_2 + 0.0025$, and the equation based on the deformations is

$$\frac{\Delta l_2 + 0.0025 - \Delta l_1}{4} = \frac{\Delta l_3 - \Delta l_2 - 0.0025}{8} \qquad \text{(g)}$$

In this case the vertical deformation of each column may be taken as

$$\Delta l = \frac{2{,}000F \times 12}{5 \times 30{,}200{,}000} = 0.000159F \qquad \text{(h)}$$

or $\Delta l_1 = 0.000159F$, $\Delta l_2 = 0.000159F_2$, and $\Delta l_3 = 0.000159F_3$. Substituting these values in equation g gives

$$2F_1 - 3F_2 + F_3 = 47.2 \qquad \text{(i)}$$

Solving equations a, b, and i simultaneously, there results

$$F_1 = 35.2 \text{ tons} \qquad F_2 = 22.2 \text{ tons} \qquad F_3 = 42.6 \text{ tons}$$

Comparison of the loads in example 2–5 with the corresponding loads of example 2–4 shows what a great difference a small change of length can make.

In example 2–6, which follows, we shall analyze a statically indeterminate structure loaded in tension.

EXAMPLE 2–6. Determine the loads on the rods shown in Fig. 2–13a. The rods are hinged at both ends and are loaded by a suspended weight of 24,000 lb. The cross-sectional area of the vertical rod *2* is $A_2 = 1$ sq in., and that of the side rods *1* and *3* is $A_1 = A_3 = 0.5$ sq in.

The forces acting on the rods will be designated F_1, F_2, and F_3. Since cos 30° = 0.866 and cos 45° = 0.707, the summation of the vertical forces is

$$0.866F_1 + F_2 + 0.707F_3 = 24{,}000 \qquad \text{(j)}$$

The relation involving the summation of horizontal forces should not be used before checking whether rod *2* remains absolutely vertical. However, the necessary second and

third equations may be obtained by considering the elongations of the rods indicated in Fig. 2–13b. If it is assumed that the end of rod *2* moves vertically downward from point *a* to *b*, the end of rod *1* must swing from *c* to *b* and the end of rod *3* must swing down from *d* to *b*. Then

$$\frac{\Delta l_1}{\cos 30°} = \Delta l_2 = \frac{\Delta l_3}{\cos 45°} \qquad \text{(k)}$$

The elongations may be expressed as follows:

$$\Delta l_1 = \frac{lF_1}{A_1 E \cos 30°} \qquad \Delta l_2 = \frac{lF_2}{A_2 E} \qquad \Delta l_3 = \frac{lF_3}{A_3 E \cos 45°} \qquad \text{(l)}$$

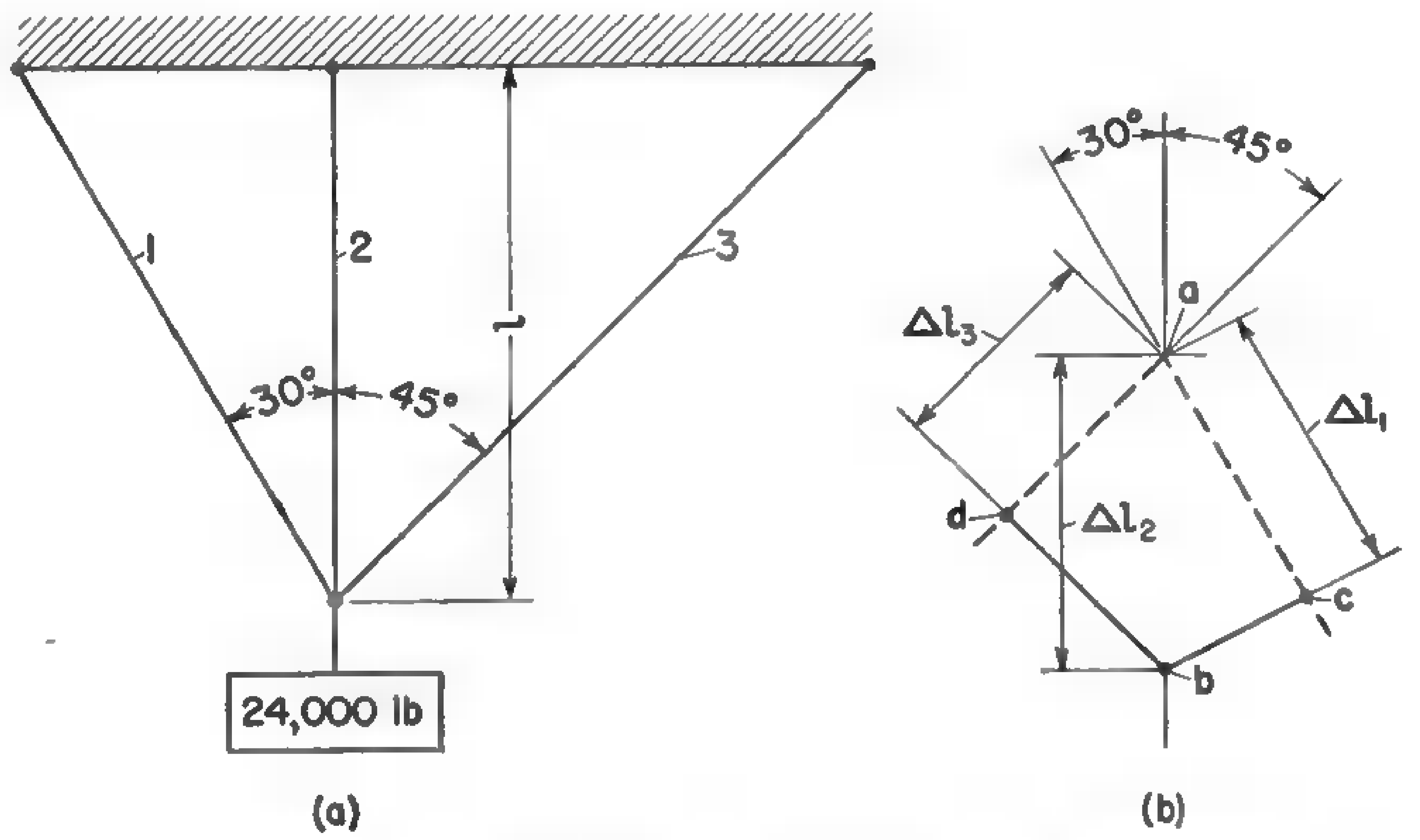


FIG. 2–13. Indeterminate structure loaded in tension.

We can now substitute these expressions in equations k. Replacing cos 30° by 0.866, cos 45° by 0.707, A_1 and A_3 by 0.5, and A_2 by 1 sq in., and simplifying the results, we obtain

$$F_1 = 0.375F_2 \qquad \text{(m)}$$

$$F_3 = 0.25F_2 \qquad \text{(n)}$$

Solving equations j, m, and n simultaneously, we get the required answers:

$$F_1 = 6{,}000 \text{ lb} \qquad F_2 = 16{,}000 \text{ lb} \qquad F_3 = 4{,}000 \text{ lb}$$

It is interesting to check the possible angle β formed by rod *2* with the original vertical position. A summation of horizontal components gives

$$6{,}000 \sin 30° - 16{,}000 \sin \beta - 4{,}000 \sin 45° = 0$$

From this

$$\sin \beta = \frac{6{,}000 \times 0.5 - 4{,}000 \times 0.707}{16{,}000} = 0.01075$$

Hence, $\beta = 0.61°$, a value so small that it did not affect the accuracy of equation j.

2–8. Indeterminate beams. Basically, the procedure for analyzing a statically indeterminate beam is the same as that used for a structure consisting of parts loaded in tension and compression. The main difference is in the greater use of moment equations, of the relations between the reactions, and of shear diagrams. The redundant elements are supports that are not necessary for stability of the beam. Again the procedure can be best illustrated by analyzing some typical cases.

Beam fixed at one end and supported at the other. In Fig. 2–14a is shown a beam fixed at one end and supported at the other, and carrying a uniformly distributed load. The full line in Fig. 2–14b represents the elastic curve. If the left-hand support were removed, the beam would become a cantilever and the elastic curve would be represented by the broken line. In this event, the loading would correspond to that for case b in Table 2–4 and the maximum deflection would be

$$y_{max} = \frac{Fl^3}{8EI}$$

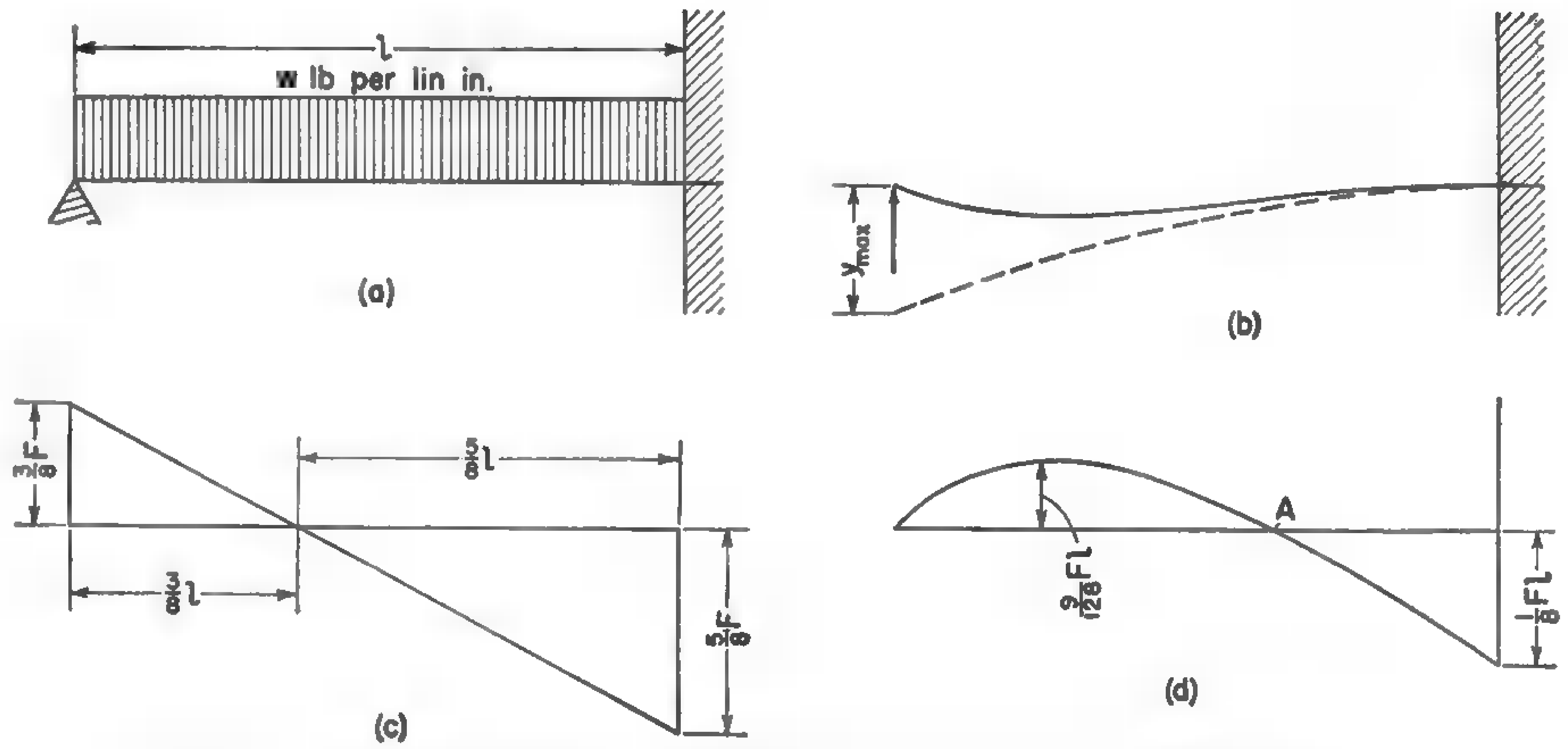


FIG. 2–14. Beam with one end fixed and the other simply supported.

The reaction of the left-hand support may be considered a concentrated load R_1 on the free end of the assumed cantilever. This reaction must deflect the end of the beam upward a distance equal to y_{max}. According to case a in Table 2–4,

$$y_{max} = \frac{R_1 l^3}{3EI}$$

Evidently

$$\frac{R_1 l^3}{3EI} = \frac{Fl^3}{8EI}$$

and

$$R_1 = \frac{3}{8}F \qquad \text{(2–32)}$$

With R_1 known, the shear and moment diagrams are constructed as shown in Fig. 2–14c and d. At point *A* in Fig. 2–14d the bending moment is zero. This point is called a *point of inflection*. The only stress here is shear.

The following example will illustrate how to take into account the condition that the supports are not on the same level.

EXAMPLE 2–7. Two 6-in. by 17.25-lb I beams with a 6-ft span are fixed at one end, are simply supported at the other, and carry a uniformly distributed load of 14,000 lb. The supports settle ⅛ in. under the action of the load. Find the reactions on the supports and the maximum fiber stress in the beams.

The moment of inertia of the I beam, ascertained from a handbook, is $I = 26.0$ in.^{4} If the supports were removed, the deflection of the ends would be

$$y_1 = \frac{7{,}000 \times 72^3}{8 \times 30{,}000{,}000 \times 26} = 0.417 \text{ in.}$$

But the reaction of the support brings the end up a distance equal to

$$y_2 = \frac{R' \times 72^3}{3 \times 30{,}000{,}000 \times 26} = 0.000159R'$$

Equating the deflections gives

$$0.417 = 0.125 + 0.000159R'$$

Hence,

$$R' = 1{,}836 \text{ lb}$$

If the support did not settle, the reaction of one beam would be

$$R_1 = 0.375 \times 7{,}000 = 2{,}605 \text{ lb}$$

Because the settling of the supports reduced their reactions, the reactions on the fixed ends are increased from

$$R_2 = 7{,}000 - 2{,}605 = 4{,}395 \text{ lb}$$

to

$$R'_2 = 7{,}000 - 1{,}836 = 5{,}154 \text{ lb}$$

The dangerous bending moment is at the fixed end and is

$$M = R_1 l - \frac{Fl}{2}$$

Without settling, this moment for one beam would be

$$M = 2{,}605 \times 72 - 7{,}000 \times \frac{72}{2} = -64{,}500 \text{ lb-in.}$$

Because of settling, it is

$$M' = 1{,}836 \times 72 - 7{,}000 \times \frac{72}{2} = -119{,}800 \text{ lb-in.}$$

The maximum fiber stress, with $Z = I/c = 8.67$ in.^{3}, is

$$s = \frac{119{,}800}{8.67} = 13{,}700 \text{ psi}$$

Beam on three supports. In Fig. 2–15a is shown a beam with two equal spans each l in. long and uniformly loaded with w lb per lin in. The supports are on the same level. The curved full line in Fig. 2–15b represents the elastic curve. If the center support is removed, the beam becomes a simple beam with a span $2l$, as shown by the broken line. Its deflection in the middle would be, by case f in Table 2–4,

$$y = \frac{5}{384} \frac{F(2l)^3}{EI}$$

where $F = 2wl$. Now, to bring the beam back to its original shape, a single force R_2 must be applied at the center. The upward deflection, by case c in Table 2–4, is

$$y = \frac{1}{48} R_2 \frac{(2l)^3}{EI}$$

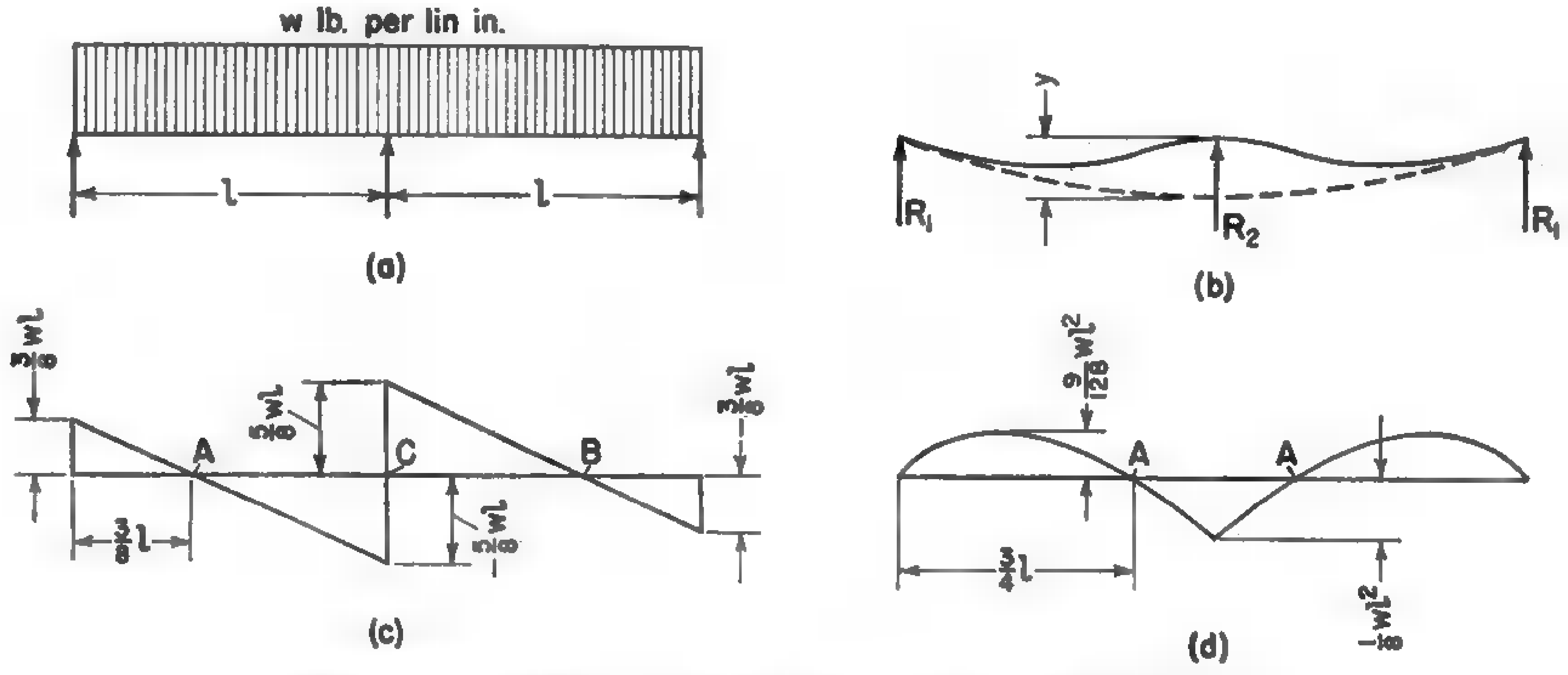


FIG. 2–15. Beam on three supports equally spaced.

Equating the expressions for the deflections and solving for R_2 gives

$$R_2 = \tfrac{5}{8}F = \tfrac{5}{4}wl \qquad (2\text{–}33)$$

Since $R_2 + 2R_1 = F$,

$$R_1 = \frac{F - R_2}{2} = \frac{2wl - \frac{5}{4}wl}{2} = \tfrac{3}{8}wl \qquad (2\text{–}34)$$

With the reactions determined, the shear diagram can be drawn as in Fig. 2–15c, and the points A and B at which the shear is zero can be located. The moment diagram is shown in Fig. 2–15d.

Continuous beams with more than three supports are seldom used in machine design and therefore will not be discussed here.

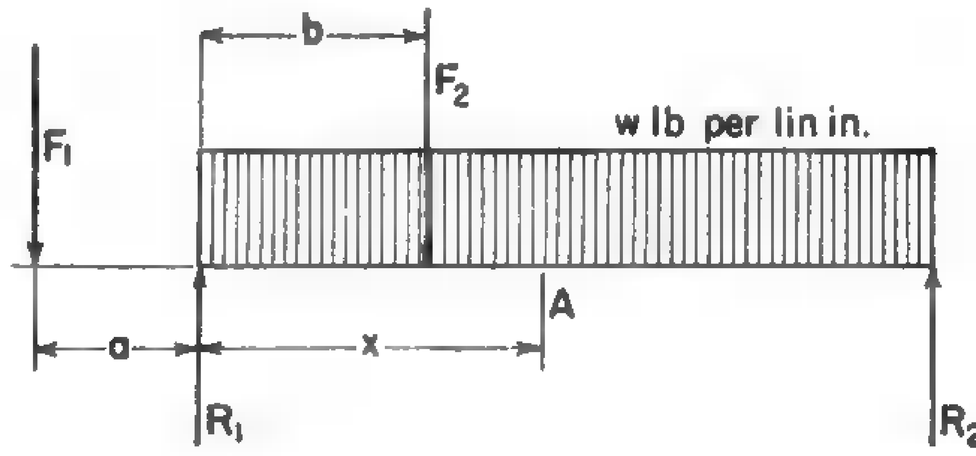


FIG. 2–16. General case of beam loading.

General moment equation. The beam shown in Fig. 2–16 rests on two supports and carries several types of loads. There is a uniformly distributed load of w lb per lin in.; and there are also two concentrated loads, F_1 to the left of the left-hand support and F_2 between the supports. The reactions on the supports from all loads are R_1 and R_2.

Taking moments about point A of the forces to the left of that point, we have

$$M_A = -F_1(a + x) + R_1 x - \tfrac{1}{2}wx^2 - F_2(x - b)$$
$$= -F_1 a + (R_1 - F_1)x - \tfrac{1}{2}wx^2 - F_2(x - b)$$

But $-F_1a$ is the moment of all loads to the left of support *1* about point *1* and may be designated M_o. Furthermore, $R_1 - F_1$ is the shear at the right-hand edge of the support. If this shear is designated V_1,

$$M_A = M_o + V_1x - \tfrac{1}{2}wx^2 - F_2(x-b) \tag{2-35}$$

This is called the *general moment equation*. By means of it, when the moment M_o at a support is known, the shear and bending moment at any point to the right of that support due to any given loading may be determined.

EXAMPLE 2–8. Using the general moment equation, find the bending moment at points A and C of the beam in Fig. 2–15. Also draw the moment diagram.

Applying equation 2–35, in which $M_o = 0$, $V_1 = R_1 = \frac{3}{8}wl$, and $x = \frac{3}{8}l$, we get for the bending moment at A

$$M_A = \tfrac{3}{8}wl \times \tfrac{3}{8}l - w\frac{\frac{3}{8}l^2}{2} = \frac{9}{128}wl^2$$

For point C, where $x = l$,

$$M_C = \tfrac{3}{8}wl \times l - \tfrac{1}{2}wl^2 = -\tfrac{1}{8}wl^2$$

By calculating the moments at several points of the beam in this manner, the moment diagram in Fig. 2–15d can be drawn.

If the beam in Fig. 2–15a were cut in two at the center support so as to form two simple beams, the maximum bending moment in each simple beam would be $M = \frac{1}{8}wl^2$. Thus, the maximum bending moment for a continuous beam on three supports equally spaced is the same [redacted] that for two simple beams on the same supports.

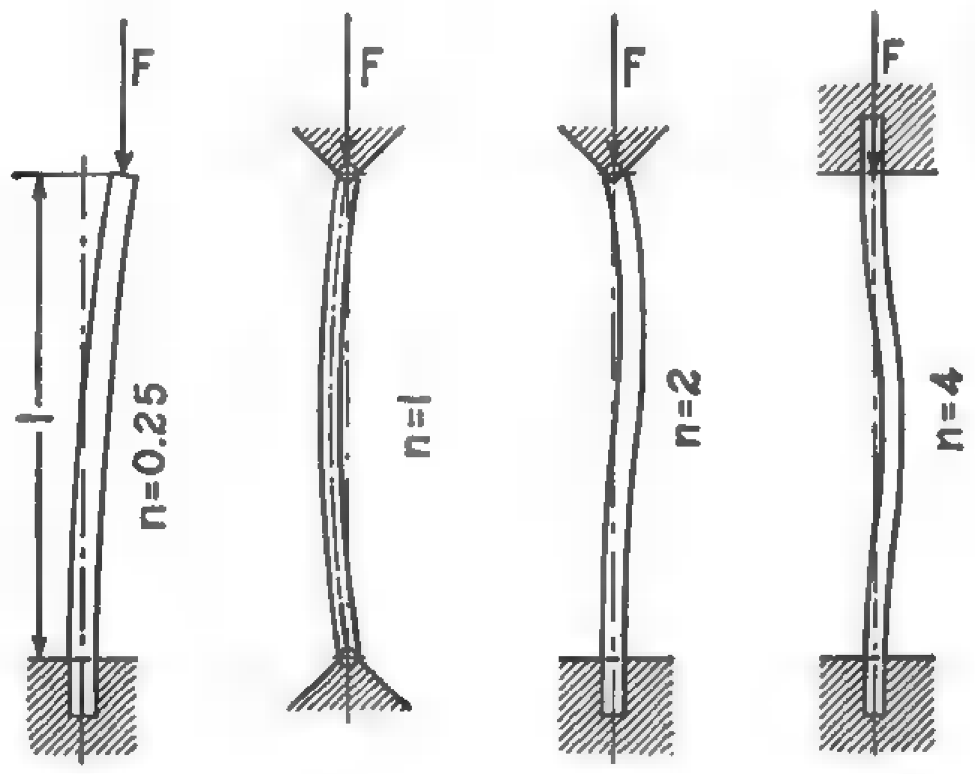


FIG. 2–17. End conditions of columns.

2–9. Stresses in columns. A cast-iron compression member whose length is more than 6 times as great as its least lateral dimension, or a compression member of a ductile material whose length is 8 times as great, is likely to buckle and should be treated as a column.

End conditions. The resistance of a column to failure depends upon the condition of its ends. The least favorable case is when one end of the column is fixed and the other is free, as in Fig. 2–17a. Smaller stresses

result when both ends are round-ended and guided or hinged, as in Fig. 2–17b. Still more favorable is the case when one end is fixed and the other is round-ended and guided or hinged, as in Fig. 2–17c. The smallest stresses are produced when both ends are fixed rigidly, as in Fig. 2–17d.

Depending upon the *slenderness ratio*, which is the ratio of the length l of the column to the least radius of gyration k of the cross section, columns are divided into two groups: *short columns* and *long columns*.

The radius of gyration of any section with regard to any axis through the center of gravity of the section can be found by the relation

$$k = \sqrt{\frac{I}{A}} \tag{2-36}$$

where I is the moment of inertia of the section with regard to the same axis, and A is the area of the section. Expressions for the radius of gyration for various sections commonly used as columns are given in the last column of Table 2–5. In determining the slenderness ratio of a column the least radius of gyration of the section must be used.

Short columns. Cast-iron columns with a slenderness ratio l/k not greater than 80, and columns of steel and other ductile materials for which l/k is not greater than 100, are considered short columns. In a short column the intensity of the simple compressive stress on its concave side is augmented by the stress that arises from bending under the axial load. In order to prevent failure the combined stress should always be below the elastic limit.

Several formulas have been proposed for determining the relation between the external load F and the induced stress s_c in a short column.

The *Ritter formula*, which is a modification of the Gordon-Rankine formula, has the advantage that it can be applied for any material for which the elastic limit S_e and the modulus of elasticity E are known. It is

$$s_c = \frac{F}{A}\left[1 + \left(\frac{l}{k}\right)^2 \frac{S_e}{\pi^2 nE}\right] \tag{2-37}$$

where, in addition to the designations already mentioned, A is the cross-sectional area of the member, in square inches, and n is the coefficient of end conditions. Values of n for various conditions are given in Fig. 2–17.

EXAMPLE 2–9. A steel bar 20 in. long, acting as a column with both ends hinged, supports a weight of 12,000 lb. The cross section of the bar is a rectangle $1\frac{3}{4}$ in. by 1 in. The elastic limit of the steel is 42,000 psi, and its modulus of elasticity is 30,000,000 psi. Determine the stress in the outer fibers.

From case a, in Table 2–5, the least radius of gyration is

$$k = 0.289 \times 1 = 0.289 \text{ in.}$$

Hence, $l/k = 20/0.289 = 69$, which is less than 100. From Ritter's formula (equation 2–37), and with the end-condition coefficient n as 1,

$$s_c = \frac{12{,}000}{1.75 \times 1}\left[1 + \left(\frac{20}{0.289}\right)^2 \times \frac{42{,}000}{\pi^2 \times 30{,}000{,}000}\right] = \frac{12{,}000 \times 1.679}{1.75} = 11{,}500 \text{ psi}$$

Long columns. Long columns are those with a slenderness ratio l/k greater than 100 for ductile materials and greater than 80 for cast iron. Long columns fail by buckling due to instability. The ultimate or breaking load for a long column may be found from Euler's rational formula, which is

$$\boxed{F_u = \frac{n\pi^2 AE}{\left(\frac{l}{k}\right)^2}} \qquad (2\text{–}38)$$

in which the coefficient n theoretically has the same values as in Ritter's formula. However, test data indicate that in long columns the end conditions do not exert so great an influence.[1]

Because of the instability of long columns when subjected to maximum loads, they are avoided in machines unless the maximum load is known very accurately and the design is based on that load.

2–10. Induced stresses. Because of the deformations produced in a machine part by the applied loads, other stresses known as *induced stresses* or *secondary stresses* are created.

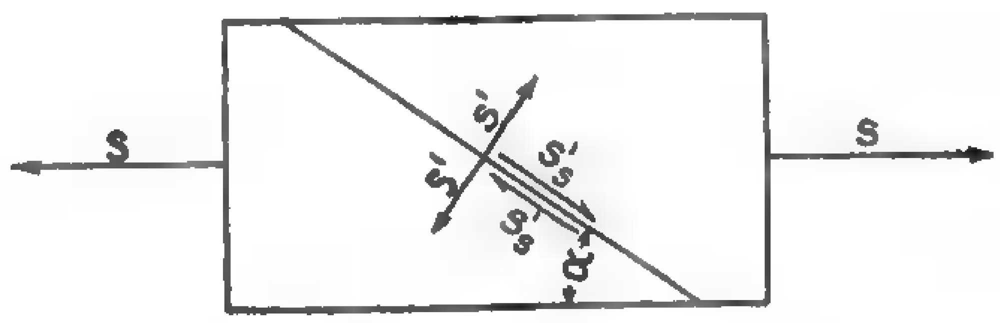


FIG. 2–18. Shear produced by a direct stress.

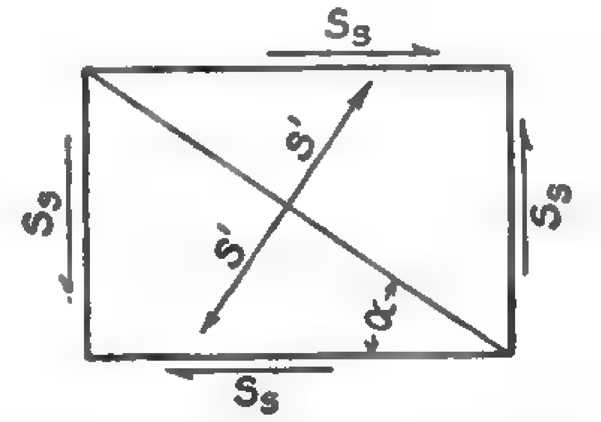


FIG. 2–19. Normal stress caused by shear.

Shear caused by tension or compression. When a force F produces a normal tensile or compressive stress $s = F/A$ in a body, it also produces on every oblique plane of the body a shear stress s_s', as indicated in Fig. 2–18. The magnitude of this shear stress is

$$s_s' = 0.5s \sin 2\alpha \qquad (2\text{–}39)$$

This stress reaches a maximum in a plane forming an angle $\alpha = 45°$ with the main plane. In this case

$$s_s' = 0.5s \qquad (2\text{–}40)$$

Normal stress caused by shear. Similarly, as indicated in Fig. 2–19, a force producing a shear stress s_s in a body produces also a normal stress s' on every oblique plane of the body. The magnitude of this stress is

$$s' = s \sin 2\alpha \qquad (2\text{–}41)$$

This stress reaches a maximum when $\alpha = 45°$. Then

$$s' = s_s \qquad (2\text{–}42)$$

[1] B. Kirsch, "Knickfestigkeit langer Stäbe," *Zeitschrift Verein Deutscher Ingenieure*, Vol. 49 (1905), p. 907.

200 for secondary members

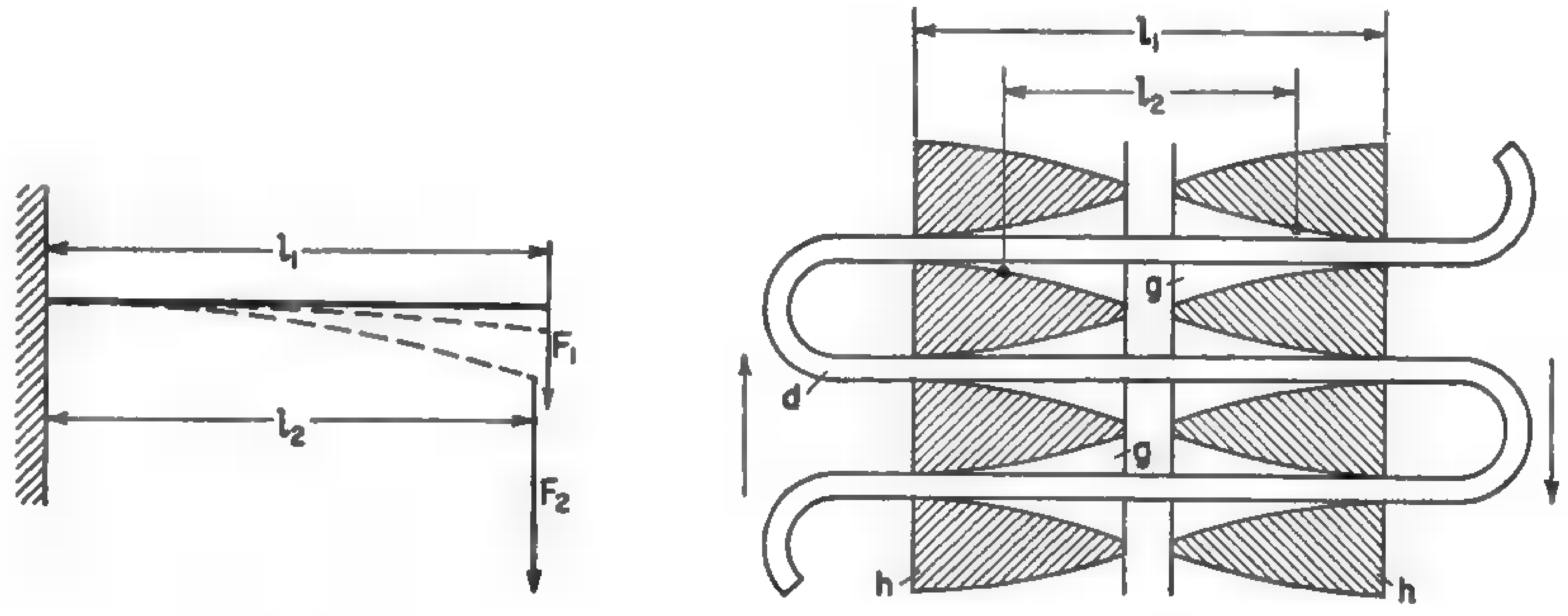
Lateral stress. When a force produces a normal stress s, it also causes a lateral strain ϵ', which may be considered as caused by a stress s' normal to s. The magnitude of this fictitious stress s' is

$$s' = -\mu s \qquad (2\text{–}43)$$

where μ is Poisson's ratio and the minus sign shows that the nature of the stress s' is opposite to that of s. If s is a tensile stress, s' is a compressive one; and vice versa.

2–11. Combination of stresses. When the loads applied to a member induce stresses of several kinds, the material is affected by the combination of all the stresses. The stress that results from the combined simultaneous action of several stresses is called the *resultant stress.*

The resultant of several stresses acting on the *same* plane at a point of a body is the geometric sum of their individual actions. But stresses acting at a point on *different* planes cannot be combined or resolved like forces.



(a) (b)

FIG. 2–20. Variable load conditions.

Superposition. Where several loads act in the same plane, their combined effect may be visualized as the separate effects of all the individual loads superimposed one on top of the other. This method, called *superposition,* can be applied to a variety of effects, such as stresses, deformations, and reactions.

It should be noted that superposition gives correct results only under two conditions: (1) The stresses and deformations must be directly proportional to the loads. (2) The deformations must not be so great as to change appreciably the configuration of the system or the point of application of the load.

Cases in which superposition *cannot* be used are shown in Fig. 2–20. A large increase of the load on the beam in Fig. 2–20a changes the horizontal distance from the support to the point of its application and hence changes the moment arm from l_1 to l_2. In Fig. 2–20b is shown the spring grid of a

Falk flexible coupling (the coupling itself is shown in Fig. 21–15). The spring d is laid in grooves g cut in the disks h. The grooves have curved sides and widen toward each other. The free beam length of the spring elements is l_1. When a torque is applied the springs are bent along the arcs of the grooves, and the span is reduced to some smaller value, such as l_2.

Normal stresses at right angles. If at a certain point there exist simultaneously a normal stress s_1 and another normal stress s_2 whose line of action is at right angles to the line of action of s_1, then there will exist an additional stress in the direction of s_1. According to equation 2–43 the magnitude of the additional fictitious stress will be $-\mu s_2$. The resultant stress, found by superposition, is

$$s' = s_1 - \mu s_2 \tag{2–44}$$

The maximum shear stress at this point is in a plane making an angle of 45° with the main plane. Its magnitude, found from equation 2–40 by superposition, is

$$s_s = 0.5(s_1 - s_2) \tag{2–45}$$

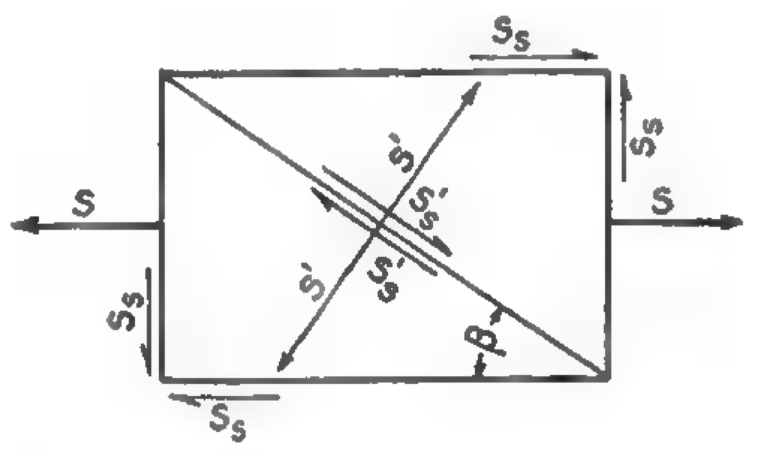


FIG. 2-21. Combined tensile and shear stresses.

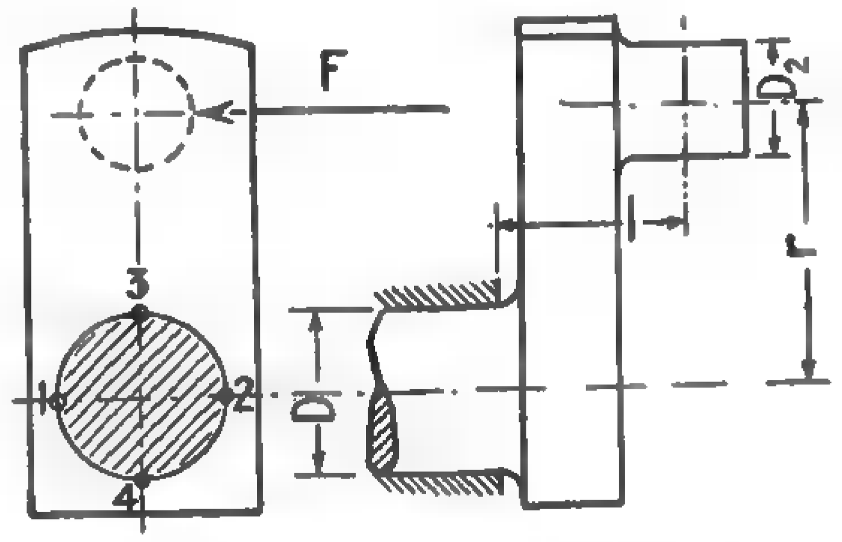


FIG. 2-22. Crankshaft with side crank.

Normal and shear stresses. The stress analysis of an element subjected to the simultaneous action of a normal stress s and a shear stress s_s shows that the combined action of these stresses produces an internal normal stress s' and a tangential stress s_s' on an interior plane, as indicated in Fig. 2-21. The ratio s/s_s determines the value of the angle β. The stress s' is maximum when $\tan 2\beta = -2s_s'/s$, and s_s' is maximum when $\tan 2\beta = +s/2s_s$. The principal direct stresses are

$$s' = \tfrac{1}{2}s \pm \sqrt{(\tfrac{1}{2}s)^2 + s_s^2} \tag{2–46}$$

The maximum shear stresses are

$$s_s' = \sqrt{(\tfrac{1}{2}s)^2 + s_s^2} \tag{2–47}$$

The two stresses s' from equation 2–46 are at right angles to each other. The two shear stresses s_s' from equation 2–47 are also at right angles to each other, but they act in planes that make an angle of 45° with the planes of the principal stresses s'.

These equations apply to all cases of a normal stress combined with shear, as when a beam is under both shear and tension or compression caused by bending, or when a bolt is under tension and shear, or when a shaft is under bending and torsion.

The greater principal stress determined by equation 2–46 is usually considered to be the maximum stress within the material. However, according to the maximum-strain theory of failure the design should be governed by the stress that corresponds to the maximum strain, which takes into account the lateral deformation. The normal stress s'' which will produce the maximum strain is

$$s'' = (1-\mu)\tfrac{1}{2}s + (1+\mu)\sqrt{(\tfrac{1}{2}s)^2 + s_s^2} \tag{2–48}$$

In practice, equation 2–46 is usually preferred to equation 2–48, although equation 2–48 gives a slightly higher value for the normal stress, especially when the shear stress s_s is relatively high.

EXAMPLE 2–10. Determine the resultant stresses created in the journal of the crankshaft shown in Fig. 2–22. The diameter $D = 3$ in.; the crank radius $r = 5$ in.; the distance $l = 3\tfrac{3}{4}$ in.; the tangential force $F = 4{,}500$ lb.

The stress in bending in fibers *1* and *2* due to the bending moment $M = Fl$, by equation 2–22, is

$$s = \frac{4{,}500 \times 3.75 \times 32}{\pi \times 3^3} = 6{,}370 \text{ psi}$$

The average shear stress due to bending is, according to Table 2–4,

$$s_s = \frac{4{,}500}{0.7854 \times 3^2} = 637 \text{ psi}$$

In fibers *1* and *2* the shear stress $s_s = 0$; in the middle section *3–4* it reaches the maximum value, which according to Table 2–3 is

$$1.33s_s = 1.33 \times 637 = 848 \text{ psi}$$

The shear stress due to torsion produced by the torque Fr is

$$s_s = \frac{4{,}500 \times 5 \times 16}{\pi \times 3^3} = 4{,}246 \text{ psi}$$

Thus in fibers *1* and *2* the principal stresses are, by equation 2–46,

$$s' = 0.5 \times 6{,}370 \pm \sqrt{(0.5 \times 6{,}370)^2 + 4{,}246^2} = 3{,}185 \pm 5{,}305 = +8{,}490 \text{ psi and } -2{,}120 \text{ psi}$$

The maximum resultant shear stress in fibers *1* and *2* is, by equation 2–47,

$$s_s = \sqrt{3{,}185^2 + 4{,}246^2} = 5{,}305 \text{ psi}$$

By equation 2–48, in which $\mu = 0.303$ (Table 2–1), the normal stress is

$$s'' = (1 - 0.303) \times 3{,}185 + (1 + 0.303) \times 5{,}305 = 2{,}220 + 6{,}920 = 9{,}140 \text{ psi}$$

The resultant shear stress in fiber *3* due to bending and torsion is

$$s_s'' = 4{,}246 + 848 = 5{,}094 \text{ psi}$$

The resultant shear stress in fiber *4*, because of the opposite directions of the shear stresses, is

$$s_s'' = 4{,}246 - 848 = 3{,}398 \text{ psi}$$

2–12. Biaxial stress condition. If in a system referred to three coordinate axes the components of all stresses parallel to one of the axes are zero, the stress condition is called *biaxial.* This condition is particularly important in machine design, because it is the most common condition on the surface of a part where the stresses are greatest.

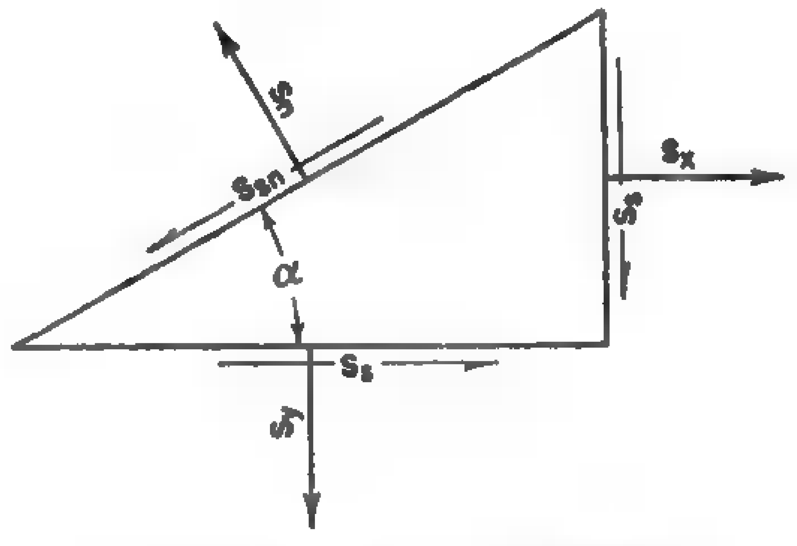


FIG. 2–23. Biaxial-stress condition.

The relationship between the stresses on various planes passing through a point may be determined by applying the equations of equilibrium to an element of the material at this point taken as a free body. For analyzing a biaxial-stress condition the most convenient free body is a triangular wedge such as that in Fig. 2–23, in which the angle α may have any value assigned to it. Since the shear stresses at right angles are equal, the resultant normal stress is found to be

$$s_n = s_x \sin^2 \alpha + s_y \cos^2 \alpha + s_s \sin 2\alpha \qquad (2\text{–}49)$$

The shear stress s_{sn} on the inclined plane is

$$s_{sn} = \tfrac{1}{2}(s_x - s_y) \sin 2\alpha + s_s \cos 2\alpha \qquad (2\text{–}50)$$

Principal stresses. Differentiating equation 2–49 with respect to α and equating the derivative to zero gives the direction of the plane on which the resultant normal stress becomes a maximum or minimum. The result is

$$\tan 2\alpha_m = -\frac{2s_s}{s_x - s_y} \qquad (2\text{–}51)$$

Equation 2–51 defines two planes which are at right angles to each other. The normal stress is a maximum on one of the planes, and it is a minimum on the other. The angle α_m is the angle between either of the axes x and y, to which the original stresses s_x and s_y are parallel, and one of the axes u and v, to which the two principal stresses are parallel.

For the value of α determined by equation 2–51, the expression for the principal stresses given in equation 2–49 becomes

$$s_{u,v} = \tfrac{1}{2}(s_x + s_y) \pm \tfrac{1}{2}\sqrt{(s_x - s_y)^2 + 4s_s^2} \qquad (2\text{–}52)$$

where s_u is the maximum principal stress, which is parallel to the axis u, and s_v is the minimum principal stress, which is parallel to the axis v.

From equations 2–50 and 2–51 it can be shown that the shear stresses on the planes of principal stresses are equal to zero.

Maximum shear stresses. By a similar procedure the angles of the planes in which the shear stresses are greatest are determined by the relation

$$\tan 2\alpha_s = \frac{s_x - s_y}{2s_s} \qquad (2\text{–}53)$$

Equation 2–53 defines a pair of planes that make angles of 45° with those determined by equation 2–50. For the angle determined by equation 2–53, the maximum shear stresses found by equation 2–51 are

$$s_s' = \pm \tfrac{1}{2}\sqrt{(s_x - s_y)^2 + 4s_s^2} \qquad (2\text{–}54)$$

Equation 2–54 shows that the two maximum shear stresses are equal in magnitude and opposite in direction.

EXAMPLE 2–11. At a point on the vertical side of a beam the horizontal stress s_x is 2,400 psi, tension; the vertical stress s_y is 900 psi, compression; and the shear stress s_s due to a positive vertical shear is 1,200 psi. Determine the maximum shear stress and the principal stresses.

From equation 2–54 the maximum shear stress is

$$s_s' = \pm \tfrac{1}{2}\sqrt{(2{,}400 + 900)^2 + 4 \times 1{,}200^2} = \pm 2{,}040 \text{ psi}$$

Also, from equation 2–52 the two principal stresses are

$$s_u = \frac{2{,}400 - 900}{2} + 2{,}040 = 2{,}790 \text{ psi}$$

$$s_v = \frac{2{,}400 - 900}{2} - 2{,}040 = -1{,}290 \text{ psi}$$

Direction of stresses. A convenient way to show the relative directions of the stresses whose magnitudes are determined by equations 2–52 and 2–54 is to compute the values of the pairs of angles defined by equations 2–51 and 2–53, and to draw the corresponding free-body diagram. The procedure is best explained by an example.

EXAMPLE 2–12. Determine the directions of the four stresses found in example 2–11.

The angles that the principal stresses make with the x axis are found from equation 2–51. For the given stresses,

$$\tan 2\alpha_m = -\frac{2 \times 1{,}200}{2{,}400 + 900} = -0.727$$

$$\alpha_{m1} = -\frac{36^\circ 2'}{2} = -18^\circ 1'; \quad \alpha_{m2} = 71^\circ 59'$$

In Fig. 2–24a is shown the original block with the acting stresses; and in Fig. 2–24b the block is shown cut along the line Oa making an angle $\alpha_{m2} = 71°59'$ with the x axis. Of the five forces, N_x, N_y, T_{xy}, T_{yx}, and N_u, acting on the block, the directions of the first four are known. From the diagram it is evident that the resultant of N_x and T_{xy} is a force that acts to the right and must be balanced by the horizontal component of N_u. Thus the force N_u must represent a tensile stress, and the stress with this direction is s_u. The directions of the axes are shown in Fig. 2–24c. The u axis is perpendicular to the line Oa, and the directions of the principal stresses are as shown in Fig. 2–24d.

The same result is obtained if the free body is assumed to be cut along the line Ob, Fig. 2–24e, making an angle $\alpha_{m1} = 18°1'$ with the x axis.

The maximum shear stresses make an angle of 45° with the principal stresses and are found in the manner just described from a diagram of the block cut along the line O_f, as in Fig. 2–24f. The components of N_u' and N_v' parallel to the line O_f act in the same direction; therefore the component of T_{ft} must be to the left to keep the free body in equilibrium, and the directions of the maximum shear stresses will be as shown in Fig. 2–24g.

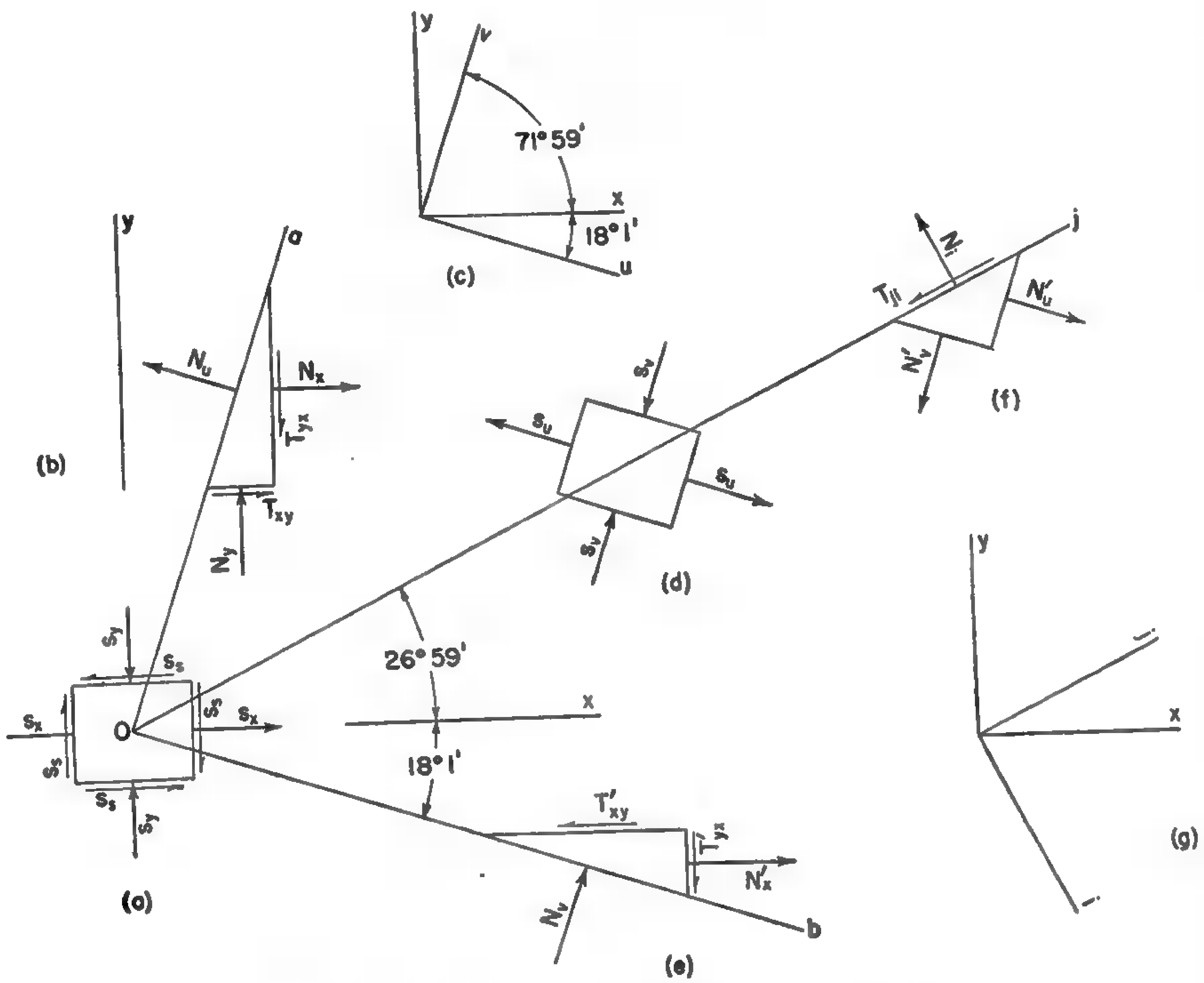


FIG. 2-24. Determination of directions of stresses.

Other stresses. Let s_u and s_v denote the principal stresses at a point. Since the shear stress in each of these planes is equal to zero, the normal stress s_n on any plane that is perpendicular to the $u-v$ plane and that intersects the $u-v$ plane in a line which makes an angle α with the u axis is, as found from equation 2–51,

$$s_n = \tfrac{1}{2}(s_u + s_v) - \tfrac{1}{2}(s_u - s_v)\cos 2\alpha \tag{2–55}$$

The normal stress s_t acting on a plane at right angles to s_n is

$$s_t = \tfrac{1}{2}(s_u = s_v) + \tfrac{1}{2}(s_u - s_v)\cos 2\alpha \tag{2–56}$$

The magnitude of the shear stress s_{sn} acting on either of two inclined planes is, from equation 2–50,

$$s_{sn} + \tfrac{1}{2}(s_u - s_v)\sin 2\alpha \tag{2–57}$$

Also, from equations 2–54 and 2–52, the maximum shear stress is

$$s_s' = \pm\tfrac{1}{2}(s_u - s_v) \tag{2–58}$$

When $\alpha = \alpha_m$, it is evident that $s_n = s_y$ and $s_t = s_x$.

Use of the Mohr circle. The Mohr circle provides a graphical relation between the biaxial stresses s_x and s_y, the shear stress s_s, the principal stresses s_u and s_v, and the maximum shear stress s_s', all acting at a certain point in a body. To explain the procedure, data from example 2–11 will be used. First the normal stresses are laid off to scale as horizontal vectors from the origin O, Fig. 2–25a. Tensile stresses, since they are considered positive, are laid off to the right from O; thus, $s_x = Oa$. Compressive stresses, considered negative, are laid off to the left; thus, $s_y = Ob$. Shear stresses are laid off from the points a and b as vertical vectors. Those which tend to rotate a free-body element clockwise are considered positive and are laid off upward; thus, $s_s = ad$. Shear stresses tending to rotate the element counterclockwise are considered negative and are laid off downward; therefore, $-s_s = be$. With the line ed as a diameter and its middle point c as the center, a circle $dfeg$ is described. From Fig. 2–25a, keeping in mind the signs of s_x and s_y, $Oc = \frac{1}{2}(s_x + s_y)$, $ca = \frac{1}{2}(s_x - s_y)$, and $cd = \frac{1}{2}\sqrt{(s_x - s_y)^2 + 4s_s^2}$. According to equation 2–52, $Of = Oc + cf = Oc + cd = s_u$ and $Og = cg - Oc = cd - Oc = s_v$. By equation 2–51, $\tan\varphi = ad/ca = s_s/\frac{1}{2}(s_x - s_y) = -\tan 2\alpha_m$.

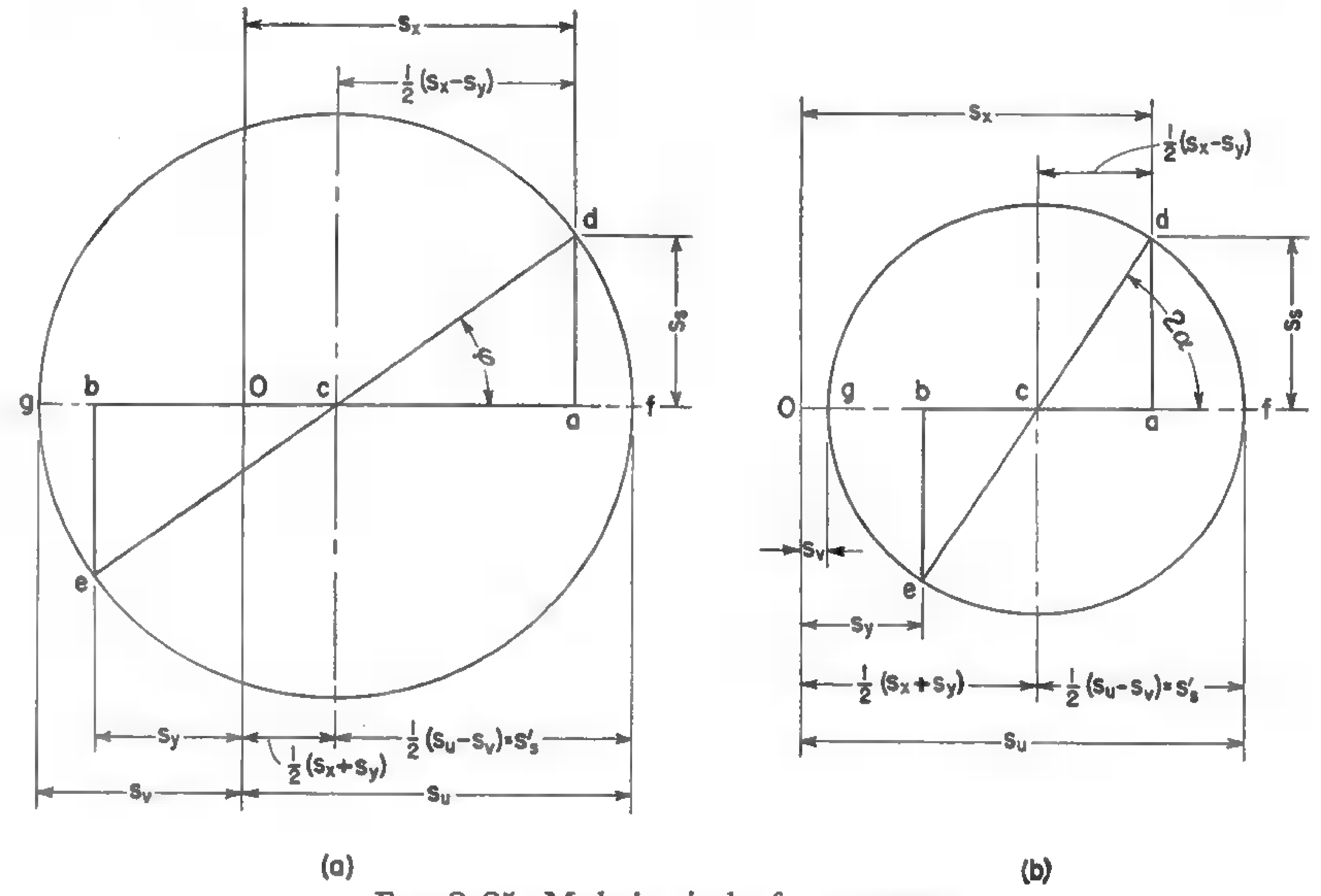


FIG. 2–25. Mohr's circle for stresses.

The foregoing explanation shows how Mohr's circle provides a simple way of finding, for a biaxial stress condition, the principal stress, the maximum shear stresses, and the angle α between the principal stresses and the biaxial stresses. On the other hand, if the principal stresses are known, Mohr's circle permits one to find very easily the maximum shear stresses and also, for any desired angle, the corresponding stresses s_x, s_y, and s_s. In

general this circle gives a clear picture of the relationship of stresses. If at any point there are biaxial stresses s_x and s_y, and the shear stress s_s is zero, these biaxial stresses are the principal stresses.

Mohr's circle also shows the influence of any change in the biaxial stresses. As an illustration let it be assumed that the sign of the vertical stress s_y in example 2–11 and Fig. 2–25a is changed. Mohr's circle for this condition is shown in Fig. 2–25b.

Angles of the Mohr's circle are double those on the actual body. The values of α_m show the angles between the biaxial stresses and the principal stresses. Thus the planes on which the maximum principal stress s_u acts make a clockwise angle α_m with the planes on which s_x acts.

2–13. Combined loads. When a machine part is subjected to several loads simultaneously, the governing stresses are best found by first determining the stresses from each load and then combining them by one of the following methods: superposition; application of equation 2–46, 2–47, or 2–48; application of equations 2–52 and 2–54; or construction of a Mohr circle. To illustrate the procedure several typical cases will be discussed.

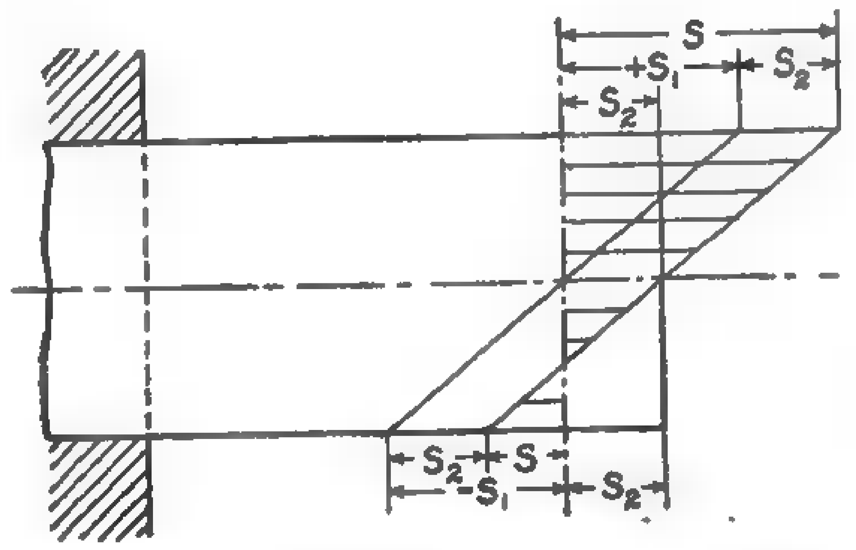


Fig. 2–26. Bending and axial load.

Bending and axial load. The main stresses in bending are direct tensile and compressive stresses, each of which is s_1; and they are simply added algebraically to the direct stress s_2 from the axial load. By superposition the resultant stress is

$$s = s_1 + s_2 \tag{2-59}$$

where a tensile stress is given a plus sign and a compressive stress is given a minus sign. In Fig. 2–26 is represented a cantilever beam that is bent and stressed in tension. Shear from bending does not affect the stresses that must be taken into account. In the outside fibers, s_s is zero; and in the center fibers, where s_s is greatest, the direct stress s is zero.

Bending by several loads. If several loads are imposed on a beam, their actions are superimposed in every respect—to find bending moments, reactions, shears, and deflections.

Example 2–13. Work example 2–2 by using the principle of superposition and taking into account the influence of the weight of the pipe on both the stresses and the deflections.

Since the specific weight of steel is 0.282 lb per cu in. (Table 4–2), the weight of the pipe is

$$0.7854 \times (4.0^2 - 3.548^2) \times 120 \times 0.282 = 90.6 \text{ lb}$$

By case f in Table 2–4, the bending moment from this weight is

$$M = \frac{90.6 \times 120}{8} = 1{,}359 \text{ lb-in.}$$

The stress produced by this moment, with $Z = 2.425$ in.^{3} from example 2–2, is

$$s = \frac{1{,}359}{2.425} = 560 \text{ psi}$$

a) The stress due to the weight of the pipe must be deducted from the allowable stress of 13,400 psi. With this correction the expression for F in example 2–2 becomes

$$F = (13{,}400 - 560) \times \frac{2.425}{30} = 1{,}038 \text{ lb}$$

b) By case f in Table 2–4, the deflection from the uniformly distributed load is

$$y = \frac{5 \times 90.6 \times 120^3}{384 \times 30{,}000{,}000 \times 4.85} = 0.014 \text{ in.}$$

The deflection from the force $F = 1{,}038$ lb is

$$y = \frac{1{,}038 \times 120^3}{48 \times 30{,}000{,}000 \times 4.85} = 0.257 \text{ in.}$$

The combined deflection is

$$y = 0.014 + 0.257 = 0.271 \text{ in.}$$

Thus, while the outside load is decreased by $100 \times (1{,}083 - 1{,}038)/1{,}083 = 4.15$ per cent, the total deflection is increased by $100 \times (0.271 - 0.268)/0.268 = 1.12$ per cent. However, the combined load is $1{,}038 + 90.6 = 1{,}128.6$ lb, which is greater than 1,083 lb.

Torsion and axial load. A combination of torsion and axial load may be encountered in a propeller shaft of a ship, in a shaft used for driving a heavy worm gear, or in a bolt when its nut is tightened. The primary shear stress s_s is found from equation 2–13 for the torsional moment involved. If the axial load is tension, the direct stress s is found by equation 2–9. If the axial load is compression and the piece is short, the stress is found by equation 2–9; if the piece must be treated as a column, equation 2–37 or 2–39 may be used. The combined normal stress is found by equation 2–46, and the combined shear stress by equation 2–47.

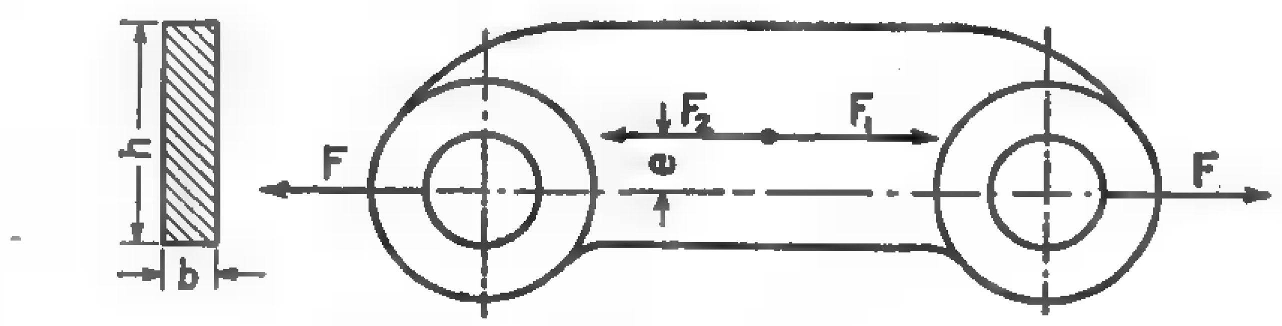


Fig. 2–27. Eccentric loading.

Torsion and bending. Torsion and bending are combined in all shafts used for transmission of power by means of pulleys, sprockets, or gears. The primary shear stress is found from equation 2–13; the bending moment causes a direct stress (tension or compression), which is found by equation 2–22. The compound stresses are found in the same way as in the case of a normal stress and a shear stress by using equations 2–46 and 2–47.

Eccentric load application. If, as in Fig. 2–27, the line of action of a force F does not go through the center of gravity of the cross section of a body but is parallel to the axis of the body, the body will be subjected simultaneously

to direct tension and bending. If we apply at the center of gravity of any cross section two imaginary opposite forces F_1 and F_2, each equal to the force F and parallel to it, the system of forces acting on the part of the body to the left of the action may be considered as the combination of a couple formed by the forces F and F_1 with an arm e and a residual axial force F_2. The stresses produced in the body by the couple with a moment Fe are

$$s_1 = \pm\frac{Fe}{Z}$$

Also, the stress produced by the force $F_2 = F$ is pure tension, and its magnitude is

$$s_2 = \frac{F}{A}$$

The combined stress, as found by superposition, is

$$s = \frac{F}{A} \pm \frac{Fe}{Z} \tag{2-60}$$

When the section is not symmetrical the proper value of Z must be used with the corresponding sign (+ or −).

In Fig. 2-27 the force F produces axial tension; but equation 2-60 applies also if the force produces compression, provided the member is short. In general an eccentric load is equivalent to simultaneous action of bending and an axial load.

EXAMPLE 2-14. Determine the maximum stress created in the offset link in Fig. 2-27 by a force $F = 1{,}000$ lb, if the height h of its cross section is 2 in., its width b is $\frac{1}{2}$ in., and the arm e of the force is $\frac{1}{2}$ in.

The section modulus, as shown in Table 2-5, is

$$Z = \frac{bh^2}{6} = \frac{0.5 \times 2^2}{6} = 0.333 \text{ in.}^3$$

and the bending stresses are

$$s_1 = \pm\frac{1{,}000 \times 0.5}{0.333} = \pm 1{,}500 \text{ psi}$$

The stress in tension is

$$s_2 = \frac{1{,}000}{2 \times 0.5} = 1{,}000 \text{ psi}$$

The tensile stress in the outer fibers nearest to the line of action of the force F is

$$s = s_1 + s_2 = 1{,}500 + 1{,}000 = 2{,}500 \text{ psi}$$

The stress in the outer fibers at the other edge is

$$s = -s_1 + s_2 = -1{,}500 + 1{,}000 = -500 \text{ psi, or 500 psi compression}$$

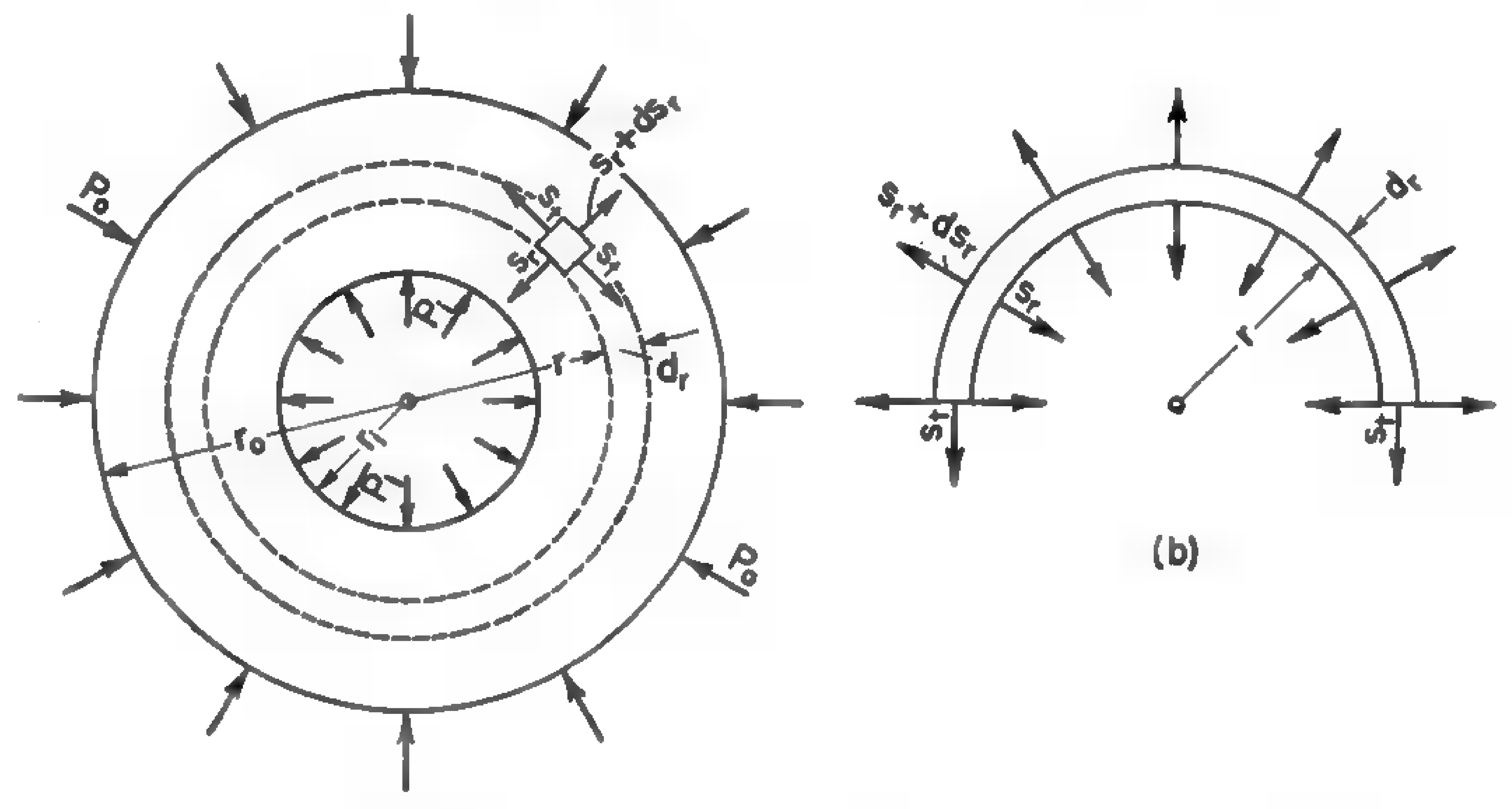
Eccentrically loaded columns. If the member in compression is of such length that it can be classed as a short column, the maximum primary stress s_2 is determined by equation 2-37 and the bending stress s_1 is added to it. To simplify the result, substitute in equation 2-60 the expression for Z in equation 2-21 and replace the moment of inertia I by its equivalent value k^2A. If

Ritter's formula (equation 2-37) is used, the maximum combined stress becomes

$$s_c = \frac{F}{A}\left[1 + \left(\frac{l}{k}\right)^2 \frac{S_e}{\pi^2 nE} + \frac{ce}{k^2}\right] \tag{2-61}$$

The influence of eccentric loading on the maximum stress in a long column is small and can usually be disregarded.

2-14. Thick-walled cylinder. In Fig. 2-28a is shown a cross section of a thick-walled cylinder subjected to uniform normal pressures p_i and p_o on the inside and outside surfaces, respectively, but not subjected to any external force parallel to its axis. Since the body and the loading are symmetrical with respect to the cylinder axis, there will not be any shear stresses in the tangential or radial directions; and any element in a thin cylindrical section



(a) (b)

FIG. 2-28. Thick-walled cylinder subjected to inside and outside pressures.

will be subjected only to normal stresses s_t and s_r as shown. In order to find the relation between these stresses and the pressures p_i and p_o, consider the equilibrium condition of a semicircular element like that in Fig. 2-28b. The axial thickness of the element is taken as unity. The vertical component of the resultant of all the inward radial stresses across the diameter of the element is equal to $2s_r r$, and the vertical component of the resultant of the outward stresses is $2(s_r + ds_r)\ (r + dr)$. Since the sum of downward stresses on the ends of the element is $2s_t\, dr$, the equilibrium equation for the element becomes

$$2s_r r + 2s_t\, dr = 2(s_r + ds_r)(r + dr)$$

After expanding the right side of this equation and neglecting the infinitesimals of higher order than the first, we obtain

$$s_t - s_r = r\frac{ds_r}{dr} \tag{2-62}$$

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If Poisson's ratio is denoted by μ and the modulus of elasticity is denoted by E, the axial deformation ϵ caused by the stresses s_t and s_r can be written as

$$\epsilon = -\frac{\mu s_t}{E} - \frac{\mu s_r}{E}$$

Hence,

$$s_t + s_r = -\frac{\epsilon E}{\mu} \tag{2-63}$$

The right-hand side of this equation is a constant, which can be conveniently designated as $2C_1$. Subtracting equation 2–63 from equation 2–62 gives

$$r\frac{ds_r}{dr} + 2s_r = 2C_1$$

Multiplying each term of the last equation by r and rearranging gives

$$\frac{d(r^2 s_r)}{dr} = 2rC_1$$

After integration we get

$$r^2 s_r = C_1 r^2 + C_2$$

where C_2 is the constant of integration. From this equation,

$$s_r = C_1 + \frac{C_2}{r^2} \tag{2-64}$$

Substituting this value of s_r in equation 2–63, we find that

$$s_t = C_1 - \frac{C_2}{r^2} \tag{2-65}$$

At the inner boundary, $r = r_i$ and the radial stress $s_r = -p_i$. Thus equation 2–64 becomes

$$-p_i = C_1 + \frac{C_2}{r_i^2} \tag{2-66}$$

At the outer boundary $r = r_o$ and the radial stress $s_r = -p_o$. So equation 2–64 now becomes

$$-p_o = C_1 + \frac{C_2}{r_o^2} \tag{2-67}$$

Simultaneously solving equations 2–66 and 2–67 results in

$$C_1 = \frac{r_i^2 p_i - r_o^2 p_o}{r_o^2 - r_i^2} \tag{2-68}$$

and

$$C_2 = \frac{r_i^2 r_o^2 (p_o - p_i)}{r_o^2 - r_i^2} \tag{2-69}$$

Substitution of these values in equations 2–64 and 2–65 gives

$$s_r = \frac{r_i^2 p_i - r_o^2 p_o - \dfrac{r_i^2 r_o^2 (p_i - p_o)}{r^2}}{r_o^2 - r_i^2} \tag{2-70}$$

and

$$s_t = \frac{r_i^2 p_i - r_o^2 p_o + \dfrac{r_i^2 r_o^2 (p_i - p_o)}{r^2}}{r_o^2 - r_i^2} \tag{2-71}$$

These equations were first developed by Lamè.

CHAPTER 3

Dynamic Stresses and Stress Concentration

3-1. Variable loads. Only a few machine parts are subjected to constant, or static, loads. The loads on most parts vary, sometimes in magnitude, sometimes in direction, and frequently both in magnitude *and* direction.

Stresses produced by variable loads are termed *dynamic stresses.* The variable, or dynamic, loads which produce such stresses may be divided into two groups: (1) those produced by outside forces; (2) those resulting from the *inertia* of the mass of the member itself when it moves with a variable speed. Loads produced by variable outside forces are also called *live loads,* as contrasted with *static loads,* or *dead loads.*

A live load may be produced either by the gradual change of a force already applied or by the sudden application of an outside force. When it is produced by a sudden outside force, it is called a *shock load,* or *impact load.*

The influences of these various types of dynamic loads upon the internal stresses of a member differ considerably and must be investigated separately.

3-2. Inertia stresses. Inertia loads are caused by acceleration that acts when a change of velocity, either linear or angular, takes place. The acceleration may be due to a change in the direction of the motion (as of a rotating crank), a change in the magnitude of the velocity (as of a reciprocating part), or a change in both direction and magnitude (as of a connecting rod, one end of which rotates while the other moves back and forth).

Centrifugal loads. The general expression for the centrifugal force created in a mass m is

$$C = \frac{mv^2}{R} \tag{3-1}$$

in which v is the circumferential velocity, in feet per second, and R is the radius of curvature of the path of the motion of the mass, in feet. We will consider the mass of 1 cu in. and we will express this mass as w/g, where w is the specific weight of the material, in pounds per cubic inch, and $g = 32.2$ fps per sec. Also, we will use the radius of the curvature r in inches and will substitute $\frac{1}{12}r$ for R. Equation 3-1 then becomes

$$C = \frac{12wv^2}{gr} \tag{3-2}$$

Centrifugal force can create tensile stresses, as in the case of the rim of a rotating pulley; or it can create bending stresses, as in the case of a locomotive coupling rod or a connecting rod.

A change of rotary speed, as in starting or stopping an engine, creates additional stresses, such as bending in the arms of a flywheel or torsion in the shaft which carries it.

Pulley or flywheel. In the case of a pulley or a flywheel the velocity can be expressed by the relation

$$v = \frac{2\pi rn}{12 \times 60} = \frac{\pi rn}{360} \tag{3-3}$$

where n is the number of revolutions per minute. If we multiply the right-hand side of equation 3-2 by the area A of the cross section, in square inches, and by the projected length of one-half the circumference, or $2r$, in order to obtain the component of the centrifugal force normal to any diameter of the pulley or flywheel, equation 3-2 becomes

$$C = w\left(\frac{\pi rn}{360}\right)^2 A \times 2r \times \frac{12}{gr} = 0.001828\,\frac{wr^2n^2A}{g} \tag{3-4}$$

This force is applied to the two rim sections at the ends of the radii of curvature. Therefore, dividing C by $2A$ gives the tensile stress in the rim. Thus,

$$s = 0.000914\,\frac{wr^2n^2}{g} \tag{3-5}$$

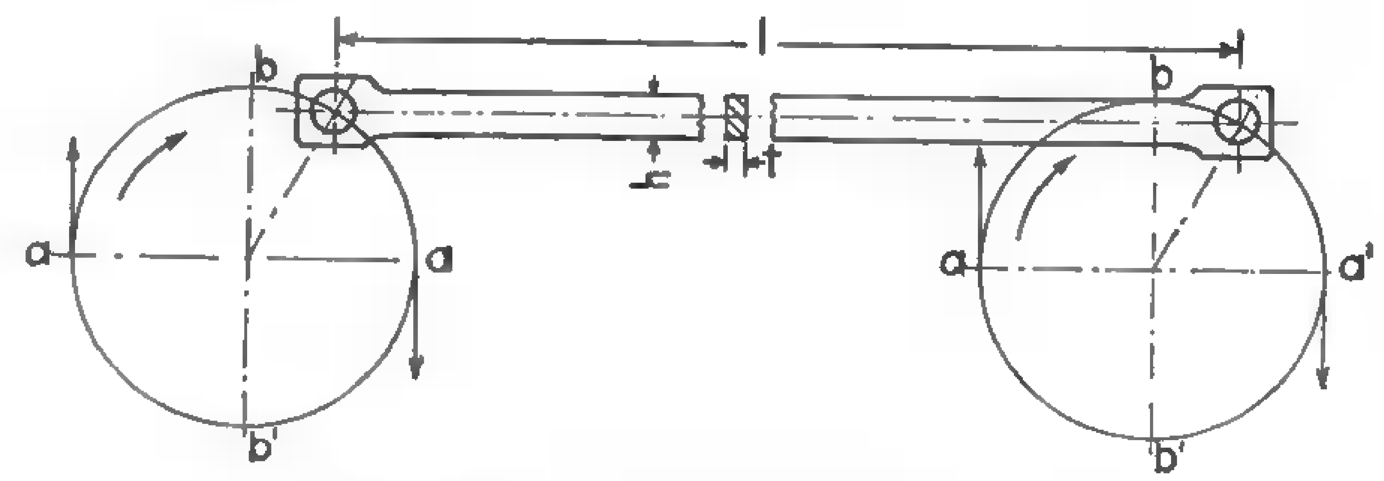


FIG. 3-1. Locomotive coupling rod.

Coupling rod. In the case of a coupling rod like that shown in Fig. 3-1 the centrifugal force acts as a load uniformly distributed on a simple beam of length l. Its bending component has a maximum value expressed by equation 3-2 when the rod ends pass through the points b or through the points b'. It becomes zero when the rod ends are at points a or a'.

The maximum uniformly distributed bending load is

$$C = \frac{12whtlv^2}{gr} \tag{3-6}$$

where t is the thickness of the rod, in inches.

The axial component of the centrifugal force is zero when the rod ends are at points b or at points b'. It has its maximum value given by equa-

tion 3–6 when the rod ends are at points a or at points a'. This force creates compressive and tensile stresses on the cross section which vary from zero at one end of the rod to a maximum value at the other end.

EXAMPLE 3–1. Find the stress due to centrifugal force in the locomotive coupling rod shown in Fig. 3–1. The wheels turn at 200 rpm, $r = 9$ in., $l = 72$ in., $h = 2\frac{1}{2}$ in., and $t = 1\frac{1}{2}$ in. Use $w = 0.282$ lb per cu in.

The circumferential velocity, by equation 3–3, is

$$v = \frac{\pi \times 9 \times 200}{360} = 15.72 \text{ fps}$$

The bending load is found by equation 3–6. It is

$$C = \frac{12 \times 0.282 \times 2.5 \times 1.5 \times 72 \times 15.72^2}{32.2 \times 9} = 784 \text{ lb}$$

If the rod is treated as a simple beam uniformly loaded, the bending moment is, according to Table 2–4,

$$M = \frac{784 \times 72}{8} = 7{,}060 \text{ lb-in.}$$

The section modulus, according to Table 2–5, is

$$Z = \frac{1.5 \times 2.5^2}{6} = 1.562 \text{ in.}^3$$

Hence, the bending stress is

$$s = \frac{7{,}060}{1{,}562} = 4{,}520 \text{ psi}$$

The direct stress is very small. It is

$$s_2 = \frac{784}{2.5 \times 1.5} = 209 \text{ psi}$$

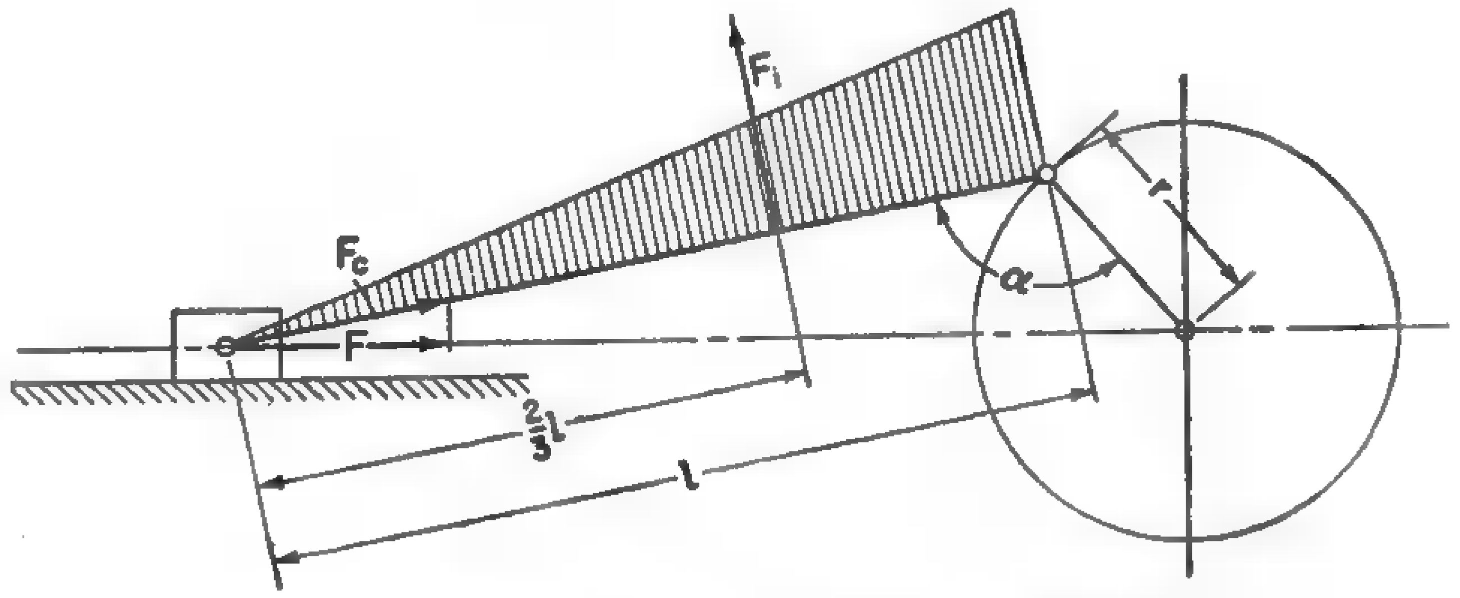


FIG. 3–2. Forces acting on a connecting rod.

Connecting rod. The motion of a connecting rod is a combination of the rotation of the crank end and the reciprocating motion of the wrist-pin end, or small end. Accordingly, the centrifugal force acting on an element of the rod gradually changes from a maximum value at the crank end to zero at the small end, as shown by the shaded triangle in Fig. 3–2. The force of inertia that produces bending acts normally to the rod axis. Without any appreciable error so far as the stress is concerned, it may be assumed that the rod has a uniform cross section and that the inertia force is represented by the shaded triangle. The magnitude of the inertia force will be one-half that given by the basic equation 3–2, or

$$F_i = \frac{12 W v^2 \sin \alpha}{2 g r} \tag{3–7}$$

where W is the weight of the rod itself, not including the ends.

This force acts at a distance of $\frac{2}{3}l$ from the wrist pin. The maximum bending moment produced by this force is at a distance of $\frac{1}{3}l$ from the wrist pin, and its magnitude is[1]

$$M = \frac{2 F_i l}{9\sqrt{3}} \tag{3–8}$$

Substituting for F_i its value from equation 3–7 gives

$$M = \frac{0.769 W v^2 l \sin \alpha}{g r} \tag{3–9}$$

The bending stress is found as usual by equation 2–22.

Naturally, in a connecting rod the main stress is caused by the force F acting on the piston (see Fig. 3–2), or more specifically by its component F_c acting along the axis of the rod. This force is also variable and depends upon the position of the piston, or angle α. The connecting rod is a short column subjected to buckling effect. The compressive stresses must be found for several values of the angle α, and these stresses must be combined with stresses caused by bending due to the inertia force for the same angles.

Reciprocating motion. The usual case of reciprocating motion is that of a crosshead produced by a uniformly rotating crank and a connecting rod. The force of inertia F in this case is expressed approximately by the formula

$$F = \frac{12 W v^2}{g r}\left(\cos \alpha + \frac{r}{l} \cos 2\alpha\right) \tag{3–10}$$

where W is the weight of the reciprocating masses, in pounds;
- v is the crank velocity, in feet per second;
- g is the acceleration of gravity = 32.2 fps per sec;
- α is the angle between the crank and the center line of the piston;
- l is the length of the connecting rod, in inches.

The angle α is measured from the head-end dead center. Substituting for v its value from equation 3–3 and carrying through the numerical calculations, we obtain

$$F = \frac{0.000914 W r n^2}{g}\left(\cos \alpha + \frac{r}{l} \cos 2\alpha\right) \tag{3–11}$$

[1] Expressions for triangle load distribution are given in Lionel S. Marks, ed., *Mechanical Engineers' Handbook*, 5th ed. (New York: McGraw-Hill Book Company, Inc., 1951), p. 427, and in R. T. Kent, *Mechanical Engineers' Handbook*, 12th ed., Vol. II, *Design and Production*, ed. by Colin Carmichael (New York: John Wiley & Sons, Inc., 1950), p. 8-13.

The maximum value of F occurs when $\alpha = 0$, or when the crank is at the head-end dead center, and is

$$F_{\max} = \frac{0.000914 Wrn^2}{g}\left(1 + \frac{r}{l}\right) \tag{3-12}$$

At the crank-end dead center, where $\alpha = 180°$, F attains the maximum negative value, acting in the opposite direction. Then

$$F_2 = -\frac{0.000914 Wrn^2}{g}\left(1 - \frac{r}{l}\right) \tag{3-13}$$

The inertia forces of reciprocating masses produce in machine parts stresses which must be added to those produced by outside forces. The inertia stresses may exceed the stresses due to the outside forces.

3-3. Impact stresses. If a moving body strikes another body, the second body is subjected to an impact which is equal to the kinetic energy of the moving body.

Impact energy. If the weight of a moving body is W lb and its velocity is v fps, its kinetic energy in foot-pounds is

$$K_i = \frac{Wv^2}{2g} \tag{3-14}$$

For stress calculations it is convenient to express the impact energy as if it were produced by a falling body. The height of fall, in feet, that would develop the velocity v is

$$h = \frac{v^2}{2g} \tag{3-15}$$

Hence, the impact energy of a body falling from a height h is

$$K_i = Wh \tag{3-16}$$

where K_i refers to the instant when the falling body strikes the other body.

The total work done by a falling body can also be computed by equation 3-16 if h is understood to include both the height of the fall before contact and the deformation of the body that is struck.

Impact stress. A general expression for impact stress may be derived by considering the normal stress created by a falling weight W, which is allowed to drop through a distance h before it strikes axially a steel rod whose cross-sectional area is A and whose length is l. When the weight W strikes the rod, the rod will be compressed a distance e', and the external work performed by the weight will be $W(h+e')$.

The stress in the rod was zero before the weight struck it; and after it had been compressed a distance e', the stress became s'. If s' is within the elastic limit, the internal work of the variable compressive force is $\frac{1}{2}As'e'$. If the internal work is equated to the external work and the equation is solved for s', the result is

$$s' = 2W\frac{(h+e')}{Ae'} \tag{3-17}$$

But $W/A = s$, where s is the static stress; and for elastic deformations

$$e' = \frac{es'}{s}$$

where e is the static deformation under the action of the weight W. Substituting these values in equation 3-17 and solving it for s', we get

$$s' = s\left(1 + \sqrt{1 + \frac{2h}{e}}\right) \tag{3-18}$$

Equation 3-18 is general, and s' is the impact stress due to the force W which, when applied as a static load, would give the deformation e.

Since the stresses and deformations are proportional, the general equation for the deformation under impact action is

$$e' = e\left(1 + \sqrt{1 + \frac{2h}{e}}\right) \tag{3-19}$$

Impact from a direct load. For tension or compression, $e = sl/E$ and the formula for the impact stress is

$$s' = s\left(1 + \sqrt{1 + \frac{2hE}{sl}}\right) = \frac{W}{A}\left(1 + \sqrt{1 + \frac{2hEA}{Wl}}\right) \tag{3-20}$$

Impact and bending. When a beam is subjected to impact, the deformation e means the deflection y, which is computed for the type of beam and the loading in the particular case. Equations 3-18 and 3-19 apply without any other changes.

Impact and torsion. Equations 3-18 and 3-19 also can be used, when a force causing torsion is applied with impact. In this case, however, the deformation e means the angular deflection θ, in radians; and the travel h of the force must also be expressed in radians. If the impact is given in energy units, the stress s' is found more easily from equation 3-31 by equating the energy K_i to the resilience U_s and noting that $s' = s_s$.

Sudden load. If the weight W is applied suddenly but it does not have an appreciable velocity before it strikes the body, the value to be substituted for h in equation 3-18 is zero and

$$s' = 2s \tag{3-21}$$

The stress produced by a suddenly applied load is therefore double that produced by the same load if it were applied statically or gradually. The deformation e' is $2e$.

Inertia effect. When a body having a weight W strikes another body that has a weight W', some of the impact energy is used to overcome the inertia

TABLE 3–1

COEFFICIENTS IN THE INERTIA EQUATION 3–27

Type of Impact	a	b
Longitudinal impact on bar	$\frac{1}{3}$	$\frac{1}{2}$
Center impact on simple beam.	$\frac{17}{35}$	$\frac{5}{8}$
Center impact on beam with fixed ends.	$\frac{13}{35}$	$\frac{1}{2}$
End impact on cantilever beam.	$\frac{4}{17}$	$\frac{3}{8}$

of the weight W' and to accelerate that weight. Therefore, because of the inertia of the body that is struck, that body is subjected to less impact and the resulting stresses and deformations in it are reduced. According to the laws of collision of two perfectly inelastic bodies, it may be assumed that the impact energy Wh is reduced to nWh. The value of n is less than 1 and may be found by the formula

$$n = \frac{1 + am}{(1 + bm)^2} \tag{3–22}$$

where $m = W'/W$, and a and b are the coefficients whose values are given in Table 3–1 for the main cases of impact action.

When the inertia effect is considered, stresses and deformations may be computed by substituting nh for h in equations 3–18, 3–19, and 3–20. However, when one body comes in contact with another, the local stress near the area of contact is increased by the inertia of the struck body; and the impact may produce a local stress that exceeds the elastic limit and thus may cause a permanent deformation.

3–4. Resilience. When external forces deform a body and create internal stresses in it, the body can absorb some of the energy. Also, the body can give up energy when the forces are removed. This property of a material is generally called *resilience*. In other words the resilience of a body is the potential energy stored up in it when it is deformed. Since resilience is measured by the amount of deformation, or strain, it is also known as *strain energy*. Resilience is a function of internal stresses, regardless of whether they are created by static loads or dynamic loads. However, since resilience is important chiefly in cases of dynamic loads, it is discussed here rather than in Chapter 2.

Numerically, the resilience U of a body is equal to the work required to produce a deformation which sets up a stress s, when s does not exceed the elastic limit. This work is equal to the product of the average applied force and the final deformation. Thus,

$$U = \tfrac{1}{2}Fe \tag{3–23}$$

If a load W is applied with impact, the force F may be taken as

$$F = \frac{We'}{e} = \frac{Ws'}{s} \tag{3–24}$$

where e' and s' have the same meaning as in the preceding section.

The resilience U indicates the amount of impact energy that a body can withstand without permanent deformation when acting as a rigid spring. Thus the resilience of a body is a measure of its ability to resist impact. Equation 3–23 is general and applies to all types of impact.

Modulus of resilience. The ability of a material to withstand impact without permanent deformation is indicated by its modulus of resilience. This modulus, which is designated by u, is a measure of the elastic energy stored up in a unit volume of the material at the elastic limit. It is usually expressed in inch-pounds per cubic inch.

Resilience in tension or compression. By substituting for F and e in equation 3–23 the values given by equations 2–9 and 2–10, we can obtain the following general formula:

$$U = \tfrac{1}{2}\frac{As \times sl}{E} = \frac{s^2V}{2E} \tag{3–25}$$

where U is the resilience, in inch-pounds;

s is the direct stress invoked by the load, in pounds per square inch;

A is the constant cross-sectional area of the body, in square inches;

$V = Al$ is the volume of the body, in cubic inches.

The modulus of resilience may be found from equation 3–25 by substituting S_e for s and 1 for V. Thus,

$$u = \frac{S_e^2}{2E} \tag{3–26}$$

For a direct stress, u may be found graphically by determining the area Oea in Fig. 2–2. For a deformation resulting in a stress S_y', the unit resilience is represented by the area $cy'b$.

It should be noted that the modulus of elasticity of a given metal is practically a constant characteristic. The elastic limit, on the other hand, depends to a great extent upon the method of shaping the piece, which may be casting, forging, or rolling; upon the addition of certain chemical elements, such as carbon, chromium, nickel, and vanadium; and finally upon the subsequent heat treatment. By proper heat treatment the elastic limit of certain alloy steels can be made 2.4 times as great as that of untreated metal, and the resilience can be made 5.7 times as great.

Table 3–2 gives moduli of resilience in tension for typical materials used in machinery. These values may serve as guides for selecting the proper material when shock loads must be dealt with.

TABLE 3–2

RESILIENCE IN TENSION

Material	Elastic Limit S_e (psi)	Modulus of Elasticity E (psi)	Modulus of Resilience u (in.-lb)	Impact Strength (Izod number)
Cast iron				
Class 20 (ordinary)....	(6,200)*	10,000,000	1.9	....
Class 25..............	(10,000)*	13,000,000	3.8	....
Nickel, Grade II	(17,000)*	18,000,000	8.0	....
Malleable...........	20,000	25,000,000	8.0	7.9
Aluminum alloy, SAE 33..	7,000	9,700,000	2.5	....
Brass, SAE 40 or SAE 41..	10,000	12,000,000	4.0	2.7
Bronze, SAE 43.........	28,000	16,000,000	24.5	....
Monel metal				
Hot-rolled..........	30,000	25,500,000	17.6	120
Cold-rolled, normalized	70,000	25,500,000	96	110
Steel				
SAE 1010...........	30,000	30,300,000	15	....
SAE 1030...........	36,500	30,000,000	22	20
SAE 1050, annealed...	48,500	29,700,000	38	....
SAE 1095, annealed...	60,000	29,700,000	60	....
SAE 1095, tempered...	75,000	29,700,000	94	....
SAE 2320, annealed...	45,000	29,700,000	34	52
SAE 2320, tempered...	100,000	29,700,000	167	40
SAE 3250, annealed...	80,000	31,000,000	193	....
SAE 3250, tempered...	200,000	31,000,000	645	30
SAE 6150, annealed...	62,000	31,000,000	62	....
SAE 6150, tempered...	160,000	31,000,000	466	....
Rubber................	300	150	300	....

* Cast iron has no well-defined elastic limit, but the values in parentheses may be safely used instead of it for all practical purposes.

EXAMPLE 3–2. A $\frac{5}{8}$-in. 6×19 construction crucible-steel rope runs at a speed of 3.5 mph between the rails of a narrow-gage road. The weight of a loaded car which must be connected to and pulled by this rope is 0.8 ton; the weight of the rope is 0.62 lb per ft; the length of the rope between the driving pulley and the point where the car is hooked in is 100 ft. Determine the stress created in the rope by the impact of hooking-in the car, assuming that the modulus of elasticity of the rope is 12,000,000 psi.

The relative velocity of the rope is

$$v=\frac{3.5\times 5{,}280}{60\times 60}=5.14 \text{ fps}$$

The impact energy, by equation 3–14, is

$$K_i=\frac{0.8\times 2{,}000\times 5.14^2}{2\times 32.2}=656 \text{ ft-lb}=7{,}872 \text{ in.-lb}$$

If it is assumed that the flexible core weighs 5 per cent, the net weight of the steel wires is $0.62\times 0.95=0.589$ lb per ft. With a specific weight of 0.282 lb per cu in. the cross-sectional area is

$$A=\frac{0.589}{12\times 0.282}=0.1741 \text{ sq in.}$$

By equation 3–25 the resilience of the cable is

$$U=\frac{s^2\times 0.1741\times 100\times 12}{2\times 12{,}000{,}000}=0.00000871s^2 \text{ in.-lb}$$

Equating this resilience to the impact energy and solving for s gives

$$s=\sqrt{\frac{7{,}872}{0.00000871}}=30{,}000 \text{ psi}$$

Resilience in bending. A cantilever beam with a concentrated load at its free end will be used as a typical case to illustrate the procedure in computing the resilience of a beam. When a load F is applied at the end of a cantilever beam having a uniform cross section and a length l, the deflection of the free end is

$$y=\frac{Fl^3}{3EI}$$

By the general equation 3–23 the resilience is

$$U=\frac{Fy}{2}=\frac{F^2l^3}{6EI}$$

If s denotes the maximum stress in the extreme fiber at the support,

$$Fl=\frac{sI}{c}$$

Substituting Ak^2 for I gives

$$U=\frac{s^2k^2Al}{6Ec^2} \tag{3–27}$$

The maximum resilience per unit volume of the beam is

$$u=\left(\frac{k}{c}\right)^2\frac{S_e^2}{6E} \tag{3–28}$$

It is easy to prove that equations 3–27 and 3–28 hold also for a simple beam loaded at the middle or for a beam clamped at the ends and also loaded at the middle. Equations for other beams have the same general form but the numerical coefficients are different.

It should be remembered that the stress in bending in a beam varies from end to end and in each transverse section from the outer fiber to the neutral plane and further on to the other outer fiber. In equation 3–27 s is the maximum stress produced in the outer fibers of the dangerous section.

Resilience in shear. By starting with the general equation 3–23 and proceeding in the manner described for deriving equation 3–25, the resilience in transverse shear of a member with the volume V can be determined. The result is

$$U_s=\frac{s_s^2V}{2G} \tag{3–29}$$

The modulus of resilience is

$$u_s=\frac{S_s^2}{2G} \tag{3–30}$$

Resilience in torsion. Assume that a round bar is held at one end and twisted at the other by a couple whose moment is Fa; and that the angular

deflection of the free end is θ. The work of the couple is then $\frac{1}{2}Fa\theta$, and the resilience of the bar is

$$U_s = \tfrac{1}{2}Fa\theta$$

If s_s denotes the maximum shear stress at the fixed end, equation 2–13 gives

$$Fa = \frac{s_s J}{c}$$

Replacing J by Ak_o^2, where k_o is the polar radius of gyration, and eliminating θ by use of equation 2–5, we can obtain the equation

$$U_s = \frac{s_s^2 k_o^2 Al}{2Gc^2} \qquad (3\text{–}31)$$

The resilience per unit volume can be found by taking Al as 1. Thus,

$$u_s = \left(\frac{k_o}{c}\right)^2 \frac{S_s^2}{2G} \qquad (3\text{–}32)$$

For a round hollow shaft with an outside diameter D_1 and an inside diameter D_2, the polar radius of gyration is $k_o = \sqrt{(D_1^2 + D_2^2)/8}$ and $c = \frac{1}{2}D_1$. Therefore the unit resilience is

$$u_s = \left[1 + \left(\frac{D_2}{D_1}\right)^2\right]\frac{S_s^2}{4G} \qquad (3\text{–}33)$$

For a round solid shaft, $D_2 = 0$ and the unit resilience is

$$u_s = \frac{S_s^2}{4G} \qquad (3\text{–}34)$$

EXAMPLE 3–3. Determine the maximum torsional impact that can be withstood without permanent deformation by a $3\frac{15}{16}$-in. cylindrical shaft 15 ft long and made of SAE 1030 steel. For this steel, $S_s = 26{,}000$ psi and $G = 12{,}500{,}000$ psi. By equation 3–34, the unit resilience is

$$u_s = \frac{26{,}000^2}{4 \times 12{,}500{,}000} = 13.5 \text{ in.-lb per cu in.}$$

The volume of the shaft is

$$V = 0.7854 \times (3.937)^2 \times 15 \times 12 = 2{,}190 \text{ cu in.}$$

Hence, the maximum torsional impact, which is equal to the resilience, is

$$K_{it} = U_s = 2{,}190 \times 13.5 = 29{,}550 \text{ in.-lb}$$

Values of the maximum resilience per unit volume for various types of loading and the more commonly used cross sections are given in Table 3–3.

3–5. Stress concentration. When the cross section of a piece changes abruptly at a certain place, as when the piece has a hole, a notch, or a groove, or where a small section joins a larger one, then the stress in the piece ceases to follow the elementary equations established in sections 2–3 to 2–5. In such a piece the fibers closest to the abrupt change in the cross

TABLE 3–3

MAXIMUM RESILIENCE PER UNIT VOLUME

Type of Loading	*Modulus of Resilience (psi per cu in.)*
Tension or compression	$\frac{S_e^2}{2E}$
Shear, simple transverse	$\frac{S_e^2}{2G}$
Bending in beams with simply supported ends:	
Concentrated center load and rectangular cross section	$\frac{S_e^2}{18E}$
Concentrated center load and circular cross section	$\frac{S_e^2}{24E}$
Concentrated center load and I beam section	$\frac{3S_e^2}{32E}$
Uniform load and rectangular section	$\frac{3S_e^2}{56E}$
Torsion:	
Solid round bar	$\frac{S_s^2}{4G}$
Hollow round bar with D_1 greater than D_2	$\left[1+\left(\frac{D_2}{D_1}\right)^2\right]\frac{S_s^2}{4G}$
Springs:	
Laminated with flat leaves of uniform strength	$\frac{S_e^2}{6E}$
Flat spiral with rectangular section	$\frac{S_e^2}{24E}$
Helical with round section and axial load	$\frac{S_s^2}{4G}$
Helical with round section and axial twist	$\frac{S_e^2}{8E}$
Helical with rectangular section and axial twist	$\frac{S_e^2}{6E}$

section are affected more than are those farther from this place. This phenomenon is called *stress concentration*, or *localized stress*, and the change of section is called a *discontinuity* or a *stress raiser*. As shown in Figs. 3–7 and 3–13, the stress in the fibers nearest to the discontinuity is increased most. This increase tapers off as the distance to the fibers from the discontinuity becomes greater until at the most remote part of the section the stress decreases even below the nominal, or average, value. This must be true because the total internal resistance of a section with a stress raiser is the same as that of an equal section without a stress raiser.

Stress concentration due to a discontinuity occurs whether the stress comes from a direct load, from bending, or from torsion.

If s_o is the nominal stress in a section with a discontinuity, as determined by an elementary formula, and if s_1 is the maximum or significant stress at the discontinuity, then

$$s_1 = s_o K' \tag{3-35}$$

where K' is called the *stress-concentration factor*. Thus the stress-concentration factor indicates the maximum increase of the stress over the nominal stress computed without the influence of the discontinuity.

The increase of the stress as expressed by the stress-concentration factor depends on three things: the type and size of the discontinuity, the material of the part, and the character of the load. Only the influence of the discontinuities as such will be discussed here. The other two factors will be discussed in Chapter 5.

Form stress factor. The theoretical magnitudes of stress-concentration factors are determined either by mathematical analysis or by special experimental methods, the more important of which are the photoelastic,[2] the plaster-model,[3] and the soap-film[4] methods.

By the nature of its determination the theoretical stress factor depends only on the type and relative size of the discontinuity or on its geometrical form. Therefore it is commonly called the *form stress factor* and is designated by K in order to distinguish it from the actual *stress-concentration factor*, which depends also on the material and type of loading.

Most engineering materials are elastic and have the ability to yield at the point of excessive stress. These characteristics spread the localized high stress and lower its intensity. As a result the actual stress-concentration factor K' is always lower than the corresponding form stress factor K. The form stress factor K may be considered as the high limit of stress concentration under the most adverse conditions.

The values of the form stress factors for various types of discontinuities given in this book, mostly in the form of curves, are based on the most nearly reliable data available at present, but should not be considered absolutely correct. New investigations are being conducted and published continuously, and as a result of these the data which follow may require revision.

3–6. Stress concentration from direct loads. In the case of a round hole the stress concentration depends on the condition of the hole, as whether it is free, empty, or filled with a bolt or pin.

[2] M. M. Frocht, "Recent Advances in Photoelasticity," *Transactions of the American Society of Mechanical Engineers*, Vol. 53, APM–53–11 (1931), p. 135.

[3] F. B. Seely and T. J. Dolan, "*Stress Concentration at Fillets, Holes, and Keyways as Found by the Plaster Model Method*," Bulletin No. 276, University of Illinois Engineering Experiment Station (June, 1935).

[4] F. B. Seely, *Advanced Mechanics of Materials* (New York: John Wiley & Sons, Inc., 1932), pp. 189 ff.

Free round holes. The stress distribution in a plate of infinite width containing a round hole, as found by a theoretical analysis, is illustrated in Fig. 3–3. At a point *1* at one edge of the hole, $K = 3$. From there the stress decreases at first very rapidly, and then gradually, asymptotically approaching s_o.

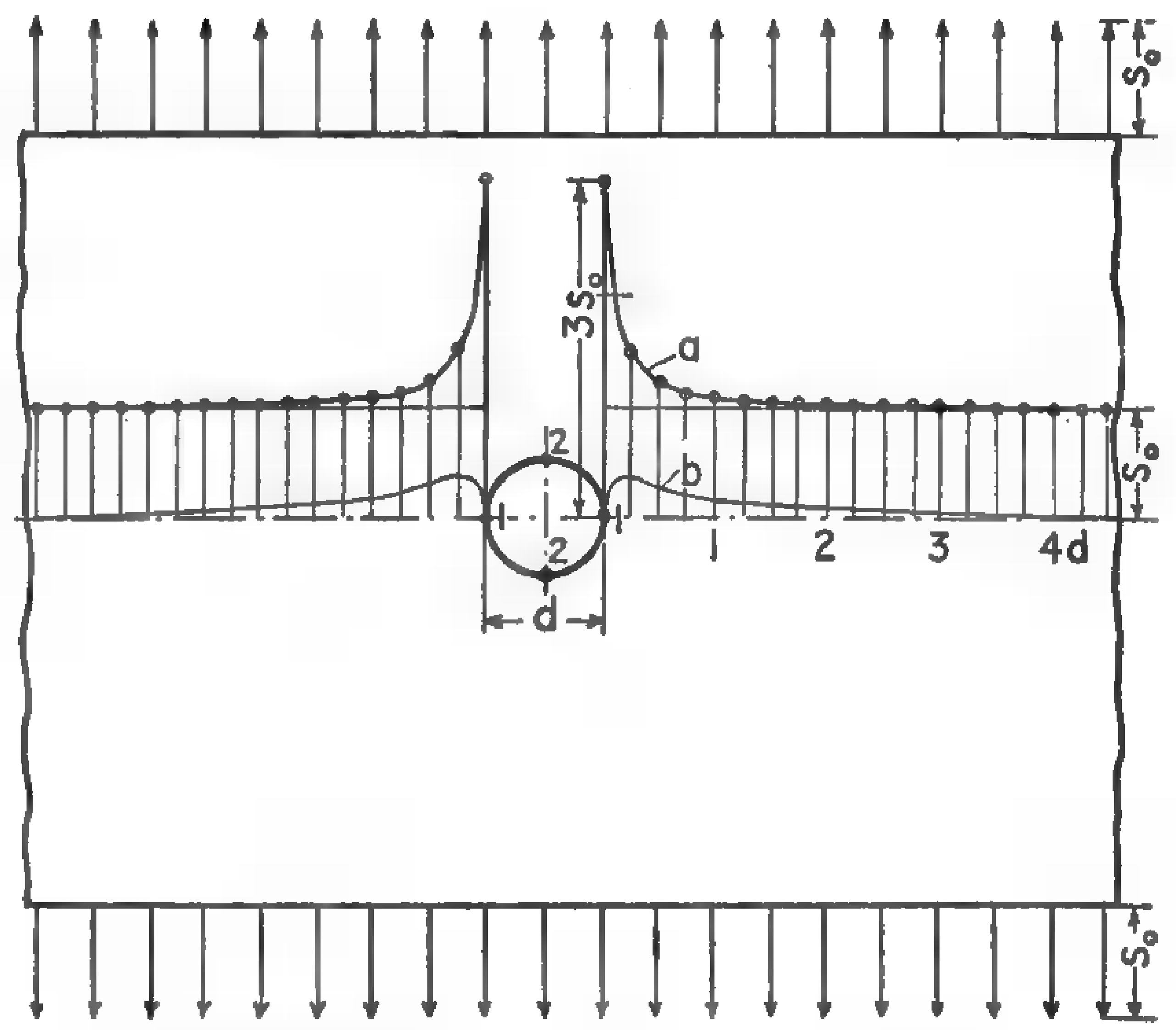


FIG. 3–3. Stress concentration due to hole in wide plate.

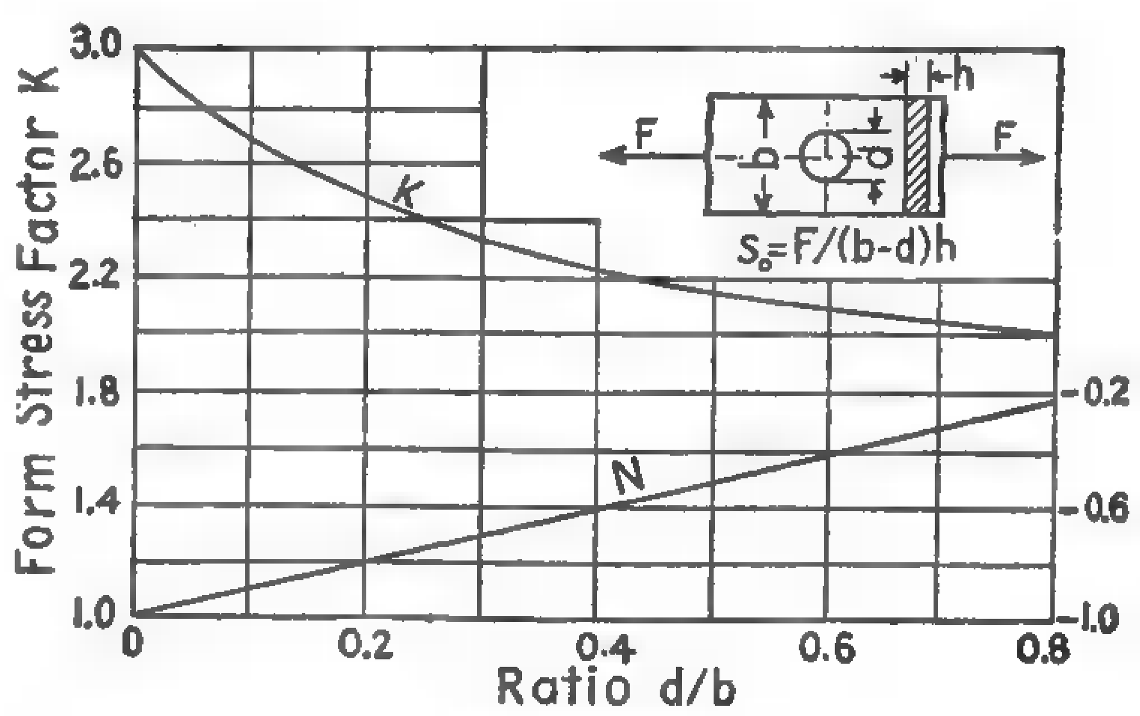


FIG. 3–4. Form stress factor due to hole in narrow plate.

If the width of the plate is b and the diameter of the hole is d, the value of K decreases with the increase of the ratio d/b, as shown in Fig. 3–4. Theoretically, a small hole has a greater effect than a large one. Actually, the effect of a small hole, by which is meant a hole less than $\frac{1}{8}$ in. in diameter, is not as great as that of a larger one.

At a point *2*, Fig. 3–3, a tranverse stress is induced. Its intensity is

$$s_2 = Ns_o \tag{3-36}$$

Values of the coefficient N are also given in Fig. 3–4, which shows that the transverse stress has a negative sign. If the direct stress is tension, the transverse stress is compression, and vice versa.

Shaft with hole. The form stress factor for a shaft containing a hole can be found from Fig. 3–5.[5] As may be seen by comparing this curve with Fig. 3–4, the values of K for a shaft are about 10 per cent lower.

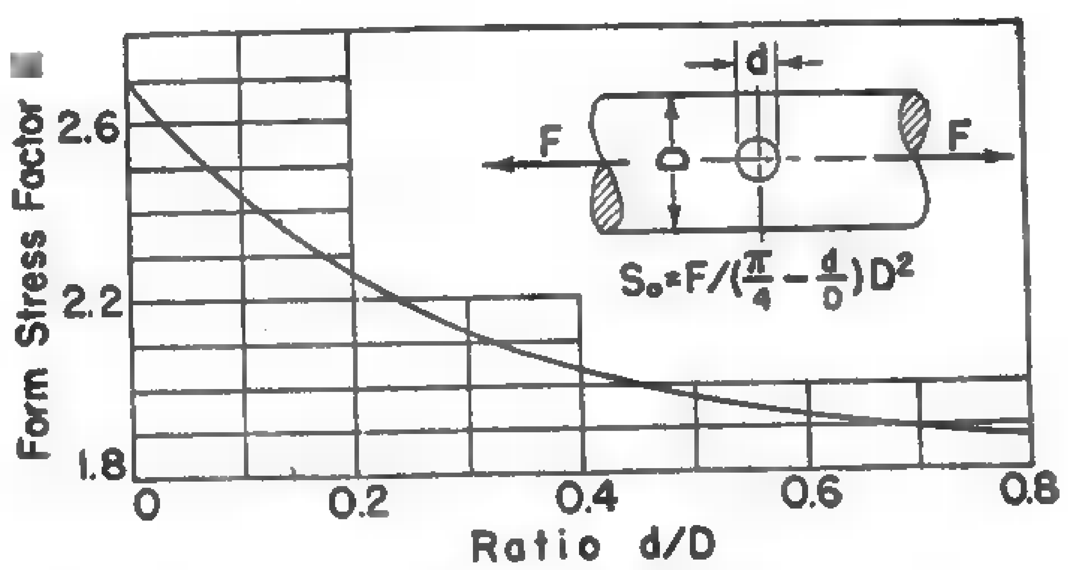


FIG. 3–5. Form stress factor due to through hole in shaft loaded in tension.

Filled holes. The relative values of the axial stresses found by the photoelastic method in a plate with a hole filled by a pin, where the load is applied at the ends of the plate, are given in Fig. 3–6. It shows that the value of K is 2.5, which is practically the same as that for a free hole when d/b is 0.2. The lower curve gives the transverse stresses s_r in the main section. At first

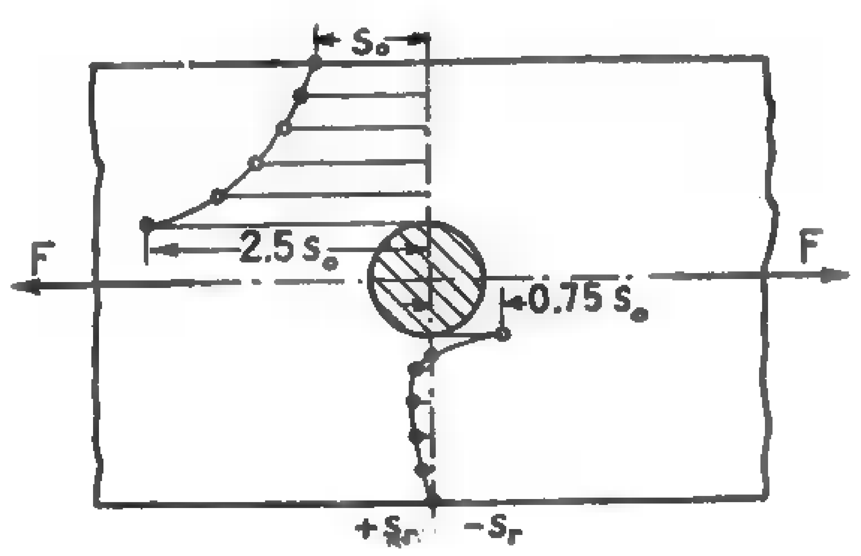


FIG. 3–6. Hole filled with pin.

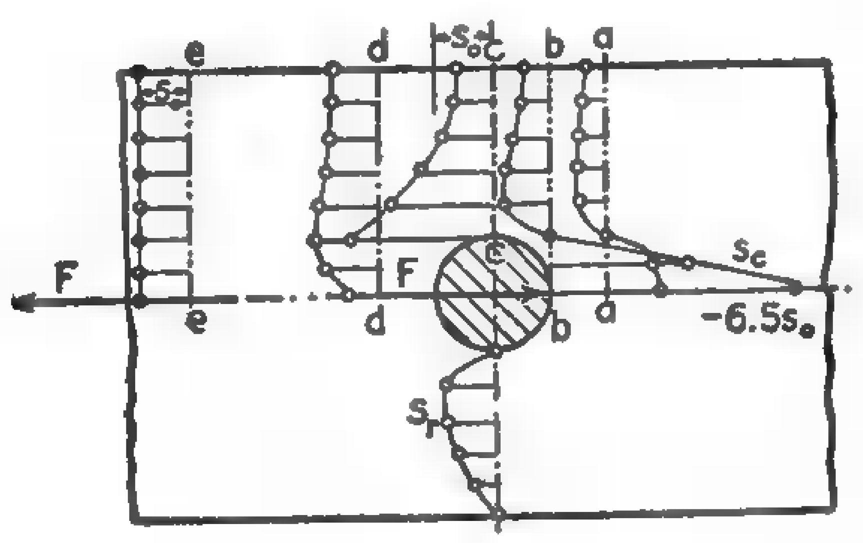


FIG. 3–7. Hole filled with pin which transmits load.

there is compression amounting to $-0.75s_o$, where the plate touches the pin. The stress then decreases rapidly to zero; and after that it reaches a maximum of about $0.25s_o$ in tension.

The stress distribution in a plate when the hole is filled by a pin and the load is applied through the pin is shown in Fig. 3–7. The maximum tensile stress in the main section parallel to the line of action of the load occurs

[5] C. Lipson, G. C. Noll, and L. S. Clock, *Stress and Strength of Manufactured Parts* (New York: McGraw-Hill Book Company, Inc., 1950), p. 68, and M. M. Frocht, "Factors of Stress-Concentration Photoelastically Determined," *Trans. ASME*, Vol. 57, APM (1935), p. A-67.

at point c and is about the same as before; thus, $s = 2.5s_o$. All the radial, or transverse, stresses s_r in this section are tension and the maximum is about $0.5s_o$. The curve for s_c shows the radial compressive stresses in the section through the center line of the plate, under the pin. The stress where the pin presses against the hole edge is very great, probably on the order of $6.5s_o$. At a distance $\frac{1}{16}$ in. lower, it is about $3s_o$, and it becomes zero about $\frac{7}{16}$ in. from the hole. All the transverse stresses in the section b–b are compressive, the maximum[6] being at point b and equal to $6.5s_o$. This gives a value of $N = -6.5$ in equation 3–36. This stress is very high; but being a compressive one, it produces a plastic deformation in actual machine parts and does not cause failure.

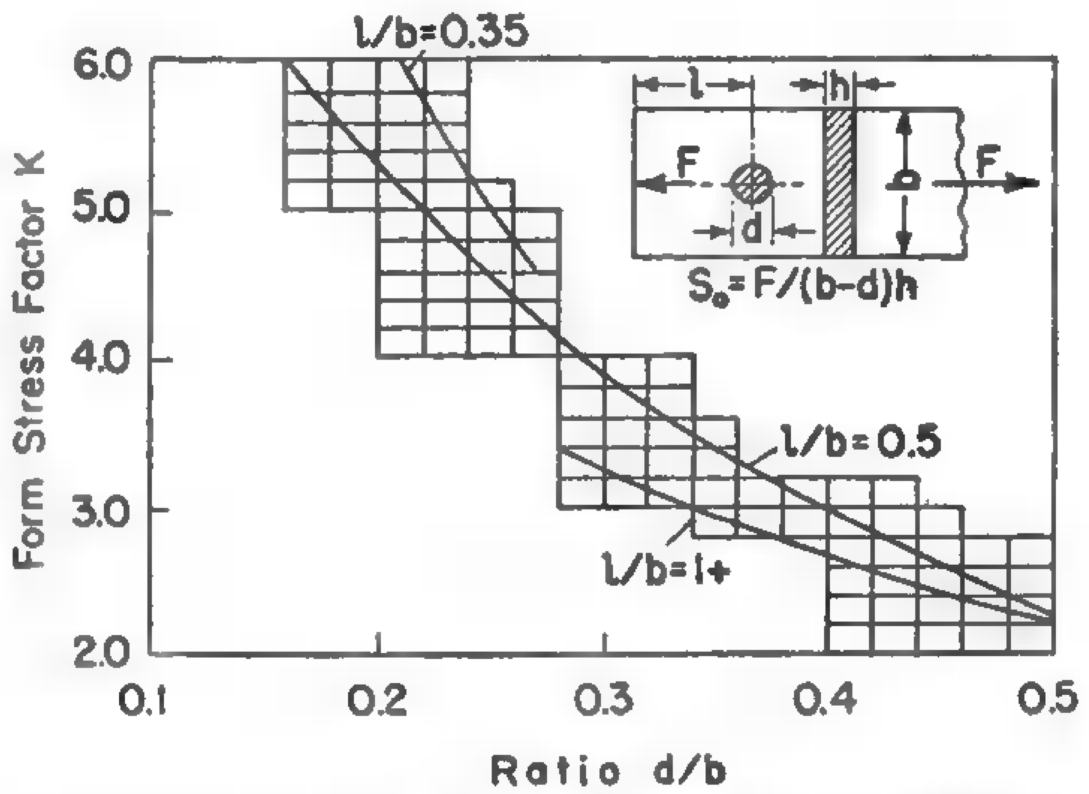


FIG. 3–8. Form stress factor for hole filled with pin which transmits load.

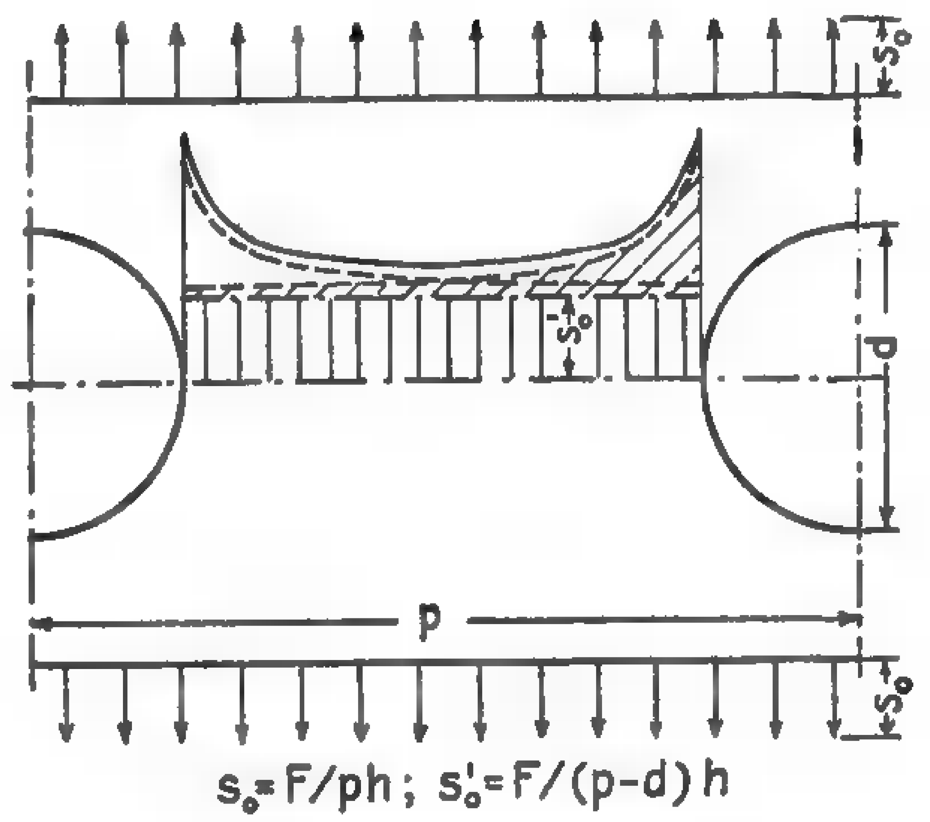


FIG. 3–9. Stresses between two holes.

The values of K for such a plate are shown in Fig. 3–8. The curves[7] show that K is affected both by the relative size of the hole, or the ratio d/b, and by the distance from the free end of the plate, or the ratio l/b.

Stresses between two holes. Stress concentrations from two adjacent holes are superimposed, as shown in Fig. 3–9 for two free holes.

Eye bar. In Fig. 3–10 are shown stresses created in the tangential direction around the edge of the hole in an eye bar due to a load applied by means of a pin which fills the opening. The stresses are those found by photoelastic measurements and are represented by the polar diagram *gbiej* with the hole edge as a base line.[8] The greatest stress is not in the section *ck*, where it could be expected, but is in the section *dl* at point *d*. It is about 4.2 times as great as the nominal stress s_o in the eye section *ck*.

Curve *mno* gives the stress variation in the section *dl* from the inner edge to the outer edge. It shows that the outer third of the eye section is under compression instead of tension.

[6] E. Lehr, *Spannungsverteilung in Konstructionselementen* (Berlin: Verein Deutscher Ingenieure, 1934), p. 43.

[7] Lipson, Noll, and Clock, *op. cit.*, p. 93.

[8] Seely, *op. cit.*, p. 220.

Curve *pqr* gives the compressive stresses normal to the inner surface of the hole. This curve shows that the maximum bearing pressure is not at the center line. Instead, because of the deformation of the eye bar, the greatest pressure occurs about 45° below the center line.

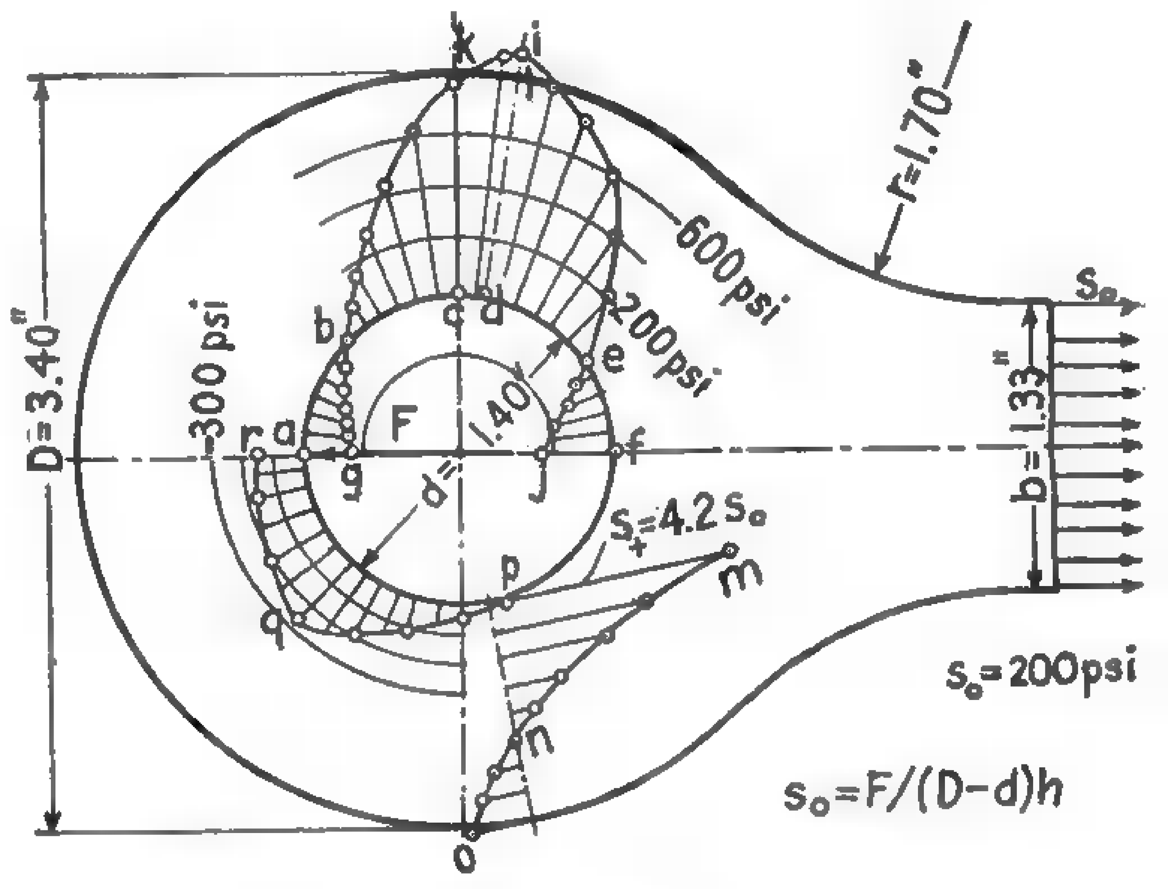


FIG. 3-10. Stresses in eye bar.

It should be noted that the stress curves and their scales apply only to a bar having the relative dimensions shown in Fig. 3–10. With other dimensions the numerical values for the stresses may change somewhat. However, dimensions used in Fig. 3–10 are typical for various machine parts of similar shape.

Biaxial loading. If a plate with a hole is subjected to stresses s_o' and s_o'' acting at right angles, the combined maximum stress is found by superposition and is

$$s'' = Ks_o' + Ns_o'' \tag{3-37}$$

EXAMPLE 3–4. Determine the stresses at the edges of a round hole in the shell of a cylindrical vessel subjected to an internal pressure. The conditions are represented in Fig. 3–11. The hoop stress in an unweakened section is s_o.

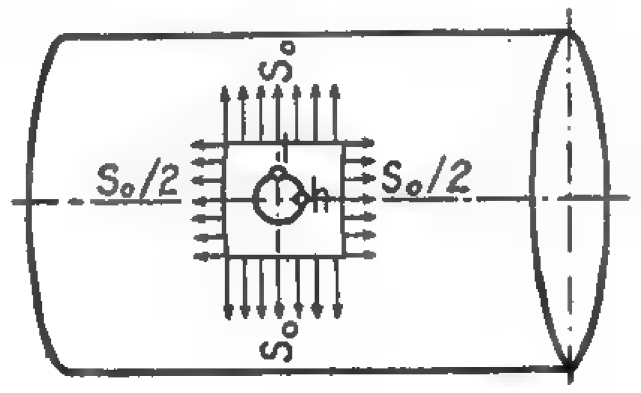


FIG. 3-11. Hole in cylindrical shell.

The stresses at points h and l can be found from equation 3–37 by using the corresponding values for s_o' and s_o''. Another method is to proceed step by step. In a closed vessel the longitudinal stress is $0.5s_o$. It invokes at point l a concentrated stress $0.5\ Ks_o$ and at point h a transverse stress $0.5\ Ns_o$. The hoop stress s_o invokes at point h a concentrated stress Ks_o and at point l a transverse stress Ns_o. From Fig. 3–4, for an infinitely wide plate, $K = 3.0$ and $N = -1$. Combining the stresses acting at the same points, we obtain for the longitudinal stress at l,

$$s_1 = 1.5s_o - s_o = 0.5s_o$$

The hoop stress at h is

$$s_1 = 3s_o - 0.5s_o = 2.5s_o$$

Inner holes. The effect of an inner blowhole in a casting or of a flaw in a forging is similar to that of a hole in a plate. Theoretically the maximum stress produced at the edge of a spherical hole is

$$s = 2s_o \tag{3-38}$$

Elliptical holes. The effect of an elliptical hole depends on the relative values of the plate width B, the major half-axis a, and the minor half-axis b, and also on the direction of the axis of the ellipse with respect to the direction of the applied force.

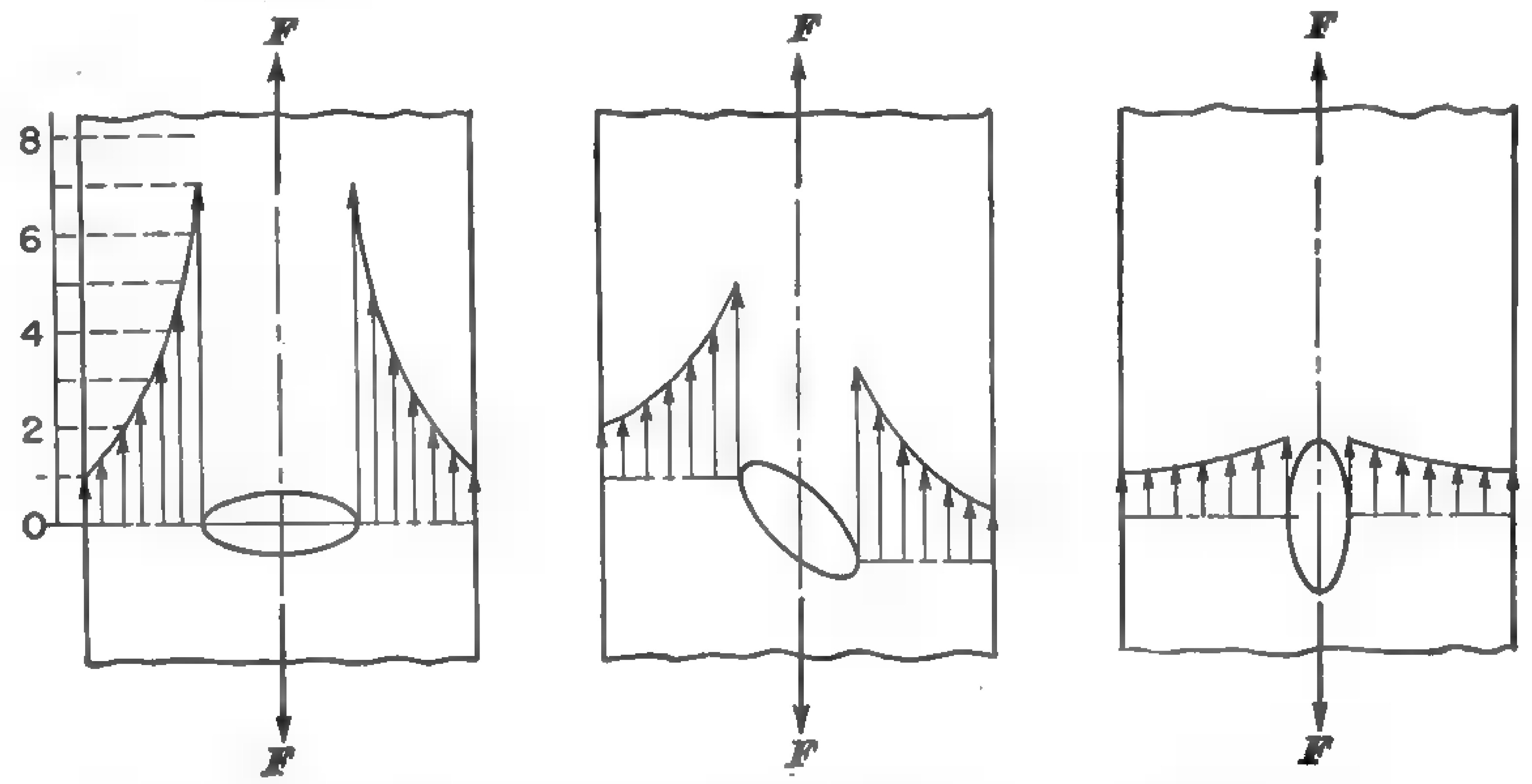


FIG. 3-12. Stress concentration caused by elliptical holes.

For a plate having a width B greater than $12a$ and containing an elliptical hole with its major axis normal to the direction of the stress, the form stress factor may be computed by the expression[9]

$$K = 1 + 2\frac{a}{b} \tag{3-39}$$

If the minor axis is normal to the direction of the stress,

$$K = 1 + 2\frac{b}{a} \tag{3-40}$$

In Fig. 3–12 is indicated the influence of the orientation of an ellipse for which $a = 3b$.[10] The general effect is similar to that of notches of different sharpness, as may be seen by comparing Fig. 3–12 with Fig. 3–13.

Notches and grooves. Notches in flat pieces and cylindrical grooves in round bars have an effect similar to that of holes. The form stress factor depends on the depth h_1 of the notch or groove as well as on its shape. The curves in Fig. 3–13 show typical stress concentration due to tension. The

[9] S. Timoshenko and J. N. Goodier, *Theory of Elasticity*, 2d ed. (New York: McGraw-Hill Book Company, Inc., 1951), p. 86.

[10] Battelle Memorial Institute, *Prevention of the Failure of Metals Under Repeated Stress* (New York: John Wiley & Sons, Inc., 1941), pp. 53, 55.

chart in Fig. 3–14 gives numerical data for various relative sizes of semicircular notches,[11] and the curves in Fig. 3–15 are for notches of various relative depths h_1 and radii r at the bottom.[12]

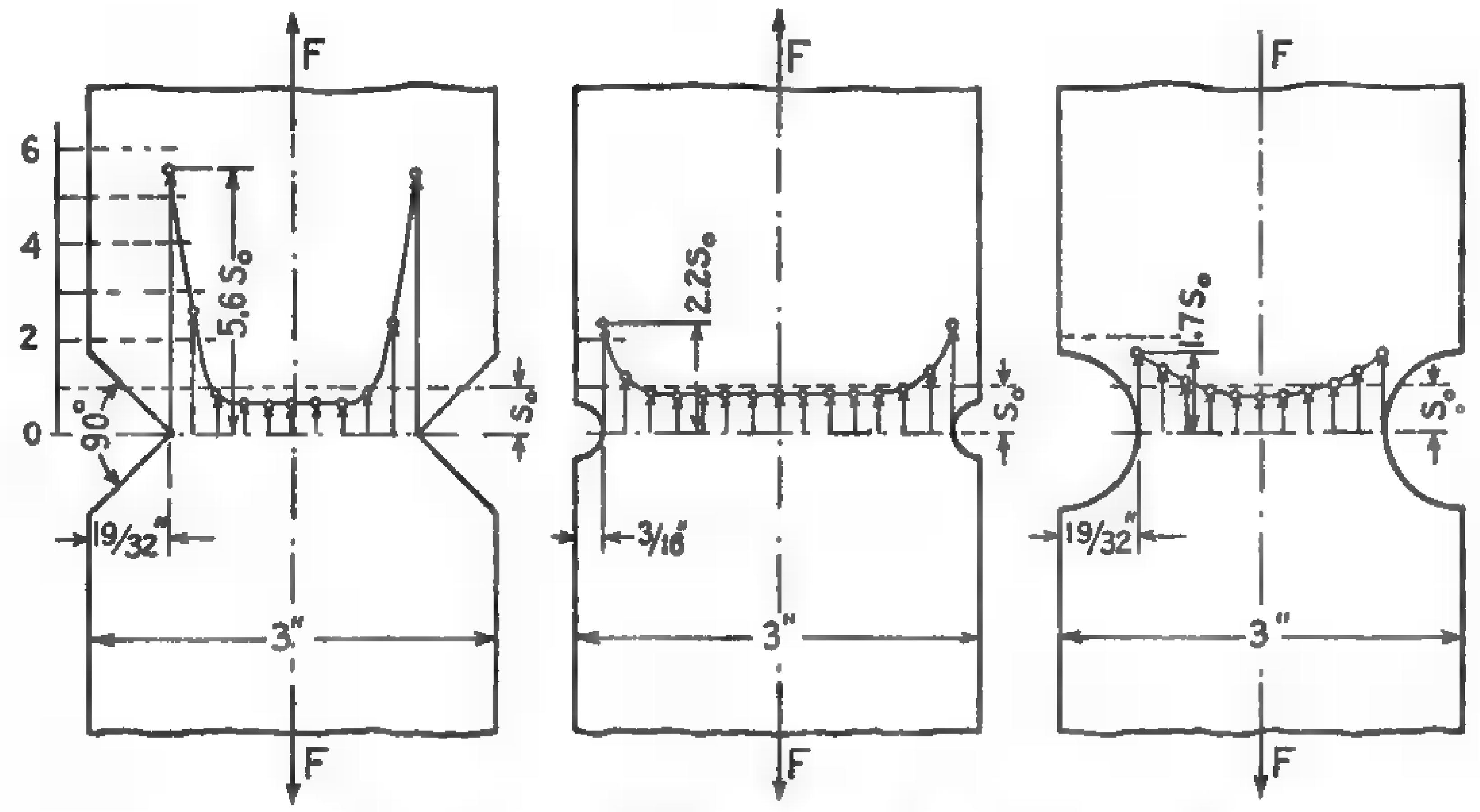


FIG. 3–13. Stress concentration caused by notches.

For small notches the form stress factor can be computed from the equation[13]

$$K = 1 + 2\sqrt{\frac{h_1}{r}} \tag{3–40}$$

Values of K for notches having various shapes, particularly different angles between the sides, are given in Fig. 3–16. These curves are based on an infinite width b but may help to estimate the correction of the value of K found from the curves in Figs. 3–14 and 3–15 when the sides of the notch are not parallel.

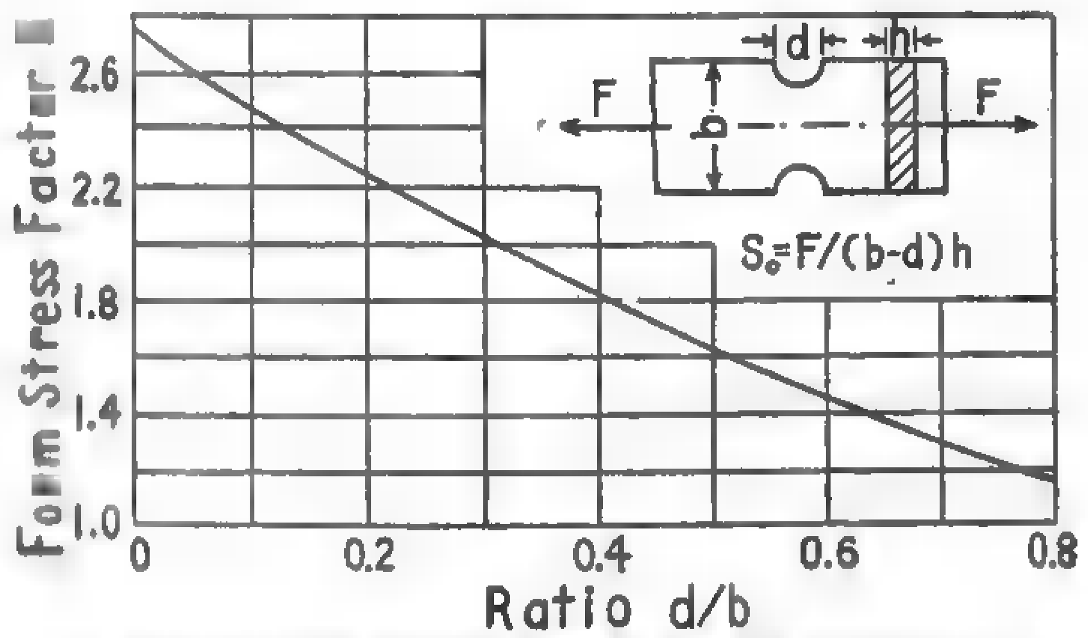


FIG. 3–14. Form stress factor due to semicircular notches.

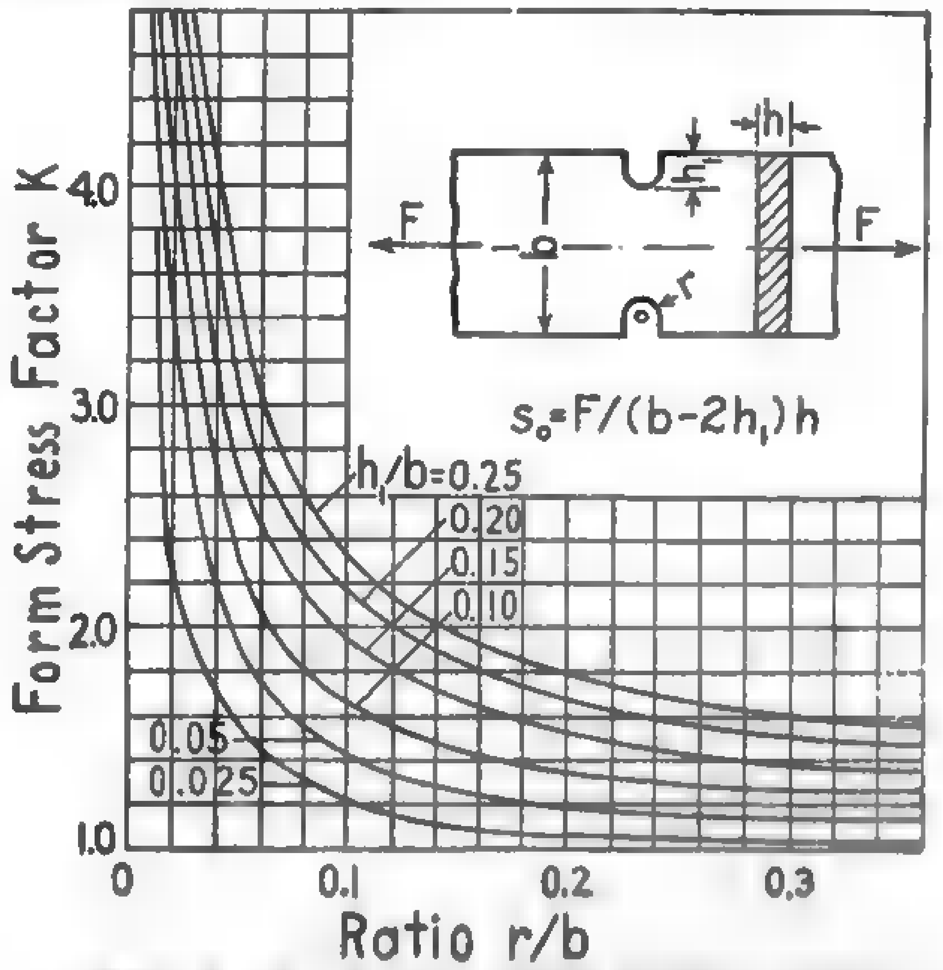


FIG. 3–15. Form stress factor due to various notches.

[11] A. M. Wahl and R. Beeuwkes, Jr., "Stress Concentration Produced by Holes and Notches," *Trans. ASME*, Vol. 56, APM–56–11 (1934), p. 621.

[12] Lehr, *op. cit.*, Table 4, Fig. 18.

[13] R. E. Peterson, "Stress Concentration Phenomena in Fatigue of Metals," *Trans. ASME*, Vol. 55, APM–55–19 (1933), p. 161.

Fillets. The influence of a fillet depends on the relative size of the fillet, as expressed by the ratio r/b, as shown in Fig. 3–17; but it does not depend on the relative size of the rib, as expressed by the ratio B/b, if r is less than $\frac{1}{2}(B-b)$.[14] The form stress factor may be determined by the curve *a*. While Fig. 3–17 shows a plate, the same values of K may also be used with sufficient accuracy for a round bar, shaft, bolt, or similar part.

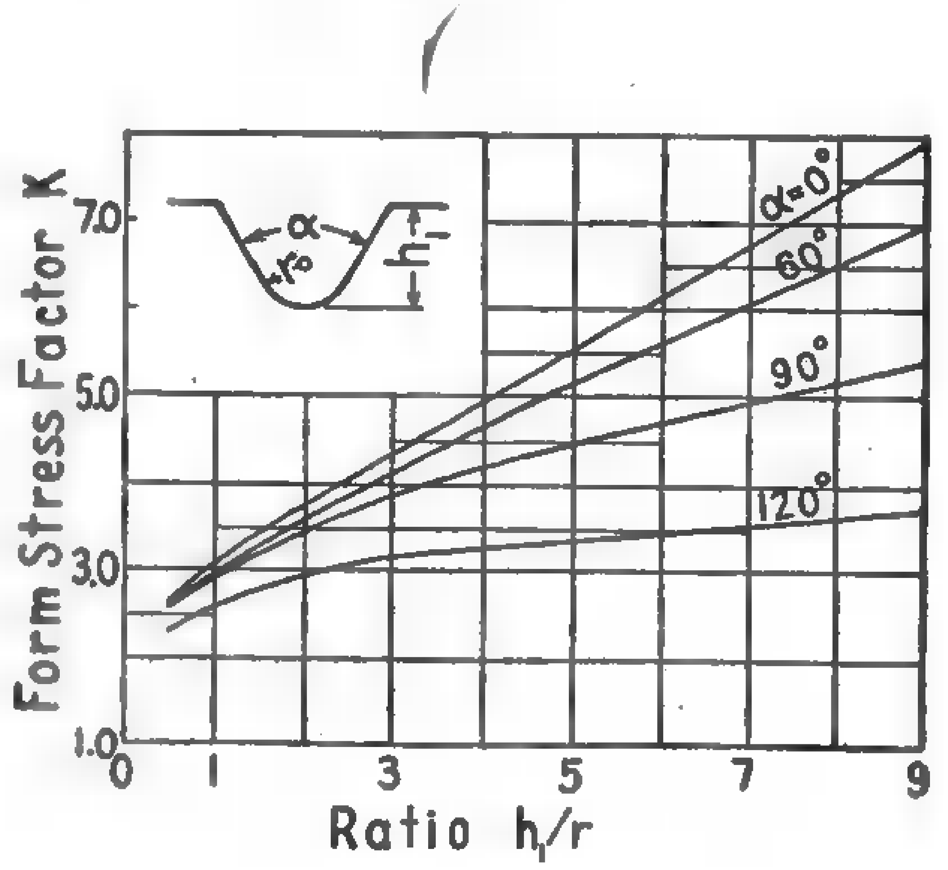


FIG. 3–16. Form stress factor due to notches of various shapes.

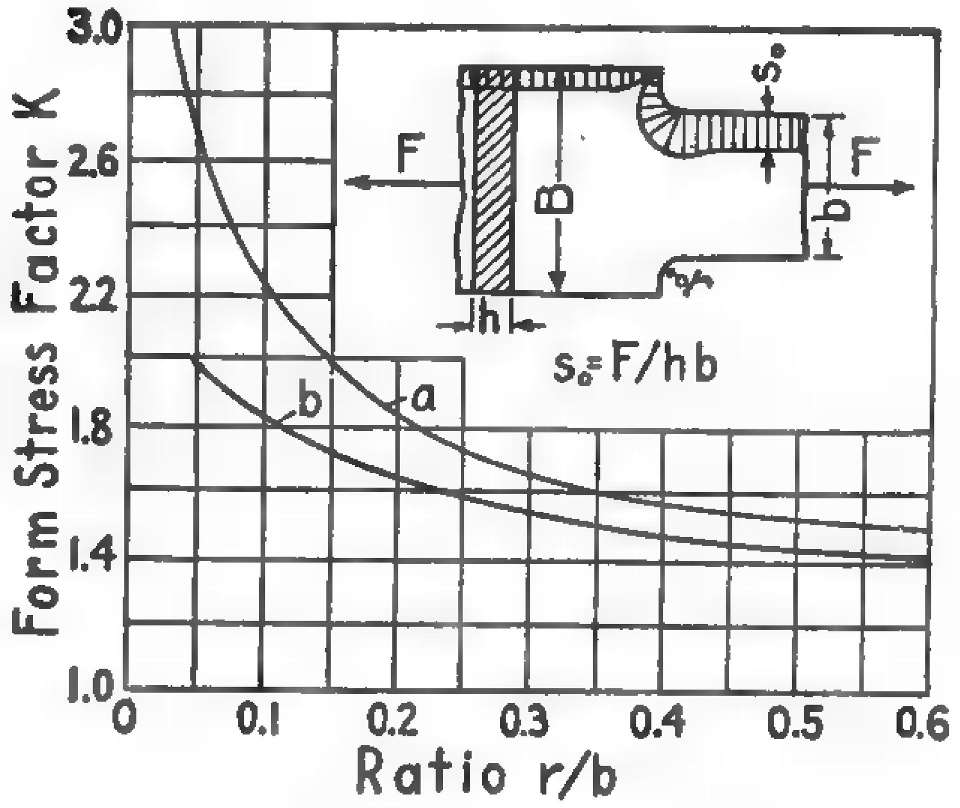


FIG. 3–17. Form stress factor due to fillets, for tension.

For a full fillet for which r is not less than $\frac{1}{2}(B-b)$, such as is encountered in a shaft or a hub of a disk, the form stress factor is lower and may be presented by the curve *b*.[15]

Screw threads. Screw threads are similar in their effect to circular grooves, but the helical shape reduces the form stress factor somewhat. The shapes of various threads were established before the effect of stress concentration was recognized. Values of the factor K are therefore rather high. Thus, for the American National thread, $K = 5.62$; and for the British Whitworth thread, $K = 3.86$.[16]

EXAMPLE 3–5. Determine the form stress factor for a plate 4 in. wide that is loaded in tension and has two symmetrical notches in the main cross section. Each notch is $\frac{1}{2}$ in. deep and has a radius of $\frac{1}{8}$ in. at the bottom, and its sides are at right angles.

From Fig. 3–15, for $h_1/b = 0.5 \div 4 = 0.125$ and $r/b = 0.125 \div 4 = 0.031$, the factor K is 3.6.

The influence of the 90° flareout can be found from Fig. 3–16. For $h_1/r = 0.5 \div 0.125 = 4$, the ratio of the values for 90° and 0° is $4.2 \div 4.9 = 0.86$. Therefore the corrected value of the form stress factor is

$$K = 3.6 \times 0.86 = 3.10$$

[14] E. E. Weibel, "Studies in Photoelastic Stress Determination," *Trans. ASME*, Vol. 56, APM–56–13 (1934), p. 641.

[15] S. Timoshenko and W. Dietz, "Stress Concentration Produced by Holes and Fillets," *Trans. ASME*, Vol. 47 (1925), p. 210. See also Lipson, Noll, and Clock, *op. cit.*, p. 59.

[16] H. F. Moore and P. K. Henwood, *The Strength of Screw Threads Under Repeated Tension*, Bulletin No. 264, University of Illinois Engineering Experiment Station (1934), p. 15.

Protrusions. Even such a discontinuity as a protrusion on the edge of a plate causes a certain stress concentration, as shown in Fig. 3–18.[17]

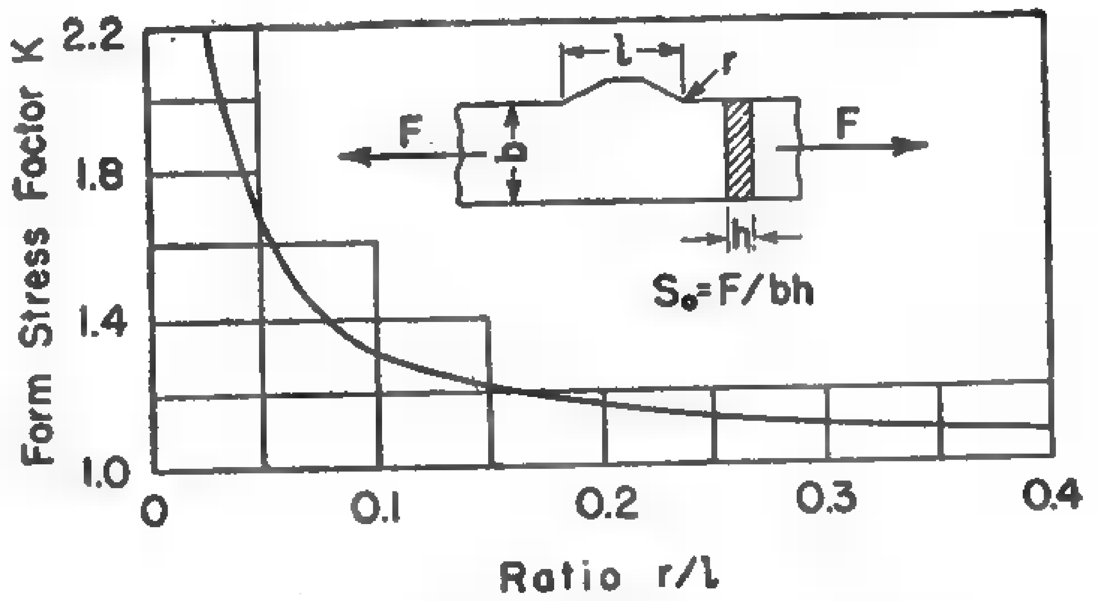


FIG. 3–18. Form stress factor due to protrusion on plate.

3–7. Stress concentration in bending. Abrupt changes in cross section may occur in various parts of beams for a number of reasons.

Holes in beams. The influence of holes on the stresses in a bar subjected to bending may be found by superimposing the hole-concentration stresses, determined by use of Fig. 3–3 or Fig. 3–6, on the bending stresses, represented in Fig. 2–5b. A hole with its axis located in the neutral plane of a beam, as in Fig. 3–19, has practically no effect on its strength. A hole located nearer the beam edge may result in high stresses at the hole edge, these stresses exceeding those in the outer fibers of the beam. Thus, holes located halfway between the neutral plane and the edge of a beam, as in Fig. 3–20, show maximum stresses about 1.5 times as great as those at the outer fibers.

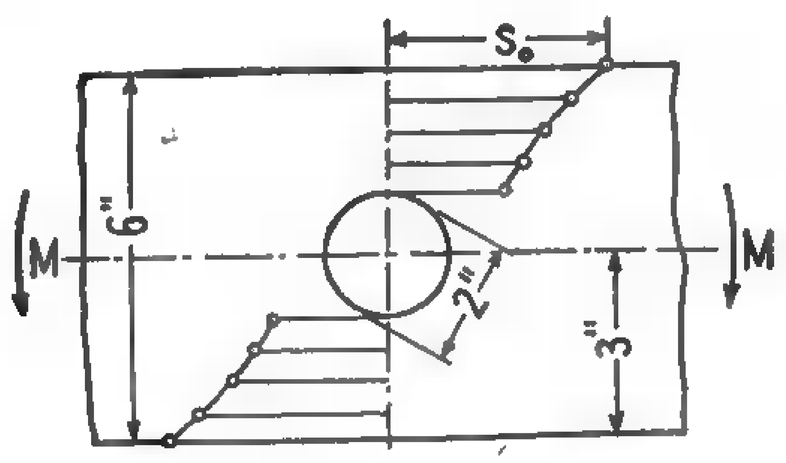


FIG. 3–19. Stresses in beam with one central hole.

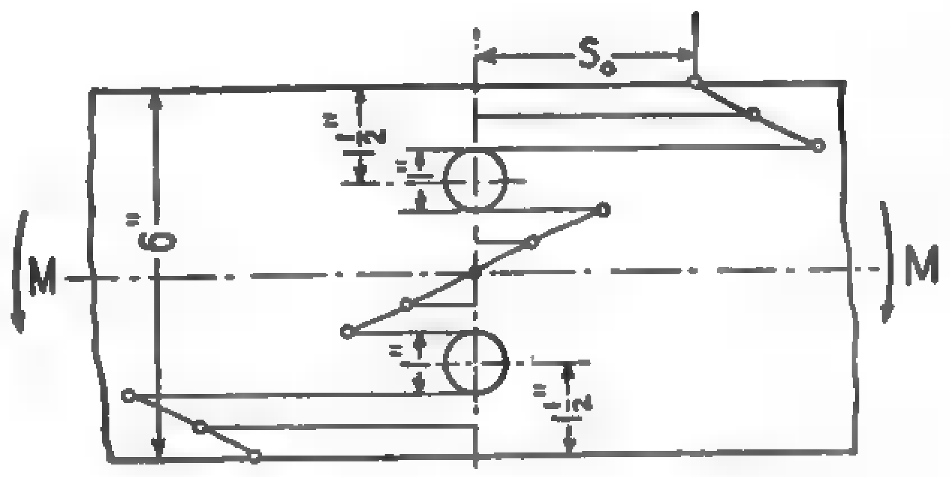


FIG. 3–20. Stresses in beam with two holes.

Notches. Values of the form stress factor, as found experimentally for rapid bending of beams with notches,[18] are given in Fig. 3–21. The curves for constant r have a parabolic shape, and those for constant h have a hyperbolic shape. With the decrease of r in sharper notches, the values of K

[17] G. H. Neugebauer, "Stress Concentration Factors and Their Effect on Design," *Product Engineering*, Vol. 14 (1943), p. 82.

[18] F. Rötscher, "Die Ermittlung der Spannungsverteilung in Konstructionsteilen durch Dehnungsmessungen," *Z. VDI*, Vol. 77 (1933), p. 375. The article does not give the depth b of the beam; h/b seemingly was very small.

increase very fast as they approach the limit values. The values from Fig. 3–21 can also be used with sufficient accuracy for tension and compression.

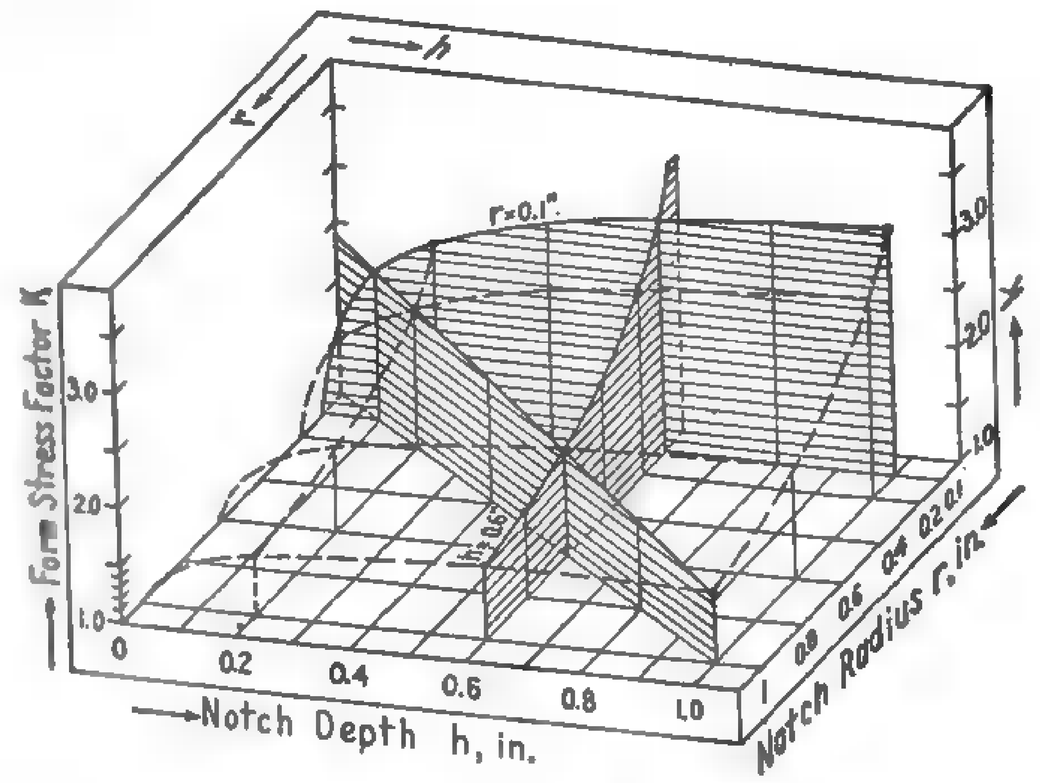


FIG. 3–21. Form stress factor due to notches, for rapid bending.

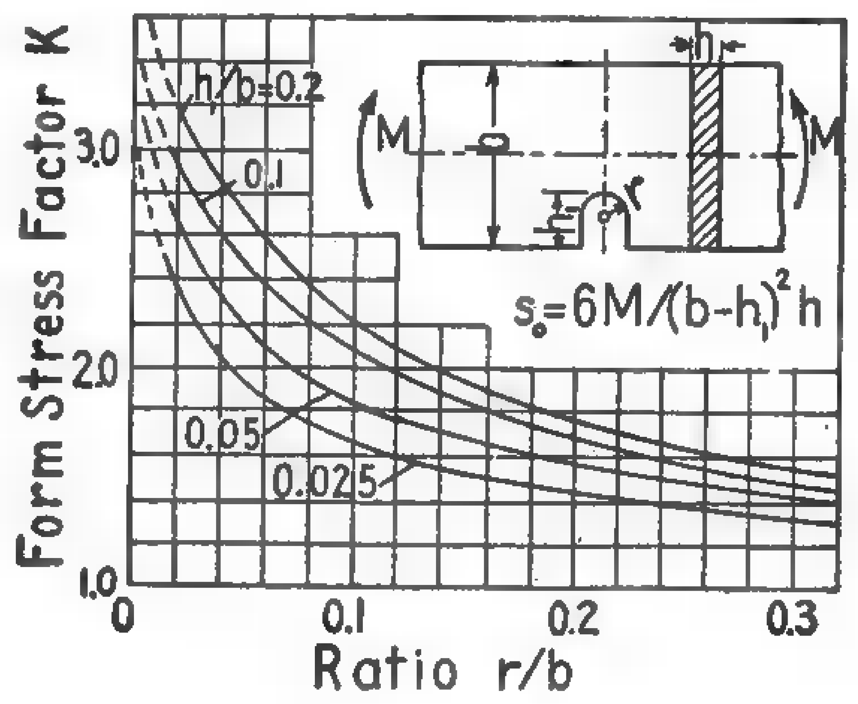


FIG. 3–22. Form stress factor due to notches in beam.

The curves in Fig. 3–22 give values of the form stress factors for beams with notches of the shape indicated. These curves take into account the dimension b of the bar, the depth h of the notch, and the radius r of its bottom. In Fig. 3–23 is pictured the effect of a sharp notch in a beam. This diagram shows both the stress distribution and the distance along the axis through which the notch exerts its influence.

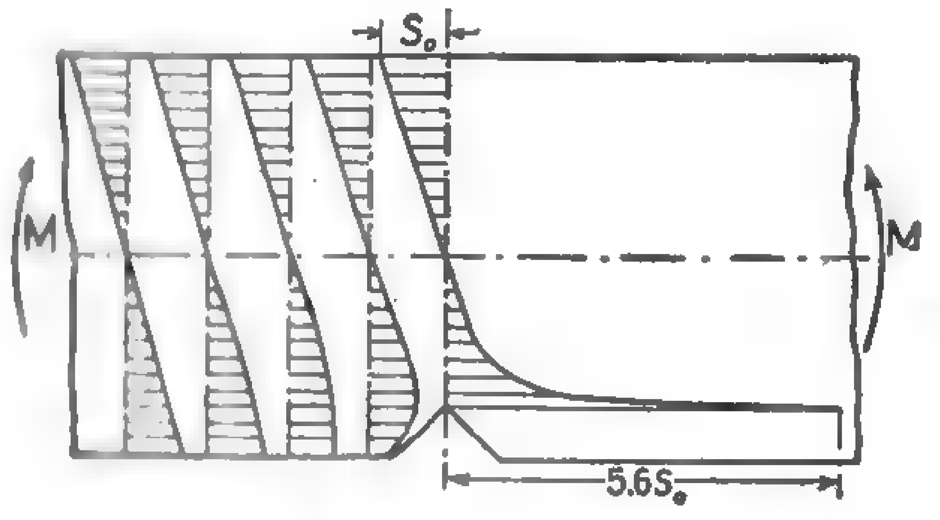


FIG. 3–23. Stress distribution in bending caused by sharp notch.

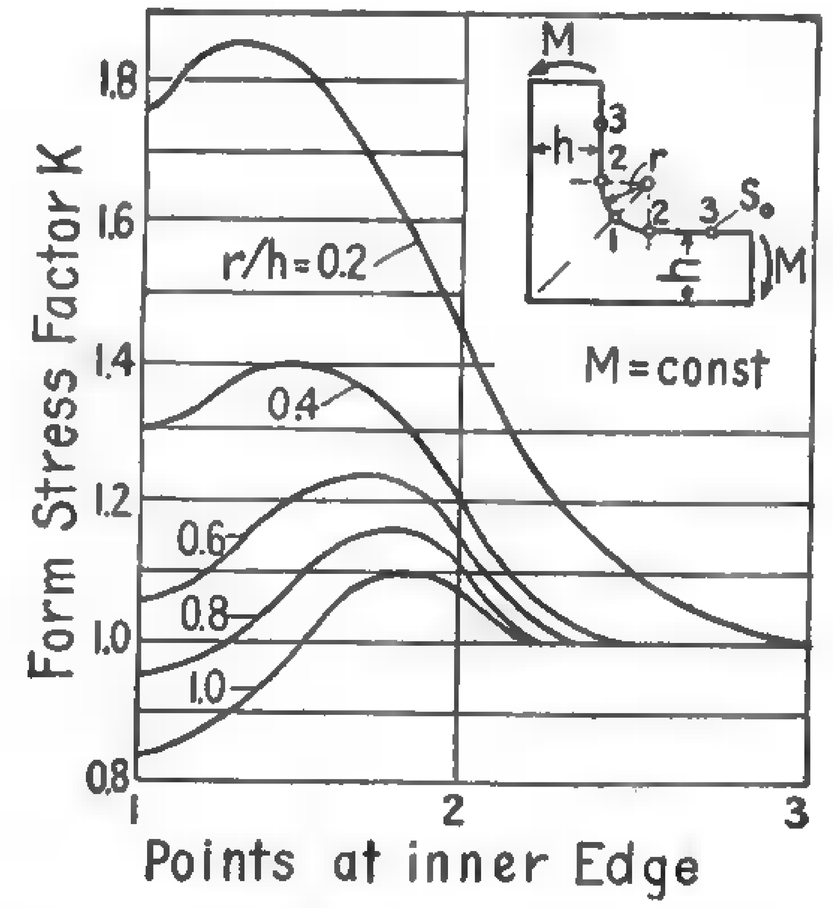


FIG. 3–24. Increase in bending stress caused by fillet.

Fillets. For a fillet in symmetrical beam sections the influence of the ratio of the fillet radius r to the plate width b is represented by curves similar to curves a and b in Fig. 3–17 for tension. For pure bending, however, the numerical values of K are about 20 per cent lower than those given in Fig. 3–17 for tension.

The form stress factor K shown in Fig. 3–17 applies both for flat sections, such as ribs, and for junctures of shafts of different diameters.

The influence of a one-sided fillet, as in an angle-shaped section, is shown in Fig. 3–24, which gives values of K found by photoelastic measurements.[19]

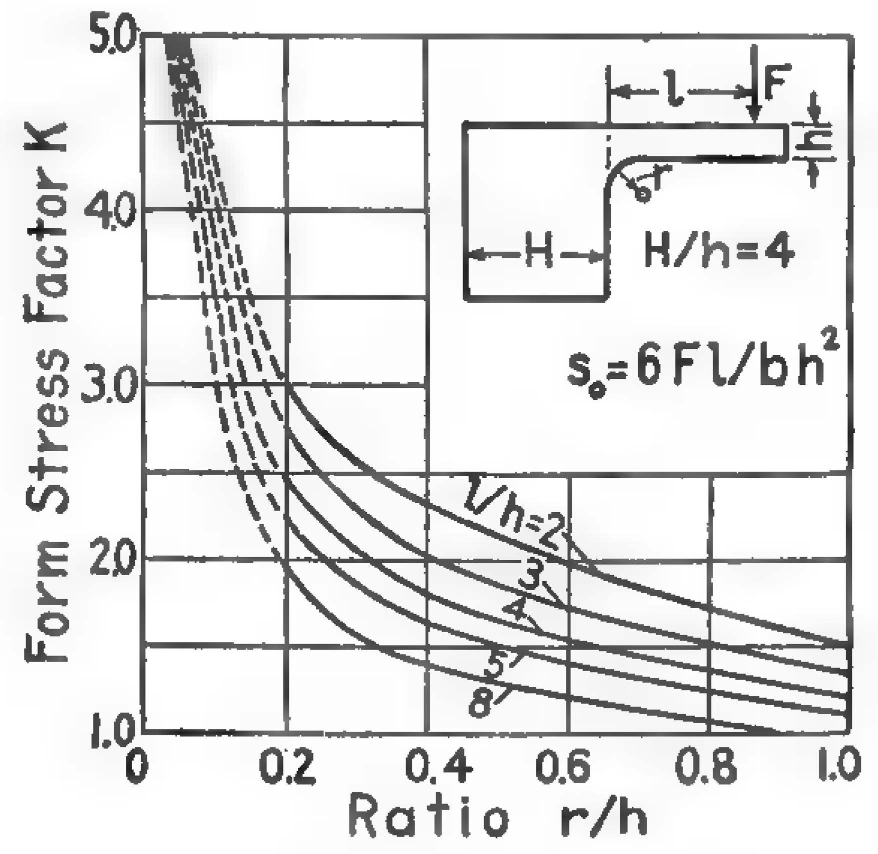


FIG. 3-25. Form stress factor for angle-shaped section bending.

The form stress factor for an angle-shaped section for which the thickness ratio H/h is 4 is given in Fig. 3–25 as a function of the ratio r/h and the ratio l/h.

Enlarged central section. The curves in Fig. 3–26[20] give values of the form stress factor for pure bending of a flat bar with an enlarged central section. The same curves may be used with sufficient accuracy for bending of a shaft with an enlarged section. It may also be added that an increase of the width to $D=3d$ raises the values of K very little; a decrease of the length to $L=D$ reduces K very little; and a decrease in length to $L=0.25D$ reduces K to the values given by the curves c and d for $D/d=2.0$ and $D/d=1.25$, respectively.

Gear teeth. The stresses at the roots of gear teeth are greatly influenced by the fillet.[21] In Fig. 3–27 is shown the stress distribution around the fillet; and the curve in Fig. 3–28 gives the factor K as a function of the fillet radius r. With a small fillet the stress concentration is considerable, as K ranges from 2 to 2.5.

Concentrated load. The application of a concentrated load produces high local stresses, as may be seen in the explanation for Fig. 3–7, where the form stress factor reached a value of 6.5.

It can be shown mathematically[22] that a concentrated load F, Fig. 3–29, applied to a beam with a narrow rectangular cross section creates radial compressive stresses s_r. The magnitude of the stress at any point a is

$$s_r = \frac{2F\cos\theta}{\pi r} \tag{3–42}$$

where F is the load per unit of beam width. This stress combined with the flexure stress $s \doteq M/Z$ will give the resultant stress in the proximity of the

[19] L. Föppl, "Fortschritte auf dem Gebiet des Spannungsoptischen Untersuchung von Konstructionen," *Z. VDI*, Vol. 76 (1932), p. 507.

[20] J. B. Hartman and M. M. Leven, "Factors of Stress Concentration for the Bending Case of Fillets in Flat Bars and Shafts with Central Enlarged Section," *Proceedings of the Society of Experimental Stress Analysis*, Vol. IX, No. 1 (1951), p. 61.

[21] S. Timoshenko and R. U. Baud, "The Strength of Gear Teeth," *Mechanical Engineering*, Vol. 48 (1926), p. 1105.

[22] Seely, *op. cit.*, p. 233.

load. The maximum value of s_r will be for the angle $\theta=0$, as at point b, where

$$s_r = \frac{2F}{\pi r} = \frac{0.637F}{r} \tag{3–43}$$

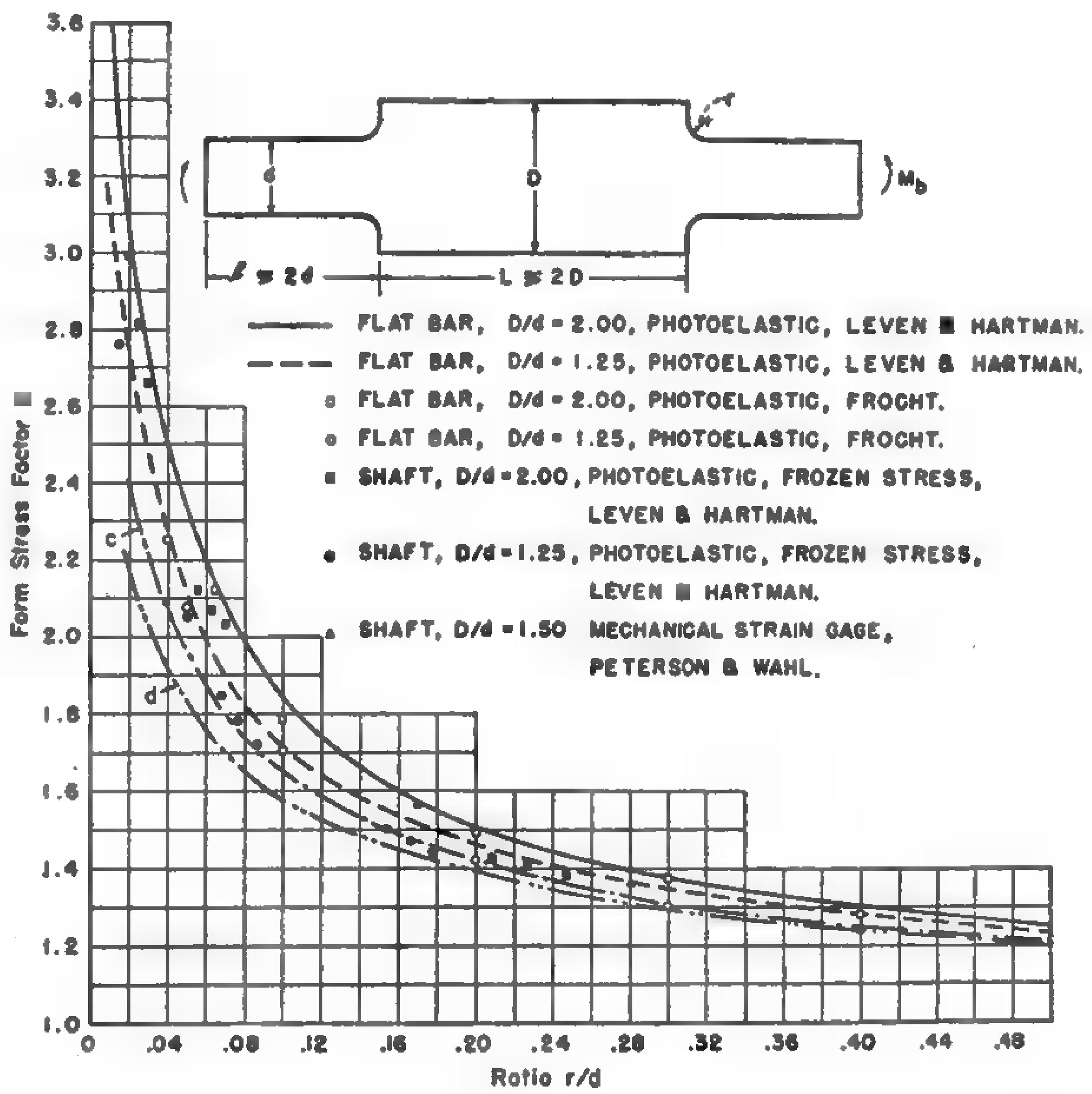


FIG. 3-26. Form stress factor for bar with enlarged section in bending.

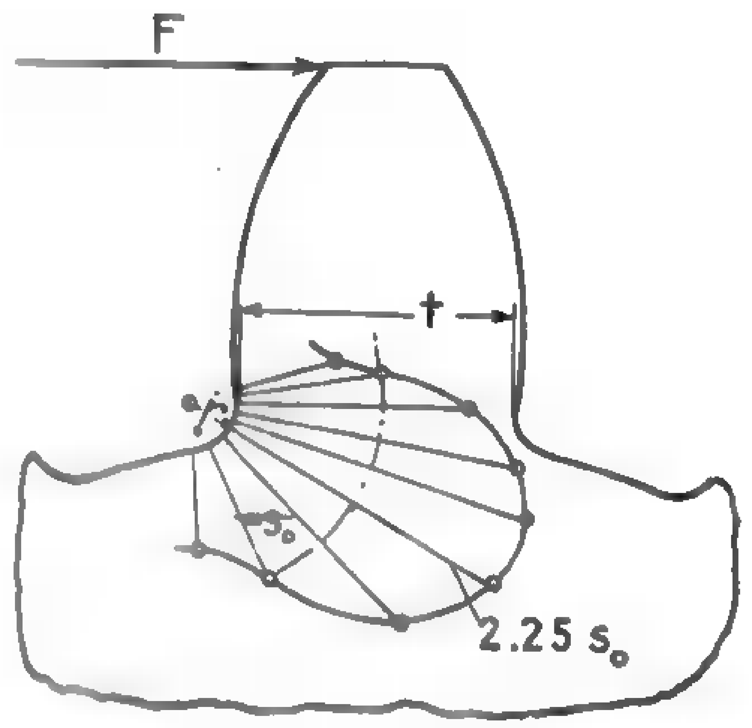


FIG. 3-27. Stress concentration at root of gear tooth.

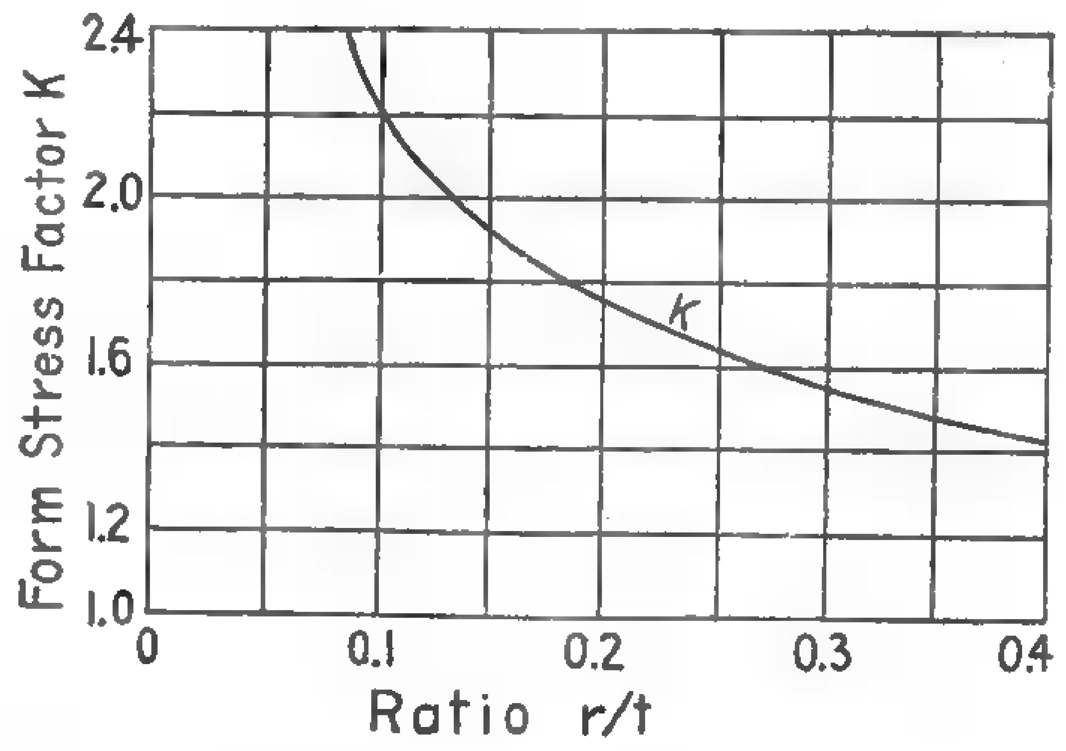


FIG. 3-28. Influence of fillet on stress concentration at root of gear tooth.

Directly under the load, where $r=0$, the stress theoretically would become infinite if the material did not yield locally and change the *line* of contact to a small *area* of contact.

In a beam loaded at the center, the maximum shear stress s_s' near the top, as at point c in Fig. 3–30, is, in accordance with equation 2–45,

$$s_s' = 0.5(s_r - s) \qquad (3\text{–}44)$$

At a point d near the support the bending stress s is negligible, but a proper bearing area must be provided to avoid a high stress concentration. The accuracy of this analysis has been confirmed experimentally.

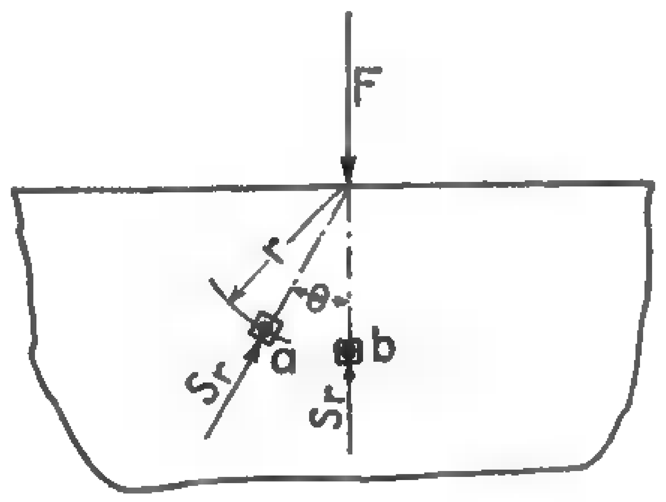


FIG. 3–29. Action of a concentrated load.

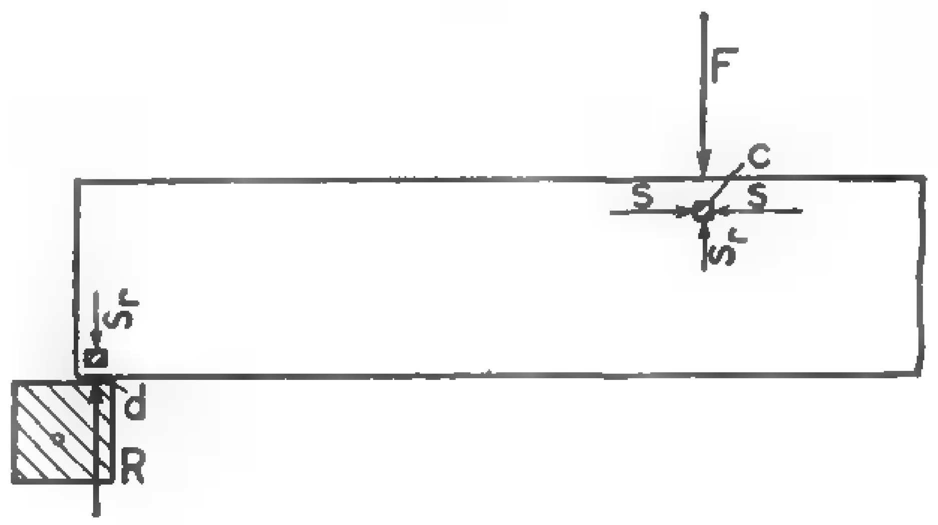


FIG. 3–30. Stresses in beam caused by concentrated load.

3–8. Stress concentration in torsion. When a shear stress is caused by torsion, the form stress factor generally is smaller than for direct stress or flexure. If the cross sections of a straight shaft are circular but vary in diameter, a plane section remains plane after twisting, but its intersection

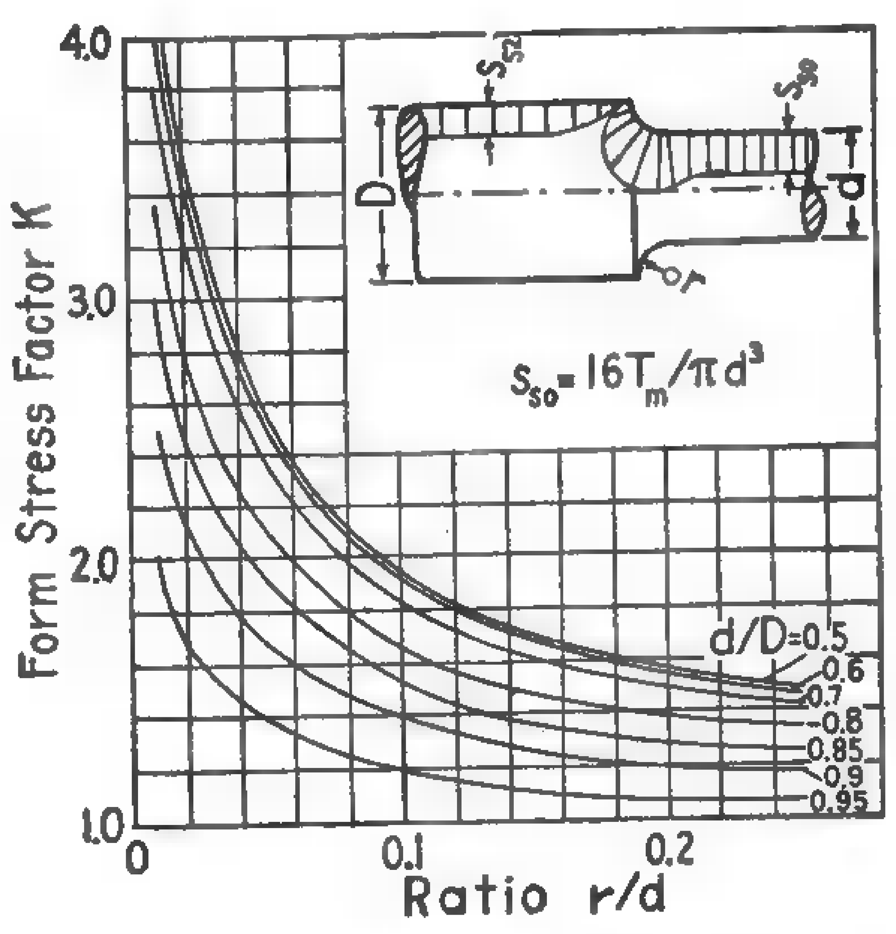


FIG. 3–31. Form stress factor due to enlargement of shaft in torsion.

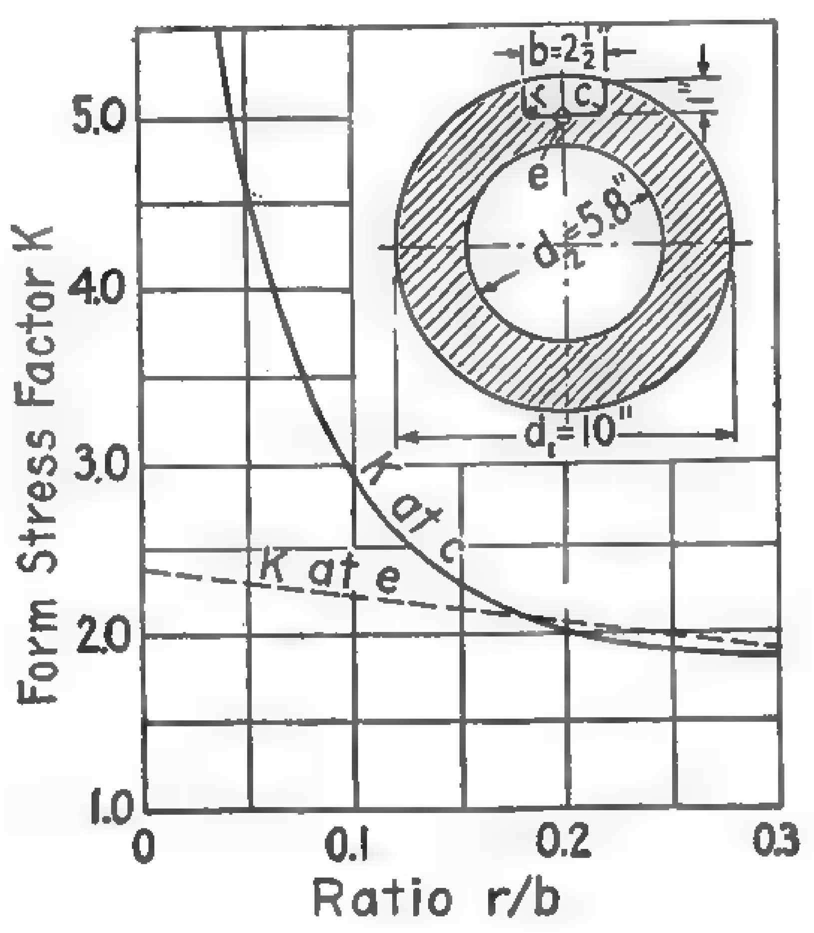


FIG. 3–32. Form stress factor due to keyway in shaft in torsion.

with an axial plane does not remain a straight line after twisting. The change in the diameter results in a concentrated shear stress, as illustrated in the upper corner of Fig. 3–31. The form stress factor is a function both of the ratio of the two adjacent diameters and of the relative size of the fillet, as shown by the curves in the same figure.

Keyways. A keyway in a shaft has a stress-concentration effect which is particularly noticeable at its end. So far very few numerical values are available. The relation between the form stress factor K in torsion and the ratio of the size of the fillet r in the keyway corner c to the width b, determined by the soap-film method, is shown in Fig. 3–32.[23] Numerically K is very high, and rectangular keyways should be avoided in highly stressed shafts. The influence of the keyway extends to the center e of the bottom of the keyway.

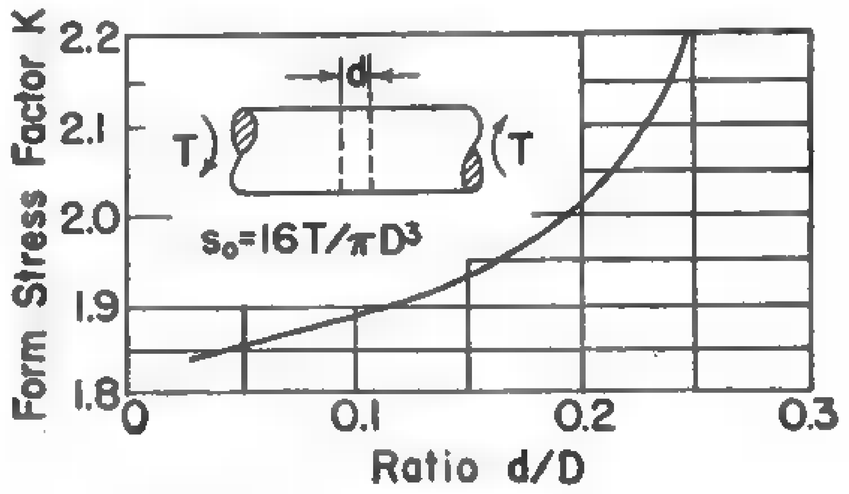


FIG. 3–33. Form stress factor due to hole in shaft in torsion.

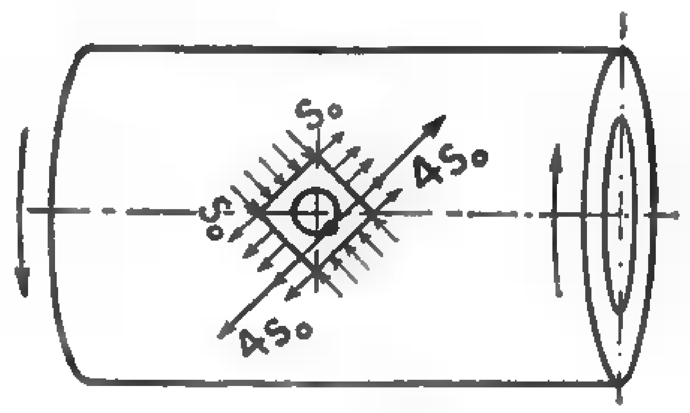


FIG. 3–34. Stress concentration due to torsion.

Tests with the plaster-model method give considerably lower values for K than do tests with other methods. Thus, for a keyway of standard proportions, for which $b = d/4$ and $h = b/2$, carefully conducted tests gave $K = 1.68$ for a profiled keyway and $K = 1.44$ for a sled-runner keyway.[24] In both cases K is referred to the full shaft section. In bending the same tests gave values for K that were lower by about 15 per cent.

Radial holes in shafts. In the case of a shaft subjected to torsion the influence of a hole passed radially through the shaft is given in Fig. 3–33. The data found by the plaster-model method are referred to the unweakened shaft.[25] Therefore the values of K cannot be compared with values given by other curves, such as those shown in Fig. 3–5. However, data furnished by Fig. 3–33 are really what a designer needs. They show directly how much a certain hole will weaken the shaft.

The influence of a radial hole on a hollow shaft subjected to torsion can be found by applying the equation of stresses for biaxial loading to a hollow shaft, Fig. 3–34, in which the two main stresses have different signs.

To determine the stress distribution shown in Fig. 3–34, we assume that $d/b = 0$ and we use equation 3–37, in which $K = 3$ and $N = -1$. The result is

$$s'' = 3s_o + (-1) \times (-s_o) = 4s_o \qquad (3\text{–}45)$$

Thus a small hole, such as an oil hole, theoretically causes a great concentration of stress.

[23] *Ibid.*, p. 228.
[24] Seely and Dolan, *loc. cit.*, p. 24.
[25] *Ibid.*, p. 21.

3–9. Qualitative evaluation of stress concentration. The discontinuities previously discussed and the resulting stress-concentration effects cover only the most typical cases. For other cases and for a clear understanding of stress concentration in general, there exist qualitative approaches that help the designer to visualize probable stress concentrations in the parts he is designing.

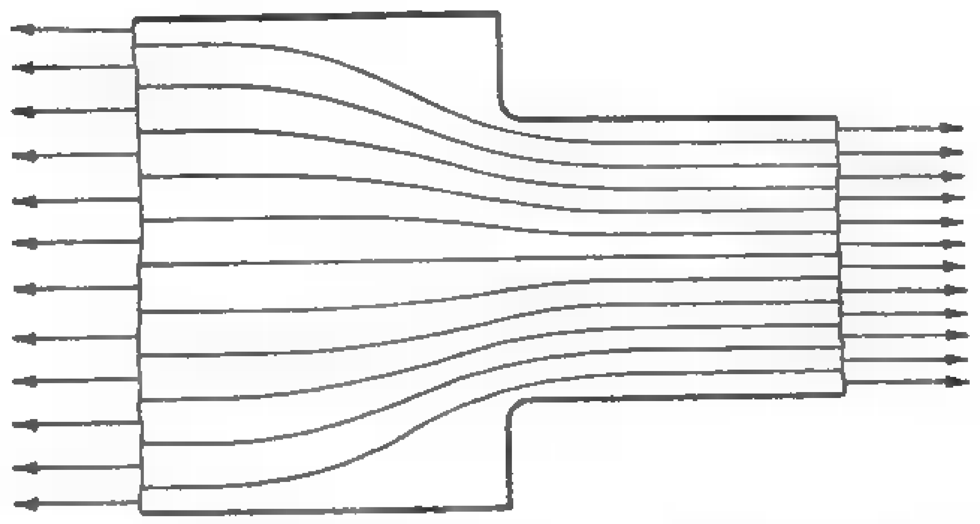
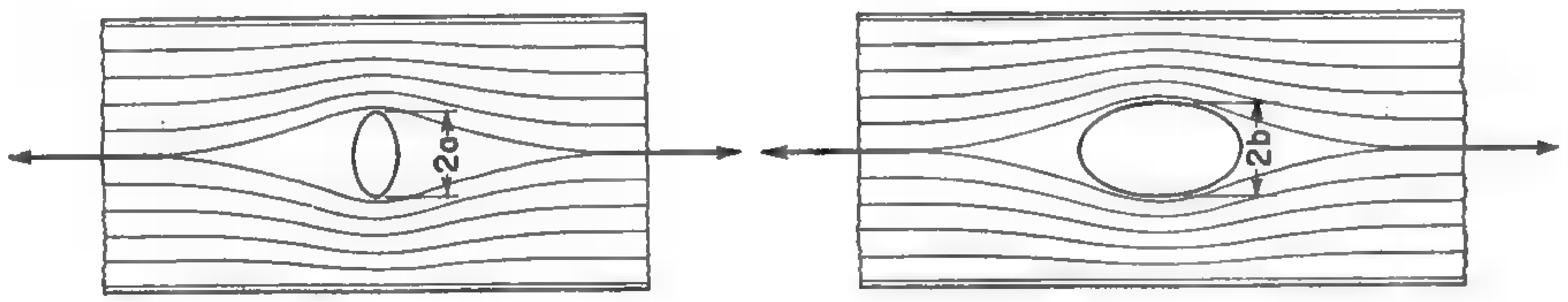


FIG. 3–35. Force flow in tension member.

Tension. Stress concentration in tension members may be represented by lines indicating the direction of the principal stresses. In Fig. 3–35 is shown an axially loaded plate with these direction lines. At both ends of the plate the lines are parallel, indicating uniform stresses; but at the right end they are closer together, indicating a higher stress. Where the lines join near the discontinuity, they are more crowded. A local stress increase is thus indicated.[26] In practice these lines are more descriptively called *force-flow lines.*



(a) (b)

FIG. 3–36. Force flow in tension member with elliptic hole.

The usefulness of the method of drawing force-flow lines can be illustrated by the example of stress concentration created by an elliptic hole, as shown in Fig. 3–36. The force-flow lines indicate that there is a lower stress concentration in Fig. 3–36b than in a, although $2b = 2a$ and the cross-sectional area of the plate is reduced by the same amount. That the stress in b is lower can be confirmed by applying equations 3–39 and 3–40. If in both cases $a/b = 2$, the form stress factor in Fig. 3–36a is $K_a = 1 + 2 \times 2 = 5$ and that in Fig. 3–36b is $K_b = 1 + 2 \times \frac{1}{2} = 2$.

Torsion. A circular shaft with two different diameters can be considered to be divided into a number of concentric hollow shafts and a solid central

[26] R. V. Baud, "Avoiding Stress Concentration by Using Less Material," *Product Engineering*, Vol 5 (1934), p. 170.

shaft of such sizes that each of these elementary shafts carries an equal share of the torque transmitted by the original solid shaft.[27] In Fig. 3–37 is shown a typical stepped shaft having both the part with the large diameter and the part with the small diameter divided into four concentric hollow shafts and a solid central shaft. The two sets of shafts of constant diameter are joined by shafts of variable diameter along smooth curves, as indicated in the drawing. Since the thickness of the outer shaft just at the discontinuity is very small, high stresses are indicated at this place. The thicknesses of the other shafts are also reduced at the discontinuity, but to a lesser degree. Using a larger fillet will give smoother junctures and will lower the localized stresses.

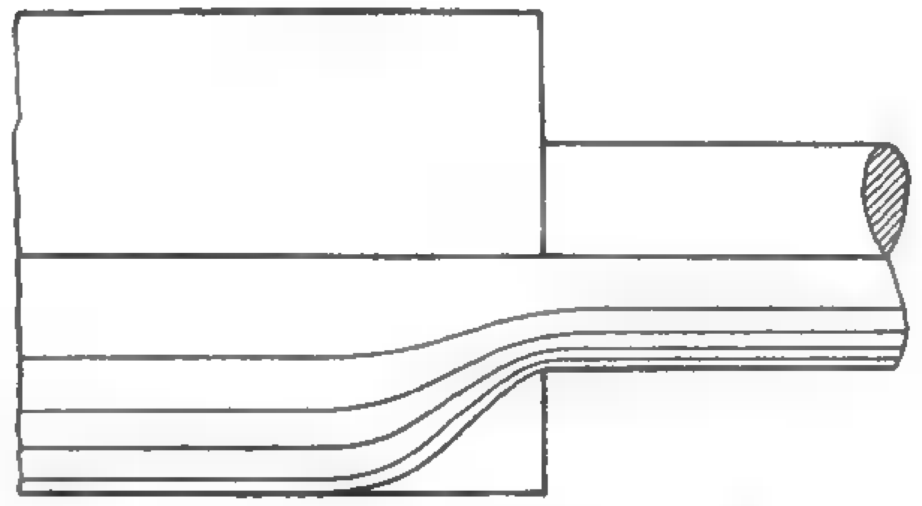


FIG. 3–37. Division of shaft into equitorque hollow shafts.

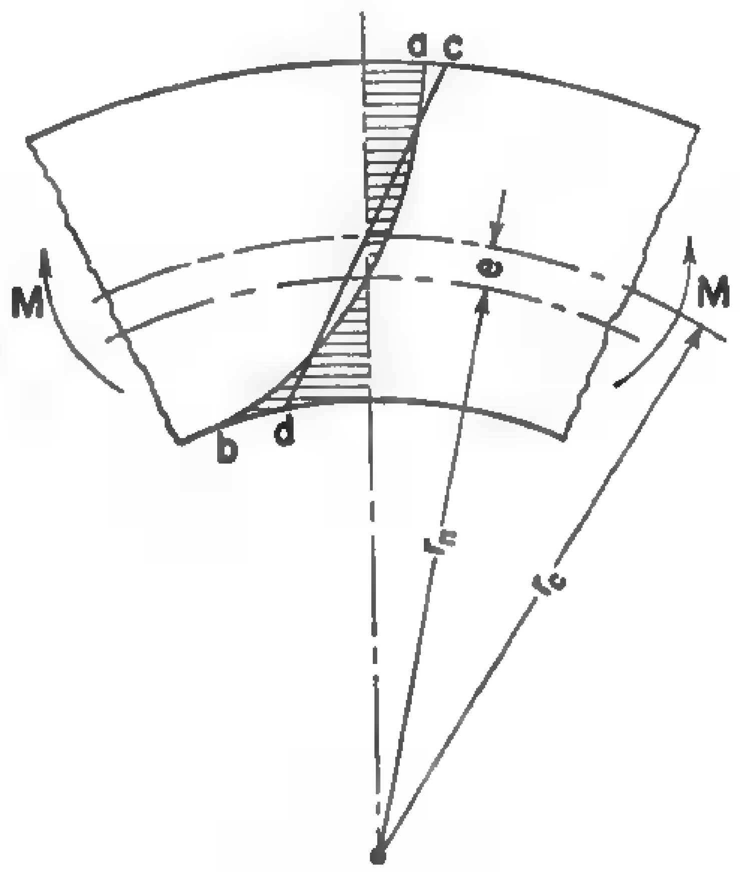


FIG. 3–38. Bending stresses in curved beam.

3–10. Curved beams. Many machine parts, such as frames of shears and punch presses, crane hooks, and various levers loaded in bending, have the basic shape of a curved beam.

The normal stresses created in a curved beam when it is subjected to bending differ from the stresses in a straight beam. The neutral axis of the cross section, instead of going through the center of gravity, or centroid, of the section, is shifted toward the center of curvature of the beam. As a result there is a nonlinear distribution of stresses, as shown by the curve ab in Fig. 3–38, whereas for a straight beam the distribution would be represented by the straight line cd. The curvature changes the force-flow lines so that there is stress concentration toward the concave side of the beam.

The equation for a curved beam can be derived by assuming, as for a straight beam, that planes normal to the centroid axis before bending remain planes and also remain normal to the centroid axis after bending. In Fig. 3–39 the lines ab and cd represent two such planes before bending, that is, when there are no stresses. When a bending moment M is applied to the beam, the plane cd rotates with respect to ab through an angle $d\theta$ to the position fg, and the outer fibers are shortened while the inner fibers are elongated. The original length of a strip at a distance y from the neutral

[27] L. S. Jacobsen, "Torsional-Stress Concentration in Shafts of Circular Cross Section and Variable Diameter," *Trans. ASME*, Vol. 47 (1925), p. 619.

line is $(r_n+y)\theta$. It is shortened by the amount $y\,d\theta$, and the stress in this fiber is, by equation 2–4,

$$s = E\epsilon = -\frac{Ey\,d\theta}{(r_n+y)\theta} \tag{3–46}$$

Then the load on a strip having a thickness dy and a cross-sectional area dA is

$$dF = s\,dA = -\frac{Ey\,d\theta\,dA}{(r_n+y)\theta}$$

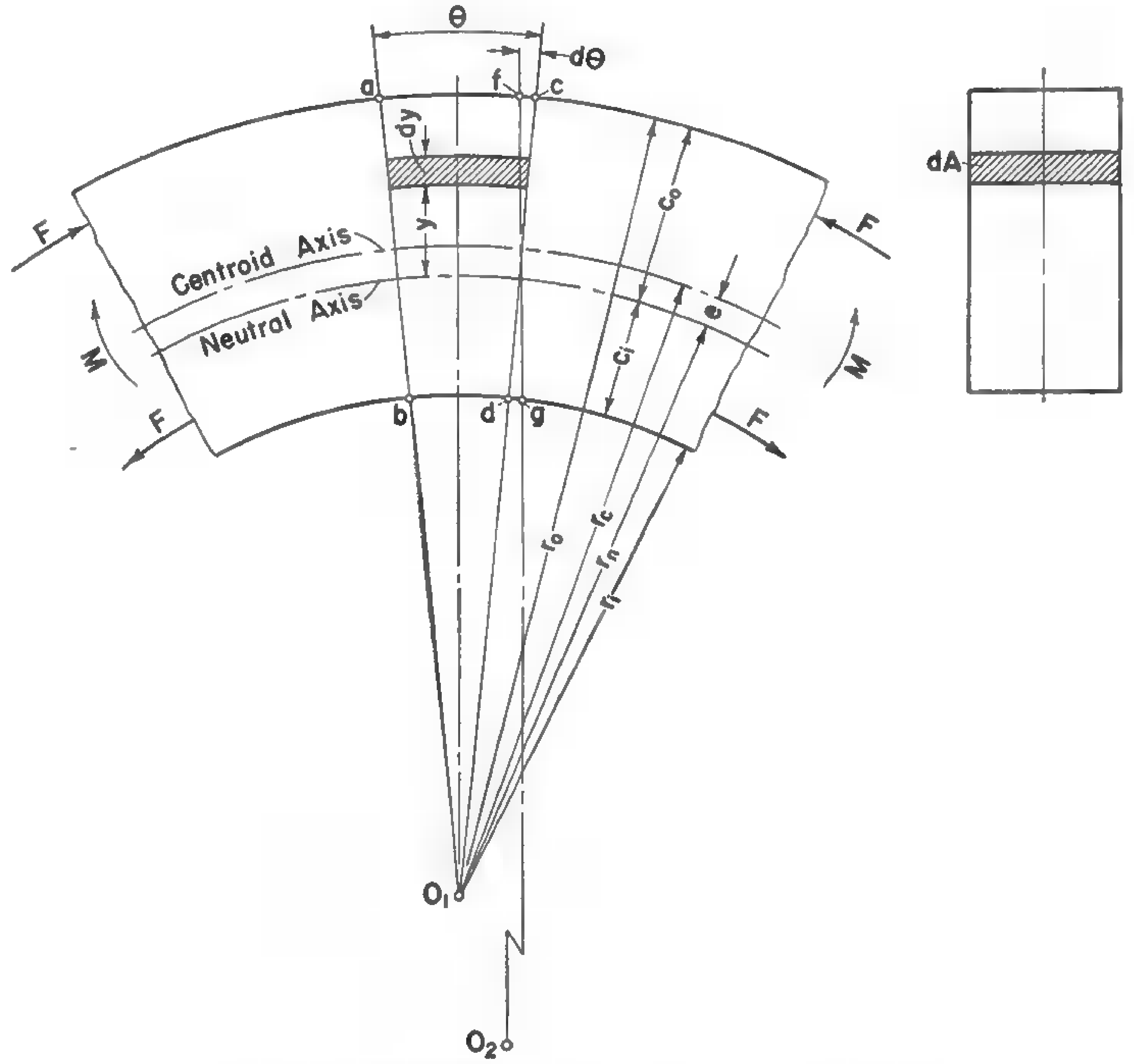


FIG. 3–39. Analysis of stresses in curved beam.

From the conditions of equilibrium, the summation of forces over the whole cross section is zero, and the summation of the moments of these forces is equal to the applied bending moment. Therefore

$$\int dF = -\frac{E\,d\theta}{\theta}\int\frac{y\,dA}{r_n+y} = 0$$

Since $E\,d\theta/\theta$ is not equal to zero, it follows that

$$\int\frac{y\,dA}{r_n+y} = 0 \tag{3–47}$$

For a given cross section, dA can be expressed in terms of y and dy, and the radius of curvature r_n can be determined from this equation.

If moments are taken about the neutral axis,

$$M = -\int y\,dF$$

By substituting for dF, we can obtain

$$M = \frac{E\,d\theta}{\theta}\int\frac{y^2\,dA}{r_n+y} = \frac{E\,d\theta}{\theta}\int\left(y - \frac{yr_n}{r_n+y}\right)dA = \frac{E\,d\theta}{\theta}\int y\,dA$$

Since $\int y\,dA$ represents the statical moment of area, it may be replaced by Ae, the product of the total area A and the distance e from the centroid axis to the neutral axis. Then

$$M = \frac{AeE\,d\theta}{\theta} \tag{3–48}$$

where e is the distance from the centroid axis to the neutral axis, or $e = r_c - r_n$, as in Fig. 3–39.

By combining equations 3–46 and 3–48 we find that

$$s = -\frac{M}{Ae}\left(\frac{y}{r_n+y}\right) \tag{3–49}$$

This is the general equation for the stress in a fiber at a distance y from the neutral axis. At the outer fiber, y is equal to c_o and the maximum compressive stress due to bending is

$$s_o = -\frac{Mc_o}{Aer_o} \tag{3–50}$$

At the inner fiber, y is $-c_i$ and the maximum tensile stress due to bending is

$$s_i = \frac{Mc_i}{Aer_i} \tag{3–51}$$

The value of r_n depends on the shape of the beam. For the more commonly used cross sections the value of r_n may be computed from data given in Table 3–4. The values of c_i and c_o in Fig. 3–39 may be determined from data given in Table 2–5, since

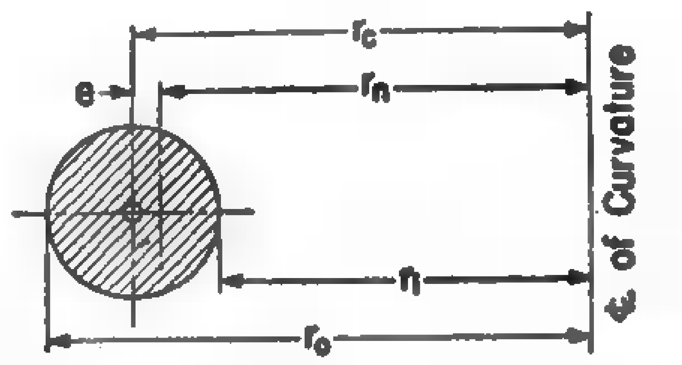
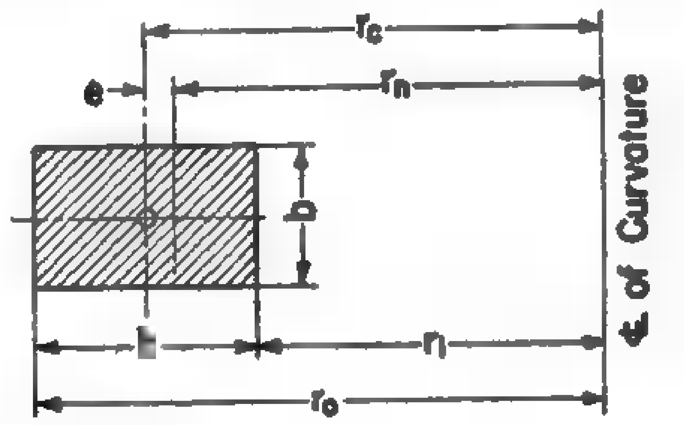
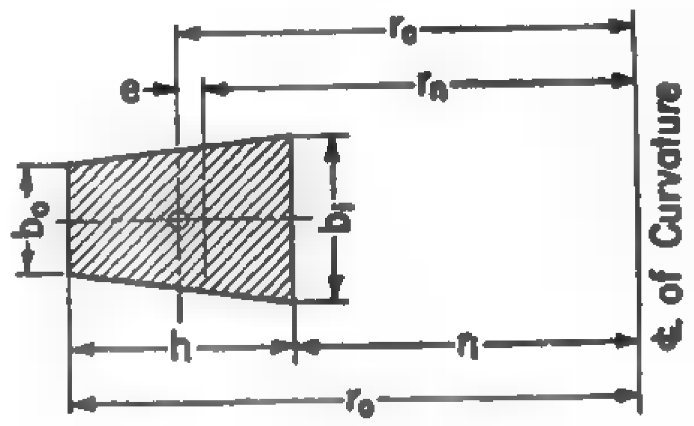
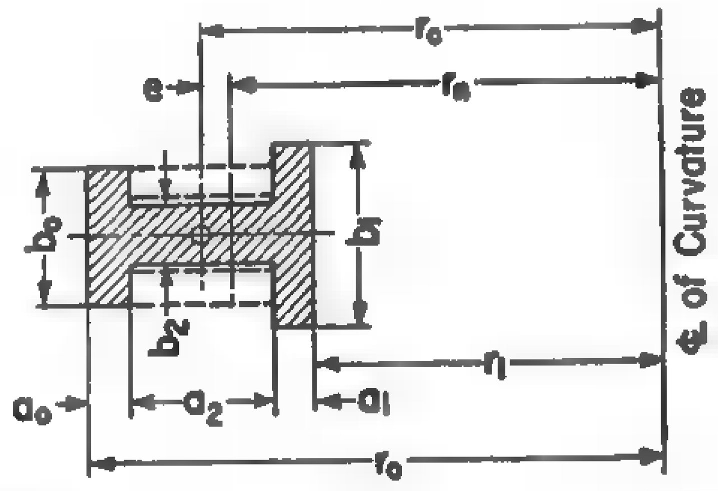
$$c_i = c_1 - e \qquad c_o = c_2 + e \tag{3–52}$$

Final remarks. The stresses determined by equations 3–49, 3–50, and 3–51 represent pure bending stresses produced by a couple. Curved machine members usually are bent by an eccentrically applied force. In such a case the normal stress from the force applied at the centroid must be added to the bending stresses in accordance with equation 2–44.

If holes must be put in a curved beam they should be located on the neutral axis rather than on the centroid axis, to decrease the effect of stress concentration as explained in section 3–7 and shown in Figs. 3–19 and 3–20.

TABLE 3–4

VALUES OF RADIUS TO NEUTRAL AXIS FOR CURVED BEAMS

Type	Section	Radius of Neutral Surface r_n
a		$r_n = \dfrac{(\sqrt{r_o}+\sqrt{r_i})^2}{4}$
b		$r_n = \dfrac{h}{\ln(r_o/r_i)}$
c		$r_n = \dfrac{\frac{1}{2}h(b_i+b_o)}{\dfrac{b_ir_o-b_or_i}{h}\ln(r_o/r_i)-(b_i-b_o)}$ If $b_o=0$, this section reduces to a triangle.
d		$r_n = \dfrac{A}{b_i \ln\dfrac{r_i+a_i}{r_i}+b_2\ln\dfrac{r_o-a_o}{r_i+a_i}+b_o\ln\dfrac{r_o}{r_o-a_o}}$ If $a_o=0$, the section reduces to a $\perp$ section; r_n is the same for a box section shown in dotted lines with each side panel $\frac{1}{2}b_2$ thick.

EXAMPLE 3–6. Determine the maximum stress in the frame of the 10-ton punch press shown in Fig. 3–40.

The radius of curvature of the neutral axis may be determined by using the expression given for case d in Table 3–4 and letting $a_o=0$. Thus

$$r_n = \frac{3\times8+4\times12}{12\ln\left(\frac{8+4}{8}\right)+3\ln\left(\frac{20}{8+4}\right)} = \frac{24+48}{12\times0.4055+3\times0.449} = 11.587 \text{ in.}$$

The distance from the centroid axis to the inner fiber may be computed by applying the formula given for case d in Table 2–5. The result is

$$c_1 = \frac{3\times12^2+9\times4^2}{2\times(3\times12+9\times4)} = 4 \text{ in.}$$

Then

$$c_2 = 12-4 = 8 \text{ in.}$$

The distance from the centroid axis to the neutral axis is

$$e = (8+4)-11.587 = 0.413 \text{ in.}$$

From equation 3–52, $c_i = 4-0.413 = 3.587$ in. and $c_o = 8+0.413 = 8.413$ in.
The bending moment referred to the neutral axis is

$$M = F(r_n+l) = 10\times2{,}000\times(11.587+36) = 952{,}000 \text{ lb-in.}$$

The compressive stress in the outer fibers due to bending is, by equation 3–50,

$$s_o = -\frac{952{,}000\times8.413}{72\times0.413\times20} = -13{,}470 \text{ psi}$$

and the tensile stress in the inner fibers due to bending is, by equation 3–51,

$$s_i = \frac{952{,}000\times3.587}{72\times0.413\times8} = 14{,}360 \text{ psi}$$

The direct stress from the force $F = 10\times2{,}000 = 20{,}000$ lb is tension. Its magnitude is

$$s_2 = \frac{20{,}000}{72} = 280 \text{ psi}$$

The combined stress in the outer fibers is

$$s = -13{,}470+280 = -13{,}190 \text{ psi}$$

and that in the inner fibers is

$$s = 14{,}360+280 = 14{,}640 \text{ psi}$$

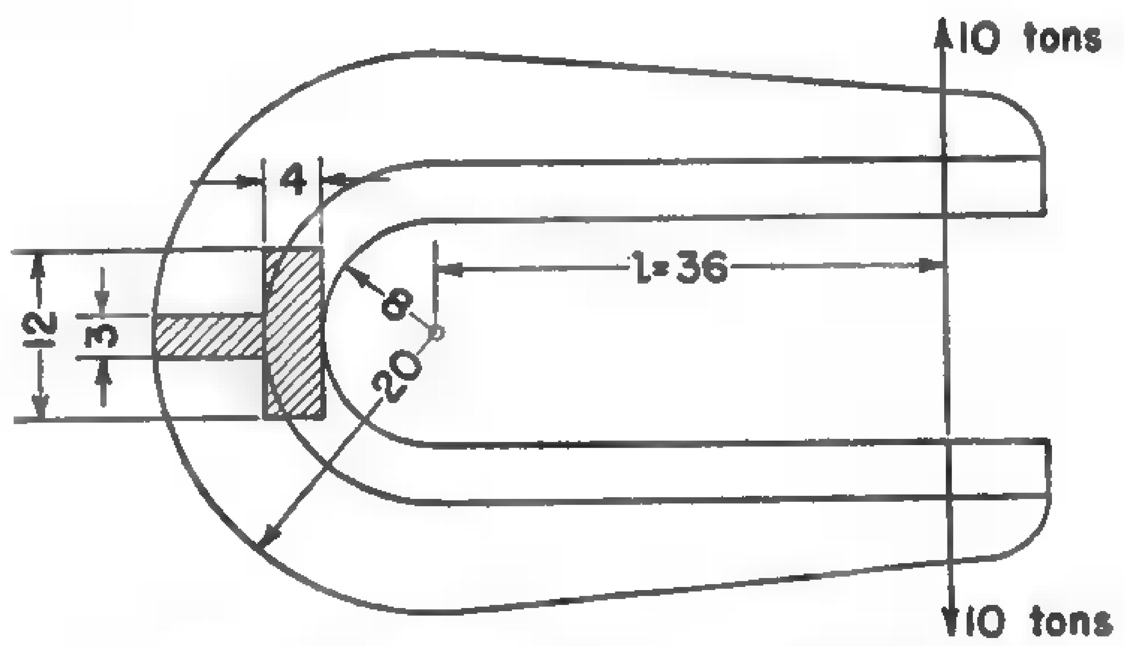


FIG. 3–40. Frame of punch press.

3–11. Repeated stresses. Experiments show that test specimens of metal fail when loads are repeated or reversed several million times, even though the unit stresses do not reach the elastic limit. Until recently this phenomenon was called *fatigue of material*.[28] It has now been established that the failure is due to a crack, which occurs at a point of the surface of the part where the highest *tensile* stress exists.[29] The fissure gradually spreads until failure takes place. Therefore the proper name is *failure by progressive fracture*.[30] This phenomenon is illustrated in Fig. 3–41 by means of lines of force flow. The dotted line b in Fig. 3–41a connects the points of maximum stresses. The beginning of a crack is shown in Fig. 3–41b; and the gradual increase of the crack depth is indicated in Fig. 3–41c and Fig. 3–41d. The fracture follows the surface of maximum stresses and is always normal to the

[28] J. B. Johnson, M. O. Withey, and James Aston, *Materials of Construction*, 8th ed. (New York: John Wiley & Sons, Inc., 1939), p. 771, and R. E. Peterson, *op. cit.*, p. 157.

[29] J. O. Almen, "Shot Peening to Increase Fatigue Resistance," *SAE Journal*, Vol. 51, No. 7 (July, 1943), p. 248.

[30] Seely, *op. cit.*, p. 56.

nearest line of force flow. Absence of elongation and of reduction of the area at the break is characteristic of a failure by progressive fracture. In general, ductile metals do not resist repeated stresses better than brittle metals.[31]

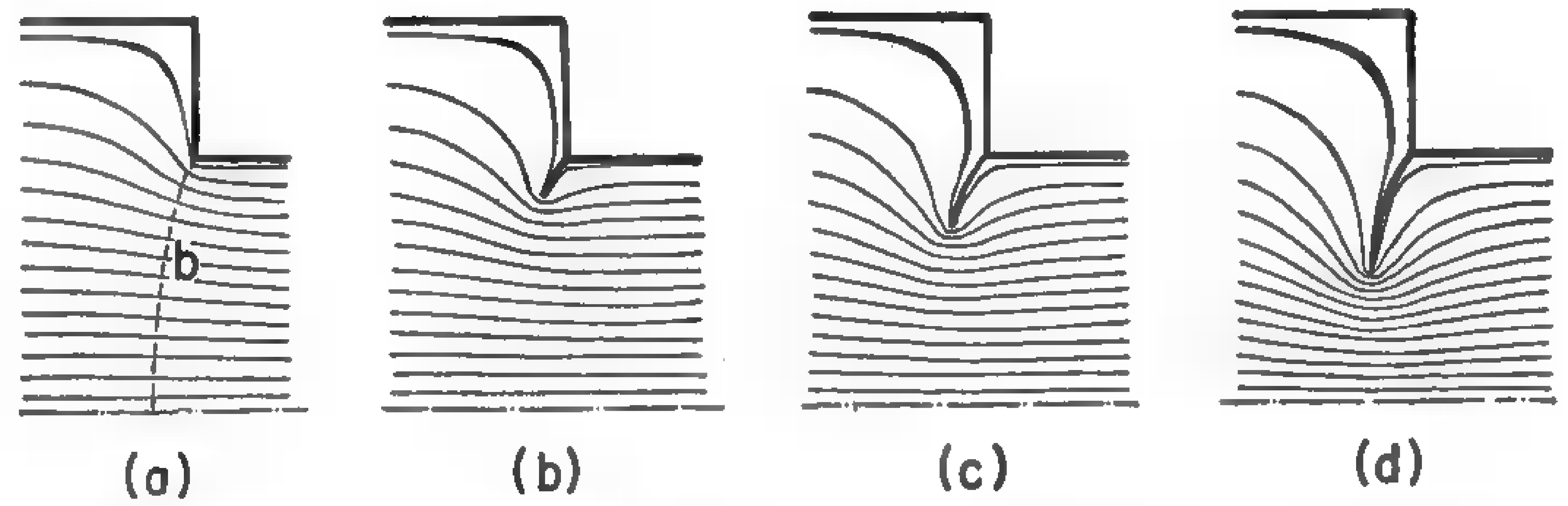


FIG. 3-41. Force flow and failure due to progressive fracture.

Repeated loading is especially dangerous for pieces with discontinuities, which always cause stress concentrations. The maximum local tensile stress may produce a minute crack which will cause a failure of the piece after a sufficiently large number of changes in the load.

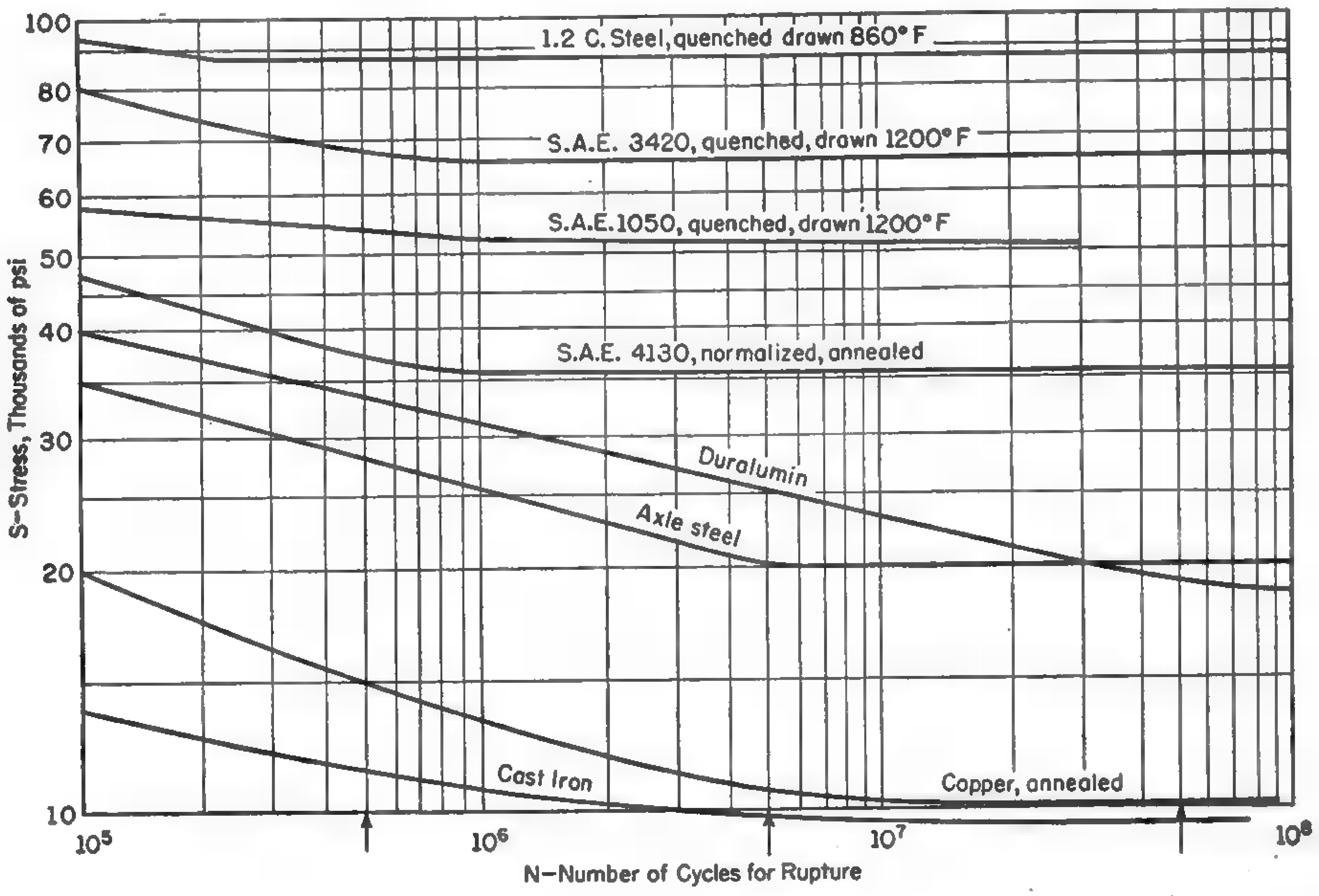


FIG. 3-42. S-N curves.

Endurance limit. The maximum stress which, with a complete reversal, can be repeated many millions of times without causing failure by progressive fracture is called the *endurance limit* of the material. There is no numerical

[31] H. W. Gillett, "Effect of Alloying and Heat Treatment on Endurance Limit of Steel," *Proceedings of the American Society for Testing Materials*, Vol. 30, Part I (1930), p. 291.

relation between the elastic limit and the endurance limit. However, there seems to exist a certain relation between the endurance limit S_{en} and the ultimate tensile strength S_u. For ferrous materials S_{en} is about $0.5S_u$, and for nonferrous materials S_{en} is between $0.25S_u$ and $0.35S_u$.[32]

S-N curves. The endurance limit of a material is determined by applying to several specimens repeated loads which cause completely reversed stresses of known values, and recording the number of stress reversals each specimen did endure before it failed. Each specimen is subjected to a lower stress than was the preceding one, and it breaks after a greater number of cycles. Finally, a stress is reached that does not cause failure, regardless of how many reversed cycles are applied. The results are plotted, as shown in Fig. 3-42, with the stresses S as ordinates and the numbers of cycles N as abscissas. Values of S_{en} and N are laid off on logarithmic scales in order to shorten the diagram and increase its accuracy. Such plots are called *S-N* curves.

For steels, the *S-N* curves flatten out abruptly and show a knee when the stress is approaching the endurance limit. With a nonferrous metal, the *S-N* curve does not show such a pronounced knee; and for some materials, such as some aluminum alloys, the curve does not level off even after 10^9 cycles. For such a material no endurance limit exists, and it is possible to give only an endurance strength indicating a stress and the number of cycles withstood at that stress.

Endurance diagrams. The maximum stress that can be created in a test specimen or a machine part by repeated loading without causing failure of the piece by progressive fracture is called its *endurance strength*. When the stress varies periodically from a maximum to a minimum but is not completely reversed, the maximum stress that can be imposed without causing failure is higher than the endurance limit of the material. The magnitude of such a stress depends on the magnitude of the mean stress, above and below which the stress fluctuates.

These conditions can be best illustrated by a diagram similar to a Goodman's diagram. The mean value S_m of the upper stress S_1 and the lower stress S_2, where the stresses are taken with the plus sign (+) for tension and the minus sign (−) for compression, is

$$S_m = 0.5(S_1 + S_2) \tag{3-53}$$

With a complete stress reversal, S_1 is the maximum tensile stress and is the endurance limit, or $S_1 = S_{en}$; and the other, or lower, stress S_2 is numerically equal to S_1 but is a compressive stress and therefore is considered negative. When $S_1 = S_{en}$ and $S_2 = -S_{en}$, it is evident that $S_m = 0$. This condition gives points a_1 and a_2 in Fig. 3-43.

[32] Supplement to *Z. VDI*, Vol. 77, No. 42 (1933), p. 1.

A test is conducted by gradually decreasing S_2 and thus permitting an increase of S_1 until S_1 becomes equal to the elastic limit S_e. The upper and lower stresses found by such a test, including their signs, are plotted as ordinates against mean stresses as abscissas. Such a plot, as Fig. 3–43, shows that the lines a_1b_1 and a_2b_2 are practically straight. It is therefore sufficient to find the lower stress S_2 (at point b_2) that corresponds to $S_1 = S_e$ (at point b_1), and to compute the corresponding mean stress $S_m' = \frac{1}{2}(S_e + S_2')$. If the test is continued until $S_1'' = S_2'' = S_u$, the rest of Goodman's diagram can be plotted as shown in broken lines. However, stresses above S_e cannot be allowed in machine design and the broken lines are replaced by the straight lines b_1c and cb_2.

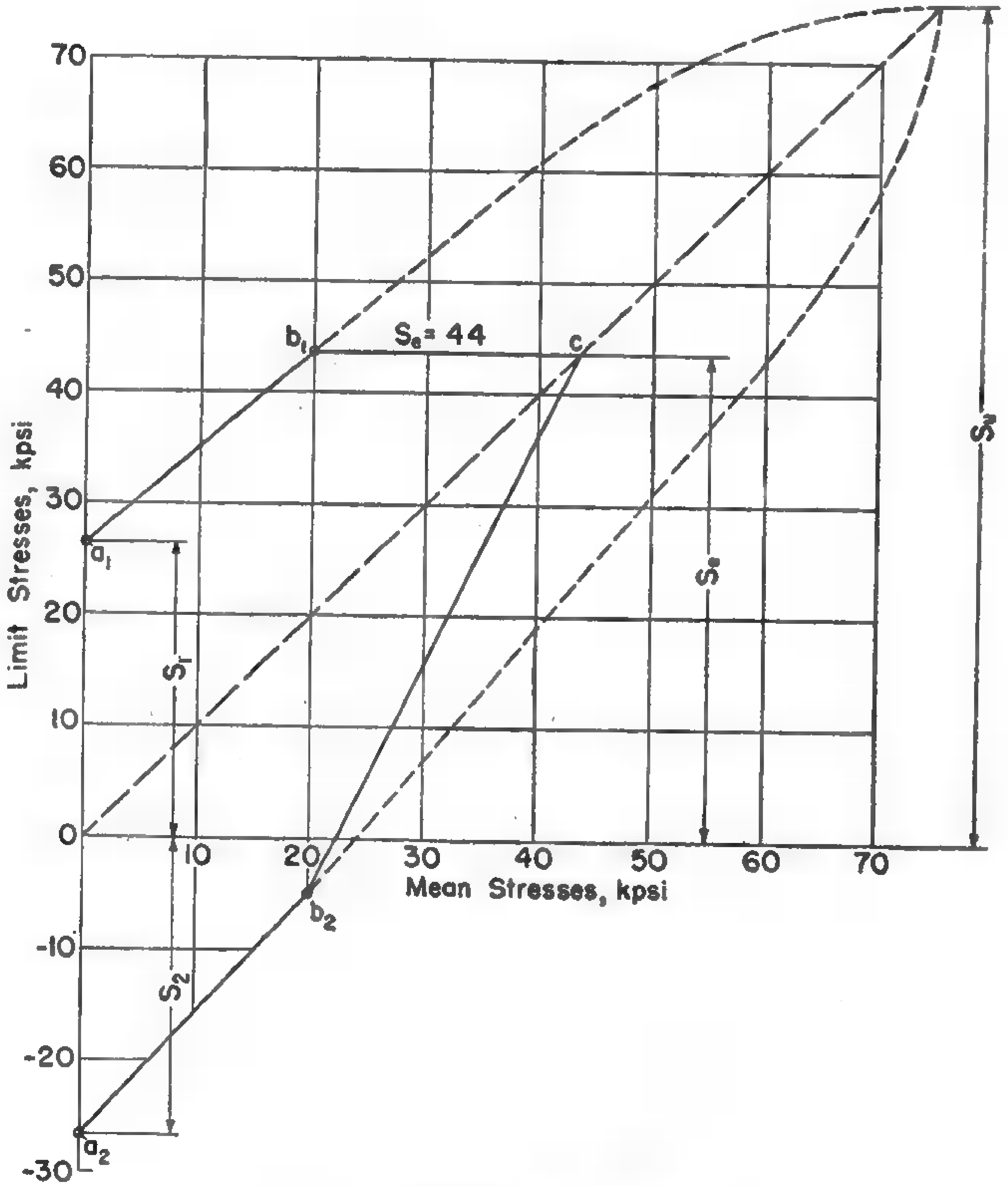


FIG. 3–43. Endurance diagram.

One-half the difference between the stresses S_1 and S_2 is called the *stress amplitude* S_a. Thus,

$$S_a = 0.5(S_1 - S_2) \tag{3–54}$$

A line drawn through the center of coordinates at an angle of 45° makes it possible to find directly the value of a stress amplitude S_a which corresponds

to a given mean stress S_m. By the definitions of S_m and S_a, the upper stress for all values of S_m is

$$S_1 = S_m + S_a \tag{3–55}$$

The endurance diagram shows that the maximum stress amplitude S_a occurs when $S_m = 0$, and then $S_a = S_{en}$. As S_m increases, S_a decreases.

It will be shown later that for design calculations the stress amplitude S_a, rather than the total stress $S_1 = S_a + S_m$, is the deciding factor.

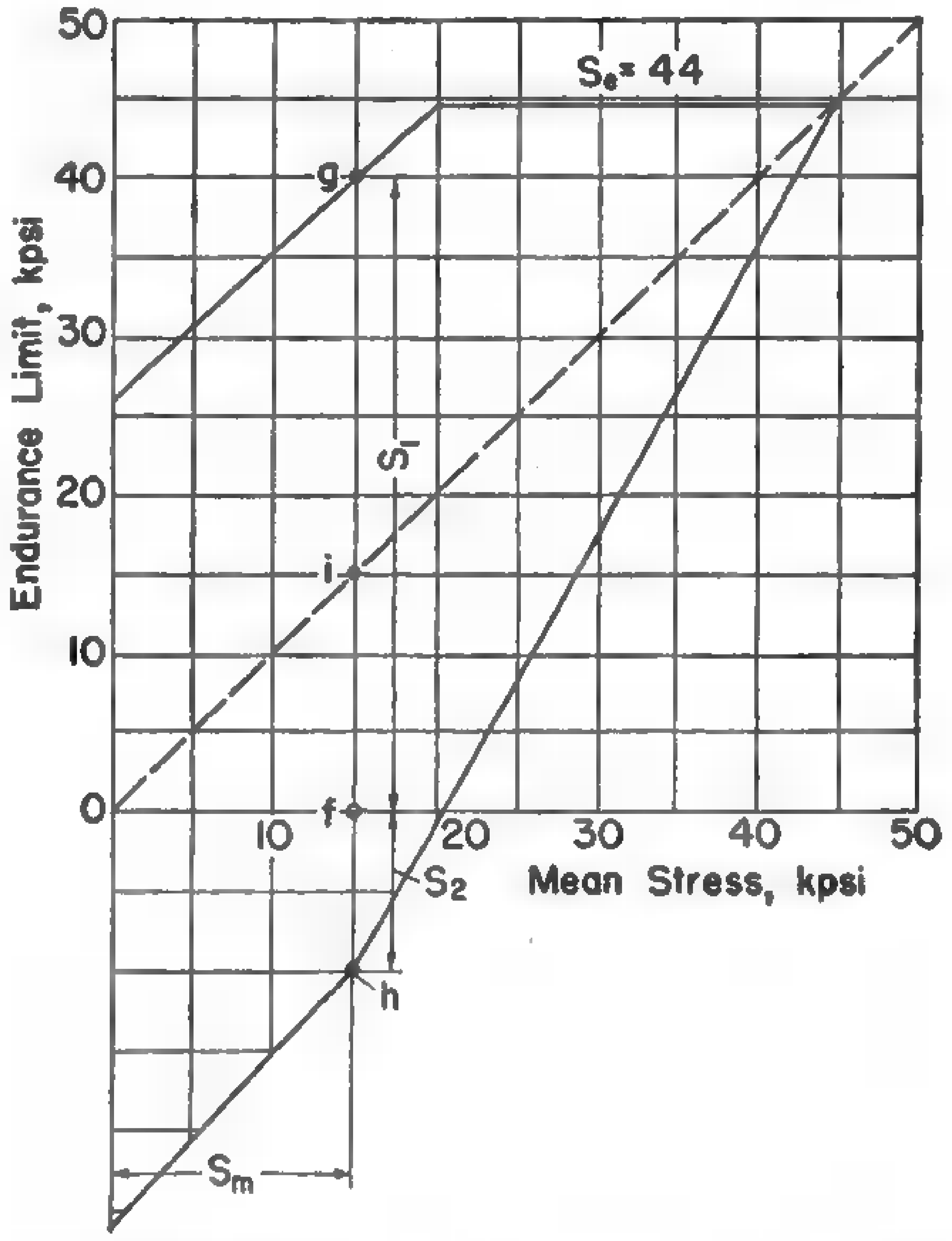


FIG. 3–44. Endurance diagram in tension or compression for SAE 1035 steel.

In Fig. 3–44 is shown an endurance diagram, as used in design practice, for tension-compression of a test specimen of SAE 1035 steel. For a mean stress $S_m = 15{,}000$ psi the stress amplitude $S_a = gi$ is found by scaling to be 25,000 psi. The diagram shows also that $S_1 = fg = 40{,}000$ psi and $s_2 = fh = -10{,}000$ psi.

Endurance diagrams similar to that in Fig. 3–44 are plotted for all three types of stresses, namely, bending, tension-compression, and shear in torsion.

Bending. During bending, only the outer fibers on both sides of the neutral plane are subjected to the maximum tensile and compressive stress. The adjoining fibers are stressed less and, in general, fibers located nearer to the neutral plane can take part of the load off the more highly stressed fibers located further from the neutral plane. This fact explains why accurate tests show that the elastic limits in tension and compression in the case

of bending are higher than the elastic limits found for direct tension and compression. This difference is about 20 per cent for carbon steels and about 11 per cent for alloy steels. However, in beam calculations in the United States these differences are neglected, as this practice is on the side of safety. On the other hand, endurance limits in pure tension and compression are probably lower by a similar amount. However, no reliable data relating to this feature are yet available, the only information being that given in the endurance diagrams of Chapter 4.

Summary. Progressive fracture can be the result of the repetition of direct loads, bending moments, or torques. The general relation of the endurance limits for different types of stresses is similar to that shown in Fig. 3–44. It depends on the range of stresses. The endurance limits in tension or compression are lower than those in bending, averaging for steels about 65 per cent of the bending limit; and those in torsion are still lower, averaging about 55 per cent of the bending limit.[33] For cast iron the endurance limit in torsion is about 92 per cent of that in bending.[34] The endurance limits given for various materials are usually found by subjecting the test specimens to repeated bending in opposite directions. At present most of the important materials have been sufficiently investigated, and diagrams similar to that in Fig. 3–44 can be drawn for them for use in design calculations.

[33] F. B. Seely, *Resistance of Materials*, 3d ed. (New York: John Wiley & Sons, Inc., 1947), p. 292.

[34] W. Herold, *Wechselfestigkeit Metallischer Werkstoffe* (Vienna: Julius Springer, 1934), p. 69.

CHAPTER 4

Engineering Materials

4–1. Technical properties. The technical properties of materials used in machinery may be classified as: the *physical*, such as composition, structure, homogeneity, specific weight, thermal conductivity and expansibility, and resistance to corrosion; the *technological*, or those relating to manufacturing, such as fusibility, forgeability, malleability, bending to shape, machinability; and the *mechanical*, which are established by tests and used in machine design. The basic mechanical properties are: elastic limits, moduli of elasticity, ultimate strengths, endurance limits, and hardness. Secondary mechanical characteristics, determined from the basic ones or simultaneously with them, are: resilience, toughness, ductility, and brittleness. For machine design the mechanical properties of a material that are of prime importance are strength, stiffness, hardness, and ductility.

Strength. The strength of a part depends on the type and nature of loading. The *static strength* of a material is expressed by the corresponding elastic limit stress S_e. The *impact strength* is measured by the corresponding modulus of resilience u. The *endurance strength* is expressed by the corresponding endurance limit S_{en}.

Stiffness, or *rigidity*, is measured by the modulus of elasticity, E being used for tension or compression and G being used for shear.

Hardness is a relative characteristic. There are several methods of measuring it, all of an arbitrary nature. The *Brinell hardness number* is obtained as follows: A hardened steel ball D mm in diameter is pressed under a load of F kg into the smooth surface of the material to be tested; the diameter d of the indentation is measured in millimeters; and the hardness number Bhn, in kilograms per square millimeter, is then computed by the relation

$$\text{Bhn} = \frac{2F}{\pi D(D - \sqrt{D^2 - d^2})} \tag{4–1}$$

The standard ball has a diameter D of 10 mm, and the standard load F is 3,000 kg for steels and irons and 500 kg for the softer, nonferrous metals.

The *Rockwell hardness number* is based upon the additional depth to which a test point is driven by a heavy major load beyond the depth to which the same penetrator has been driven at first by a lighter, minor load. The indentor is either a hardened steel ball or a spherical-tipped conical diamond, called a *Brale*. The hardness number is read directly from a scale which indicates penetration.

Several combinations of indentor and load are possible. Those most commonly used are for softer materials: the R-B scale, which assumes the use of a $\frac{1}{16}$-in. steel ball and a major load of 100 kg; for harder metals, the R-C scale, which assumes the use of a Brale and a major load of 150 kg.

The *Vickers method* of determining hardness is similar in principle to the Brinell method. It expresses the pressure under the indentor in kilograms per square millimeter. The indentor is a diamond having the shape of a square pyramid, the loads are much lighter, and the indentation is measured with a medium-power microscope.

When the *Shore Scleroscope* is used, a small cylinder of steel with a hardened point is allowed to fall upon the smooth surface of the material, and the height of the rebound of the cylinder is taken as the measure of hardness.

Comparison of hardness determinations. The Brinell method of determining hardness is used quite commonly because it gives accurate results at reasonable cost. However, it cannot be applied to brittle materials; neither can it be applied to a finished product without leaving a mark.

The Rockwell test is simple, fast, and accurate. It can be used with either flat or round surfaces, and the indentor leaves a smaller mark than does that of the Brinell tester. Its use is gradually increasing, especially in production work.

The Vickers test is considered to be more accurate than either the Brinell test or the Rockwell test, but the equipment is almost four times as expensive.

The Scleroscope test is simple, rapid, and definite for materials for which it is suited, but the results vary somewhat with the size and thickness of the sample. It is sufficiently accurate as a comparative measure for different specimens of the same material, but it is not reliable for comparing different metals.

There is no exact relation between the scales of different hardness tests, because each measures a somewhat different kind of hardness. The Brinell and Vickers numbers are identical up to 250. Beyond 250 the Brinell numbers begin to be increasingly lower because of the flattening of the steel ball. At 420 Vhn the difference is about 5 per cent; at 540 it is 8.3 per cent; at 675 it is 14.3 per cent; and at 960 it is 26 per cent.

American industry uses both the Brinell and Rockwell tests. Therefore both figures are given in tables in this book where hardness data are shown.

The following expressions[1] may be useful for changing from a Brinell hardness number B to a Rockwell-C number R_C or to a Rockwell-B number R_B:

$$R_C = 88B^{0.162} - 192 \tag{4-2}$$

[1] I. H. Cowdrey and R. G. Adams, *Materials Testing*, 2d ed., rev. ptg. (New York: John Wiley & Sons, Inc., 1944), p. 75.

and

$$R_B = \frac{(B-47)}{0.0074B+0.154} \tag{4-3}$$

Ductility. A material is ductile if it is capable of undergoing a large permanent deformation without rupturing. There is no absolute measure of ductility. The percentage of *elongation* or the percentage of *reduction of area* during a tensile test carried to rupture is used as a relative measure. Ductility helps to relieve localized stress concentration through local yielding. It is a necessary characteristic of a material used to sustain live loads, especially where concentrated stresses may occur.

Brittleness is a characteristic opposite to ducility and toughness. A material may be considered brittle if its elongation at rupture through tension is less than 5 per cent in a specimen 2 in. long. Usually brittleness and hardness are closely associated, and very hard materials are brittle.

Typical properties. Some mechanical properties have already been discussed in the preceding chapter. Tables given in this chapter contain some of the physical properties and most of the basic mechanical properties, as far as they are established, of the materials used in modern machine construction. The values given in the tables are average values, not minimum ones. Average values are more representative; and with the use of proper safety factors, they give entirely satisfactory results. At the same time, the use of average values is an incentive for manufacturers of material to improve the quality of their products, while the use of minimum values would be favorable only to manufacturers who do not keep abreast of progress.

4–2. Cast iron. Ordinary cast iron is the most used of all materials employed in machines. The reasons are its strength, particularly in compression; the ease with which it can be cast into any desired shape; the facility with which its strength and hardness can be varied; its machinability; and its cheapness. Because of its structure it also has the valuable characteristic of *damping out* vibration.

Composition. Cast iron is iron containing so much carbon—from about 2 to 4 per cent—that normally it is not malleable at any temperature. In addition, cast iron contains other elements in varying quantities, the more important ones being silicon, manganese, phosphorus, and sulfur.

Classes. Cast irons are produced with various strengths to suit different design requirements. According to ASTM specifications, gray-iron castings are listed by classes. The class number gives the minimum tensile strength of the cast test bar: No. 20 cast iron has a strength of 20,000 psi; No. 60, the cast iron of the highest test specifications, has a strength of 60,000 psi. Only a few representative classes are listed in Table 4–1.

Meehanite. Meehanite is the name of several strong, uniform cast irons produced by a special patented process and having different combinations

TABLE 4-1

MECHANICAL PROPERTIES OF FERROUS CAST ALLOYS

No.	Material: Name	Material: Characteristic[a]	Ultimate Tensile Strength S_u (kpsi)	Elastic Limit, Average: Tension S_e (kpsi)	Elastic Limit, Average: Compression S_e (kpsi)	Elastic Limit, Average: Shear S_{es} (kpsi)	Modulus of Elasticity: Direct, or Young's, E (kpsi)	Modulus of Elasticity: Transverse, or Shear, G (kpsi)	Endurance Limit S_{en} (kpsi)	Hardness Minimum: Brinell (Bhn)	Hardness Minimum: Rockwell B	Hardness Minimum: Rockwell C
1	Cast iron, Class No. 20 (ASTM)	Medium section, ½ in.	20	6.2	50.0	6.2	10,000	4,200	6.5	163	85	3
		Light section, <½ in.	24	7.5	60.0	7.5	12,000	5,000	7.5	180	89	8
2	Cast iron, Class No. 25	Medium section, ½ in.	25	10.0	75.0	10.0	13,000	5,500	9.5	180	89	8
		Light section, <½ in.	30	12.0	90.0	12.0	14,000	6,000	11.0	200	94	14
3	Cast iron, Class No. 35	Semisteel, ½-in. section	35	14.0	100.0	14.0	16,000	6,500	13.0	200	94	14
4	Cast iron, Class No. 50	Electric-furnace, ½ in.	50	30.0	120.0	30.0	20,000	8,000	18.0	230	98	21
5	Malleable iron, ASTM 32310	Standard	50	32.5	32.5	23.0	25,000	10,000	24.0	120	70	..
6	Malleable iron, ASTM 48005	Pearlite	70	48.0	48.0	35.0	27,000	11,000	30.0	190	91	11
7	Iron-copper alloy	Special heat treatment	57	30.0	40.0	24.0	24,000	10,000	30.0	280	104	29
8	Meehanite, M40, Type B	General use	45	19.0	68.0	18.0	19,000	7,500	19.0	196	93	13
		Heat-treated	60	40.0	90.0	....	....	....	....	237	99	22
[illegible]	Meehanite, N50, Type A	General use	50	21.0	75.0	20.0	21,000	8,000	22.0	207	95	16
		Heat-treated	75	50.0	100.0	....	28,000	11,000	28.0	250	102	24
10	Nickel cast iron, Grade II	Ni, 1.25	35	17.0	90.0	15.0	16,000	6,500	13.0	180	89	8
11	Nickel cast iron, Grade III	Ni, 1.75	44	18.0	100.0	16.0	18,000	7,500	16.0	180	89	8
12	Hi-Test cast iron	Ni, 1.8 ± 0.5; Mo, 0.3	45	10.0	95.0	25.0	23,000	8,000	20.0	220	96	19
13	Ni-Tensyliron	Ni, 2.1 ± 0.2; Mo, 0.3	50	14.0	140.0	30.0	23,000	8,000	22.0	320	108	34
14	NiResist cast iron	As cast	25	8.0	90.0	8.0	12,000	5,000	8.0	120	70	..
15	Nitralloy cast iron	Nitrided, heat-treated	65	35.0	175.0	35.0	23,000	9,500	30.0	1,050	...	68
16	Ductile iron, 90–65–02	As cast	90	65.0	....	58.0	25,000	9,300	40.0	225	97	20
17	Ductile iron, 80–60–05	Heat-treated	80	60.0	....	54.0	25,000	9,300	34.0	195	93	13
18	Ductile iron, 60–45–15	Annealed	60	45.0	....	40.0	25,000	9,300	24.0	140	78	..
19	Ductile iron, 80–60–00	As cast	80	60.0	....	54.0	25,000	9,300	34.0	230	98	21

[a] Specific weight of all cast irons is about 0.26 lb per cu in.

of physical and mechanical properties. These products are of four main types: (a) general engineering, (b) wear-resisting, (c) heat-resisting, and (d) corrosion-resisting. Tensile strengths vary from 25,000 to 55,000 psi; and with oil-quenching and tempering 75,000 psi can be obtained.

4–3. Heat-treatment of cast iron. Several types of heat treatment are used to alter or enhance some properties of cast irons and thus to increase their usefulness.

Aging is applied to relieve casting stresses without materially affecting physical properties. It is carried on during 1 to 5 hr in the temperature range of 800 F to 1,000 F.

Annealing is carried out for 1 to 5 hr at 1,100 F to 1,600 F, the temperature depending on the size of the part. It is intended to reduce hardness and to facilitate machining. However, annealing is accomplished at the expense of some strength.

Baking is applied to castings that have been pickled in acid to remove sand and scale. Pickling makes castings brittle, but baking for a few hours at 300 F removes this brittleness.

Quenching cast iron in oil or water after it has been heated above the critical range increases its hardness and also its brittleness.

Drawing, or *tempering*, accomplished by reheating the quenched metal to a temperature below the critical temperature, reduces the brittleness but still leaves an increase of hardness. By such a treatment a Brinell hardness number from 200 to 400 can be attained, the value depending on the quenching and drawing temperatures.

Malleableizing is also a heat-treating process, but the final product has properties entirely different from those of the original product.

Malleable-iron parts are made of clean, white-iron castings, preferably having a low sulfur content and a certain manganese content. The castings are heated in an annealing furnace in contact with some iron oxide, which absorbs part of the combined carbon from the cast iron. The annealing process converts the combined carbon of the original hard casting into a free nodular form called *temper carbon*.

4–4. Alloy cast irons. In order to obtain a material stronger than ordinary gray cast iron but having all its desirable characteristics, certain metals may be added, such as nickel, vanadium, molybdenum, copper, and aluminum. An alloy cast iron can be improved by heat-treating, just as can ordinary cast iron.

The addition of from 1 to 2.5 per cent of *nickel* increases the tensile strength of cast iron to 60,000 psi. At the same time, the nickel makes the cast iron easier to machine but tougher and harder, thus increasing its resistance to wear.

The addition of *aluminum* to cast iron allows it to be nitrided like nitralloy steel. This process gives the iron a very hard surface.

Adding *molybdenum*, even in small amounts (0.2 to 1.5 per cent) raises the strength and the elastic and endurance limits of cast iron, as may be seen from Table 4–1. It also increases creep resistance.

Tests show that *copper* can be used instead of nickel with practically the same results. Another interesting development is an alloy of cast iron and copper which is used for making automobile-engine crankshafts.[2] Its composition is C, 1.3; Si, 2.0; Mn, 0.5; Cu, 2.6; Cr, 3.5. As cast the metal is hard, white, and brittle. After a special heat treatment, the material obtains the properties of open-hearth steel and has even a higher endurance limit in bending than steel.

Semisteel is a term often applied to a cast iron produced in the cupola by adding to the usual cast-iron mixture from 20 to 40 per cent of low-carbon steel scrap. This mixture, when properly handled in the cupola and while being poured into the mold, produces strong, tough, close-grained castings with good machinability. However, the metal has none of the characteristics of steel and is only a high-grade gray cast iron.

Cast iron produced in an *electric furnace*, from mixtures similar to those used in a cupola, has considerably better mechanical properties, as well as greater hardness and good machinability. It is particularly suitable for impact loads.

4–5. Ductile iron. Recently it has been found that small amounts of magnesium added to cast iron convert flake graphite to spherical, or nodular, graphite. This conversion results in a great increase of strength, elastic limit, hardness, and elongation. Since the new material is ductile and on the basis of its mechanical properties resembles steel, it has received the name *ductile iron*.[3] Ductile iron is not a single material, but is rather a group of materials. As in steel, the matrix structure can be modified by alloys and heat treatment. From an industrial viewpoint the most interesting grades are the perlitic and ferritic ductile irons.

The four main types of ductile iron produced at present are shown in lines 16 to 19 of Table 4–1. The 90–65–02 grade has a perlitic structure, is used as-cast, and has good resistance to mechanical wear. The 80–60–05 grade has a perlitic-ferritic structure and is used to obtain a combination of strength and toughness. The 60–45–15 grade has a fully ferritic structure and is obtained by a short anneal of either of the first two grades. It has exceptional machinability and is very tough. By proper heat treatment its

[2] P. Dwyer, "Ford Foundry Casts V-8 Engine Crankshaft," *Foundry*, Vol. 62, No. 4 (April, 1934), p. 14.

[3] Based on information furnished by the International Nickel Company. See also A. P. Gagnebin, "The Industrial Status of Ductile Iron," *Mechanical Engineering*, Vol. 73 (1951), pp. 101–18, and A. P. Gagnebin, K. D. Millis, and N. B. Pilling, "Engineering Application of Ductile Iron," *Machine Design*, Vol. 22, No. 1 (January, 1950), pp. 108–14.

mechanical properties S_u, S_e, and its Brinell hardness number can be more than doubled; however, there is a loss of elongation down to 1 per cent. The 80–60–00 grade contains more manganese and phosphorus than the other grades. It is used for parts that require high strength and toughness but are subject only to moderate shock.

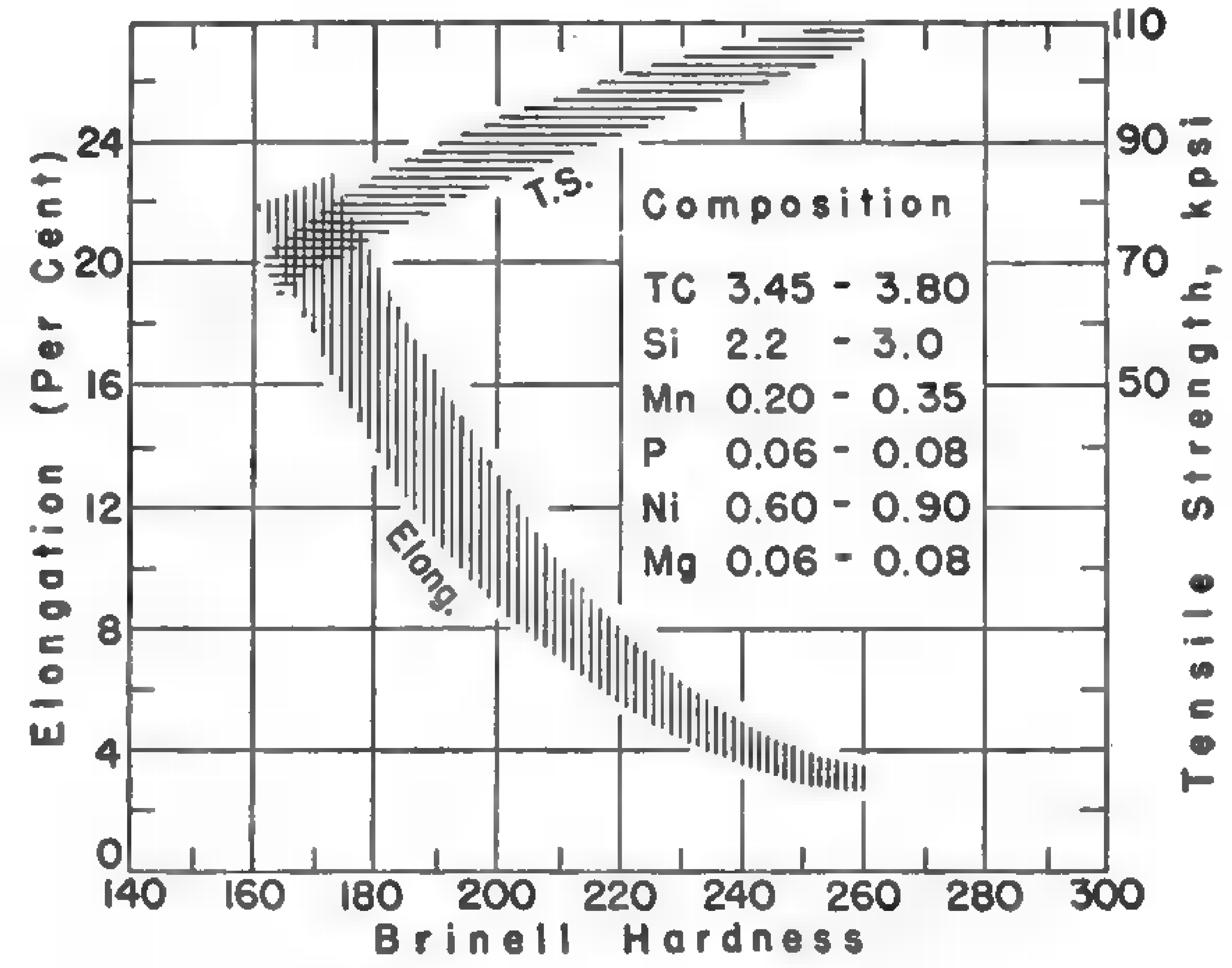


FIG. 4–1. Properties of ductile iron in as-cast condition.

The relationship between tensile strength, elongation, and hardness in the as-cast condition is shown in Fig. 4–1. A very important feature of ductile iron is that it combines the fluidity and castability of gray iron with properties resembling—and in some instances exceeding—those of steel. It also has an exceptional resistance to wear. Ductile iron can be used for parts which are too intricate to be cast of steel and which cannot be made strong enough when cast in the best grades of gray iron. In addition there are a great number of other parts that so far have been made of other materials but will have better properties or will be lighter if made of ductile iron. Among these are frames, gears, sprockets, diesel-engine pistons, crankshafts, generator shafts, dies for press forming, and various intricate castings. It can be expected that in time this new material will become the third ranking industrial metal on a tonnage basis, exceeding all other materials except steel and gray iron. In many cases it will take the place of cast steel and malleable iron.

4–6. Carbon steel. Simple steel, often called *carbon steel*, consists chiefly of iron, carbon, and manganese. Other elements, such as silicon, phosphorus, and sulfur, are always present but are not essential to the formation of the steel. A steel that contains one or more elements other than carbon and manganese, such as nickel, chromium, vanadium, molybdenum, or

tungsten, in sufficient quantity to modify or noticeably improve some of its useful properties, is called an *alloy steel*.

SAE specifications. The Society of Automotive Engineers has adopted specifications for various steels based on their chemical composition.[4] Each specification is given a four-figure or five-figure number. The first two digits indicate the class, while the last two or three indicate the carbon content in hundredths of one per cent. Thus, a carbon steel the carbon content of which is 0.50 per cent has 1050 as its specification number, 10 being the class number of carbon steels.

Ordinary steel castings are designated also by carbon content, with 00 in front. An example is SAE 0030—a general-purpose steel with a maximum carbon content of 0.30 per cent. High-strength carbon steels and alloy steels are designated by the tensile strength with one zero in front, as in SAE 090—a steel which has $S_u = 90$ kpsi.

AISI specifications. The American Iron and Steel Institute has developed specifications which are intended to limit the number of standard grades. A capital-letter prefix indicates the steel-making process. The symbol B indicates a Bessemer carbon steel; C, an open-hearth carbon steel; and E, an electric-furnace alloy steel. An index of four numerals, identical with the index of the SAE specifications, indicates the composition. The first two show the type of alloy and the last two give the average carbon content. Thus C1020 is an open-hearth carbon steel with a carbon range from 0.18 to 0.23 per cent; E2515 is a 5 per cent nickel steel having a carbon content of 0.13 to 0.18 per cent and is made in an electric furnace.[5]

ASTM specifications. Specifications of the American Society for Testing Materials for steel forgings divide all steels into 12 classes numbered from A to M.[6] The ultimate strength is taken as the basis of classification.

Methods of producing steel. Steel is made from pig iron by burning out the carbon, silicon, manganese, and other impurities, then restoring the carbon content to the desired percentage, and in the case of alloy steels, adding the desired additional elements. The three main processes of producing steel are: (a) the *Bessemer method*, (b) the *open-hearth method*, and (c) the *electric-furnace method*.

Bessemer and open-hearth steels do not differ materially in composition and properties. Bessemer steel was formerly used for all rolled products, such as plates, rails, shafting, and structural shapes. Open-hearth steel at present has largely superseded Bessemer steel for all rolled products and forgings because of its lower production cost.

[4] *SAE Handbook*, published each year with changes and additions.

[5] Data on chemical composition and mechanical properties of AISI steels are given in Lionel S. Marks, ed., *Mechanical Engineers' Handbook*, 5th ed. (New York: McGraw-Hill Book Company, Inc., 1951), pp. 540–54.

[6] *Ibid.*, pp. 565, 582–84.

Electric furnaces are of the induction type and the arc type. The induction furnace produces steel similar in quality to crucible steel obtained from a charge of pig iron and pure scrap steel. The arc furnace produces a high-grade steel from a charge of steel scrap covered with a slag containing a high percentage of lime. The reaction eliminates the impurities, and the proper composition is obtained by introducing the necessary alloys.

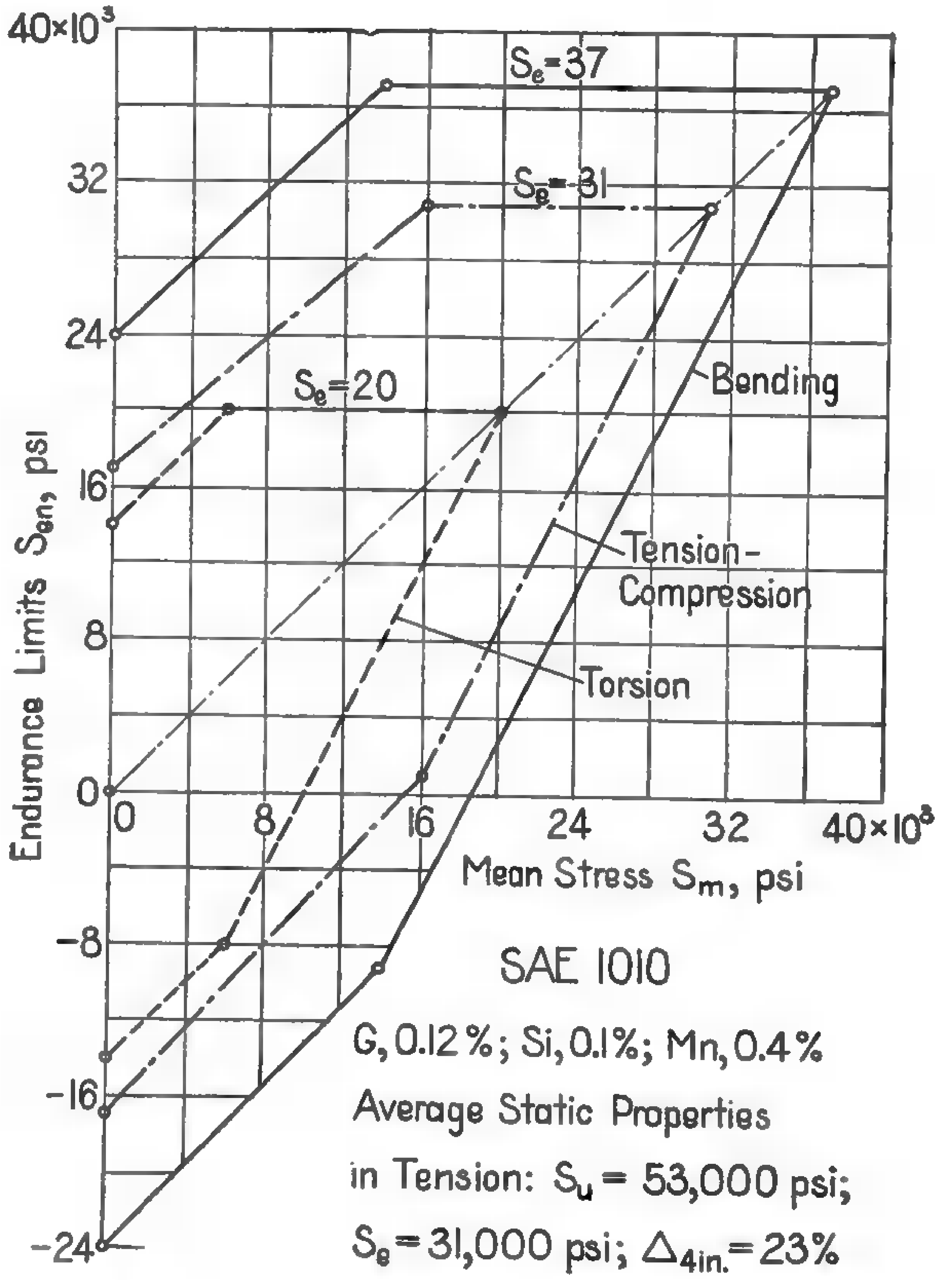


FIG. 4–2. Endurance diagram for SAE 1010 steel.

The main advantages of electric furnaces are the purity of the product and the ease of controlling its composition. Their disadvantage is a higher production cost.

Methods of manufacturing. Regardless of the process by which steel products are originally obtained, they can be divided into hot-rolled bars, beams, and plates; hot-rolled ingots used for forgings; cold-rolled bars; and castings.

Cold-rolled steel bars and strips are rolled hot to approximately the required size. The surface is then cleaned, usually by chemical means, and

the material is rolled cold to very accurately gaged dimensions. The characteristics of cold-rolled bars are: (a) The surface is hard, smooth, and bright. (b) The dimensions are very accurate. (c) The elastic limit and ultimate strength are increased, but the ductility is decreased. (d) Inner stresses are set up in the surfaces that cause a twisting or warping of the bar if its skin is removed, especially on one side only, as by cutting a keyway.

Composition. In commercial rolled and forged steels the carbon content varies from 0.05 to 1.50 per cent; the manganese content is from 0.25 to 0.90 per cent; silicon, from 0.08 to 0.16 per cent; phosphorus, from 0.06 to 0.13 per cent; and sulfur, from 0.05 to 0.16 per cent.

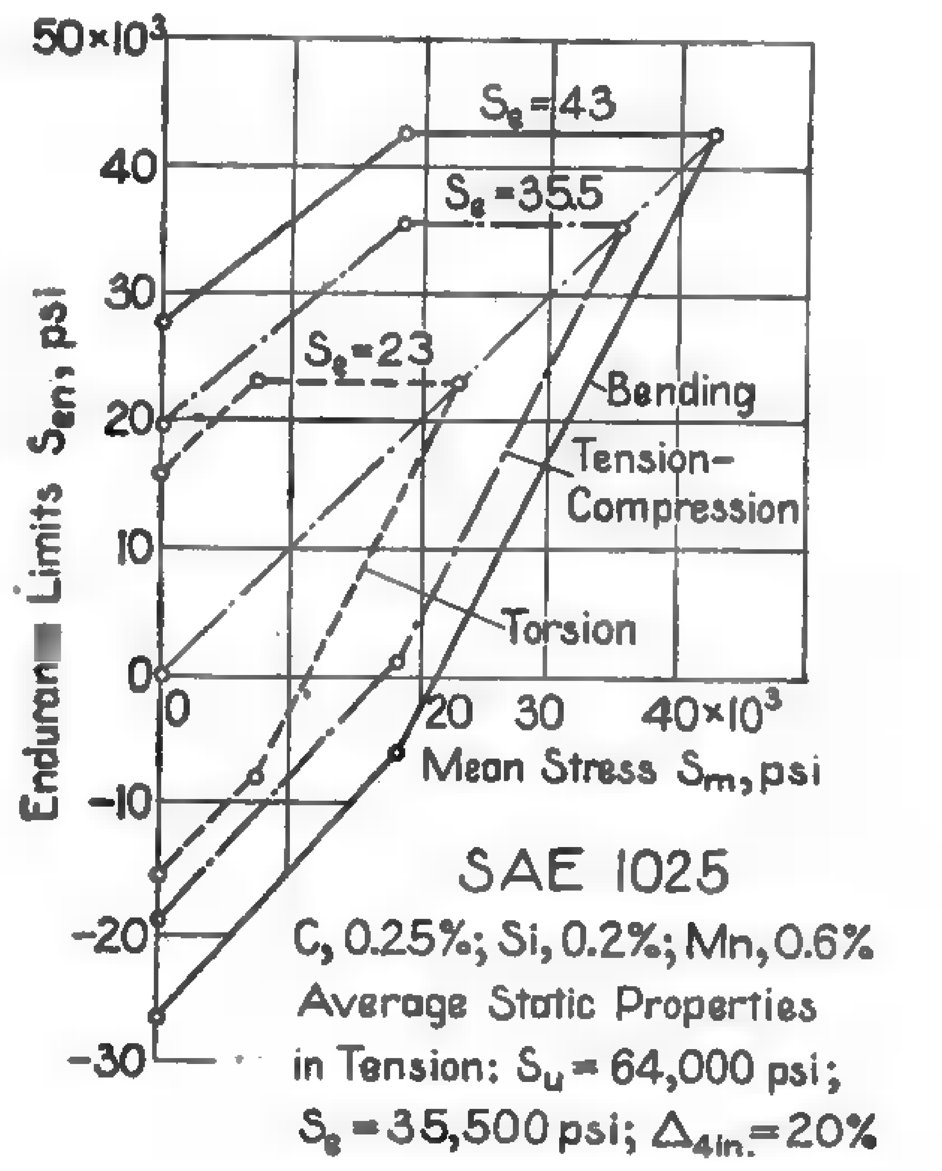


FIG. 4-3. Endurance diagram for SAE 1025 steel.

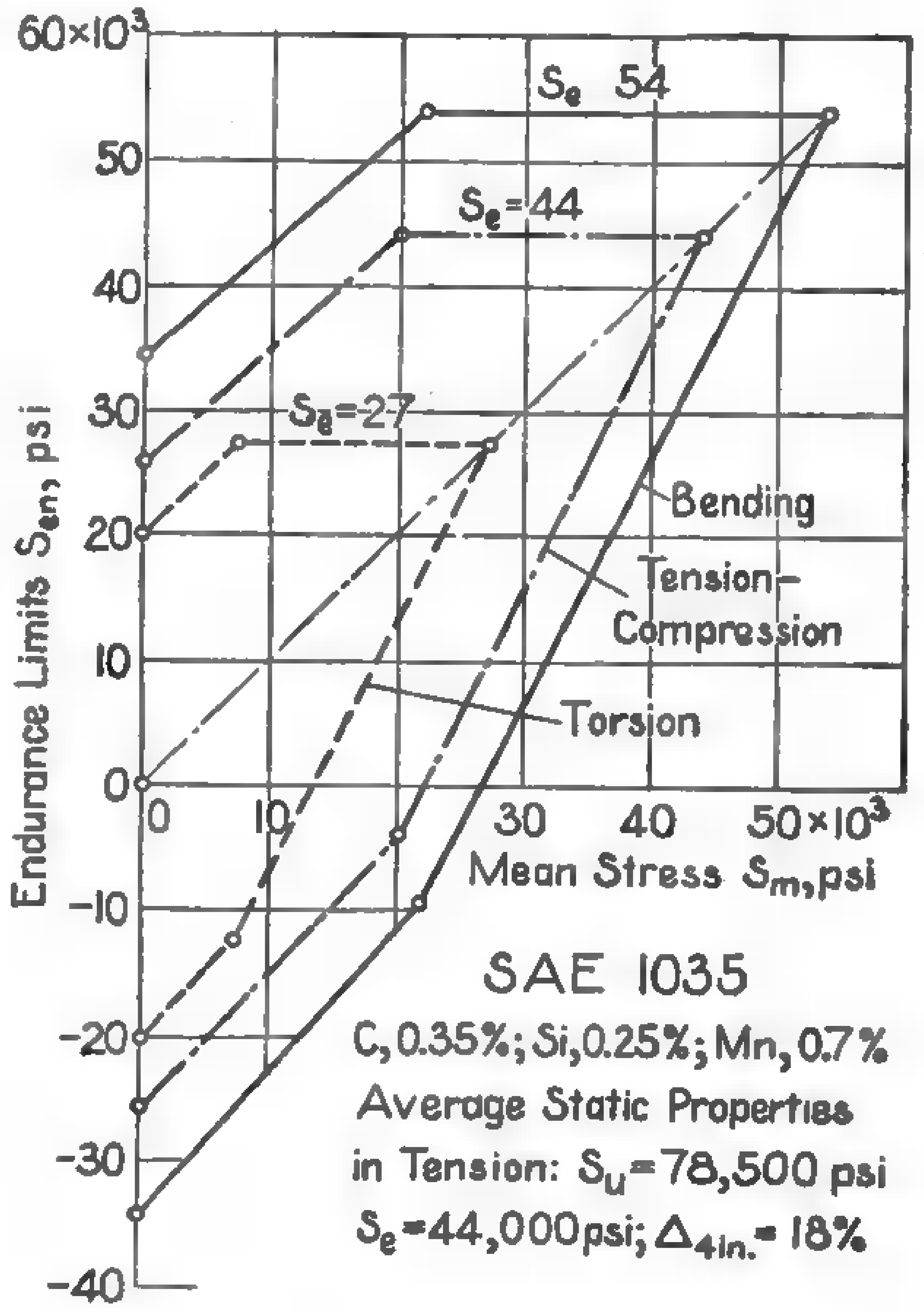


FIG. 4-4. Endurance diagram for SAE 1035 steel.

Carbon content. As may be seen from Table 4-2 and Figs. 4-2 to 4-6,[7] an increase of the carbon content of steel raises its ultimate strength, elastic limit, endurance limit, and hardness, both in the annealed state and the heat-treated state. At the same time, an increase of the carbon content decreases the ductility, as is indicated by the lowering of the percentage of elongation.

Carbon steels can be subdivided into three main groups: (a) low-carbon steels, having a carbon content of 0.05 to 0.25 per cent and suitable where only moderate strength but considerable ductility is required; (b) machinery steels, which have a carbon content of 0.30 to 0.55 per cent and can be heat-treated to develop high strength; and (c) high-carbon steels, which contain

[7] Supplements to *Zeitschrift Verein Deutscher Ingenieure*, Vol. 77, No. 42 (October 21, 1933), p. 3, and Vol. 77, No. 50 (December 16, 1933), p. 1.

0.60 to 1.30 per cent and are widely used for tools and springs, mostly with heat treatment.

Steel castings. Steel castings are used for parts that require strength and can be molded to shape so that machining is required only on surfaces where the casting is assembled with other machined parts. There are several classes of commercial steel castings, which differ in carbon content and in alloy content. However, castings of medium-carbon steel, with a carbon content of 0.25 to 0.50 per cent, represent the bulk of the steel-casting output, and this class is considered the regular-grade product.

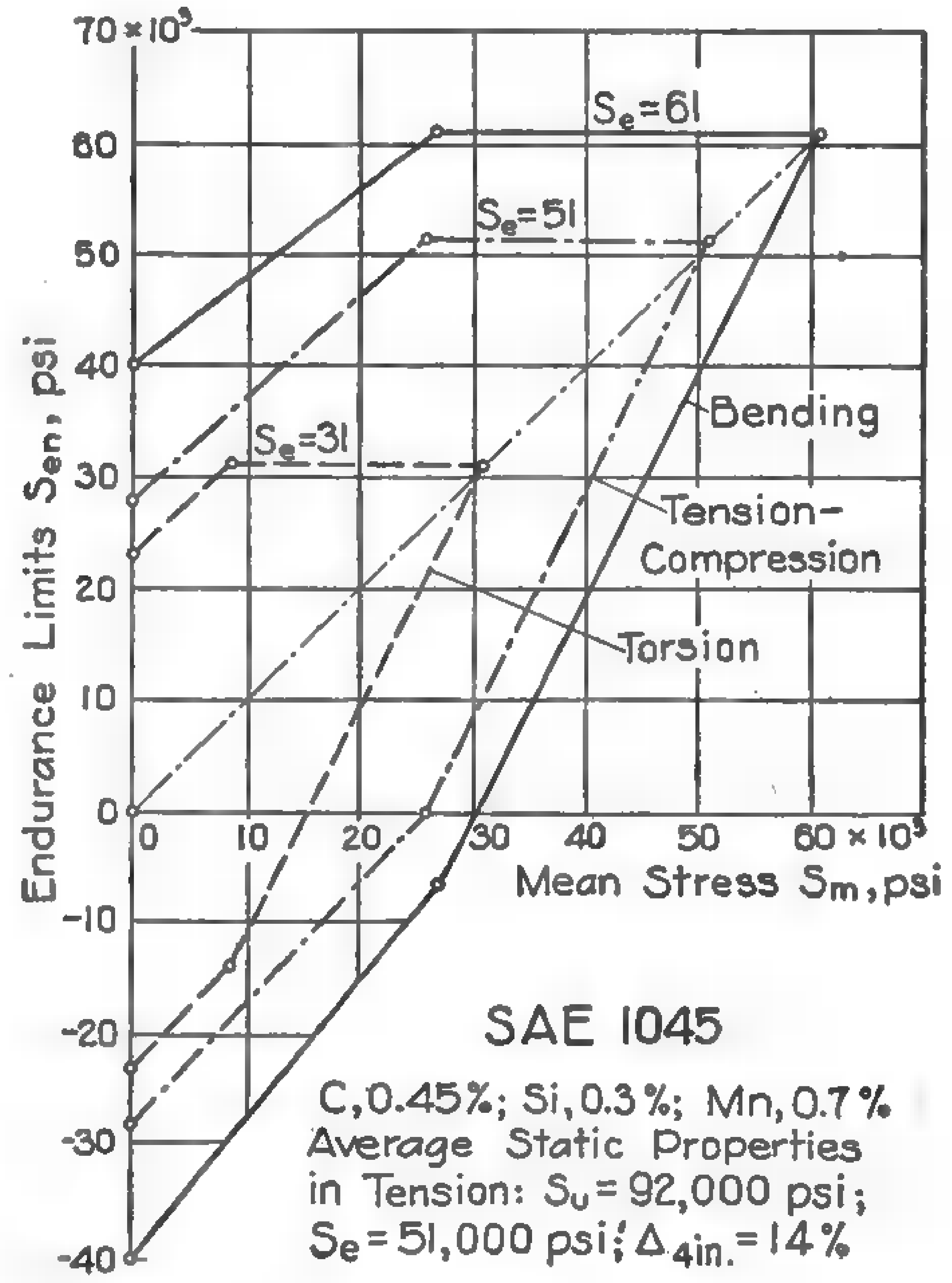


FIG. 4-5. Endurance diagram for SAE 1045 steel.

Steel in the cast form does not differ much in its properties from rolled and forged steel. In comparison with gray cast iron, steel castings have higher strength, much higher ductility, and greater toughness. However, they cannot be used for intricate shapes because of lower fluidity in the molten state. They are also more expensive. They are used practically in every industry, mostly for medium-weight and heavy parts where strength is one of the main requirements.

4-7. Heat-treatment of carbon steel. The elastic limit, ultimate strength, and hardness of carbon steel can be changed by heating the steel and then allowing it to cool rapidly or slowly. The various processes that may be used are the following:

TABLE 4-2

MECHANICAL PROPERTIES OF STEELS AND WROUGHT IRONS

No.	Material: Name	Material: Characteristic	Specific Weight (lb per cu in.)	Ultimate Tensile Strength (kpsi)	Elastic Limit*: Tension (kpsi)	Elastic Limit*: Compression (kpsi)	Elastic Limit*: Shear (kpsi)	Modulus of Elasticity: Direct, or Young's (kpsi)	Modulus of Elasticity: Transverse, or Shear (kpsi)	Endurance Limit, Bending (kpsi)	Elongation in 2 In. (%)	Hardness Minimum: Brinell (Bhn)	Hardness Minimum: Rockwell B	Hardness Minimum: Rockwell C
1	Steel casting, 0.20% C (SAE 0022)	Annealed	0.28	60	25	33	15	29,000	11,200	24	30	120	70	..
2	Steel casting, 0.30% C (SAE 0030)	Annealed	0.28	72	30	39	17	29,000	11,200	29	27	140	78	..
[illegible]	Steel casting, 0.40% C (SAE 0050)	Annealed	0.28	80	32	43	20	29,000	11,600	32	22	160	84	2
4	Alloy-steel casting (SAE 090, ASTM A-142)	Normalized	0.28	90	60	60	36	29,000	11,300	...	20	187	...	10
5	Stainless steel: C, 0.10; Cr, 12; Ni, 1	Drawn at 1,400 F	0.281	90	55	55	33	29,000	11,200	40	..	180	89	8
		Drawn at 800 F	0.281	190	130	130	80	29,000	11,200	70	..	380	112	40
[illegible]	Stainless steel: C, 0.10; Mn, 0.4; Si, 0.35; Cr, 12; Ni, 0.6	Quench 1,750 F, draw 1,200 F	0.283	105	60	60	36	30,000	12,000	40	..	200	94	14
7	Stainless steel, SAE 30905	Nonhardenable	0.282	96	48	48	30	30,000	12,000	40	24	190	92	11
8	Carbon steel, SAE 1010	Hot-rolled	0.282	54	31	31	20	30,300	11,700	24	36	110	65	..
9	Carbon steel, SAE 1020	Hot-rolled	0.282	62	35	35	22	30,200	11,600	26	30	125	71	..
10	Carbon steel, SAE 1030	Soft	0.282	75	42	42	26	30,000	11,500	32	26	150	81	..
[illegible]	Carbon steel, SAE 1040	Annealed	0.282	90	50	50	30	29,800	11,400	37	22	180	89	[illegible]
12	Carbon steel, SAE 1050	Annealed	0.282	95	52	52	35	29,700	11,400	42	20	190	92	11
13	Carbon steel, SAE 1095	Annealed	0.282	120	60	60	36	29,700	11,400	46	20	240	100	23
		Drawn at 900 F	0.282	150	80	80	50	29,700	11,400	60	16	300	107	32
14	Carbon steel, SAE 1120	Screw stock	0.282	62	34	34	22	30,200	12,600	26	20	125	72	..
15	Nickel steel, SAE 2320	Annealed	0.282	70	45	45	27	29,700	12,000	40	29	140	78	..
		Drawn at 1,000 F	0.282	120	80	80	50	29,700	12,000	60	25	260	103	26
16	Nickel steel, SAE 2340	Annealed	0.282	95	55	55	32	30,000	12,100	45	26	190	32	11
		Drawn at 1,000 F	0.282	120	95	100	60	30,000	12,100	50	22	220	96	19
17	Cr-Ni steel, SAE 3140	Not treated	0.282	85	60	60	36	30,500	12,500	50	26	170	87	6
		Drawn at 1,000 F	0.282	155	95	100	57	30,500	12,500	80	17	270	104	28
18	Cr-Ni steel, SAE 3240	Not treated	0.282	110	80	80	48	30,500	12,500	50	26	235	99	22
		Drawn at 1,000 F	0.282	160	120	140	72	30,500	12,500	80	19	325	109	35
19	Cr-V steel, SAE 6150	Oil-temper, draw 800 F	0.282	200	170	190	100	31,000	13,000	80	13	390	112	41
20	Cr-Ni-V steel	Oil-tempered	0.282	160	130	130	80	30,500	12,500	70	..	340	109	36
[illegible]	Nitralloy steel	Quench, draw 1,000 F	0.279	125	90	120	55	29,000	11,600	80	13	400	113	42
22	Wrought iron (A41-30)†	Double-refined	0.277	47	26	24	16	27,000	10,000	20	25	100	60	..
[illegible]	Armco ingot iron: Fe, 99.94%	Hot-rolled	0.286	44	25	25	15	30,000	12,000	24	30	100	60	..

* For sections ½ in. in diameter; gradually decrease with size. † ASTM.

Annealing consists in heating steel to a certain temperature and cooling it by a relatively slow process. Annealing may be used to remove stresses that are produced in forgings and castings, to refine the crystalline structure of steel castings, or to alter ductility or toughness.

Normalizing is a special kind of annealing in which the material is heated to a temperature above the critical range and is subsequently cooled to a temperature below that range, in still air at room temperature.

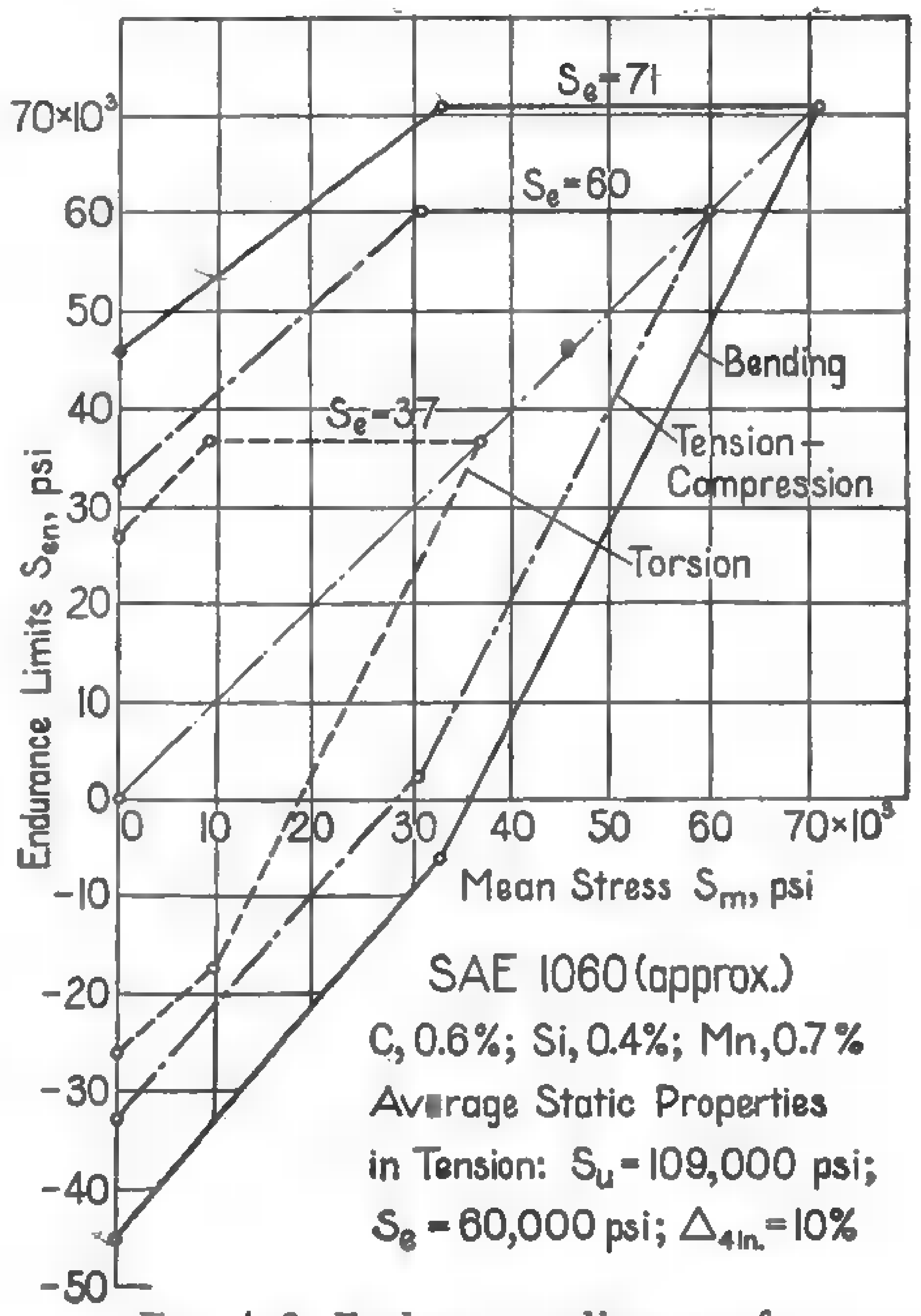


FIG. 4-6. Endurance diagram for SAE 1060 steel.

Hardening consists in heating steel to a temperature above its critical range and cooling it suddenly by *quenching* it in water, oil, or some other cooling medium that absorbs heat rapidly. Quenching increases the hardness of steel having a carbon content of 0.20 per cent or higher. It also raises the elastic limit and ultimate strength and reduces the ductility. However, it induces internal stresses, and the metal is apt to become brittle.

Drawing, also called *tempering*, consists in reheating hardened or quenched steel to a temperature below the critical range in order to remove the internal stresses and restore some of its ductility. The elastic limit and ultimate strength are slightly reduced by drawing, but they are still higher than they were before hardening.

Carburizing is a process for increasing the carbon content of soft steel by heating it below its melting point in contact with carbonaceous material, such as charcoal, leather, or barium carbonate.

Casehardening consists in carburizing a metal product and subsequently heating and quenching it to harden all or part of its surface. The *case* is that outside portion of a piece in which the carbon content is increased by carburizing; the *core* is the inner portion of the piece in which the carbon content has not been markedly increased. Casehardening is applied to soft steels with a carbon content of 0.20 per cent or less which cannot be hardened by simple heating and quenching.

Cyaniding is hardening of all or part of the surface of a steel piece by heating it at a suitable temperature in contact with a cyanide salt and then quenching it. Cyaniding gives a thin but very hard case in a very short time.

Nitriding is the introduction of nitrogen into the outer surface of a part made of any one of several special steel alloys called *nitralloys.* The treatment consists in heating the part to a temperature of about 1,040 F inside a chamber through which a stream of ammonia gas is passed. The part is rough-machined before being nitrided, and is then heated to the nitriding temperature without ammonia, in order to produce whatever distortion may occur. After that the part is finish-machined and nitrided. The nitrided surface has a hardness from 730 to 1,100 Bhn and resists corrosion. The hardness is not lost when the nitrided surface is heated up to 1,000 F. Nitriding also raises the endurance limit of the steel.

Flame hardening is a process in which a steel part is heated locally above the critical temperature and then quenched. The depth of the flame-hardened layer can be varied from $\frac{1}{16}$ in. to about $\frac{1}{8}$ in., the exact depth depending on the service requirements. Since flame hardening produces only a very small distortion, this process is useful for hardening surfaces of large steel parts. After being quenched the piece must be stress-relieved by tempering; a temperature of 400 F is usually sufficient. Plain carbon steels with a carbon content of 0.35 to 0.70 per cent are best suited for this method. Flame hardening may be used with castings, forgings, and rolled sections, irrespective of their size. It is often applied to gear teeth and cams.

Local hardening is similar to flame hardening, but the heating is done by an electric current. High-frequency induction heating is well adapted for surface-hardening of cylindrical parts. Crankshaft journals and crankpins are hardened in this manner by the *Tocco process.* The extent of the heated zone can be so closely controlled that the fillets will remain soft while the bearing surface is hardened. Such control reduces the danger of failure by progressive fracture.

SAE heat treatments. The Society of Automotive Engineers has worked out detailed specifications for steels used for practically all purposes. These specifications can be divided into two main groups: casehardening heat treatments I to V, and hardening-and-toughening heat treatments VI to VIII. The various heat treatments in the two main groups differ by the number of reheatings and quenchings found necessary for obtaining various

stages of refinement of the stock. The casehardening specifications apply to steels with a carbon content of 0.20 per cent or less. They can be subdivided also into two groups. In the first group, I to III, carburizing is followed by quenching; in the second group, IV and V, carburizing is followed by cooling in the box, and the other operations then follow. Specifications VI to VIII apply to steels with a carbon content of 0.20 or over. Some steels, such as SAE 2320 nickel steel, may be treated either by method III to V or by method VI or VII. More details may be found in the SAE Handbook.

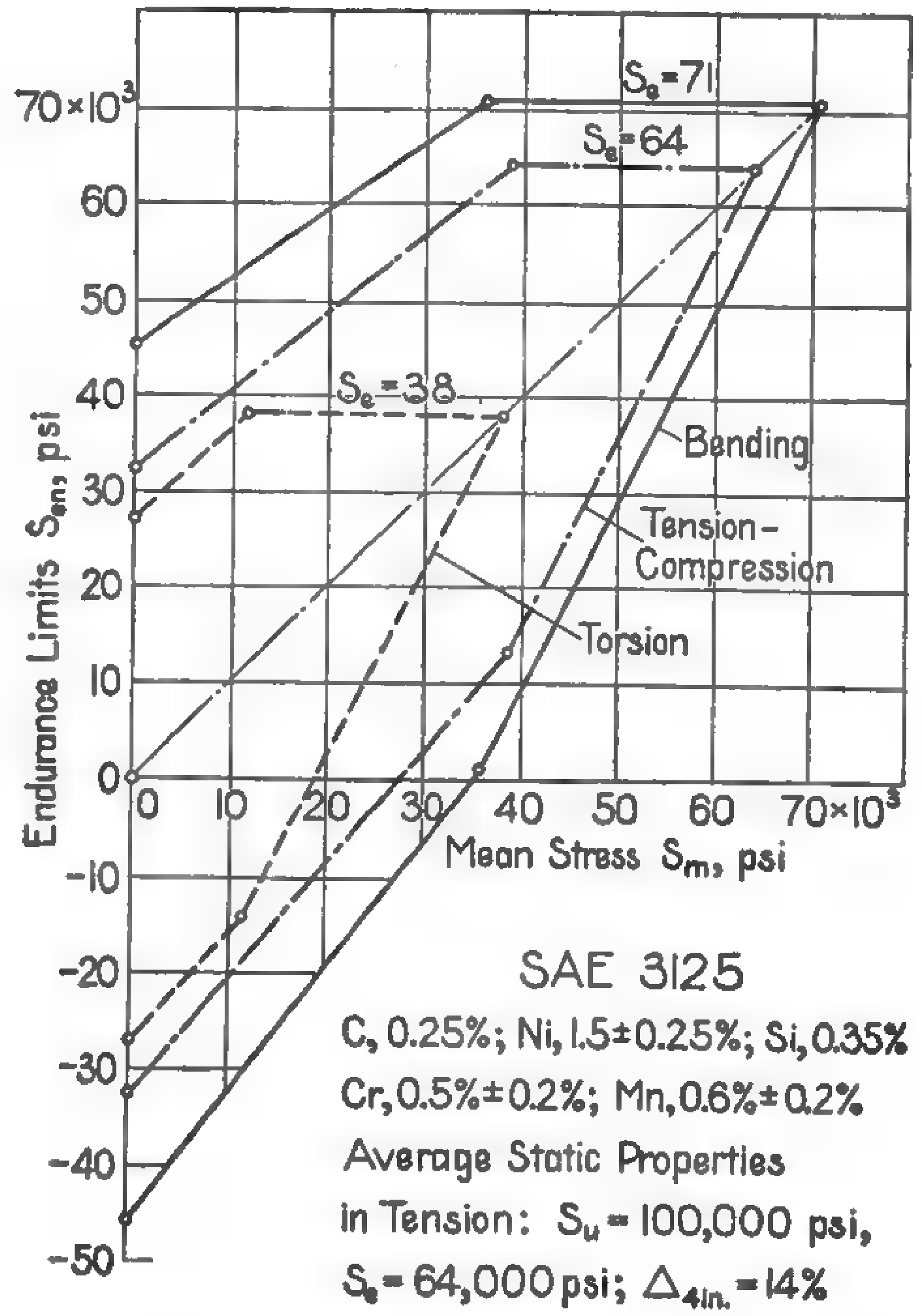


FIG. 4-7. Endurance diagram for SAE 3125 steel.

4-8. Alloy steels. The elements most often alloyed with steel, singly or two or more together, are—besides carbon—nickel, chromium, silicon, manganese, molybdenum, vanadium, tungsten, and aluminum.

Carbon increases the hardness and strength of steel but decreases its ductility.

Nickel increases the hardness, toughness, corrosion resistance, and (up to a 12 per cent content) also the elastic limit of steel, but decreases its ductility slightly.[8] The ratio of elastic limit to ultimate strength increases

[8] J. W. Sand, "Nickel in Steel," *Metals Handbook* (Cleveland: The American Society for Metals, 1948), p. 473.

gradually with increasing nickel content up to about 4 per cent, and with a further increase it begins to decrease.

Nickel steels are the most important of the commercial alloy steels. Their nickel content varies from 0.50 to 5.25 per cent, while the usual carbon content ranges from 0.30 to 0.60 per cent. The chief uses of nickel steels are for structural shapes, rails, steel castings, engine forgings, and automotive parts.

Chromium increases the elastic limit and hardness of steel. It is added either alone or in conjunction with nickel or vanadium. It also increases the resistance to corrosion.

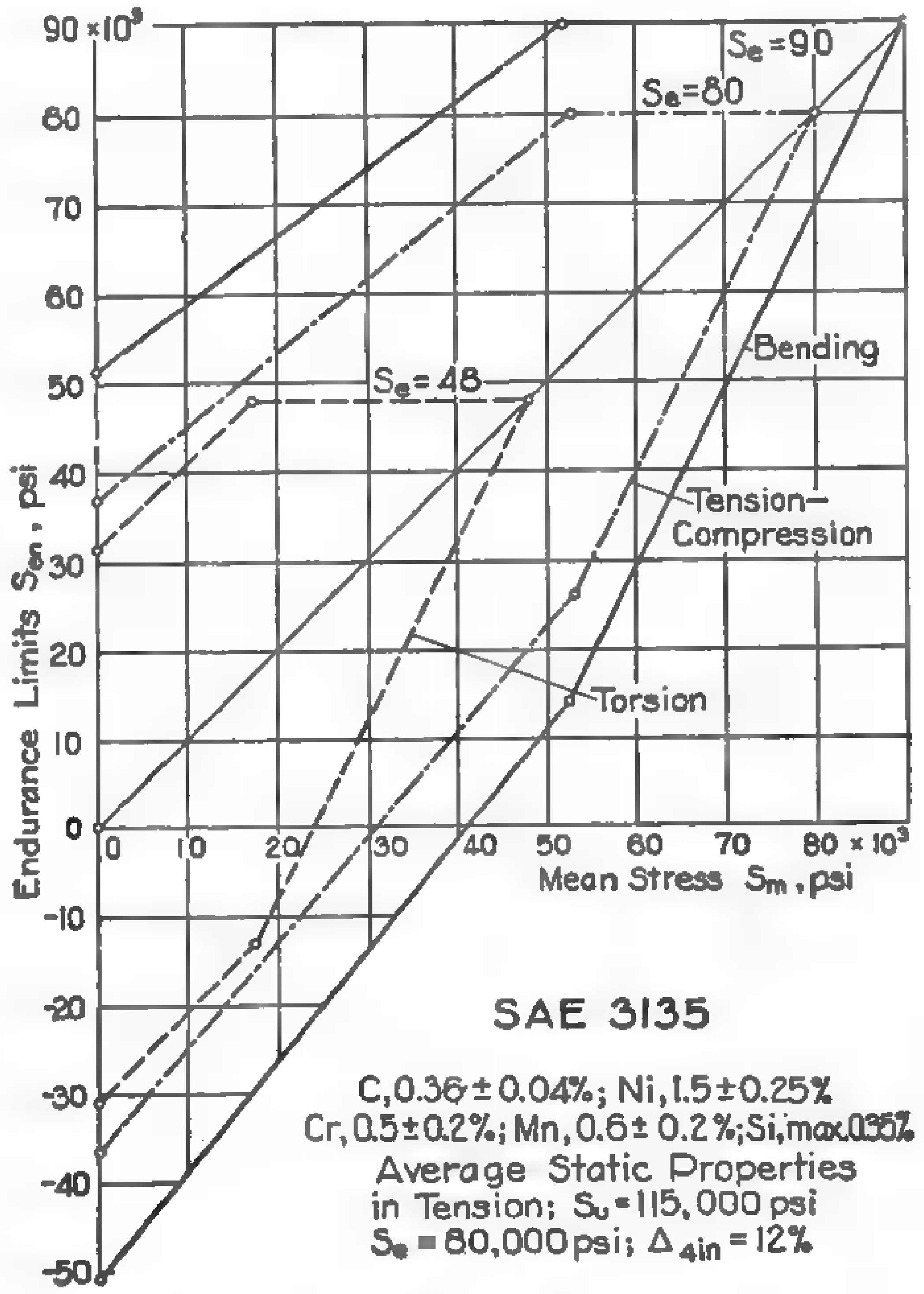


FIG. 4–8. Endurance diagram for SAE 3135 steel.

Chrome steels are used for ball and roller bearings, gears, and other machine and automotive parts where hardness is essential. Chrome steels are always heat-treated.

Steels with a chromium content of 11 per cent or more—in some cases over 20 per cent—are known as *stainless steels* because of their resistance to corrosion. Steel No. 6, Table 4–2, is used for bolts subjected to high temperatures and for large studs in steam turbines.[9] At 800 F its elastic limit is still 27,000 psi.

[9] W. J. Kerr, *et al.*, *Symposium on Effects of Temperature of Metals* (New York: McGraw-Hill Book Company, Inc., 1932), p. 37.

Chrome-nickel steels combine high strength with great hardness. They are produced with a chromium content from 0.6 to 1.2 per cent, a nickel content from 1.5 to 3.5 per cent, and various carbon contents. Steels with a carbon content up to 0.2 per cent are used only when casehardened; those having a content of 0.25 to 0.6 per cent are used for structural parts of automobiles; and those having a content of 0.5 per cent and more are used for gears and automotive parts in place of plain chrome steel.

Physical and mechanical properties of chrome-nickel steels corresponding to various SAE specifications are given in Figs. 4–7 to 4–10.[10]

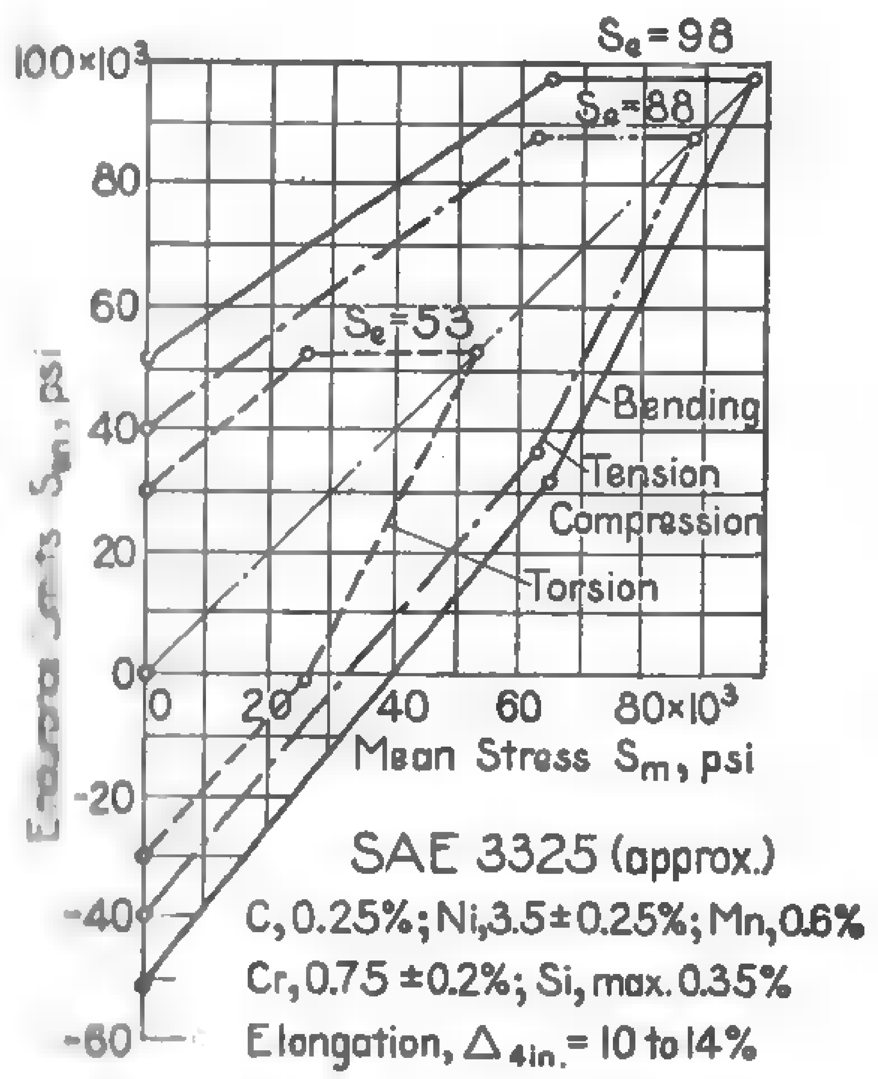


FIG. 4–9. Endurance diagram for SAE 3325 steel.

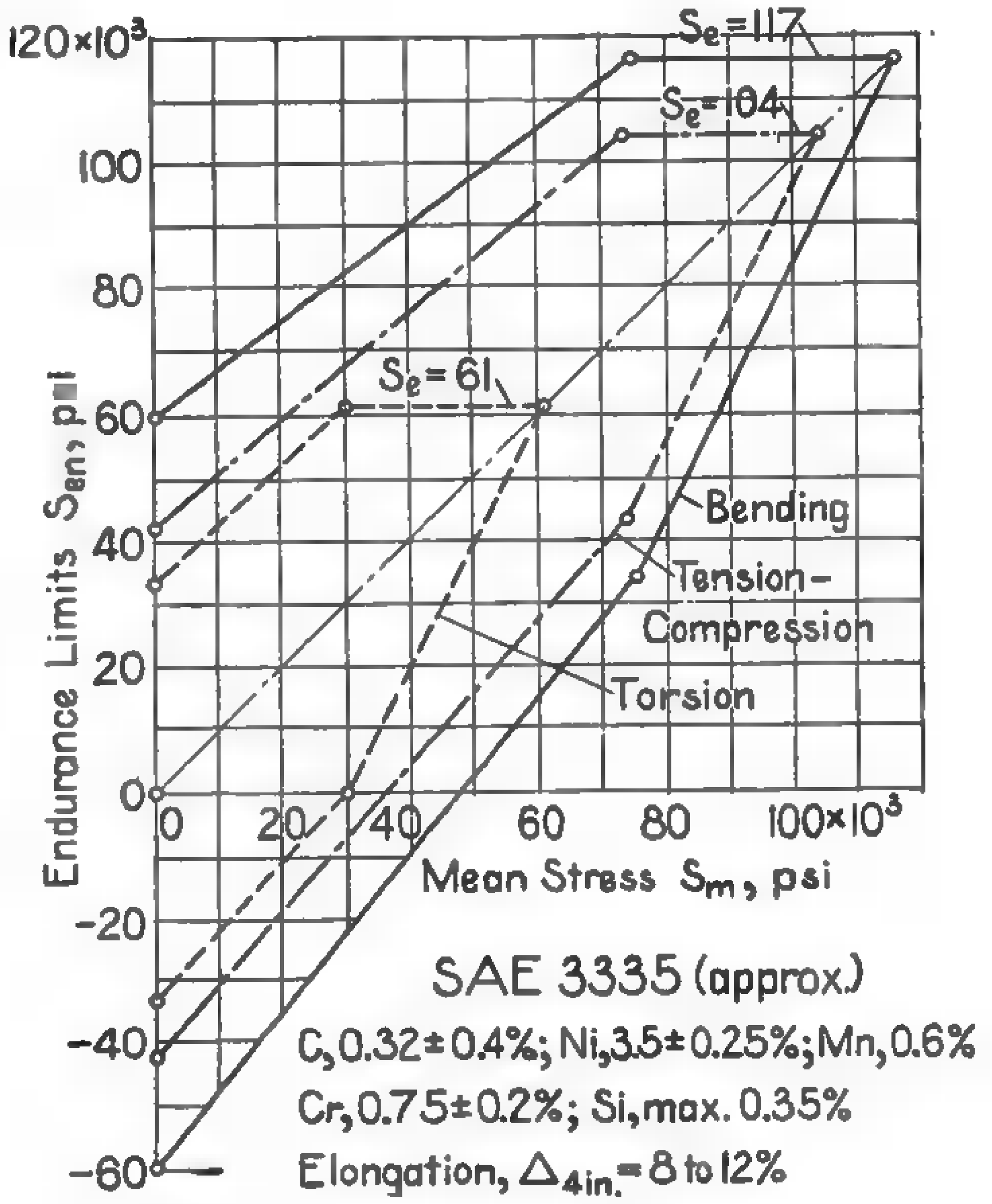


FIG. 4–10. Endurance diagram for SAE 3335 steel.

Silchrome steel contains 0.5 per cent C, 0.30 per cent Mn, 3.50 per cent Si, 8.0 per cent Cr, not more than 0.02 per cent P, and not more than 0.02 per cent S. This steel is used for exhaust valves in internal combustion engines. It is tough, is hard (even at high temperatures), and has a high resistance to scaling and corrosion. Its drawback is the difficulty of machining it.

Silicon-manganese steel contains 0.45 to 0.65 per cent C, 0.60 to 0.90 per cent Mn, and 1.8 to 2.0 per cent Si. It has a high elastic limit and is used extensively for springs and gears. This steel must be given a suitable heat treatment, the type depending on the kind of service for which the steel is intended.

[10] Supplements to *Z. VDI*, Vol. 78, No. 7 (February 17, 1934), p. 1, and Vol. 78, No. 12 (March 24, 1934) p. 1.

Vanadium added to carbon steel, nickel steel, or chrome steel, even in such a small amount as 0.15 to 0.25 per cent, increases the elastic limit and resilience. *Chrome-vanadium steel* is used especially for automotive springs. Steel that has an average composition of 0.47 per cent C, 0.84 per cent Mn, 0.032 per cent S, 0.026 per cent P, 0.10 per cent Si, 1.06 per cent Cr, and 0.15 per cent V and has undergone a special heat treatment and drawing has the following properties: If drawn to 500 F, the tensile strength is 380,000 psi and the elastic limit is 250,000 psi; if drawn to 900 F, the values are 200,000 and 185,000 psi, respectively.

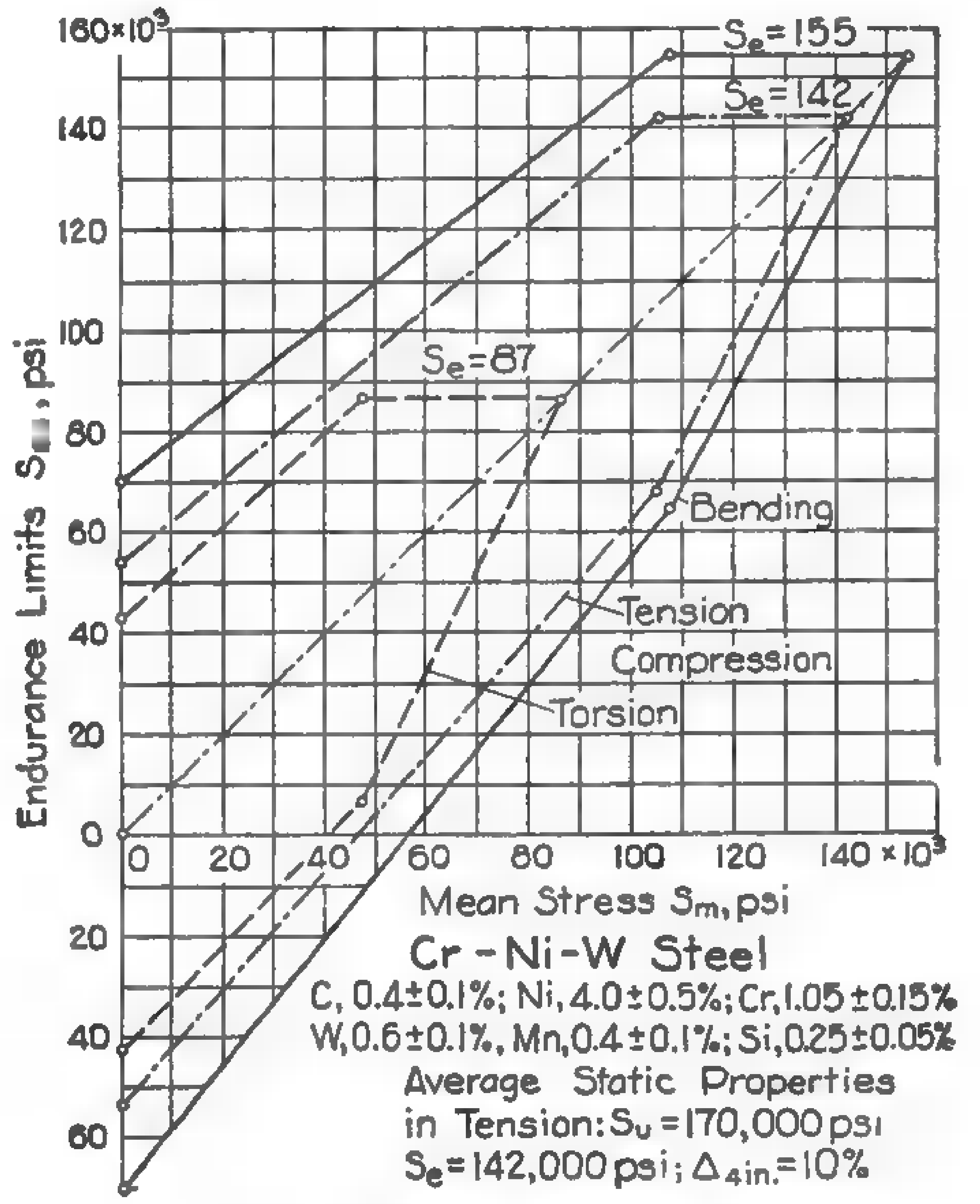


FIG 4–11. Endurance diagram for a special crankshaft steel.

Tungsten and *molybdenum* are sometimes added to machine steels for strength and toughness, but their chief use is in the production of high-speed cutting tools. Physical and mechanical properties of a chrome-nickel-tungsten steel, particularly suitable for highly stressed crankshafts, are given in Fig. 4–11.[11]

Aluminum in small amounts, about 0.10 per cent, increases the fluidity of steel. Aluminum is also added to a ferrous alloy used as stock for nitriding. This alloy, known as *nitralloy*, has an approximate composition of 0.2 to 0.4 per cent C 0.5 per cent Mn, 0.2 to 0.5 per cent Si, 0.5 to 0.6 per cent Ni, 1.5 to 1.7 per cent Cr, 0.2 per cent Mo, 0.9 to 1.3 per cent Al, and about 95.5 to 96 per cent Fe.

[11] *Ibid.*

4–9. Mechanical properties of steel. The ultimate tensile strength of steel is a very important general characteristic. As shown in Fig. 4–12, it determines within sufficiently close limits the hardness of either carbon or alloy steel that has not been heat-treated.

The ratio S_e/S_u of the elastic limit or yield point in tension to the tensile strength also increases. However, it is expressed by a rather wide band, as shown in Fig. 4–12.

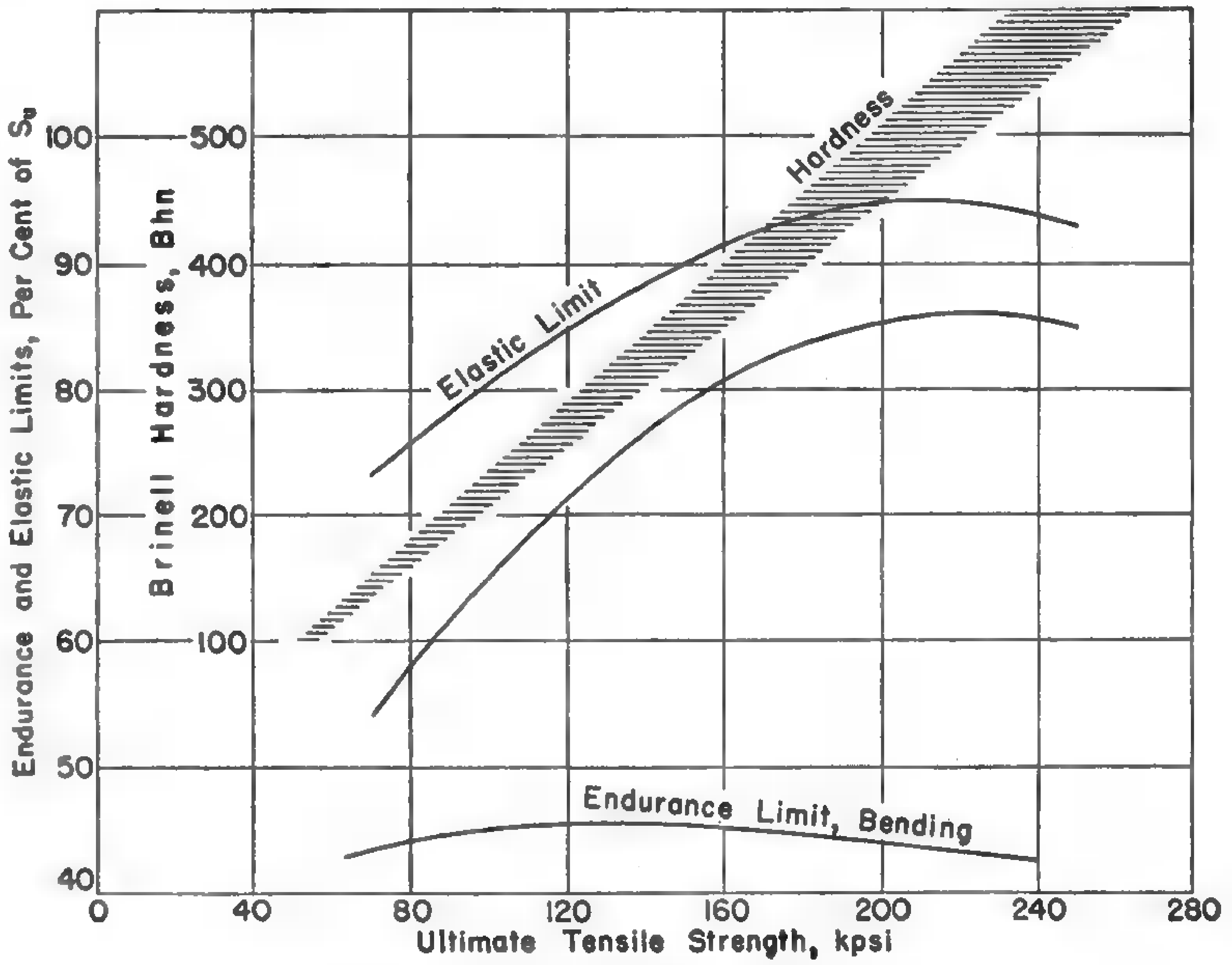


FIG. 4–12. Relation between mechanical properties and tensile strength of steel.

The endurance limit in bending for most steels is equal to about 0.45 S_u. However, this factor decreases slightly for high-strength, hard steels.

Average mechanical properties of various steels are given in Table 4–2.

4–10. Wrought iron. Wrought iron is a malleable ferrous metal which has been produced from a pasty condition. It contains a small proportion of manganese, 0.07 to 0.30 per cent. When single-refined it has an ultimate tensile strength from 40,000 to 45,000 psi; when double-refined its strength ranges from 47,000 to 54,000 psi. It is more resistant to corrosion than steel. Being almost pure iron, it cannot be hardened. It is used in the manufacture of pipes, boiler tubes, rivets, staybolts, and crucible steel; for general forging purposes, especially where welding is required; and for parts of electrical machinery. At present wrought iron is not used so extensively, steel having taken its place to a great extent. Table 4–2 contains data pertaining to its mechanical properties.

Armco ingot iron is made in a basic open-hearth furnace similar to that used in producing steel, but it is purified to a much greater degree by the addition of a very pure iron ore. Its composition is especially uniform, and freedom from inclusions insures mechanical uniformity. Its chief use in machinery is for boilers, tanks, large pipes, tubing, and welding rods.

4–11. Copper alloys. Copper used in machine parts is alloyed with zinc, tin, lead, aluminum, and nickel. Copper-zinc alloys are as a rule termed *brasses*, although some brasses are called *bronzes*. A *true bronze* is an alloy of copper and tin only, the tin content being 4 to 25 per cent. Copper alloys with tin, lead, or aluminum usually are also termed bronzes.

Industrial brass may be divided into two classes: cast brass and wrought brass.

Cast brass has a zinc content from 30 to 40 per cent, the most desirable proportion being 35 per cent. A small percentage of tin is added to increase the hardness, and 1 to 2 per cent of lead gives good machinability. This alloy is commonly called *yellow brass*.

Wrought brass has a zinc content of 15 to 44 per cent. Alloys of copper and zinc, with a zinc content of 38 to 44 per cent, become plastic when heated to redness. The properties of these brasses can be altered by heat treatment. *Muntz metal*, composed of 60 per cent copper and 40 per cent zinc, is rolled at a temperature above 1,100 F and is hardened by quenching.

Modified hot-working brass is brass with small amounts of tin and iron. *Tobin bronze* (SAE 73) is practically the same as *naval brass*. It consists of 59 to 62 per cent Cu, 0.5 to 1.5 per cent Sn, not more than 0.10 per cent Fe, and not more than 0.3 per cent P, the remainder being zinc. It is very resistant to corrosion.

Delta metal is similar in composition and properties to naval brass but it contains from 1 to 3 per cent iron and has a tensile strength of 44,000 psi.

Cold-working brass is an alloy of copper and zinc with a zinc content of 28 to 38 per cent. It has considerable strength and great ductility, and it can be worked cold to such forms as rods, tubes, and wires. The addition of a small percentage of tin increases its hardness and resistance to corrosion, and the addition of 1 to 2 per cent of lead increases its machinability.

Cold-rolled admiralty metal, consisting of 71 per cent Cu, 28 per cent Zn, and 1 per cent Sn, has a tensile strength of 50,000 psi to 90,000 psi, the actual value depending upon its hardness.

Red brasses, so called because of their color, are composed of 76 to 88 per cent Cu, 4 to 16 per cent Zn, 2 to 6 per cent Sn, and 1.5 to 6 per cent Pb. They are easily cast and machined.

Aluminum brass consists of 76 per cent Cu, 22 per cent Zn, and 2 per cent Al. Its tensile strength ranges from 40,000 to 50,000 psi, and its elastic limit is between 30,000 and 38,000 psi. Aluminum, when added to brass in the form of aluminized zinc, makes the brass flow freely, so that castings made from it have smooth surfaces and are free from blowholes. Such castings are used in marine service because of their resistance to impingement pitting.

Gun metal is copper combined with 8 to 11 per cent tin and sometimes small percentages of zinc, lead, and iron. It is hard and has a tensile strength of 30,000 psi; but its strength decreases very rapidly if the temperature exceeds 500 F. By proper heat treatment, with quenching above 930 F, this alloy, like other true bronzes, can be made stronger and more malleable.

Phosphor bronze in castings contains from 80 to 90 per cent Cu, from 20 to 10 per cent Sn, and up to 1 per cent P. Being hard and tough, it is particularly suitable for gears. In the wrought form it contains only 3 to 9 per cent tin and not over 0.5 per cent of phosphorus. It is hard, tough, and corrosion-resistant and is used for springs and wire.

Manganese bronze is in reality a brass, consisting of copper and zinc. Manganese is used only as a deoxidizer, and the finished product often contains only traces of it. The term bronze is applied generally because of the alloy's red color.

Manganese bronze has high strength, is ductile, and resists corrosion in salt water. Cast manganese bronze is used for such parts as ship propellers, heavy gears, and heavy-duty bearings. Manganese bronze can be wrought like Tobin bronze, and when drawn it has a tensile strength of 75,000 psi.

Super-strength bronze is also a special brass, similar to manganese bronze, in which iron and aluminum have been substituted in part for zinc. When sand-cast, it has a tensile strength of 110,000 to 115,000 psi and an elastic limit of 30,000 to 32,000 psi in tension and about 5 per cent less in compression. Heat treatment and forging or rolling do not change its properties materially.

Aluminum bronze is copper with 2 to 11 per cent aluminum. Its tensile strength increases, as the aluminum content becomes greater, from 30,000 to 95,000 psi. A 10 per cent aluminum bronze is used in cast and hot-worked condition for parts requiring high tensile strength and resistance to corrosion, wear, and alternating stresses. Its mechanical properties are improved by quenching from 1,650 F and reheating to 1,100 F. Iron up to 0.5 per cent increases the strength and hardness without reducing the ducility appreciably.

An alloy containing 11 per cent Al, 5 per cent Ni, 5 per cent Fe, and 79 per cent Cu has extreme hardness and wear resistance at high temperatures, a quality essential in aircraft-engine valve seats.

TABLE 4–3

MECHANICAL PROPERTIES OF COPPER ALLOYS

No.	Metal: Name	Metal: Characteristic	Specific Weight (lb per cu in.)	Ultimate Tensile Strength S_u (kpsi)	Elastic Limit: Tension (kpsi)	Elastic Limit: Compression (kpsi)	Elastic Limit: Shear (kpsi)	Modulus of Elasticity: Direct E (kpsi)	Modulus of Elasticity: Transverse G (kpsi)	Endurance Limit, Bending (kpsi)	Elongation in 2 In. (%)	Hardness Minimum: Brinell (Bhn)	Hardness Minimum: Rockwell B
1	Admiralty metal	Soft to hard	0.308	30–90	25.0	...	...	...	...	...	65–14	53–157	15–83
2	Aluminum brass	Free-flowing	0.306	40–50	32.0	...	...	13,000	5,000	...	17–52	86	49
3	Aluminum bronze	SAE 68	0.280	65	14.0	14.0	12.0	17,000	6,500	14.0	20	70–190	35–91
4	Ambrac	Soft	0.319	55	20.0	...	...	...	...	...	...	71	35
5	Ambrac	Hard	0.320	80	45.0	...	...	19,000	7,000	...	...	160	84
6	Brass, red	SAE 40	0.306	28	10.0	9.0	9.0	13,000	5,000	6.0	16	53	15
7	Brass, yellow	SAE 41, cast	0.306	25	10.0	9.0	9.0	12,500	4,500	7.0	20	52	15
8	Brass, yellow	SAE 41, rolled	0.307	45	25.0	20.0	15.0	12,800	6,000	12.0	...	90	54
9	Brass, yellow	SAE 41, hard-drawn	0.307	75	31.0	...	25.0	16,000	5,800	20.0	...	130	74
10	Bronze, bearing	SAE 62	0.306	30	11.0	9.0	7.0	12,500	5,000	9.0	14	60	22
11	Beryllium copper	Soft	0.315	60	7.0	...	...	18,500	7,500	...	...	100	60
12	Beryllium copper	Heat-treated	0.315	155	60.0	...	...	19,000	8,000	40.0	...	360	110
13	Bronze	SAE 64	0.313	25	12.0	9.0	8.0	14,000	5,500	8.0	8	70	35
14	Bronze, superstrong	Sand-cast	0.270	115	32.5	30.0	...	15,000	5,800	23.0	...	240	100
15	Copper	Soft	0.321	32	2.8	2.8	2.0	15,500	6,000	12.5	...	42	5
16	Copper	Cold-drawn	0.322	50	12.0	...	8.5	17,000	6,200	17.0	...	107	64
17	Gear bronze	SAE 65	0.315	35	18.0	20.0	13.0	13,500	5,300	12.0	10	100	60
18	Gun metal	Cast	0.318	40	12.0	12.0	10.0	26,000	9,500	...	...	60	12
19	Manganese bronze	SAE 43	0.280	65	25.0	...	22.0	16,000	...	16.0	25	90	54
20	Monel metal	Cast	0.318	72	25.0	22.0	23.0	25,000	9,000	20.0	...	100	60
21	Monel metal	Hot-rolled	0.323	85	30.0	26.0	24.0	25,500	9,200	30.0	...	120	70
22	Monel metal	Cold-rolled	0.322	120	70.0	60.0	40.0	25,500	9,200	50.0	20	145	80
[redacted]	K-Monel metal	Grade C	0.310	150	90.0	80.0	55.0	25,500	9,200	60.0	30	300	107
24	Muntz metal	Soft, cast	0.303	45	22.0	...	...	12,800	5,000	15.0	30	80	45
25	Muntz metal	Hard, rolled	0.303	70	45.0	...	...	12,800	5,000	25.0	30	158	54
[redacted]	Phosphor bronze	SAE 77, rolled	0.321	55	18.0	16.0	15.0	15,500	6,200	12.0	...	60	22
27	Tobin bronze	SAE 73 Hard	0.304	65	23.0	10.0	18.0	15,000	6,000	21.0	25	165	86
[redacted]	Tobin bronze	SAE 73 Soft	0.304	54	13.0	8.0	18.0	15,000	6,000	...	35	90	54

Monel metal is the most used of the copper-nickel alloys, all of which are very strong and highly resistant to corrosion. Monel metal is a natural alloy composed of 68 to 70 per cent Ni, 1.5 per cent Fe, and copper. It retains its strength at temperatures up to 1,100 F to a greater degree than most brasses and bronzes, and even steels.

Ambrac is another copper-nickel alloy, with properties similar to those of Monel metal. An alloy composed of 65 per cent Cu, 30 per cent Ni, and 5 per cent Zn, and either rolled or drawn, has a tensile strength of 65,000 psi when soft and 80,000 to 120,000 psi when hard.

Other copper alloys. The number of copper alloys adapted to special purposes is very great. One of the latest developments is *beryllium copper.* Alloys containing from 1.5 to 2.5 per cent beryllium take on exceptionally high characteristics of strength, endurance, and hardness through cold working and heat treatment. Thus the tensile strength of untreated beryllium copper in a soft state is about 60,000 psi; but its strength can be raised to 110,000 psi by cold working, to 145,000 psi by proper heat treatment, and to 175,000 psi by heat treatment after cold reduction. The main use of this alloy in machinery is for springs, particularly in internal combustion engines, high-duty gears, valve sleeves, and valve seats.

The mechanical properties of the most frequently used copper alloys are given in Table 4–3.

4–12. Bearing metals. Bearing metals may be divided into five classes. Each is named to correspond to the base, or main constituent, which makes up 50 per cent or more of the alloy. The bases are (1) copper, (2) tin, (3) lead, (4) zinc, and (5) aluminum.

Copper-base metals are stronger than those with tin, lead, or zinc bases. Also, being harder, they have a lower coefficient of friction. It is usually a misnomer to designate an alloy with a tin, lead, or zinc base as an anti-friction metal. The explanation of the use of the term lies in the fact that copper-base bearing metals are more likely to heat under abnormal operating conditions because they do not have enough plasticity. The presence of any irregularities of the journal or bearing, or of any foreign particles between them, therefore creates points of high specific pressures, which are absent in the softer alloys. The properties of some typical bearing alloys are given in Table 4–4.

Copper-base metals are used for bearings where heavy pressures and shock action occur, as in the bearings for connecting rods of explosion engines. The Brinell hardness number of *Nida bronze*, a drawn phosphor bronze, ranges from 200 when hard to 80 when annealed.

Copper-lead alloys, known as plastic bronze and Allan red metal do not

TABLE 4–4

PROPERTIES OF BEARING METALS

Metal	Percentages of Ingredients: Copper Cu	Lead Pb	Tin Sn	Miscellaneous	Compressive Elastic Limit (psi)	Hardness at 82 F (Bhn)	Class of Use
Nida bronze, Hard	91–92	...	8–9	P, 0.5	37,000	150–200	High shock load
Nida bronze, Annealed	91–92	...	8–9	P, 0.5	15,000	80–90	High shock load
Bronze, SAE 62	86–89	>0.2	9–11	Zn, 1–3	12,000	60	Shaft bearings
Phosphor bronze	80–82	9–10	9–11	P, 0.7	15,000	55–80	Shock load
Aluminum-nickel bronze*	80	[Al, 10]	[Ni, 5]	5	45,000	180	High shock load
Aluminum alloy 750	1.0	[Al, 91.5]	6.5	Ni, 1.5	8,500	35–50	Severe loads
Aluminum alloy XA80S	1.0	[Al, 90.5]	6.5	Ni, 0.5, Si, 1.5	15,000	40–55	General automotive
Plastic bronze	65	30	5	...	3,500	30	Crank-pin bearing
Allan red metal	50	50	...	...	2,500	13.5	Crank-pin bearing
Babbitt							
SAE 10	4–5	>0.35	90–92	Sb, 4–5	2,500	17	Light loads
SAE 12	2.75–3.25	25	60–62	Sb, 10.5	1,300	22	Moderate loads
SAE 14	...	75	10	Sb, 15	1,300	23	Crank-pin bearing
Parsons' white brass	2.5–5	>0.2	64–65	Zn, 33	12,000	18	Light load
Lumen bronze	10	...	[Zn, 86]	Al, 4	15,000	116	Light load, high speed
Asarcoloy, No. 7	[Cd, 98.5]	...	...	Ni, 1.3	37,000	33	Severe load

* *Nickel Topics*, Vol. 4, No. 1 (January, 1951), p. 2.

score or cut a journal, even if they become red-hot. Nor do they melt and run out, as do the babbitts.

Tin-base metals are commonly known as *babbitts*. Copper and antimony are added to increase the hardness of the alloy. For light bearings, lead may replace copper in order to lower the cost of the metal. Table 4–4 contains only three babbitts, but there are a great number which are adapted to various special operating conditions.

Parsons white brass is a tin-base alloy with a large amount of zinc. It is used for marine and automobile bearings. It is hard and tough; but being sluggish to pour, it cannot be cast into thin sections. After being cast it is peened to increase its resistance to wear.

Lead-base bearing metals contain antimony and a certain amount of tin, which diminishes the brittleness and increases the compressive strength of the alloy. The antifriction metals of this class are cheapest, and are satisfactory in many services. Various widely advertised brands of babbitt come within this class.

Zinc-base metals are used when a low coefficient of friction is essential. *Lumen bronze* can be cast in sand, is easily machined, and is used for crane,

motor, and pivot bearings. It is particularly suitable for high-speed bearings with low specific pressures.[12]

Aluminum alloys developed by Alcoa for bearings are of two types. One type, designated as alloy 750, is recommended for permanent-mold casting. The other, alloy XA80S, is obtainable in the form of flat-rolled sheets. The composition and mechanical properties of these alloys are given in Tables 4–4 and 4–5.[13] These aluminum alloys were originally developed for use in bearings of internal combustion engines, but they gradually found application for other heavy-duty services, including shoes of reciprocating crossheads. Aluminum bearings have long life, withstand high pressures, resist corrosion, and have high thermal conductivity and low cost of manufacture.

4–13. Powder metals. The commonly used powdered metals are copper, tin, and iron. They may be used singly or mixed in certain proportions. The powder, mixed with a volatile binder, is molded under high pressure. The molded piece is sintered and becomes a metal part with a desired density or porosity. A product of powdered metal can be made with such close tolerances that no machining is required, except possibly drilling or tapping of very small holes. Small and medium-size bushings made of powdered metal are used for light duty. Because of the porosity of the material, the bushing does not need an oil reservoir but is self-lubricating. Small gears, ratchets, sprockets, levers, and similar parts can be made of powdered metal. Powder-molded parts are often used as an alternative to zinc die-castings when greater strength and hardness are desired.

4–14. Aluminum alloys. Light alloys have wide use at present in light portable machines and particularly in various types of transportation machinery. The three main constituents of light alloys are aluminum, which has a specific weight of 0.098 lb per cu in.; magnesium, with a weight of 0.052 lb per cu in.; and silicon, which weighs 0.090 lb per cu in.

Pure aluminum is seldom used in machinery, being too weak and not sufficiently machinable. Copper in proportions from 0.15 to 12 per cent is the main constituent of aluminum alloys, as it gives strength, hardness, and machinability. Other metals, chiefly silicon, manganese, and nickel, are also added for the same purposes.

The number of aluminum alloys in use is very great, but only a few that are typical and more widely used will be discussed.[14]

[12] Data about various materials can be found in J. B. Johnson, M. O. Withey, and James Aston, *Materials of Construction*, 8th ed. (New York: John Wiley & Sons, Inc., 1939); Lionel S. Marks, ed., *Mechanical Engineers' Handbook*, 5th ed. (New York: McGraw-Hill Book Company, Inc., 1951); and R. T. Kent, *Mechanical Engineers' Handbook*, 12th ed., Vol. II, *Design and Production*, ed. by Colin Carmichael (New York: John Wiley & Sons, Inc., 1950).

[13] H. Y. Hunsicker, in *Sleeve Bearing Materials* (New York: American Society for Metals, 1949), p. 98.

[14] Based on Aluminum Company of America, *Alcoa Aluminum and Its Alloys* (Pittsburgh: 1944) and *SAE Handbook*.

TABLE 4–5

Composition and Properties of Aluminum Alloys*

SAE Num-[illegible]	Alcoa Number	Per Cent of Alloy (Balance Aluminum and Normal Impurities)					Specific Weight (lb per cu in.)	Ultimate Strength		Yield Point, 0.2% Set		Endurance Limit, Bending (kpsi)	Elongation in 2 In. (%)	Hardness, 500 kg (Bhn)	Manufacturing Process	General Information
		Cu	Si	Mn	Mg	Ni		Tension (kpsi)	Shear (kpsi)	Tension (kpsi)	Compression (kpsi)					
38	195	4.5	1.5	0.3	...	...	0.101	32	24	16	16	6.0	8.5	60	Sand-cast†	Structural castings requiring high strength and shock resistance
								36	30	24	25	6.5	5.0	75	Sand-cast‡	
								40	31	30	38	7.0	2.0	95	Sand-cast§	
39	142	4.0	0.7	0.3	1.5	2.0	0.098	32	27	28	34	8.0	0.5	85	Sand-cast	Air-cooled cylinder heads, pistons in high-perf. i-c engines
								40	26	33	46	9.5	0.5	110	Permanent mold	
322	355	1.3	5.0	0.5	0.5	...	0.095	32	30	25	29	8.5	2.0	80	Sand-cast	General use for high strength and pressure tightness
								37	30	27	26	9.0	1.5	90	Permanent mold	
35	43	0.6	5.0	0.3	...	...	0.097	19	14	9	10	6.5	6.0	40	Sand-cast	Intricate castings with thin sections; corrosion resistance
								24	18	9	9	...	9.0	45	Permanent mold	
321	A132	0.9	12.0	0.1	1.1	2.5	0.096	31	24	28	30	...	0.5	105	Permanent mold	Pistons in i-c engines, low heat expansion
[illegible]	B195	4.5	2.5	0.3	...	0.3	0.101	40	30	22	22	9.5	10.0	75	Permanent mold†	Same as SAE 38, modified for use in permanent molds
								45	32	33	33	10.0	5.0	90	Permanent mold‡	
								39	...	20	...	...	4.5	90	Permanent mold§	
305	13	0.6	12.0	0.3	...	...	0.096	37	...	18	...	15	1.8	...	Die-cast	Excellent casting characteristics
306	380	3.5	8.5	0.5	0.1	0.5	0.099	45	...	25	...	...	2.0	...	Die-cast	Good mechanical properties
...	750	1.0	...	[Sn, 6.5]	...	1.0	0.104	20	14	8.5	8.5	9.0	10.0	35–50	Permanent mold	Bearing alloy
24	24S	4.5	0.5	0.6	1.5	...	0.100	27	18	11	...	12	20	42	Rolled‖ and extruded#	Sheets, plates, tubes, rivets with high strength
								68	41	46	...	18	20	120		
260	14S	4.4	0.8	0.8	0.4	...	0.101	27	18	14	...	11	18	45	Rolled,‖ extruded,‡ and forged#	Highly stressed forgings, alternate for SAE 24
								62	34	40	...	18	25	100		
								70	42	60	...	18	13	135		
280	A51S	0.35	1.0	[Cr, 0.3]	0.6	...	0.097	44	32	34	[Shear 26]	11	12	90	Forged#	Complicated shapes
290	32S	0.9	12.5	0.2	1.1	0.9	0.097	52	38	42	...	16	5	115	Forged#	Press-forged pistons in i-c engines
...	XA80S	1.0	1.5	...	...	0.5	0.102	21	...	15.0	(15.0)	...	15.0	40–55	Rolled sheet	Bearing alloy

* Modulus of elasticity: $E = 10{,}300{,}000$ psi; $G = 3{,}850{,}000$ psi. † Solution heat-treated and naturally aged. ‡ Solution heat-treated and artificially aged. § Solution heat-treated and stabilized. ‖ Annealed. # Heat-treated to stable temperature.

Casting alloys. Castings are made in sand, in permanent molds, and in die-casting machines. Castings made in permanent molds have better mechanical properties than those made in sand. The properties of some casting alloys of aluminum are listed in Table 4–5. The strength of most cast alloys can be increased by heat treatment.

Alloy 195 is one of those most widely used for sand castings requiring high strength, particularly for engine crankcases and other parts in motor vehicles and aircraft. It has good resistance to corrosion and is used extensively in marine outboard motors. It is used in the heat-treated state.

Alloy 142, also known as Y-alloy, has a high strength when heat-treated. Moreover, the rate at which its strength decreases as the temperature rises is slower than that of most other alloys. For this reason it is used for piston and cylinder heads of internal combustion engines, as well as in general castings.

Alloy 355 has excellent foundry characteristics, high resistance to corrosion, and great strength at elevated temperatures. It is used also for leakproof castings of intricate design, such as cylinder heads of water-cooled automotive engines.

Alloy 43 has a comparatively low casting shrinkage and high fluidity at pouring temperature. These properties render it suitable for complicated castings with thick sections joined by thin webs. It is very resistant to corrosion and is free from porosity. It is therefore suitable for marine work and for pressure and vacuum apparatus.

Alloy A132 has a low coefficient of thermal expansion, retains its strength at elevated temperatures, is hard, and resists wear well. Its main use is for pistons of automotive engines.

Alloy B195 is a modification of alloy 195 for use in permanent molds. It has the same characteristics, but its strength and hardness are slightly higher and it is less ductile.

Alloy 13 has excellent casting characteristics and is widely used for complicated die castings with thin sections. It is very resistant to corrosion.

Alloy 380 has good foundry characteristics but is not well suited for thin sections. It has good mechanical properties and fair resistance to corrosion. It is used only in cold-chamber casting machines.

Silumin. Alloys A132 and 13 are also known as silumin because of the high content of silicon.

Wrought alloys. A great number of aluminum alloys depend for the attainment of their mechanical properties upon heat treatment, after the cast structure has been changed by hot or cold working. These alloys are used in various forms, including extruded structural shapes, sheets, plates, rods, and tubing, as well as press forgings and hammer forgings.

Alloy 24S is used for all forms of rolled, drawn, and extruded products where high strength is desired. It has very good formability immediately

after quenching. It retains this formability for a considerable time if it is stored at about 0 F immediately after quenching and is then formed soon after it is taken from the cold storage. Annealing facilitates its use for difficult forming. The formed pieces must be heat-treated before they are used.

Alloy 14S is used for high-strength rolled and extruded shapes and forgings. Its strength depends on proper heat treatment and aging.

Alloy A51S, because of its formability when hot, is used in the production of complicated forgings of moderate strength and for impact extrusion.

Alloy 32S retains high strength at elevated temperatures and has a lower coefficient of thermal expansion than other wrought aluminum alloys. For these reasons it is particularly suitable for press-forged pistons of heavy-duty internal combustion engines.

TABLE 4–6

MECHANICAL PROPERTIES OF MAGNESIUM ALLOYS*

Designation		Ultimate Strength			Yield Point		Endurance Limit, Bending (kpsi)	Elongation in 2 In. (%)	Hardness, 500 Kg (Bhn)	Manufacturing Process
ASTM Number	American Magnesium Corporation	Tension (kpsi)	Compression (kpsi)	Shear (kpsi)	Tension (kpsi)	Compression (kpsi)				
AZ92	AM260	20	46	48	11	11	7	5	54	Permanent mold or sand-cast
AZ10	AM240	21	47	16	11	11	9	1	50	Sand-cast
		35	54	18	18	18	9	0.5	78	Sand-cast†
AZ63	AM265	27	47	16	12	10	10	6	49	Sand-cast
		39	51	18	19	18	10	5	68	Sand-cast†
M1	AM3S	12	..	..	11	17	..	..	..	Cast
		28	..	14	17	7	5	5	40	Extruded
		30	..	..	18	7	..	3	44	Forged
AZ31	AM52S	37	(54)	(18)	22	12	12	12	51	Press-forged
		40	59	20	29	14	14	16	47	Extruded
AZ61	AM57S	43	69	20	30	16	17	17	54	Extruded
AZ80	AM58S	43	..	19	28	18	..	9	54	Extruded
		46	(54)	22	31	21	15	6	72	Forged†

* Specific weight about 0.065 lb per cu in.; $E=6{,}500{,}000$ psi; $G=2{,}500{,}000$ psi.
† Heat-treated and aged.

4–15. Magnesium alloys. High-magnesium alloys with small amounts of aluminum, manganese, and zinc have desirable physical properties and good corrosion resistance except to marine atmospheric conditions. These alloys have two-thirds the weight of aluminum and they machine easily and smoothly. They are finding increasing use, especially in aviation, in the form of castings, forgings, and rolled sheets. Magnesium pistons and connecting rods combine strength with light weight.

In Table 4–6 are given the mechanical properties of the alloys most used

in the United States. The alloys AZ63, AZ61, and AZ80 are also known under the trade name *Electron.* They are used for crankcases, gear boxes, pistons, and cylinder heads.

Alloy AZ63 and *alloy AZ92* are those most used for sand castings.

Alloy M1 castings can be readily welded to sheets and forgings of the same alloy.

Alloy AZ92 and *alloy AZ10* are used for permanent-mold castings.

Magnesium-alloy forgings are used where greater strength is required than is obtainable from castings. They are generally press-forged.

Alloy AZ61 is a general-purpose forging alloy, while *alloy AZ80* is used for forgings of highest strength and simple design. Alloy M1 may be either press-forged or hammer-forged. It is used for parts of low cost with moderate strength requirements.

A wide range of extruded shapes is available in many alloys. Alloy M1 is used for low-cost extrusions with moderate strength. Extrusions made of alloys AZ31, AZ61, and AZ80 increase in strength, but also in cost, in the order named.

4–16. Zinc alloys. Alloys containing 3.5 to 4.5 per cent of aluminum and small amounts of copper, iron, and magnesium, with the balance zinc, are used rather extensively for die-casting parts of various mechanisms and equipment (often of very intricate shape) in which strength is not the most important requirement. Both the American Society of Testing Materials and the Society of Automotive Engineers have established specifications for three types of alloys the physical properties of which are given in Table 4–7.

TABLE 4–7

PHYSICAL PROPERTIES OF ZINC ALLOYS*

Trade Name	ASTM Specification	SAE No.	Tensile Strength (psi)	Compressive Strength (psi)	Shear Strength (psi)	Hardness (Bhn)	Elongation in 2 In. (%)
Zamak-2....	XXI	921	47,900	93,100	45,800	83	5.1
Zamak-3....	XXIII	903	40,300	60,500	30,900	74	4.7
Zamak-5....	XXV	925	45,400	87,300	38,400	76	3.0

*Specific weight of all zinc alloys is about 0.24 lb per cu in.

4–17. Nonmetallic materials. Properties of nonmetallic materials used in machines are given in Table 4–8. Most of these materials are either synthetic phenolic resin, which is widely used under the name of *Bakelite*, or laminated sheets obtained by compressing layers of paper or canvas impregnated with phenolic resin. In machinery the main use of such materials as Celoron, Formica, or Micarta is for manufacturing silent gears and bearing shells. Rawhide and vulcanized paper fiber are also used for silent gears, but these materials absorb moisture more readily.

TABLE 4–8

MECHANICAL PROPERTIES OF NONMETALLIC MATERIALS

Trade Name	Material	Specific Weight (lb per cu in.)	Tensile Strength (psi)	Compressive Strength (psi)	Shear Strength (psi)	Modulus of Elasticity E (psi)	Hardness, 500 Kg (Bhn)
Bakelite	Phenolic resin	0.0487	4,500	32,000	12,600	1,000,000	4[illegible]–54
Celoron	Phenolic resin, paper	0.049	10,000	25,000	15,000	2,300,000	40
Celoron	Phenolic resin, canvas	0.050	8,000*	30,000	10,000	1,100,000	38
Formica	Phenolic resin	0.050	10,000*	25,000	15,000	1,100,000	65
Micarta	Phenolic resin	0.050	12,000	31,000	12,000	1,050,000	38
....	Rubber, hard	0.050	3,500	....	....	....	...
....	Rubber, soft	0.054	....	....	....	150	...
....	Hardwood	0.024	20,000,† 900‡	1,100	1,100	1,700,000	...
....	Leather, rawhide	0.035	8,000	8,000	....	18,000	...
....	Vulcanized fiber	0.054	6,000	8,000	11,000	....	10

* Endurance limit in bending approximately 5,000 psi. † Parallel to grain. ‡ Across grain.

CHAPTER 5

Machine Design Calculations

5–1. Definitions and designations. The elementary equations for stresses in Chapter 2 are based on the assumption that there is equilibrium between the external forces and the internal stresses. For stresses in machine parts these equations give correct values only for special cases of static loading.

In most cases, in order to determine the true stress conditions in various sections, it is also necessary to consider other factors, such as the following:

a. The type of load: constant, shock, or repeating
b. The maximum and minimum values for repeated loads
c. The number of cycles expected for repeated loads
d. Discontinuities
e. Certain properties of the material, in addition to those determined by static tests
f. Internal stresses set up by manufacturing or operating conditions

Types of stresses. The actual stresses in a certain section are seldom uniform, and the determination of the maximum stress and of the point of its action is of the greatest importance for a safe and economical design.

The stress computed by one of the elementary equations is called the *nominal stress* and is designated s_n.

The *significant stress* s_{sg} is the greatest stress that exists in a section of a part. This stress determines the dimensions of the section, and it is found by considering the factors mentioned above.

The term *limit stress* will be applied to the maximum stress to which a machine part can be subjected without being damaged. The limit stress S_l is a characteristic of the material, but its value and the method of determining it depend on the type of loading, the thickness of the section, the method of manufacturing, and the surface conditions of the piece.

The *design stress*, also called the *working stress* or *allowable stress*, is the maximum stress that should never be exceeded in a properly designed machine part. To allow for inaccuracies in assumptions, workmanship, and qualities of the material, the design stress S_d always must be lower than the limit stress S_l.

Factor of safety. The amount by which S_d is kept below S_l is expressed by their ratio, which is called the *design factor of safety* and is designated as n.

Thus,

$$n=\frac{S_l}{S_d} \tag{5-1}$$

This definition of the safety factor differs from the older one, still sometimes used, by which the nominal stress was compared with the ultimate strength of the material. However, the present definition is more logical and has decided advantages with respect to economy of material and actual safety.

The *actual safety factor* is the ratio of S_l to s_{sg} and is designated by n'. Thus,

$$n'=\frac{S_l}{s_{sg}} \tag{5-2}$$

The factor n' differs from n when the dimensions that are found by using n are changed, either by rounding them off or by using standard sizes.

Types of loads. Three main types of loads must be distinguished. They are (1) *steady loads*, called *static loads* or *dead loads;* (2) *impact loads*, or *shock loads;* and (3) *repeated loads*, which either may vary gradually or may have the characteristics of repeated impact. So-called *inertia loads* are special cases of single impact loads, or of repeated, gradually changing loads, or of repeated impact loads.

The methods of designing parts subjected to different types of loading differ greatly and must be considered separately.

5–2. Strength under combined stress. A machine part is generally subjected simultaneously to several different stresses, whose actions are combined. Therefore it is necessary to establish the combined stress that will cause failure of the part. It should be noted that a machine part may fail either by having undergone a permanent deformation or by actually breaking at the dangerous section.

Failure by yielding, when a part is subjected to steady or gradually applied loads, occurs in materials that are classified as ductile, whereas failure by fracture usually occurs in materials that are classified as brittle. However, as already mentioned in section 4–1, there is no sharp dividing line between ductile and brittle materials. Also, it was shown how a part made of ductile material may fail by fracture when subjected to repetitive loading.

Theories of failure. Four theories of failure are used at present. They are known as the maximum-normal-stress theory, the maximum-strain theory, the maximum-shear theory, and the shear-energy theory.

The *maximum-normal-stress theory*, also called *Rankine's theory*, assumes that yielding at a point begins only when the maximum principal stress on a certain plane passing through the point reaches a value equal to the elastic

limit as found in a simple tension test, regardless of any other stresses that occur on other planes passing through the point. The condition of failure is

$$s'=S_e \tag{5-3}$$

The *maximum-strain theory*, or *Saint-Venant's theory*, assumes that the elastic limit of a material is reached when a certain deformation has been produced. Therefore a member is safe as long as the maximum normal stress s'' found by equation 2–48 is below the elastic limit S_e of the material.

The value of Poisson's ratio μ for all materials used in machinery is close to 0.3. By inserting this value in equation 2–48 and simplifying the terms, the condition of failure for the combination of a direct stress s and a shear stress s_s is expressed by the equation

$$0.35s+0.65\sqrt{s^2+4s_s^2}=S_e \tag{5-4}$$

The *maximum-shear theory* of Guest assumes that the elastic limit of a material is reached when the greatest resultant shear stress reaches a certain limit as determined under the condition of pure shear. According to this theory a member subjected to tensile stress fails when the maximum shear stress determined by equation 2–40 exceeds $\frac{1}{2}S_e$, where S_e is the elastic limit in tension. In the case of a combined load the member will fail when its maximum shear stress s_s determined by either equation 2–47 or 2–54, as the case may be, exceeds the elastic limit in shear. In general, if a member is subjected to the combination of a direct stress s and a shear stress s_s, the condition of failure is expressed by the equation

$$\sqrt{s^2+4s_s^2}=S_e \tag{5-5}$$

The *shear-energy theory*, or the constant-energy-of-distortion theory, also referred to as the Hencky-von Mises theory, gives the following equation[1] for the condition of failure for a combination of a direct tensile stress s and a shear stress s_s:

$$\sqrt{s^2+3s_s^2}=S_e \tag{5-6}$$

Comparison of theories. For pure tension, $s_s=0$, and all four theories give the same results. For pure shear, the maximum-normal-stress theory gives, in accordance with equation 2–42, $s_s=0.5S_e$; the maximum-strain theory, equation 5–4, gives $s_s=0.77S_e$; the maximum-shear theory, equation 5–5, gives $s_s=0.5S_e$; and the shear-energy theory, equation 5–6, gives $s_s=0.58S_e$. For combinations of tension and shear the differences are less than for pure shear.

At present it is difficult to determine definitely which of these theories should be given preference. The available data seem to indicate that for

[1] C. R. Soderberg, "Working Stresses," *Transactions of the American Society of Mechanical Engineers*, Vol. 55, No. 3 (1935), p. A-107, and *Code for the Design of Transmission Shafting* (New York: American Society of Mechanical Engineers, 1927), p. 3.

hard, brittle materials, such as cast iron or hardened steel, the maximum-normal-stress theory or the maximum-strain theory should be preferred. For ductile materials experimental data are in favor of the shear-energy theory. However, it is much simpler to use the maximum-shear theory, and

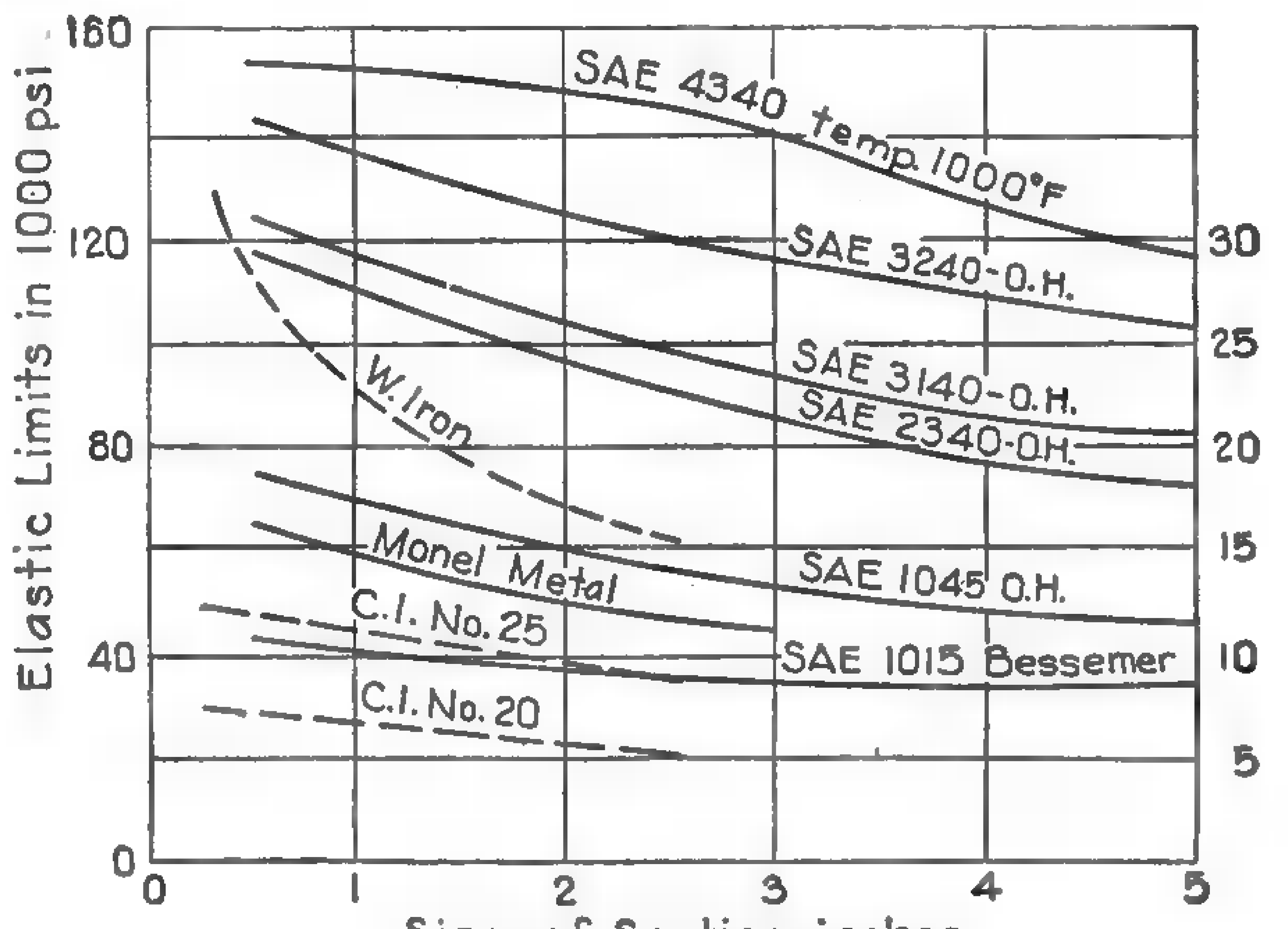


FIG. 5–1. Influence of size on elastic limits.

the differences resulting from application of this theory rather than the shear-energy theory are very slight and on the safe side. Therefore the maximum-shear theory is often used as a basis for design calculations.[2] However, the trend in design practice is toward the wider use of the Hencky-von Mises theory for ductile materials. The safe course to follow is to design a member so that the maximum principal stress does not exceed some selected proportion or fraction of the tensile or compressive elastic limit; and so that the maximum principal strain does not exceed the same proportion of strain corresponding to this elastic limit; and also so that the maximum shear stress does not exceed the same proportion of the elastic limit in shear.[3]

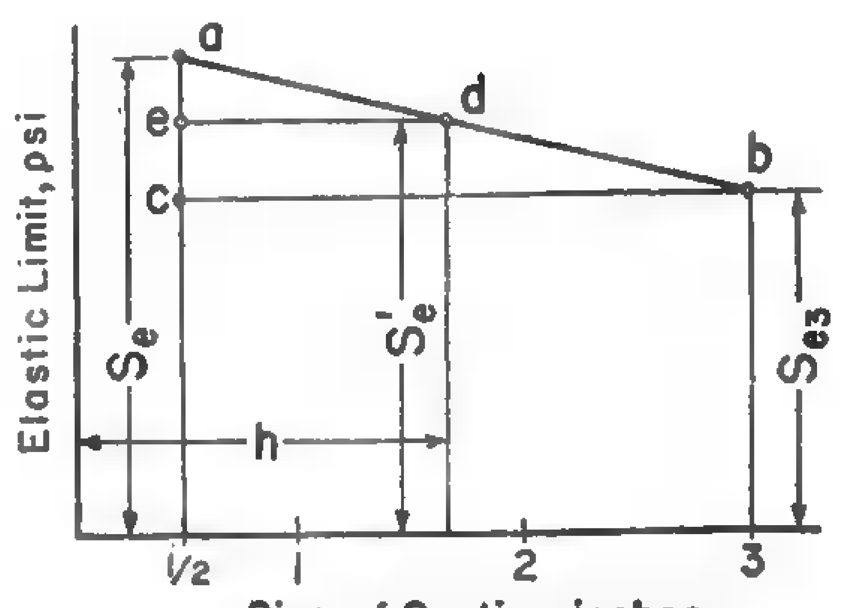


FIG. 5–2. Change of elastic limit with size of section.

5–3. Influence of size. The values for elastic limit or yield point given in various tables of Chapter 4 were obtained with test specimens of small cross sections, about $\frac{1}{2}$ in. in diameter. Tests show that, because of decreased uniformity of structure, pieces of material with larger sections have mechanical properties inferior to those with smaller sections. The influence of size upon the elastic limits in tension of various materials may be seen in Fig. 5–1, where the scale of ordinates for the three dotted curves is indicated on the right-hand side of the diagram.

[2] *Ibid.*

[3] F. B. Seely, *Resistance of Materials*, 3d ed. (New York: John Wiley & Sons, Inc., 1947), p. 292.

TABLE 5–1

SIZE FACTORS FOR MECHANICAL PROPERTIES OF MATERIALS

Material	Values of S_{e3}/S_e				
	Natural State	Annealed	Drawn at 1200 F	Drawn at 1000 F	Drawn at 800 F
Aluminum, strong, wrought	0.93	...	...	...	...
Tobin bronze	0.90	...	...	...	...
Monel metal, forged	0.80	...	...	...	...
Ductile iron	0.80	0.98	...	...	...
Low-carbon steel, C<0.20%	0.84	...	...	...	...
Medium-carbon steel, 0.30 to 0.50% C	...	0.85	0.72	0.59	0.53
Nickel steel, SAE 2340	...	0.86	0.80	0.74	...
Cr-Ni steel, SAE 3140	...	0.80	0.75	0.70	0.65
Cast iron, Class No. 20	0.55	...	...	...	...
Cast iron, Class No. 25	0.73	...	...	...	...
Cast iron, Class No. 35	0.60	...	...	...	...
Wrought iron	0.55	...	...	...	...

As a first approximation the decrease of the elastic limit for all metals may be represented by an inclined straight line, as shown in Fig. 5–2. In this case, if the elastic limit given in the tables is S_e, approximately corresponding to a section $\frac{1}{2}$ in. thick, and the elastic limit of a 3-in. section S_{e3} is known, the elastic limit S_e' for any thickness h between $\frac{1}{2}$ in. and 3 in. can be determined from the relation

$$S_e' = S_e - \frac{(S_e - S_{e3})(h - 0.5)}{3 - 0.5}$$

This equation may be written in the simple form

$$S_e' = e_{sz} S_e \tag{5-7}$$

where the *size coefficient* e_{sz} may be found from Fig. 5–2 as

$$e_{sz} = 1 - 0.4\left(1 - \frac{S_{e3}}{S_e}\right)(h - 0.5) \tag{5-8}$$

Values of the ratio S_{e3}/S_e for a few of the more important materials are given in Table 5–1.

A further decrease of the elastic limit with the increase in the size above 3 in. in most metals is less pronounced, as may be seen from Fig. 5–1. For actual design calculations the proper data should be obtained from the manufacturer or some other reliable source.[4]

[4] International Nickel Company, Inc., *Nickel Alloy Steels*, Sec. II, Data Sheet No. 4 (New York, 1934, revised to March, 1941), pp. 64 ff.

Although data are meager, it appears that an increase in size has the same influence on the elastic limit in shear as on the elastic limit in tension. Hence, the values in Table 5–1 can be used also for shear.

It should be noted that the use of the size factor is a refinement necessary in designing light, highly stressed parts. Many ordinary parts are designed without considering this refinement.

5–4. Stress concentration. As explained in section 3–5, the presence of a discontinuity increases the stress at and near the discontinuity. The maximum theoretical increase in stress, compared with the average or nominal stress, is represented by the form factor K. The magnitude of K may be obtained by using the formulas and diagrams given in sections 3–6, 3–7, and 3–8. The actual stress increase in a machine part is smaller than that given by the factor K, because of the elasticity and plasticity of the metals. In general, however, the actual increase is along the same lines as the theoretical increase.

The actual weakening effect is the effective stress increase, which is called the *stress-concentration factor* and is designated as K'. The best way to find the stress-concentration factor in a machine part is to determine the ratio of the stresses both with and without stress concentration by testing the machine part under actual or simulated service conditions. Such experimental data exist, but not for all parts, materials, and types of loading.

A simpler, although less accurate, method is to find the relation between the theoretical form stress factor K and the real stress-concentration factor K' for a certain set of conditions, and to use this relation to compute K' from known values of K for the design conditions.

Index of sensitivity. The relation between K' and K may be expressed as follows:

$$K' = 1 + q(K-1) \tag{5–9}$$

where q is called the *index of sensitivity* of the material to abrupt changes of section. From equation 5–9,

$$q = \frac{K'-1}{K-1} \tag{5–10}$$

The magnitude of q depends on the material and on the character of loading. In general, q is higher for hard, brittle materials and decreases with an increase of ductility. Also, q is highest for repeated heavy-shock loads, decreases with a decrease of the intensity of the shocks, and has the smallest value when the load is more or less steady, as when it is static.

5–5. Static loads. The design stress for a static load can be found from the equation

$$S_d = \frac{S_e'}{n} = \frac{e_{ss}S_e}{n} \tag{5–11}$$

where n is the safety factor. This factor should never be less than 1.25, and it should usually be at least 1.5. If the material is not of the very best quality or if there is no definite information about its quality, the value of n should be taken as 2. For a brittle material it is advisable to use a value of n between 2 and 3.

Sensitivity index. Tests on various ductile metals subjected to static loads have shown that discontinuities do not affect the significant stress. For a ductile material, therefore, the sensitivity index q is zero and the stress-concentration factor K' is 1. Values of q for other materials may be taken as follows: for brittle materials, such as hardened steel, 0.10; for very brittle materials, such as steels that are quenched but not drawn, 0.20. Since cast iron has so many internal discontinuities in its structure, it may be assumed that q is zero for this metal.

In an eye bar the effect of the hole is to produce compressive stress in the outer fibers, as indicated in Fig. 3–10. Therefore, the equalizing yield cannot take out all of the localized stress; and the value of K' based on the reduced normal section ck should be taken as 1.5.[5] The corresponding value of q is 0.16.

Significant stress. A discontinuity increases the stress to an effective value that is K' times as great as the nominal stress. This increase takes place only at one point or in a limited region. However, since the purpose of a correct design is to prevent the stress from exceeding the allowable design stress S_d at *any* place, the highest effective stress caused by stress concentration becomes the significant stress. To find this significant stress, the nominal stress s_n is first computed by applying the proper elementary stress equation and it is then multiplied by K'. Thus,

$$s_{sg} = K's_n \tag{5–12}$$

Bearing stress. A special case of compressive stress encountered in machine parts is the stress caused at the surface of contact of two members that are relatively at rest, as at the surface between a rivet and a plate or between a key and a shaft. This kind of stress is called *bearing stress.* This term should not be confused with bearing pressure, which exists at the contact surfaces of two parts in relative motion.

When there are no friction forces at the surfaces in contact, it has been found both theoretically and experimentally that the magnitude of the bearing stress at which yielding starts is[6]

$$S_b = \frac{\pi}{\sqrt{3}} S_e = 1.81 S_e \tag{5–13}$$

[5] F. B. Seely, *Advanced Mechanics of Materials* (New York: John Wiley & Sons, Inc., 1932), p. 221.

[6] A. Nadai and A. M. Wahl, *Plasticity* (Engineering Societies Monographs. New York: McGraw-Hill Book Company, Inc., 1933), p. 247.

Actually friction exists and increases the factor 1.81 materially. For practical applications the elastic limit of bearing stress may be assumed as

$$S_b = 2S_e \tag{5-14}$$

Combined loading. Where there are combined stresses the value of the maximum significant stress s_{sg} must be determined in compliance with the theory of failure applicable to the material used.

Safety Factor. For static loading the limit stress is $S_l = S_e'$. Therefore the actual safety factor is

$$n' = \frac{S_l}{s_{sg}} = \frac{e_{sz}S_e}{K's_n} \tag{5-15}$$

In Fig. 5-3 are illustrated the various relations between the stresses which have to be considered for a machine part under static loading. The difference between S_d and s_{sg} is the additional safety margin Δs.

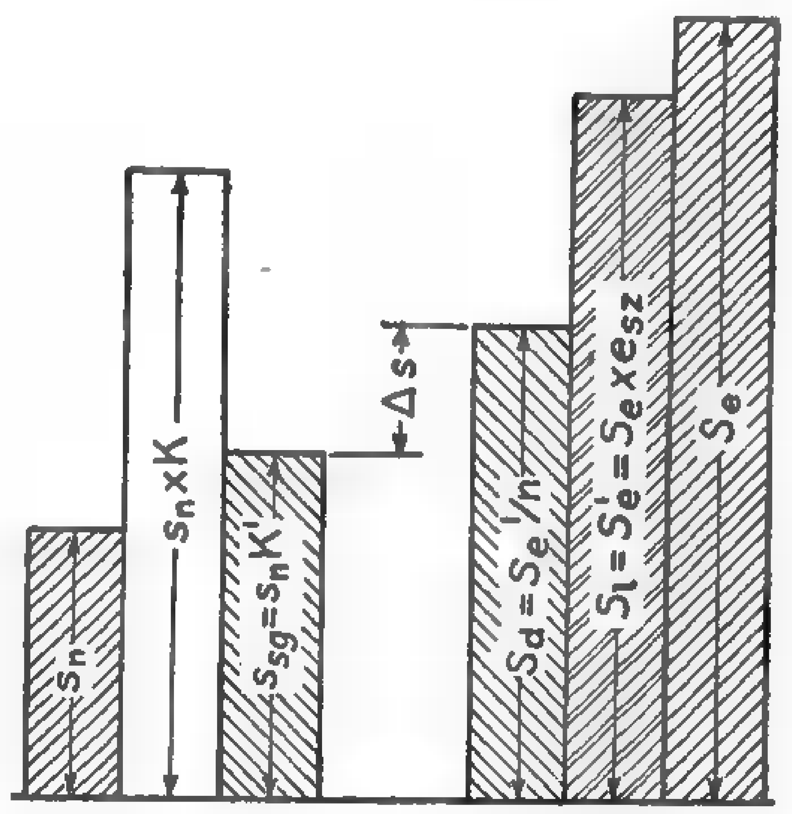


Fig. 5-3. Relation between stresses for static loading.

The recommended values of the actual safety factor n' for static loads are as follows:

a) For reliable high-grade materials, when loads and stresses can be determined very accurately and a low weight is desired, n' is taken between 1.25 and 1.5.

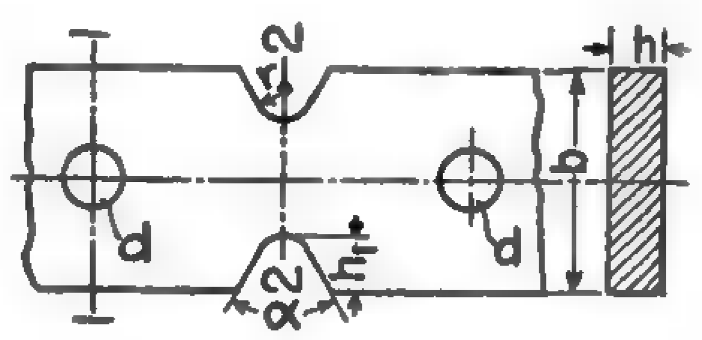


Fig. 5-4. Tie rod.

b) For the same conditions, when a low weight is not important, n' ranges from 1.5 to 2.

c) For ordinary materials, when loads and stresses can be determined with sufficient accuracy, n' ranges from 2 to 2.5.

d) For brittle materials under ordinary conditions, n' ranges from 2.5 to 3.

e) For ordinary materials, when loads and stresses cannot be determined accurately, n' ranges from 3 to 4.

Example 5-1. Find the safety factor for the flat tie rod shown in Fig. 5-4, if it is made of SAE 2320 nickel steel drawn at 1,000 F and is subjected to a steady pull of 120 tons. The dimensions are as follows: $b = 6$ in., $h = 1\frac{1}{4}$ in., $d = 1\frac{1}{2}$ in., $h_1 = \frac{3}{4}$ in., $r = \frac{3}{4}$ in., and $\alpha = 45°$.

The elastic limit in tension for this steel, from Table 4-2, is 80,000 psi. If the value of the ratio S_{es}/S_e is taken as that given in Table 5-1 for SAE 2340 nickel steel, or 0.74, the size coefficient is, by equation 5-8,

$$e_{sz} = 1 - 0.4 \times (1 - 0.76) \times (1.25 - 0.5) = 0.922$$

Then the corrected elastic limit, by equation 5-7, is

$$S_e' = 0.922 \times 80{,}000 = 73{,}760 \text{ psi}$$

The nominal stress created at section *1-1*, Fig. 5-4, by the pull of 120 tons is

$$s_{n1} = \frac{120 \times 2{,}000}{(6 - 1.5) \times 1.25} = 42{,}700 \text{ psi}$$

From Fig. 3-4, since d/b is $1.5/6 = 0.25$, the form stress factor for the hole is $K = 2.40$. If it is assumed that the index of sensitivity is 0.10, the stress-concentration factor found by equation 5-9 is

$$K' = 1 + 0.10 \times (2.40 - 1) = 1.14$$

By equation 5-12 the significant stress is

$$s_{sg1} = 1.14 \times 42{,}700 = 48{,}700 \text{ psi}$$

The nominal stress in section 2-2, Fig. 5-4, is

$$s_{n2} = \frac{120 \times 2{,}000}{(6 - 2 \times 0.75) \times 1.25} = 42{,}700 \text{ psi}$$

From Fig. 3-15, for $r/b = 0.125$ and $h_1/b = 0.125$, the form stress factor is $K = 1.7$. Hence, the stress-concentration factor is

$$K' = 1 + 0.10 \times (1.7 - 1) = 1.07$$

According to Fig. 3-16 the influence of the angle α can be neglected. Therefore the significant stress is

$$S_{sg2} = 1.07 \times 42{,}700 = 45{,}700 \text{ psi}$$

Since section *1-1* is subjected to the greater significant stress, that is the dangerous section. By equation 5-2 the actual safety factor is

$$\frac{73{,}760}{48{,}600} = 1.51$$

Design procedure. The first step in the design of a machine part is to select a suitable material by considering such factors as the shape of the part, the operating conditions, and the cost. Next, the expected thickness of the sections involved is assumed, and the corrected elastic limits S_e' is estimated. The basic value of S_e may be taken from one of the tables of Chapter 4. In actual industrial design of an important part the value of S_e or S_e' may be obtained from the manufacturer of the material, and in important cases S_e' may be determined by special tests. Then the safety factor n is selected and the design stress S_d is determined.

After that the relation between the acting force F and the allowable stress S_d is expressed as a function of the dimensional characteristic of the part, such as its area A or its section modulus Z or Z_o. The equation is then solved for this characteristic, and the linear dimensions are determined. Any discontinuities causing stress concentration may require an increase of the preliminary dimensions.

Example 5-2. Determine the dimensions b and h of a tie rod having discontinuities like those in Fig. 5-4, to sustain a dead load F of 30 tons with a safety factor of 2. The two holes on the center line have a diameter $d = 1$ in. and are 5 in. apart. The two notches are midway between the holes and are so shaped that $r = \frac{1}{4}$ in., $h_1 = \frac{1}{2}$ in., and $\alpha = 90°$.

A suitable material would be medium-carbon steel SAE 1030, with an elastic limit S_e of 42,000 psi. This value of S_e may be decreased slightly, say by 5 per cent, to take care of a thickness over $\frac{1}{2}$ in. If the steel is annealed, no stress concentration has to be taken into account from either the holes or the notches. Hence the design stress is

$$S_d = \frac{S_e'}{n} = \frac{42,000 \times 0.95}{2} = 20,000 \text{ psi}$$

The minimum cross-sectional area is

$$A = \frac{F}{S_d} = \frac{30 \times 2,000}{20,000} = 3.0 \text{ sq in.}$$

If it is assumed that $b'/h = 4$ or $A = 4h^2$, the required depth is

$$h = \sqrt{\frac{A}{4}} = \sqrt{\frac{3}{4}} = 0.867, \text{ or } \frac{7}{8} \text{ in.}$$

If 1 in. is added for the hole or $2 \times \frac{1}{2} = 1$ in. for the notches, the resulting total width becomes

$$b = \frac{3.0}{0.875} + 1 = 4.43, \text{ or } 4\tfrac{1}{2} \text{ in.}$$

From Table 5-1, the ratio S_{e3}/S_e may be taken as 0.85. The corrected elastic limit, by equations 5-7 and 5-8, is

$$S_e' = 42,000 \times [1 - 0.4 \times (1 - 0.85) \times (0.875 - 0.5)] = 42,000 \times 0.977 = 41,000 \text{ psi}$$

Therefore the actual safety factor is

$$n' = \frac{41,000 \times (4.5 - 1) \times 0.875}{30 \times 2,000} = 2.09$$

This is only slightly greater than that which was originally specified.

Columns. In designing a new column Ritter's formula may be used. If the column has a length l and must carry a load F, the cross-sectional area A and the radius of gyration k are expressed by one of the dimensions of the section, the proper design stress S_d is substituted for S_c, and equation 2-37 is solved for this dimension. The same procedure is followed if the column is loaded eccentrically, but in this instance the modified equation 2-61 is used.

EXAMPLE 5-3. Find the dimensions of the rectangular cross section of a steel bar 20 in. long which must act as a column with the ends hinged and must support a weight of 18,000 lb. The width b of the bar must be twice its thickness h. The properties of the steel used are: $S_e = 34,000$ psi and $E = 30,200,000$ psi. Use a safety factor $n = 2$.

To find the area A needed to carry the load, it is necessary first to determine the design stress. Considering the stress as pure compression and allowing 3 per cent as a deduction for the expected influence of size, we get

$$S_d = \frac{e_{sz} S_e}{n} = 0.97 \times \frac{34,000}{2} = 16,500 \text{ psi}$$

Then

$$A = \frac{18,000}{16,500} = 1.083 \text{ sq in.}$$

Since $b = 2h$, the cross-sectional area is $hb = 2h^2$. Hence,

$$h = \sqrt{\frac{1.083}{2}} = 0.736 \text{ in.}$$

According to Table 2-5 the least radius of gyration is $k = 0.289h = 0.289 \times 0.736 = 0.213$ in. Since the ratio of slenderness is $l/k = 20/0.213 = 94$, the bar must be treated as a short column.

According to Table 2-5 the least moment of inertia is $I = bh^3/12$. When $b = 2h$, then $I = h^4/6$. The least radius of gyration is

$$k = \sqrt{\frac{h^4}{6 \times 2h^2}} = \frac{h}{2\sqrt{3}}$$

Ritter's formula (equation 2-37), in which $n = 1$ for the specified end conditions, gives

$$16,500 = \frac{18,000}{2h^2}\left[1 + \frac{(20 \times 2\sqrt{3})^2}{h} \times \frac{34,000 \times 0.97}{\pi^2 \times 30,200,000}\right]$$

This reduces to

$$h^4 - 0.545h^2 - 0.289 = 0$$

from which $h = 0.935$ in. To the nearest $\frac{1}{16}$ in., $h = \frac{15}{16}$ in., and $b = 1\frac{7}{8}$ in.

To find the actual safety factor, the size influence is determined by equation 5-8. Thus,

$$e_{sz} = 1 - 0.4 \times (1 - 0.85) \times (0.937 - 0.5) = 0.974$$

Equation 2-37 gives for the actual stress

$$s_c = \frac{18,000\left[1 + \left(\dfrac{20 \times 1.875\sqrt{3}}{0.937}\right)^2 \times \dfrac{34,000 \times 0.974}{\pi^2 \times 30,200,000}\right]}{1.875 \times 0.937} = 16,500 \text{ psi}$$

The actual safety factor is

$$n' = \frac{34,000 \times 0.974}{16,500} = 2.0$$

Long columns. In designing a long column, Euler's formula (equation 2-38) may be used. The load F that the column must carry is multiplied by a safety factor n_c, and the product n_cF is substituted for the critical load F_u. The area A and the radius of gyration k of the cross section are expressed as functions of one of the dimensions b of the section, the corresponding values of E and the end-condition factor n are taken, and the equation is solved for b. Since Euler's formula gives the load that corresponds to the ultimate strength S_u, which is about twice S_e, the safety factors given before must be doubled. Thus 4 to 5 should be used for ductile materials, and 6 to 8 for brittle ones. Eccentric loading does not affect a long column appreciably; but to be on the safe side, it can be taken into account by increasing the safety factor slightly.

5-6. Impact loading. When a force F acts on a machine part with impact, the magnitude of the maximum stress s' in the part is greater than the static stress s, as shown by equation 3-18. The stress s' thus becomes the effective stress and must not exceed the permissible stress, or design stress, S_d. Therefore the static stress s is the significant stress which must be used in determining the stressed cross-sectional area of the part. On the other hand, since the energy of the impact must be absorbed by the body, the resilience of the body must be at least equal to the impact energy. Therefore either of two approaches is possible. One is based on the significant static stress s, while the other is based on the resilience U of the body.

Safety factor. Since a low safety factor means higher stresses and hence greater resilience, the safety factor n is normally taken as 1.5.

Index of sensitivity. The value of q may be taken as 0.4 for those materials which are ductile and very soft, and increased up to 0.6 for other ductile materials. For hard and brittle materials, such as hardened high-carbon steel, q should be taken as 1. In this case the stress-concentration factor K' becomes equal to the theoretical form-stress factor K. Cast iron, because of its structure, can be classified among the less-sensitive materials, q having a value of about 0.5.

Design procedure. After the material has been selected, the size coefficient e_{sz} is estimated by assuming the probable thickness of the section involved and by applying equation 5–8. The corrected elastic limit S_e' can then be found by equation 5–7. The next step is to establish the value of the form-stress factor K as explained in Chapter 3 and to find the stress-concentration factor K' by equation 5–9. The design stress can then be found by the equation

$$S_d = \frac{e_{sz} S_e}{nK'} \tag{5–16}$$

This value of S_d is substituted in equation 3–18 for s', and the significant stress is then found by the relation

$$s = \frac{S_d}{1 + \sqrt{1 + \dfrac{2h}{e}}} \tag{5–17}$$

For tension or compression,

$$s = \frac{S_d}{2\left(\dfrac{hE}{S_d l} + 1\right)} \tag{5–18}$$

The dimensions of the part are now found by the usual elementary stress equations in which this value of s is taken as the maximum allowable stress.

After a part is thus designed for an impact load, its dimensions should be checked to make sure that its resilience U is sufficiently greater than the impact energy K_i. The resilience U may be determined by the general equation 3–23 or by one of the special equations 3–25, 3–30, 3–32, or 3–34.

In finding the impact energy that must be absorbed by the part, it is necessary to take into account the deformations caused by the impact. Therefore the energy caused by the additional travel of the impact force, computed by equation 3–16, must be added to the main impact energy found by equation 3–14 or equation 3–16.

Because energies are proportional to the second power of stresses, the actual safety factor n' is, in terms of energies,

$$n' = \sqrt{\frac{U}{K_i}} \tag{5–19}$$

EXAMPLE 5–4. Determine the main dimensions of a round rod of SAE 1030 steel stressed in tension by a weight of 325 lb falling from a height of 2 in. The free length of the rod is 60 in., and the ends of the rod are fastened as shown in Fig. 5–5.

In the preliminary calculations S_e' may be assumed = $0.93S_e$, or $0.93 \times 42{,}000 = 39{,}100$ psi. Then the design stress, with $n = 1.5$, is

$$S_d = \frac{39{,}100}{1.5} = 26{,}000 \text{ psi}$$

The significant stress, found by equation 5–18, is

$$s = \frac{26{,}000}{2\left(\dfrac{2 \times 30{,}000{,}000}{26{,}000 \times 60} + 1\right)} = 330 \text{ psi}$$

The required cross-sectional area is

$$A = \frac{325}{330} = 0.985 \text{ sq in.}$$

and the diameter is

$$d_1 = \sqrt{\frac{0.985}{0.7854}} = 1.112 \text{ in.}$$

In static loading the actual dimension would probably be the next larger one, which is $1\frac{1}{8}$ in. For dynamic loading, in order not to decrease the deformation and resilience, it is better to keep the computed diameter.

At the ends the diameter must be increased to take care of the weakening caused by the cross-pin holes and to take care of the stress concentation due to these holes.

Assume the pins to be made of the same SAE 1030 steel. The elastic limit in shear for this steel is 26,000 psi. Since there are two pins with four shearing areas, the pins will be as strong as the rod if the cross-sectional area of the pins is

$$A_3 = \frac{0.985 \times 42{,}000}{26{,}000 \times 4} = 0.398 \text{ sq in.}$$

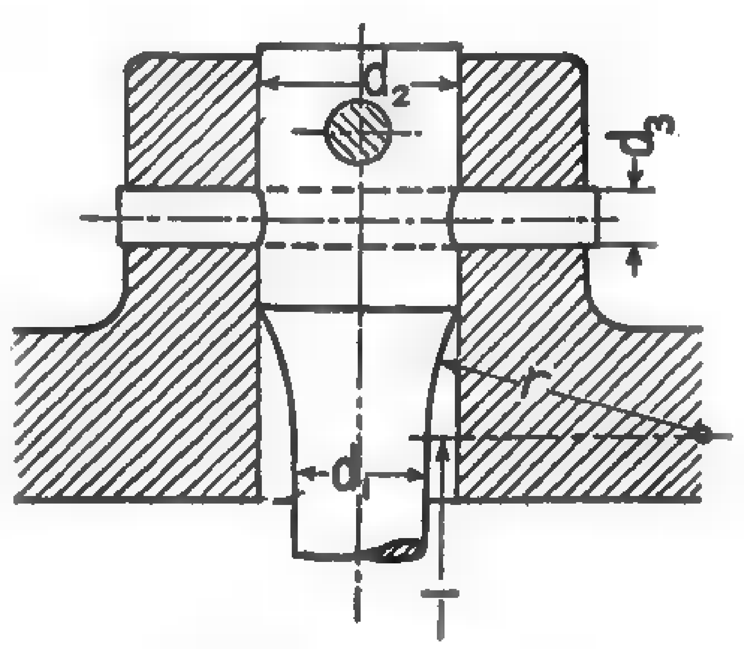


FIG. 5–5. Fastening of end of rod.

The diameter should be

$$d_3 = \sqrt{\frac{0.398}{0.7854}} = 0.712 \text{ in.; use } \frac{23}{32} \text{ in.} = 0.719 \text{ in.}$$

The stress-concentration factor for the holes may be estimated at first as $K' = 1.4$, and the diameter d_2, Fig. 5–5, can be computed by equating the cross-sectional area of the end weakened by the pinhole and by stress concentration to the area of the body of the rod. Thus,

$$\frac{0.7854d_2^2 - 0.719d_2}{1.4} = 0.7854 \times 1.125^2$$

which gives $d_2 = 1.87$ in., or $1\frac{7}{8}$ in.

Now the stress concentration can be computed more accurately by using the form stress factor from Fig. 3–5. For $d/D = 0.719/1.875 = 0.383$, the value of K is 2.02. With $q = 0.4$, the stress-concentration factor is, by equation 5–9,

$$K' = 1 + 0.4 \times (2.02 - 1) = 1.408$$

This is practically the same as was estimated. Thus no changes in the design are required. Naturally the change from d_2 to d_1 must be very gradual in order not to introduce a new stress concentration.

It is advisable to check the final dimensions by computing the resilience of the rod. The impact stress, by equation 3-20, is

$$s' = 330\left[1 + \sqrt{1 + \frac{2\times 2\times 30{,}000{,}000}{330\times 60}}\right] = 26{,}000 \text{ psi}$$

According to equation 3-25 the absorbed energy is

$$U = \frac{26{,}000^2\times 0.985\times 60}{2\times 30{,}000{,}000} = 665 \text{ in.-lb}$$

The impact energy which must be absorbed is, by equation 3-16,

$$K_i = 325\times\left(2 + \frac{26{,}000\times 60}{30{,}000{,}000}\right) = 665 \text{ in.-lb}$$

This check indicates that the calculations are correct.

To find the actual safety factor the corrected elastic limit must be determined. The size coefficient, by equation 5-8, is

$$e_{sz} = 1 - 0.4\times(1 - 0.85)\times(1.875 - 0.5) = 0.917$$

which gives $S_e' = 42{,}000\times 0.917 = 38{,}500$ psi. The actual safety factor is

$$n' = \frac{S_e'}{s'} = \frac{38{,}500}{26{,}000} = 1.48$$

Another method of calculating n' is by finding the maximum stress S_{max} that can be created in the main part of the rod without exceeding the elastic limit in the section weakened by the pinhole and stress concentration. Since the stresses are inversely proportional to the cross-sectional areas,

$$S_{max} = \left(\frac{38{,}500}{1.41}\right)\times\frac{0.7854\times 1.875^2 - 0.719\times 1.875}{0.994} = 38{,}700 \text{ psi}$$

Since S_{max} is greater than S_e', the latter value must be used. The maximum energy that could be absorbed without exceeding the elastic limit at any point is

$$U = \frac{38{,}500^2\times 0.985\times 60}{2\times 30{,}000{,}000} = 1{,}462 \text{ in.-lb}$$

and the actual safety factor, by equation 5-19, is

$$n' = \sqrt{\frac{1.462}{665}} = 1{,}482$$

Increase of impact strength. Equation 3-18 shows that the impact stress can be decreased by decreasing the impact travel h. If h is a clearance between two reciprocating parts, as between a wrist pin and its bearing, then the proper procedure is to reduce h to the minimum value permissible from the standpoint of a running fit.

The influence of the deformation e is in the opposite direction; that is, an increase of the deformation lowers the impact stress. Therefore machine parts subjected to impact should not be made more rigid than is necessary for proper operation.

The same conclusion may be reached by considering resilience. Equation 3-23 shows that the resilience of a member is increased if its deformation is increased; and this in turn requires an increase of stress. Whatever the nature of the stress is, the deformation of a member will be greatest

when all its sections have the same maximum significant stress. Thus, to best resist shock a member must have the shape of *uniform strength.*

Increase of resilience in bending is illustrated by the shape of springs in automobiles and railroad cars, Fig. 14-3. The springs have approximately the same strength in bending over the whole length. This characteristic makes them flexible and at the same time capable of absorbing heavy shocks.

In many instances the impact strength can be increased by reducing local stress concentration through a lowering of the form stress factor K.

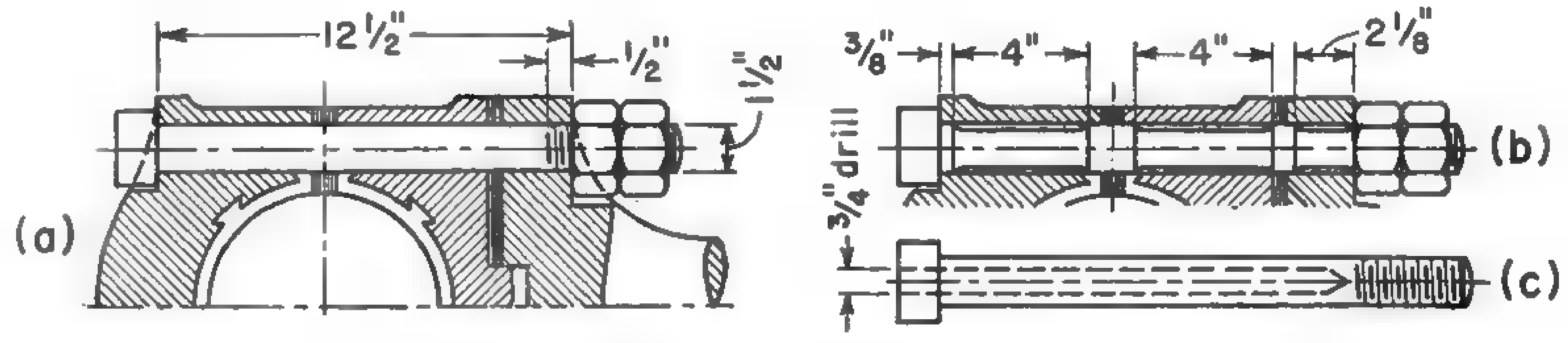


FIG. 5-6. Connecting-rod bearing bolt.

EXAMPLE 5-5. Find the impact strength in tension of the connecting-rod bolt, Fig. 5-6a, if SAE 3140 chrome-nickel steel is used and the safety factor is 1.5.

The elastic limit in tension of this steel, drawn at 1,000 F, is 95,000 psi. If the stock size is taken as the thickness of the head, which is found to be 2¼ in. by scaling the diagram in Fig. 5-6, the size coefficient is

$$e_{sz} = 1 - 0.4\times(1 - 0.70)\times(2.25 - 0.5) = 0.79$$

The design stress then is

$$S_d = \frac{95{,}000\times 0.79}{1.5} = 50{,}000 \text{ psi}$$

The form stress factor of the thread, according to section 3-6, is $K = 5.62$. The elongation of heat-treated SAE 3140 steel is 17 per cent, and $q = 0.4$. Hence the stress-concentration factor is

$$K' = 1 + 0.4(5.62 - 1) = 2.85$$

Therefore the nominal impact stress should not exceed

$$s_n = \frac{S_d}{K'} = \frac{50{,}000}{2.85} = 17{,}500 \text{ psi}$$

Since the area of the dangerous section of the thread, or the stress area, is 1.4041 sq in. (Table 11-1), the stress in the unthreaded solid part will be

$$s = \frac{17{,}500\times 1.4041}{0.7854\times 1.5^2} = 13{,}900 \text{ psi}$$

As $l = 12$ in. and $E = 31{,}000{,}000$ psi, the resilience is, by equation 3-25,

$$U = \frac{13{,}900^2\times 1.767\times 12}{2\times 31{,}000{,}000} = 66.0 \text{ in.-lb}$$

In order to increase the resilience by increasing the stress, the bolt can be turned down to a diameter of 1¼ in., leaving 1½ in. at the center and at the ends, as shown in Fig. 5-6b. At these places the bolt must fit accurately to secure the position of the bearing. The stress in the reduced section is increased to

$$s_1 = \frac{13{,}900\times 1.5^2}{1.0^2} = 31{,}300 \text{ psi}$$

The resilience now becomes

$$U=\frac{31{,}300^2\times 0.7854\times(4+4)+13{,}900^2\times 1.767\times(12-8)}{2\times 31{,}000{,}000}=121.3 \text{ in.-lb}$$

Thus the resilience, or impact strength, is almost doubled without changing the maximum stress. This stress occurs in the threaded part and is $S_d=50{,}000$ psi.

Second solution. The cross-sectional area of the bolt can be reduced to 0.982 sq in. by drilling a $\frac{3}{4}$-in. hole at the center of the bolt all the way down to the threaded part, as shown in Fig. 5-6c. The stress in this part will then be increased to

$$s=\frac{13{,}900\times 1.767}{0.982}=25{,}000 \text{ psi}$$

The resilience becomes

$$U=\frac{25{,}000^2\times 0.982\times 12}{2\times 31{,}000{,}000}=120.0 \text{ in.-lb}$$

To avoid a stress concentration at the bottom of the drilled hole, the bottom of the hole should have a gradual change of section, as shown in the illustration. The disadvantage of this method is that the bolt is more rigid in bending than if turned down as in Fig. 5-6b, and a high bending stress may be created if the seat is not quite normal to the bolt axis.

5-7. Repeated loads. The term endurance limit is commonly applied to the highest stress to which a material can be repeatedly subjected an infinite number of times without failing. The method of applying repeated loading when testing the material is usually by bending, and the stress changes from a maximum value in tension to the same value in compression. In a more general sense the endurance limit depends on the following factors:

a. The amplitude of the stress variation
b. The mean value of the stresses
c. The type of stresses invoked
d. The method of manufacturing the material, including its heat treatment
e. The size of the section
f. The condition of the surface
g. Discontinuities in the sections

Endurance diagram. The influence of the first three factors in the foregoing list is best expressed by a diagram like one of those shown in Figs. 4-2 to 4-10. According to present practice the design calculations of pieces made of steel and subjected to repeated loads are based on endurance diagrams. In these diagrams each endurance stress S_{en}' can be considered to be the algebraic sum of the stress amplitude S_a and the mean stress S_m, as in Fig. 5-7. Thus,

$$S_{en}'=S_a+S_m \tag{5-20}$$

where $S_a=0.5(S_1-S_2)$ and $S_m=0.5(S_1+S_2)$. The upper and lower stress limits, S_1 and S_2, must be taken with their signs. Evidently the limit stress for a given mean stress $S_{en}'=S_1$.

The influence of both S_a and S_m upon the endurance strength are taken into account automatically when the endurance diagram is used. As

explained in section 5-8, the calculations are based on stress amplitudes. The nominal stress amplitude S_{na}, which is S_a/K, is added to the allowable mean design stress S_{dm}; and this sum must be less than the endurance stress S_{en}'. The proper stress amplitude S_a is found from the endurance diagram of the material. The relation between S_{na} and S_a is illustrated in Fig. 5-14.

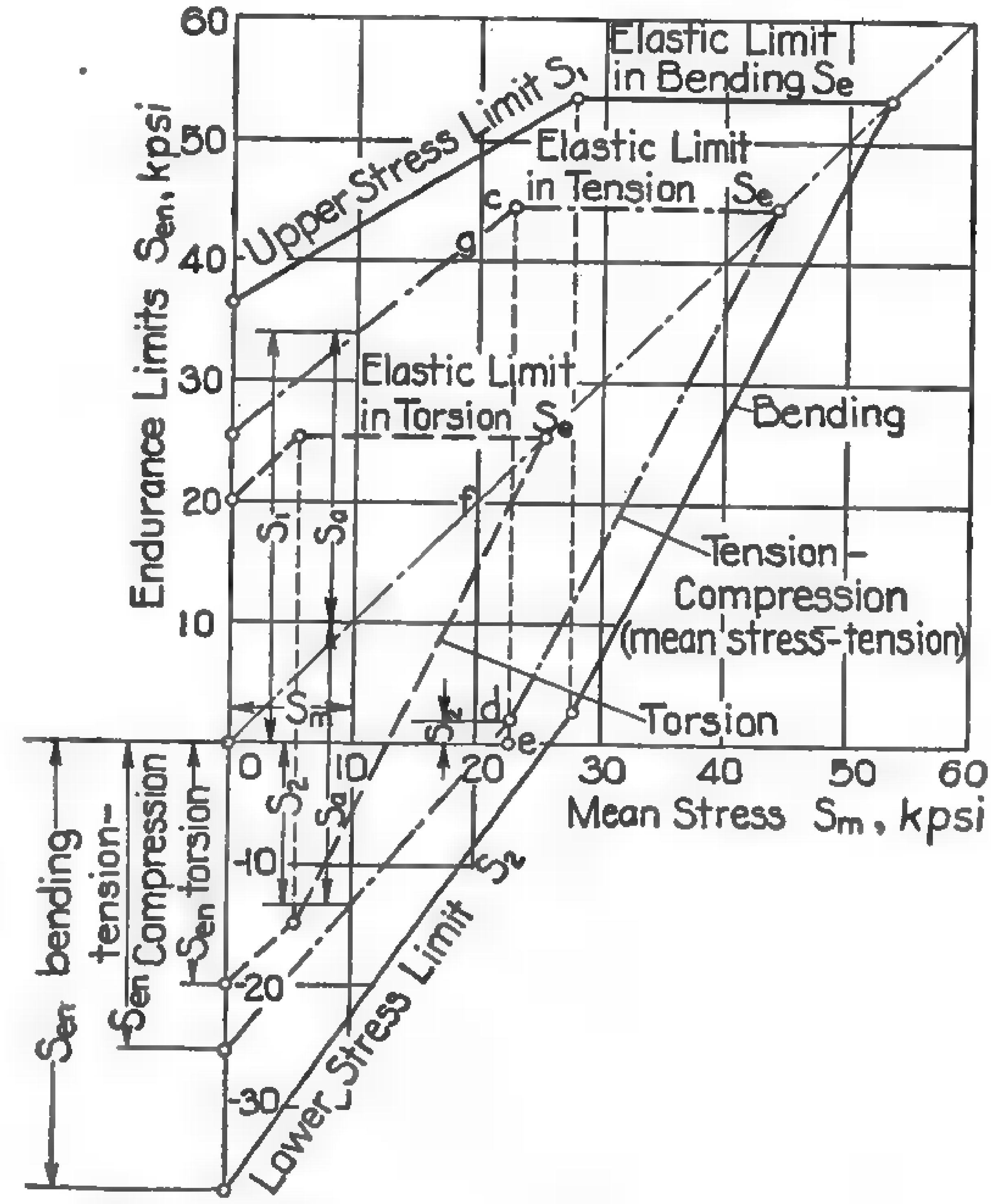


FIG. 5-7. Endurance diagram for three types of stresses.

Auxiliary diagrams. If a material is used for which an endurance diagram is not established, a preliminary diagram can be laid out by using the endurance limit S_{en} as found for alternating bending, and taking for the elastic limit in bending $S_e''=1.1\ S_e$, where S_e is the elastic limit in tension. The slopes of the upper-stress line and other lines should be those in established diagrams for which S_{en}/S_e is about the same. The endurance limits in tension and torsion must be taken lower than in bending, as explained at the end of section 3-10.

Safety factor. The usual procedure in designing parts subjected to repeated loads is to use endurance limits S_{en} instead of elastic limits S_e as limit stresses and to apply the same safety factors that are given in section 5-5 for static loads.

5-8. Endurance limits. Methods of manufacture and of heat treatment influence the endurance limits of a metal, but this influence can be established only experimentally and is best expressed by an endurance diagram.

The influence of size on the endurance limit is as pronounced as its influence on the elastic limit,[7] and the same size coefficient e_{sz}, determined by equation 5–8 and Table 5–1, can be used for each limit.[8] For shafts over 3 in. in diameter, e_{sz} may be taken equal to 0.75 until additional experimental data become available. The size correction is referred to the value of the stress amplitude S_a, and not to the whole endurance stress S_{en}'. The corrected stress amplitude is

$$S_a' = e_{sz}S_a \tag{5–21}$$

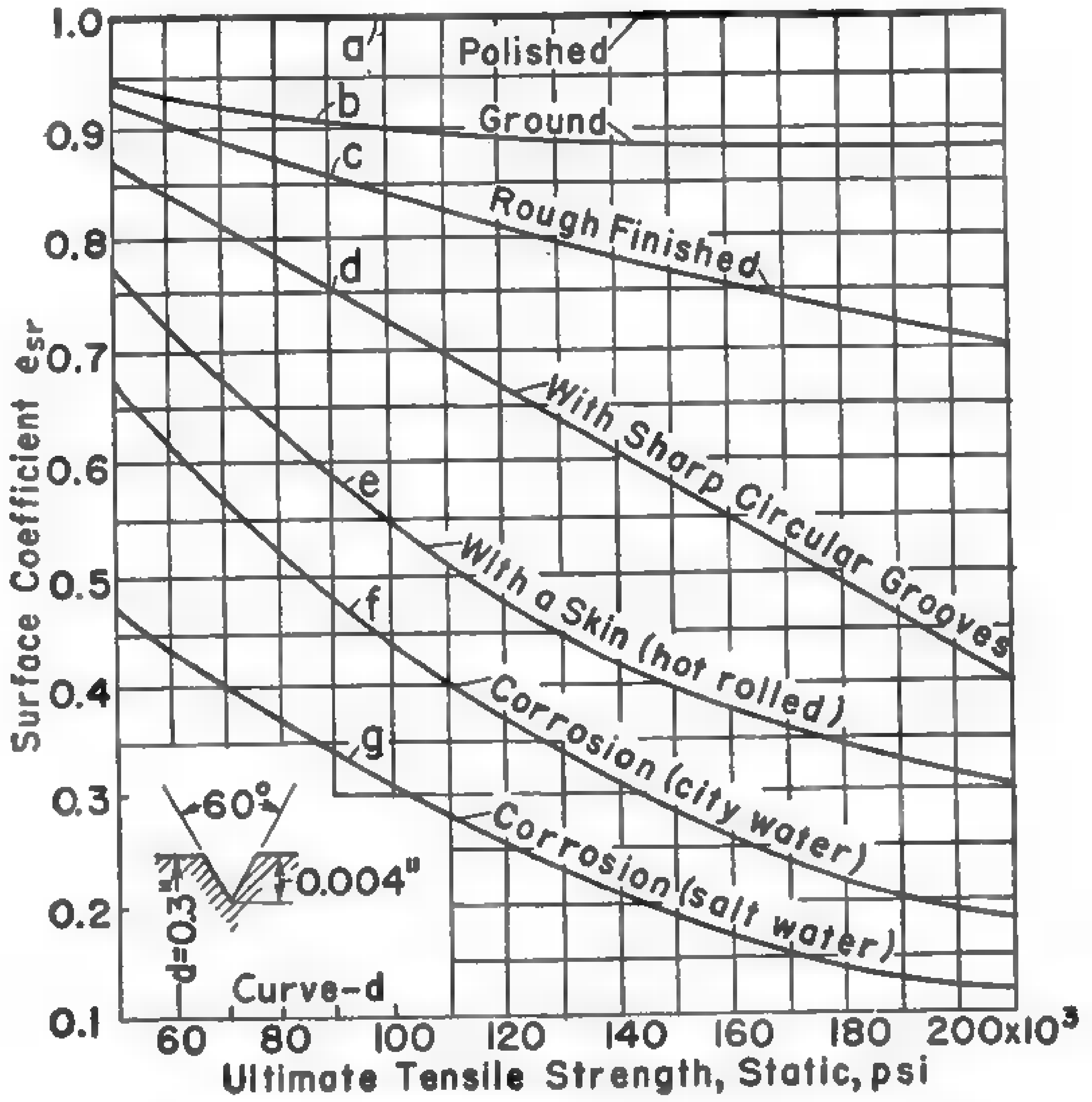


Fig. 5–8. Reciprocals of stress-concentration factors caused by surface condition.

Surface conditions. Experiments show that the nature of the surface has a very great influence upon the endurance strength of metals. The surface influences have greater effect for materials having high static ultimate strengths. The highest values of endurance strength are obtained with a perfectly smooth polished surface. Slight scratches, such as are produced by grinding, reduce the endurance strength by 5 to 12 per cent; a rough finish reduces it still more. A sharp, circular V groove only 0.044 in. deep may reduce the endurance strength of a shaft under certain circumstances as much as 63 per cent. The curves in Fig. 5–8 show these influences for materials with various ultimate strengths, the influence being expressed as a coefficient e_{sr}. This coefficient must be referred to the stress amplitude S_a and does not

[7] Battelle Memorial Institute, *Prevention of the Failure of Metals Under Repeated Stress* (New York: John Wiley & Sons, Inc., 1941), p. 123.

[8] Supplement to *Zeitschrift Verein Deutscher Ingenieure*, Vol. 78, No. 34 (August 25, 1934), pp. 2, 4.

affect the mean stress S_m. The limit value of the stress amplitude thus becomes

$$S_a'' = e_{sz}e_{sr}S_a \tag{5–22}$$

The values of e_{sr} from Fig. 5–8 can be used for bending, tension, and compression. For torsion the surface influence is smaller, and the corresponding value of e_{sr}' can be computed by the relation[9]

$$e_{sr}' = 0.425 + 0.575e_{sr} \tag{5–23}$$

Superficial plastic distortions, such as cold-chisel marks, hammer-blow marks, or center-punch marks, do not affect the endurance appreciably; but any scratches, such as those that come from vise jaws or from press fits, are almost as harmful as sharp grooves, the surface coefficient for which is represented by curve *d*, Fig. 5–8. Particularly detrimental is the corrosion effect, which is indicated by curves *f* and *g*.

Surface rolling increases the endurance limit of mild annealed steel by 24 to 32 per cent.[10]

Discontinuities. Any discontinuity affects the endurance strength of a machine part. However, the effect is considerably less than indicated by the corresponding value of the form stress factor K. For repeated stresses this influence is expressed by the ratio S_{en}/S'_{en}, where S_{en} is the endurance limit without a discontinuity and S_{en}' is the endurance limit with the discontinuity. This ratio may be called the *endurance stress-concentration factor*.[11] As it has definite relation to the form stress factor K, but refers to repeated stresses, the designation K_r will be used. Thus,

$$K_r = \frac{S_{en}}{S_{en}'} \tag{5–24}$$

5–9. Stress amplitudes. A more refined procedure may be used for parts in which the repeated stress is not completely reversed and which are made of materials for which endurance diagrams exist. This procedure is based on stress amplitudes.

Significant stresses. If the nominal stress amplitude S_{na} is multiplied by the stress-concentration factor K_r, the product is the significant stress amplitude; or

$$s_{sga} = K_r s_{na} \tag{5–25}$$

Design stress. The design stress amplitude can now be determined as follows:

$$S_{da} = \frac{S_a''}{n} = \frac{e_{sz}e_{sr}S_a}{n} \tag{5–26}$$

[9] Supplement to *Z. VDI*, Vol. 77, No. 42 (October 21, 1933), p. 4.

[10] O. J. Horger, "Increase of Endurance Strength of Steel," *Journal of Applied Mechanics*, Vol. 2, No. 4 (December, 1935), p. A-128.

[11] Seely, *Advanced Mechanics of Materials*, p. 229.

where the safety factor n should be not less than 1.25 for aircraft design and should be 1.5 or slightly greater in all other cases.

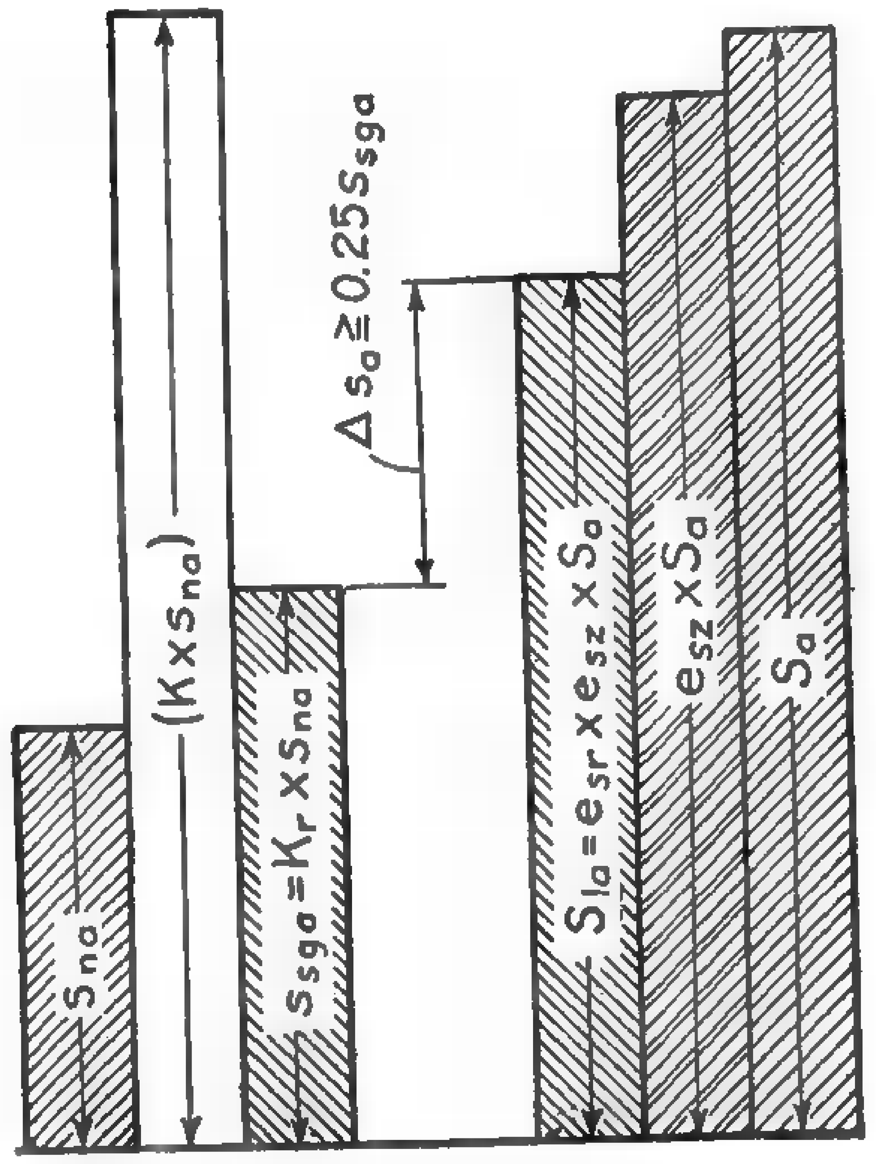


FIG. 5-9. Relations between various stress amplitudes.

The actual safety factor for a part whose dimensions are established is found by considering the general definition and applying the equation

$$n' = \frac{e_{sz}e_{sr}S_a}{K_r s_{na}} \tag{5-27}$$

The relations between the various stress amplitudes that must be considered in a machine part subjected to repeated stresses may be illustrated by Fig. 5-9. The difference Δs_a between the limit stress amplitude S_{la} and the significant stress amplitude S_{sga} represents the margin of safety of the design. This difference should be not less than 0.25 S_{sga} for aircraft design and should be at least twice as much for all other cases.

Stress-load relations. For the sake of simplicity a case will be considered in which the piece is subjected to repeated direct loads, causing tension and compression. If the maximum load is F_1, the minimum load is F_2, and the cross-sectional area is A, then the nominal stresses are

$$s_{n1} = \frac{F_1}{A} \qquad s_{n2} = \frac{F_2}{A} \tag{5-28}$$

The nominal stress amplitude determined by equations 3-54 and 5-28 is

$$s_{na} = \frac{s_{n1} - s_{n2}}{2} = \frac{F_1 - F_2}{2A} \tag{5-29}$$

The maximum, or significant, stress amplitude, from equations 5-25 and 5-29, is

$$s_{sga} = K_r \frac{F_1 - F_2}{2A} \tag{5-30}$$

If the value of s_{sga} given by equation 5-30 is equated to the value of S_{da} given by equation 5-26, the result is

$$K_r \frac{F_1 - F_2}{2A} = \frac{e_{sz}e_{sr}S_a}{n} \tag{5-31}$$

Hence the endurance stress amplitude is

$$S_a = nK_r \frac{F_1 - F_2}{2Ae_{sz}e_{sr}} \tag{5-32}$$

Similarly, it can be established that the maximum mean stress, which is not affected by the surface factor, is

$$S_m = nK_r \frac{F_1 + F_2}{2Ae_{sz}} \tag{5-33}$$

The ratio of the stress amplitude to the corresponding mean stress is

$$\frac{S_a}{S_m} = \frac{F_1 - F_2}{(F_1 + F_2)e_{sr}} \tag{5-34}$$

Graphical solution. The magnitudes of S_a and S_m can be easily found from a given endurance diagram. In the endurance diagram in Fig. 5-10, the vertical distances ab and bc represent the denominator and the numerator, respectively, of the right-hand member in equation 5-34. From the point d, at which the auxiliary line Oc intersects the endurance-limit line, draw a vertical line df. This line intersects the 45° line at e. Then de represents S_a, and ef or Of represents S_m.

Bending and torsion. In the case of bending, the stress amplitude can be expressed by the equation

$$S_a = nK_r \frac{M_1 - M_2}{2Ze_{sz}e_{sr}} \tag{5-35}$$

where M_1 is the moment producing the upper stress, M_2 is the moment producing the lower stress, and Z is the section modulus. Also, the maximum mean stress is

$$S_m = nK_r \frac{M_1 + M_2}{2Ze_{sz}} \tag{5-36}$$

The ratio of the stress amplitude to the corresponding mean stress is

$$\frac{S_a}{S_m} = \frac{M_1 - M_2}{(M_1 + M_2)e_{sr}} \tag{5-37}$$

In the same manner, for torsion,

$$S_a = nK_r \frac{T_1 - T_2}{2Z_o e_{sz}e_{sr}'} \tag{5-38}$$

where T_1 is the torsional moment producing the upper stress, T_2 is the moment producing the lower stress, and Z_o is the polar section modulus. Also, the maximum mean stress is

$$S_m = nK_r \frac{T_1 + T_2}{2Z_o e_{sz}} \tag{5-39}$$

Then

$$\frac{S_a}{S_m} = \frac{T_1 - T_2}{(T_1 + T_2)e_{sr}'} \tag{5-40}$$

The values of S_a and S_m can be determined graphically by the construction shown in Fig. 5-10 if the bending moments or torsional moments are used in place of the loads F_1 and F_2 and the corresponding limit lines in the endurance diagram.

Simultaneous action of repeated and static loading. When a part is subjected to both a static load and a repeated load, the static load F' is simply added algebraically to the maximum and minimum loads F_1 and F_2. According to equation 5–33 this operation changes the magnitude of S_m; and according to the relations between S_m and S_a on the endurance diagrams, it also changes the stress amplitude S_a and consequently the design stress amplitude S_{da}. The absolute influence of the static load F' depends on the signs F', F_1, and F_2, and that load may either increase or decrease the maximum stresses in the part.

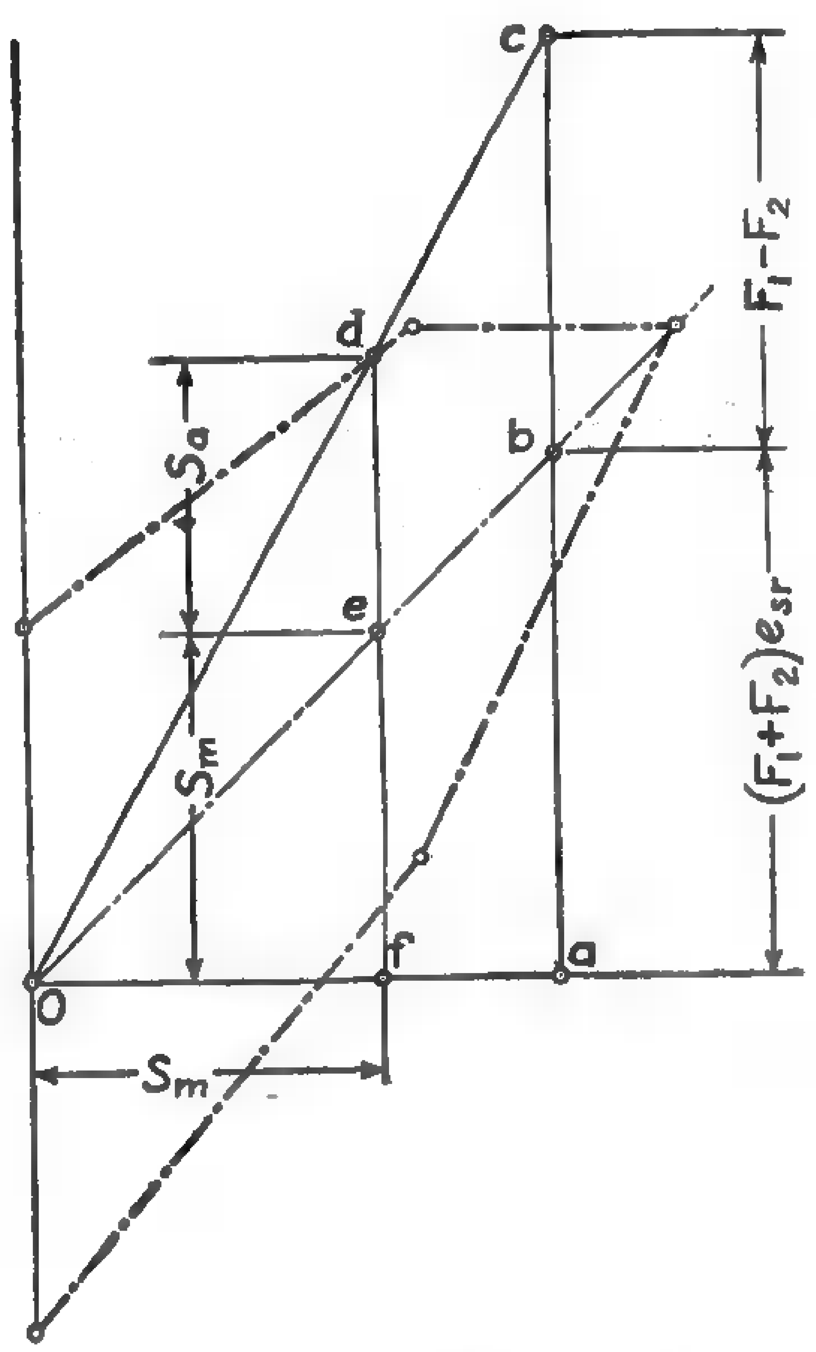


Fig. 5–10. Finding stress amplitudes.

The same principle applies if the loads produce a bending moment or a torsional moment.

Example 5–6. Determine the diameter of a turned and ground steel shaft which transmits a torque fluctuating from $T_1 = 80{,}000$ lb-in. to $T_2 = -40{,}000$ lb-in.

A suitable material is SAE 1035 steel, for which the endurance diagram is shown in Fig. 4–4. To solve the problem graphically, the procedure described here for the typical diagram in Fig. 5–10 may be followed. In this example, $T_1 + T_2 = 40{,}000$ lb-in. and $T_1 - T_2 = 120{,}000$ lb-in.; and from Fig. 4–4 the endurance limit for torsion is found to be 26,000 psi when $S_a = 20{,}000$ psi and $S_m = 6{,}000$ psi.

For a material for which $S_u = 78{,}500$ psi and for a ground surface, curve *b* in Fig. 5–8 gives $e_{sr} = 0.91$. By equation 5–23, for torsion,

$$e_{sr}' = 0.425 + 0.575 \times 0.91 = 0.95$$

For preliminary calculations a size factor $e_{sz} = 0.75$ and a safety factor $n = 1.5$ may be assumed. Since $K_r = 1$, equation 5–38 gives

$$Z_o = \frac{1.5 \times 120{,}000}{2 \times 0.75 \times 0.95 \times 20{,}000} = 6.32 \text{ in.}^3$$

The corresponding shaft diameter is

$$D = \sqrt[3]{\frac{16 \times 6.32}{\pi}} = 3.18, \text{ or } 3\tfrac{3}{16} \text{ in.}$$

Since $D > 3$ in., the size factor $e_{sz} = 0.75$ remains unchanged.

If the shaft diameter were calculated without considering the endurance diagram, equation 2–13 would be applied for finding the polar section modulus. Based on the maximum torque $T_1 = 80{,}000$ lb-in., the endurance limit in torsion for complete stress reversal $S_{en} = 20{,}000$ psi, $e_{sz} = 0.75$, and $n = 1.5$, the value of the section modulus would be

$$Z_o = \frac{T}{S_{en} e_{sz}} = \frac{80{,}000 \times 1.5}{20{,}000 \times 0.75} = 8.0 \text{ in.}^3$$

Then

$$D = \sqrt[3]{\frac{16 \times 8}{\pi}} = 3.44 \text{ in., or } 3\tfrac{1}{2} \text{ in. at least}$$

This result would be on the safe side, but the shaft would be unnecessarily heavy.

5–10. Endurance stress-concentration factor. The magnitude of the stress-concentration factor K_r for endurance limits is found from tests, as shown by equation 5–24. These tests show that K_r depends on the discontinuity, on the form stress factor K, and on the material of which the specimen is made.

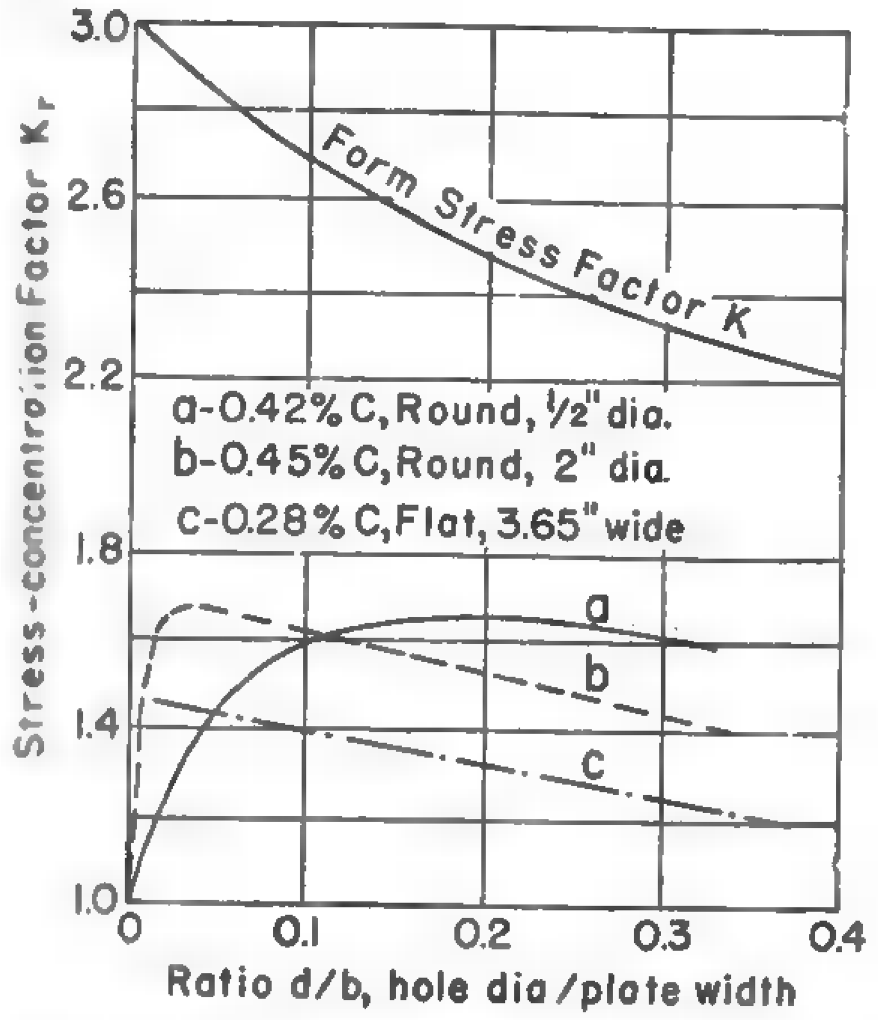


Fig. 5–11. Stress concentration factors for bars with transverse holes in bending.

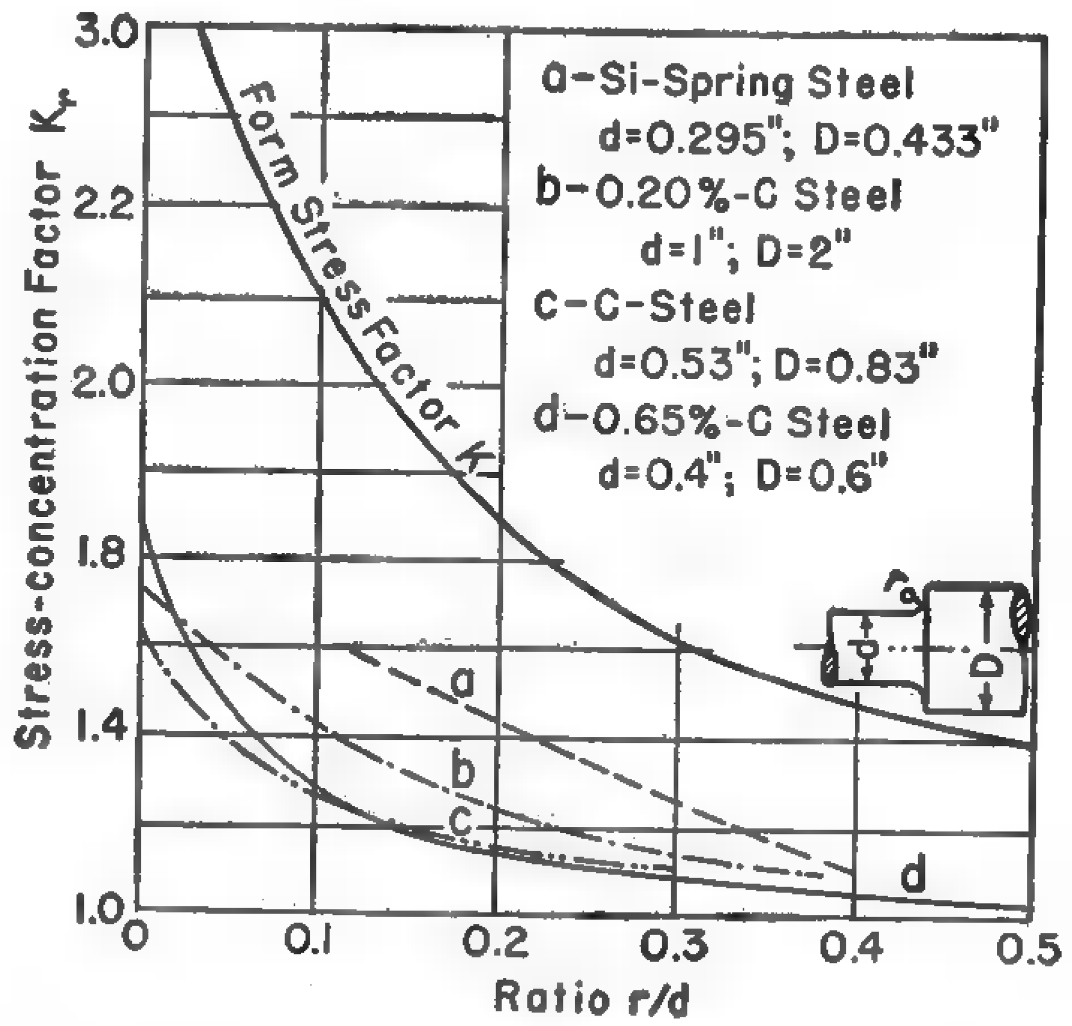


Fig. 5–12. Stress-concentration factors for shafts with fillets in bending.

It will take many years to determine K_r for all materials and all possible types of discontinuities. In the meantime K_r may be determined with sufficient accuracy for practical purposes by the relation

$$K_r = 1 + q_r(K - 1) \tag{5–41}$$

The values of K and q_r are based on data already accumulated. The sensitivity index (see equation 5–10) is

$$q_r = \frac{K_r - 1}{K - 1} \tag{5–42}$$

This index is a measure of the sensitivity of the material to repeated stresses in the presence of various discontinuities.

In Figs. 5–11 and 5–12 are shown values of K_r found experimentally for the bending of bars with transverse holes and for the bending of shafts with fillets. These curves show the influence of the relative size of the discontinuity on the value of K_r.[12] The influence of the actual size of the tested

[12] R. E. Peterson, "Stress Concentration Phenomena in Fatigue of Metals," *Trans. ASME*, Vol. 55, APM-55-19 (1933), p. 157.

bars and shafts is shown in Fig. 5–13. The curves in Fig. 5–14 show the influence upon K_r of the size of round bars tested in reversed bending and also of the fillet radius r and material. Curves a and b were obtained with

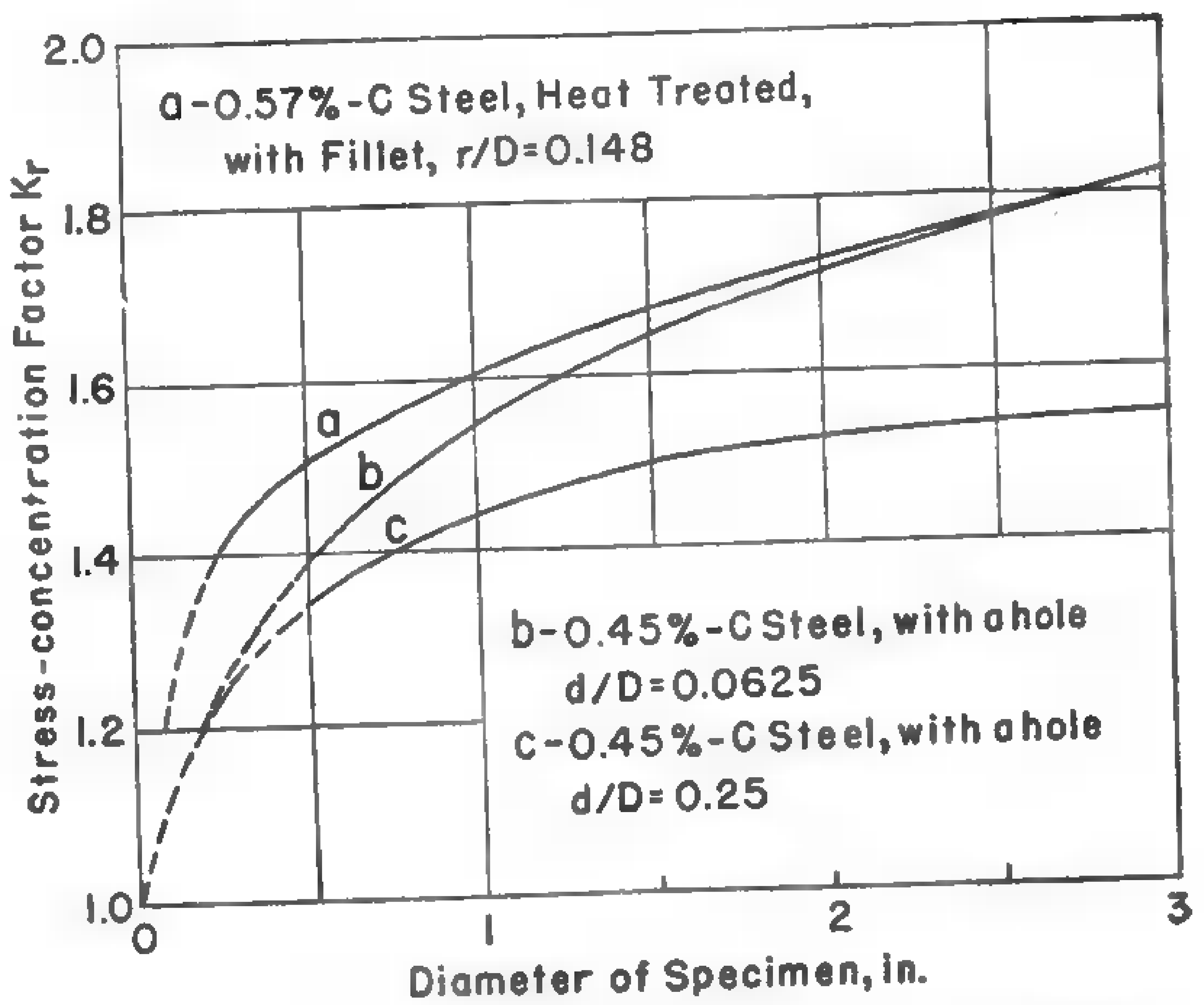


FIG. 5–13. Influence of size of hole on stress concentration.

specimens turned from steel having a carbon content of 0.45 per cent and normalized, whereas curve c was obtained with specimens made from Ni-Mo steel (C, 0.52; Ni, 2.96; Mo, 0.38; Mn, 0.68; Si, 0.19) heat-treated and drawn.[13]

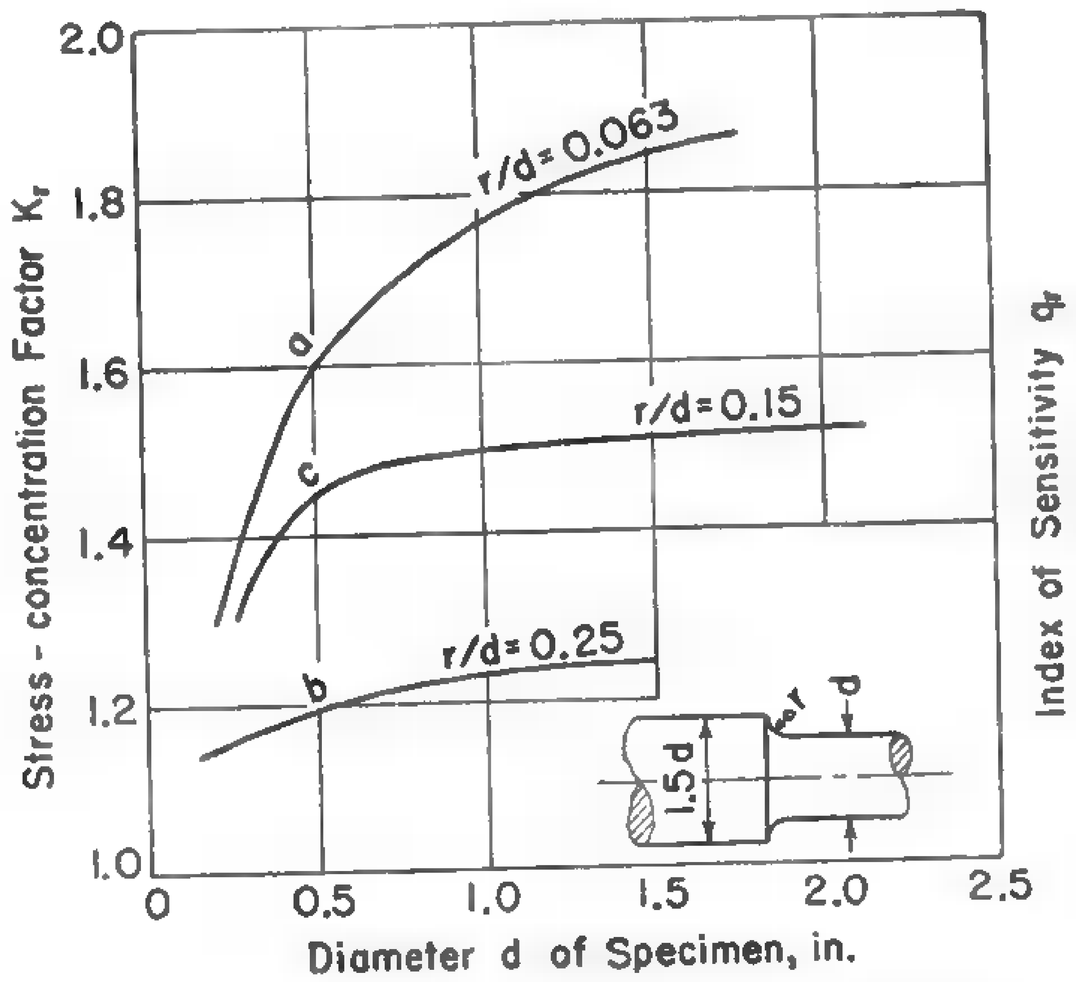


FIG. 5–14. Influence of size on stress concentration in bending.

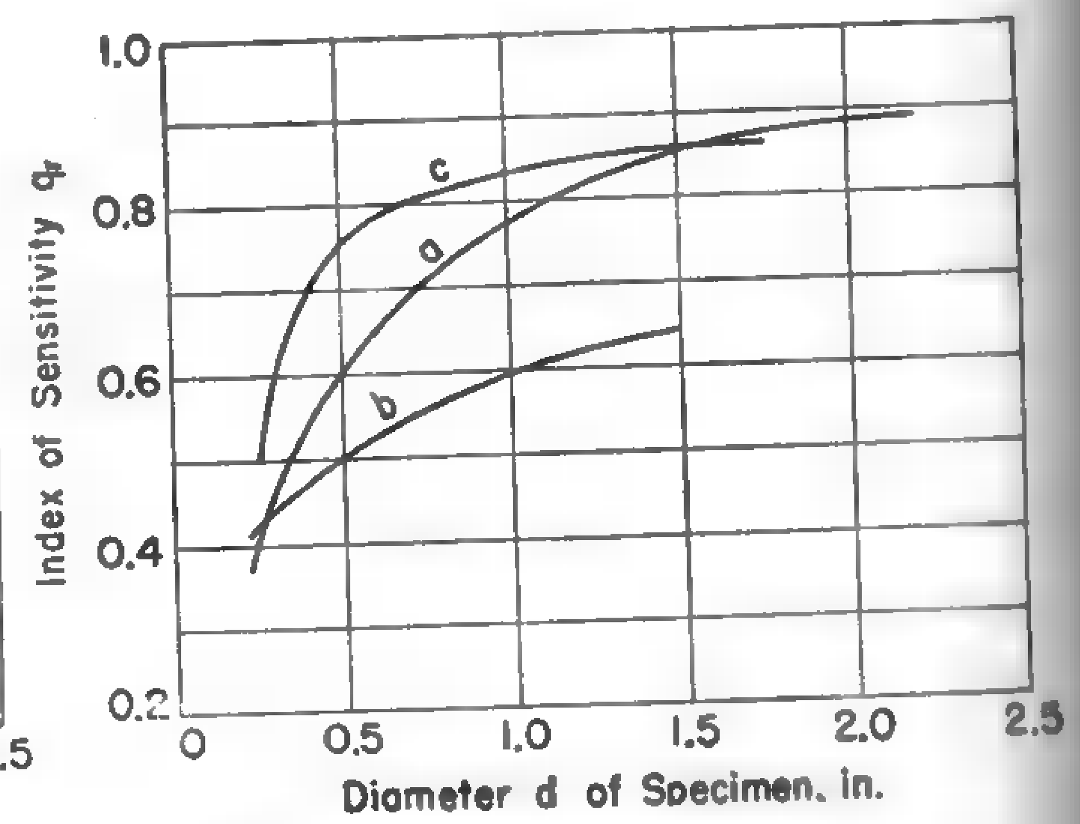


FIG. 5–15. Influence of size on index of sensitivity.

By using data from Figs. 5–14 and 3–17 and allowing a reduction of 20 per cent for bending, values of the corresponding sensitivity index q_r were

[13] R. E. Peterson and A. M. Wahl, "Fatigue of Shafts at Fitted Members with a Related Photoelastic Analysis," *JAM*, Vol. 2, No. 1 (March, 1935), p. A-20.

TABLE 5–2

INDEX OF SENSITIVITY FOR REPEATED STRESSES

Material	Average Index of Sensitivity q_r		
	Annealed or Soft	Heat-treated and Drawn at 1200 F	Heat-treated and Drawn at 900 F
Armco iron, 0.02% C	0.15–0.20	...	...
Carbon steel			
0.10% C	0.05–0.10	...	...
0.20% C (also cast steel)	0.10	...	...
0.30% C	0.18	0.35	0.45
0.50% C	0.26	0.40	0.50
0.85% C	...	0.45	0.57
Spring steel, 0.56% C, 2.3 Si, rolled	...	0.38	...
SAE 3140, 0.37 C; 0.6 Cr; 1.3 Ni	0.25	0.45	...
Cr-Ni steel, 0.8 Cr; 3.5 Ni	...	0.25	0.70
Stainless steel, 0.3 C; 8.3 Cr; 19.7 Ni	0.16	...	...
Cast iron	0–0.05	...	...
Copper, electrolitic	0.07	...	...
Duraluminum	0.05–0.13	...	...

computed by equation 5–42. The results are presented in Fig. 5–15. Although the curves of Fig. 5–15 should not be considered as giving accurate data for q_r because they cover a limited number of tests, they show the trends clearly and give at least the order of the values of q_r.

In Fig. 5–16 are shown values of the sensitivity index for carbon steels compiled from tests published in technical literature. The heavier, lower lines refer to average values, while the upper lines represent approximately maximum values. Finally, Table 5–2, compiled from the same sources, gives data for a wider range of materials. These data should be considered only as a first approximation, the possible discrepancies being about ± 10 per cent. For this reason it is useless to segregate the influence of different types of discontinuities, such as transverse holes, grooves, or fillets, or for different types of load. Table 5–2 shows that for various materials q_r ranges from zero up to 0.70. The index is smaller for ductile materials, and it increases as the ductility is decreased, as by heat treatment. Armco iron, however, is rather sensitive to stress concentration, in spite of its ductility.

Information from Fig. 5–15 and Fig. 5–16, combined with the data of Table 5–2, furnishes a fair basis for estimating q_r for any particular case.

Screw threads. The few data available for screw threads indicate high values for K_r. The following values are found by repeated tension:[14] 2.84 for an NC screw and 1.76 for a Whitworth screw, both made of SAE 1030 steel; 3.85 and 3.32 for the same threads, respectively, on screws made of SAE 2320 heat-treated steel.

[14] H. F. Moore and P. K. Henwood, *The Strength of Screw Threads Under Repeated Tension*, Bulletin No. 264, University of Illinois Experiment Station (1934), p. 10.

Keyways. Endurance tests with keyways cut with very sharp corners have shown that the stress-concentration factors are not high. For very ductile steel $K_r = 1.14$; for hard SAE 1065 steel, $K_r = 1.27$.[15] With a form

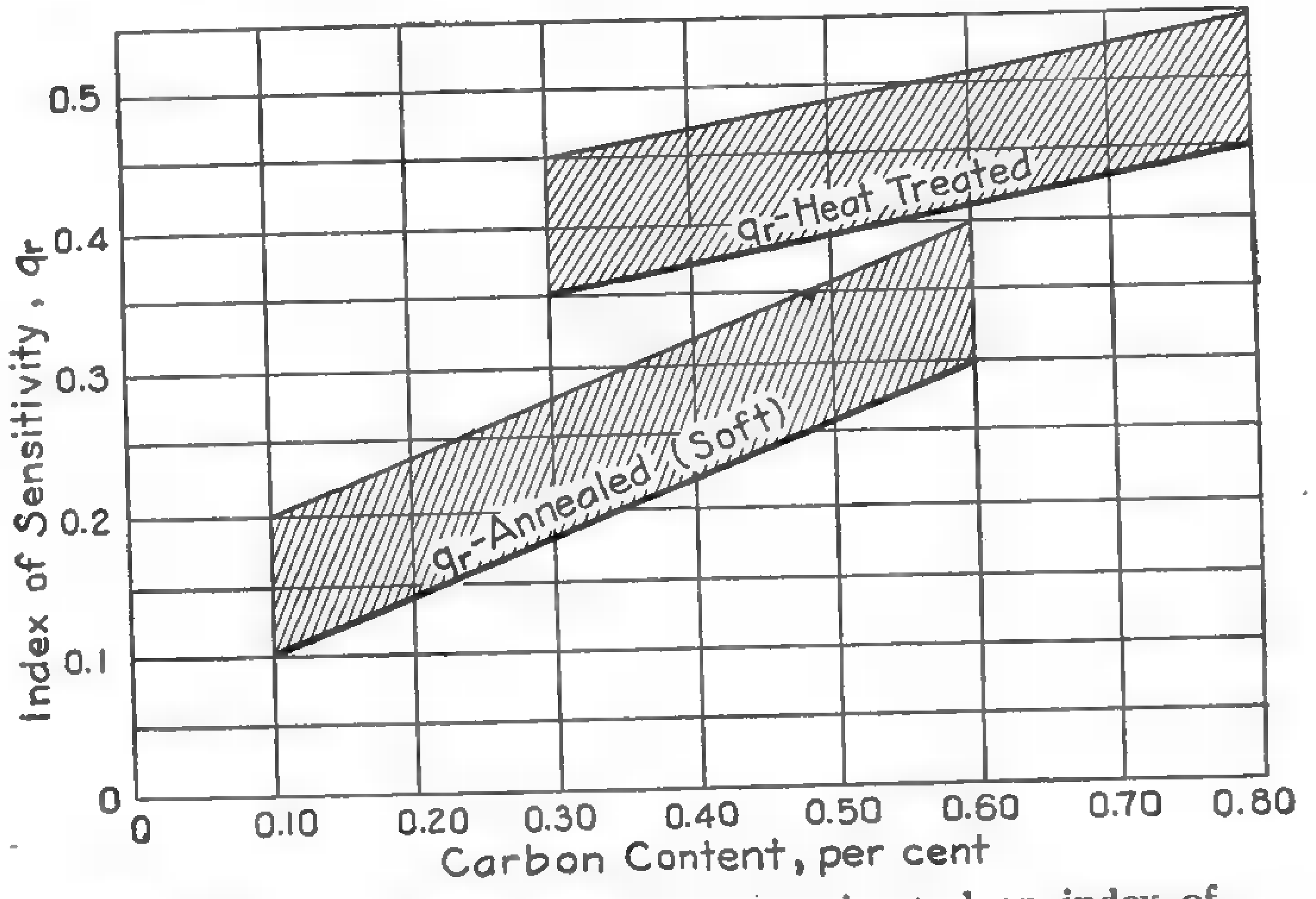


FIG. 5-16. Influence of carbon content in steel on index of sensitivity.

stress factor $K = 4.8$, the corresponding values of the index of sensitivity q_r are 0.04 and 0.07, respectively. The width of the keyway does not affect K_r.

Other tests have shown that sled-runner keyways, cut with an ordinary milling cutter having the same width as the keyway, weaken shafts much less than profiled keyways cut by a cutter having a diameter equal to the keyway width. In these tests, failure occurred at the end of the keyway.[16]

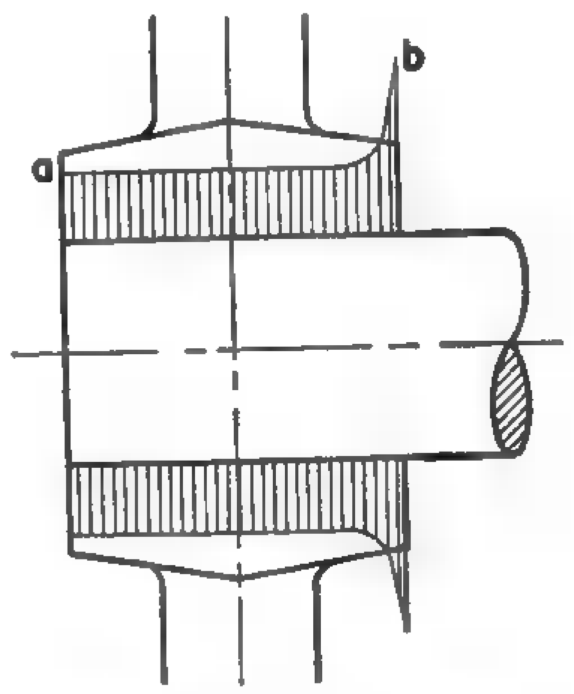


FIG. 5-17. Press-fit stress concentration.

Press and shrink fits. When a shaft is pressed or shrunk into a hub, the compressive stress in the shaft increases near the end of the hub, as indicated by curve *ab* in Fig. 5-17, because of the resistance of the fibers of the uncompressed end of the shaft. The resulting stress-concentration factor K_r may reach a considerable magnitude, as shown in Fig. 5-18.[17] With a mild-steel shaft shrunk into a hardened hub or bushing, K_r may become as high as 2.12. At the same time the surface of a highly stressed shrink fit has a tendency to oxidize. Oxidation, in turn, causes rust, and its action increases progressively with rust. In a part subjected to repeated

[15] Seely, *Advanced Mechanics of Materials*, p. 229.
[16] Peterson, *loc. cit.*, p. 161.
[17] A. Thum and F. Wunderlich, "Der Einfluss von Einspann—und Kraftangriffstellen auf die Dauerhaltbarkeit der Konstructionen," *Z. VDI*, Vol. 77 (1933), p. 851.

stresses rusting must be prevented by all available means, such as applying rust-preventing grease, making frequent inspections of exposed places, and cleaning such places and covering them with oil paint or grease.

5-11. Increase of endurance strength. Since the stress-concentration factor K_r is a function of the form stress factor K, a lowering of the latter will lower K_r and will thus increase the strength of the piece. Another way of neutralizing or diminishing the localized stresses is to set up stresses of opposite action.

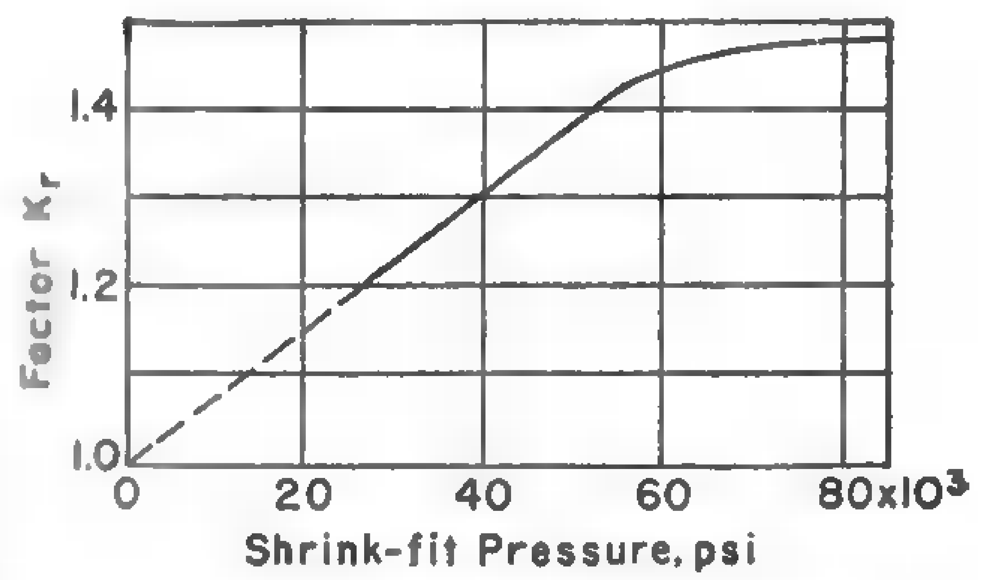


FIG. 5-18. Stress-concentration factor due to shrink-fit pressure.

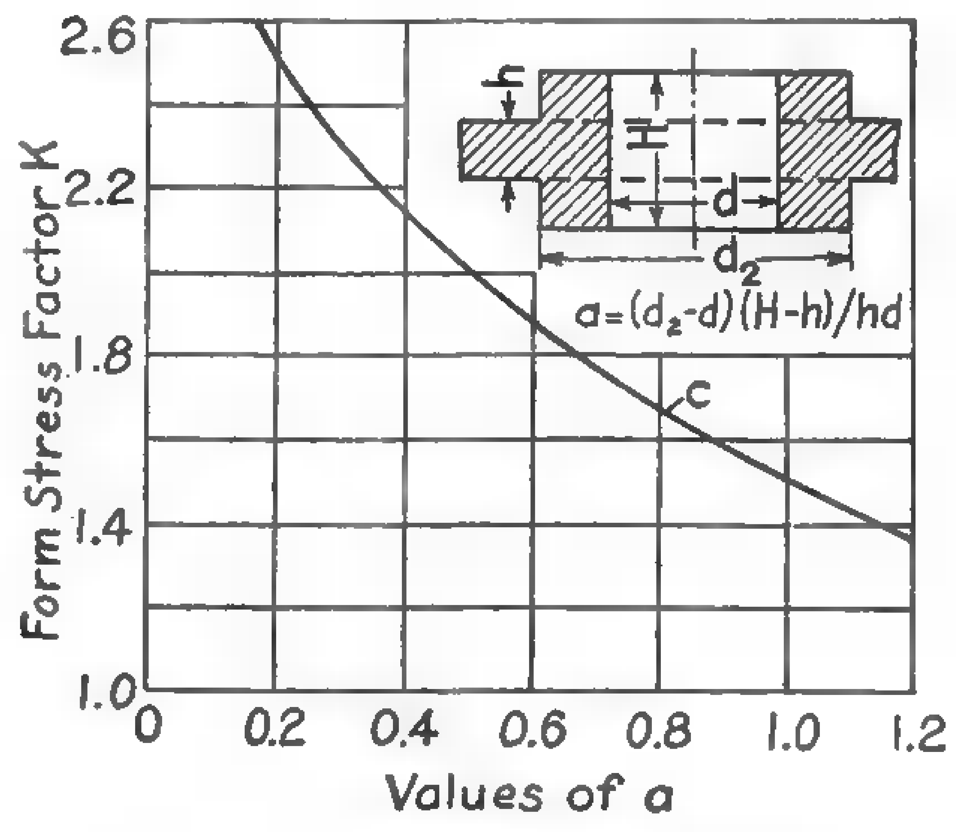


FIG. 5-19. Reinforcing bead to lower stress concentration.

Reduction in form-stress factor. The effect of a hole in a disk can be decreased by a symmetrical reinforcing bead, as shown in Fig. 5-19. The reduction of the form stress factor K is a function of the ratio of the cross-sectional area of the bead proper, which is $(d_2 - d)(H - h)$, to the area of the diametrical section taken out by the hole, or hd. Designating the ratio $(d_2 - d)(H - h)/hd$ as a, and plotting the values of K against those of a, curve c is obtained. This shows that with $a = 1$ the factor K can be reduced to about one-half of its maximum value of 3.[18]

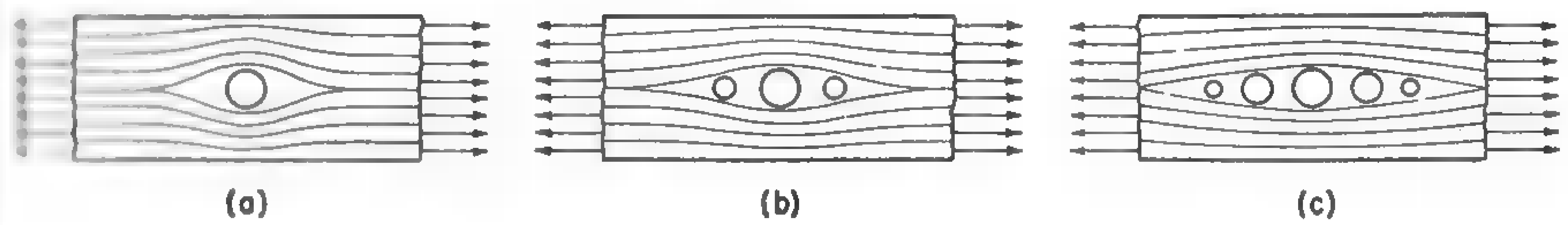


FIG. 5-20. Reduction of stress concentration in tension.

The problem of reducing the magnitude of the form stress factor is greatly simplified by using the method of force-flow lines as indicated in section 3-9.

Holes. Stress concentration due to a hole in a tension member, such as that in Fig. 5-20a, can be reduced by drilling additional holes which will give smoother force-flow lines, as indicated in Figs. 5-20b and c.

[18] S. Timoshenko and W. Dietz, "Stress Concentration Produced by Holes and Fillets," *Trans. ASME*, Vol. 47 (1925), p. 207.

The injurious effect of a transverse hole in a shaft subjected to bending or torsion can be reduced by filing tangential notches near the edge of the hole, as shown in Fig. 5–21a at *c*. This spreads the stress concentration over a larger section and, by decreasing its intensity, reduces K and K_r. Pressing in the stress-relieving grooves instead of filing them gives still better results. A similar effect is obtained by drilling holes crosswise, as in Fig. 5–12b at *e*.

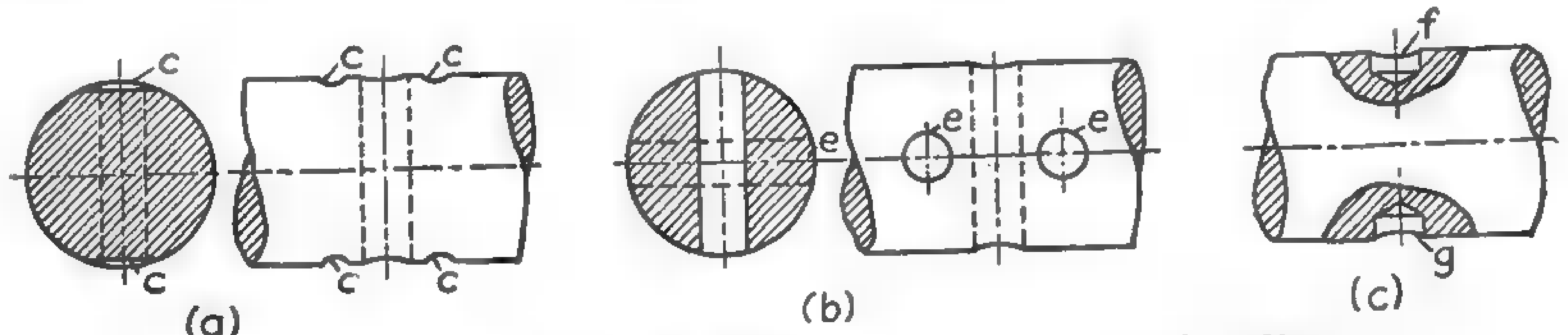


FIG. 5–21. Reduction of stress concentration in bending.

Asymmetry. Stress concentration is also caused by unsymmetrical shapes. The endurance strength of a shaft with one radial hole, as in Fig. 5–21c at *f*, can be raised by drilling another hole *g* symmetrical with the hole *f*.

Fillets. The best method of lowering stress concentration near a fillet is to increase the radius of the fillet. The advantage of a larger fillet is illustrated in Figs. 5–22a and b. If the fillet radius cannot be increased in the usual way, the fillet may be cut into the shoulder, as in Fig. 5–22c, or even in the side of the smaller shank, as in Fig. 5–22d. Another procedure is to smoothen the force-flow lines by cutting notches, as in Fig. 5–22e, or by drilling holes, as in Fig. 5–22f.

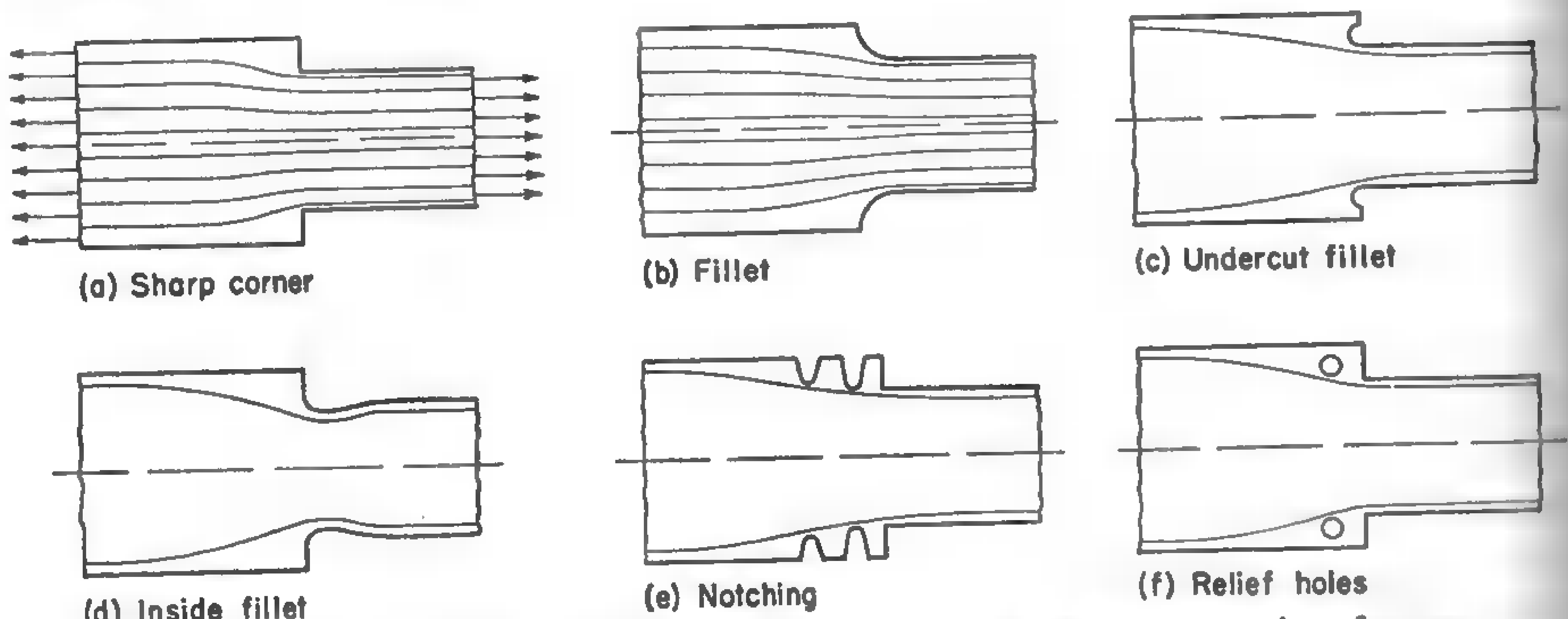


FIG. 5–22. Test samples of steel for determining stress-concentration factors.

In Fig. 5–23a is shown a valve of an internal-combustion engine that broke at *f* by progressive fracture because of the stress concentration due to an insufficient fillet *r*. The trouble was overcome by increasing the fillet *r*, as shown in Fig. 5–23b. The material from the top is taken out to obtain a more uniform thickness and thus to reduce stress concentration. In Fig. 5–23c is shown another solution which makes it possible to bring the top of the guide *g* closer to the valve seat.

Stress concentration due to square shoulders in a shaft of SAE 1080 steel having the dimensions shown in Fig. 5–24a lowered the endurance strength from $S_{en} = 48{,}000$ psi to 23,000 psi, and thus gave a stress-concentration factor $K_r = 2.09$. A fillet with a $\frac{1}{4}$-in. radius, as in Fig. 5–24b, raised S_{en} to 44,000 psi and gave $K_r = 1.09$; and a 1-in. radius as in Fig. 5–24e, raised S_{en} to 47,500 and gave $K_r = 1.01$.[19] For ingot iron the values of K_r were 1.86 for a square shoulder and 1.18 for a $\frac{1}{4}$-in. radius.

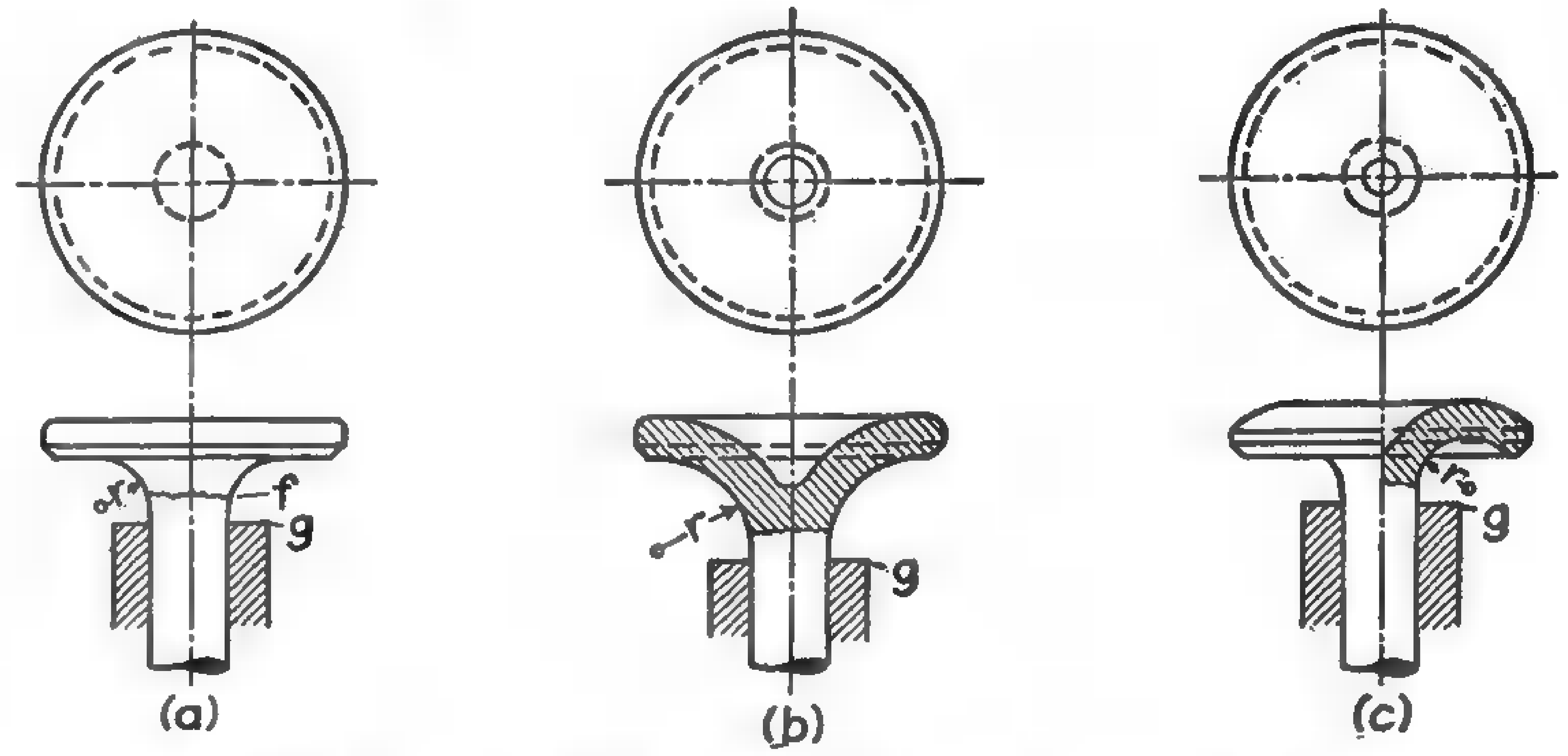


FIG. 5–23. Exhaust valve for an internal combustion engine.

If a shaft must be turned down to a smaller diameter, as in Fig. 5–25a, and the radius of the fillet cannot be increased, stress concentration can be reduced in several ways.[20] A flat groove reducing the larger shaft diameter from $\frac{7}{8}$ in. to $\frac{25}{32}$ in., as in Fig. 5–25b, increases the endurance limit by 13.5 per cent. A slightly increased fillet radius with a slight reduction of the small shaft diameter, as in Fig. 5–25c, gives an even greater endurance increase

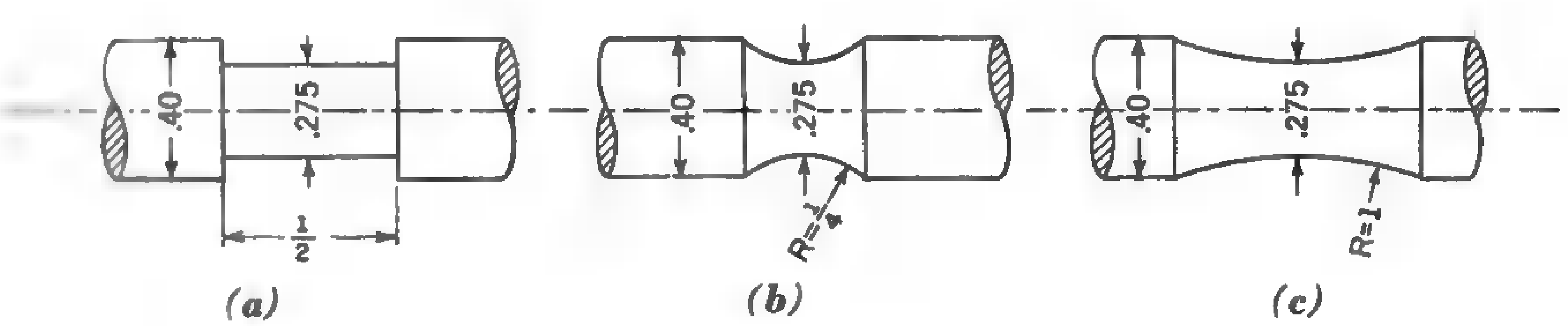


FIG. 5–24. Test samples of steel for determining stress concentration factors.

(14.8 per cent), probably because of a lower rigidity of the shaft juncture. As indicated in Fig. 5–25d, a combination of both methods shown in Figs. 5–25b and c gives only a slight additional increase (to 15.8 per cent). The explanation of the beneficial effect of the additional grooves may be found by visualizing the force-flow lines in a shaft subjected to bending or in concentric hollow shafts subjected to torsion.

[19] Battelle Memorial Institute, *op. cit.*, p. 59.

[20] O. J. Horger and T. V. Buchwalter, "How to Increase Endurance Strength," *Product Engineering*, Vol. 12, No. [illegible] (February, 1941), p. 78.

In another case the stress concentration was lowered by cutting eccentric circular grooves to the depth c, Fig. 5–26.[21] This operation removed the most highly stressed material and increased the elastic length l. The torsional shear stresses that existed before the grooves were cut are shown in a polar diagram by the curve a, and those that occurred after the grooves were cut are shown by the dotted curve b. This method could be applied in this particular case because the crankshaft, originally designed for plain bearings, was used with ball bearings.

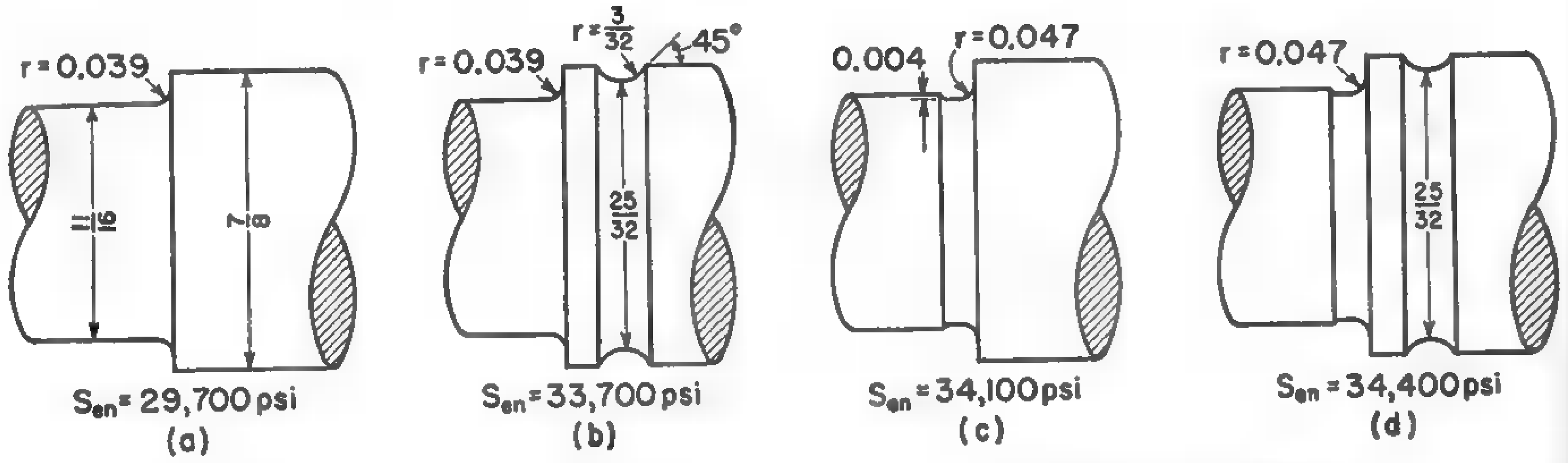


Fig. 5–25. Methods of increasing endurance of a shaft in bending.

Decrease of local stresses. Parts fractured through repeated loading show that the fracture always starts at the point where the tensile stress has a maximum value.[22] Any process that can lower the tensile stress in the outer fibers will reduce the stress-concentration factor K_r and will increase the endurance strength of the piece.

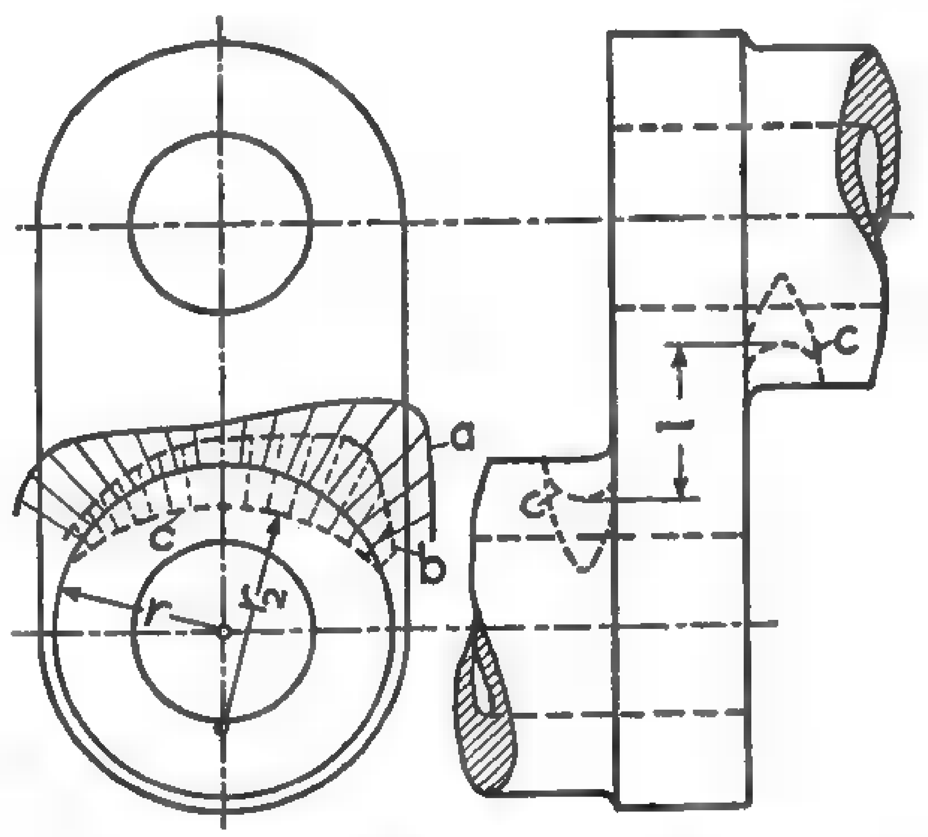


Fig. 5–26. Reducing stress concentration by removing material.

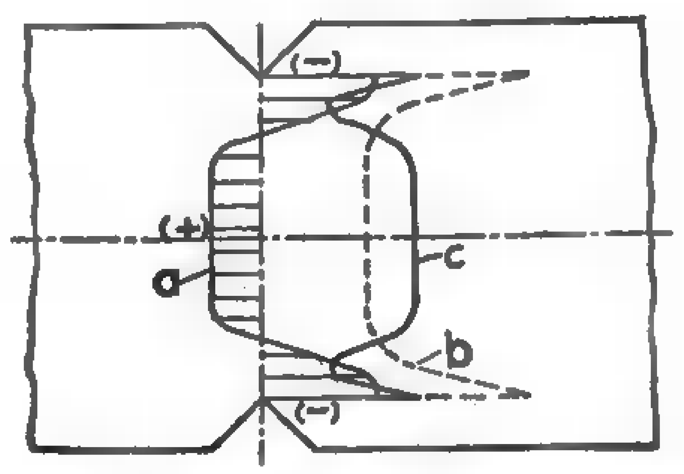


Fig. 5–27. Stress redistribution after a permanent set.

The injurious effect of notches and circular grooves in a piece subjected to repeated loading can be reduced by prestretching the piece so as to produce a permanent set in the groove or notch. As a result, internal tensile stresses will be set up in the center of the piece, causing compressive stresses in the fibers next to the notch, as shown by curve a, in Fig. 5–27. The distribution of stresses due to tension is represented by curve b, and the redistribution after a small permanent set in compression is created in the outer fibers is shown by curve c. The tensile stress near the notch is reduced, and the strength of the section is thus increased. Putting a plastic deformation into the notch by local pressure accomplishes the same result. Thus the beneficial effect of the equalization grooves c, Fig. 5–21a, is materially increased if the grooves are pressed in instead of being filed.[23]

[21] O. Dietrich and E. Lehr, "Das Dehnungslinienverfahren," *Z. VDI*, Vol. 76 (1932), p. 981.

[22] J. O. Almen, "Shot Peening to Increase Fatigue Resistance," *SAE Journal*, Vol. 51, No. 7 (July, 1943), p. 248.

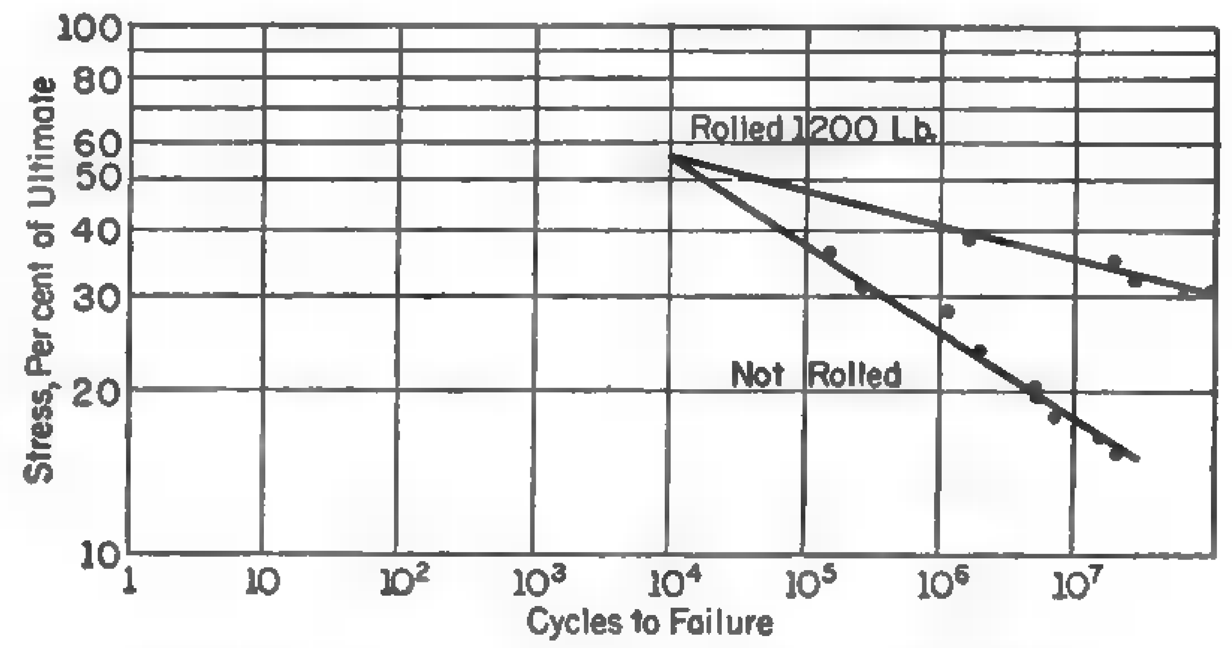


Fig. 5–28. Effect of rolling on railway axles.

Cold working. The endurance strength of a machine part can be increased if a thin layer of the part, subjected to tensile stress during its operation, is prestressed in compression by a cold-working operation such as peen hammering, swaying, shot blasting, or pressing by balls or rollers. Such an increase in endurance strength is shown by the S-N curves in Fig. 5–28.[24] Tests show that prestressing the surface in compression increases the endurance strength, regardless of whether the prestress is applied to highly finished parts or to parts with rough surfaces and regardless of whether the surface is soft or hard, as when it is casehardened. The decrease of the slope of the S-N curves gives a measure of the increase in endurance strength.

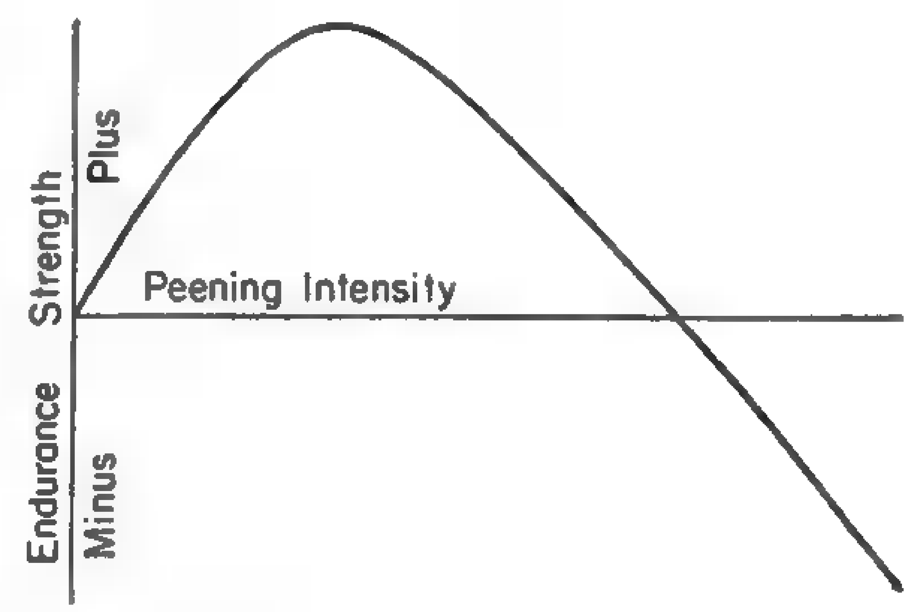


Fig. 5–29. Effect of peening.

The amount of prestressing and the stress magnitude and depth depend on the size of the part, the material, and the hardness of the surface. Excessive prestressing may decrease the endurance strength, as may be seen from Fig. 5–29, which gives only a qualitative picture. The proper amount of prestressing for any particular case must be found experimentally.

Other methods of reducing stress concentration. Shrink-fit stress concentration can be materially reduced by making the gripping part conical. The

[23] H. Ude, "Steigerung der Dauerhaltbarkeit der Konstructionen," *Z. VDI*, Vol. 79 (1935), p. 48.

[24] Almen, *loc. cit.*, pp. 249–52.

elasticity of material, even in a cast-iron hub, gives a gradually increasing shrink pressure. Such a change in design may lower K_r from 1.87 in a hub of the type shown in Fig. 5–30a to 1.38 in a hub of the type shown in Fig. 5–30b. Still better is a parabolic hub, as shown in Fig. 13–2. Another method of lowering the stress-concentration factor is to put a compressive stress in the outer fibers of the shaft by cold rolling before shrinking the hub on. If a shaft with an ordinary hub of the type shown in Fig. 5–30a is given such a plastic deformation and is afterward ground to the proper shrink dimension, the stress-concentration factor may be as low as 1.07. A shaft with a conical hub of the type in Fig. 5–30b that is so treated may become even stronger than an untreated shaft without stress concentration.

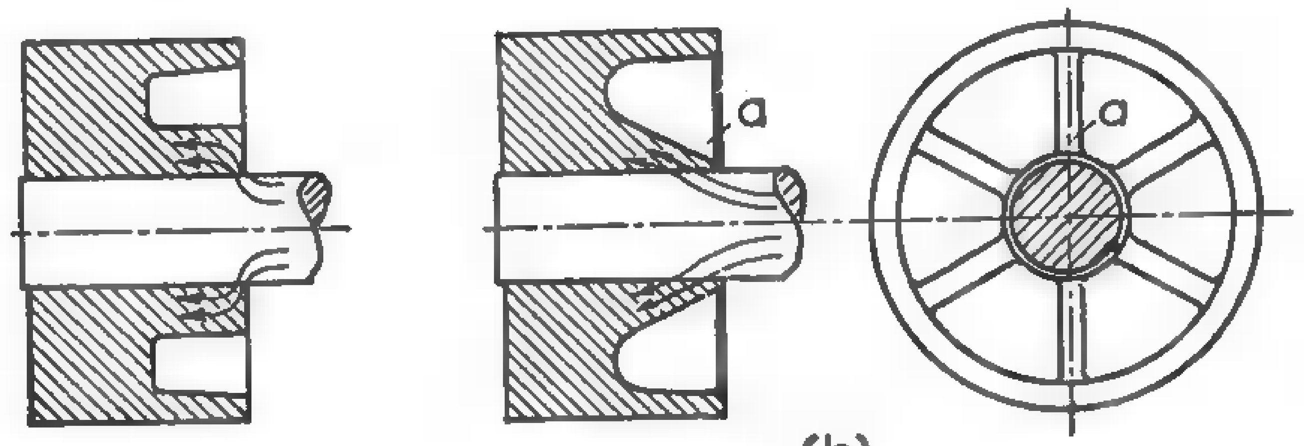


FIG. 5–30. Press-fit stress concentration.

Casehardening a mild-steel shaft practically eliminates the shrink-fit stress concentration. Nitriding does it entirely, and it also eliminates the injurious pressure oxidation that occurs in a press fit. Nitriding greatly reduces the injurious effect of notches and grooves by setting up compressive stresses in the surface layers similar to those caused by peening or some other cold-working process.

Oxidation stress concentration can be decreased by grinding off the outer fibers affected by oxidation to a depth of 0.004 to 0.008 in.

Corrosion effect upon steel is reduced very materially when the piece is sherardized or cadmium-plated. Nitriding eliminates corrosion effect entirely. The reduction of endurance limits caused by corrosion effect is less for stainless steels than for other steels.

5–12. Vibration. If an outside force changes the shape of a body without producing stresses beyond the elastic limit, it invokes restoring, internal forces which will act as soon as the outside force ceases to act. The body will return to its original shape, but because of its mass it will pass through and beyond the position of equilibrium and will create a restoring force in the opposite direction. These pendulum-like movements through the position of equilibrium will continue until the energy imparted by the outside force is absorbed by being damped out by internal friction. A series of such movements is called *vibration* or *oscillation*.

Vibration may be caused by longitudinal forces, by transverse forces, or by torsional moments. Forces that cause vibration of certain parts of a machine or of an adjoining structure usually are of two kinds: first, those due to the inertia of reciprocating parts; second, those due to centrifugal forces created by unbalanced revolving parts.

The *natural period of vibration* is the time T, in seconds, during which a body or system, set into free vibration, will complete one cycle. The reciprocal of the natural period of vibration is called the *natural frequency*. This is designated as f and is expressed in vibrations per second. The natural period of a system depends on the mass of the system and on the force required to produce a unit deflection. It can be computed by the relation

$$T = 2\pi\sqrt{\frac{W}{Fg}} \tag{5–43}$$

where W is the weight of the oscillating system, in pounds;
F is the force necessary to deflect the body 1 in., in pounds per inch;
g is the acceleration of gravity, which is 386 in. per sec per sec.

If the deflection produced by the load W is designated by y, then $y = W/F$ and the frequency becomes

$$f = \frac{1}{T} = \frac{3.125}{\sqrt{y}} \tag{5–44}$$

Resonance. If a body is set in vibration by a force recurring at regular intervals, and if these intervals happen to be equal to the natural period of the body or to some simple fraction or a multiple of that period, then the amplitude of vibration of the body may gradually increase, even though the force is small. Such conditions produce what is termed *resonance*, and the resulting large deflections may create dangerously high stresses in the parts involved.

Vibrations that have the same natural period of vibration and therefore the same frequency are called *synchronous vibrations*.

The *critical speed* of a machine with respect to a certain part is the speed at which synchronous vibrations are developed in the part by resonance. Resonance may produce synchronous vibrations in long rods, crankshafts, springs, brackets, beams, and in the whole structure supporting a piece of machinery.

Synchronous vibrations can be developed in a body not only at the main critical speed but also at one-half, one-third, or some other simple fraction of the main critical speed. In this event each such fraction of the main critical speed also becomes a critical speed. However, such critical speeds are less dangerous than the main critical speed, because they are set up by harmonics of higher orders, which have smaller amplitudes, and the impulses are therefore smaller.

In order to avoid critical speeds in a machine, the natural frequency of vibration of every part should be considerably higher than the number of impulses which the part receives during the operation of the machine.

Vibration damping. The rate of absorption of the energy of vibration of a part depends on the structure of the material, which governs its inner friction. Cast iron has a higher damping capacity than steel, which fact makes cast iron a more suitable material for parts where resonance may be expected. High-carbon steels and alloy steels have a greater damping capacity than low-carbon steels; Electron, a magnesium alloy, has the smallest damping capacity known.[25]

Damping does not affect the natural frequency of a part; it affects only the amplitude of the vibration.

Engine supports. In a high-speed engine operating at variable speeds, the range of speeds may be so great that it becomes very difficult, and sometimes impossible, to avoid synchronous vibration of the supports at all speeds. Often there are several critical speeds. In this case the transmission of vibration impulses can be stopped by inserting flexible members, such as rubber pads or springs, between the engine and the supporting structure.

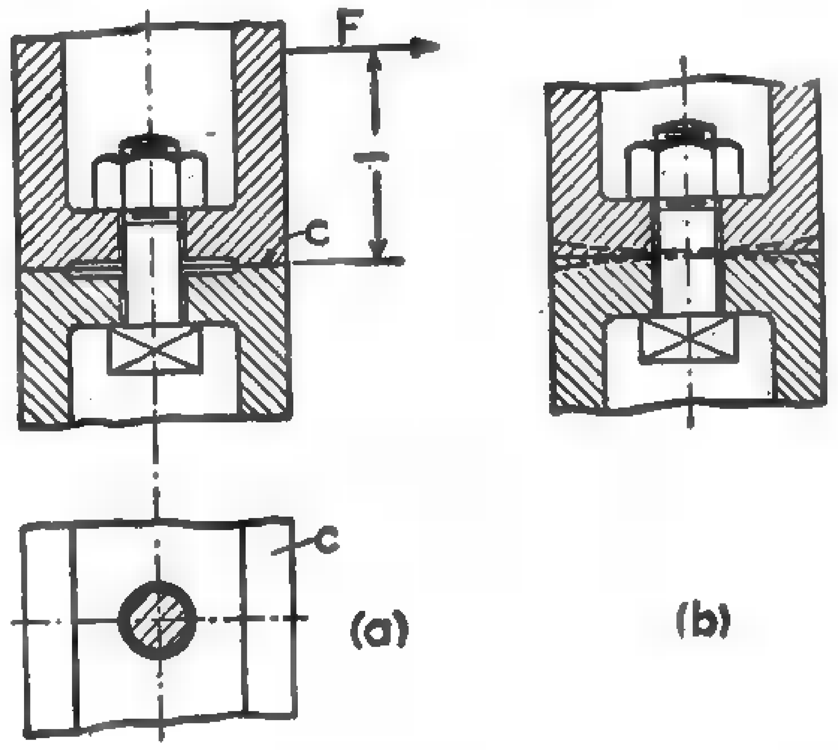


FIG. 5-31. Rigidity of junctures.

Rigidity. According to equation 5–44, the natural frequency of a body or system can be raised by decreasing the deflection y; that is, by making it more rigid. In the case of a structure formed by bolting together several pieces, the presence of working strips c, Fig. 5–31a, makes the lever arm of the resistance to the bending moment Fl greater than the arm with an unrelieved contact area, as in Fig. 5–31b. The contact in Fig. 5–31b can be considered, with exaggeration, as that of two cylindrical surfaces. The numerical change in the natural frequency can be computed if the conditions are known.[26]

EXAMPLE 5–7. A small steam turbine operates at speeds from 1,800 to 4,800 rpm. The turbine weighs 250 lb and is fastened to two American standard 5-in. 9-lb channels laid flat side up. The distance between the supports of the channels used as simple beams is 26 in. Determine the possibility of resonance and the corresponding critical turbine speed.

If the channels are considered simply supported at the ends and loaded uniformly over the full length, the deflection is

$$y = \frac{Fl^3}{77EI}$$

The load on each beam is $F = \frac{1}{2} \times 250 + \frac{1}{12} \times 26 \times 9 = 144.5$ lb and $I = 0.632$ in.^{4} Therefore,

$$y = \frac{144.5 \times 26^3}{77 \times 30{,}000{,}000 \times 0.632} = 0.00174 \text{ in.}$$

[25] A. Esan and H. Kortum, "Die Veränderlichkeit der Werkstoffdämpfung," *Z. VDI*, Vol. 77 (1933), p. 1133.

[26] K. Schönfelder, "Die Bedeutung der Arbeitsleisten in Teilfugen für die Steifigkeit der Konstructionen," *Z. VDI*, Vol. 77 (1933), p. 1070.

The frequency of vibration found by equation 5–44 is

$$f = \frac{3.125}{\sqrt{0.00174}} = 75.3 \text{ vibr per sec}$$

The number of vibrations per minute is $75.3 \times 60 = 4{,}518$. Therefore the critical turbine speed would be about 4,518 rpm. The channels are too flexible. By turning them 90°, the moment of inertia would be increased to $I = 8.8$ in.^{4}, and the frequency would be increased to the safe value

$$f' = 4{,}518\sqrt{\frac{8.8}{0.632}} = 16{,}850 \text{ vibr per min}$$

5–13. Torsional vibration. When a couple twists a shaft and then ceases to act, the internal stresses will bring the shaft sections back beyond their positions of equilibrium and will twist the shaft in the opposite direction because of the inertia of the shaft. The shaft will thus be twisted back and forth with respect to the center line of the shaft. A series of such angular movements of one section relative to another is called *torsional vibration.* Torsional vibration is created in a crankshaft of a reciprocating engine because of the periodic impulses to which it is subjected; and it is created in a straight shaft when the shaft is either coupled to such a crankshaft or otherwise subjected to periodic torque variations.

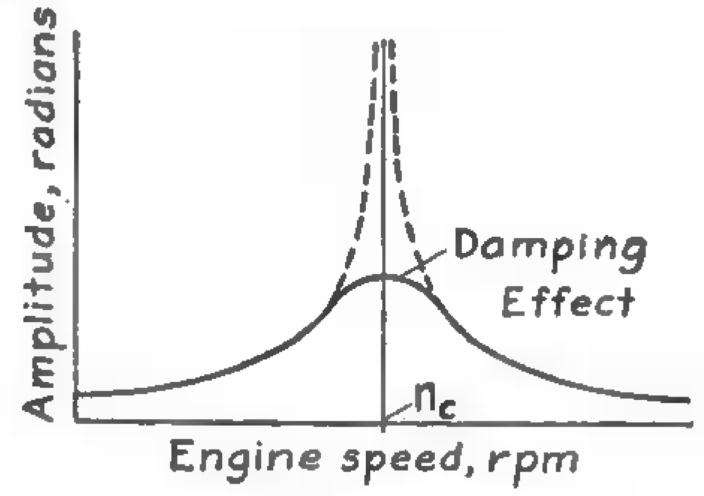


FIG. 5-32. Damping of vibration.

Critical speeds. The natural frequency f_n of torsional vibration of a shaft is a function of its dimensions, of the characteristics of its material, and of the size and arrangement of the masses attached to it. If f_n is an even multiple of the frequency f_f of the disturbing forces which come at regular time intervals, that is, if f_n/f_f is a whole number, a condition of resonance will exist. The amplitude α of vibration, expressed in radians, may be found by the relation

$$\alpha = \frac{Q}{T_m\left(1 - \frac{f_f}{f_r}\right)} \tag{5-45}$$

where Q is the disturbing torque and T_m is the shaft constant, or the torque which will twist the shaft 1 radian.

The critical speed occurs when $f_f = f_n$. Theoretically, α could then become infinitely large, as shown in Fig. 5–32; but actually it is stopped by damping due to internal friction of the material. However, if the amplitude—even with damping—is large, torsional stresses may exceed the endurance limit and the shaft may fail by progressive fracture.

Node. That section in which the torsional stresses reach their maximum value rotates without oscillation and is called the *nodal section* or the *node.*

The angular deflections are measured with reference to the node. The location of the node depends on the inertia masses causing torsional vibration. If one rotating mass is fastened to the shaft, the shaft will have a node at section n-n, Fig. 5–33, where the most distant of the tangential forces acts. If there are several inertia masses, as in Fig. 5–34, the node will be located somewhere between the first and last rotating masses. The location of the node can be found from the equation of dynamic equilibrium, the flywheel effects of the rotating masses being considered as though they were weights and the node being located at the center of gravity of the system.[27]

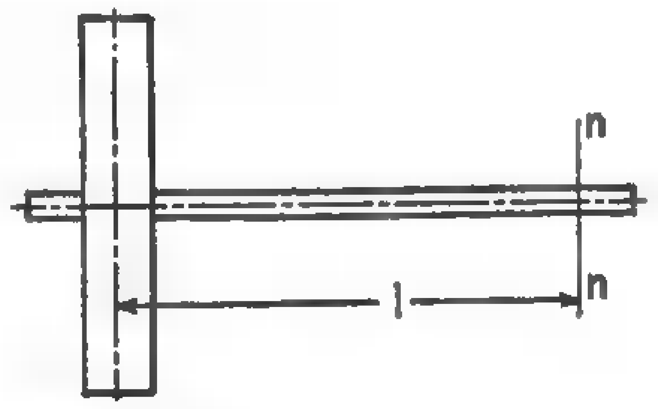


FIG. 5–33. Node location with one mass.

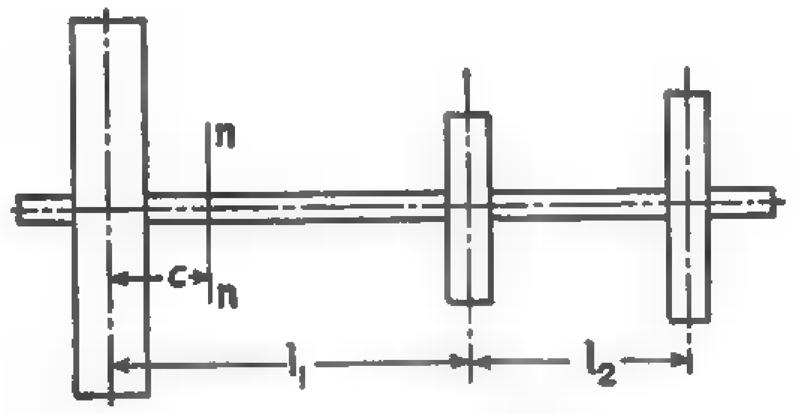


FIG. 5–34. Node location with several masses.

For three rotating masses, as a flywheel and two cranks shown schematically in Fig. 5–34, the equation of equilibrium is

$$I_1c = I_2(l_1 - c) + I_3(l_1 + l_2 - c)$$

The distance c to the node from one end is

$$c = \frac{I_2l_1 + I_3(l_1 + l_2)}{I_1 + I_2 + I_3} \tag{5–46}$$

Both equation 5–46 and Fig. 5–34 show that a node divides a shaft into two parts, each behaving as a shaft with one oscillating mass.

Frequency. The natural period of torsional vibration T, in seconds, can be determined by a formula similar to equation 5–43. Thus,

$$T = 2\pi\sqrt{\frac{I}{T_m}} = 2\pi\sqrt{\frac{Wk^2_ol}{gGJ}} \tag{5–47}$$

where I is the moment of inertia of the rotating mass, which is Wk^2_o/g lb-in.-sec²;

T_m is the shaft constant, or the torque which produces in the oscillating shaft a twist of 1 radian in a length of 1 in., in inch-pounds;

W is the weight of the rotating mass, in pounds;

k_o is the radius of gyration of this mass, in inches;

l is the oscillating length of the shaft, in inches;

[27] H. F. P. Purday, *Diesel Engine Design* (London: Constable & Company, Ltd., 1928), p. 108. This method gives only approximate values and is used because of its simplicity. Accurate methods involve lengthy calculations and may be found in special books on vibrations.

g is the acceleration due to gravity, which is 386 in. per sec per sec;

G is the modulus of elasticity in shear, in pounds per square inch;

J is the polar moment of inertia of the shaft section, in inches⁴.

When equation 5–47 is applied to a shaft with several rotating masses, the length l must be considered as the distance from the application of the inertia mass to the node for each mass. The equation for the period becomes

$$T = 2\pi\sqrt{\frac{\Sigma(Wk^2_ol)}{gGJ}} \tag{5–48}$$

When the constant coefficients have been combined, the frequency is

$$f = 3.125\sqrt{\frac{GJ}{\Sigma(Wk^2_ol)}} \tag{5–49}$$

Because of the approximate nature of equation 5–46 the values of T and f determined by equations 5–48 and 5–49 are also only approximate, being within 3 or 4 per cent of the actual values.

A shaft or a crankshaft rigidly connected to other shafts with rotating parts, such as generator rotors, pump impellers, or pulleys, cannot be considered alone; it must be considered as a part of the whole system of interconnected shafts and rotating masses. On the other hand, a flexible coupling connecting two shafts divides them into two separate systems as far as vibration is concerned.[28]

The determination of the frequency of torsional vibration of a shaft system, as well as the elimination of resonance, will be discussed in greater detail in connection with the design of crankshafts.

5–14. Temperature effects. The effects of temperature and particularly of its variation upon metals are very numerous and important for the design of certain machine parts. These effects may be divided into two groups: those produced by temperature changes during the various manufacturing processes through which a part goes; and those caused by temperature variations in a part during operation of the machine. Although the first group is very important, its discussion belongs to courses in metallurgy and technology of metals. Only the second group will be considered here.

The main effects of temperature changes in machine parts are:

a. Expansion with the increase of temperature
b. Stresses due to temperature differences
c. Deterioration of surfaces
d. Change of ultimate strength
e. Change of elastic and endurance limits
f. Change of the modulus of elasticity

[28] A comprehensive treatment of the subject may be found in W. K. Wilson, *Practical Solution of Torsional Vibration Problems* (New York: John Wiley & Sons, Inc., 1942).

g. Change of hardness
h. Change of ductility and impact strength
i. Creep phenomenon
j. Growth or bulk change

TABLE 5–3

HEAT PROPERTIES OF METALS

Metal or Alloy	Mean Coefficient of Linear Expansion Between 170 and 500 F $\times 10^4$	Coefficients in Equation 5–51			Average Coefficient of Heat Conductivity k (Btu-in. per hr per sq ft per deg F)	
		$a \times 10^3$	$b \times 10^3$	Temperature Range (deg F)	At 68 F	At 212 F
Aluminum alloy	0.141	12.58	3.0	32–1,130	1,290	1,330
Brass, yellow	0.104	9.40	1.35	32–1,100	620	740
Bronze, aluminum	0.098	9.28	1.22	32–1,100	618	710
Cast iron, gray	0.063	5.44	1.75	32–1,100	340	320
Copper	0.099	9.28	1.24	32–1,160	2,670	2,640
Malleable iron	0.071	6.50	1.62	32–1,300	480	400
Monel metal	0.083	7.70	1.22	32–1,100	180	180
Nickel	0.081	7.65	1.02	32–1,830	415	400
Steel, mild	0.065	5.80	1.40	32–1,400	315	310
Steel, ingot	0.070	6.21	1.62	32–1,380	335	330

Expansion. The coefficient of thermal expansion increases with the temperature; but for ordinary ranges of temperature, mean values such as those given in Table 5–3 may be used. The length l at a temperature t is

$$l = l_o[1 + \alpha(t - 32)] \tag{5–50}$$

where l_o is the length at +32 F and α is the mean linear coefficient of expansion.

At higher temperatures it is better to use more accurate data and to compute the length from the equation

$$l = l_o[1 + a(t - 32)10^{-3} + b(t - 32)^2 10^{-6}] \tag{5–51}$$

where values of the coefficients a and b are given in Table 5–3.

Expansion due to a temperature increase must be taken into consideration in designing parts of various heat engines, such as piston rods, crankshafts, and bedplates, and particularly in determining the diameters of the pistons of steam engines and internal combustion engines.

Temperature stresses. The stress s set up by a change of temperature from t_1 to t_2 if the part cannot change its length is, according to equation 2–4,

$$s = \alpha(t_1 - t_2)E \tag{5–52}$$

If $t_1 > t_2$, the stress is negative, or compression; if $t_1 < t_2$, it is positive, or tension. As can be readily seen, even a moderate change of temperature may result in very high stresses.

This restriction of change of length exists in a casting with sections of varying thickness. The smaller ones cool faster than the larger ones, and the stresses thus set up may cause a fracture even without an external load. Such temperature stresses can be eliminated, or at least reduced, by a uniform heating and cooling of the casting, and also by a careful design that avoids nonuniform and rigid sections as much as possible.

Temperature stresses of a second type are set up when the temperatures at different points of a part are not the same and the part cannot change its shape, as in a cylinder liner of an oil engine with hot gases inside and cooling water outside. In such a case the stress may be determined from equation 5–52, but the meaning of the terms will be slightly different. Under these conditions t_1 and t_2 will designate the coexistent temperatures at certain points, and s will be the difference between the stresses at these points. The existence of a temperature difference $(t_1 - t_2)$ is possible only because of a continuous flow of heat from the points with the highest temperature to those with the lowest one. The relation between the flow of heat through a wall and the temperature difference may be expressed by the equation

$$Q = A(t_1 - t_2)\tau\frac{k}{b} \tag{5–53}$$

where Q is the flow of heat, in Btu per hour;
A is the area through which Q flows, in square feet;
τ is the ratio of time of heat flow to total time;
k is the heat conductivity, in Btu per hr per sq ft per deg F;
b is the thickness of the wall, in inches.

Combining equations 5–52 and 5–53, to eliminate $(t_2 - t_2)$, gives

$$s = \frac{\alpha EQb}{A\tau k} \tag{5–54}$$

This equation shows that for given conditions fixing Q, A, and τ and for a certain material with known properties α, E, and k, the stress is directly proportional to the thickness b of the wall. Thus, if the stress is too high, an increase of the dimension b will only make the wall weaker. If other forces acting on the wall set up a tensile or compressive stress s_2 which decreases with an increase of b, then it is necessary to find the thickness b for which the algebraic sum $(s + s_2)$ is least. If $(s + s_2)$ is greater than the allowable stress for the selected material, then the next procedure is to select a material with a higher k or a lower α. When temperature stresses are present, it is better to use ductile metals. In such metals excessive local stresses are not so dangerous as they are in brittle metals, and the excess may disappear after the metal assumes a permanent set in the overstressed section.

Deterioration of surfaces. In parts subjected to high and fluctuating temperatures, such as the tops of the pistons in internal combustion engines, temperature stresses near the surface may exceed the elastic limit. Then destruction will start in the shape of minute cracks, which gradually increase in size and number. As the cracks deepen, the strength of the part is decreased until its failure occurs.

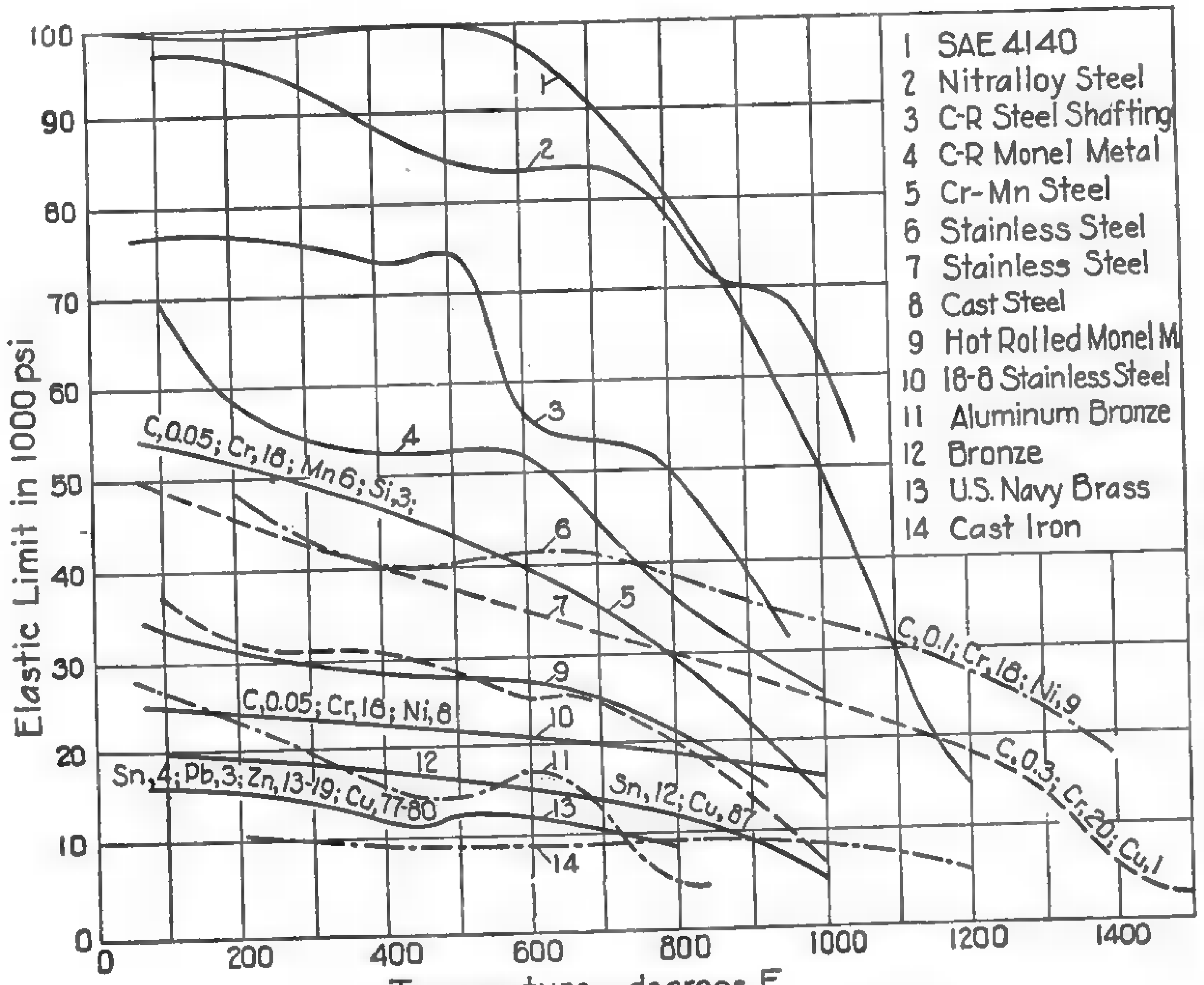


FIG. 5–35. Influence of temperature on elastic limits of metals.

Often such deterioration cannot be eliminated and can only be slowed down by using higher-grade materials and by making the piece which is subject to deterioration a separate and easily replaced part or insert.

Ultimate strength. The ultimate strength of most metals and alloys increases slightly as the temperature increases up to 400 to 500 F. With further increase in the temperature, however, the strength decreases under all loads—tension, compression, bending, or torsion.

Elastic limits. The decrease of elastic limits with increase in temperature is more pronounced than that of strength, as may be seen from Fig. 5–35. The endurance limits decrease approximately at the same rate as the elastic limits.

Modulus of elasticity. The modulus of elasticity decreases with elevated temperatures, very slowly up to 500 F and thereafter progressively faster because of creep.

Hardness. An increase of the temperature of a metal part up to 300 F does not affect its hardness. With a further temperature increase the hardness begins to decrease, at first rather slowly. The Rockwell C hardness of heat-treated steels used in ball bearings decreases from 66 to 62 at 600 F. A fully hardened SAE 1050 steel has a Rockwell C hardness of 55 at 400 F, 51 at 600 F, and about 47 at 800 F.

Impact strength. In Fig. 5–36 is shown the change of impact strength of various steels at comparatively low temperatures due to the change that takes place in the ductility.[29] Curve *a* is for high-carbon steel; *b* is for medium-carbon steel; *c* is for low-carbon steel; and *d* and *e* are for normalized nickel-chrome steel, *d* being for specimens prepared for large ingots and *e* for lengthwise specimens of bars.

Creep. At high temperatures, such as are encountered in steam boilers, turbines, and piping, the deformation of materials ceases to be elastic and becomes plastic with a continuous increase under a constant load. The equilibrium between stress and load is not established, even after a very long time. The material under tensile stress continues to stretch, or creep. Creep is measured in terms of plastic deformation during a certain time. The limiting creep stress for a certain temperature is the maximum stress under which the material will not fail during a prescribed length of time.

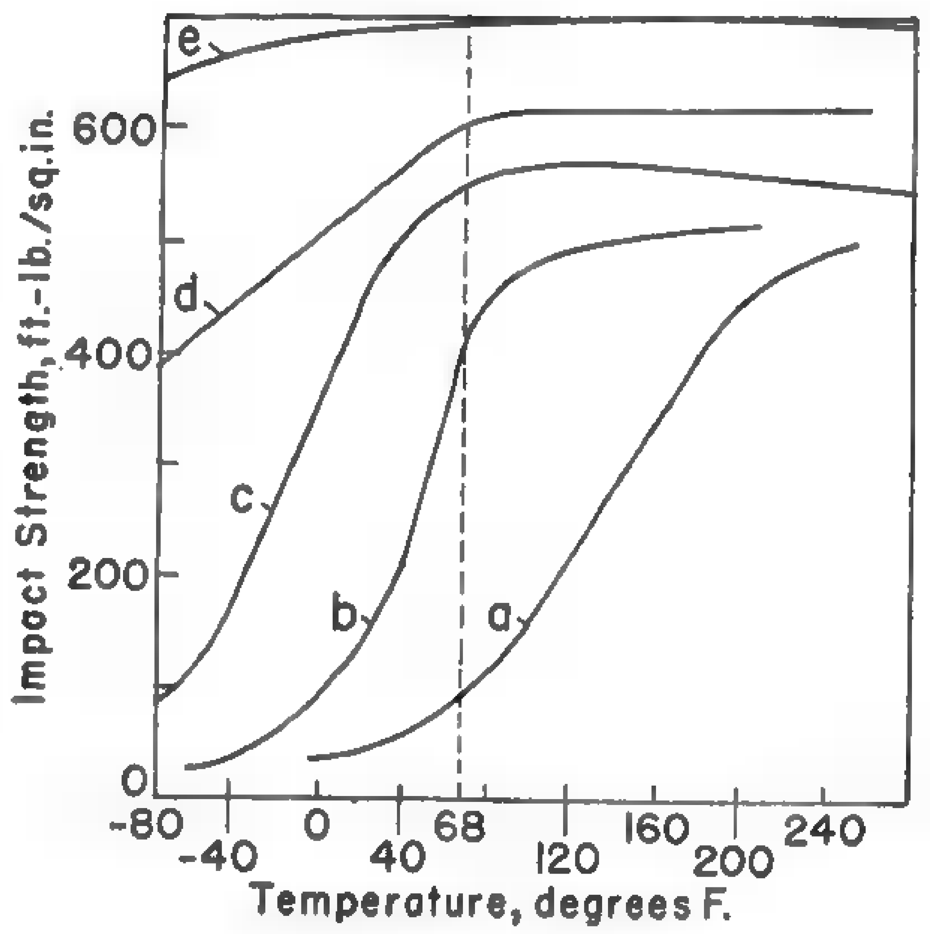


FIG. 5–36. Impact strength at low temperatures.

Creep is a function of temperature, stress, and time. It should therefore be represented by curves in a three-dimensional system of coordinates. But in practice creep is usually of interest in connection with a certain temperature, in which case a diagram such as that shown in Fig. 5–37 serves the purpose even better.[30] The influence of temperature and stress on the total creep can be studied by means of a diagram like that in Fig. 5–38. Chromium-molybdenum steels show great resistance to creep.

Growth of cast iron. When cast iron is heated repeatedly, its specific volume begins to increase. The dangerous temperature may vary from 550 to 1,000 F, the actual value depending on the composition of the iron. This

[29] W. Schwining, "Die Festigkeitseigenschaften der Werkstoffe bei tiefen Temperaturen," *VDI*, Vol. 79 (1935), p. 35.

[30] P. G. McVetty, "Working Stresses for High Temperature Service," *Mechanical Engineering*, Vol. 56, (1934), pp. 149–54; "Failure from Creep as Influenced by the State of Stress," *Sulzer Technical Review*, No. 1 (1943), pp. 1–16.

increase in volume, called *growth*, may be due to any or all of the following causes:

a. The conversion of combined carbon to free carbon
b. The action of the silicon and phosphorus in the cast iron
c. The expansion of particles of gases dissolved in the metal
d. The penetration of oil and gas under pressure into the graphite particles of cast iron, as it occurs in pistons of heat engines

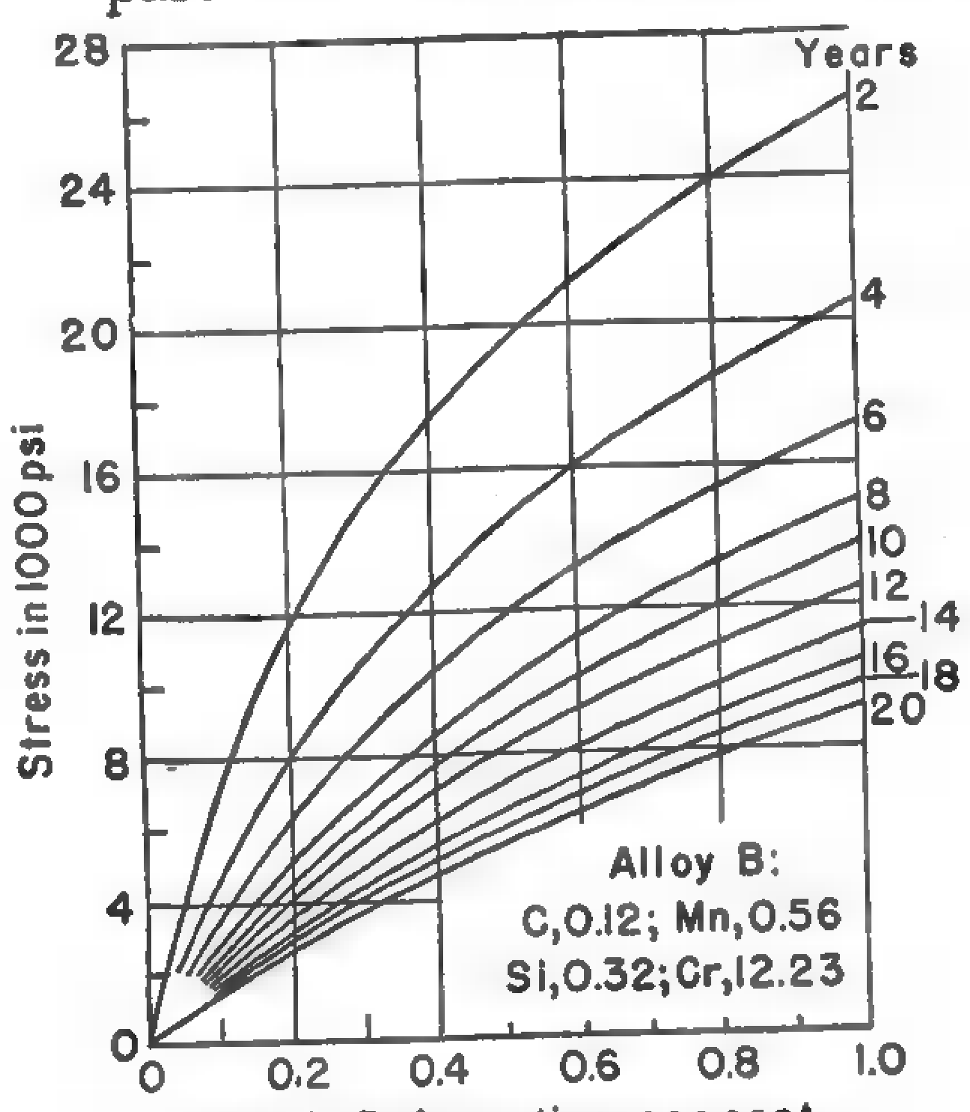


FIG. 5–37. Stress-strain curves at constant temperature of 800 F

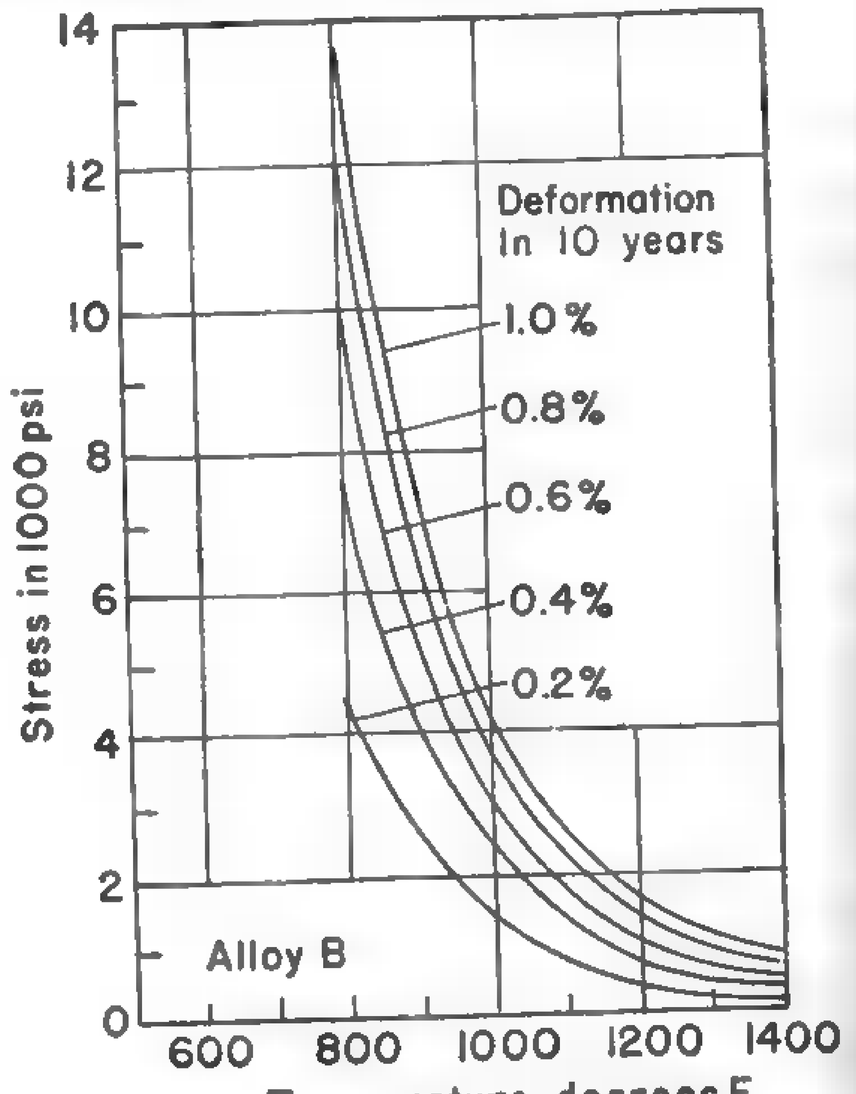


FIG. 5–38. Stress-temperature curves for various creep values.

This growth is dangerous in machine parts because it provides local compressive stresses similar to excessive heat stresses. It also reduces the clearance between moving parts and may result in excessive wear or seizure. The designer of a machine subjected to high temperatures should provide ample clearances and should specify the use of cast iron with a minimum growth. The addition of steel to cast iron, making so-called semi-steel, and the addition of about 1.5 per cent of nickel are the best methods for reducing the growth to practically harmless amounts. Ni-Tensyliron and Ni-Resist have particularly small growth and comparatively high strength at high temperatures. So has molybdenum cast iron.

Design for creep conditions. When it becomes impossible to keep the stresses in a part subjected to high temperatures within elastic limits, the allowable stresses must be determined from a diagram, such as that in Fig. 5–37, obtained by creep tests with the material to be used for the temperature involved. A maximum plastic deformation at the end of a certain number of years is selected to conform to the working conditions, and the corresponding stress S_{cr} is taken from the diagram. The design stress S_d is then found by dividing S_{cr} by the safety factor n, which should be at least 1.25 and preferably 1.5.

PART II: MANUFACTURE OF MACHINE PARTS

CHAPTER 6

General Manufacturing Considerations

6–1. Manufacturing methods. Designing a machine part means putting down on paper the dimensioned shape that the part must have to properly fulfill its functions. In order to be able to lay out and draw this shape the designer must know how the shape can be produced from different raw materials. Such a knowledge requires a thorough understanding of the various manufacturing methods. These methods may be advantageously divided into two groups: (1) preliminary shaping of machine parts, mostly, although not always, by using heat; and (2) final shaping by means of cold machining.

Under the first grouping come the processes of *casting*, *welding*, *riveting*, and *forging*. Each of these methods of forming machine parts has different possibilities, but it also has different limitations which influence the design of a part. The designer must know and keep in mind these possibilities and limitations. The main features of each method from the standpoint of machine design will be discussed in separate chapters in the order indicated above. The first group includes also such processes as rolling, drawing, extruding, and stamping. However, their purpose is not to produce machine parts as such, but to produce stock material used widely in industry.

Among the methods under the second grouping are a great number of different operations which give the piece the exact dimensions required and produce the surface conditions necessary for its functioning. The main methods in this group are *turning*, *boring*, *milling*, *planing*, *shaping*, *drilling*, *reaming*, *spot facing*, *broaching*, *grinding*, *honing*, and *polishing*. There are additional machining processes, such as screw-cutting, tapping, and gear-cutting, which are special adaptations of the basic methods. Some of the machining operations fulfill the same object and are often interchangeable. Examples are milling, planing, and shaping. Other operations, such as boring and drilling, are somewhat similar but are really different. The majority are intended for different purposes.

A good knowledge of the various operations is very helpful to a machine designer. However, such knowledge can be acquired only by working in a machine shop, and the type of machining operation does not have too much effect on the shape of a part during its design. In practice the proper operation often is selected after the designer consults the man in charge of the machine shop. Therefore no attempt will be made here to give any information about the differences between the various types of machining

operations, and no suggestions will be given for selecting the best type of operation for a specific case. The information that must be given to a beginning designer is how to make simpler and easier the machining of the parts he is designing and how to determine the degree of accuracy in machining that he should prescribe on his drawings.

Production conditions. The design of a machine part depends on the facilities of the shop where the part will be built. The facilities in a small jobbing machine shop are naturally different from those in a large plant manufacturing some special machinery.

Also, the design will not be the same when the part will be produced in quantities as when only one piece, or at most a few pieces, must be made. For instance, for quantity production it may be proper to make a part as a die casting, whereas a single piece may be machined from a block. As another example, when it is necessary to replace a large sand-molded casting that was produced in quantity, the single piece may be produced more quickly and more cheaply by welding.

No rules can be given for deciding how to act in every case; the designer must be guided by his former experience and his personal judgment.

6–2. Machining. Removing metal by hand in order to obtain a desired dimension or to fit one part to another is much more expensive than doing the work by a metalworking machine. Therefore each machine part should be designed so that it can be finished and assembled without special fitting after it has been machined in the manner indicated on the drawing by the designer.

Limitations. The designer must know the limitations of the machine shop in which the part that he is designing will be machined. Such limitations are the biggest diameter and the greatest length or height that can go in the lathe and boring mill and the greatest length and width that can be handled by the planer. He must know for what pitches hobs are available in the shop for cutting gear teeth, as very few shops have all standard hobs in stock. He must have similar information concerning other tools, such as taps, reamers, and broaches.

The designer should know which sizes of cold-rolled and hot-rolled steel material are standard and which standard sizes are kept in stock in the shop where his design will be executed. However, if the design of his part really requires a standard size which the shop is not carrying in stock, he should not hesitate to call for such material. He should not try to use available stock sizes that involve extra machining.

Design for machining. The area of machined surfaces should be kept to a minimum both in castings and weldments. If a part can be left rough, except for surfaces where it must be in contact with other parts, it must be provided with special strips that are raised above the unmachined surfaces.

Such strips are needed along the edges of a cover and on the parts on which the cover rests. If in Fig. 6–1 the thickness of the cover a must be t_1, the thickness at the edges must be increased to t_2. Similarly, if the required thickness of the main casting b is t_3, the thickness must be increased to t_4 where the casting must be machined. Instead of machining the outside edges of the cover a in order to prepare a true surface for the nut, it is quicker and less expensive to spot-face the surface around the hole drilled for the stud to a diameter d slightly greater than that of the nut.

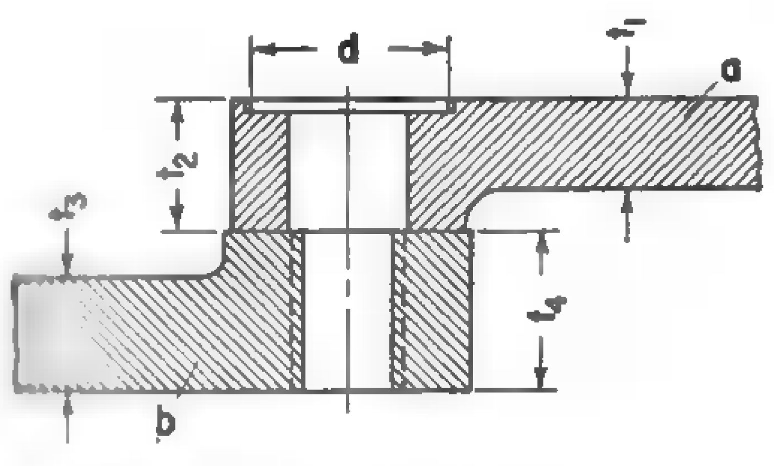


FIG. 6–1. Size of machined surfaces.

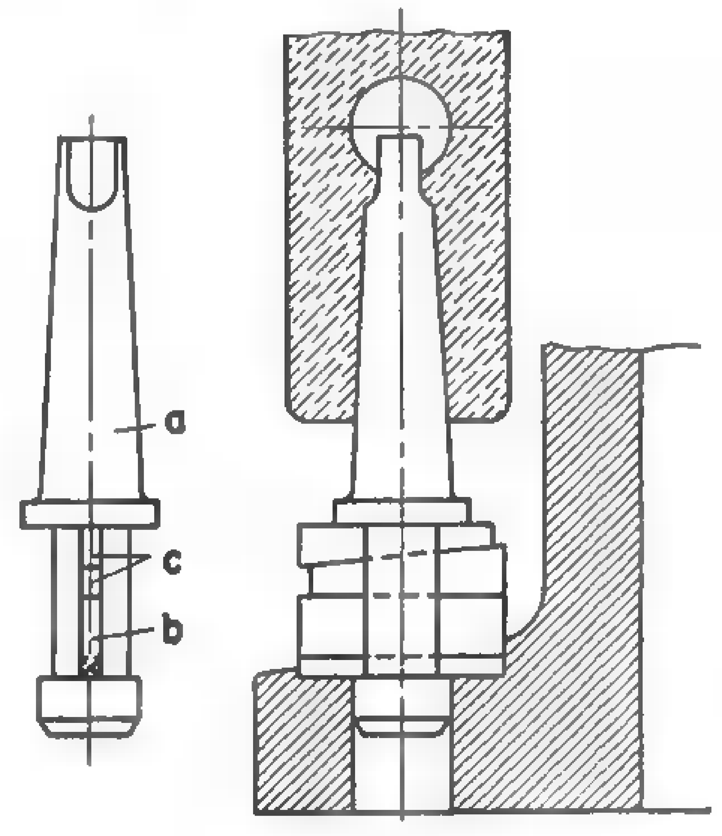


FIG. 6–2. Spot facing.

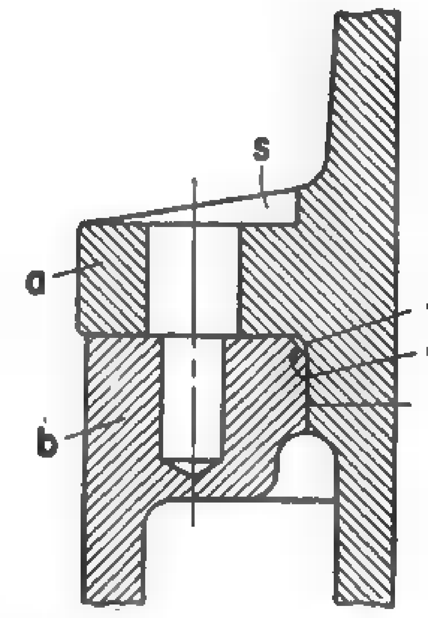


FIG. 6–3. Fitting of a flanged cylinder.

Spot facing is especially necessary if the surface is not normal to the axis of the bolt or stud, as shown in Fig. 6–2. This illustration also shows the spot-facing tool—a rod a with a slot into which is inserted a cutter b held by two tapered keys c. The upper end of the rod a fits into the head of a drill press, while the lower end fits into the bolt hole and acts as a guide.

In Fig. 6–3 is shown another example to indicate how the cost of machining may be reduced in several ways. The flange a has a larger diameter than the lower part and is left rough, since a slight difference in the overhang will not attract attention when both part a and part b are left rough. In this case the register at r is made short, since its length is immaterial and shortening it reduces the cost of machining each casting. When the opening in part b is bored, the sharp edge at c is cut off at an angle of 45° to a depth of about $\frac{1}{8}$ or $\frac{3}{16}$ in. This cut permits the register and the underside of the flange to be turned so as to leave a fillet at d. Providing such a juncture of a cylindrical surface and a flat surface is cheaper than bringing them together to a sharp corner and also gives a stronger flange. The spot facing for the stud nut is shown at s.

The cost of actually removing material is not always the main cost item in machining a part. Sometimes the fastening of the piece to the machine is even more expensive, requiring considerable time or special clamping devices. Thus it is difficult to fasten for machining a plate with the cross section in Fig. 6–4a, whereas it is easy to fasten a plate having the section in

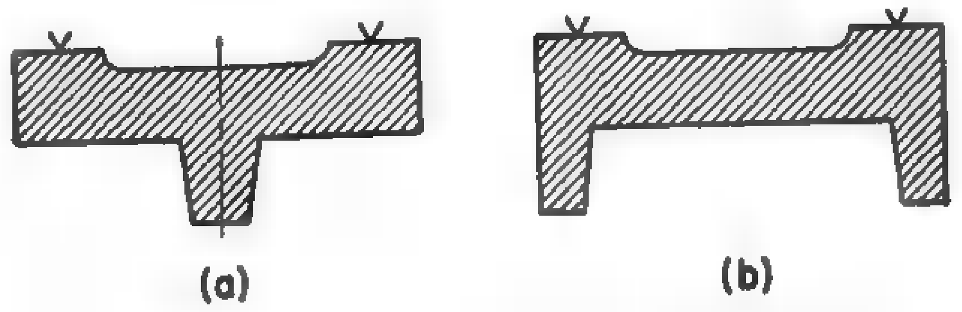


FIG. 6–4. Poor and good shapes for machining.

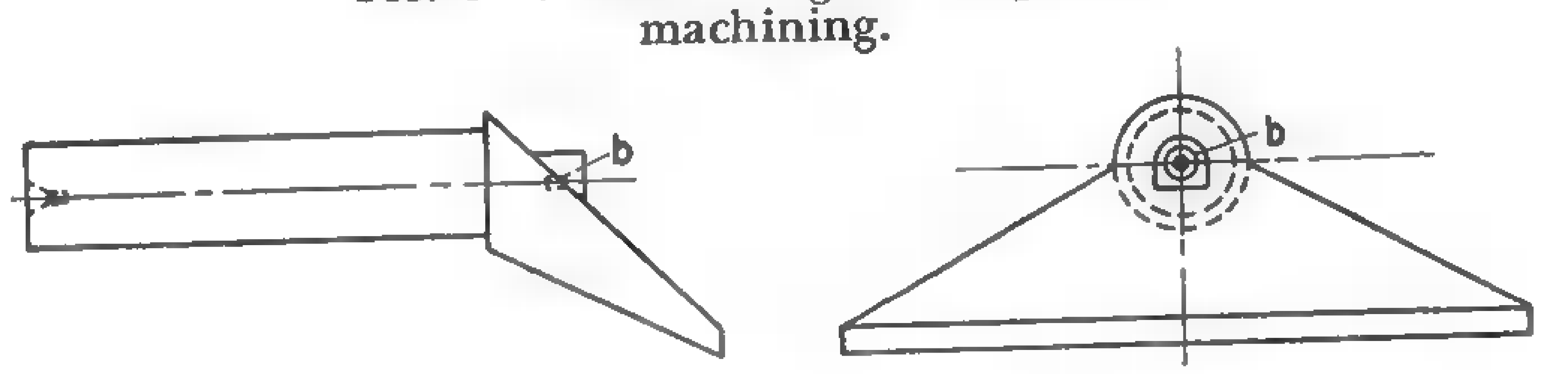


FIG. 6–5. Tool rest with boss for centering.

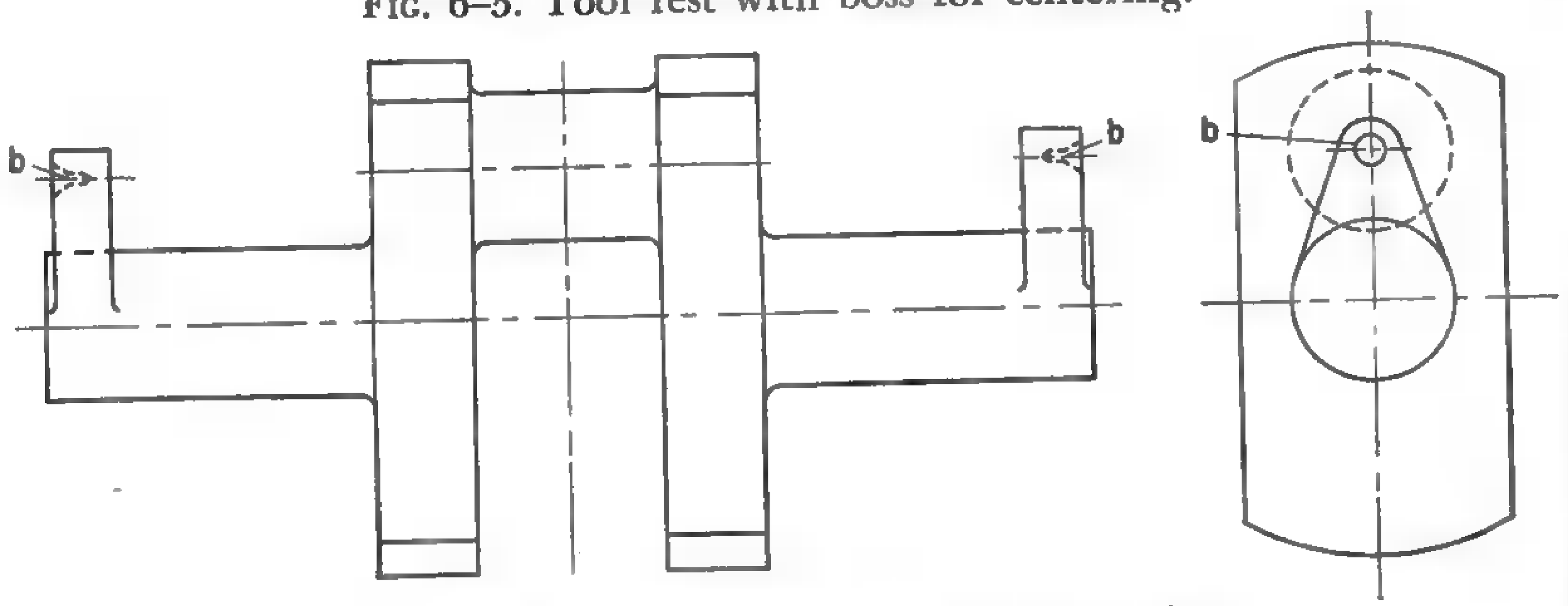


FIG. 6–6. Crankshaft with bosses for centering.

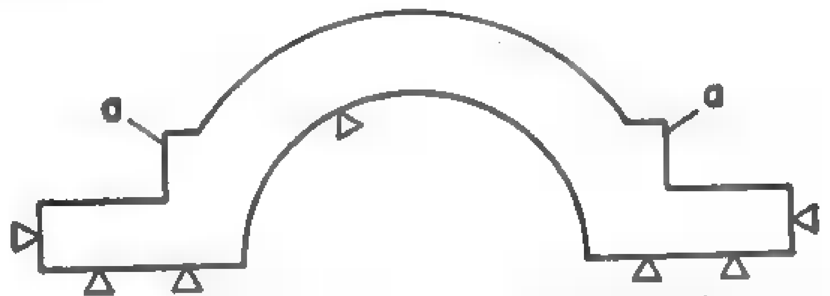


FIG. 6–7. Bearing cap with lugs for chucking.

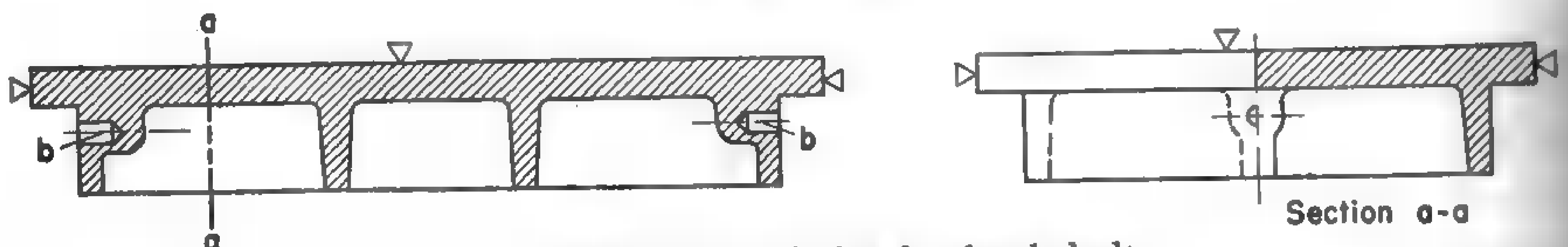


FIG. 6–8. Plate with holes for hook bolts.

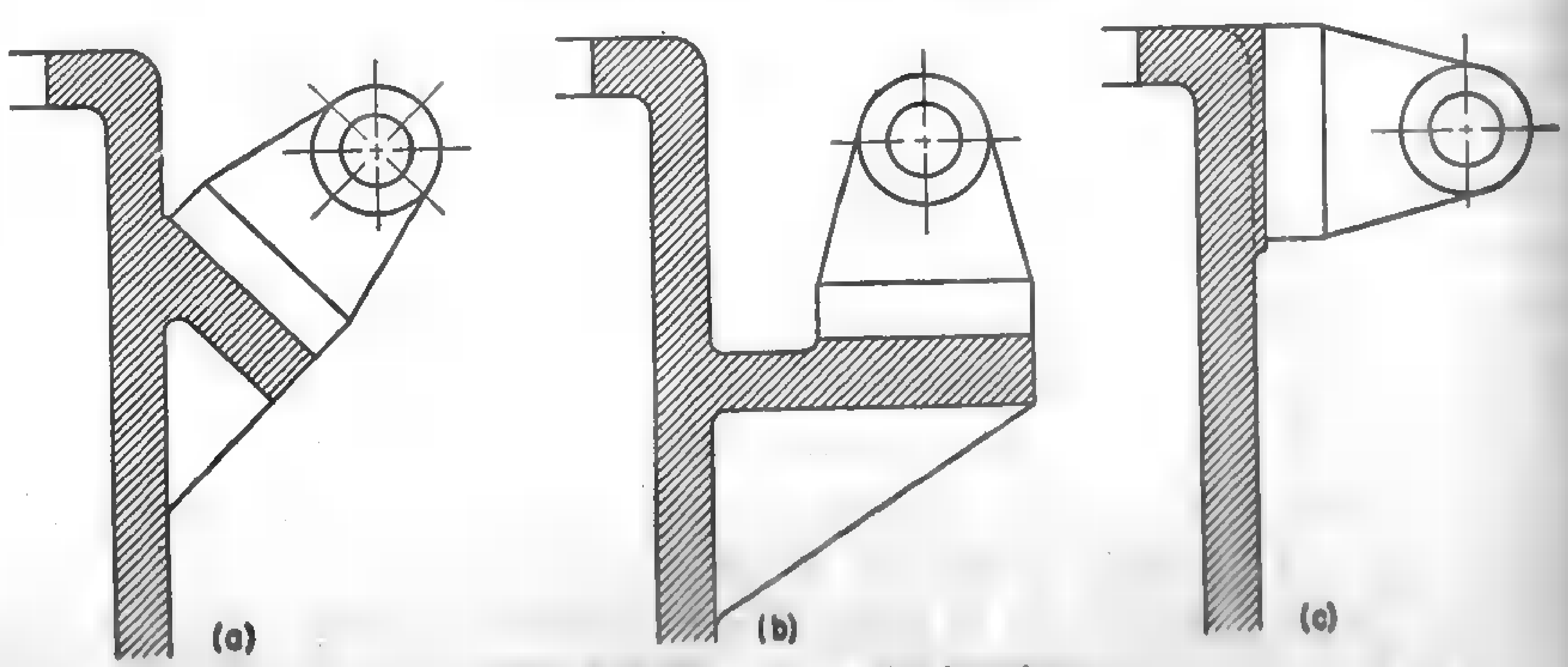


FIG. 6–9. Fastening of a bracket.

Fig. 6–4b; and the casting cost is the same. Special clamping devices and fixtures are permissible and advisable only if a sufficient number of pieces must be made. If only one piece or a few pieces must be made, the designer should make provision for simple and accurate clamping. He can add special bosses or extensions to support a piece during machining. In Fig. 6–5 is shown a boss *b* for centering; and in Fig. 6–6 are shown bosses *b* on a cast compressor crankshaft. It is also possible to provide openings in the casting for inserting clamping bolts. The bearing cap in Fig. 6–7 is provided

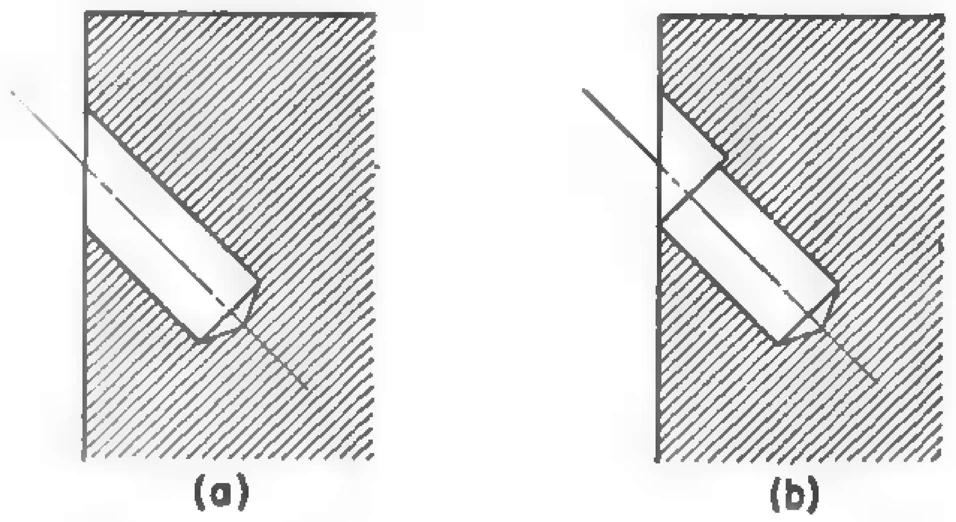


FIG. 6–10. Hole at an angle.

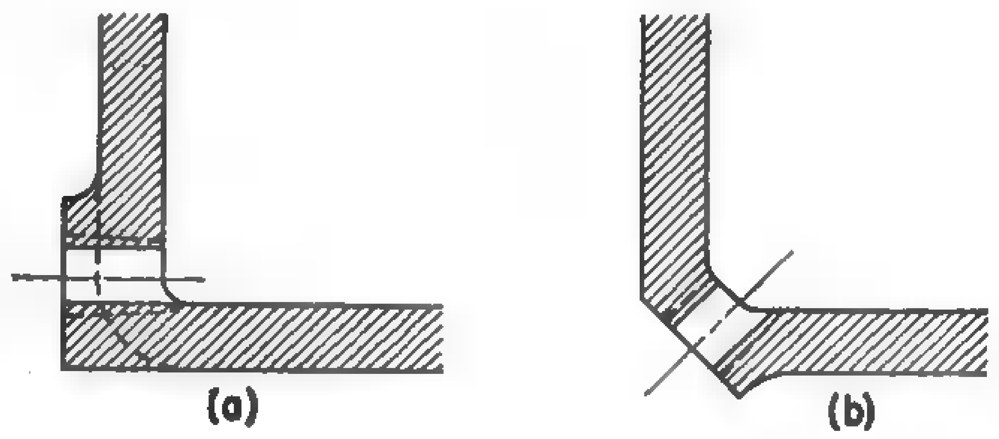


FIG. 6–11. Location of a drain hole.

with lugs *a* to facilitate chucking during machining. In some cases the special projection must be cut off after the part has been machined, in order not to interfere with the assembling or the functioning of the part. For example, it will be necessary to remove the bosses *b* in Fig. 6–6. Finally, Fig. 6–8 shows holes *b* drilled in bosses specially provided for hook bolts. These holes help to handle the heavy ribbed plate while it is being machined and when it is in service.

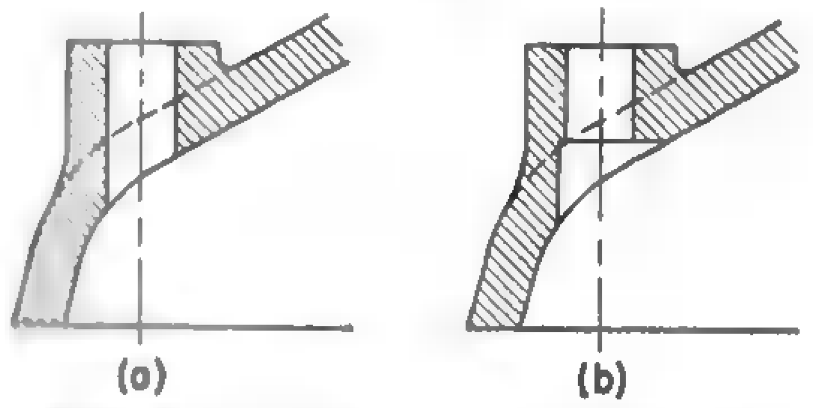


FIG. 6–12. Improving a casting for machining.

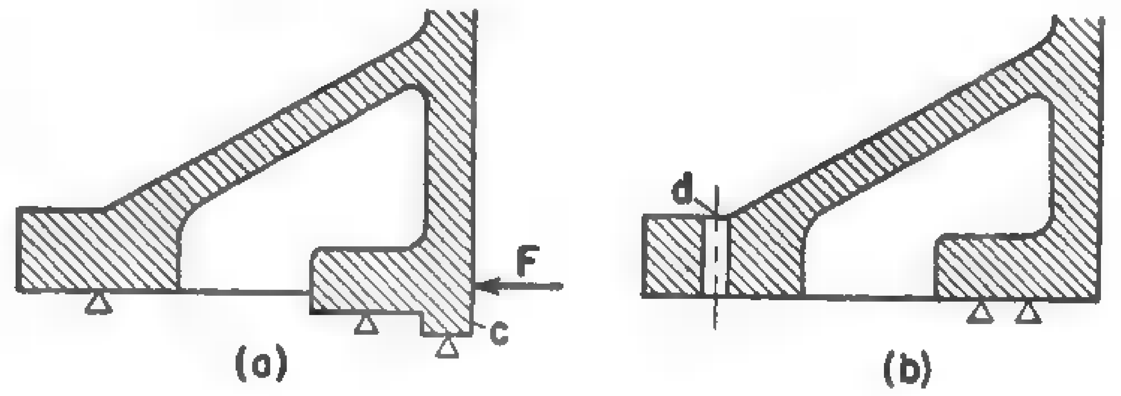


FIG. 6–13. Position of machined surfaces.

A machine part should be designed so that it can be machined without reclamping it and so that the surface to be machined will be parallel or normal to the surface to which the part is clamped. An oblique or inclined machined surface requires an inclined clamping, which always takes extra time and therefore is expensive. Arrangement of a bracket as in Fig. 6–9a requires expensive machining. The scheme in Fig. 6–9b facilitates machining and alignment in assembly. The scheme in Fig. 6–9c is still better, as it removes the danger of breaking off a bracket from a large casting. An inclined hole such as that in Fig. 6–10a cannot be drilled, because a drill cannot be started. A recess must therefore be provided in the casting, as shown

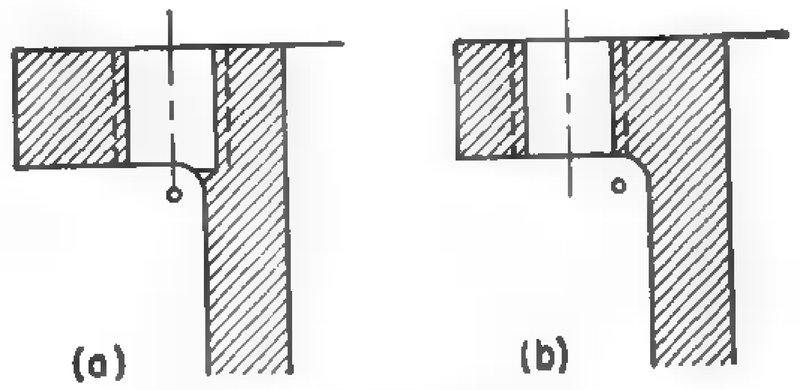


Fig. 6-14. Tapping holes in a flange.

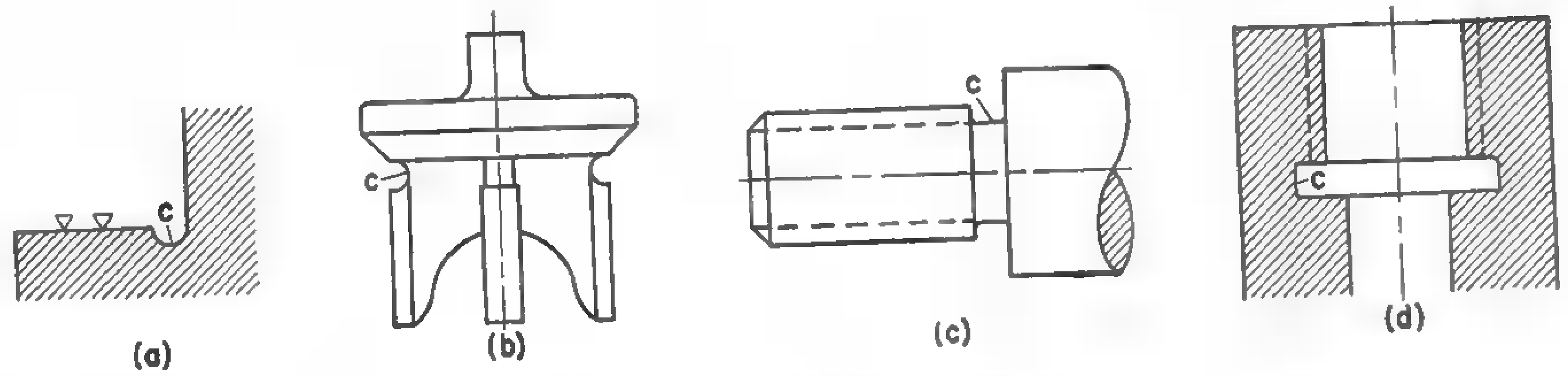


Fig. 6-15. Tool clearance.

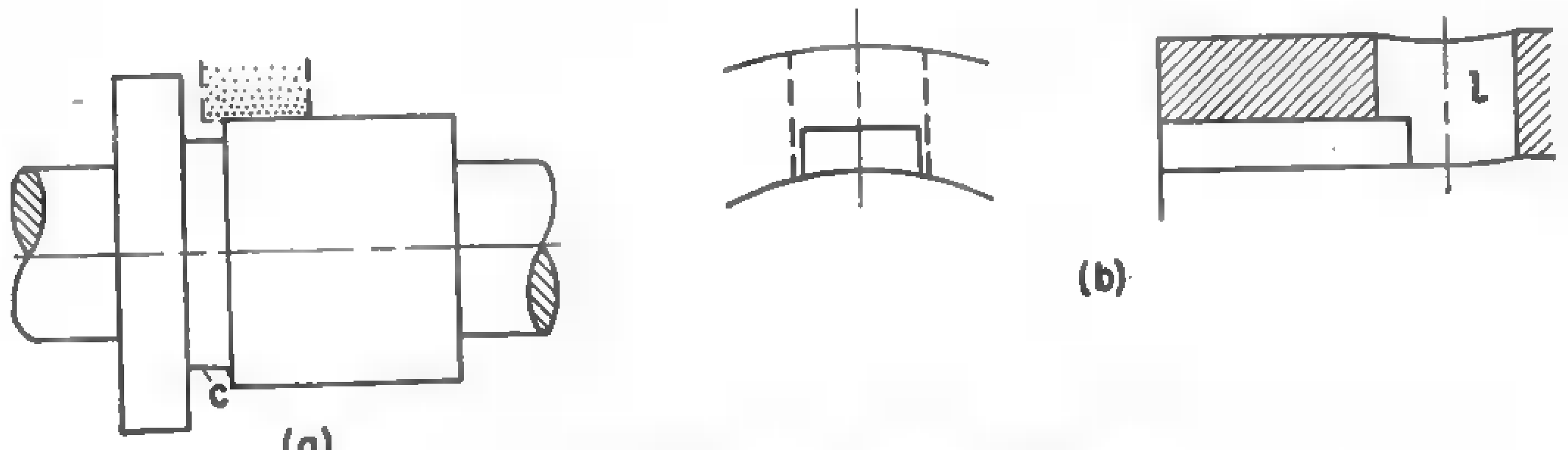


Fig. 6-16. Providing tool clearance.

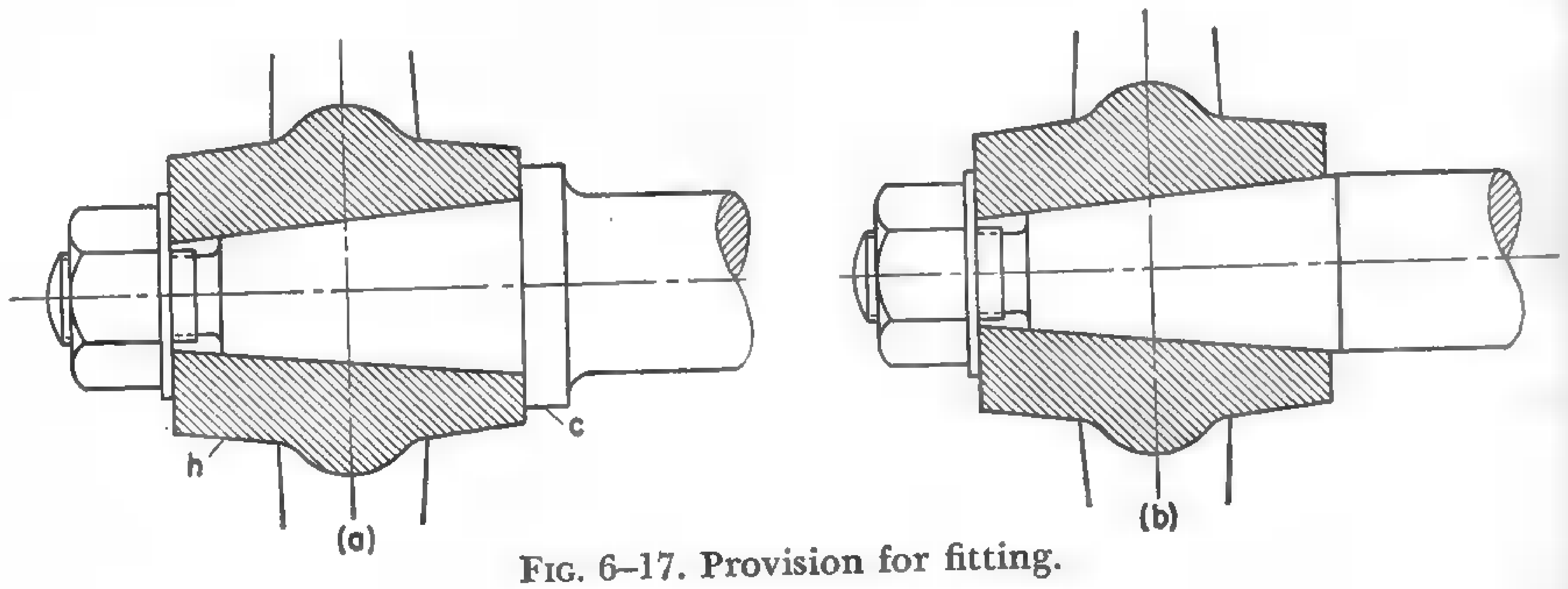


Fig. 6-17. Provision for fitting.

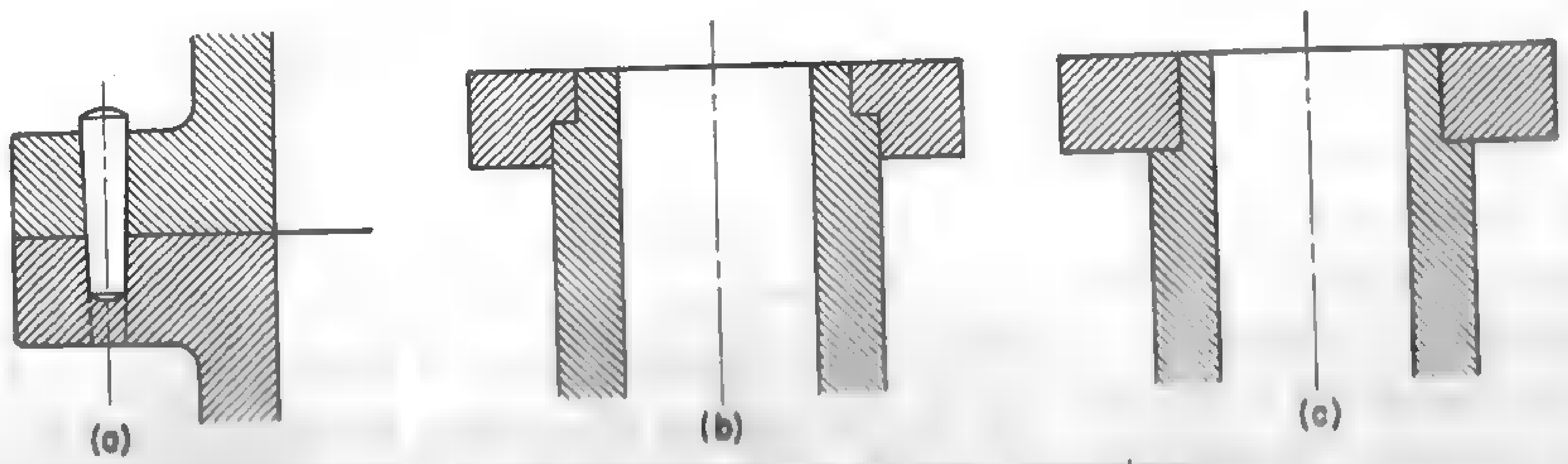


Fig. 6-18. Improved machining operations.

in Fig. 6-10b. Even if there is a recess for the drilling, a special fixture will be needed to hold the piece so that the hole will be vertical or horizontal. For the same reason a tapped hole for a drain plug should be located as in Fig. 6-11a, rather than as in Fig. 6-11b. If a hole is to be drilled in a casting shaped as shown in Fig. 6-12a, the drill may become snagged when breaking through. The shape of the casting should be changed as shown in Fig. 6-12b. Use of milled, planed, or turned surfaces in several parallel planes, as in Fig. 6-13a, is not good practice. If possible, all machined surfaces should be in one common plane, as in Fig. 6-13b. If projection *c* in Fig. 6-13a was provided to locate the part in the assembly or to resist a lateral force *F*, dowel pins *d* in Fig. 6-13b can be used instead. However, if the projection *c* is obtained by turning and must fit into a bored recess and serve as a register in assembling, this design is better than one with dowel pins.

The designer should always provide clearance for the runout of a cutting tool. The tapping of the hole in Fig. 6-14a is impossible. However, by moving the hole a little, as shown in Fig. 6-14b, the drilling operation is improved and tapping is made possible. Other examples are shown in Fig. 6-15. In Fig. 6-15a the clearance *c* must be provided in the rough casting; in Fig. 6-15b the clearance *c* in the comparatively small valve should be turned before turning the conical valve seat; in Fig. 6-15c the clearance *c* must be turned to permit cutting of the thread; and in Fig. 6-15d clearance *c* is provided for a tapped hole. Figure 6-16a shows a runout clearance for grinding. When the keyway stops inside of the bore, as in Fig. 6-16b, a hole *l* drilled at the end of the keyway will provide the necessary clearance for the keyseater.

In order to insure correct and inexpensive assembling of concentric parts, they must have registers, as shown in Fig. 6-3 at *c*. Screw threads can never be produced accurately enough to serve as a register.

Simultaneous fitting on more than one surface must be avoided. If the gear hub *h* in Fig. 6-17 must be a tight fit on the tapered end of the shaft, it should not be in contact with the collar *c*, as shown in Fig. 6-17a. Actually the collar in Fig. 6-17a is not necessary, and the correct design is shown in Fig. 6-17b. In some cases a certain design may be feasible, but it can be improved by an additional analysis. Here are a few examples: In Fig. 6-18a the upper flange is shown fastened to the lower one by a tapered dowel pin. However, drilling and reaming of the hole in the lower flange will be easier if the hole is drilled through as shown by the dotted lines. Also, a

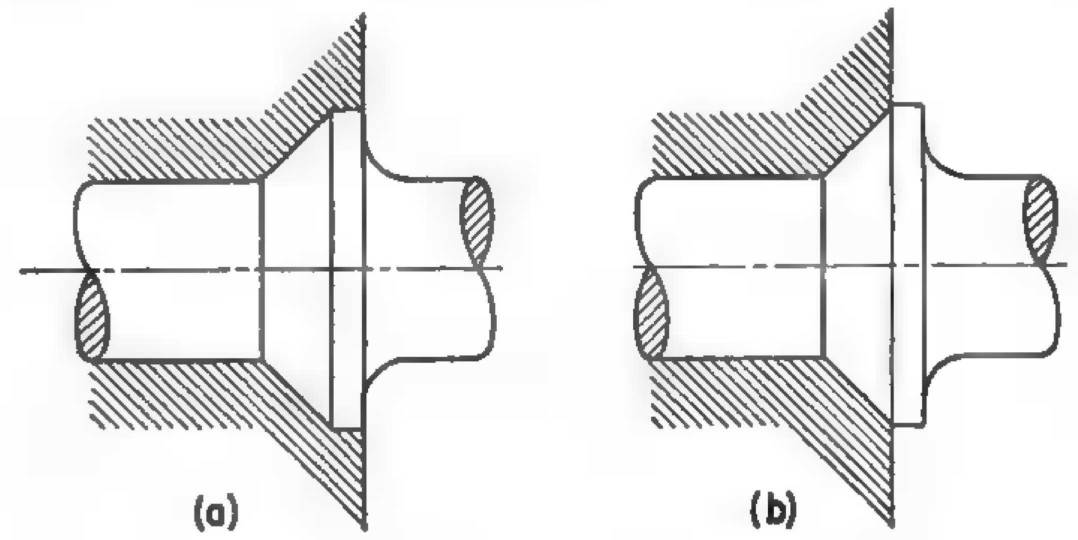


Fig. 6-19. Piston-rod fitting.

through hole will permit the pin to be knocked out when the parts are disassembled. In Fig. 6–18b is shown a register of a tubular part in a flange; and in Fig. 6–18c is shown an easier method of machining the flange without any loss of accuracy in assembly or otherwise. The design in Fig. 6–19a requires the use of a special, accurate counterboring tool, whereas the design in Fig. 6–19b is obtained by use of a simple chamfer tool and is just as good. In Fig. 6–20a is shown the conventional design of a key in a tapered shaft end. By using keyseating, as shown in Fig. 6–20b, a less expensive machine setup and easier fitting of the key are obtained.

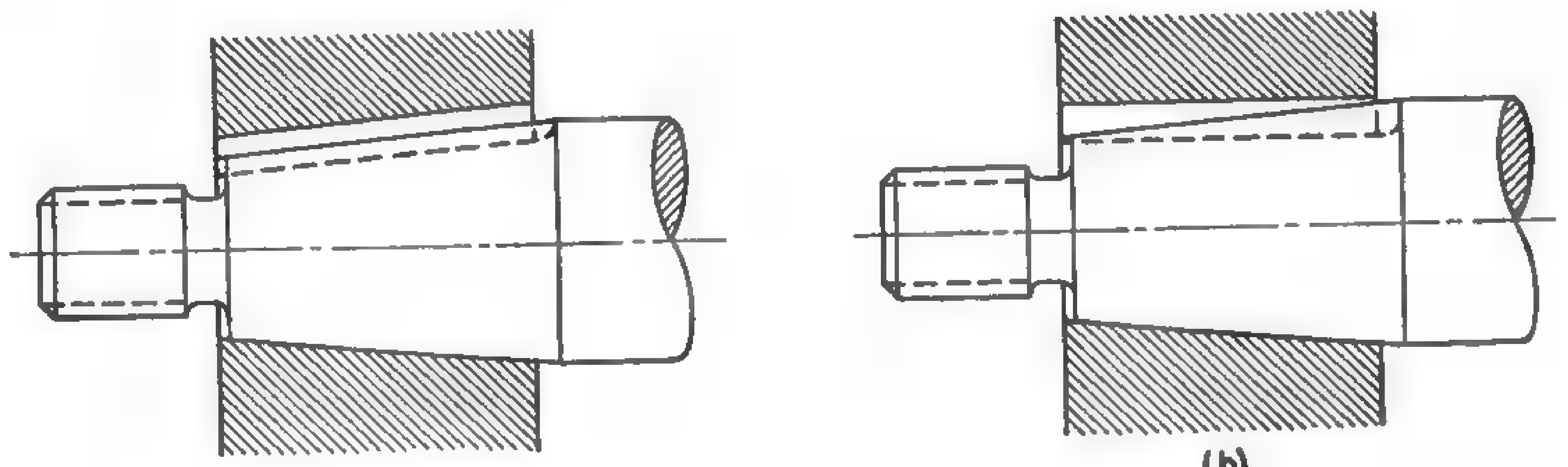


FIG. 6–20. Key in tapered shaft.

Grinding. A surface that is finished by grinding must be first machined by turning or boring. When giving the dimensions to which the piece is machined before grinding, the designer should not forget to leave or allow enough stock to be removed by grinding. Some manufacturers recommend the following allowances for grinding: 0.003 to 0.008 in. for pieces less than 3 in. in diameter; and 0.006 to 0.010 in. for pieces over 3 in. in diameter. The allowances recommended by Landis Tool Company are given in Table 6–1.

6–3. Dimensioning and tolerances. Even with the best machines and workmanship it is impossible to obtain a certain dimension of a metal part exactly. The actual dimension will be a few thousandths of an inch larger or smaller than the dimension called for, the difference depending on the procedure by which the part is finished and on the skill of the machinist.

The actual dimension of a part can deviate from the nominal dimension by a certain amount. The difference between the nominal dimension and either the largest or smallest permissible dimension is called the *tolerance*. Its amount for any particular part depends on the duty of the part.

The designer must specify the tolerance for each dimension on a drawing. He should remember that the smaller the tolerance, the more accurately a machine part will be finished and the smoother will be the operation of the mechanism that he is designing—at least within certain limits. On the other hand, the smaller the tolerance, the more expensive the machining of the part will be. As a first approximation one may assume that the product of the cost of machining and the tolerance is constant. When this

TABLE 6–1

ALLOWANCES FOR GRINDING

Diameter (Inches)	Allowance				
	0.010 In.	0.015 In.	0.020 In.	0.025 In.	0.030 In.
	Length (Inches)				
½–¾	3–12	15–24	30–48	.....	.....
1	3–9	12–24	30–48	.....	.....
1¼	3–6	9–24	30–48	.....	.....
1½	3	6–18	24–48	.....	.....
2	....	3–15	18–42	48	.....
2¼	....	3–12	15–36	42–48	.....
2½	....	3–9	12–30	36–48	.....
3	....	3–6	9–24	30–48	.....
3½	....	3	6–18	24–48	.....
4	....	....	3–15	18–42	48
4½	....	....	3–12	15–36	42–48
5	....	....	3–9	12–30	36–48
6	....	....	3–6	9–24	30–48
7	....	....	3	6–18	24–48
8	....	....	.....	3–15	18–48
9	....	....	.....	3–12	15–48
10	....	....	.....	3–9	12–48
11	....	....	.....	3–6	9–48
12	....	....	.....	.....	3–48

statement is expressed graphically, as in Fig. 6–21, you can see how fast the cost goes up with a decrease of tolerance. Therefore the designer should carefully analyze what tolerance is permissible for every dimension and should not specify a tolerance smaller than that really necessary.

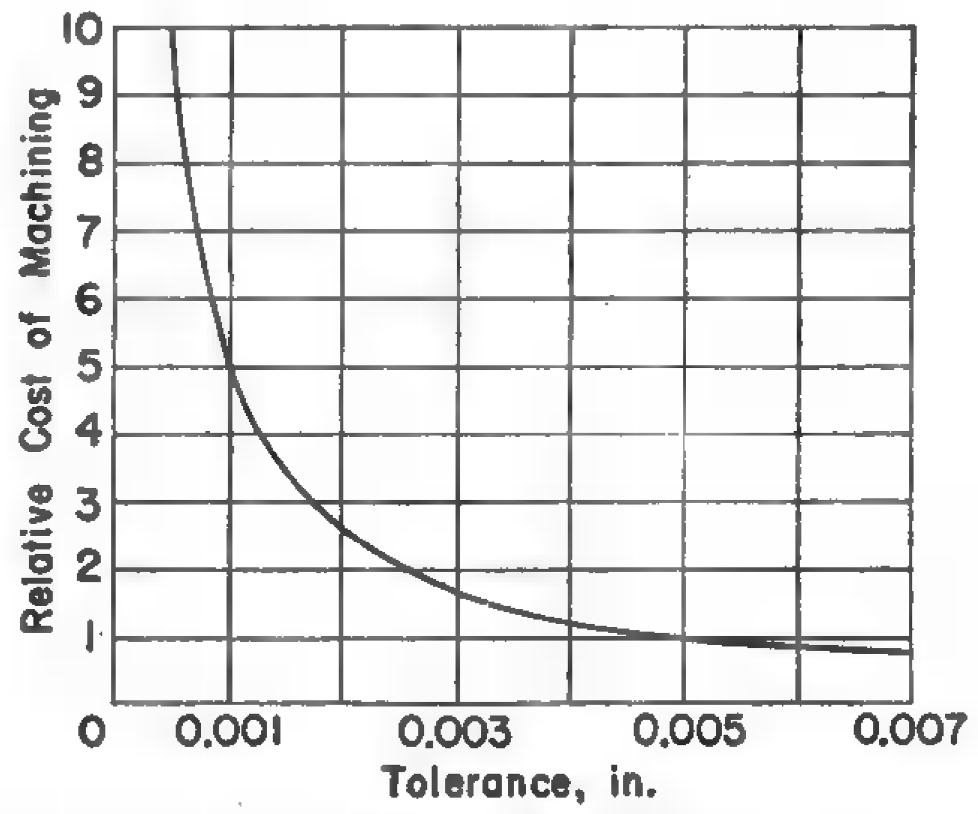


FIG. 6–21. Influence of tolerance on cost of machining.

In the shop it is much more difficult to obtain a certain tolerance with a large dimension than with a small one. It has been found that the probable error due to both the machining operation and the measuring process is proportional to $\sqrt[3]{d}$. The numerical relation between the basic international tolerance and the size is given by the ISA formula.[1] However, this formula gives the tolerance in ISA units, one unit being equal to 0.00001 in. Converted to English units, this formula is

$$i = 0.000052\sqrt[3]{d} + 0.000001d \tag{6–1}$$

[1] ISA stands for International Federation of the National Standardizing Associations.

where i is the tolerance, in inches, and d is either the diameter or the length (whichever is larger), in inches. The second term in equation 6–1 is introduced to take into account inaccuracies due to temperature and elasticity of the gages. Usually it is below the accuracy of shop measurements, except for very large dimensions. This relationship is shown graphically in Fig. 6–22.

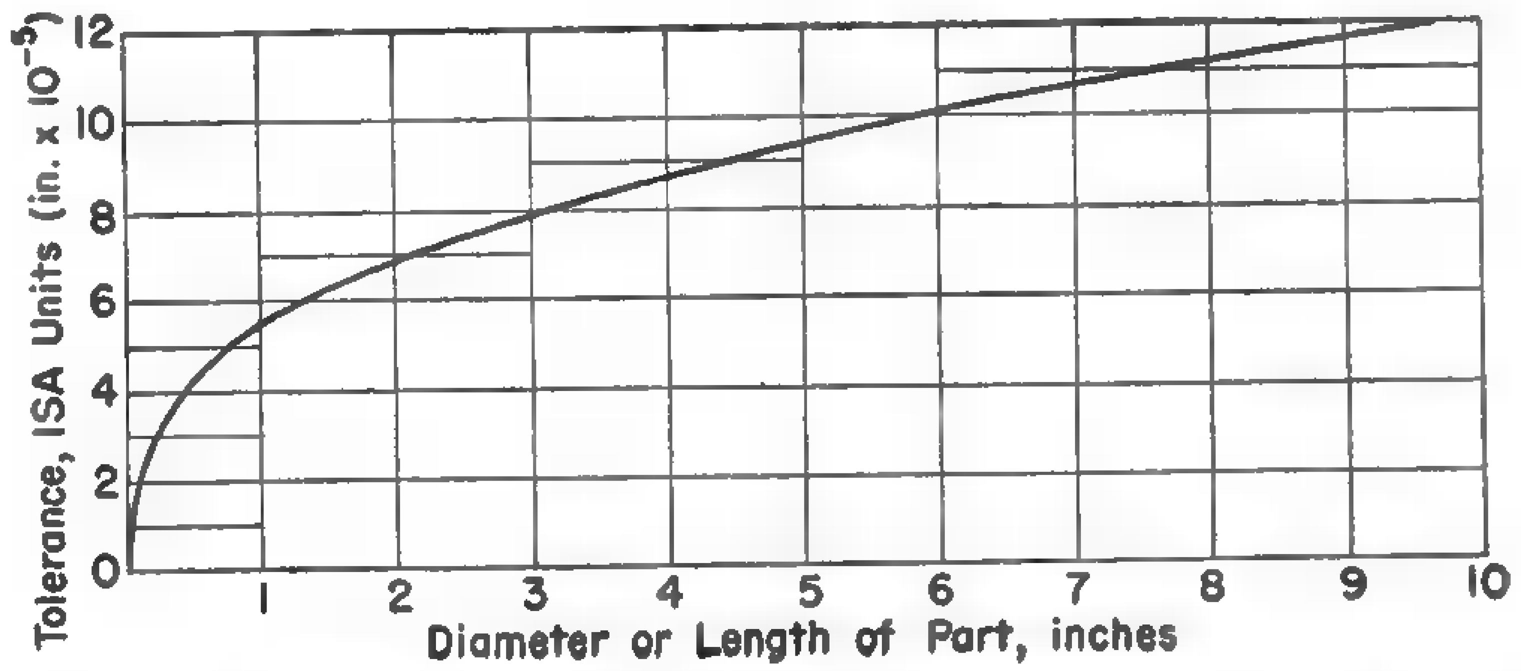


FIG. 6–22. Relation between size of part and manufacturing tolerance.

The actual shop tolerance t is made several times as great as the basic international tolerance. Thus,

$$t = ni \tag{6–2}$$

where n usually is taken as 12 for a very fine fit, 16 for a fine fit, 24 for a free fit, and 48 for a loose fit.

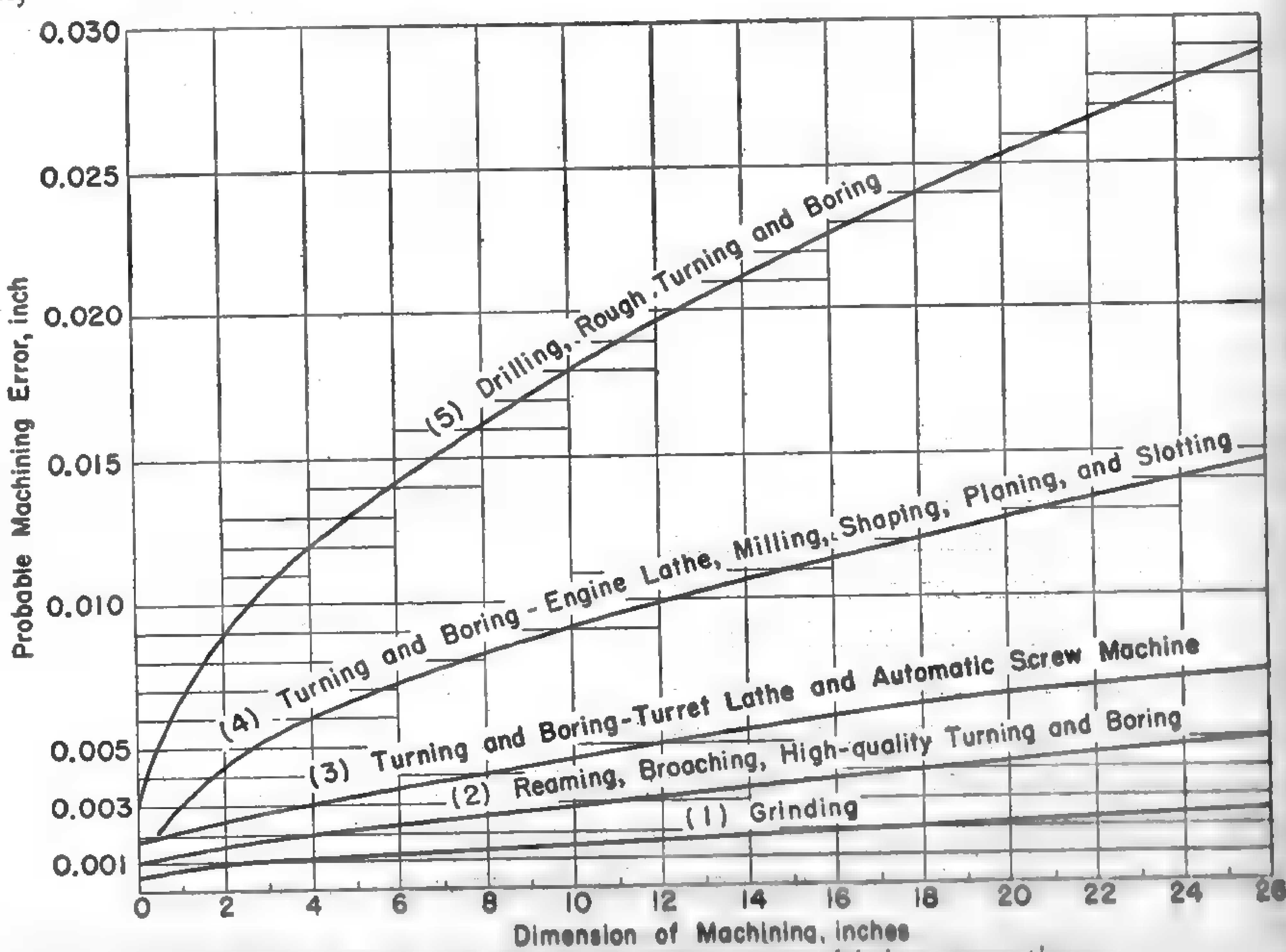


FIG. 6–23. Probable errors of different machining operations.

In specifying a certain tolerance the designer should also remember that the various machining operations do not have the same accuracy. The curves in Fig. 6–23 give the average probable errors of different machining operations and may serve as guides in specifying tolerances. If necessary the specified tolerances can be smaller than the probable errors. However, reducing the tolerance will increase the cost of machining.

Tolerance designation. The tolerance for a dimension may be put on a drawing in several different ways. Under some conditions either the minimum permissible dimension or the maximum permissible dimension must be equal to the nominal dimension. The tolerance is then said to be *unilateral.* If the actual dimension may be either greater or less than the nominal size, the tolerance is *bilateral.*

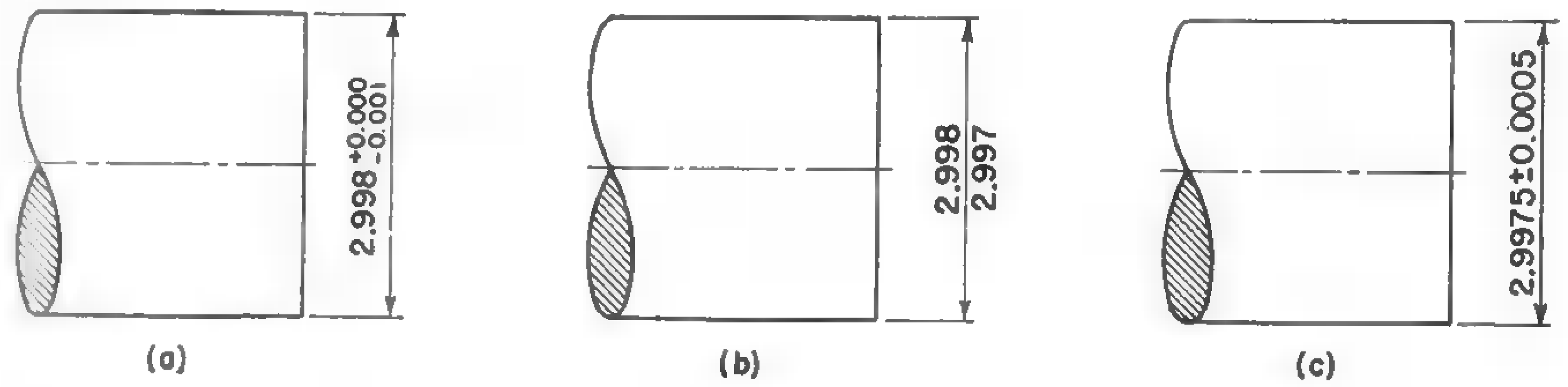


FIG. 6–24. Methods of indicating tolerance for a shaft.

When the tolerance is unilateral, it is good practice to indicate the nominal, or basic, dimension for the *maximum* metal condition. The dimension of a hole or any other inside dimension indicates the minimum permissible size; and the dimension for a diameter or an outside length shows the maximum permissible size. The machinist then aims to obtain the basic dimension, and he has the tolerance as leeway for inaccuracy of his work. One method of indicating a unilateral tolerance on a drawing is to write the tolerance after the basic dimension with its proper sign and also to give the opposite sign followed by a number of ciphers. An alternate method with the same meaning is to indicate the basic dimension first and to place under it another dimension that includes the tolerance.

When the tolerance is bilateral, the nominal dimension is indicated as the average between the maximum and minimum sizes, and the tolerance is given as one-half the difference between the maximum and minimum sizes with a plus-and-minus sign.

In Fig. 6–24 are illustrated the three different methods of indicating tolerance for a shaft, and Fig. 6–25 illustrates the dimensioning of a hole. In each case the first method described for a unilateral tolerance is shown in Fig. 6–24a and Fig. 6–25a; the alternate method is shown in Fig. 6–24b and Fig. 6–25b; and the method for a bilateral tolerance is shown in Fig. 6–24c and Fig. 6–25c.

Basic shaft and hole systems. The most common case of assembling two machine parts is when one of the parts is a rotating shaft and the other is either a hub that must fit tightly on the shaft or a bearing in which the shaft rotates. Cold-rolled or ground shafting comes in certain standard sizes with close tolerances. In such a case it is logical to fit the hole to the standard or basic shaft size and to indicate the required hole dimension and tolerances on the drawing. This method of dimensioning is called the *basic shaft system* and is illustrated in Fig. 6–24a and b.

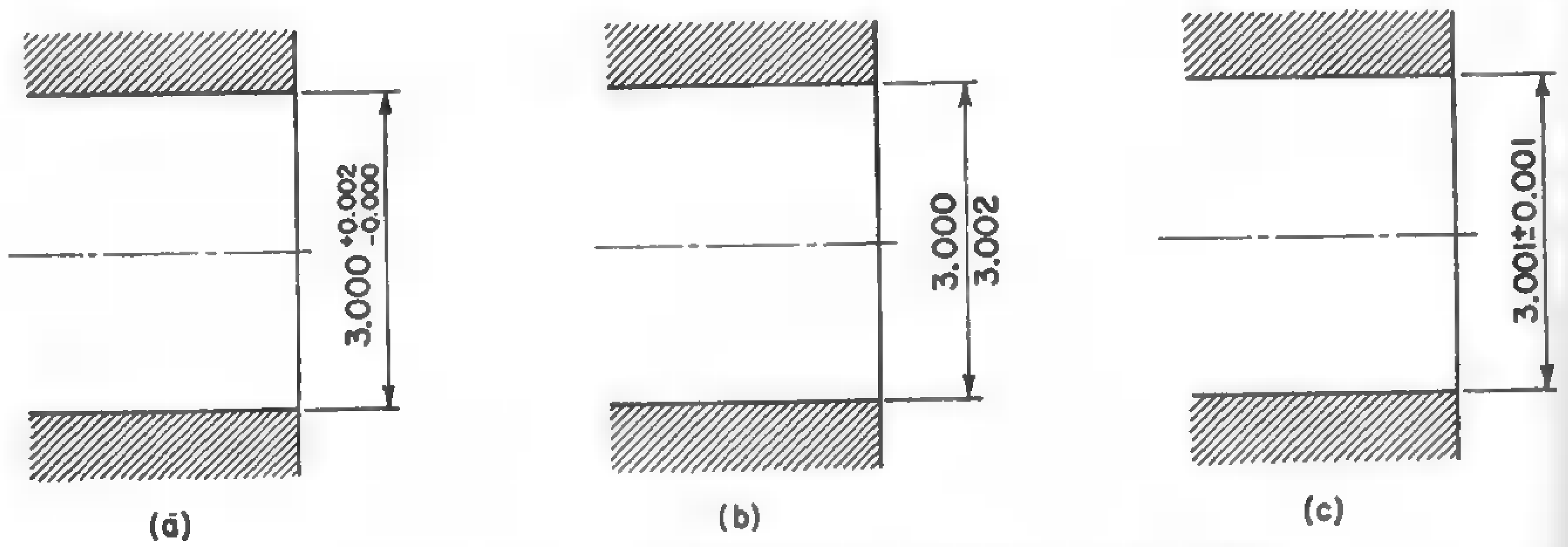


Fig. 6–25. Methods of indicating tolerance for a hole.

On the other hand, when a shaft or some other male part must be machined anyway, it is convenient to use standard-size reamers for the holes and to machine the male parts to fit the reamed holes. In this case the dimensioning is said to be done on the *basic hole system.* This method is illustrated in Fig. 6–25a and b.

In the United States the American Standards Association has adopted the unilateral method of expressing tolerances, with the hole as the basis of the system.[2] However, for reasons just explained, both the bilateral method and the basic shaft system are still used in industry.

Comparison of tolerance methods. The unilateral system of dimensioning possesses distinct advantages in making interchangeable parts. Since both male and female basic dimensions are selected so as to produce the desired fit, the tolerances may be slightly changed without affecting the interchangeability of the mating parts. Also, when the unilateral system is used, fewer spoiled parts are rejected after inspection, because the machinist has the full tolerance as leeway for his accuracy. Moreover, it is easier to check drawings made with unilateral tolerances than those made with bilateral tolerances.

In certain cases, however, the bilateral method of tolerances may be better. An example is in the location of a hole center when the deviation from the basic dimension is equally critical in both directions. Also, for dimensions where large tolerances are permissible, it may be more convenient

[2] ASA B4A-1925.

to give the mean dimension and the allowable deviations as plus and minus values. For the same reason, welded assemblies are often dimensioned with bilateral tolerances.

Standard tolerances. Some engineering departments use drawing paper with a blanket note printed or rubber-stamped stating: "Tolerances not specified to be ±0.010 in." A note used by a manufacturer of precision equipment reads: "All finished dimensions ±0.007 in. unless otherwise specified." Another concern uses a note that reads: "Unless otherwise specified, dimensions up to 3 in. to be machined with a tolerance ±0.003 in.; larger dimensions, up to 12 in., with a tolerance ±0.005 in.; dimensions over 12 in., with a tolerance ±0.010 in."

Such a note simplifies the work of the designer to some extent. However, it does not relieve him of the responsibility of specifying smaller tolerances if the duty of the part really requires them; or of specifying larger tolerances if the duty of the part permits them, in order to cut down the machining cost.

Decimals and fractions. Because of the increasing use of small tolerances, which must be shown in decimals of an inch, many manufacturers and U. S. Government agencies have decided to use decimal designations for all main dimensions also, even where a simple fraction could be used instead. It is good practice, if the decimal system is to be used at all, to use it consistently and to refrain entirely from the use of fractions. Care should be observed in selecting the number of digits to be used when substituting decimal values for simple fractions. The number of digits should correspond to the accuracy desired. Thus, $\frac{5}{16}$ in. must be designated as 0.313 in. if a high degree of accuracy is desired, as indicated by a small tolerance. If accuracy is not so important, as where a tolerance of ±0.010 in. is permissible, the same dimension should be written as 0.31. If a variation of ±0.015 in. will not interfere with the duty of the part, the dimension may be given as 0.3.

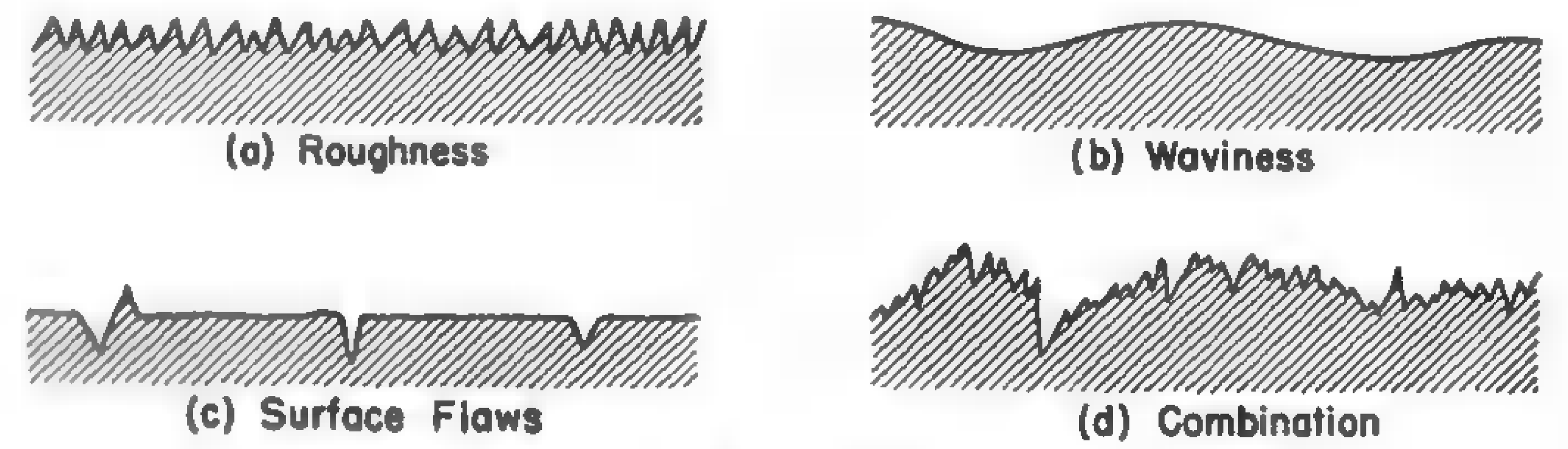


Fig. 6–26. Types of surface irregularities.

6–4. Surface finishes. The possible types of deviations of the actual surface of a machine part from its nominal surface are shown in Fig. 6–26 in greatly magnified profile sections. These types are classified as follows:[3]

[3] *Surface Roughness, Waviness, and Lay,* Proposed American Standard ASA 1346.1 (July, 1947).

Roughness, shown in Fig. 6–26a, is characterized by small irregular unevennesses, such as may be felt when drawing a fingernail over a ground surface. *Waviness*, in Fig. 6–26b, is characterized by more or less regular and repeating unevennesses that are in the nature of waves. *Surface flaws*, shown in Fig. 6–26c, comprise scratches and checks, and similar defects which occur at irregular intervals and are often caused by heat treatment. Combination of the three basic types of irregularities is often found, as shown in Fig. 6–26d.

Lay, or the direction of the prevailing surface pattern, often affects the performance of the part.

Measurement of irregularities. With the improvements in finishing operations a demand has gradually developed for a more exact method of specifying surface finish than simply by marking on the drawing "finish," "grind," or "hone." The surface smoothness can be specified by indicating the permissible height of the irregularities above and below the mean plane.

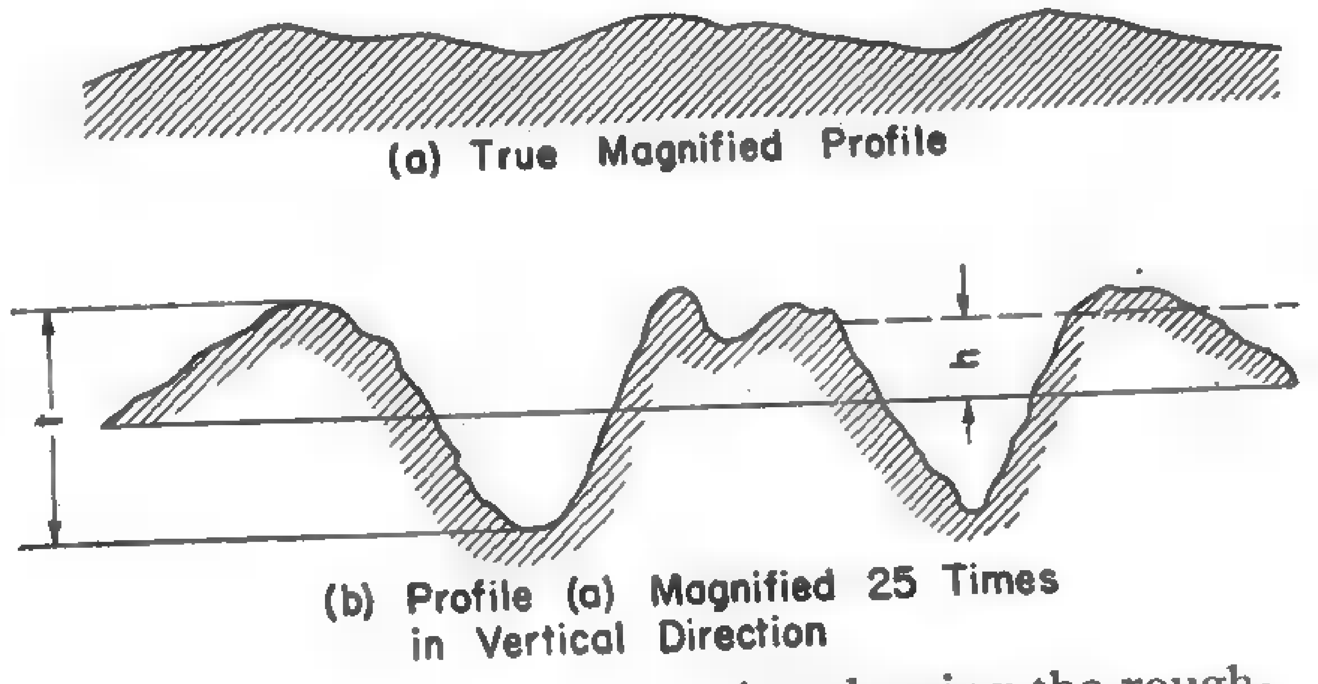


FIG. 6–27. Magnified cross section showing the roughness of a surface.

A true magnified profile of a surface, or the contour of a section through the surface, is shown in Fig. 6–27a. In order to study the surface conditions better, Fig. 6–27b shows the same profile when an additional magnification of 25 times is given to the vertical dimensions without any change in the horizontal dimensions. This additional magnification is obtained by preparing the specimens for viewing under the microscope by a process called *taper sectioning*. In this process the surface is ground at a small angle with the surface, and the heights of the irregularities are thus accentuated. Evidently the magnification is equal to $1/\sin \alpha = \csc \alpha$, and a magnification of 25 is obtained with $\alpha = 2°17'$.

In Fig. 6–27b the distance t is the total roughness, or the distance from a peak to a valley; the mean surface is at the mean profile height; and h is a special kind of height, called the ***root mean square height***, which is the average distance above and below the mean surface. This height h is found by adding the squares of ordinates at equidistant points on the profile, dividing the sum by the number of ordinates measured, and then taking the square root

TABLE 6–2

RANGE OF SURFACE ROUGHNESS PRODUCED BY MACHINING

Machining Operation	Roughness, rms Values (μin.)	Machining Operation	Roughness, rms Values (μin.)
Superfinish	1– 10	Boring	20– 250
Polishing, buffing*	2– 20	Radial cutoff saw	40– 100
Honing, lapping	2– 20	Turning, milling, shaping	40– 500
Cylindrical grinding	10–100	Disk grinding, filing	100– 500
Surface grinding	20–250	Hand grinding	250–1000
Reaming	20–100	Chipping, hand sawing	500–1000
Drilling	20–250	Flame cutting	500–1000

* Depending on previous machine operation and grit and grade of abrasive.

of this average. The ordinates are measured from the mean surface in millionths of an inch, or microinches, abbreviated as μin. The rms (root mean square) method of calculating gives more weight to the higher peaks of the surface, since high, narrow peaks seem to have considerable influence on the surface qualities but have only a small effect upon the position of the mean-surface line. The rms height is thus slightly larger than the arithmetical average of the ordinates. Sometimes the arithmetical average, or even the peak-to-valley height, is used as measure of roughness. The drawing must indicate which one is used.

A symbol often used for surface finish is a check mark with a bar on top, as shown in Fig. 6–28. The number indicates the allowable roughness in microinches.[4] Table 6–2 gives the range of surface roughness that can be produced by various machining operations. It may serve as a guide for specifying the surface finish on the drawing of a designed part. The numbers recommended for specifying the rms height of irregularities are 1, 2, 4, 8, 16, 32, 63, 250.[5]

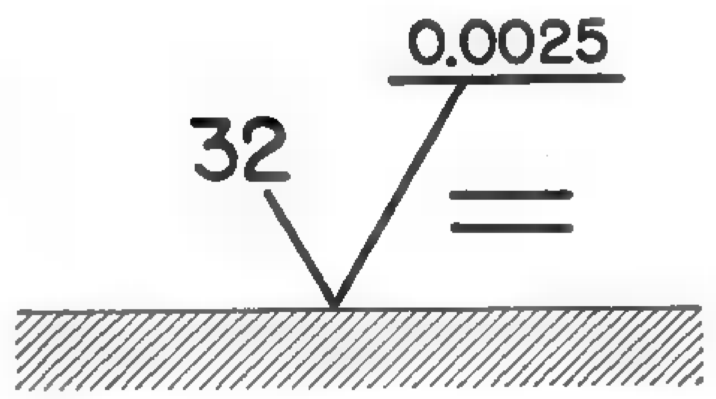


FIG. 6–28. Marking for surface finish.

6–5. Cost. One of the most important things that the designer must keep in mind at all times is the cost of manufacturing the machinery that he is designing. The cost of any part and assembly of parts depends on the material used; the amount of labor involved in machining and assembling; and, in any multiple reproduction, features of standardization and sales appeal.

[4] F. M. Mallet and R. U. Lundberg, "Tolerances and Allowances for Interchangeable Assembly," *Product Engineering*, Vol. 15 (1944), p. 479.

[5] M. R. Norton, "Development of Standards for Army Ordnance Finishes," *Mechanical Engineering*, Vol. 64 (1942), p. 703.

Materials. As a result of the metallurgical developments of the last years a great number of materials are available for every machine part. These materials vary in quality and in price. A good designer will select the least expensive material that will be satisfactory for the duty of the part. For example, he will use more-expensive alloy steels only when ordinary low-carbon steel cannot give satisfactory service. When forced to use alloy steel he should compare the properties and costs of different kinds and should again follow the same principle.

In selecting materials the designer must consider not only their strength but also their rigidity and their resistance to wear. The permissible wear of a part depends on its duty and also on the length of service that it must give. If a part is subjected to severe wear when in operation but is operated intermittently and not very often, a comparatively inexpensive material may be satisfactory; whereas a similar part in continuous operation will require a more wear-resistant material. The expected life of a part or a machine must also be taken into account. The longer the intended life, the better should be the grade of the material used.

The same consideration applies to parts purchased from other manufacturers, such as bolts, setscrews, bushings, and ball bearings. For instance, if the intended life of a certain machine is 3,000 hr, its cost will be unnecessarily increased by using in it ball bearings with an expected life of 10,000 hr. Whenever possible, standard stock sizes of rolled and extruded materials should be used without changing their cross-sectional dimensions.

Machining. Any kind of machining should be specified only where it is necessary to permit the part to function properly. In former years it was considered necessary to machine the outsides of parts that were to be put together in contact, such as the top of a cylinder and the outside of the cylinder head, in order to match them accurately. This is not necessary, and the method shown in Fig. 6–3 is less expensive and just as good. Finishing cover plates on the edges and from the top is another waste of machining. All that is necessary is to spot-face around the holes for the nuts.

Where machining is necessary it should be done by the least expensive method that is consistent with the purpose of the machining. If, for example, a part must be turned for the sake of balancing, rough turning is satisfactory, and specifying finish turning would be a mistake. Similarly, if planing is satisfactory, more expensive milling should not be specified on the drawing, and reaming should not be specified if simple drilling is satisfactory.

Where fitting of parts requires tolerances, the specified tolerances should not be closer than those absolutely necessary, as explained in section 6–3. Where surface finish is considered, do not specify a smaller rms roughness than that actually necessary.

In order to reduce the cost of assembling a machine, the parts of the machine should as far as possible be so designed and built that they will place and align themselves automatically when brought together. Usually, the more complicated a machine is, the more important it is that the ease and cost of assembling be given careful consideration. If bench work and hand fitting cannot be avoided, they should be reduced to a minimum.

The number of machines to be built has an important bearing on the design of a machine and its parts. If only one machine or a few machines of a certain kind and size are to be built, the limitations of the available plant equipment should be kept in mind in the design. The capacity of the equipment of the foundry or machine shop may require a large casting to be made in two or three parts, and the available machine tools at hand may limit the methods of machining and thus influence the design. In certain cases it may be found more economical to weld a frame than to make a large, expensive pattern for it.

On the other hand, mass production or interchangeable manufacture justifies the use of special molding machines in the foundry, special jigs and fixtures, special production and inspection gages, and special tools, dies, and machines in the welding and machine shops.

Standardization. If a part is made in lots, especially if the part is manufactured on a mass-production basis, it is very important to follow certain standards. Once a part is designed and developed, it should be considered standardized; and no changes should be made that would make the part not interchangeable with the original design. Standard stock parts should be used without any additional machining.

Preferred numbers. When a machine is to be made in several sizes having different powers or capacities, it is necessary to decide what capacities will cover a certain range efficiently with a minimum number of sizes. When a larger similar machine is built, its relation to the original smaller machine is complicated by the fact that lengths of parts are proportional to the first powers of the linear dimensions; areas are proportional to the second powers, or squares, of the linear dimensions; volumes and section moduli are proportional to their third powers, or cubes; and moments of inertia are proportional to their fourth powers. A certain range can be covered efficiently with a minimum number of sizes by the use of a geometrical progression with a constant ratio. With our decimal system it is convenient to select a series of numbers from 10 to 100. Then the series can be extended to 1,000, 10,000, and so on, simply by multiplying the base sizes by 10, 100, etc., or the series can be reduced in size by dividing the base sizes by 10. The numbers in such a series are called *preferred numbers.* The following series have been established:[6] the coarse series with the ratio $r = \sqrt[5]{10} = 1.6$, the next series with $r = \sqrt[10]{10} = 1.25$, the next with $r = \sqrt[20]{10} = 1.12$, and the

[6] *Preferred Numbers,* ASA Z17.1–1936 (New York: American Standards Association, 1936).

finest with $r = \sqrt[40]{10} = 1.06$. The first series consists of the rounded numbers 1, 1.6, 2.5, 4.0, 6.3, and 10.

When preferred numbers are used, fewer stock sizes can cover certain ranges. Thus in Germany, manufacturers of power-transmission equipment formerly carried 3,600 patterns of belt pulleys. After the introduction of standard sizes selected in accordance with preferred numbers, this number was reduced to 600.[7] Catalogues of American manufacturers of power transmission machinery list 3,400 different sizes of stock belt pulleys. This number could be reduced to about 570 sizes by use of preferred numbers. Such a reduction would mean a great saving in inventory and probably in manufacturing cost too. There is a wide field for the application of preferred numbers in various fields, and machine designers can contribute a good share to our economy by using them for serial designs in proper places.

[7] M. Ten Bosch, *Vorlesungen über Maschinenelemente*, 2d ed. (Berlin: Julius Springer, 1940), p. 10.

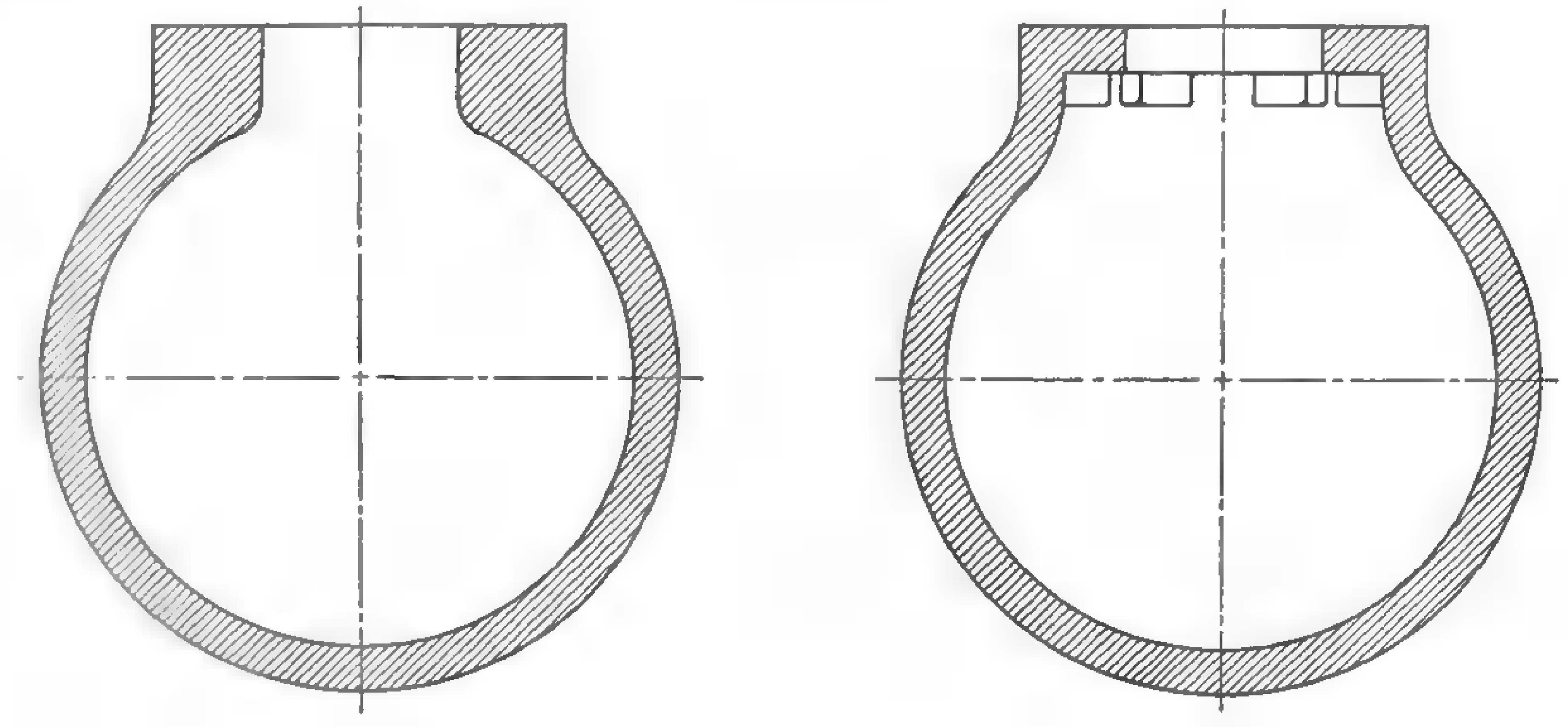
CHAPTER 7

Design of Castings

7–1. General considerations. Both the art and the science of casting the various metals have been developed and specialized to such an extent that it is impossible for the average designer to grasp all the requirements and details which are conducive to the best results. When designing a more or less important part the designer should always consult the foundryman and the patternmaker, whose cooperation is a prerequisite of ultimate success. However, every designer should know and observe the general rules which are given below.

Basic rules. The two main rules that should be observed in the design of castings are the following: (1) The section thickness should preferably be uniform. (2) Where section uniformity is not possible, light sections should be blended into heavy ones.

In the conventional design in Fig. 7–1a the section thickness is not uniform. By removing excess metal, as in Fig. 7–1b, the design is considerably improved, and the casting is made sounder, stronger, and lighter.



(a) (b)

FIG. 7–1. Cylinder with opening for a pipe flange.

Blending. An abrupt change from a thin section to a heavy one, as in Fig. 7–2a, will produce a high stress concentration in the juncture and may result in shrinkage cracks or in cracking when a tensile load is applied. On the other hand, a big fillet, as in Fig. 7–2b, is apt to produce a large local accumulation of metal which will still be liquid when the outside fibers have solidified, with the result that there will be a porous inside or even a blow

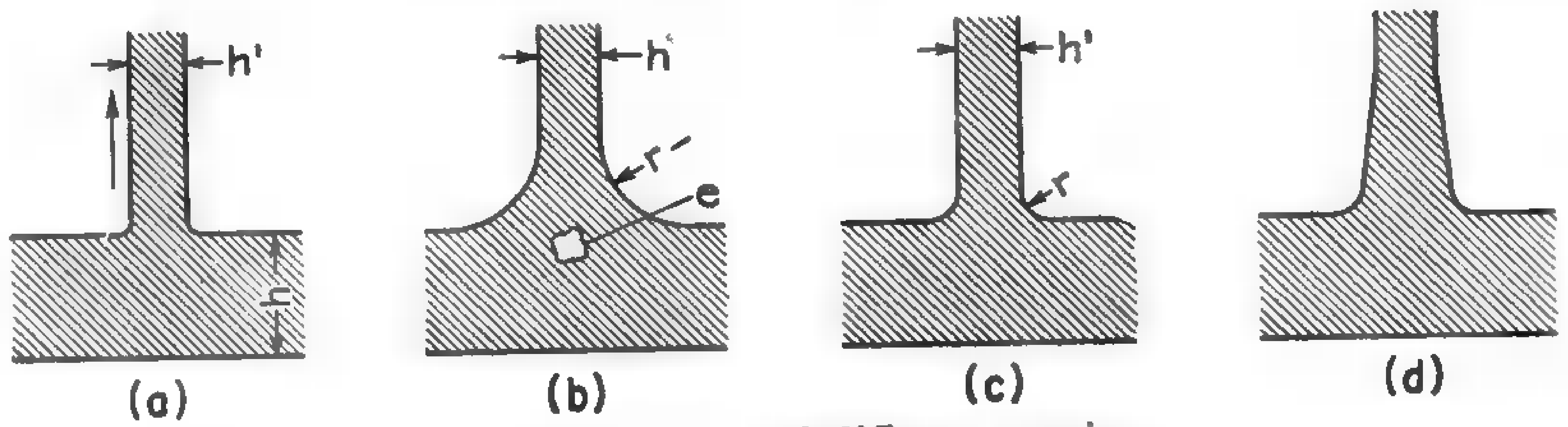


FIG. 7-2. Junctures of different sections.

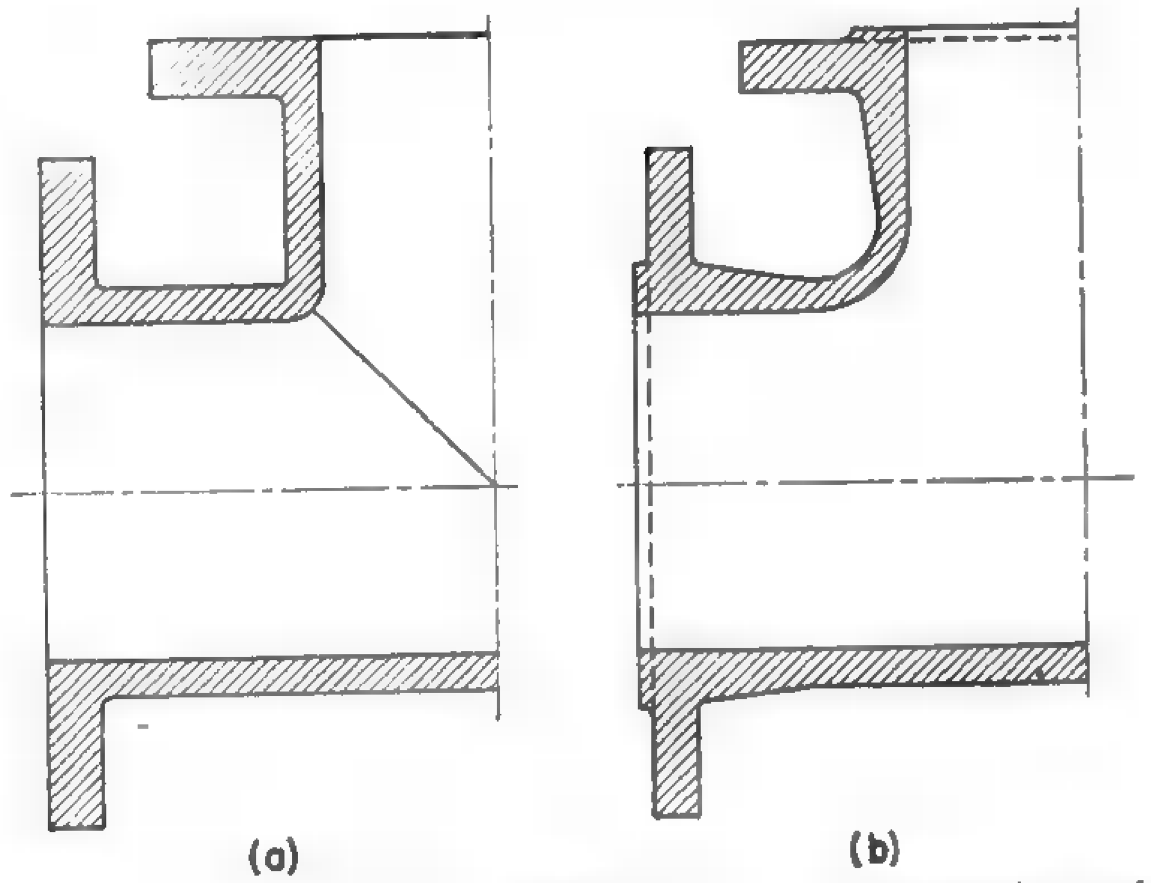


FIG. 7-3. Poor and good design of a tee-shaped casting.

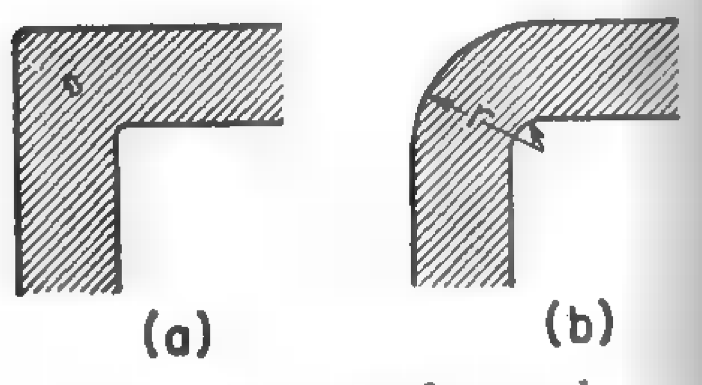


FIG. 7-4. Faulty and correct corners.

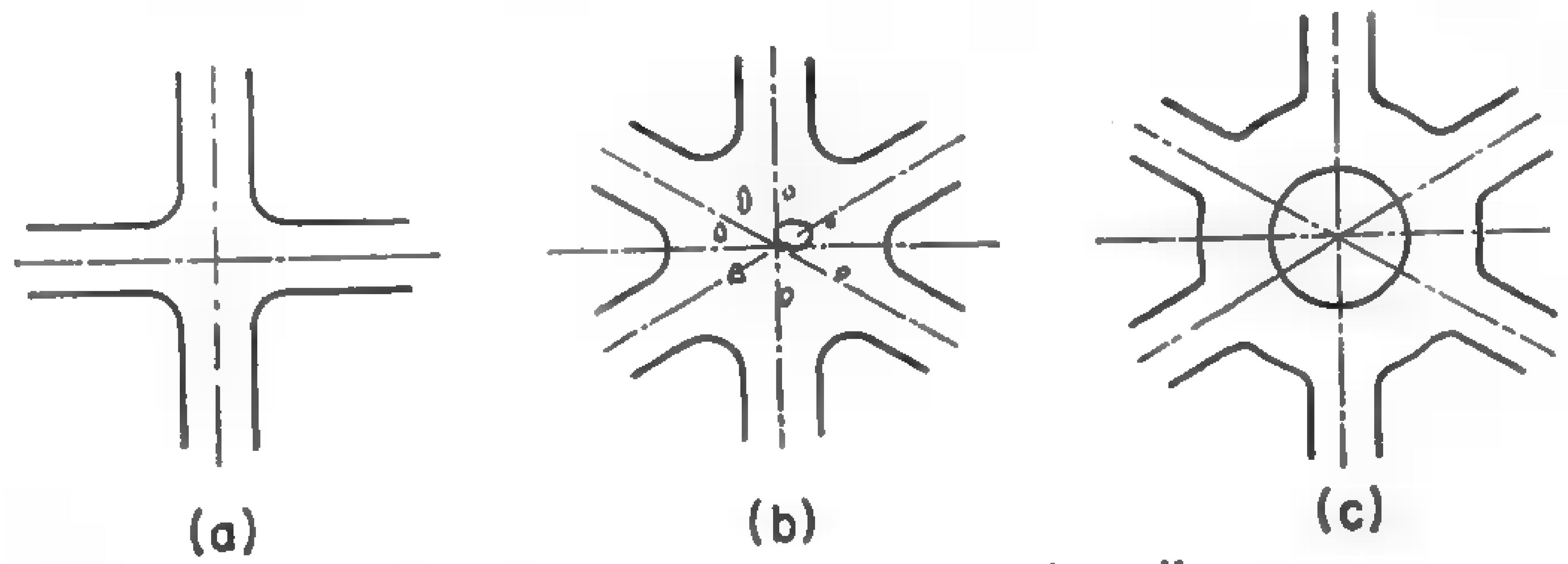


FIG. 7-5. Junctures of intersecting ribs.

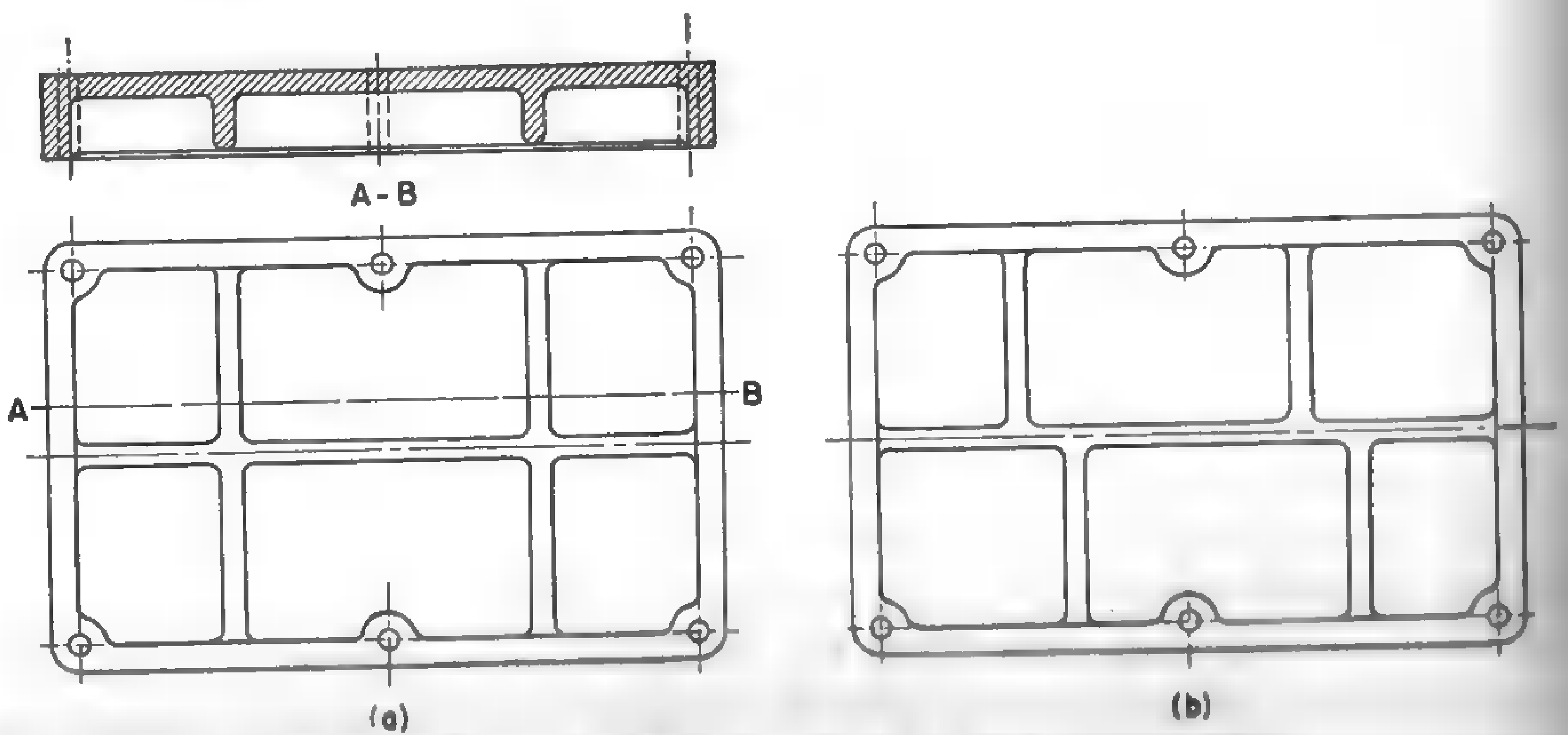


FIG. 7-6. Ribbing of a flat surface to make it more rigid.

hole *e*. A moderate fillet, as in Fig. 7–2c, with a radius *r* approximately equal to $0.5h'$, will avoid an excessive stress concentration without producing an unsound casting. A gradual taper, as in Fig. 7–2d, will give still better results.

In Fig. 7–3a is shown a poor design that can be improved in several respects as shown in Fig. 7–3b. There is a gradual change from a thin wall to the heavy flanges; a generous radius is inserted at the juncture of the two pipes; and the inside portions of the flange faces are raised as working surfaces to decrease the amount of machining.

Mass distribution. When a heavy section or a boss is surrounded by light sections it is important to secure proper feeding of the boss by placing ribs with a sufficiently large cross section on the light parts. This will prevent unsound casting.

The main sections should be made sufficiently heavy, or thick, to permit the addition of a gate of such size as to secure good feeding of all sections.

At a sharp corner, as in Fig. 7–4a, the casting will be unsound and there will be high stress concentration. Whenever possible a rounded corner should be used, as in Fig. 7–4b.

Consideration must be given to possible constraining effects which may prevent complicated castings from responding freely to necessary changes of form during cooling in the mold.

Intersecting ribs of the types shown in Fig. 7–5a should be avoided, as they are apt to produce an unsound casting. Six ribs, as in Fig. 7–5b, make a poor design because of shrinkage cavities, which can be avoided by the arrangement in Fig. 7–5c.

Unsoundness due to excessive rigidity resulting from intersecting ribs on a flat surface, as in Fig. 7–6a, can be avoided by staggering the ribs as shown in Fig. 7–6b.

Working surfaces should be provided, as in Fig. 7–3b, to cut down the cost of machining. Properly arranged working surfaces also contribute to the rigidity of a construction.

Minimum thickness. The required thickness of a casting determined by calculations based on strength alone is often too small to permit the production of a good casting. Other sections carry either very small loads or no loads at all. In every such case the thickness of the section must not be less than a certain practicable minimum value, which depends on the size and intricacy of the part and on the kind of metal used for the casting.

Small gray-iron castings may have sections as thin as $\frac{1}{8}$ in. However, the average minimum thickness for gray iron is about $\frac{1}{4}$ in. for parts up to about 18 in. in length or diameter; and it gradually increases to $\frac{3}{4}$ in. for large and heavy castings. For malleable- and ductile-iron castings the minimum thickness may be taken as $\frac{1}{8}$ in., but the average minimum thickness of

larger castings should be about $\frac{1}{4}$ in. For steel the minimum thickness is usually regarded as $\frac{1}{4}$ in. In a small casting, however, it can be lowered to $\frac{3}{16}$ in. For brass and bronze the minimum thickness is $\frac{3}{32}$ in.; and for aluminum it is $\frac{1}{8}$ in., with $\frac{3}{16}$ in. as an average minimum.

7–2. Ribs. Ribs are added to make a construction either stronger or more rigid. While they always increase the rigidity, they may fail to increase the strength. If the addition of ribs lowers the maximum stress in a part, the ribs strengthen it. If the maximum stress in the main part of the original design is lowered, but the stress in the added rib is equal to or higher than the stress in the original design, the rib either is useless or weakens the part. A crack may develop in the highly stressed rib, and it will eventually spread and cause the part to fail. It should be remembered that rigidity is measured by deflection, which is inversely proportional to the moment of inertia I of a section; whereas the stress is inversely proportional to the section modulus Z. The resulting conditions can be best illustrated by a numerical example.

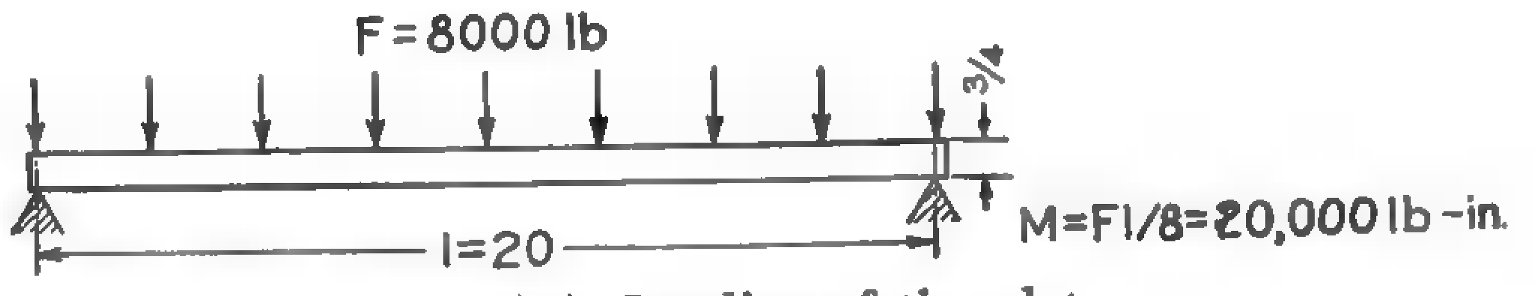


FIG. 7-7. Loading of the plate.

EXAMPLE 7-1. A cast-steel plate 20 in. long, 15 in. wide, and $\frac{3}{4}$ in. thick is supported at the ends and carries a uniformly distributed load of 8,000 lb, as indicated in Fig. 7-7 and Fig. 7-8a. Find the influence of conventional ribs upon the rigidity and strength of the plate.

The bending moment due to the load is

$$M = \tfrac{1}{8}\,Fl = \tfrac{1}{8} \times 8{,}000 \times 20 = 20{,}000 \text{ lb-in.}$$

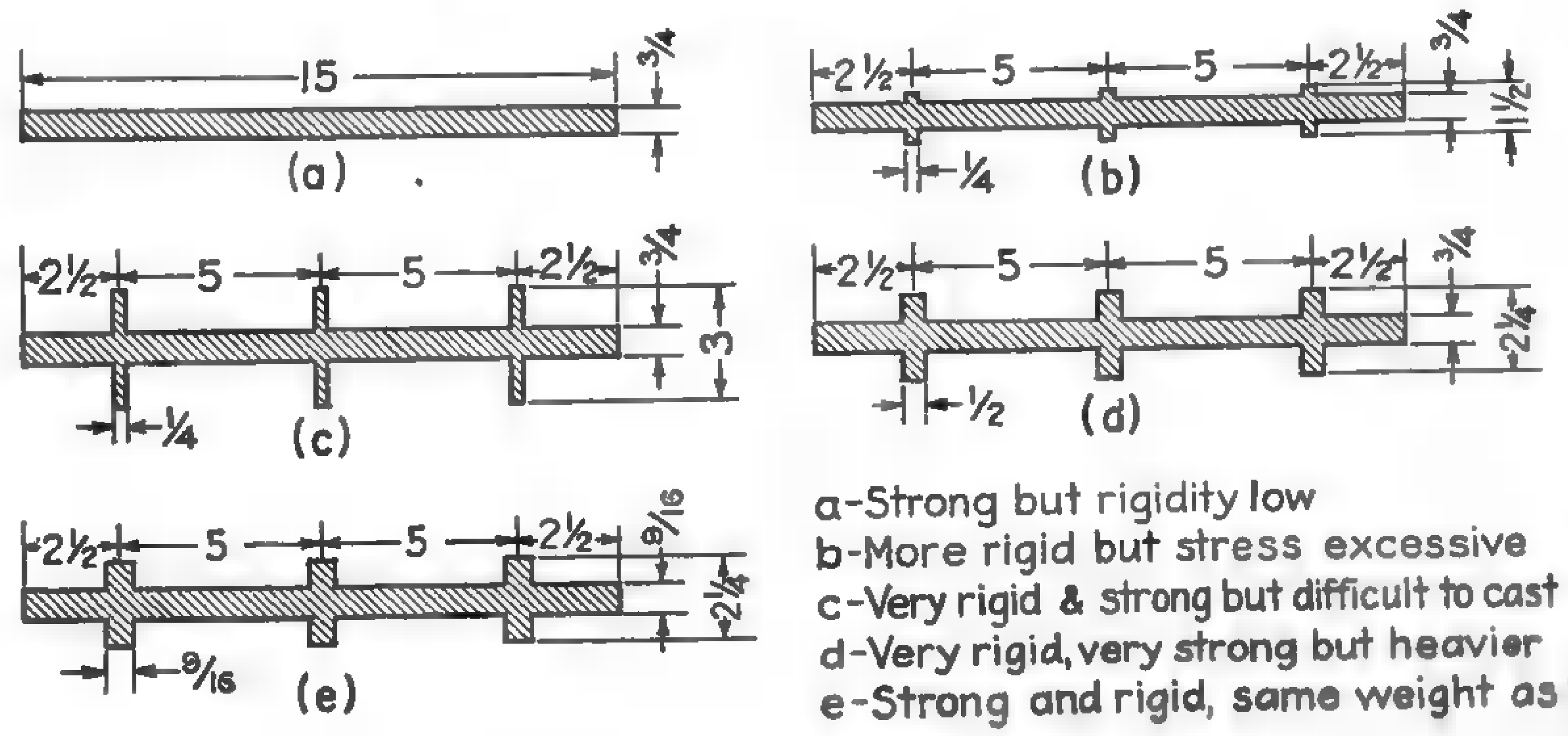


FIG. 7-8. Various ribs added to a plate.

Naturally the ribs must extend in the direction of the greatest dimension, or lengthwise. Dividing the width into three strips, each 5 in. wide, and adding $\frac{1}{4}$-in. ribs to double the thickness of the plate, we obtain the section in Fig. 7-8b. In Table 7-1 are given I, Z, W,

TABLE 7-1

RIGIDITY AND STRENGTH OF A PLATE WITH RIBS

Section of Plate (Fig. 7-8)	Moment of Inertia I (in.^{4})	Section Modulus Z (in.^{3})	Weight W (lb)	Highest Stress s (psi)	Relative Rigidity	Relative Strength	Relative Weight
a....	0.53	1.41	63.0	14,200	1	1	1
b....	0.71	0.95	66.2	21,000	1.34	0.68	1.05
c....	2.19	1.46	72.5	13,700	4.13	1.04	1.15
d....	1.90	1.69	75.6	11,800	3.58	1.20	1.20
e....	2.20	1.96	62.1	10,200	4.15	1.39	0.99

and s of the flat plate in a, and of the plate in b, after the small ribs are added. Since the moment of inertia is increased from 0.53 to 0.71, the rigidity is increased by about 34 per cent. However, as the maximum stress is increased from 14,200 to 21,000 psi, the strength is decreased by 32 per cent.

The types of ribs in Fig. 7-8c were provided in order to obtain at least the same strength as in Fig. 7-8a. By increasing the height to 3 in. the plate became 4.13 times as rigid with a slight decrease of the stress from 14,200 to 13,700 psi. These results are also shown in Table 7-1. However, it will actually be very difficult to cast such thin and high ribs, and they will cause additional stresses because of the abrupt change of the thickness from $\frac{3}{4}$ in. to $\frac{1}{4}$ in.

A better type of rib is shown in Fig. 7-8d. The height of each rib is equal to the thickness of the plate, and the width of the ribs is twice that of the two previous designs. As shown in Table 7-1, the rigidity in Fig. 7-8d is only slightly smaller than in Fig. 7-8c, while the strength in Fig. 7-8d is about 20 per cent above that of the flat plate. The weight W is not appreciably affected by the ribs.

Finally, in Fig. 7-8e is shown a design which has still greater rigidity and greater strength, and is even slightly lighter than the flat plate. In this case the body of the plate is made thinner and the ribs are of the same thickness as the plate. This plate will have only very small internal casting stresses and little stress concentration.

A quicker and more precise method than cut-and-try for determining the proper dimensions of ribs exists.[1]

Impact loading. It has already been shown that to decrease the danger of failure of a part subjected to impact loads, the resilience of the part involved must be increased. Such a change in design, in turn, requires greater deformations. Since ribs have an opposite effect, it is obvious that they should be avoided where impact loads are anticipated.

General conclusions. Ribs should be used chiefly for static loads. Wide and low ribs are safer than thin and high ones.

7–3. General remarks about shape. There are many features of a good design which cannot be expressed by figures or formulas. A statement

[1] A. Thum and S. Berg, "Über die Festigkeit von Rippen bei ruhender, wechselnder und Stossartigen Belastung," *Zeitschrift Verein Deutscher Ingenieure*, Vol. 77 (1933), pp. 281–287; V. L. Maleev, "Why Add Ribs That Don't Add Strength?" *Machine Design*, Vol. 8, No. 9 (September, 1936), pp. 29–32.

which will be found helpful in designing, perhaps more valuable to an experienced designer, is that although a thing which looks wrong usually is wrong, the reverse is not always true. The following advice is intended to be helpful in shaping machine parts.

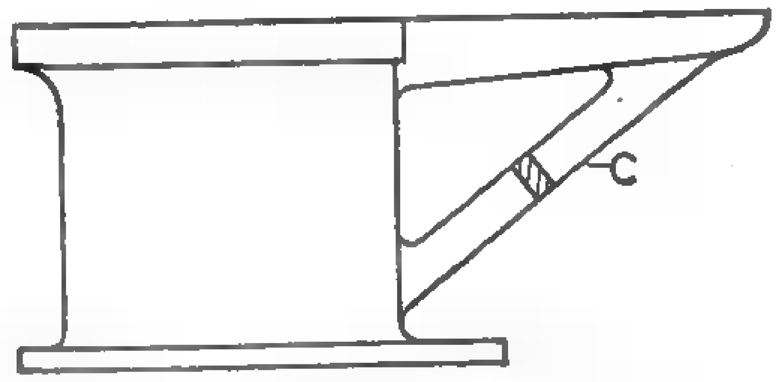


FIG. 7-9. Bed cast with an extension.

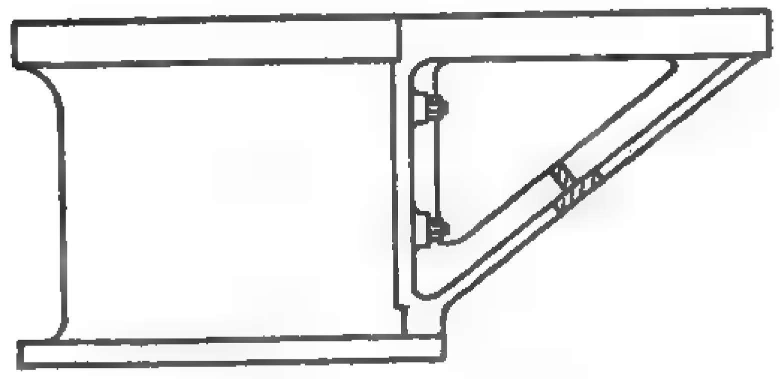


FIG. 7-10. Bed with a separate extension.

Light sections and protruding extensions should not be cast in one piece with a heavy section or part, in order to avoid scrapping of the heavy piece through breakage of an extension. The design shown in Fig. 7-9 is incorrect because brace c is out of proportion to the rest of the bed. A correct, two-piece design is shown in Fig. 7-10.

Hollow or box sections should not be combined with ribbed sections, since this design complicates the casting unnecessarily.

Straight lines in most cases look better than curved lines and are cheaper to obtain in a pattern. However, the parabola is the proper shape for a heavy pedestal which resists a bending moment, such as the pedestal in Fig. 7-11, because its strength is more nearly uniform. A curved reinforcing rib a, Fig. 7-12, is better than a straight one b, as it is less rigid and causes less stress concentration. However, the maximum bending stress in rib a will be higher than that in rib b, and care must be taken to put in a sufficient number of comparatively heavy ribs.

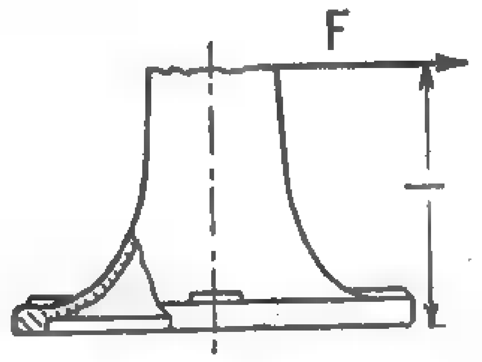


FIG. 7-11. Pedestal loaded in bending.

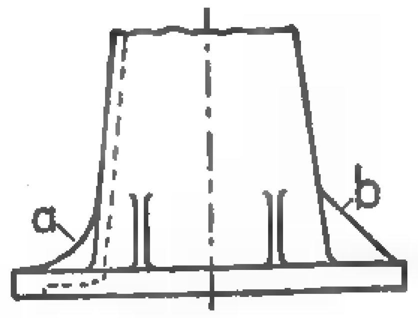


FIG. 7-12. Reinforcing ribs.

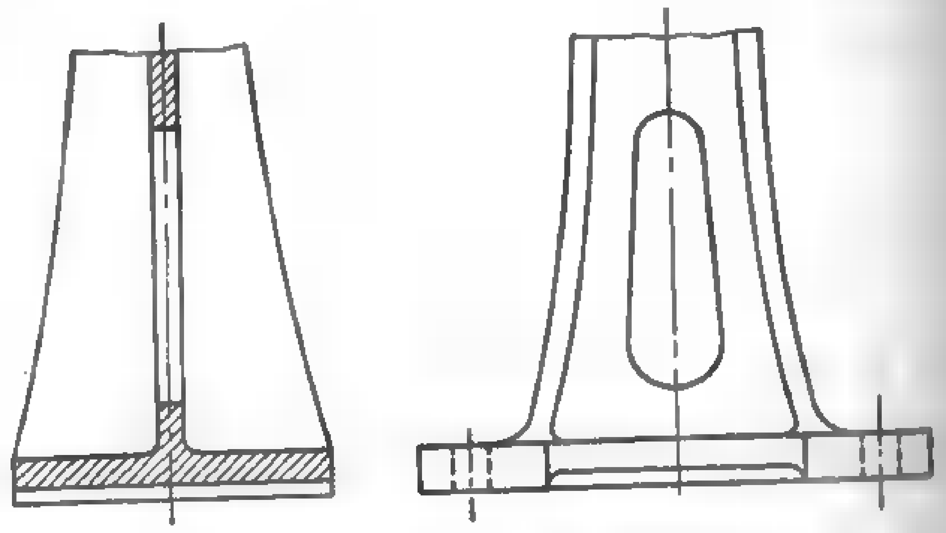


FIG. 7-13. Pedestal cast without a core.

A casting that can be molded in green sand without cores, such as that shown in Fig. 7-13, is less expensive than a casting requiring a core, such as that in Fig. 7-11 or in Fig. 7-12. If a casting must have a core, it should be designed so as to provide support for the core without the use of chaplets and so as to permit easy removal of the core from the casting. Since chaplets may cause leakage, they are especially objectionable for cast pieces that must be under fluid pressure.

The old practice of hiding the reinforcing bead, as shown in Fig. 7-14 at a, is wrong. To lower the stress concentration a symmetrical bead b should be used. The old advice to make a web between flanges as thin as it can be cast in order to save material, is not correct. Stress concentration at the juncture of sections that differ widely in thickness, as in Fig. 7-15a or b, is likely to cause failure. A heavy web, as in Fig. 7-15c, with a more even distribution of material, is much to be preferred.

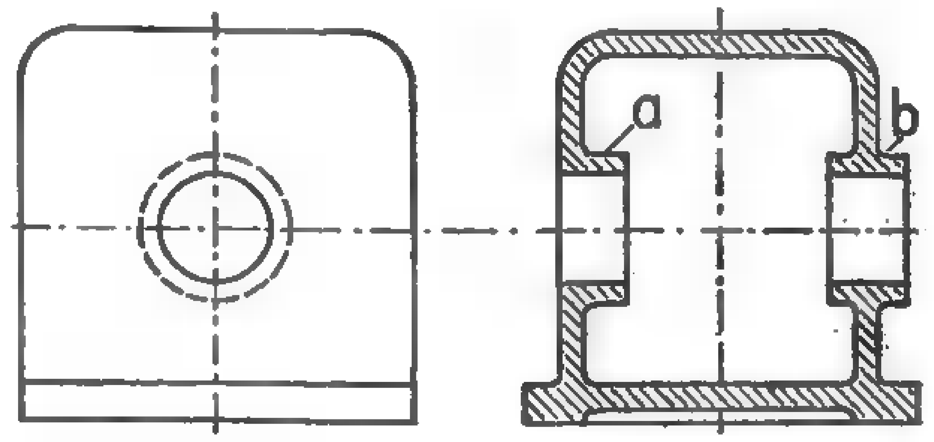


FIG. 7-14. Reinforcing an opening.

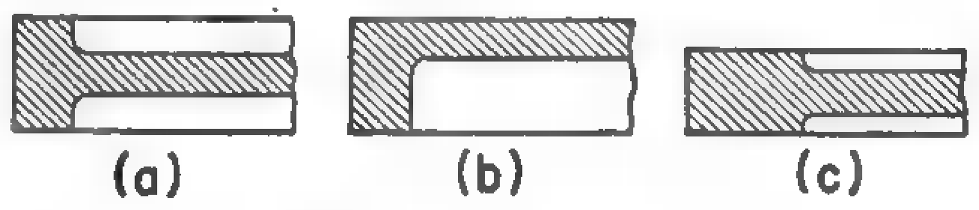


FIG. 7-15. Shape of web between flanges.

Pattern shaping. Parting lines should be made as even as possible. The design should provide for ample draft for easy molding. While the patternmaker undoubtedly will put in the necessary minimum draft, even when the design does not show any, as in Fig. 7-16a, it will be considerably cheaper to mold a casting from a pattern with ample draft, as shown in Fig. 7-16b, and there will be fewer spoiled and rejected castings.

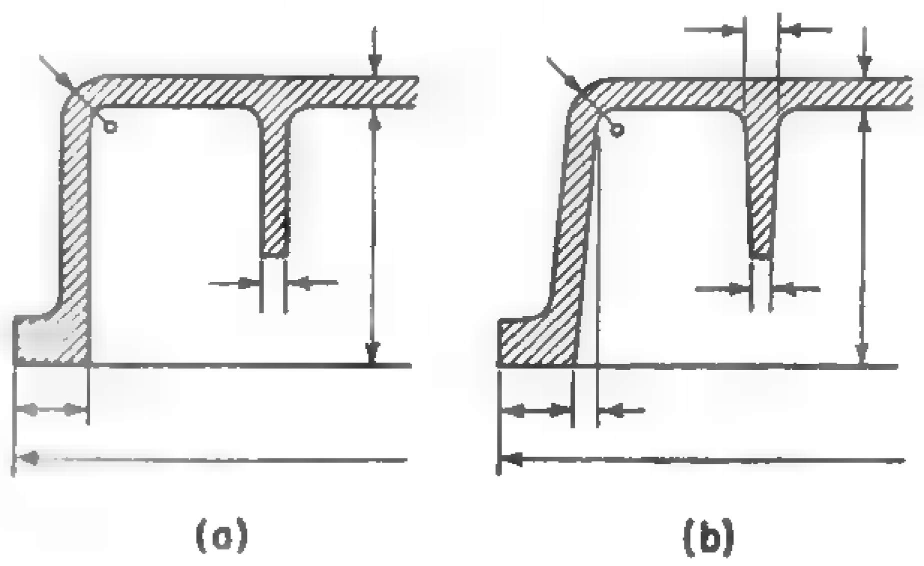


FIG. 7-16. Indication of draft.

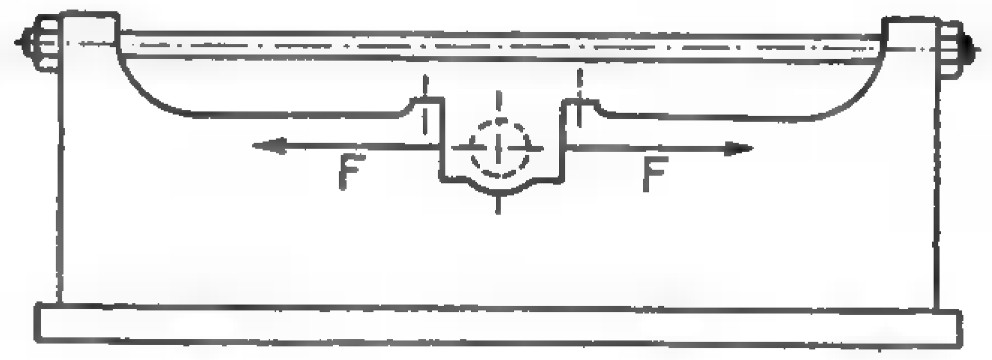


FIG. 7-17. Bedplate with a tie rod.

7-4. Special requirements. Because of certain distinctive properties of the various metals used for castings, different precautions should be taken with each material.

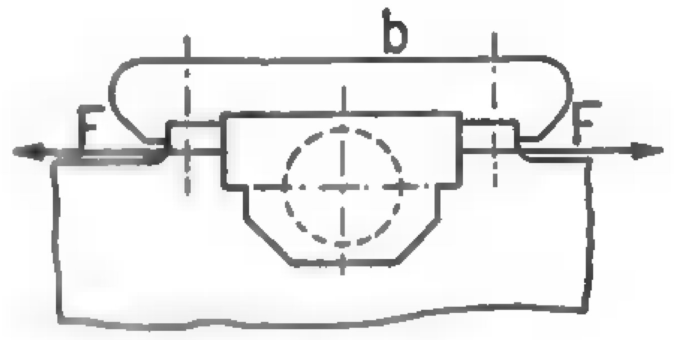


FIG. 7-18. Steel bearing cap.

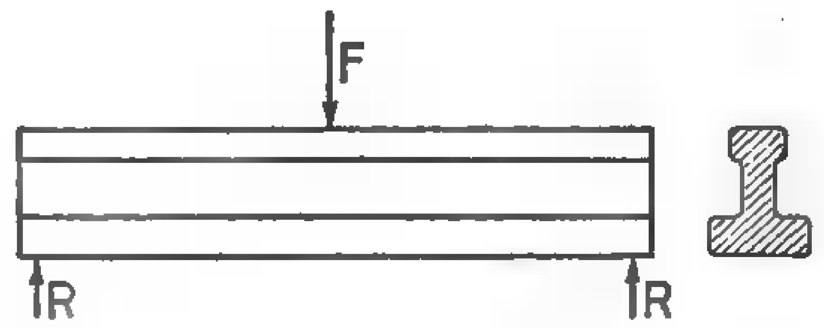


FIG. 7-19. Cast-iron beam.

Cast iron. Both the ultimate strength and the yield point of cast iron in compression are several times as great as those in tension. Cast-iron parts should therefore be so designed that tensile stresses will be avoided as much

as possible. Sections subjected to tensile stress should be either reinforced or relieved of the stress by appropriate tie rods, as in Fig. 7–17, or by clamp-shaped devices such as forged-steel bearing caps *b* in Fig. 7–18.

For the same reason, sections of parts subjected to bending in one direction should not be made symmetrical with respect to the neutral plane. In Fig. 7–19 is given a typical section for a cast-iron beam.

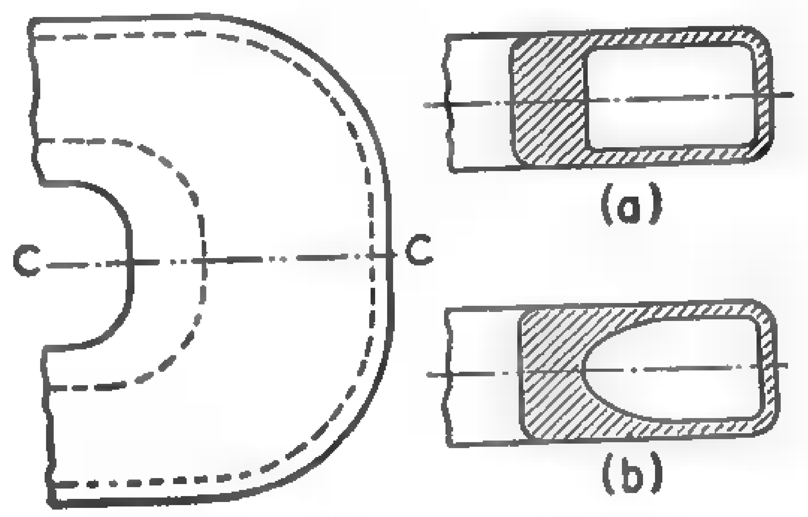


Fig. 7–20. Punch-press frame.

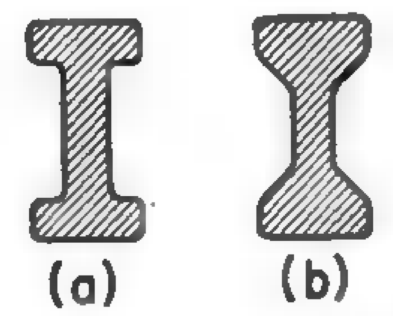


Fig. 7–21. Beam sections of cast iron and steel.

In the case of a heavy load, as in the punch frame in Fig. 7–20, the box section is made with a heavy wall in tension. If the shape at section *c-c* is as shown in Fig. 7–20a, stress concentration may cause the frame to break; whereas the shape shown in Fig. 7–20b, because of the gradual change of wall thickness, gives a satisfactory frame. However, in this case a welded steel construction is a still better solution, being stronger, more rigid, and less expensive.

Malleable iron. Dry-sand cores for malleable iron castings should be avoided wherever possible, especially where the casting, in cooling, contracts around a core.[2]

Steel castings. Steel is not as fluid as cast iron. Therefore complicated shapes, sharp corners, and thin sections cannot be obtained. If a cast-iron rocker arm with the cross section shown in Fig. 7–21a is to be cast in steel for greater strength, the pattern must be changed to produce the shape shown in Fig. 7–21b. If made of phosphor bronze, which is almost as strong as cast steel and is more fluid, the section need not be changed.

All steel castings must be annealed to relieve them of internal stresses. Their strength can be increased by subsequent heat treatment. This fact should be borne in mind when designing a piece.

Light alloys. It should be remembered that the mechanical properties given in Table 4–5 and Table 4–6 refer to small test sections. In castings the values are 10 to 20 per cent lower.

The comparatively low modulus of elasticity E of light alloys, particularly magnesium alloys, requires the use of sections with large section moduli if a certain degree of rigidity is desired. Thus the engine bed in Fig. 7–22a,

[2] F. A. Halsey, *Handbook for Machine Designers*, 2d ed. (New York: McGraw-Hill Book Company, Inc., 1916), p. 480.

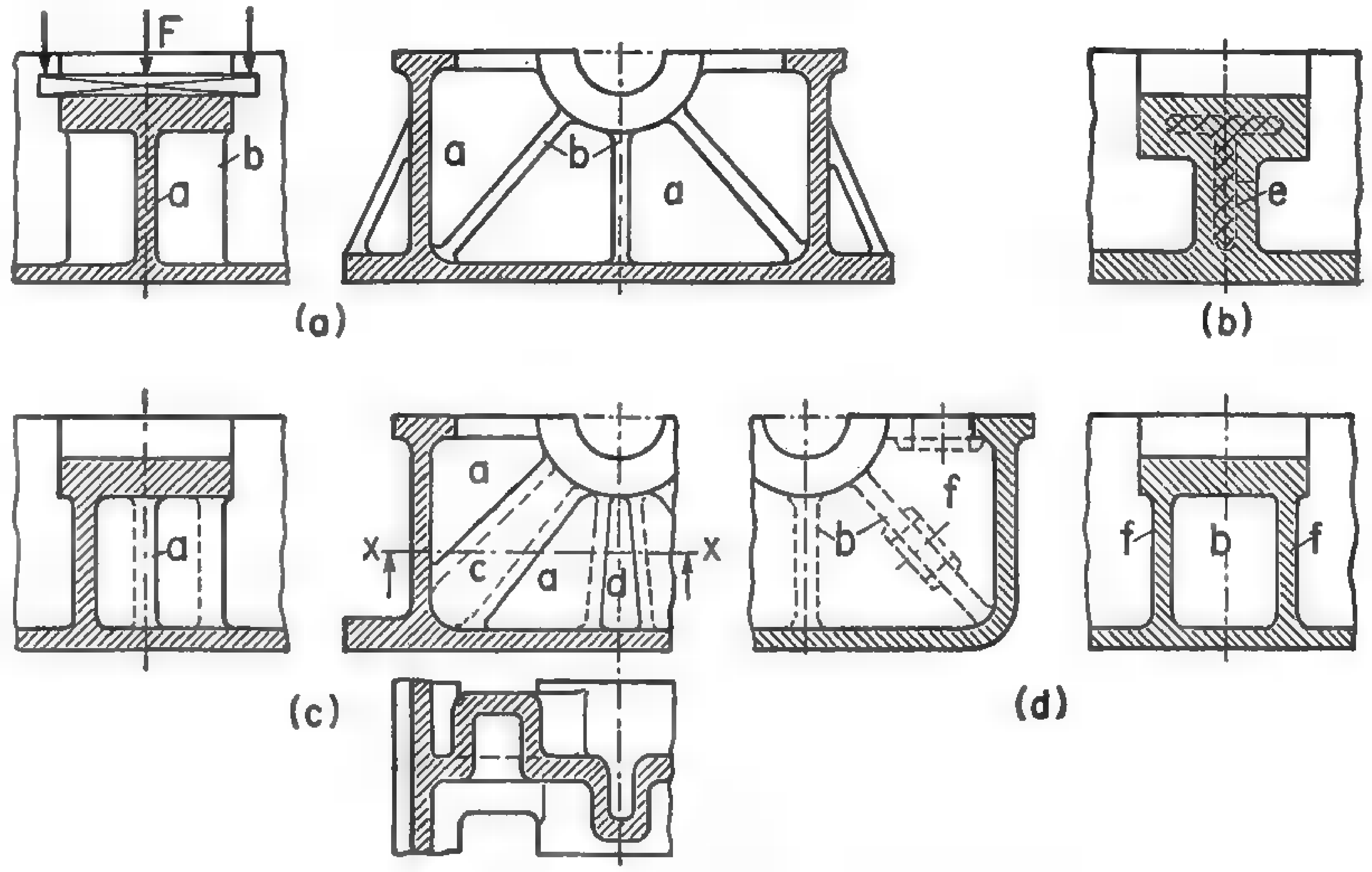


Fig. 7–22. Bearing bridge in a bedplate cast of aluminum alloy.

which is designed along the lines of a cast-iron casting, would be strong but would not be rigid enough to take the load F. The change to the design in Fig. 7–22c increased the rigidity, but not sufficiently. Finally the box-shaped construction in Fig. 7–22d proved to be entirely satisfactory. The same result could be attained by using a heavy T-section, as in Fig. 7–22b. However, this construction is heavier and presents the probability of a porous center *e* because of an excessive accumulation of material. The use of lower allowable stresses is therefore necessary.

In Fig. 7–23a is shown the proper section for a light-alloy beam bent in one direction, as compared with the section of a cast-iron beam in Fig. 7–23b.

Light alloys are more sensitive to stress concentration produced by fillets and notches than is cast iron. Fillets should be made larger; notches should be avoided; and as much uniform wall thickness as possible should be provided.

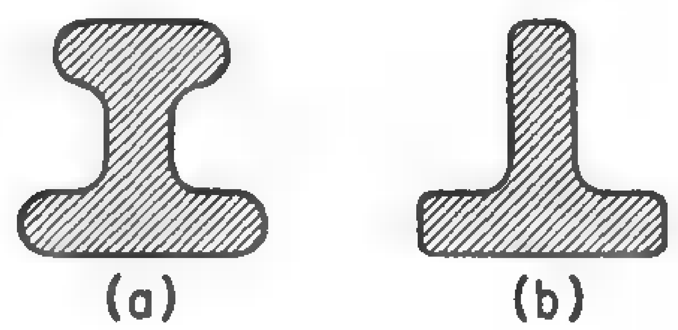


Fig. 7–23. Beam sections of aluminum and cast iron.

The strength of aluminum sections in tension can be increased by cast-in steel reinforcing rings or anchors.

The high coefficient of linear expansion of a light alloy with a change in temperature should be kept in mind. Also the designer must not overlook the fact that light alloys lose a considerable part of their strength at temperatures above 300 F; and that they should not be used at all for parts whose operating temperature is over 500 F.

With proper distribution of material an aluminum casting will show a saving of weight up to 50 per cent.

A screw joint should be designed so as to divide the load over a comparatively large number of small screws. Large washers should be used under boltheads and nuts to lower the specific bearing pressures, which should not exceed 2,000 psi for static loads and should never be more than 3,000 psi. The bosses under such washers must be proportionately large. Holes can be tapped in the alloy if the screws are not unscrewed frequently. The length of thread should be 2.5 d. If the screws must be unscrewed often, the holes should be drilled and tapped in cast-in buttons of a harder material, such as brass or Monel metal, with which aluminum castings form a good bond. In Fig. 7–24 is shown the use of cast-in, transverse steel buttons a for distributing the load of a stud b. Another example of a cast-in tapped bushing is given in Fig. 11–30.

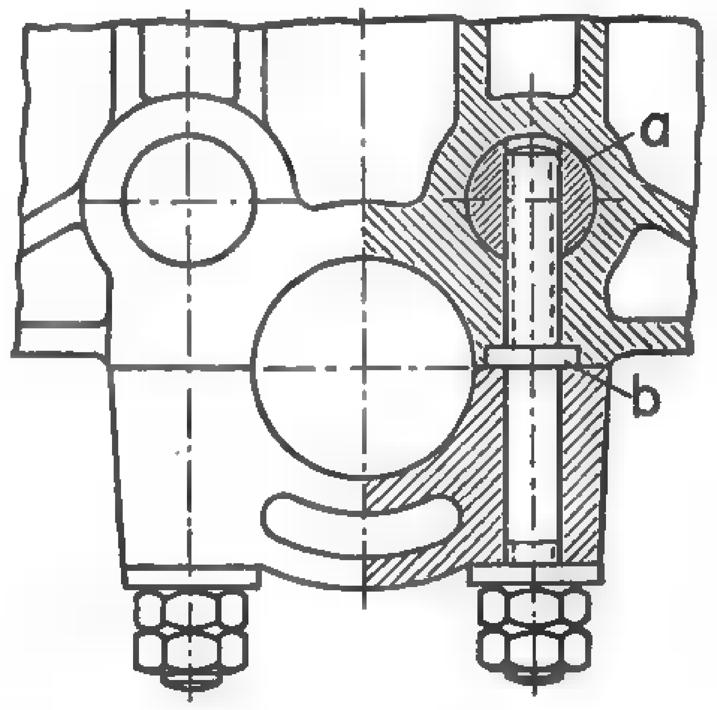


FIG. 7–24. Cast-in steel button for a stud.

Silumin: A heat-treated aluminum-silicon alloy, known as silumin-gamma, has such high strength characteristics that parts cast of it can be made with the same wall thickness as when cast iron is used.[3] However, the other design requirements must be observed. Working stresses in tension for silumin-gamma can be up to 10 per cent higher than those given in Table 8–3 for shielded-arc steel welds. The exact increase depends on the character of the loading. However, the allowable crushing stress is 5,700 psi for a static load and 4,300 psi for variable loads.

[3] J. Dornant, *Silumin-Gamma* (Frankfurt am Main: Metallgesellschaft A. G., 1938).

CHAPTER 8

Design of Weldments

8–1. Methods of welding. A machine part or structure whose component parts are joined by welding is called a *weldment.*

Welding is defined as the localized intimate union of metal parts in the plastic or plastic and molten states with the application of blows or mechanical pressure, or the union of parts in the molten state without any pressure. There exist three main methods of welding: forge welding, pressure welding, and fusion welding.

Forge welding is the oldest process known. It is accomplished either by hand hammering, as practiced by blacksmiths, or by machines, as in the manufacturing of wrought-iron pipes and pressure vessels. Forge welding can be applied only to wrought iron or low-carbon steel.

Pressure welding, or *resistance welding*, utilizes the heat created by an electric current which passes through the two pieces to be welded. The current meets with a much higher resistance at the surfaces of contact than in the body of the metal, and it raises their temperature to a molten state; and the applied pressure upsets the edges, which become united. This welding, if properly done, results in a joint which is practically as strong as the body of the metals joined. Because it is difficult to obtain a uniform area contact and to secure a uniform heat distribution, the method is not suitable for large sections—larger than about 1×14 in.

Spot welding is another form of resistance welding. It is sometimes used instead of riveting. Two sheets of metal are put together as for a lap joint, the overlapped portions being gripped between two electrodes. Pressure is applied to the electrodes and a heavy, low-voltage current is passed from one electrode to the other through the overlapping plates. Heat is developed at the spot of contact between the two plates until the necessary welding temperature is reached, when the pressure is increased. As a result the pieces are tacked together. Spot welding is usually done on thin plates not over $\frac{3}{8}$ in. thick, since it is difficult to develop pressures required to tack heavier plates together.

Seam welding is a development of spot welding. In this process the electrodes are in the form of two rollers, between which the overlapping edges of the thin plates are passed. A continuous strip of welded surfaces is thus produced.

Fusion welding does not require any pressure to form the weld. The places of contact of the two metal pieces to be joined are heated to the fusion

temperature of the metal; additional metal is usually applied in the corner of the joint by melting a filler rod of suitable composition, and the joint is allowed to cool. The heating of the metals is produced in one of the following ways: by a burning gas, by an electric arc, in an electric furnace, or by thermit.

Gas welding is often called *autogenous welding*. The *oxyhydrogen process*, the older of the gas-welding processes, uses a blowpipe flame produced by the combustion of oxygen and hydrogen, a temperature of about 4,000 F being obtained. It is employed chiefly for welding nonferrous metals of low fusibility.

The *oxyacetylene process* uses the flame produced by the combustion of oxygen and acetylene, and a temperature up to 6,300 F is attainable. This process is used for welding steel, steel castings, wrought iron, cast iron, copper and its alloys (including Monel), aluminum, and other commercial alloys. It is also used to weld a metal to a dissimilar metal, such as steel to cast iron or copper to steel.

Arc welding was developed about the end of the last century but has been employed extensively in production only during the last twenty years. The necessary temperature is produced by an electric arc formed between the pieces to be welded and an electrode held by the operator or moved by an automatic machine. The electrode may be either a carbon rod with a separate filler rod furnishing the additional molten metal or a metal rod which acts as a filler rod.

In *metallic-arc welding* globules of metal from the tip of the electrode are carried across the arc and are deposited in the joint, where they solidify. The globules can be deposited upward against the force of gravity; this property is a great advantage in structural and bridge work. In *carbon-arc welding* the heat of the arc forms a small pool of molten metal in the joint to which is being added metal from the filler rod. This process cannot be used for vertical or overhead work.

Deoxidizing and shielding. When the weld metal from the bare filler rod is being deposited, it absorbs oxygen and nitrogen from the atmosphere. These form oxides and nitrides in the completed weld that lower its strength, ductility, and resistance to corrosion. By introducing into the arc stream a *deoxidizing agent*, which has a greater affinity for oxygen and nitrogen than has the weld metal, or by *shielding* the arc and the molten metal from contact with the atmosphere, a weld can be produced which is free from impurities and is therefore much stronger than when the metal is deposited by an ordinary arc.

One of the deoxidizing methods uses a jet of hydrogen which is projected through an arc formed between two tungsten electrodes and produces a reducing atmosphere. The shielding of the arc is accomplished by employing electrodes consisting of a metal core with a coating of a material which

fluxes out the impurities as it burns away. Shielded-arc welds are free from slag inclusion and flow holes and have a finer crystalline structure and a greater integration of weld and base metal in the fusion zone. At present all arc welds where strength is important are made with a shielded arc.

In *electric-furnace welding* the parts to be welded are assembled to give a snug fit by inserting one part into the other, by spot welding, or by pinning the parts together. Copper in the form of wire or paste is applied at the joints, and the assembled piece is put into the hydrogen-filled heating zone of an electric furnace. The parts are gradually brought up to a temperature of 2,100 F. This process melts the copper, which is drawn by capillary attraction both downward and upward into the seams. The hydrogen atmosphere completely reduces all oxides, scale, and other foreign matter on the surface of the parts and secures chemically clean surfaces for the welds. After the welding is completed, the piece is gradually cooled in the same reducing atmosphere. This procedure insures a complete absence of internal stresses, and surfaces that are clean and free from scale.

Although resembling brazing, this process should nevertheless be classified as welding because the iron and copper form an alloy. Some of the copper goes into a solid solution in the steel, and some of the iron is dissolved by the copper, to produce an integral copper-iron alloy bond between the parts. The weld thus obtained is actually stronger than the steel itself, as proved by a number of tensile, compressive, shearing, and bursting tests.

The main field of application of electric-furnace welding is for manufacturing machine parts of intricate shape, such as diesel cylinder heads. Such parts produced by copper-hydrogen electric welding not only are stronger, lighter, and of better appearance but with proper design can be produced at a lower cost.

Thermit welding is based on the generation of heat by the chemical combination of iron oxide and powdered aluminum. When the mixture is ignited, the aluminum reduces the oxide to molten steel, which unites with the parts to be joined. The temperature of the reaction reaches almost 5,000 F. This process is of great value in repairing heavy cast-iron and steel parts but is not practical for production manufacturing.

8-2. Field of application. Of the many types of welding, the one most used in the manufacture of machinery is arc welding; next comes gas welding. Only these two methods will be discussed in connection with design of weldments.

Welding of steel. Most weldments are made of low-carbon steel. There are two general fields of application for welding steel: (a) when welding is substituted for riveting and (b) when welding is used as an alternate method for casting or forging.

TABLE 8-1

COMPARISON OF FABRICATION METHODS FOR A BEARING SUPPORT (FIG. 8-1)

Basis for Comparison	(a) Cast Iron (per cent)	(b) Cast Steel (per cent)	(c) Forged Steel (per cent)	(d) Welded Steel (per cent)	(e) Welded Steel (per cent)
Static strength	100	100	100	100	100
Thickness	100	60	54	54	74
Deflection	100	191	259	259	100
Vibration damping	100	28	26	26	36
Weight	100	76	72	71	85
Cost	100	110	300	45	54

The main advantages of welded joints over riveted ones are:

a) Lighter weight, due to elimination of straps, gusset plates, clip angles, etc.

b) Greater strength. With proper care the joint can be made with an efficiency of 100 per cent, and with the development of X-ray and other methods of inspection the strength of a welded joint can be determined without destroying the piece.

c) Lower cost, chiefly because of the lighter weight.

d) Noiselessness. Since it has been established that noise is not only disagreeable but harmful to both the efficiency and health of workers, noiselessness has become a rather important factor.

An important development is the arc-welding of low-creep Cr-Mo-alloy steels used for making boilers and superheater tubes subjected to pressures of 1,060 psi and temperatures of 930 F.[1]

The main advantages of welded machine parts over cast parts are:

a) Lighter weight, because the metal is utilized better.

b) Greater strength. The metal is put exactly where it is needed and the danger of stress concentration due to discontinuities can be eliminated.

c) Lower cost, due to elimination of patterns and to lighter weight.

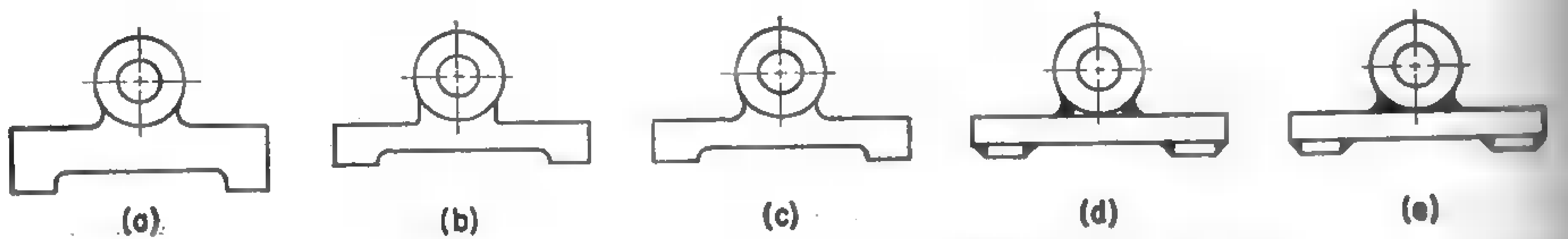


FIG. 8-1. Shaft support fabricated by different methods.

Table 8-1 in connection with Fig. 8-1 gives some idea of how different fabrication methods affect the characteristics of a certain machine part. The five alternative designs of a bearing support in Fig. 8-1a, b, c, d, and e have the same static strength but differ in rigidity, vibration-damping

[1] H. Schottky, "Das Schweissen der warmfesten und hitzbeständigen Stahllegierungen," *Zeitschrift Verein Deutscher Ingenieure*, Vol. 79 (1935), pp. 41-46.

capacity, weight, and cost. If rigidity and damping capacity are not important, the welded design in Fig. 8-1d will result in a large saving in cost. If rigidity is essential, the welded design in Fig. 8-1e will again give a substantial saving in cost with the same rigidity as a casting and will give a greater strength. However, if vibration damping is important, the best results are obtained by using a gray-iron casting.

Welding of aluminum. Aluminum and its alloys can be welded by any of the various methods used in welding steel plates. Since many aluminum shapes and castings are heat-treated, it should be remembered that welding anneals the joint and thus decreases its strength almost to the untreated values. The weld itself has a cast structure, and to increase its strength it must be reinforced in the same manner as a steel weld. More detailed information may be found in special trade literature.[2]

8-3. Welded joints. There are five basic forms of welded joints: *edge*, *butt*, *lap*, *corner*, and *tee*. These forms are illustrated in Fig. 8-2, where the weld is shown in black. The joint shown by full lines in Fig. 8-2a is a plain edge joint; and the dotted lines show a so-called *leaf edge joint* obtained by bending the plates after the weld has been made. In Fig. 8-2b is shown a single-welded butt joint; that in Fig. 8-2c is a double-welded butt joint; those in Fig. 8-2d and e are lap joints; those in Fig. 8-2f and g are corner joints; and those in Fig. 8-2h and i are tee joints.

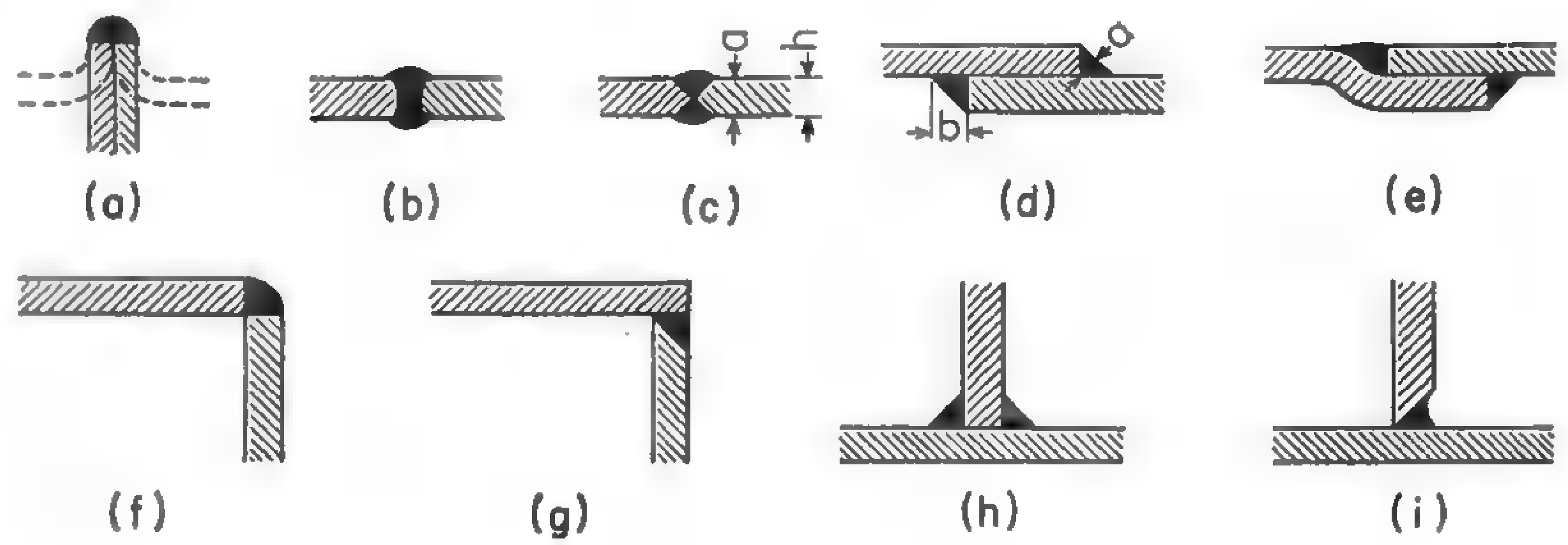


FIG. 8-2. Forms of welded joints.

A joint in which the base metal plates are brought close together, as in Fig. 8-2e or h, is called a *closed joint*. A joint with a gap between the plates prior to the welding, as in Fig. 8-2b, is called an *open joint*.

In an *edge weld* the weld metal is deposited along the edges of the plates, as in Fig. 8-2a. In a *butt weld* the weld metal is deposited between the edges to be joined, as in Fig. 8-2b, c, g, or i. In a fillet weld the weld metal is deposited in a corner formed by the two surfaces to be joined, as in Fig. 8-2d, e, f, or h.

[2] Aluminum Company of America, *Welding and Brazing Alcoa Aluminum* (Pittsburgh: 1944).

Spot-welded joints are formed by so-called *tack welds.*

From the standpoint of strength, welds are classified as reinforced, flush, or concave. The welds in Fig. 8–2a, b, c, and f are reinforced; those in Fig. 8–2d, e, g, and h are flush; and that in Fig. 8–2i is concave. The usual amount of reinforcement is one-fourth the depth of a flush weld. A *calk weld* has for its main purpose the tightness of the joint. The main requirement of the majority of joints is tensile strength, but tightness is also important in some cases. The dimension *a*, in Fig. 8–2, is called the *throat*, and *b* is the *leg* of a weld.

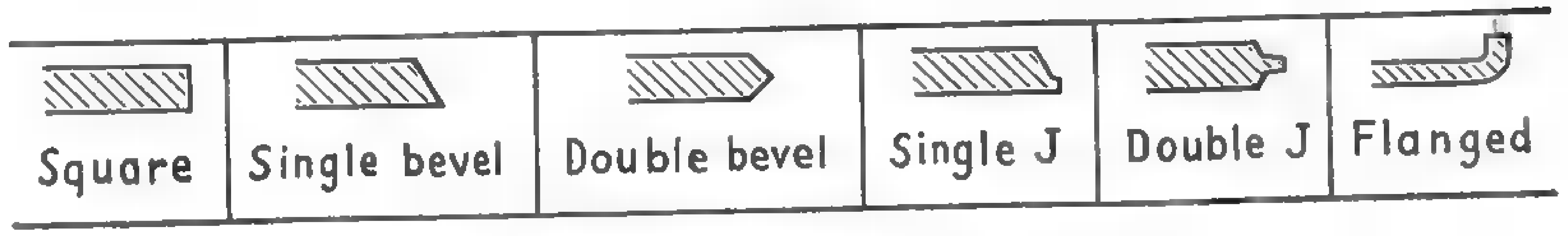


FIG. 8–3. Forms of edges for welding.

The edges of the plates to be welded may have various forms some of which are shown in Fig. 8–3. The selection of the edge form depends upon the welding procedure, the plate thickness, and whether both sides of the joint or only one can be welded.

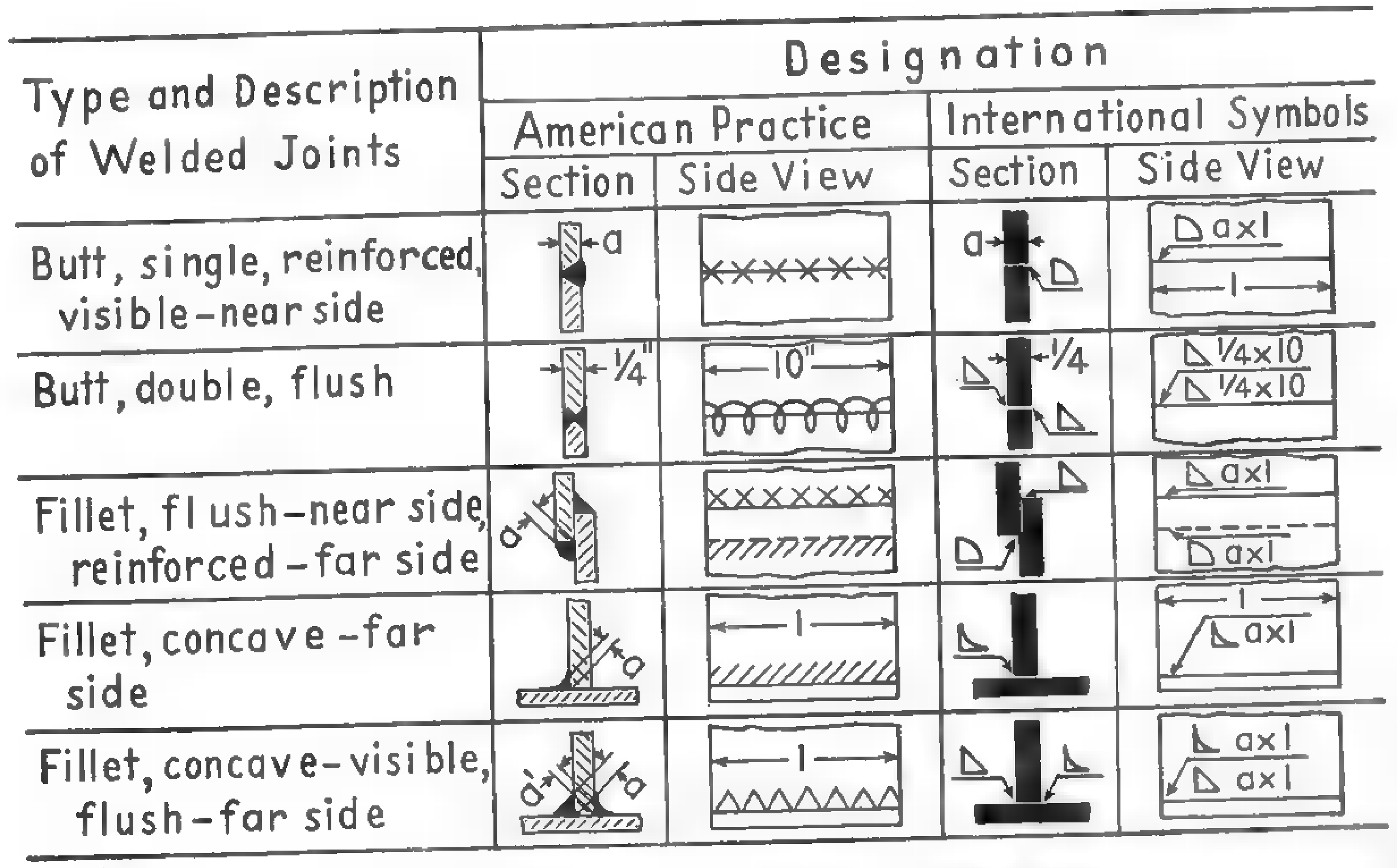


FIG. 8–4. Designations of welds on drawings.

Designations on drawings. In Fig. 8–4 are shown samples of the designations used for welds on drawings. According to American practice, a weld that appears in cross section is shown filled in and is given its true shape as it is to be made—reinforced, flush, or concave. A weld that is visible in a side view, or on the near side, is represented by a row of crosses; and a weld that is invisible, or on the far side, is represented by a row of short lines inclined at an angle of 45°. A butt joint with double welds is indicated by a zigzag line when reinforced and by a coil when flush.

The International Committee on Standardizations recommends the use of the following symbols in a cross section: a full quadrant for a reinforced fillet weld, a triangle for a flush weld, and an angle with an inscribed quadrant for a concave weld. In a side view the type of weld, the throat dimensions *a*, and the length *l* are given *over* the leader line when the weld is *visible*, and *under* the leader line when the weld is *not visible*. The American Welding Society advocates the use of this system with a few additional refinements.

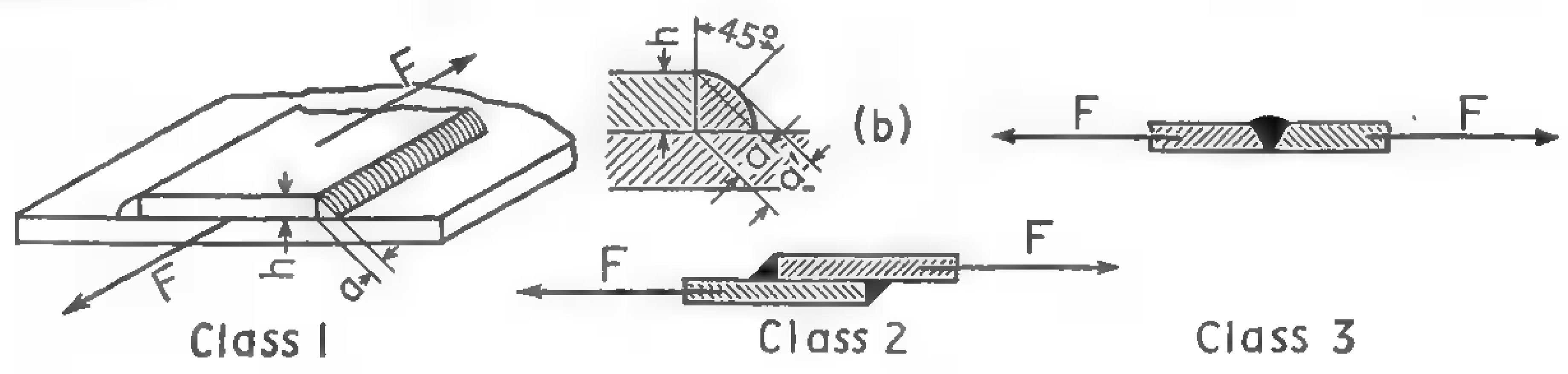


FIG. 8–5. Force action in various classes of welds.

8–4. Strength of welded steel joints. Welded joints subjected to tension may be divided into three classes with respect to the internal stresses set up by the external load. These are shown in Fig. 8–5. In class 1 the weld is subjected only to longitudinal shear; in class 2 the weld is in transverse shear and in tension; in class 3 the weld is in pure tension. It should be noted that in a weld of class 1 the shear area is $2al$, where l is the effective length of each weld. In a weld of class 2 the shear area is $2hl$, where h is the thickness of the weld and is often made equal to the thickness of the plate. In a flush weld, $a = 0.707h$. Therefore a class 2 joint should be about 40 per cent stronger than a class 1 joint. Repeated tests have shown that the length of weld and all other factors being equal, welds of class 2, in which the lineal dimension is *normal* to the lines of stress are about 30 per cent stronger than welds of class 1, in which the lineal dimension is parallel to the lines of stress.

TABLE 8–2

ALLOWABLE LOADS ON MILD-STEEL SHIELDED-ARC WELDS IN SHEAR

LOCATION OF WELD	SIZE OF FILLET WELD (IN.)							
	$\frac{1}{8}$	$\frac{3}{16}$	$\frac{1}{4}$	$\frac{5}{16}$	$\frac{3}{8}$	$\frac{1}{2}$	$\frac{5}{8}$	$\frac{3}{4}$
	Allowable Static Load per Lineal Inch (lb)							
Transverse weld, AWS Code...	1,325	1,985	2,650	3,310	3,975	5,300	6,620	7,950
Transverse weld, Fusions Code, Structural..............	1,200	1,800	2,400	3,000	3,600	4,800	6,000	7,200
Longitudinal weld, Class 1.....	1,000	1,500	2,000	2,500	3,000	4,000	5,000	6,000

TABLE 8-3

STRENGTH OF SHIELDED-ARC FLUSH-STEEL WELDS

KIND OF STRESS	LIMIT STRESS (PSI)			RECOMMENDED DESIGN STRESS (PSI)		
	Base Metal	Deposited Metal		Safety Factor 2	Safety Factor 2	Safety Factor 2.75
	Elastic Limit	Elastic Limit	Endurance Limit	Static Load	Load Varies from 0 to F	Load Varies from $+F$ to $-F$
Tension	32,000	40,000	(22,000)	16,000	14,500	8,000
Compression	35,000	44,000	....	18,000	16,000	8,000
Bending	35,000	44,000	26,000	18,000	16,000	9,000
Shear	20,000	24,000	....	11,000	10,000	5,000
Shear and tension	....	....	....	11,000	10,000	5,000

Approximate strength calculations. In the design of new machine parts and structures under moderate static loads, the required lengths of joints with shielded-arc welds may be determined in accordance with values found satisfactory in practice. These values are given in Table 8-2. For bare-electrode welds the values in Table 8-2 should be reduced by 20 per cent.

In arc-welded joints, about $\frac{1}{2}$ in. should be added to the calculated length of each weld to allow for starting and stopping of the weld, because of the weakening by the magnetic blowing of the arc.

EXAMPLE 8-1. Determine the total load that a shielded-arc-welded joint of class 1 (Fig. 8-5) can carry if it joins $\frac{3}{8}$-in. plates and each fillet is $4\frac{1}{2}$ in. long.

The effective length of the two $\frac{3}{8}$-in. welds is

$$l = 2 \times (4.5 - 0.5) = 8 \text{ in.}$$

From Table 8-2 the safe load per 1-in. length is 3,000 lb. Therefore

$$F = 8 \times 3{,}000 = 24{,}000 \text{ lb}$$

8-5. Design of steel welds. For more accurate design the values of design stresses for shielded-arc flush welds are given in Table 8-3.[3] These values apply for painstaking workmanship, proper supervision, and careful inspection; with less skill or less care, the values should be lowered. For bare-electrode welds the allowable stresses in Table 8-3 should be multiplied by 0.8. For gas welds these values should be multiplied by 0.8 to 0.85, the factor depending on the skill of the welder.

In computing the area of the cross section of a reinforced fillet, the throat dimension a', Fig. 8-5b, should be taken as $1.2a$.[4] For a concave weld it is safe to take a' as $0.5a$.

The *efficiency* of a welded joint is the ratio of the resistance of the joint to the resistance of the weakest section of the base metal. Since the lengths of

[3] O. Graf, "Dauerfestigkeit von Schweissverbindungen," *Z.VDI*, Vol. 78 (1934), pp. 1423–27; G. D. Fish, *Arc-Welded Steel-Frame Structures* (New York: McGraw-Hill Book Company, Inc., 1933), p. 89.

[4] Graf, *loc. cit.*

the welds in many cases are not limited, an efficiency of 100 per cent or even higher can be easily obtained.

Initial stresses. Initial stresses are unavoidable in welds, but they are not very dangerous for static loading because if the yield point is passed at the most stressed point, the metal starts to creep at this point and the load then redistributes itself more evenly over the cross section.[5] After several local overloadings the stress distribution in the structure approaches that which would exist without initial stresses. However, this is true only for ductile materials. Therefore the most important requirement for a weld material is high ductility.

With variable loads initial stresses become dangerous and must be relieved by annealing. Especially dangerous is the presence of initial stresses with impact loads. The weldment must be carefully stress-relieved. During the process of welding, each layer of deposited metal should be peened; and the finished weldment should be heat-treated.

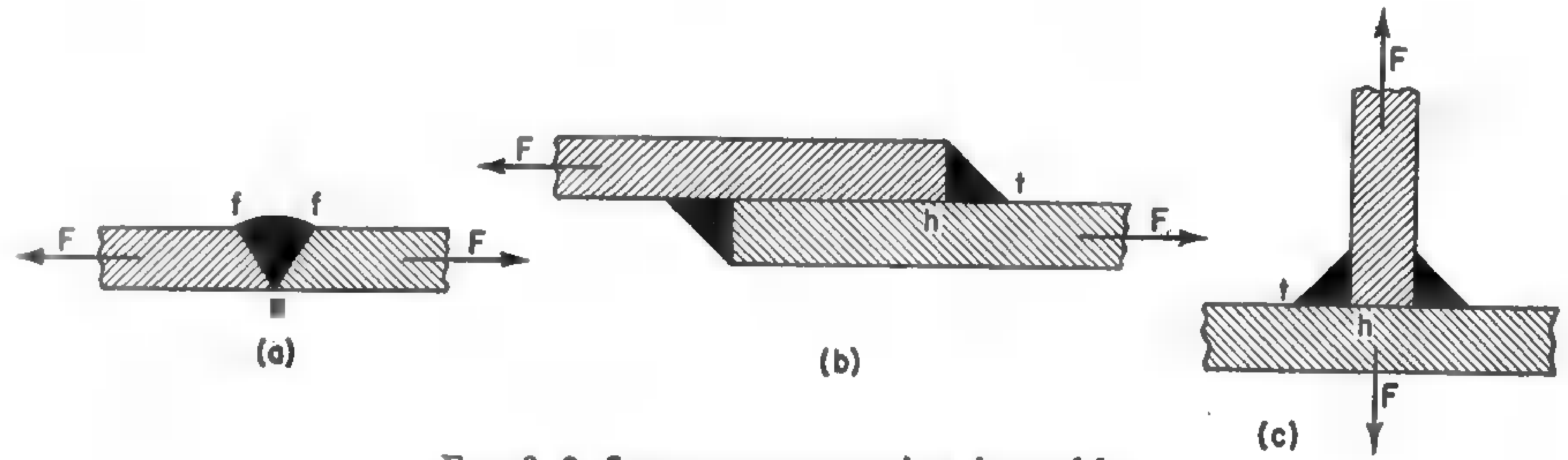


FIG. 8-6. Stress concentration in welds.

Stress concentration. Since abrupt changes in cross sections at welds are unavoidable, stress concentration will develop at these places. For static or steady loads, stress-concentration effects may be neglected; for variable loads, and particularly for shock loads, they must be taken into account in the usual way. Values of the stress-concentration factor K' are given in Table 8-4.

TABLE 8-4

STRESS-CONCENTRATION FACTOR K' FOR WELDS

Type of Weld	K'
Reinforced butt weld	1.2
Toe of transverse fillet weld	1.5
End of longitudinal weld	2.7
T-butt joint with sharp corners	2.0

In a weld, stress concentration depends to a great extent on workmanship. Thus an insufficient fusion at e in Fig. 8-6a or a hump at f would create serious stress concentration. In the welds of Fig. 8-6b and c, point h

[5] J. Mather, "Determination of Initial Stresses by Measuring Deformations around Drilled Holes," *Transactions of the American Society of Mechanical Engineers*, Vol. 56 (April, 1934), IS-56-2, p. 254.

is called the *heel* and *t* is the *toe*. Imperfections in the weld at either the heel or the toe cause stress concentration.

When a welded joint is subjected to bending or reversed stresses, the weld should be so proportioned as to be stiffer than the adjacent material, in order to throw the deflection into the base-metal parts.

EXAMPLE 8–2. Determine the load which an arc-welded joint of class 1, Fig. 8–5, can safely carry if it joins $\frac{3}{8}$-in. plates and each fillet is $4\frac{1}{2}$ in. long and reinforced. The load varies from zero to a certain maximum value.

The cross-sectional area of a flush fillet weld resisting shear would be $a(l-0.5)$, and the area of a fully reinforced fillet may be taken as $1.2\,a(l-0.5)$. Thus the total cross-sectional area for both fillets is

$$A_1 = 0.707h \times 1.2(l-0.5)\times 2 = 0.707\times 0.375\times 1.2\times(4.5-0.5)\times 2 = 2.55 \text{ sq in.}$$

The working stress being taken from Table 8–3 as $S_s = 10{,}000$ psi, the safe load is

$$F_1 = A_1S_s = 2.55\times 10{,}000 = 25{,}500 \text{ lb}$$

EXAMPLE 8–3. Determine the loads which welded joints of class 2 and class 3, Fig. 8–5, can safely carry, using the conditions and data of the previous example; and compare them with the strength of the strip.

In the joint of class 2 the section area in tension is equal to that in shear. Therefore the strength in shear is the smaller. The area of the weld is

$$A_2 = 0.375(4.5-0.5)\times 2 = 3.00 \text{ sq in.}$$

and the safe load, with $S = 10{,}000$ psi, is

$$F_2 = A_2S_s = 3.00\times 10{,}000 = 30{,}000 \text{ lb}$$

Thus one may say that a class 2 joint is considerably stronger than a class 1 joint, although they are not quite comparable.

In the joint of class 3 the area of the dangerous section with a reinforced joint is

$$A_3 = 0.375\times 1.2\times 4.8 = 1.8 \text{ sq in.}$$

From Table 8–4 the allowable stress in tension is $S = 14{,}500$ psi, and the safe load is

$$F_3 = A_3S = 1.8\times 14{,}500 = 26{,}100 \text{ lb}$$

The class 1 joint is the weakest one, although its strength is only 2 per cent below that of class 3.

Assume that the base metal is SAE 1010 steel with the endurance diagram of Fig. 4–2. For a force amplitude $F_a = \frac{1}{2}F_{max}$, and $F_m = \frac{1}{2}F_{max}$, the stress amplitude is $S_a = 14{,}500$ psi. The safe load amplitude for a strip $\frac{3}{8}\times 4\frac{1}{2}$ in., with a safety factor $n = 2$, is then

$$F_a = \frac{0.375\times 4.5\times 14{,}500}{2} = 12{,}200 \text{ lb}$$

Hence

$$F_{max} = 2F_a = 24{,}400 \text{ lb}$$

Thus all arc-welded joints theoretically are stronger than the strip.

EXAMPLE 8–4. (a) Compare the strength of the joints in the two preceding examples with class 1 and class 2 joints made with flush welds and a class 3 joint with a reinforced weld ground flush afterward. All welds are to be made by the oxyacetylene process by a first-class welder. (b) Find the efficiencies of the joints.

a) In a class 1 joint with a flush weld the area of the dangerous cross section is

$$A_1 = 0.707\times 0.375\times 2\times 4.5 = 2.38 \text{ sq in.}$$

With the stress coefficient of 0.85 for gas welds, the safe load is

$$F_1 = A_1S_s\times 0.85 = 2.38\times 10{,}000\times 0.85 = 20{,}200 \text{ lb}$$

In a class 2 joint the area of the dangerous cross section is

$$A_2 = 0.375\times 4.5\times 2 = 3.37 \text{ sq in.}$$

and the safe load for a gas weld is

$$F_2 = A_2S_s\times 0.85 = 3.37\times 10{,}000\times 0.85 = 28{,}600 \text{ lb}$$

In a class 3 joint the area of the dangerous section is

$$A_3 = 0.375\times 4.5 = 1.685 \text{ sq in.}$$

and the safe load, with the coefficient 0.85, is

$$F_3 = 1.685\times 14{,}500\times 0.85 = 20{,}800 \text{ lb.}$$

b) The efficiencies of the three types of gas welds are:

$$e_1 = \frac{20{,}200}{24{,}400} = 0.828, \text{ or } 82.8 \text{ per cent}$$

$$e_2 = \frac{28{,}600}{24{,}400} = 1.172, \text{ or } 117.2 \text{ per cent}$$

$$e_3 = \frac{20{,}800}{24{,}400} = 0.85, \text{ or } 85.0 \text{ per cent}$$

With flush gas welds, joints of class 1 and class 3 have a strength lower than that of the strip, and the strength of a class 2 joint is higher than that of the strip. However, it must be remembered that Table 8–4 assumes the best workmanship, which is not always available.

Eccentric loads. When the load on a welded joint is applied eccentrically, the welds will be subjected to a combination of shear caused by the direct load and shear caused by torque. The state of stress in such a joint is complicated; and in order to determine the value of the significant stress even approximately, it is necessary to assume that the torsional shear stress at any point is proportional to its distance from the centroid of all weld areas.

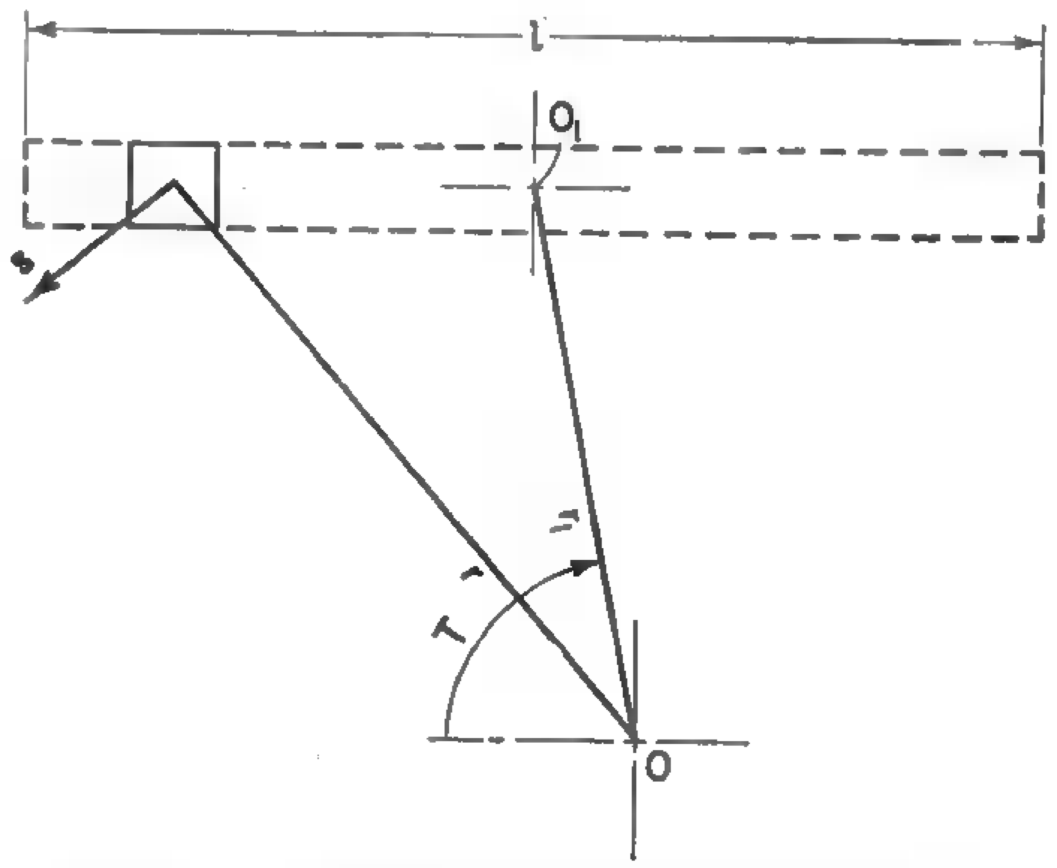


FIG. 8–7. Element of an eccentrically loaded weld.

If the weld shown in Fig. 8–7 is one of several forming a joint with the centroid of the weld areas at O, the torsional shear stress s on an element dA of the weld will be perpendicular to r and can be presented as

$$s = nr$$

where n is a constant of proportionality and r is the distance from the element to O. The external torque T is equal to the torque resistance of the element, or $s\,dA\,r$, integrated over all the welds in the joint. Thus,

$$T = \int s\,dA\,r$$

or

$$T = n\int r^2\,dA = nJ$$

where J is the polar moment of inertia with respect to O for all elements of the weld.

The stress in any element can then be found by the relation

$$s = \frac{Tr}{J} \tag{8-1}$$

The maximum stress will be found by using for r the distance to the point farthest away from the center of gravity O.

The significant stress will be found by adding geometrically the maximum torsional stress and the direct shear stress. For static loads it is standard practice to assume that the direct stress in a weld is uniformly distributed throughout the area.

The procedure for finding the polar moment of inertia J of all welds in a joint is as follows: First the polar moment of inertia of each weld about its own center of gravity, as O_1 in Fig. 8-7, is computed from the relation

$$J_1 = \frac{Al^2}{12} \tag{8-2}$$

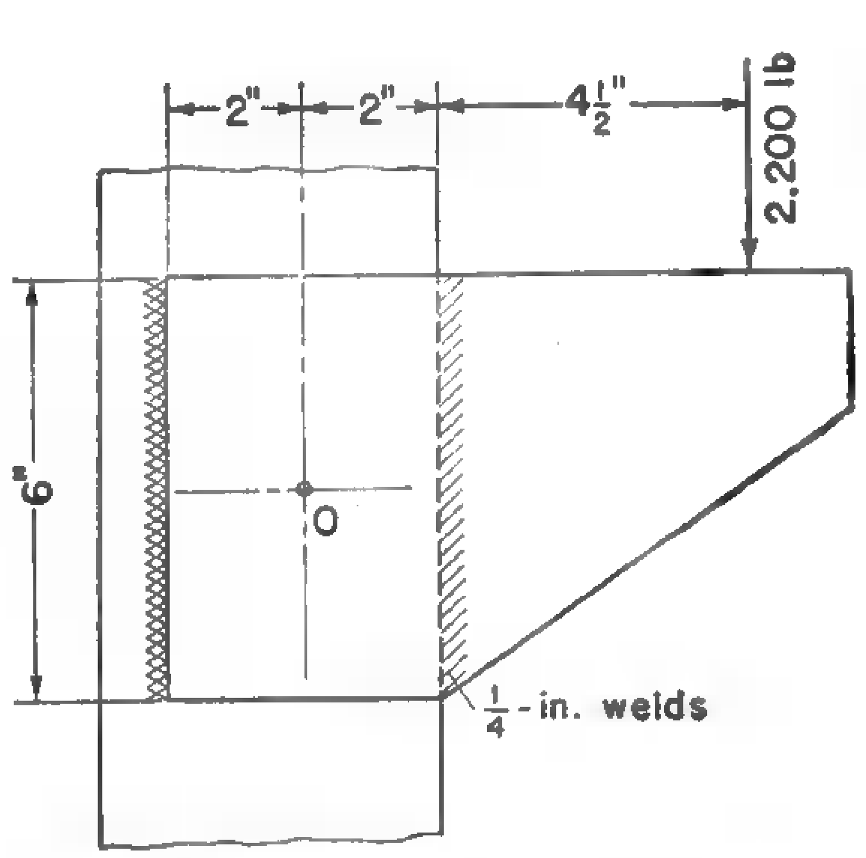


FIG. 8-8. Bracket plate welded to a column.

After this the polar moment of each weld J_1' with respect to the common center of gravity O is found by using the relation for parallel axes, which is

$$J_1' = J_1 + Ar_1^2 \tag{8-3}$$

The sum of all these moments of inertia J' will then give the numerical value of J to be used in equation 8-1.

The method described for finding the significant stress is only a simplified approach, but it gives satisfactory results if a proper safety factor—$n = 1.75$ or higher—is used.

EXAMPLE 8-5. Determine the maximum stress in the reinforced weld of the bracket plate in Fig. 8-8. Assume that the load varies from zero to the maximum value.

The throat area of each weld is

$$A = 0.707 \times 0.25 \times (6 - 0.5) \times 1.2 = 1.16 \text{ sq in.}$$

For each weld, $r_1 = 2$ in. and the total polar moment of inertia is

$$J = 2A\left(\frac{l^2}{12} + r_1^2\right) = 2 \times 1.16 \times \left(\frac{5.5^2}{12} + 2^2\right) = 15.1 \text{ in.}$$

The torque is

$$T = 2{,}200 \times 6.5 = 14{,}300 \text{ lb-in.}$$

The distance r in equation 8-1 is

$$r = \sqrt{2.75^2 + 2^2} = 3.35 \text{ in.}$$

Hence, the maximum torsional stress, by equation 8-1, is

$$s = \frac{14{,}300 \times 3.35}{15.1} = 3{,}190 \text{ psi}$$

This stress is resolved into a vertical component

$$s_v = \frac{3{,}190 \times 2}{3.35} = 1{,}900 \text{ psi}$$

and a horizontal component

$$s_h = \frac{3{,}190 \times 2.75}{3.35} = 2{,}620 \text{ psi}$$

The direct stress has only a vertical component, which acts upward and is

$$s_v = \frac{F}{2A} = \frac{2{,}200}{2 \times 1.16} = 950 \text{ psi}$$

The total vertical stress is

$$s_v = 1{,}900 + 950 = 2{,}850 \text{ psi}$$

Therefore the resultant stress is

$$s = \sqrt{2{,}850^2 + 2{,}620^2} = 3{,}870 \text{ psi}$$

With a stress-concentration factor $K' = 2.7$, from Table 8-4, the significant stress is

$$s_{max} = 3{,}870 \times 2.7 = 10{,}450 \text{ psi}$$

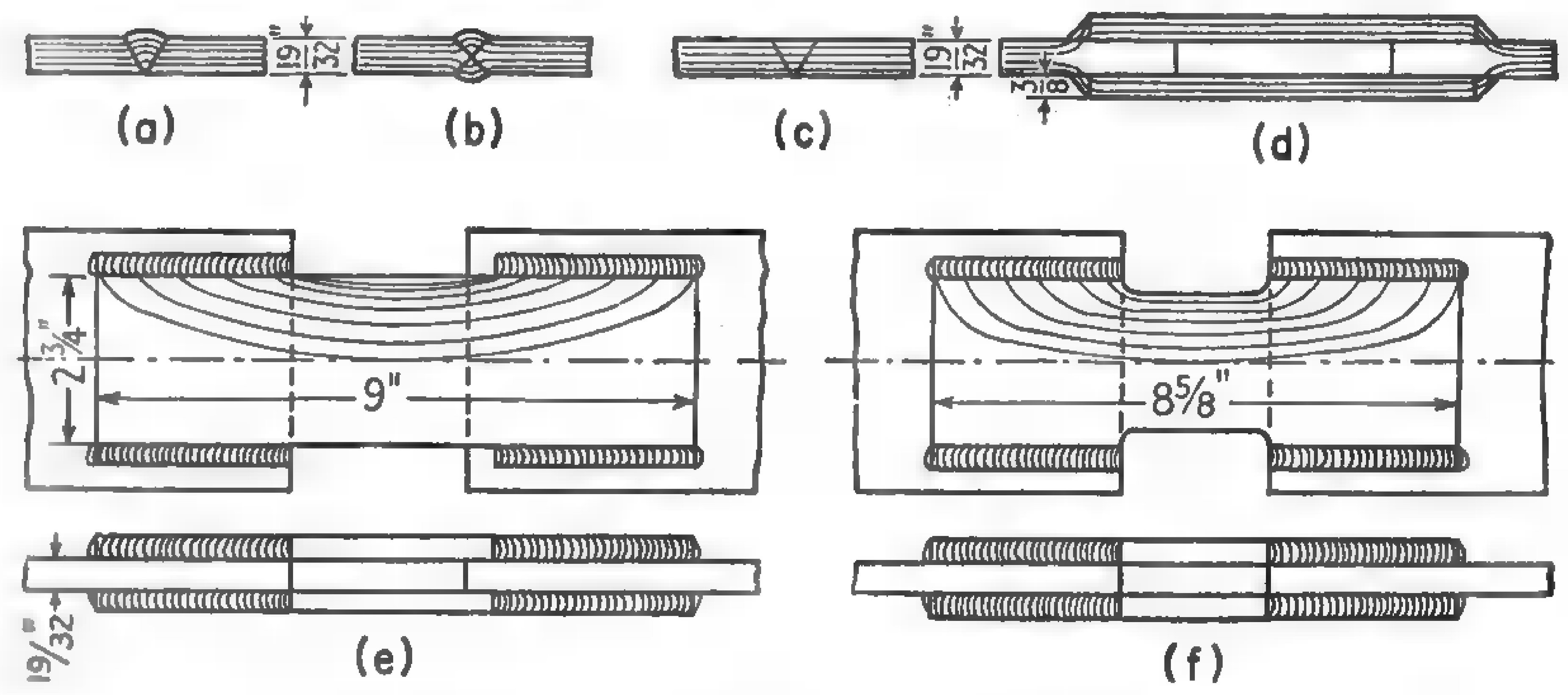


FIG. 8-9. Force-flow lines in welded joints.

8-6. Repeated stresses. Experiments with repeated tension stresses have shown that the endurance limit of butt-welded joints, Fig. 8-9a and b, is $S_{en}' = 0.85S_{en}$, where S_{en} is the endurance limit of a whole plate.[6] For the joint in Fig. 8-9c, with a ground-off weld, $S_{en}' = 0.87S_{en}$. Joints with cover plates, if of class 2 as in Fig. 8-9d, have $S_{en}' = 0.75S_{en}$; those of class 1, as in Fig. 8-9e, have $S_{en}' = 0.85S_{en}$; and those with notched cover plates, as in Fig. 8-9f, have $S_{en}' = 0.81S_{en}$. The cause of such a variation is the difference in the force, or stress flow, which is shown diagrammatically in these sketches.

[6] A. Thum and W. Schick, "Dauerfestigkeit von Schweissverbindungen bei vershiedener Formgebung," *Z.VDI*, Vol. 77 (1933), p. 493.

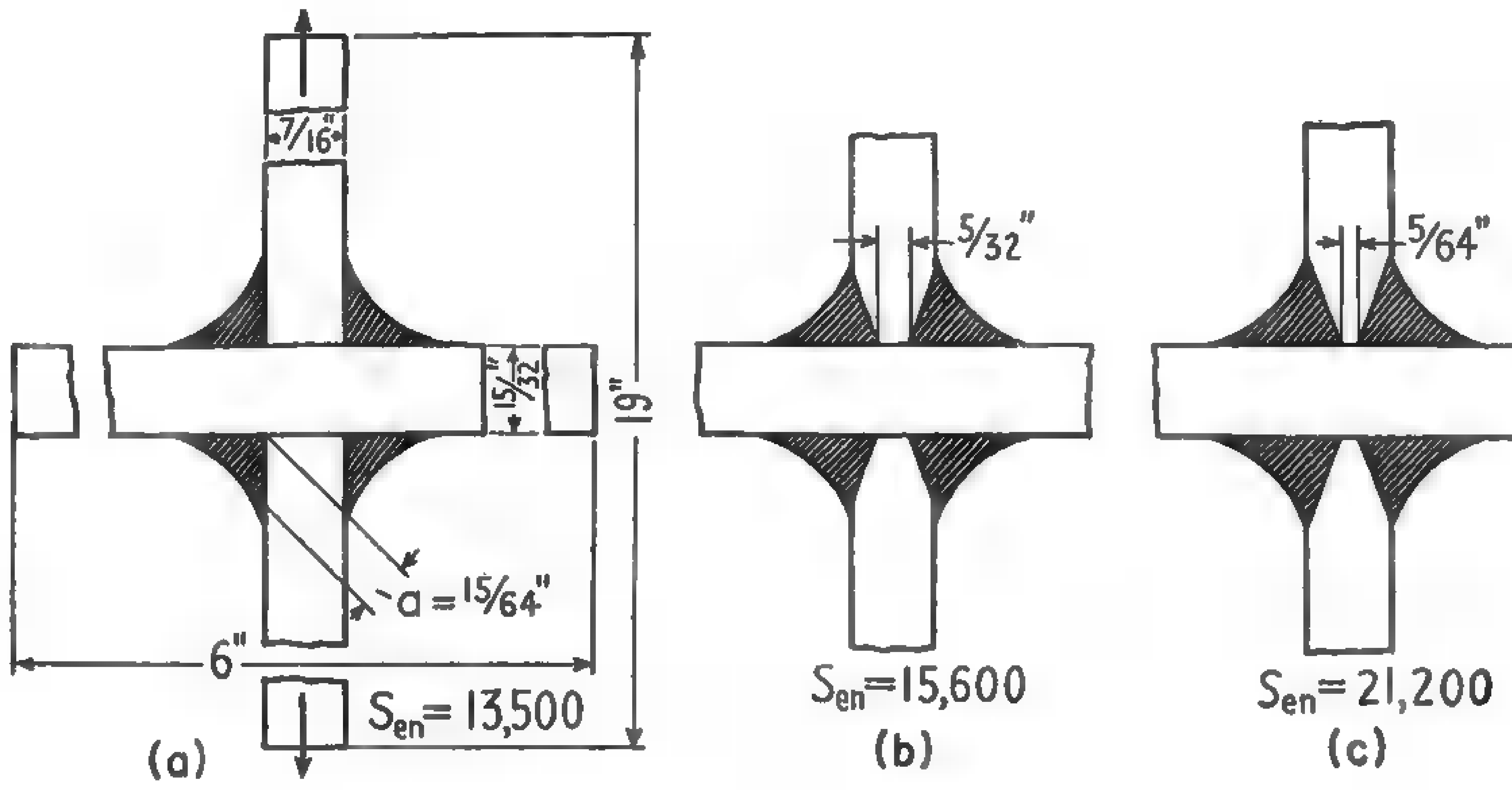


FIG. 8–10. Endurance of various types of welds.

The more uniform the flow is, the higher can be the magnitude of S_{en}'. This consideration should guide the designer in laying out the connections.

Additional data can be obtained from the results of tests conducted with various types of connections.[7] The connections were subjected to tensile loads repeated 2 million times for each specimen. The stress amplitude ranged from 700 psi to the indicated endurance limit. The material corresponded to SAE 1010 steel. As shown, changing the blunt edge in Fig. 8–10a to a sharper one, as in Fig. 8–10b, raised the endurance limit from 13,500 to 15,600 psi. By making the unwelded edge still smaller, as in Fig. 8–10c, the endurance limit is raised to 21,200 psi.

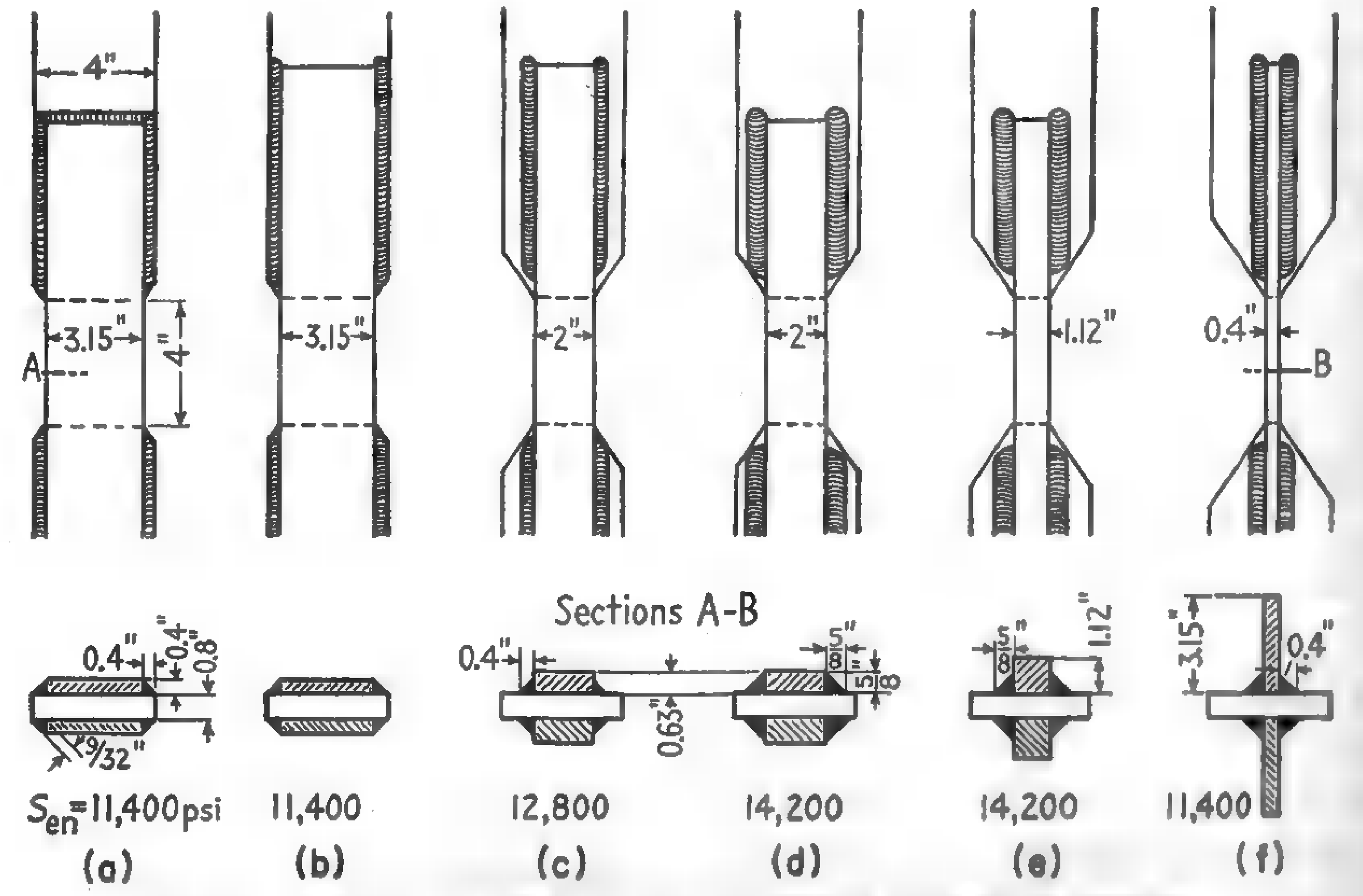


FIG. 8–11. Endurance of various connections stressed in tension.

[7] Graf, *loc. cit.*, and *Dauerfestigkeit mit Schweissverbindungen* (Berlin: VDI-Verlag, 1935).

In Fig. 8–11 it is shown that with the same cross-sectional area the thicker and narrower straps have a higher endurance limit than the thinner and wider ones, regardless of their location.

As shown in Fig. 8–12, the endurance limit of channel-iron straps can be raised from 12,000 to 17,000 psi by cutting the ends in a V shape and welding the ends as well as the sides. This increase of S_{en}' is due to a more direct force flow.

The grinding off of porous welds, and welds with cracks, increases the endurance limit, since these imperfections act as notches and produce stress concentration.

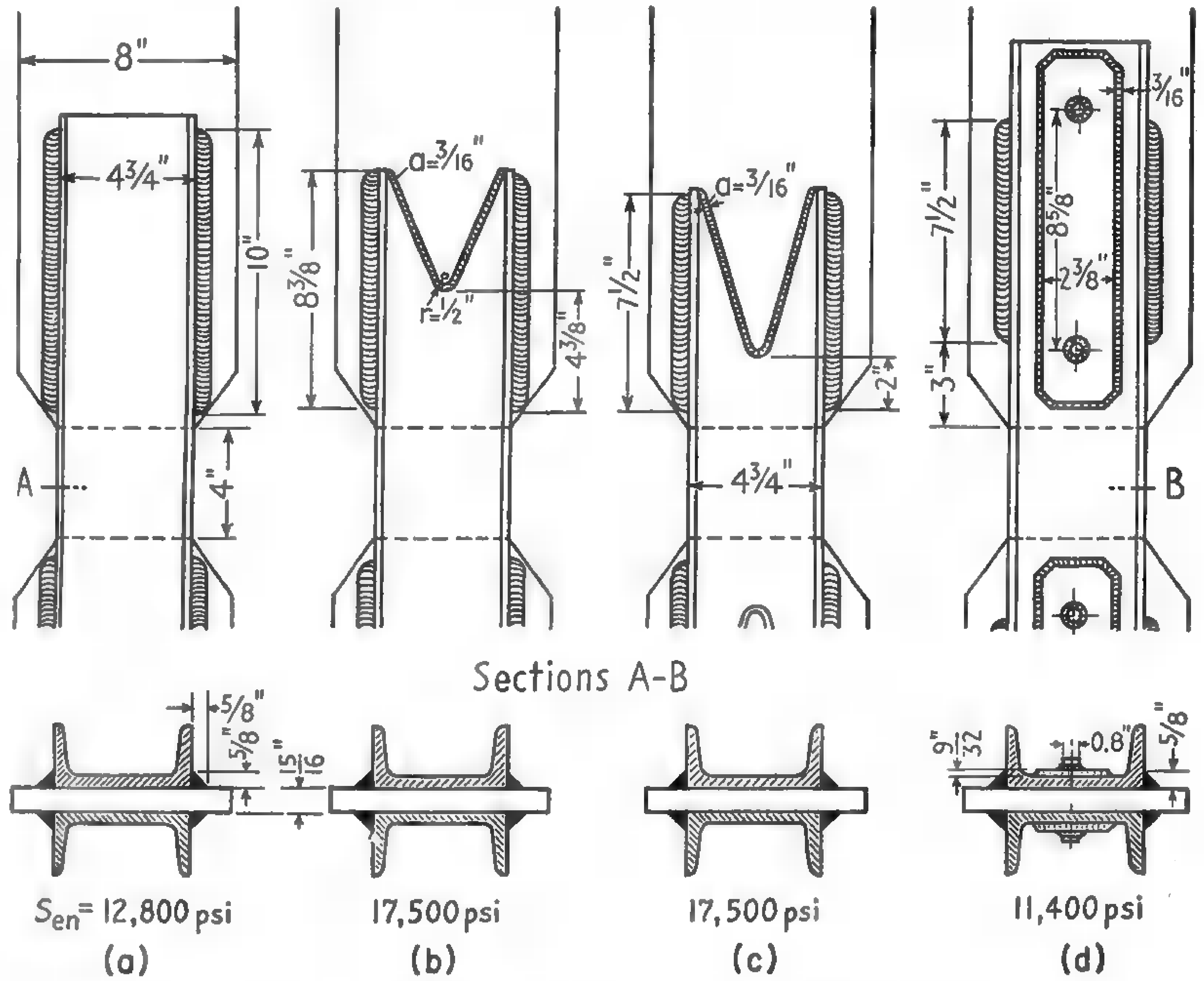


FIG. 8–12. Endurance of class 1 welds connecting channels to a plate.

Design stress for structures and machine parts subjected to repeated loads may be based on the values in the last two columns of Table 8–3 or may be taken from Table 8–5,[8] which is more conservative. Values in Table 8–3 take into consideration repeated stresses, according to Figs. 3–43 and 4–3 and actual tests.[9] All statements made previously in connection with the use of Table 8–3 for static loads apply also for repeated loads.

[8] Based on *Standard Specifications for Welded Highway and Railway Bridges* (D2.0–47) and *Standard Code for Arc and Gas Welding in Building Construction* (D1.0–46) (New York: American Welding Society, 1946).

[9] G. E. Thorton, *Study of Welded Metals*, Bulletin No. 34, Washington State College Engineering Experiment Station (September 1930); C. H. Jennings, "Welding Design," *Trans. ASME*, Vol. 58 (October, 1936) MSP-58-1, p. 504.

TABLE 8–5

STRENGTH OF SHIELDED-ARC STEEL WELDS SUBJECTED TO REPEATED LOADS

Type of Joint	Kind of Stress	Recommended Stress (psi): Load Varies from 0 to F	Recommended Stress (psi): Load Varies from $+F$ to $-F$
Butt-welded from both sides.	Tension or compression.......	13,500	9,000
Butt-welded from both sides.	Shear......................	9,000	6,000
Fillet-welded............	Tension, compression, or shear .	7,200	4,800

If single-V, backed-up welds are used, the values in Table 8–5 must be reduced by 15 per cent, and the stress-concentration factors given in Table 8–4 should be used.

8–7. Application of welding. Welding is becoming more and more popular for various kinds of construction.

Pressure vessels. The latest editions of the ASME Boiler Construction Code and its Section VIII on Unfired Pressure Vessels devote considerable attention to welding.[10] For boilers the maximum design stress is prescribed as 5,600 psi for fusion welding and 8,000 psi for forge welding. For unfired pressure vessels the maximum design stresses for fusion welding may vary from 5,600 to 8,000 psi, the value depending on the type of joint.

Fusion-welding of boiler joints may be produced either by one of the autogenous processes or by shielded-arc welding. The physical properties of the welding material used in autogenous welding correspond more closely to those of the plate than in the case of arc welding, which gives a weld that is stronger than the boiler plate. If it is desired to have the finished work as uniform throughout as possible, autogenous welding should be used. If a high tensile strength or high elastic limit of the weld is desired, arc welding is to be preferred.[11] Fusion welding should not be used where bending stresses may occur. Longitudinal seams of high-pressure boiler drums are forge-welded.

Fusion welding is extensively used as calk weld for riveted joints, screwed-in or riveted nipples, and other connections.

Tests on sample welds and full-size welded boiler drums have shown that metallic arc welding, properly done, has an efficiency of almost 100 per cent.[12] The ASME Code allows a maximum efficiency of 80 per cent that can be increased by 10 per cent if the joint is radiographed and can be increased by

[10] *ASME Code for Unfired Vessels* (New York: American Society of Mechanical Engineers, 1950).

[11] *Sulzer Technical Review*, No. 1. (1928), pp. 1–6.

[12] Linde Air Products, *Engineering and Management Phases of Oxyweld Connections* (New York: 1935), p. 63.

an additional 5 per cent if the joint is thermally stress-relieved. Thermal stress-relieving by annealing is beneficial also for other parts of the structure, such as flanged or cold-bent plates.

A good weld for a longitudinal joint is shown in Fig. 8–13. Welded girth joints for the connection of a tank bottom to the drum are shown in Fig. 8–14. That in Fig. 8–14a is good but requires accurate fitting; that in Fig. 8–14b is much superior but is feasible only for a tank with a manhole;[13] that in Fig. 8–14c is a very common method; that in Fig. 8–14d is another common method which, however, reduces somewhat the tank capacity and is not advisable for high inside pressures because of its convex shape; that in Fig. 8–14e is excellent but is limited to a tank with a manhole.

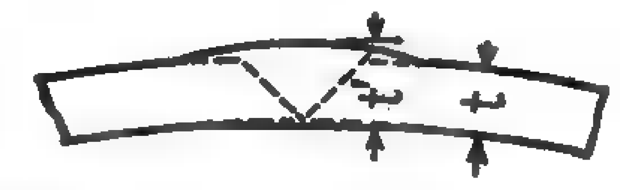


FIG. 8–13. Reinforcement of a welded joint.

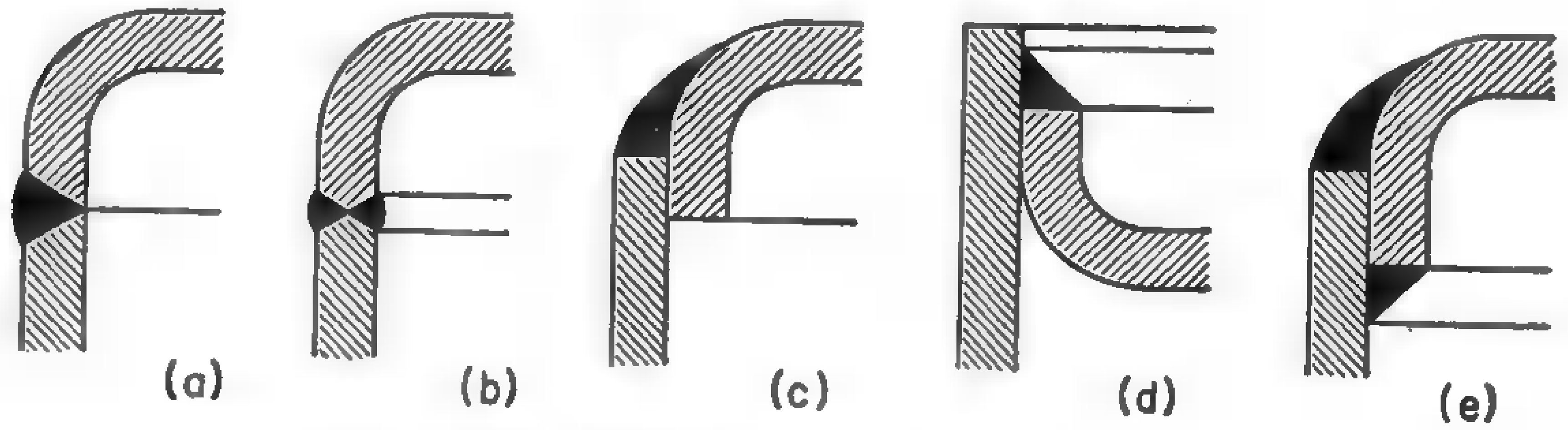


FIG. 8–14. Welding of tank heads to drum.

In Fig. 8–15 is shown a seamless press-forged steel nozzle (produced commercially), which is welded to a pressure-vessel shell by a butt weld *a*.[14] The reinforcing *b* counteracts the weakening of the shell by the large hole.

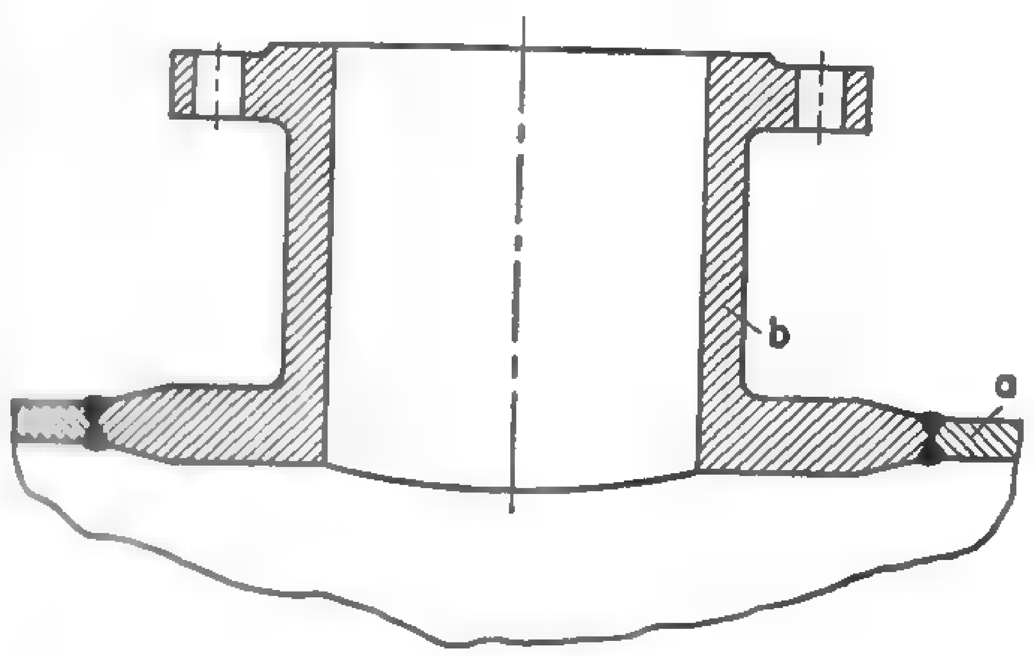


FIG. 8–15. Forged nozzle welded to a shell.

Steel structures under static loads. The design of welded steel structures subjected to static loads does not present any peculiarities.

Solid beams under dynamic loads. In the design of a solid beam the following requirements should be observed:[15]

a) Only continuous welds should be used.

b) Cross welds in a shelf working in tension should be avoided; where they cannot be avoided, they should be made as small as possible.

[13] A. J. Moses, "Tests on Welded Boiler Drums," *Combustion*, Vol. 2, No. 5 (November, 1930), p. 23.

[14] *ASME Rules for Construction of Unfired Pressure Vessels* (New York: American Society of Mechanical Engineers, 1950).

[15] A. Schaper, "Die Dauerfestigkeit der Schweissverbindungen," *Z.VDI*, Vol. 77 (1933), p. 560.

c) An accumulation of welds should be avoided by all means.

d) Shelf plates superimposed on other plates should be made narrower toward the ends in order to obtain a gradual change from a heavier cross section to a lighter one.

e) Welds used at the intersection of shelf and web plates should always be butt welds.

Frame beams under dynamic loads. Complete data for the economical design of frame beams under dynamic loads are not yet available. However, many valuable pointers may be obtained from the test results discussed in connection with Figs. 8–9 to 8–12.

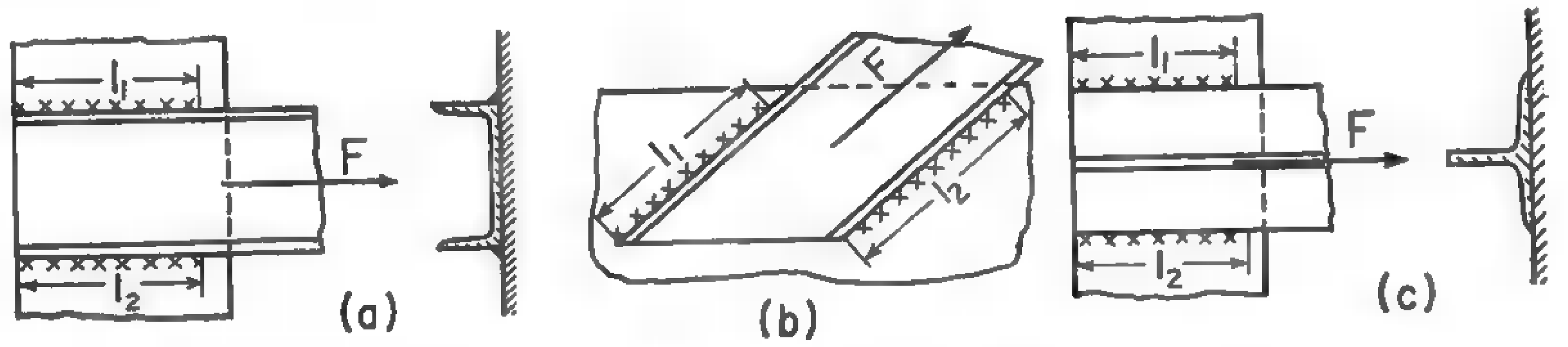


FIG. 8–16. Typical channel and tee connections to a plate.

Allowable stresses. For building construction the code of the American Welding Society recommends the following stresses in a section through the throat of a steel weld for bare or lightly coated electrodes: in shear, 11,300 psi; in tension, 13,000 psi; in compression, 18,000 psi. When a shielded arc is used, the allowable shearing and tensile stresses may be increased by 20 per cent, or to 13,600 psi and 15,600 psi, respectively.

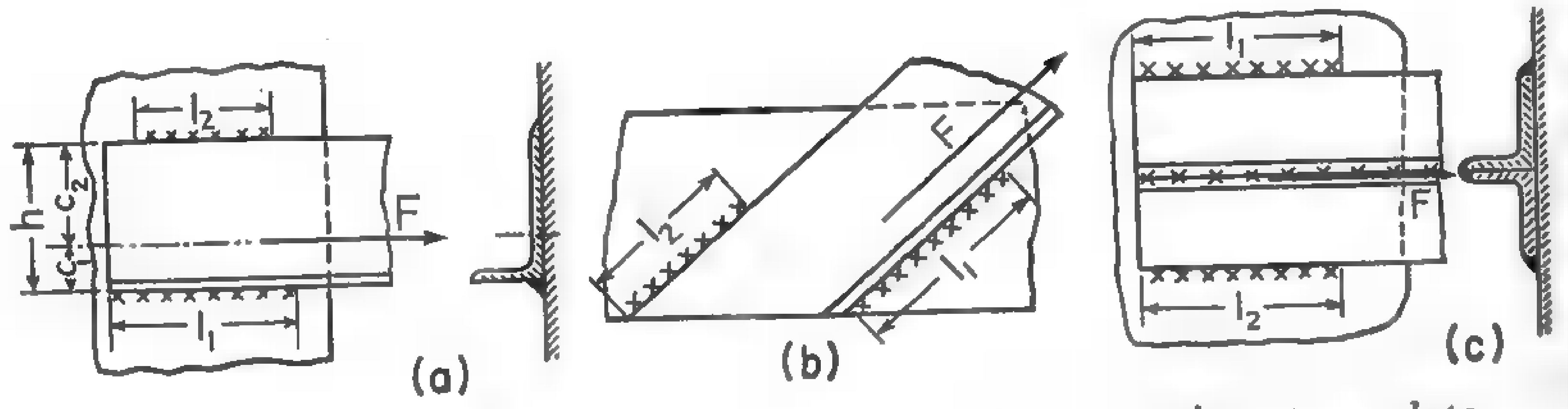


FIG. 8–17. Typical single-angle and double-angle connections to a plate.

Typical connections. In Fig. 8–16a, b, and c and Fig. 8–17c are shown some typical connections of *symmetrical structural shapes* in which the load is applied halfway between the two edges. Regardless of whether the end is cut off at a right angle, as in Fig. 8–16a or c and Fig. 8–17c, or on a bias, as in Fig. 8–16b, the length of each fillet weld must be the same, or $l_1 = l_2$. This length may be found from the equation

$$F = 2a(l_1 - 0.5)S_s \tag{8–4}$$

The T shape in Fig. 8–16c may be obtained by splitting an H beam along its center line by a gas- or arc-welding flame.

In the case of an *unsymmetrical member*, as in Fig. 8–17a or b, the load F is applied at the center of gravity; and the lengths l_1 and l_2 should be selected so that they give equal moments with respect to the center of gravity. Thus they can be found, when the $\frac{1}{4}$-in. imperfect ends are taken into account, from the following simultaneous equations:

$$F = (l_1 + l_2 - 2 \times 0.5)S_s a \tag{8–5}$$

and

$$(l_1 - 0.5)c_1 = (l_2 - 0.5)c_2 \tag{8–6}$$

EXAMPLE 8–6. Determine: (a) the size of the single angle iron in Fig. 8–17a, loaded in tension by a static load $F = 33{,}200$ lb; and (b) the dimensions of the welds.

a) For structural steel, which is approximately SAE 1020, $S_e = 34{,}000$ psi. With a safety factor $n = 2$, the allowable stress is $S_d = 17{,}000$ psi, and the necessary cross-sectional area is

$$A = \frac{F}{S_d} = \frac{33{,}200}{17{,}000} = 1.95 \text{ sq in.}$$

From a structural-steel handbook, the nearest size that is not too thick is an angle $4 \times 3\frac{1}{2} \times \frac{5}{16}$ in., for which $A = 2.25$ sq in. and $c_1 = 1.18$ in.

b) The thickness of the throat of the fillet weld is

$$a = 0.707h = 0.312 \times 0.707 = 0.221 \text{ in.}$$

According to Table 8–3, $S_s = 11{,}000$. From equation 8–5,

$$l_1 + l_2 = \frac{F}{S_s a} + 1.0 = \frac{33{,}200}{11{,}000 \times 0.221} + 1.0 = 13.66 + 1.0 = 14.66 \text{ in.} \tag{a}$$

Substituting the corresponding values in equation 8–6 gives

$$1.18(l - 0.5) = (4 - 1.18)(l_2 - 0.5) \tag{b}$$

from which

$$l_2 = \frac{1.18l_1 + 0.82}{2.82} = 0.418l_1 + 0.29 \tag{c}$$

By substituting the value of l_2 from equation c and solving the latter for l_1, we get

$$l_1 = \frac{14.37}{1.418} = 10.13, \text{ or } 10\tfrac{1}{8} \text{ in.}$$

Then,

$$l_2 = 14.66 - 10.13 = 4.53, \text{ or } 4\tfrac{5}{8} \text{ in.}$$

8–8. Design of welded machine parts. Welding is coming into wide use in the manufacture of frames, bases, crankcases, flywheels, gears, various jigs, and many other machine parts. Rolled plates and structural shapes are then used instead of cast iron and cast steel, with a considerable saving of weight and cost. Various welding processes are employed, the best one depending on the thickness of the metal. The component parts are usually cut to size with shears or a cutting torch and are assembled in a preliminary way by tacking in a few spots, and the welds are completed when all parts are accurately assembled. Some component parts can be prepared advantageously of weldable cast steel.

Welding of machine parts is particularly economical when only a limited number of them is to be made and also when comparatively heavy pieces

have thin or medium-thick walls, or when the pieces have unusually heavy walls, as in the frames of large presses.

Welded parts can be fabricated in large quantities more economically if the component parts are stamped or die-punched. Before a designer decides whether a piece should be cast or welded, or welded of cast component parts, he should prepare careful designs of the part by the different methods. He should also take into account not only the cost but also various other considerations, such as rigidity, resistance to corrosion, and even pleasing appearance.[16] There is much literature on the design of welded parts.[17]

In making a layout of a weldment the designer should remember that the cost of deposited metal in the welds is from thirty to fifty times as great as the cost of the mild steel from which the component parts are made. Therefore it is usually advisable to shape the component parts by bending wherever possible, and thus to reduce the amount of welding.

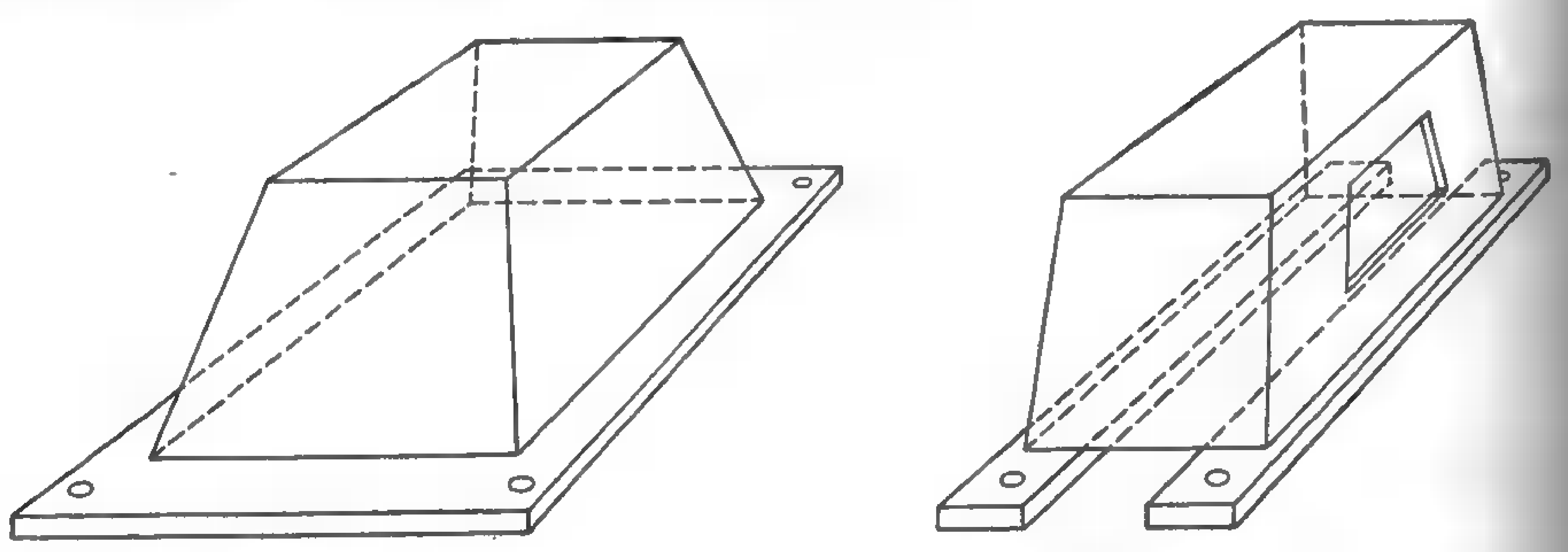


Fig. 8–18. Welded pedestal.

Also, in designing a piece to be made by welding, it is advisable to forget how this piece looks when it is cast or riveted; to keep in mind only its functions; to obtain these functions with the simplest combination of available stock material—plates, angle irons, and channels, and occasionally to use special steel castings or forgings.

In Fig. 8–18a is shown a pedestal which was designed to replace a casting and which looks like a casting. In Fig. 8–18b is shown a design in which better advantage is taken of the possibilities presented by welding. The top, front, and back are made of one sheared piece bent to conform with the sides, and the sides have flame-cut openings which facilitate the handling of the piece and also add to its appearance. The design in Fig. 8–18b permits a considerable cost saving over the design in Fig. 8–18a because of a reduction of 30 per cent in the length of welds and about 30 per cent in weight. This design costs much less than a casting shaped like the design in Fig. 8–18a.

[16] J. L. Brown, "Casting or Welding in Machine Design," *Trans. ASME*, Vol. 58 (October, 1936), MSP-58-9, pp. 553 ff.

[17] E. J. Charlton, "Trends in the Use of Welded Machinery Parts," *Mechanical Engineering*, Vol. 67 (1945), pp. 109 ff; Lincoln Electric Company, *Procedure Handbook of Arc Welding—Design and Practice, 9th ed.* (Cleveland: 1950); also articles in *Machine Design*, *Product Engineering*, *Steel*, *Welding Journal*, and other periodicals.

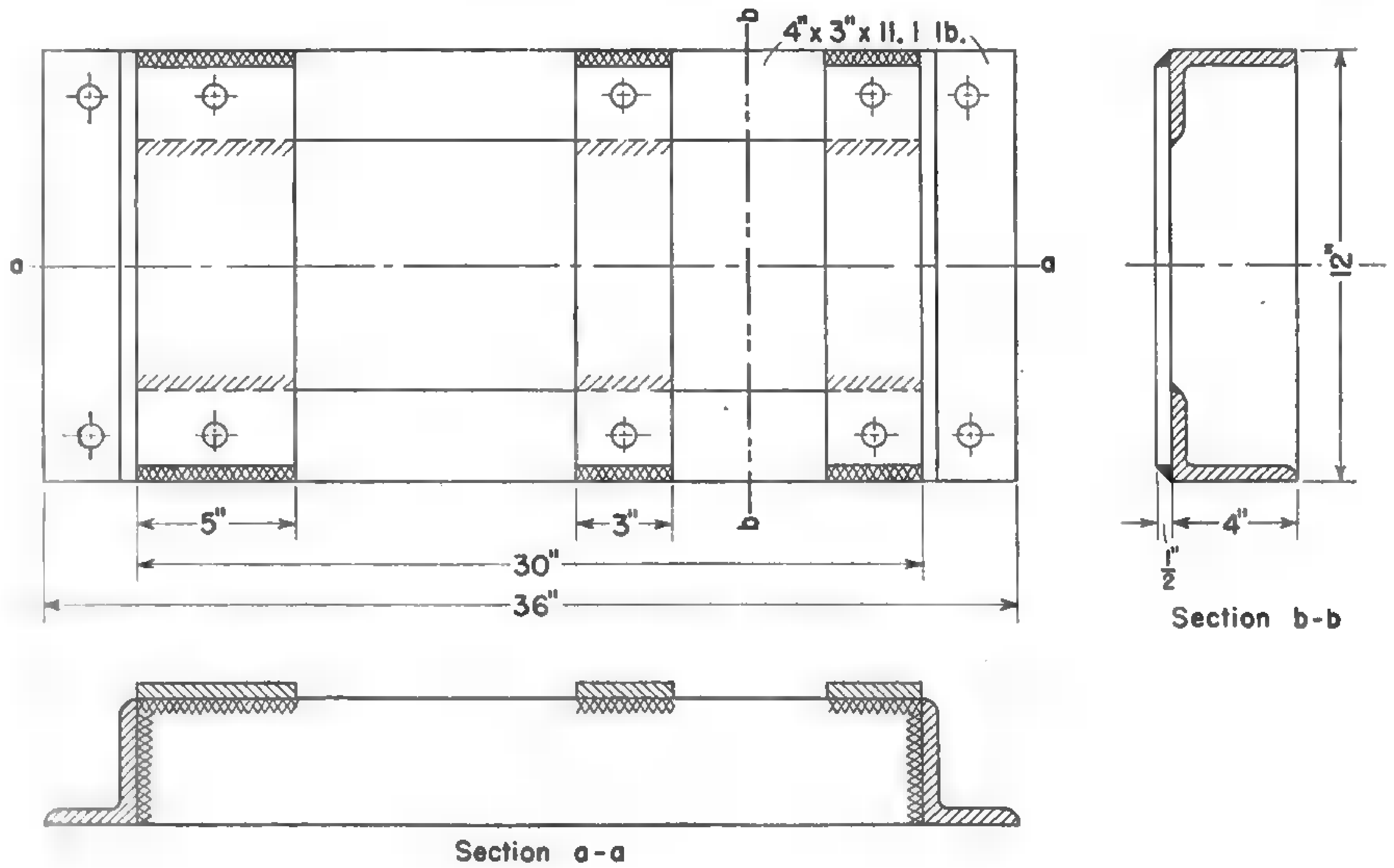


Fig. 8–19. Welded bedplate.

A bedplate for a motor-driven pump appropriately made from three sheared plates and four angle irons is shown in Fig. 8–19. The weight of this weldment is only 45 per cent of that of a casting, and its cost is about one-half that of a cast bed because it does not require expensive machining of the supporting surfaces.

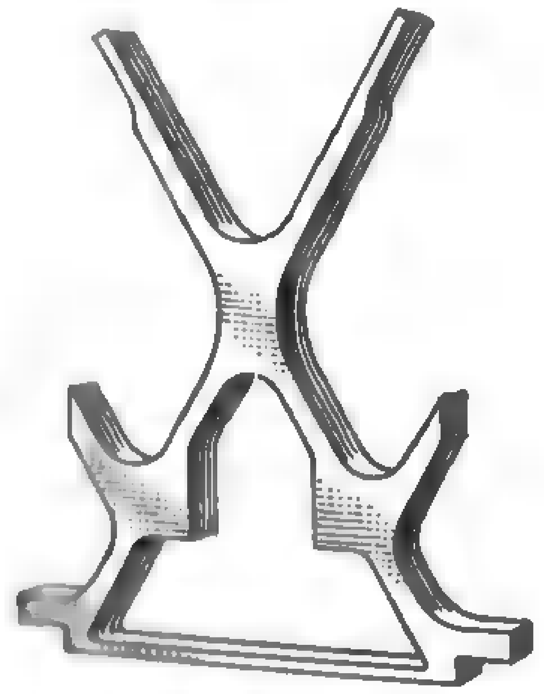


Fig. 8–20. Flame-cut part of a crankcase.

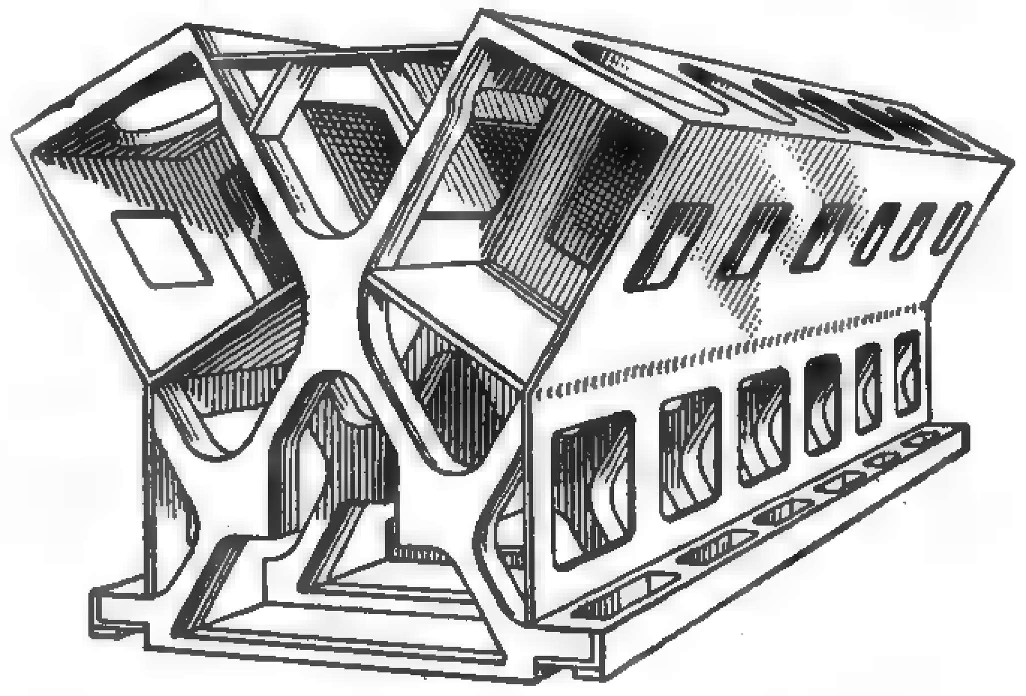


Fig. 8–21. Crankcase in process of fabrication by welding.

The drawings in Figs. 8–20, 8–21, and 8–22 illustrate the construction of a crankcase for a 12-cylinder, lightweight, 1,000-hp diesel engine.[18] In Figs. 30–20 and 30–21 are shown large welded gears.

[18] E. Chapman, "Welded-Steel Diesel Structures," *Motorship*, Vol. 18 (1933), pp. 404–9.

Stress distribution. In designing a welded machine part the chief aim should be to secure a proper distribution of material in order to obtain a body of uniform strength as well as to avoid the discontinuities of section which result in stress concentration and endurance failure. Arrangement of the material is particularly important in parts subjected to repeated high loads, as in diesel-engine crankcases. Eccentric action should be avoided, and the joints should be symmetrical. A gear hub that was welded to I-shaped structural-steel arms, as in Fig. 8–23a, with one-sided welds at *c*, failed because of progressive fracture cracks at *b*. The modified construction in Fig. 8–23b, with symmetrical welds *c* from both sides of the flange, proved satisfactory.

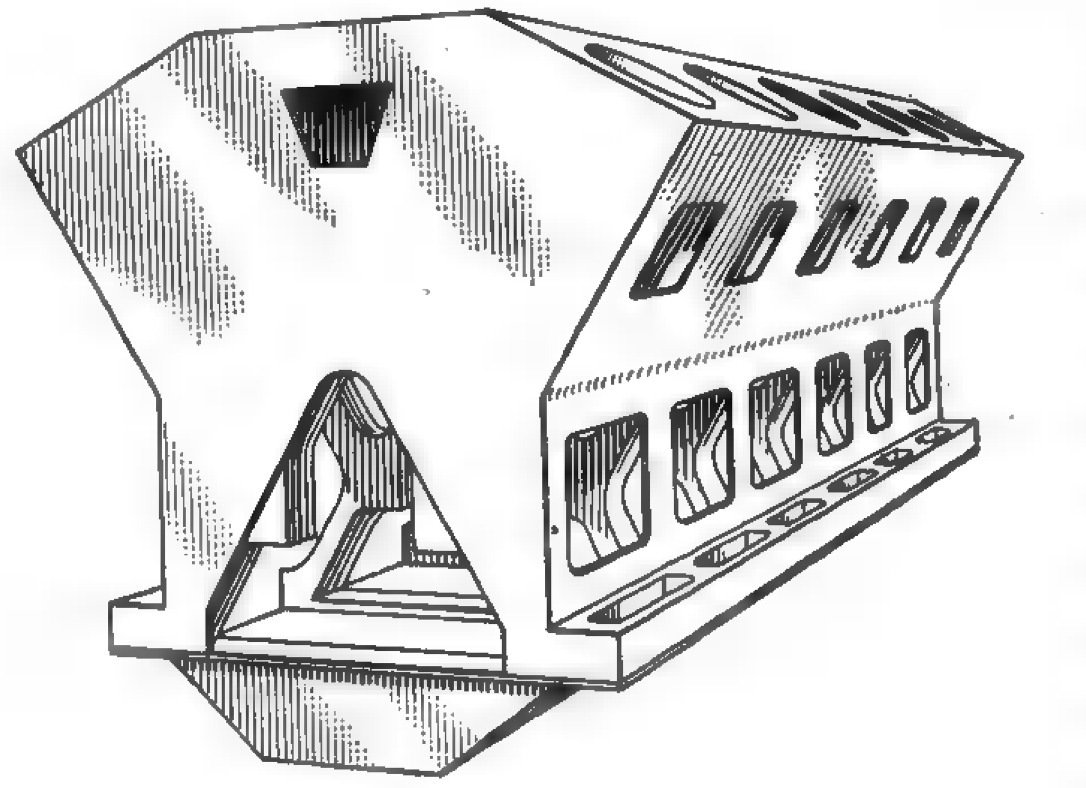


FIG. 8–22. Welded crankcase.

An effective method of determining points of maximum stress in a three-dimensional structure is by painting it with a fast-drying varnish, called *stress-coat*, which possesses a low modulus of elasticity and a low yield point. When a static load is applied to the structure, the varnish cracks at the points of maximum strain while the structure is only lightly loaded. The varnish will crack first at those points where the fillet radius is not large enough, at the contours of an improper weld, or at the root of an undercut. These cracks will give valuable information as to how to improve the structure in order to give it more uniform strength throughout. Similar although not as accurate results may be obtained by using lime wash, which peels off from points subjected to strain.

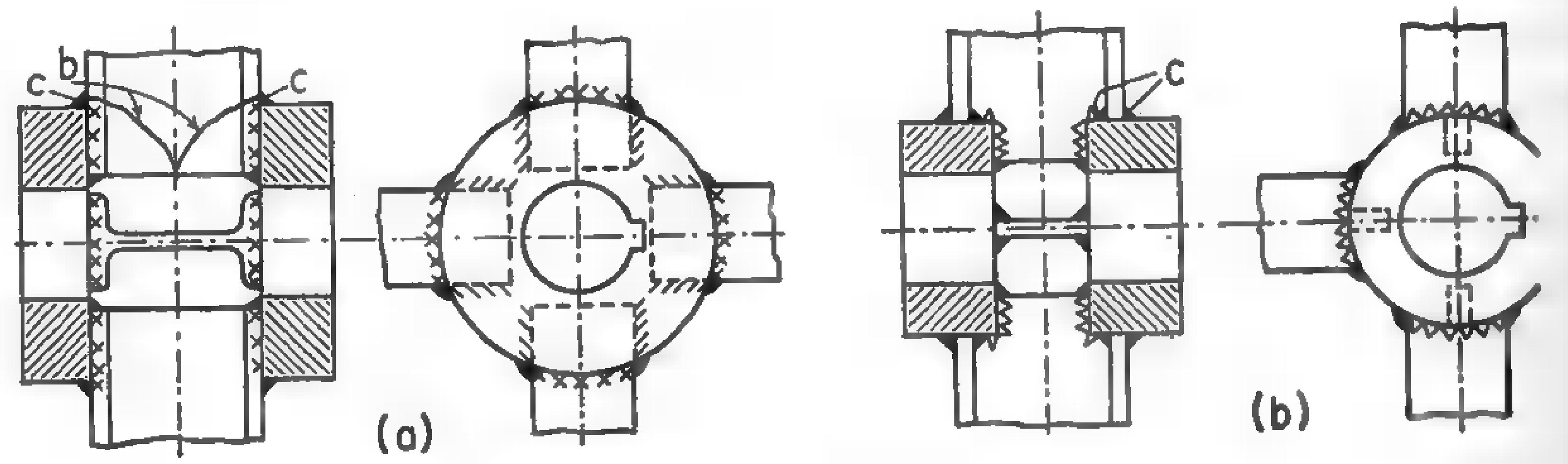


FIG. 8–23. Welded gear hub with arms.

Rigidity. In machine construction the rigidity or flexibility is often of great importance. Proper arrangement of weld joints gives a control in this respect. In Fig. 8–24a is shown a connection of two columns that is conveniently made but is not very rigid. The rigidity, and the strength also, can be materially increased by adding the inside welds *e*, as in Fig. 8–24b, if the distance l allows such construction, but this is seldom the case. In Fig. 8–24c is shown a rigid connection which is easy to make.

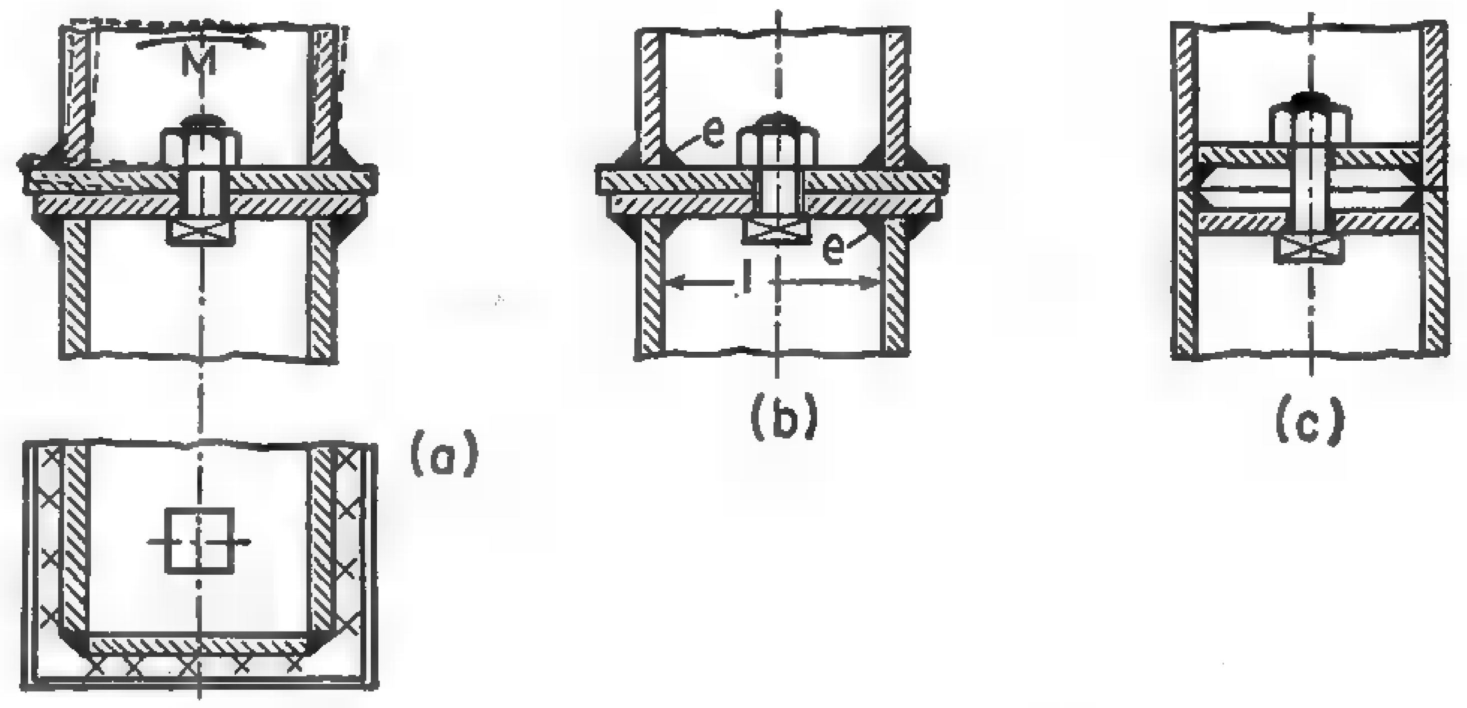


FIG. 8–24. Welded joints with different rigidity.

Other considerations. Welding also presents the advantage of employing different materials, each most suitable for the particular service, to form an integral part. In Fig. 8–25, for example, is shown a horizontal planer support with hardened cast-steel guides *g* welded to a ductile-steel U-shaped box *b*.

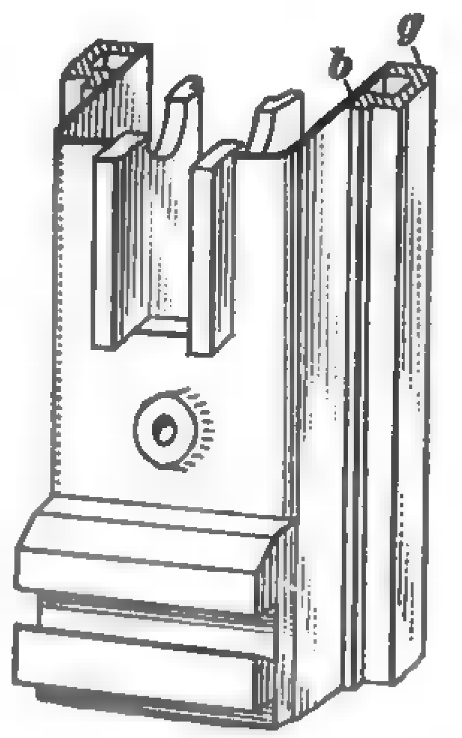


FIG. 8–25. Welded frame of plates and castings.

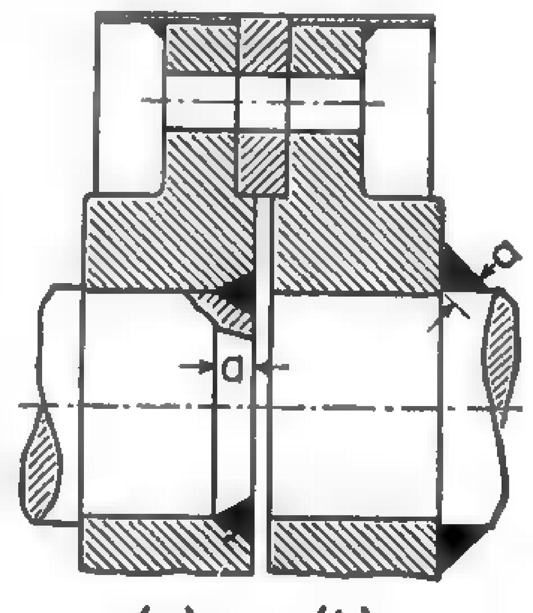


FIG. 8–26. Flange coupling.

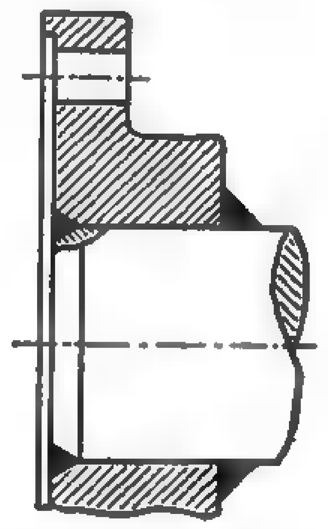


FIG. 8–27. Stub-shaft flange.

Two methods of welding used with flange couplings to replace tapered keys if the flanges do not have to be removed are shown in Fig. 8–26.[19] The simultaneous use of both methods, as in Fig. 8–27, reduces the throat dimension *a*; this is desirable with a large torque.

[19] L. v. Roessler, "Schweissungen statt Keilbefestigungen," *Electroschweissungen*, Vol. 7 (1936), p. 209; also *Z.VDI*, Vol. 81 (1937), p. 305.

CHAPTER 9

Design of Riveted Constructions

9–1. General remarks. Riveting was the standard method of joining plates and structural parts before welding began to replace it with increasing rapidity.

Rivets. A rivet is a round bar consisting of an upset end called the *head* and a long part called the *shank.* The rivet blank is heated to a red glow and inserted into one of the holes; and while the head is held firmly against the plate by a heavy sledge, the projecting end is formed into a second head, called the *point,* by means of a hand hammer and a forming tool called a *set,* or by a press. In Fig. 9–1 are shown various shapes of heads and points, with the length of shank necessary to form the corresponding point in each case. The head *a* is a button head; *b* is a double-radius button, or conoidal, head; *c* is a pan head; and *d* is a countersunk head. The point *e* is a cone point; *f* is a steeple point; and *g* and *h* are countersunk points. Button heads are used for small rivets up to $\frac{5}{16}$ in. in diameter, which are driven cold; pan heads are used chiefly in ship work; countersunk heads are used only in special cases, chiefly in structural work and below the water line in ships. The countersunk points weaken the plate so much that they should be used only when unavoidable; the others, including points similar to button heads, are used in boiler and structural work.

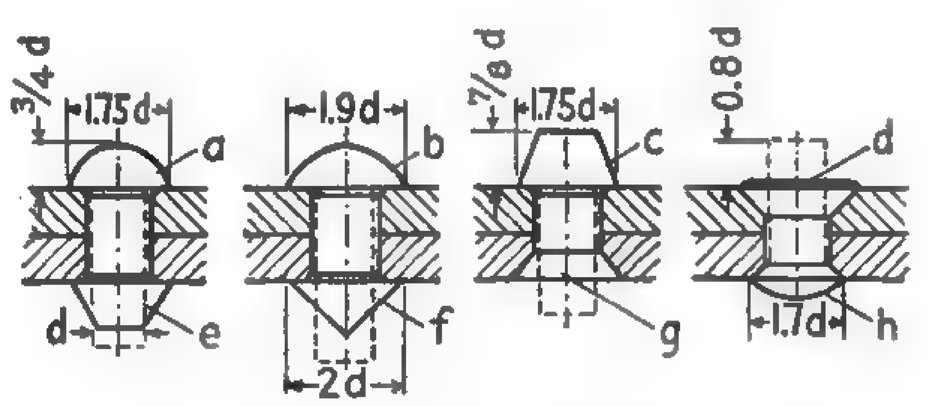


FIG. 9–1. Rivet heads and points.

Rivet diameters in common use increase from $\frac{3}{8}$ in. to $1\frac{1}{2}$ in. by eighths of an inch.

Material. Rivets are made of tough and ductile low-carbon steel or nickel steel. According to the ASME Boiler Construction Code, the rivet material must have a tensile strength between 45,000 and 55,000 psi and a minimum yield point equal to one-half the tensile strength.

Brass rivets are used only cold and in small sizes.

Rivet holes. Holes for rivets are either punched or drilled. Punching injures the metal around the holes. Therefore they should be punched at least $\frac{1}{8}$ in. undersize, and then be reamed. Simultaneous reaming of the matching holes through the plates to be connected by a rivet straightens the holes and eliminates offsets, which weaken the rivet. Holes of a diame-

ter smaller than the thickness of the plate cannot be punched, since the punch is likely to crush.

The maximum force F needed to punch a hole in a plate may be estimated conservatively as follows:[1]

$$F = 1.25\pi d t S_u \tag{9–1}$$

where d is the diameter of the hole, in inches;
t is the thickness of the plate, in inches;
S_u is the ultimate strength in shear of the plate material, in pounds per square inch, and is about 0.75 times the tensile strength S_u.

The work of punching a hole, in inch-pounds, may be taken as

$$W = 0.68Ft \tag{9–2}$$

Drilling the holes is the best method. With the present improved machinery it can be accomplished almost as cheaply as punching and reaming.

Rivet holes are made $\frac{1}{16}$ in. larger than the shank of the cold rivet. When the red-hot rivet is driven and set, it fills out the full size of the hole. In order to reduce the stress concentration at the junctures of the stem with the head and the point, the holes should be countersunk about $\frac{1}{16}$ in. deep.

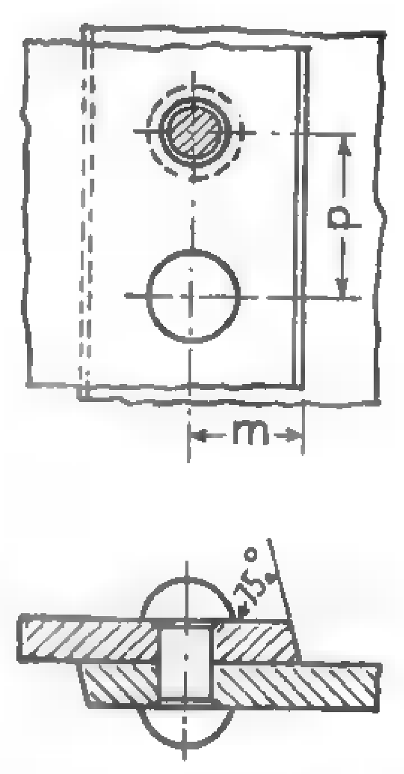


FIG. 9–2. Single-riveted lap joint.

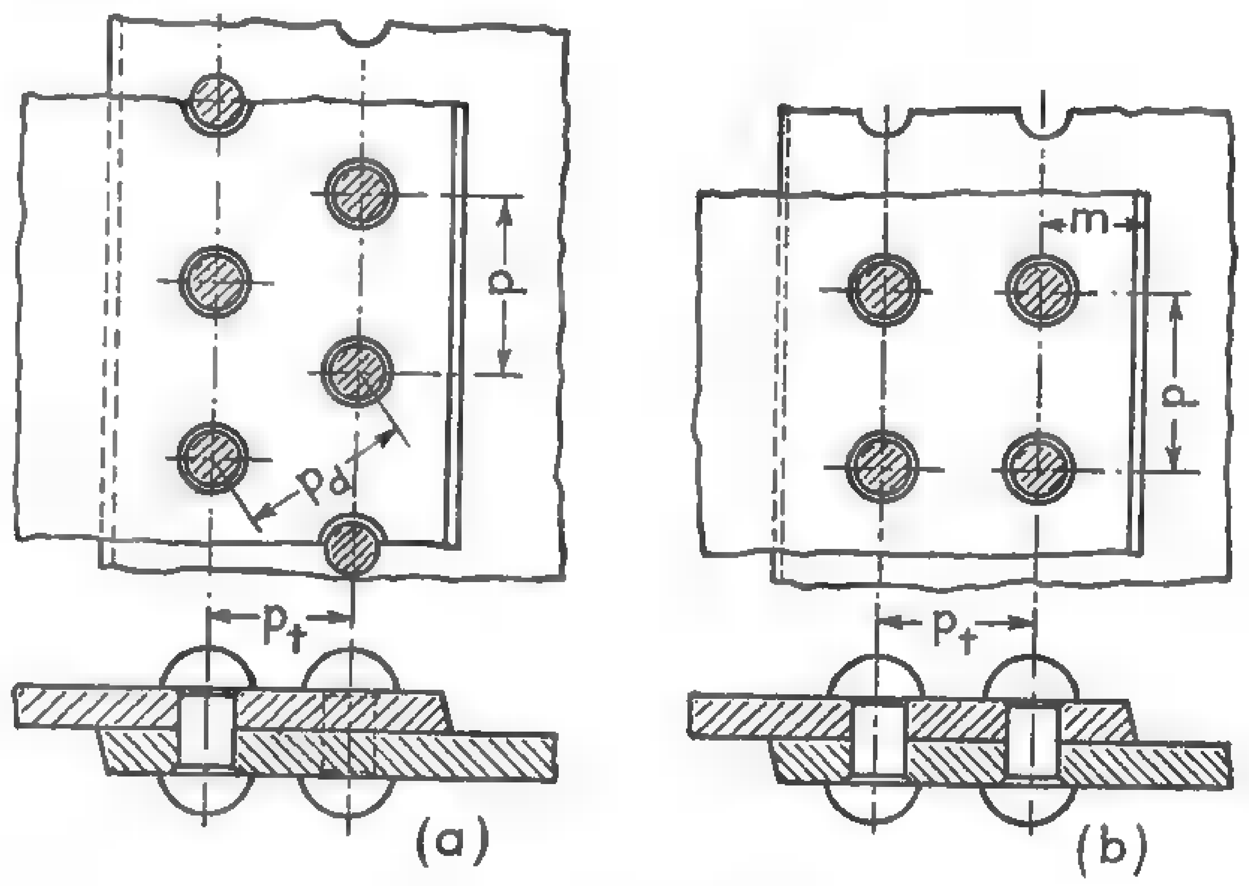


FIG. 9–3. Double-riveted lap joints.

9–2. Riveted joints. Structures in which riveted joints or connections are used may be grouped as follows:

a. Tanks and pressure vessels
b. Bridges, buildings, cranes, and machinery in general
c. Hulls of ships

In structures in the first group, the joints must be leakproof. In structures in the second group the connections must resist given outside loads and

[1] C. D. Albert, *Machine Design Drawing Room Problems,* 4th ed. (New York: John Wiley & Sons, Inc., 1949), p. 212.

have sufficient rigidity. In structures in the third group it is important to consider strength, rigidity, and durability, as well as security against leakage.

Types of joints. Two arrangements used in joining plates by means of rivets are equally well adapted to all three groups of structures: namely, *lap joints* and *butt joints.*

Lap joints consist of overlapping plates held together by one or more rows of rivets. A single-riveted lap joint is shown in Fig. 9–2. A double-riveted lap joint can have rivets arranged either staggered, as in Fig. 9–3a, or in a chain form, as in Fig. 9–3b. In triple-riveted lap joints the rivets are staggered.

The eccentric force action in a lap joint produces bending; and the resulting distortion, shown exaggerated in Fig. 9–4, tends to produce a concentrated crushing load on the corners of the plates, a tensile stress in the rivet shank, and a shear stress in the heads. This action thus tends to decrease the frictional resistance. However, tests have shown that a lap joint has a 30 per cent greater frictional resistance than a butt joint with two straps,[2] like that shown in Fig. 9–5. The explanation seems to be in the pinching of the lap joint at the edges and in an unavoidable difference in the thickness of the two plates in a butt joint. Also, it is more difficult to keep lap joints tight, because of the bending that occurs.

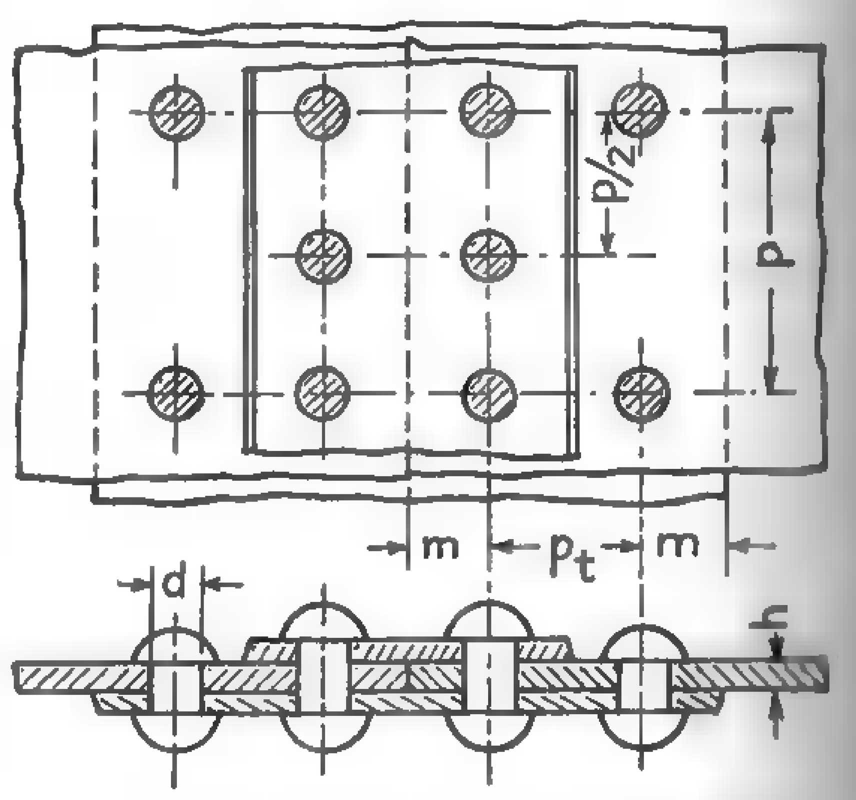


FIG. 9–5. Double-riveted chain butt joint.

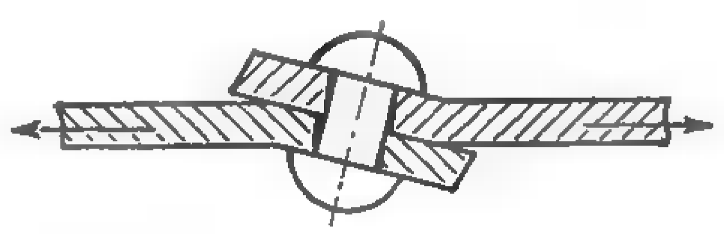


FIG. 9–4. Bending in a lap joint.

Butt joints are formed by butting the plates against each other and joining them by overlapping straps, or cover plates. A butt joint may have one strap on the outside, but most butt joints have one strap on the outside and another on the inside. The cover plates may be of the same width, or the outside plate may be narrower, as in Fig. 9–5. Butt joints are made single-, double-, triple-, quadruple-, and sometimes even quintuple-riveted.

Terminology. The line through the centers of a row of rivets, parallel to the edge of the plate, is called the *gage line.* The distance between the centers of adjacent rivets measured on the gage line is called the *pitch*; it is denoted by p in the illustrations and tables of this book. If the spacing on one gage line is greater than on another, as in Fig. 9–5, the pitch is the maximum rivet spacing. A *unit strip* or *unit length* is equal in width to the pitch. The distance between two adjacent gage lines in the same plate is called the *back pitch*, or *transverse pitch*; it is denoted by p_t, as in Fig. 9–3 and Fig. 9–5. In staggered riveting the distance between the centers of adjacent rivets on adjacent gage lines is called the *diagonal pitch;* it is denoted by p_d, as in Fig. 9–3. The distance from the gage line to the edge of the plate is called the *lap distance, marginal distance,* or *margin;* it is denoted by m, as in Fig. 9–2, Fig. 9–3, and Fig. 9–5.

[2] C. A. Norman, *Principles of Machine Design* (New York: The Macmillan Company, 1925), p. 57.

9–3. Stresses in joints. Rivets should always be placed at right angles to the acting forces, and the maximum stress induced in them should be either shear or crushing. In a long rivet the initial stress set up at the junction of the shank and the point, when the rivet cools, is dangerous. This initial stress increases with the relative length of the rivet, and in a very long rivet it may cause the head to snap off without any load. For this reason the length of the rivet between the heads should not exceed four or five times its diameter. Longer rivets can be used if the head end is cooled before the rivet is placed in its hole.[3]

Calking was developed primarily for the purpose of making riveted joints leakproof. It is good practice to calk not only the plate but also around the rivet heads, as shown in Fig. 9–6.

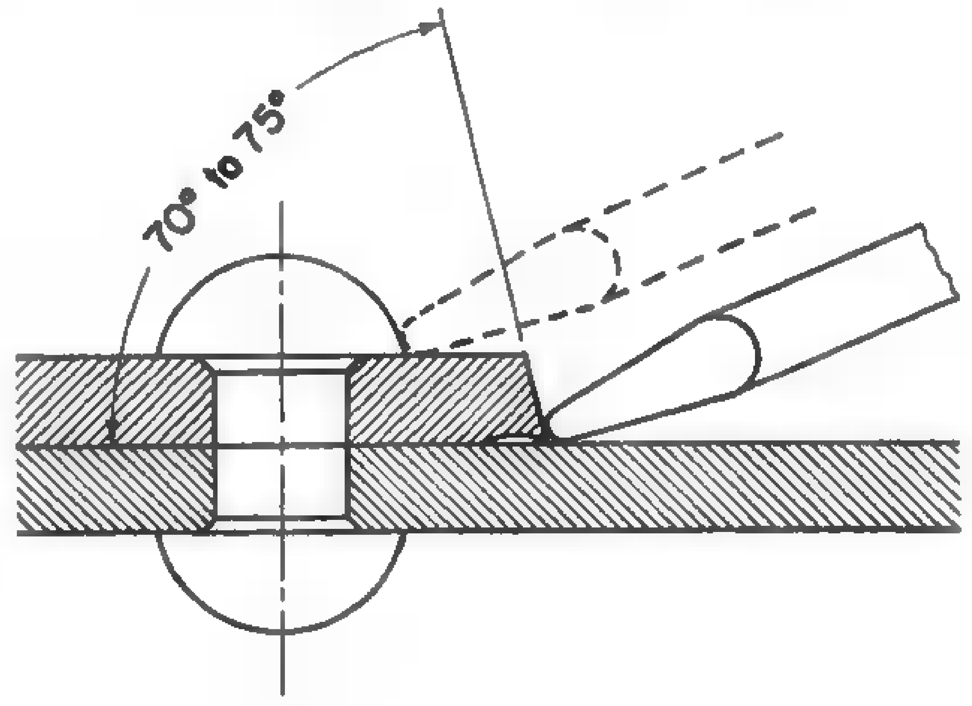


FIG. 9–6. Calking.

Failure of a riveted joint. The conventional assumption is that the load is distributed equally among all rivets. This assumption does not approximate even roughly the actual conditions, particularly for multiple-riveted joints. However, the load distribution upon the rivets depends on so many factors and is so indefinite that no better theory exists. According to the conventional theory, the failure of joints may occur in one of the following ways:

a. Shearing of the rivets, as in Fig. 9–7a
b. Rupturing of the plate by tension, as in Fig. 9–7b
c. Crushing of the plate or of the rivets, as in Fig. 9–7c
d. Shearing of the margin, as in Fig. 9–7d
e. Tearing of the margin, as in Fig. 9–7e
f. Rupturing of the plate by tension in a zigzag line passing diagonally between the rivet holes in staggered riveting.

In a multiple-riveted joint ultimate failure may be due to one or more of the causes just cited.

[3] H. J. Spooner, *Machine Design*, 6th ed. (London: Longmans, Green & Company, 1925), p. 127.

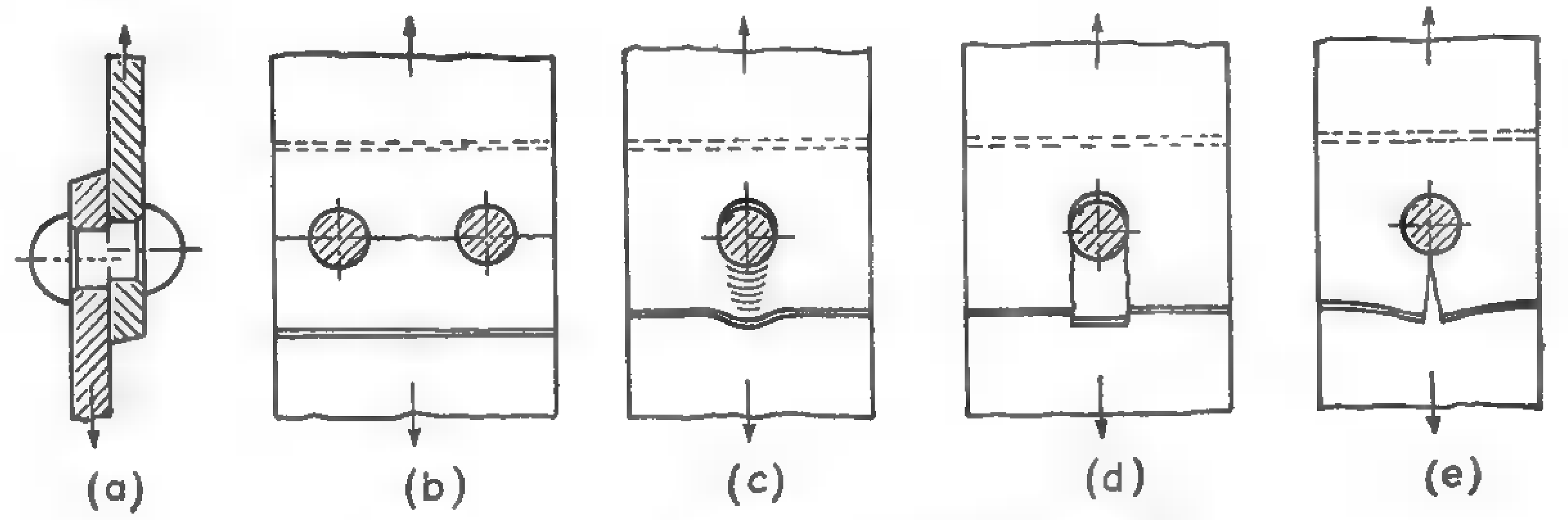


FIG. 9-7. Ways of failure of a riveted joint.

Strength of rivets. It should be noted that in designing riveted joints of *pressure vessels* the diameter of the hole is used in calculating the resistance of the rivets both in shear and in crushing. In *structural joints* the rivets sometimes are driven cold and sometimes are replaced by bolts; it is therefore customary to use the nominal rivet diameter in calculating the resistance of rivets.

Efficiency. The ratio of the strength of the weakest section of a unit length of the joint to the tensile strength of an equally wide unperforated plate is called the *efficiency* of the joint. A well-designed joint should have the same efficiency in regard to every possibility of failure. If this equality does not exist, the governing lowest efficiency of the joint can be raised by strengthening the corresponding weaker element at the expense of the stronger one.

Margin. The width of the margin m is independent of other elements. With the usual proportions, however, if the margin is equal to $1.5d$, it will be safe against both shearing and tearing by the rivet pressure. For the sake of additional safety, in single-riveted joints m is given the width $2d$. On the other hand, in boiler work the margin should not be made larger than necessary, as it is then more difficult to calk the joint and to make it steam-tight.

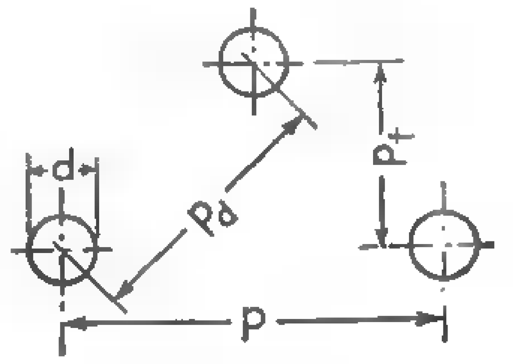


FIG. 9-8. Pitch relations.

Transverse pitch. In chain riveting the transverse pitch p_t, in Fig. 9-3b, may be determined by the requirement of room for the heading tool. It should not be less than $2d$ and preferably should be $2.5d$.

In staggered riveting, as in Fig. 9-8, the minimum value of p_t is determined by the condition that the plate will not rupture along the diagonal pitches p_d. Then

$$2(p_d - d) \geqq p - d$$

The elimination of p_d by means of the geometric relation in this diagram gives

$$p_t \geqq \sqrt{0.5pd + 0.25d^2} \tag{9-3}$$

With the usual proportions of joints, p_t is greater than or equal to $1.7d$, and a safe value of p_t is $2d$.

If one of the cover plates is narrower, as in Fig. 9-5, the transverse pitch must be made longer to give room for the riveting tool. A practical value for the transverse pitch in this case is $2.75d$.

9-4. Theoretical strength analysis. In a theoretical analysis of strength the problem is to find the general relation between the thickness of the plates, the rivet diameter, and the pitch when a riveted joint is equally strong against failure in any of the six possible ways. Since the margin and the transverse pitch do not affect the longitudinal pitch p and can be either determined independently or assigned values based on the diameter of the rivet, it follows that the last three ways of failure, designated d, e, and f in section 9-3, may be omitted from the general analysis. Hence the problem narrows down to the analysis of three types of failure, designated a, b, and c.

In this analysis Fig. 9-9 will be used only as an illustration of a complex joint with various rivet spacings, some of the rivets working in double shear and some in single shear. In accordance with the ASME Boiler Code, the shear stress s_s will be considered the same, whether induced in single shear or in double shear. Also, elasticity will be disregarded and the crushing stress s_c will be considered to be the same in all rivet rows. In addition to the designations in Fig. 9-9, n_2 will designate the number of rivets working in double shear in a unit length, whose width is p, and n_1 will designate the number of rivets in single shear. For the joint in Fig. 9-9, $n_2 = 8$ and $n_1 = 3$.

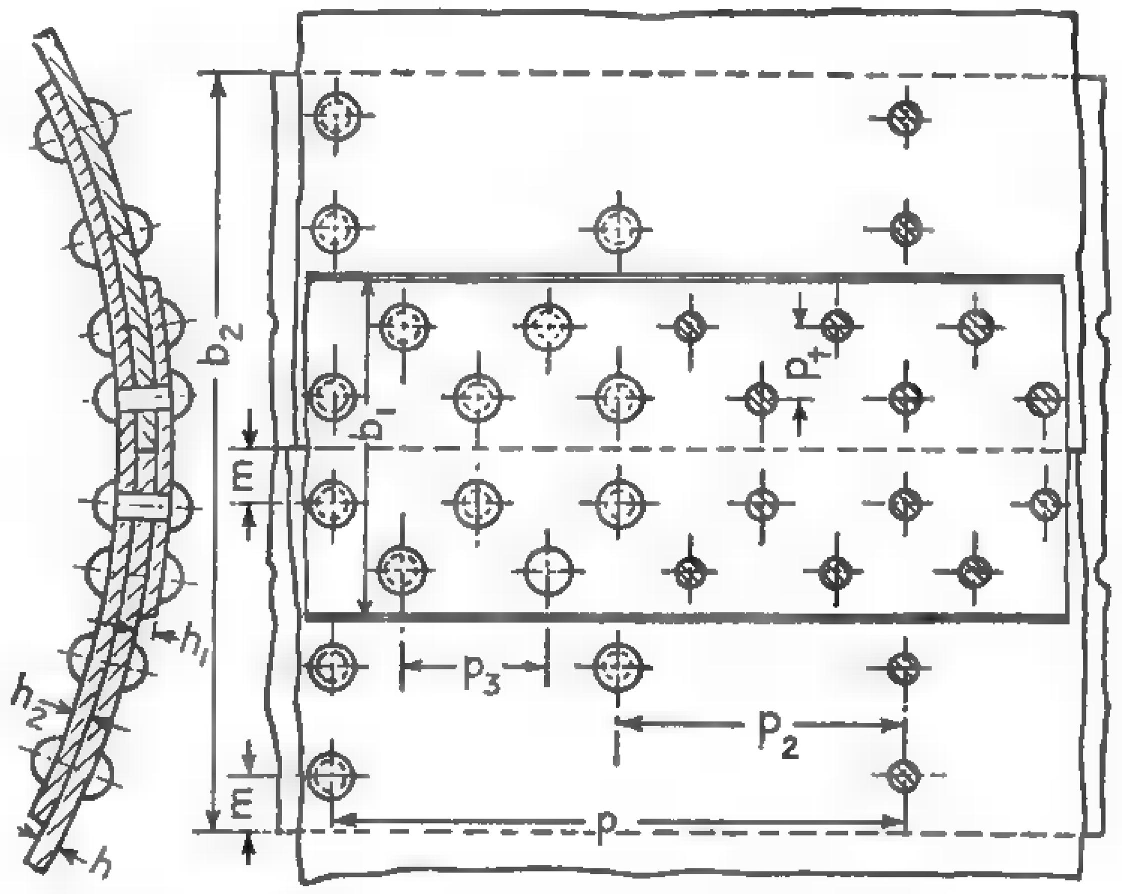


FIG. 9-9. Quadruple-riveted double-strap butt joint.

The tensile strength of the solid plate is

$$F_p = phS \tag{9-4}$$

The tensile strength of the perforated strip along the outer gage line is

$$F_t = (p - d)hS \tag{9-5}$$

The general expression for the resistance to shear of all the rivets in a unit strip is

$$F_s = (2n_2 + n_1)\tfrac{1}{4}\pi d^2 S_s \tag{9-6}$$

The general expression for the resistance to crushing of the rivets is

$$F_c = (n_2h + n_1h_2)dS_c \tag{9-7}$$

where h_2 is the thickness of the wider strap.

If the joint is designed for maximum efficiency, $F_t = F_s = F_c$ and each of these three quantities divided by F_p gives the theoretical efficiency e of the joint. Thus, dividing equation 9–6 by equation 9–4 gives

$$e = \frac{(2n_2 + n_1)\pi d^2 S_s}{4phS} \tag{9-8}$$

If the values of F_t and F_s given by equations 9–5 and 9–6 are equated and the resulting equation is solved for p,

$$p = \frac{(2n_2 + n_1)\pi d^2 S_s}{4hS} + d \tag{9-9}$$

If this expression for p is substituted in equation 9–8, the result is

$$e = \frac{(2n_2 + n_1)\pi d^2 S_s}{(2n_2 + n_1)\pi d^2 S_s + 4dhS} \tag{9-10}$$

If the values of F_s and F_c given by equations 9–6 and 9–7 are equated and the equation thus obtained is solved for dh, the result is

$$dh = \frac{(2n_2 + n_1)\pi d^2 S_s}{4\left(n_2 + \frac{n_1h_2}{h}\right)S_c} \tag{9-11}$$

When this value of dh is substituted in equation 9–10 and the resulting equation is simplified, the result is

$$e = \frac{\left(n_2 + \frac{n_1h_2}{h}\right)S_c}{\left(n_2 + \frac{n_1h_2}{h}\right)S_c + S} \tag{9-12}$$

Equation 9–12 applies to any form of riveted joint and gives the limit value of efficiency for any form and material. The actual proportions adopted may give a lower efficiency, but they can never give a higher efficiency. It is interesting to note that the resistance in shear does not affect the theoretical efficiency.

In a multirow joint, if one of the resistances F_t, F_s, or F_c is not sufficiently high, failure may occur through combined action, instead of in one of the simple ways. For example, tearing of the plate in Fig. 9–9 between the rivet holes in the second row with the pitch p_2 may be combined with failure of the rivets in the first row. These rivets may fail either by shearing or by crushing. For the first case the permissible load is

$$F_{s1} = (p - 2d)hS + 0.7854d^2S_s \tag{9-13}$$

TABLE 9–1

EFFICIENCY OF RIVETED JOINTS

Type of Joint		Efficiency (per cent) Normal Range	Maximum
Lap joints	Single-riveted	50–60	63
	Double-riveted	60–72	77
Butt joints with two cover plates	Double-riveted	76–84	87
	Triple-riveted	80–88	95
	Quadruple-riveted	86–94	98

For the second case the permissible load is

$$F_{c1} = (p - 2d)hS + dhS_c \tag{9-14}$$

Another way of failing can be through the shearing of the rivets in the outer row and the crushing of the rivets in the two inner rows. If their number in a unit is designated by n, the allowable load is

$$F_{sc} = 0.7854d^2S_s + ndhS_c \tag{9-15}$$

The efficiency of the joint will be the smallest of the values F_t, F_s, F_c, F_{s1}, F_{c1}, or F_{sc} divided by F_p. If the efficiency thus determined happens to be lower than the value assumed before, it should be raised by changing the pitch or the size of the rivets or by changing their distribution.

Table 9–1 gives the ranges of values of the efficiency for the various types of joints used in boiler work and may serve as a guide in making the necessary assumptions when starting to design a certain joint.

Actual stresses. It is most difficult to evaluate the straining actions to which the different elements of a riveted joint are subjected, because of the elasticity of the parts through which the loads are transferred and the unavoidable occurrence of bending stresses. This is particularly true for multiriveted joints. Tests show[4] that while actual efficiencies of lap joints are 1 to 3 per cent higher than the theoretical values, the actual efficiencies of double-strap butt joints are from 2 to 12 per cent lower than the theoretical values.[5] Furthermore, the method of riveting—by hand or by a machine—and the workmanship have a great influence upon the actual efficiency of a riveted joint. To take care of the unknown factors a sufficiently large factor of safety, as 2.5 to 3.0 referred to the elastic limit, is used.

9–5. Boiler and tank joints. The stress set up by the pressure on the ends of a circumferential section or joint of a boiler shell is only half as great

[4] J. Considère, *European Experiments on Riveted Joints*, Bulletin No. 62, American Railway Engineering Association (Chicago: April, 1905), p. 149.

[5] W. M. Wilson, J. Mather, and C. O. Harris, *Tests of Joints in Wide Engineering Plates*, Bulletin No. 239, University of Illinois Engineering Experiment Station (November, 1931), p. 44.

as the stress set up by the radial pressure in a longitudinal joint. Therefore a circumferential, or girth, joint does not need to be more than half as strong as a longitudinal joint. Even single-riveted lap joints, when properly designed, have an efficiency over 50 per cent. Therefore single-riveted or double-riveted lap joints, being simpler and cheaper to make than butt joints, should be used for girth joints. For longitudinal joints the more efficient although more expensive double-strap butt joints are more suitable.

Rivet diameter. The diameter of the rivets in a boiler shell or a tank should be between $1.2\sqrt{h}$ and $1.4\sqrt{h}$, where h is the thickness of the plate.

Pitch. Equating the values of F_t and F_s given by equations 9–5 and 9–6, and solving for p, gives

$$p=\frac{0.7854(2n_2+n_1)d^2S_s}{hS}+d \tag{9–16}$$

In a lap joint the pitch is taken as a multiple of $\frac{1}{4}$ in. In a quadruple-riveted butt joint the long pitch p_1, Fig. 9–9, is taken as a multiple of $\frac{1}{2}$ in., so that the middle pitch p_2 will be a multiple of $\frac{1}{4}$ in. and the short pitch p_3 will be a multiple of $\frac{1}{8}$ in. In order to obtain a tight joint that is suitable for calking, the pitch p_2 should not be more than $8h$.

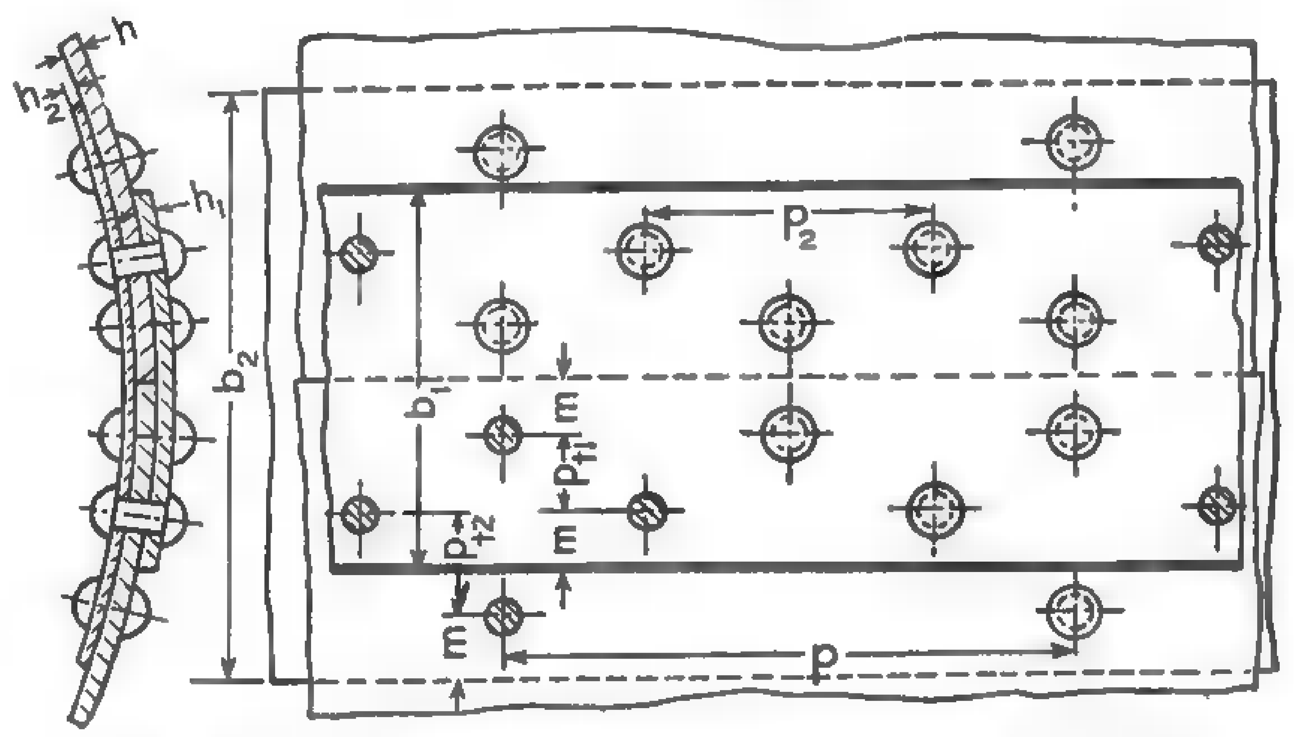


FIG. 9–10. Triple-riveted double-strap butt joint.

Cover plates. When only one cover plate, or strap, is used, its thickness should be at least equal to that of the main plates. A butt joint with a single strap is equivalent to two lap joints and is used mainly where a smooth surface is desired. When double straps are used, each should be slightly thicker than half the thickness of the main plate, about $\frac{5}{8}h$ to $\frac{3}{4}h$. Where the outer strap is narrower than the inner one, as in Fig. 9–10, the narrower strap is often made of the same thickness as the main plates, and the thickness of the inner one should be from $\frac{5}{8}h$ to $\frac{3}{4}h$.

Calking. Each joint is made tight against leakage by calking—hammering the edge of the plate and rivet into contact with the plate underneath it by blows on the calking tool. The edge of the plate must be beveled as shown in Fig. 9–6. Calking cannot be applied to plates less than $\frac{1}{4}$ in. thick.

Joining of plates. The form of the joint at the intersection of a longitudinal joint and a circumferential joint may be as shown in Fig. 9–11.

The methods of joining plates at right angles are shown in Fig. 9–12. The types in Fig. 9–12a, b, and c are used mostly for cylindrical vessels; that in Fig. 9–12d is used for connecting flat plates.

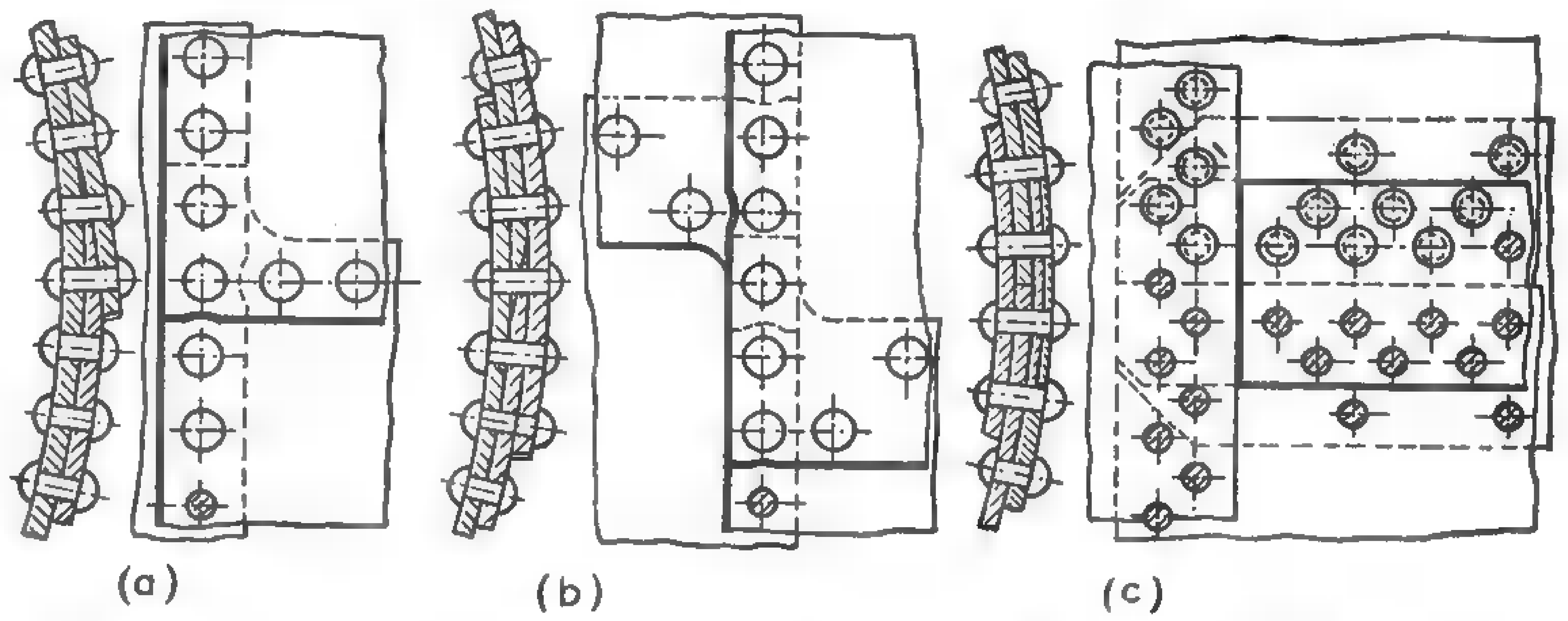


FIG. 9–11. Forms of riveted joints at junction of three or four plates.

Materials. The ASME Boiler Construction Code specifies definite physical properties for the materials used and a safety factor $n_u = 5$, based on ultimate strength. Wrought iron being practically no longer used, the allowable stresses are commonly assumed as 11,000 psi in tension, 19,000 psi in crushing, and 8,800 psi in shear, for both steel plates and rivets. These values correspond approximately to data of Table 4–2 when the factor of safety n is taken as 3 and when SAE 1010 steel is used for the rivets and SAE 1020 steel is used for the plates. With such low allowable stresses, since the materials of the rivets and plates are very ductile and the loads are fairly constant, the influence of stress concentration from the holes can be neglected. For grade A alloy steel the ASME Code allows the use of a crushing strength of 120,000 psi and a shear strength of 60,000 psi, which correspond to allowable stresses of 24,000 and 12,000 psi respectively.

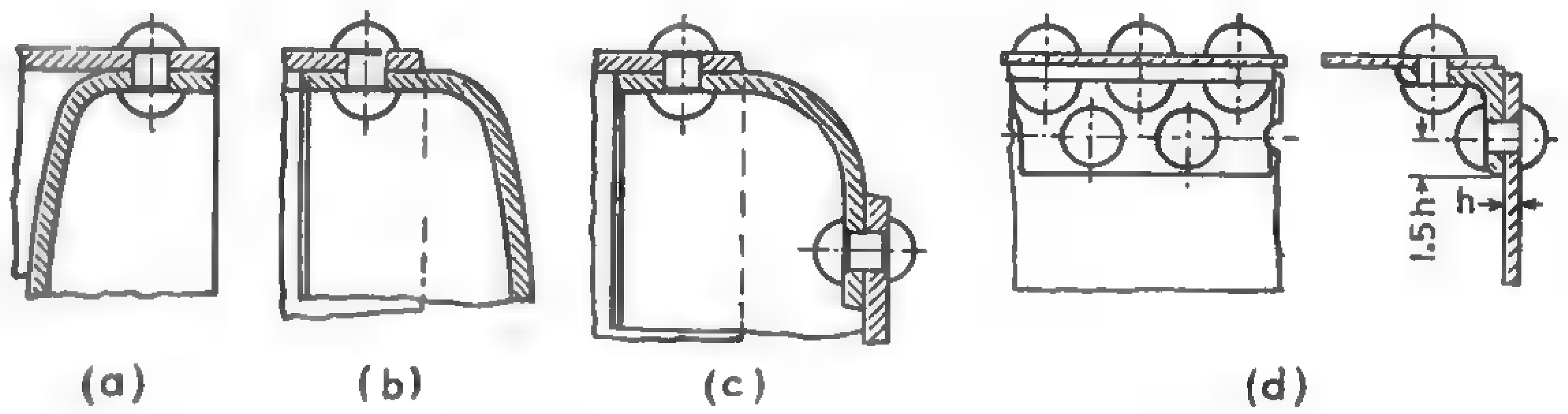


FIG. 9–12. Joining plates at right angles.

9–6. Design procedure for vessels. The first step in designing a cylindrical vessel is to select the type of longitudinal joint to be used, the choice depending on the diameter D of the vessel and the internal, or fluid, pressure p_f. The ASME Boiler Code permits lap joints for power boilers 36 in. or less in diameter, provided the steam pressure does not exceed 100

psi; for unfired pressure vessels not over 24 in. in diameter with a maximum pressure of 200 psi; and for unfired pressure vessels not over 48 in. with a maximum pressure of 150 psi. In all other cases butt joints must be used.

The next step is to determine the number of rivet rows. A greater number of rows gives a higher efficiency and permits the use of thinner plates, but it increases the cost of manufacturing. The materials selected for the plates and rivets determine the allowable stresses S, S_c and S_s.

Next, the thickness h of the plates can be determined by equating the fluid pressure upon a ring 1 in. wide, or $p_f D$, to the tensile stress of the material in two sections of the ring on opposite ends of a diameter and solving the equation for h. Thus,

$$h = \frac{p_f D}{2Se} \tag{9-17}$$

With the type of joint and the number of rivet rows selected, an expected efficiency e is assumed, based on the data in Table 9–1.

Plate thicknesses commonly used vary from $\frac{1}{4}$ in. upward by $\frac{1}{16}$-in. increments, but $\frac{1}{32}$-in. increments may be obtained if necessary.

Thereafter the size of the rivets is selected so that it will be between $1.2\sqrt{h}$ and $1.4\sqrt{h}$.

Then the main pitch is determined by equation 9–16, and the efficiency of the joint is computed, all possible cases of failure being considered. This efficiency must be equal to or greater than the one assumed in computing h by equation 9–17.

Finally, the margins and transverse pitches are determined chiefly from practical considerations, as explained in section 9–3.

An example will illustrate the method of procedure.

EXAMPLE 9–1. Design the longitudinal joint of a boiler 60 in. in diameter for a working pressure of 200 psi.

It is obvious that only a double-strap butt joint can be used. In view of the pressure and diameter, no smaller than a triple-riveted joint should be selected. Suppose that the upper strap goes over only two rows of rivets on each side and every other rivet is omitted in the third row, as in Fig. 9–10, to increase the efficiency. From Table 9–1 an average efficiency $e = 0.84$ is selected. The design stresses may be taken as prescribed by the ASME Boiler Code: shear $S_s = 8{,}800$ psi, tension $S = 11{,}000$ psi, crushing $S_c = 19{,}000$ psi.

By equation 9–1, the thickness of the plates is

$$h = 200 \times 60 \div 2 \times 11{,}000 \times 0.84 = 0.649 \text{ in.}$$

The nearest larger size is $\frac{11}{16}$ in., which will be used.

The diameter of the rivets should be between $1.2\sqrt{0.688} = 1.0$ and $1.4\sqrt{0.688} = 1.16$ in. Select the nearest more-common size of $1\frac{1}{8}$ in. After the rivet is driven in, the size will be equal to the hole diameter of $1\frac{3}{16}$ in.

The pitch, computed by equation 9–16, is

$$p = \frac{0.7854(2 \times 4 + 1) \times 1.188^2 \times 8{,}800}{0.687 \times 11{,}000} + 1.188 = 11.61 + 1.188 = 12.798 \text{ in.}$$

As stated in section 9–5, the pitch p_2 should not be greater than $8h$, or $8 \times \frac{11}{16} = 5\frac{1}{2}$ in., and the corresponding value of p is $2 \times 5\frac{1}{2} = 11$ in. This pitch will be used in order to insure a tight joint. The thickness h_2 of the wider plate can be made $\frac{5}{8}h = 0.430$, or $\frac{7}{16}$ in.

The thickness h_1 of the narrow cover plate may be taken equal to $h = \frac{11}{16}$ in. With these values the strength of the solid plate, by equation 9–4, is

$$F_p = 11.0 \times 0.688 \times 11{,}000 = 83{,}300 \text{ lb}$$

By equation 9–5 the strength of the plate along the outer gage line is

$$F_t = (11.0 - 1.188) \times 0.688 \times 11{,}000 = 74{,}200 \text{ lb}$$

The resistance to shear, by equation 9–6, is

$$F_s = (2 \times 4 + 1) \times 0.7854 \times 1.188^2 \times 8{,}800 = 87{,}800 \text{ lb}$$

The resistance to crushing, by equation 9–7, is

$$F_c = (4 \times 0.688 + 1 \times 0.437) \times 1.188 \times 19{,}000 = 72{,}000 \text{ lb}$$

The resistance against failure of the plate through the second row and simultaneous shearing of the rivet in the first row, by equation 9–13, is

$$F_{s1} = (11.0 - 2 \times 1.188) \times 0.688 \times 11{,}000 + 0.7854 \times 1.188^2 \div 8{,}800 = 65{,}350 + 9{,}750 = 75{,}100 \text{ lb}$$

Similarly, the resistance F_{c1}, by equation 9–14, is

$$F_{c1} = 65{,}350 + 1.188 \times 0.688 \times 19{,}000 = 65{,}350 + 15{,}500 = 80{,}850 \text{ lb}$$

and the resistance F_{sc}, by equation 9–15, is

$$F_{sc} = 4 \times 1{,}188 \times 0.688 \times 19{,}000 + 9{,}750 = 62{,}000 + 9{,}750 = 71{,}750 \text{ lb}$$

Evidently the deciding minimum efficiency is

$$e = \frac{F_{sc}}{F_p} = \frac{71{,}750}{83{,}300} = 0.862$$

For a triple-riveted joint this efficiency is good and is higher than the assumed value of 0.84. However, it can still be improved. The simplest way to increase the efficiency is by decreasing the pitch. For instance, making $p = 10\frac{1}{2}$ in. gives

$$F_p' = 10.5 \times 0.688 \times 11{,}000 = 79{,}500 \text{ lb}$$

The other resistances are also affected by the change of p. Their new values are:

$$F_t' = (10.5 - 1.188) \times 0.688 \times 11{,}000 = 70{,}500 \text{ lb}$$
$$F_{s1}' = (10.5 - 2 \times 1.188) \times 0.688 \times 11{,}000 + 9{,}750 = 61{,}500 + 9{,}750 = 71{,}250 \text{ lb}$$
$$F_{c1}' = 61{,}500 + 15{,}500 = 77{,}000 \text{ lb}$$

Thus the new efficiency is

$$e' = \frac{F_t}{F_p'} = \frac{70{,}500}{79{,}500} = 0.887$$

The margin for a multiriveted joint is

$$m = 1.5d = 1.5 \times 1.188 = 1.783, \text{ or } 1\tfrac{13}{16} \text{ in.}$$

The inner transverse pitch is taken as

$$p_{t1} = 2 \times 1\tfrac{3}{16} = 2\tfrac{3}{8} \text{ in.}$$

The outer transverse pitch p_{t2} must be wider, and as stated in section 9–3, it is made equal to $2d$, or

$$p_{t2} = 2.75 \times 1\tfrac{3}{16} = 3.26, \text{ or } 3\tfrac{1}{4} \text{ in.}$$

From Fig. 9–10, the width of the upper cover plate is

$$b_1 = 4m + 2p_{t1} = 4 \times 1\tfrac{13}{16} + 2 \times 2\tfrac{3}{8} = 12 \text{ in.}$$

and the width of the lower cover plate is

$$b_2 = b_1 + 2p_{t2} = 12 + 2 \times 3\tfrac{1}{4} = 18\tfrac{1}{2} \text{ in.}$$

9–7. Structural joints. The design of riveted joints for structural work is the subject of a separate branch of engineering. Here it will be discussed very briefly in order to give the starting points for forms encountered in machine design.

Rivets. Structural rivets are available in sizes from $\frac{1}{2}$ in. to $1\frac{1}{4}$ in. in $\frac{1}{8}$-in. increments. The size of the rivet depends chiefly on the thickness of the connected members, but the commonly used sizes are: $\frac{1}{2}$ in. or $\frac{5}{8}$ in. for channels or built-up sections up to 6-in., and for 2-in. angles; $\frac{5}{8}$ in. for 8-in. sections, and for $2\frac{1}{2}$-in. angles; $\frac{3}{4}$ in. for sections up to 12 in., and for 3-in. angles; and $\frac{3}{4}$ in. or $\frac{7}{8}$ in. for larger sections. Rivets are driven either by machines or by power-operated hand tools—only seldom by hand hammers. The maximum size of rivets to be used with a certain structural shape is given in handbooks.[6] The rivet diameter may be approximately determined by the equation

$$d = \sqrt{2h} - \tfrac{1}{16} \text{ in.} \tag{9–18}$$

Rivet holes. Holes for rivets are either drilled or punched $\frac{1}{16}$ in. larger than the nominal rivet diameter for $d \leqq \frac{3}{4}$ in. and are made $\frac{1}{8}$ in. larger for $d \geqq \frac{7}{8}$ in. However, in all cases the computed resistance of rivets is based on their nominal size.

Spacing. Practical rules give for the pitch the limits $16h \geqq p \geqq 3d$, where h is the thickness of the thinnest plate used in the joint. An excessively large pitch prevents intimate contact between the members. Water may then collect in the crack and deteriorate the joint by rusting.

If the force is acting parallel to the edge, the distance from the center of a rivet to the edge should be $m_1 \geqq 1.5d$. If the tensile force is acting normal to the edge, the edge distance should be $m_2 \geqq 2d$.

Types of joints. The lap and butt joints used in structural work are similar to those used in pressure vessels. In addition the structural shapes are fastened to each other directly or by means of intermediate members, such as connection angles, clip angles, splice plates, or gusset plates. In Fig. 9–13 are shown two connection angles in a typical construction—the fastening of a beam to another beam or to a column. The size of connection angles have been standardized, the handbooks giving their detailed dimensions and the maximum safe load to be borne by the rivets in the parallel legs a, Fig. 9–13, as well as by the rivets in the outstanding legs b. If the standard connection angles for a given beam are not sufficiently strong, a

[6] Lionel S. Marks, ed., *Mechanical Engineers' Handbook*, 5th ed. (New York: McGraw-Hill Book Company, Inc., 1951), pp. 1558–76; *Steel Construction Manual*, 5th ed. (New York: American Institute of Steel Construction, 1951); and handbooks published by the major steel companies.

special connection must be designed with a greater number of rivets. This may be accomplished either by increasing the size of the angles or by adding a seat angle c, Fig. 9–13.

Splice plates are used to connect two members placed end to end in a straight line, as a in Fig. 9–14.

Gusset plates, as g in Fig. 9–14, are used to connect members that intersect or meet at an angle.

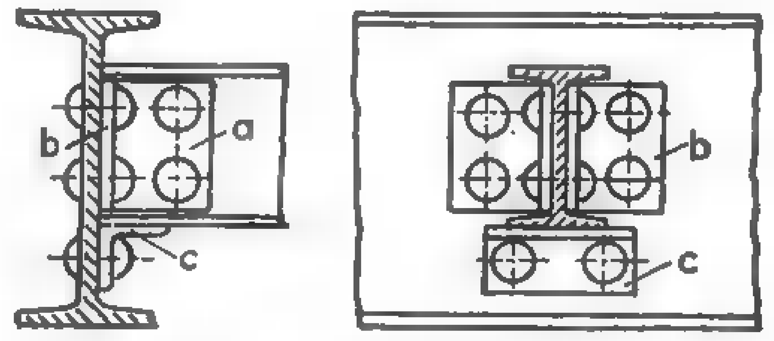


FIG. 9–13. Connection angles.

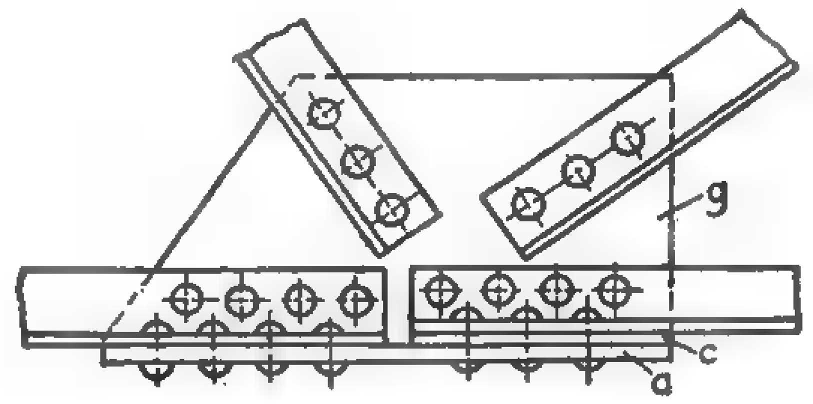


FIG. 9–14. Gusset, splice, and filler plates.

Filler plates, as c, are used to line up members of different thicknesses.

Clip angles, as i in Fig. 9–15b, are intended to secure a central loading of angle-shaped members.

Stress determination. In computing the stress in a tension member it is customary to determine the net area, by considering the size of the rivet holes as $\frac{1}{8}$ in. larger than the rivet diameter. For compression members the area of the rivet hole is not deducted from the total cross section.

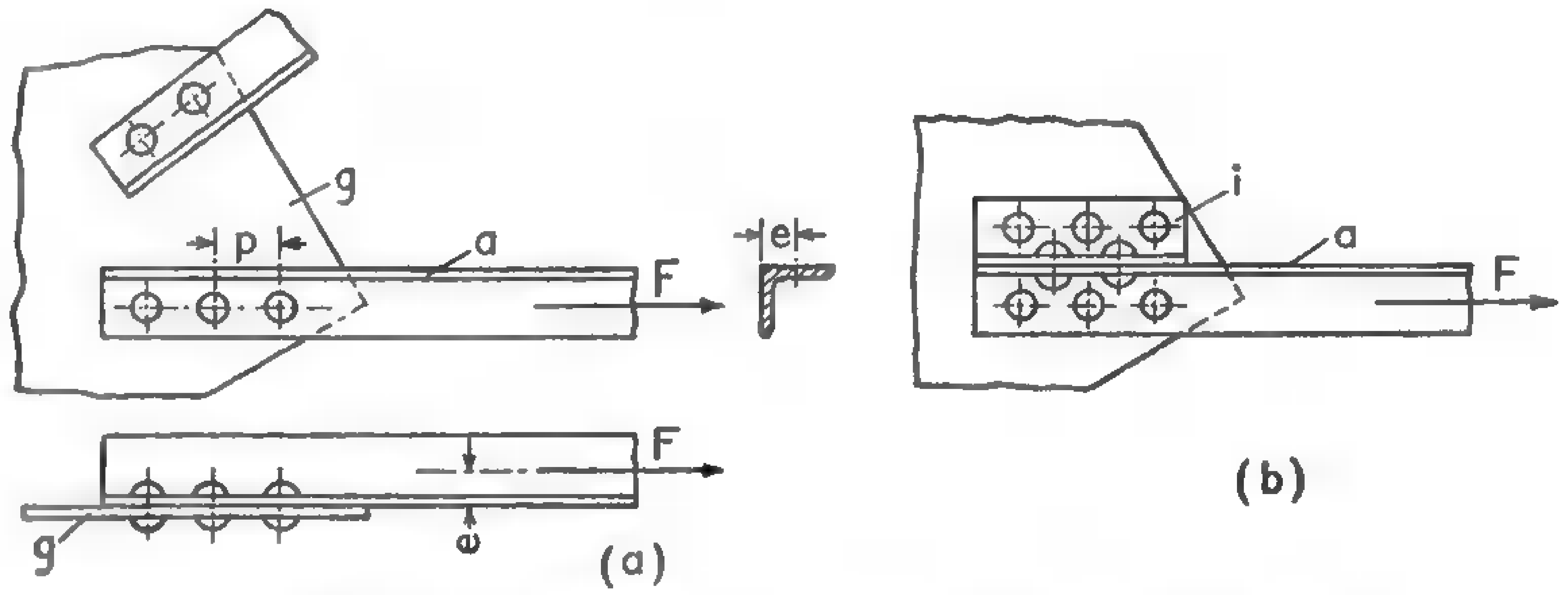


FIG. 9–15. Riveting of an angle to a gusset plate.

The connection of structural shapes to other members is often accompanied by eccentric loading, as shown in Fig. 9–14. This eccentricity increases the stress considerably over that due to central loading. The determination of the additional stress due to the bending moment does not complicate the problem a great deal and the analysis should be made as complete as possible. Thus, for the conditions in Fig. 9–15a, the total stress is

$$s = \frac{F}{A} + \frac{Fe}{Z} \tag{9–19}$$

TABLE 9–2

ALLOWABLE STRESSES IN STRUCTURAL RIVETING (PSI)

Load-carrying Member	Type of Stress	Rivet-driving Method	Rivets Acting in Single Shear	Rivets Acting in Double Shear
Rolled steel, SAE 1020	Tension	...	18,000	18,000
Rivets, SAE 1010	Shear	Power	13,500	13,500
	Shear	Hand	10,000	10,000
	Crushing	Power	24,000	30,000
	Crushing	Hand	16,000	20,000

where A is the cross-sectional area of the angle iron a, in square inches;
e is the eccentricity of the force, which may be considered to be applied at the center of gravity of the section, in inches;
Z is the section modulus of the angle iron, taken from a reference book, in. ^{3}

The allowable stresses are selected in the manner described for pressure vessels, but the safety factor is taken lower—about 2.0. The allowable stresses also depend on the method of driving the rivets. Recommended values are given in Table 9–2.

Aluminum structures. The design of aluminum riveted structures is similar to that of steel structures, the main difference being in the greater variety of rivet alloys available. The safe working stresses for the stock alloys are given in Table 9–3. In selecting the rivet material the general rule is to use the rivet with about the same properties as those of the structure to be riveted. Aluminum rivets can be obtained with the same heads as steel rivets and in sizes from $\frac{1}{4}$ in. to 1 in. in $\frac{1}{8}$-in. increments. Additional information may be found in special trade literature.[7]

TABLE 9–3

ALLOWABLE STRESSES FOR ALUMINUM RIVETS

RIVET ALLOY	PROCEDURE OF DRIVING	ALLOWABLE STRESS*	
		Shear (psi)	Bearing (psi)
2S (pure aluminum)	Cold, as received	3,000	7,000
17S	Cold, immediately after quenching	10,000	26,000
17S	Hot, 930 F to 950 F	9,000	26,000
61S–T6	Cold, [illegible] received	8,000	15,000
53S	Hot, 960 F to 980 F	6,000	15,000

* Factor of safety, 1.5.

[7] Aluminum Company of America, *Riveting of Alcoa Aluminum and Its Alloys* (Pittsburgh: 1950).

9–8. Design procedure for structural joints. The following sequence of steps applies to the calculations for structural joints:

a) The load transmitted by each member of the structure is determined analytically or graphically.

b) The shape and size of each member is determined from the magnitude of the load that it takes.

c) The diameter of the rivets is determined by the thickness of the structural shapes.

d) The number of rivets required in each member is based upon the shearing or crushing stress, whichever determines the cause of failure.

e) The rivets in the joint are spaced and arranged in such a manner as to utilize the material in the most economical way, avoiding eccentric loading as far as possible.

An example will illustrate the sequence of calculations.

EXAMPLE 9–2. Determine the size of the angle and the size and number of rivets required in the connection shown in Fig. 9–15a, if the force F acting on the member a is 12,500 lb.

Step a. This step is omitted in this case, since the load F is given.

Step b. From Table 9–2, the stress S for the angle is 18,000 psi; and for power-driven rivets, $S_s = 13{,}500$ psi and $S_c = 24{,}000$ psi. Hence, the necessary area of the cross section for axial loading is

$$A = \frac{12{,}500}{18{,}000} = 0.70 \text{ sq in.}$$

However, because of the comparatively large eccentricity of the load, a section somewhat more than twice as large will be assumed, and a $3\frac{1}{2} \times 2\frac{1}{2} \times \frac{1}{2}$ in. angle will be tried. After deduction is made for the hole for one $\frac{7}{8}$-in. rivet, the net area of this angle is 2.25 sq in. It can be assumed that the force F is applied at the center of gravity of the angle and that the reaction acts in the plane where the longer leg is in contact with the gusset plate. Therefore the eccentricity e in equation 9–19 is equal to the distance from the center of gravity of the angle to the outer surface of the longer leg. This is the distance designated as x in handbook tables and is 0.70 in. Also, the section modulus is $Z = 0.76$ in.^{3} By equation 9–19,

$$s = \frac{12{,}500}{2.25} = \frac{12{,}500 \times 0.70}{0.76} = 5{,}550 + 11{,}500 = 17{,}050 \text{ psi}$$

The stress is slightly lower than the permissible value, but the stress in a $3 \times 2\frac{1}{2} \times \frac{1}{2}$ in. angle, which is the next-smaller size, would be excessively high.

The thickness of the gusset plate may be taken slightly less than the thickness h of the angle. So $\frac{3}{8}$ in. would be a suitable value.

Step c. The rivet diameter, according to equation 9–18, is

$$d = \sqrt{2 \times \tfrac{1}{2}} - \tfrac{1}{16} = \tfrac{15}{16} \text{ in., or } \tfrac{7}{8} \text{ in.}$$

This is the maximum size recommended for the angle in the handbooks, and the size previously assumed in determining the net area of the angle.

Step d. To determine the number of rivets, it is necessary first to find out whether the rivet is stronger in shear or in crushing. In shear the resistance is

$$F_s = 0.7854 \times 0.875^2 \times 13{,}500 = 8{,}110 \text{ lb}$$

In crushing, the resistance is

$$F_c = 0.375 \times 0.875 \times 24{,}000 = 7{,}870 \text{ lb}$$

The number of rivets required is

$$n = \frac{F}{F_c} = \frac{12{,}500}{7{,}870} = 1.59, \text{ or } 2$$

Step e. The arrangement of the rivets is simple—in a straight line and as close as possible to the vertical leg of the angle, that is, to the center of gravity of the section. In accordance with the recommendations given previously, the minimum and maximum values for the spacing p are

$$p_{min} = 3 \times \tfrac{7}{8} = 2\tfrac{5}{8} \text{ in.} \qquad p_{max} = 16 \times \tfrac{3}{8} = 6 \text{ in.}$$

A spacing of $2\frac{3}{4}$ in., or 3 in., will be satisfactory.

Theoretically it would seem to be useful to connect both legs of the angle to the gusset plate by means of a clip angle, as in Fig. 9–15b, in order to obtain a more central loading of the angle a. However, tests made with such a clip-angle connection indicate that very little is gained by it, evidently because of the elastic deflection of the connection.[8]

[8] O. A. Leutwiler, *Elements of Machine Design* (New York: McGraw-Hill Book Company, Inc., 1917), p. 66.

CHAPTER 10

Design of Forgings

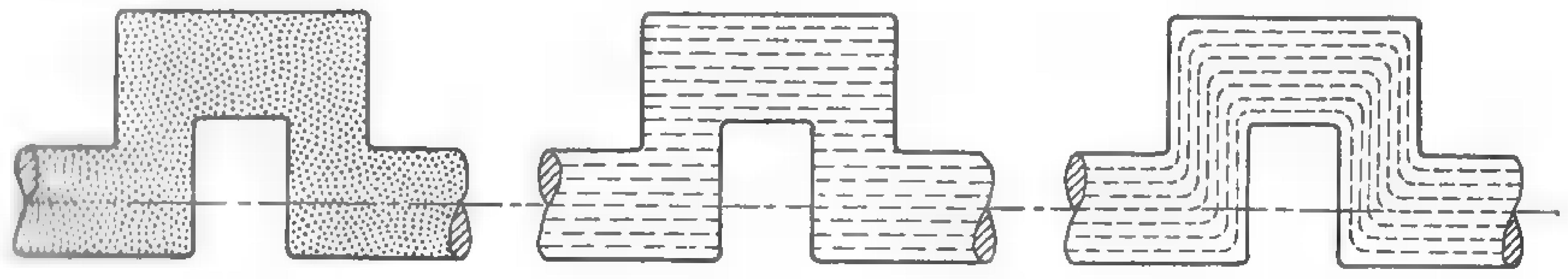
10–1. General considerations. Manufacturing machine parts by forging is more expensive than by most other methods. Nevertheless the process is used quite extensively because forged pieces have great strength and resistance to shock.

Raw material for forgings is usually in the shape of bars, plates, or billets. Before being forged the material is heated to a plastic state and is then subjected to several of the following operations: bending, slitting, cutting out, drawing down (thinning), swaging, upsetting (thickening), punching and drifting, welding, and swaying.

By a combination of these primary operations it is possible to produce parts of comparatively complicated shape. Since forging is an expensive process the designer should know what shapes and forms can be produced without undue difficulty, and should always strive to use the simplest forms in order to keep down the cost of production.

In spite of the wide use of forgings, comparatively little specific information is available for the designer. The main reason for this is probably the fact that with a certain skill the blacksmith shop can reproduce without great trouble any reasonable shape worked out by the designer.

Forging methods may be divided into the following groups: (a) hand forging, (b) machine forging, (c) die forging, or drop forging, and (d) press forging.



(a) (b) (c)

FIG. 10–1. Structure of material in a piece produced by different methods.

Fiber flow. A great advantage of producing a part by forging, rather than by some other method, is the possibility that the fiber lines in a piece may be kept unbroken. This statement may be illustrated by Fig. 10–1. In Fig. 10–1a is shown a piece made of steel by casting, the casting having a granular structure; in Fig. 10–1b is shown a similar piece produced by cutting from bar stock, in which case the fiber lines are cut; and in Fig. 10–1c is shown another similar piece produced by forging, the bar being bent hot

and the fiber lines remaining unbroken. If the pieces are made of about the same metal but by using the three different methods, then (as indicated in Fig. 10–1c) the grain slip planes in the forging are arranged in the most favorable way and the forging can withstand greater stresses, particularly those due to impact or repeated loads. This important fact must be kept in mind in designing every forging, regardless of whether it is made by hand or by any type of machinery.

10–2. Hand forging. By using proper combinations of the primary operations, a skilled blacksmith can shape a piece of metal into very intricate forms. An example of hand forging is the agitator propeller shown in Fig. 10–2. First one blade is drawn out from the side of a cylinder 7 in. long and about 14 in. in diameter. Then the opposite side is split, and the other two blades are formed by spreading and drawing them out from the split parts. Finally the hub is formed and a hole is punched in the center.

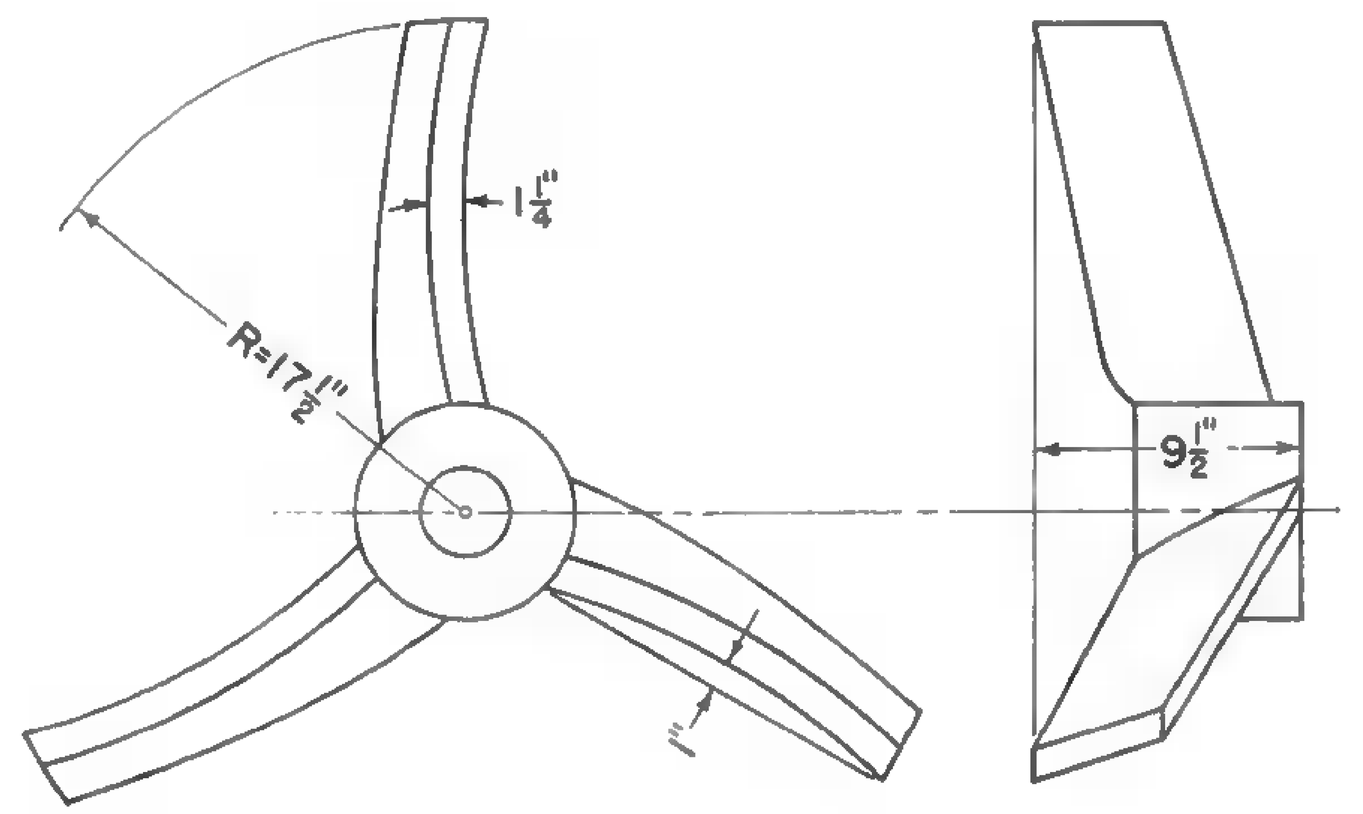


Fig. 10–2. Forged agitator propeller.

It may be interesting to know that a propeller like that in Fig. 10–2 can be produced by a less-skilled worker, and probably more cheaply, by forging the three blades separately to conform to appropriate templates and then welding them to the hub. However, the direct forging of the whole piece should be preferred if a high-carbon alloy steel is to be used, as welding gives best results only with low-carbon steel.

10–3. Machine forging. By machine forging we mean forging of large, heavy pieces by using in general the same primary operations as in hand forging, but by striking the metal with some kind of power hammer such as a steam or air hammer or a friction-operated board-drop hammer. These hammers are used for such forgings as heavy crankshafts, connecting rods, and gear blanks.

The designer of such heavy forged pieces should strive to place as much material as possible in one plane and to avoid undercut surfaces and deep

TABLE 10–1

Forging Draft Angles

Forging Machine To Be Used	External Surfaces: Normal Angle (deg)	External Surfaces: Commercial Limits (deg)	Internal Surfaces: Normal Angle (deg)	Internal Surfaces: Commercial Limits (deg)	Remarks
Board drop hammer	7	0–10	10	0–13	Modern machines permit reduction to 5 deg
Steam drop hammer	5½	0–8	7	0–10	Modern machines permit reduction to 2 deg
Upsetter	3	0–4	5	0–6	Draft required only on deep dies
Press	0–3	...	0–3	...	No draft needed if knock-out pins eject part

recesses. Deep recesses can be obtained, if actually necessary, but they increase the cost of production very much.

10–4. Die forging. Die forging is often called *drop forging*, the selection of the name depending on whether the speaker desires to emphasize the use of dies or the method of applying the energy that forces the metal to assume the form determined by the shape of the dies.

Before deciding to use die forging the designer should take into account the fact that because the dies must be made of steel or steel castings, and also must be heat-treated, they are very expensive. Moreover, most die forgings require a series of dies, usually three sets, for a gradual change of the formless piece of steel into an accurately shaped machine part; therefore die forging is economical only in mass production, that is, when the cost of dies is absorbed by a large number of forged pieces.

Die forgings should be so designed as to (a) permit the use of cheaper, longer-lasting dies; (b) obtain stronger forgings; and (c) permit subsequent machining operations to be performed efficiently.

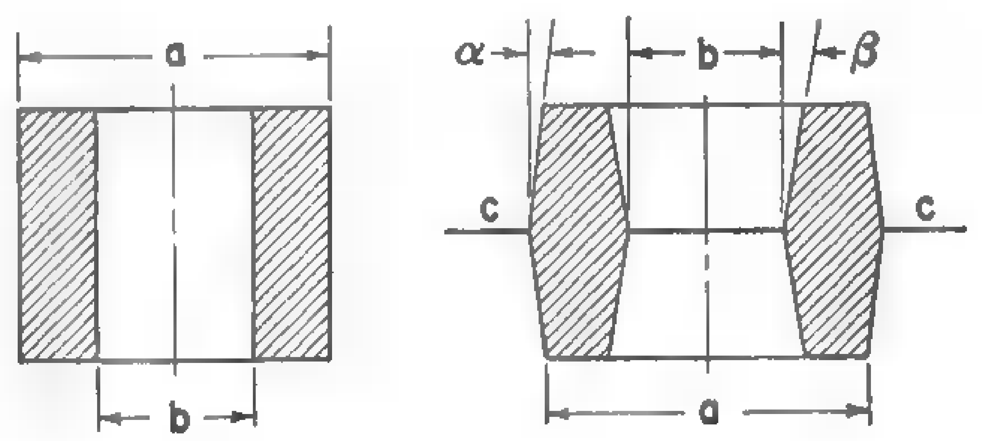
10–5. Dies. The main points to be considered on a die are the draft angles, the partings, the recesses, and the fillets.

Draft angles. The two dies shown in Fig. 10–3a and b will have practically the same weight and the same strength if the dimensions a and b are made alike. However, a die should be provided with sufficient draft angles α and β, Fig. 10–3b, as specified in Table 10–1. The draft angle β for internal surfaces must be made larger than the angle α for external surfaces because the forging, which shrinks while cooling, tends to grip the projecting part of the die.

Parting. The parting line should be a straight line; all parting surfaces preferably should be in one plane, as plane c–c in Fig. 10–3b; and the part-

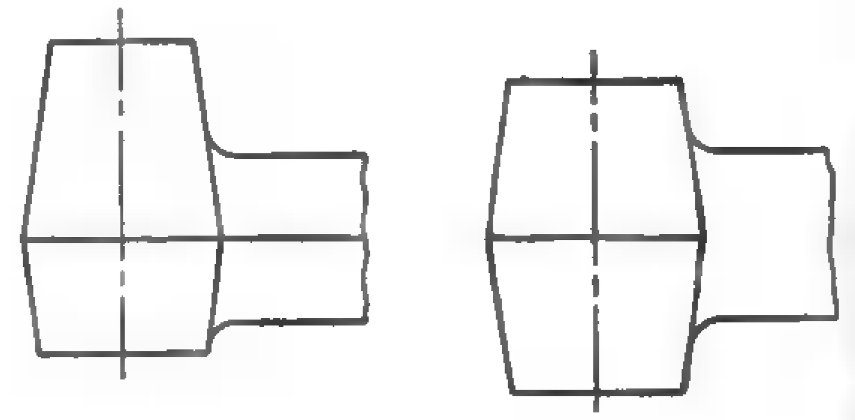
ing surfaces should divide the volume of metal into approximately two equal amounts. In this respect Fig. 10–4a shows a poor design and 10–4b shows a good design.

If a one-plane parting is not feasible, the designer should select the simplest irregular parting that will produce the required results.



(a) (b)

Fig. 10–3. Modification of a part for die forging.



(a) (b)

Fig. 10–4. Incorrect and correct parting of a die.

Recesses. Deep recesses should be avoided, and recesses should be as simple in shape as possible. The relatively sharp corners in the web recess, in Fig. 10–5a, will result in rapid wear of the die and in difficulty in removing scale; a simple recess with well-rounded ends, as in Fig. 10–5b, gives a cheaper and a longer-lasting die.

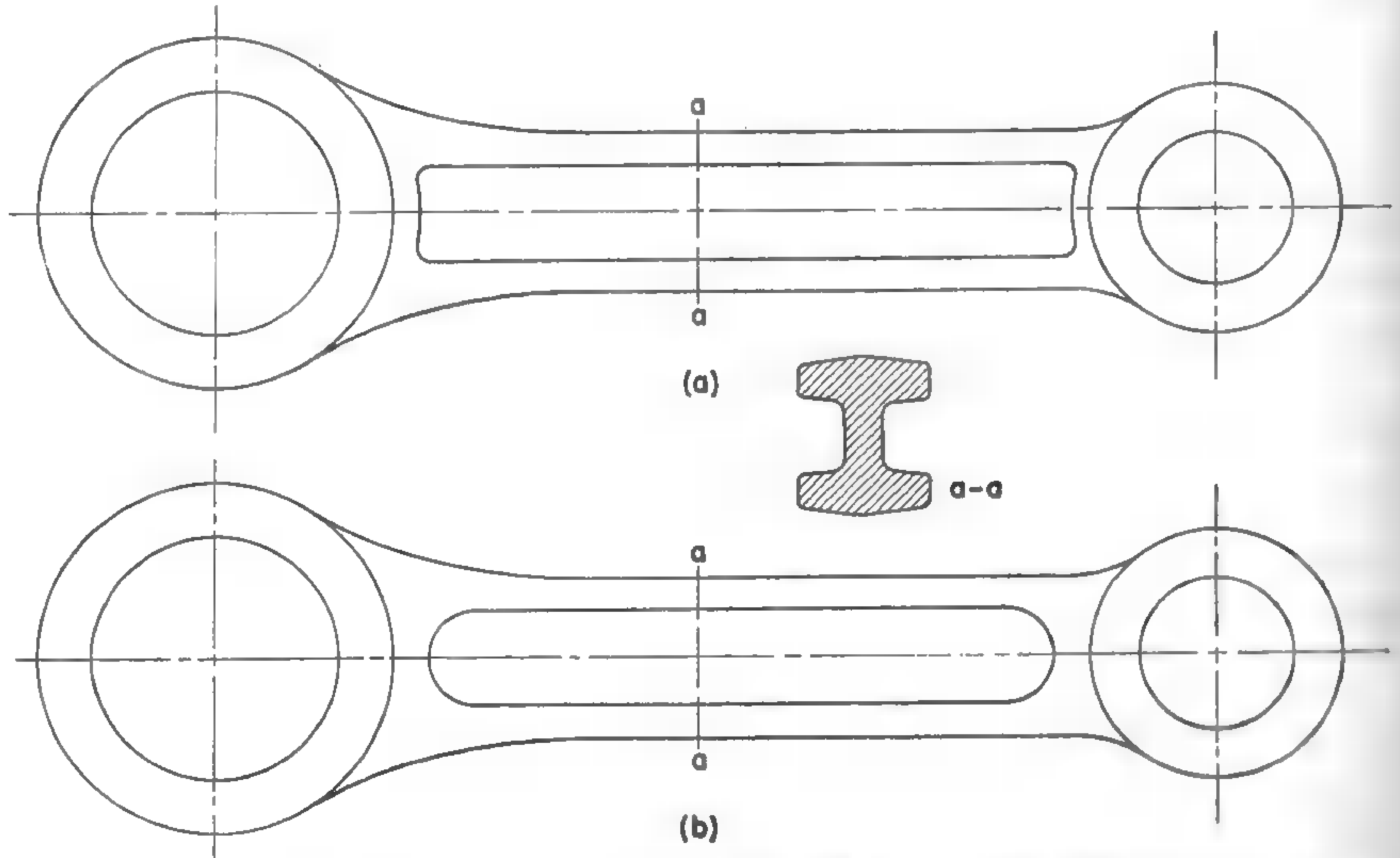
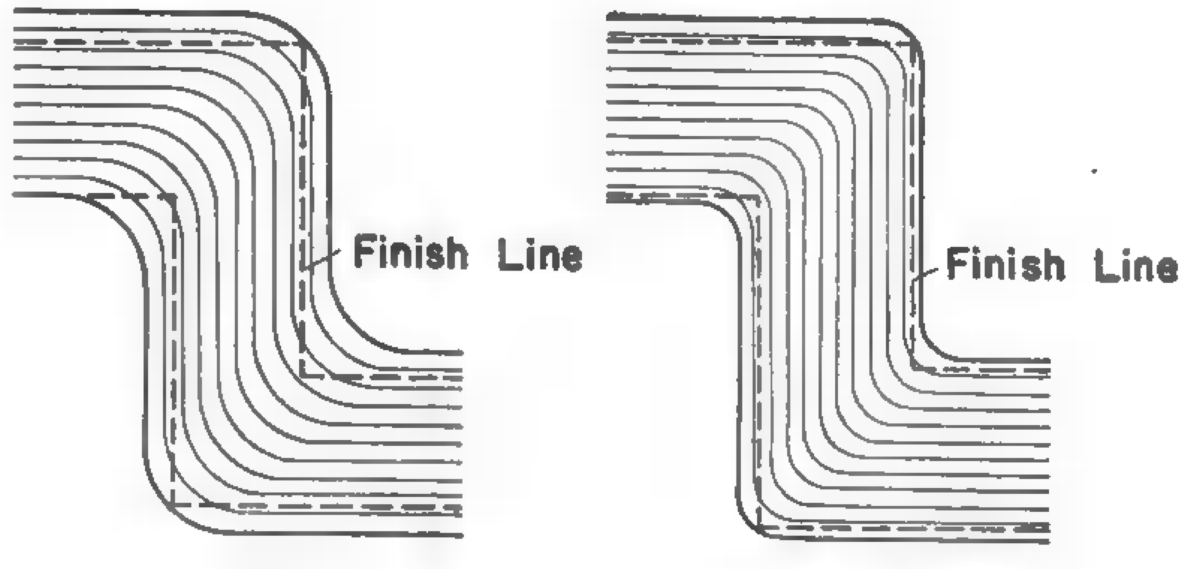


Fig. 10–5. Poor and good designs of a connecting rod.

Fillets should be provided to avoid sharp inside corners and sharp exterior edges and corners. Fillets that are too small result in die cracks and reduce the life of a die. The minimum fillet runs from $\frac{3}{16}$ in. for a small part to $\frac{1}{2}$ in. for a die forging weighing 100 lb.

However, if the forging must be machined, an excessively large fillet is also undesirable, as illustrated by Fig. 10–6. If there is a very large fillet as in Fig. 10–6a, the machined surfaces will cut many more fiber lines than when the fillet in the forging is of the same order as the fillet after machining, as shown in Fig. 10–6b.



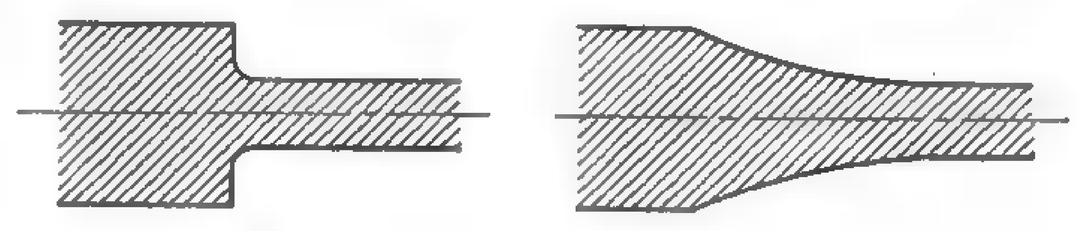
(a) (b)

Fig. 10–6. Effect of fillet size in a machined part.

Practical advice. After having completed the preliminary design of a die forging, a designer who did not specialize in this field should consult a manufacturer of die forgings, who may have valuable suggestions with respect to possible improvements. The suggestions may pertain to possible changes in the design of the dies in order to increase their life or to lower their cost or to produce a stronger or less expensive forging.

10–6. Strength of forgings. The important factors affecting the strength of a forging are its thickness and freedom from scale on its surface.

Thickness. Abrupt changes in thickness should be avoided. Since thin sections cool quicker than thicker ones, heavy shrinkage stresses are likely to be created at the places of abrupt changes. A poor design is illustrated in Fig. 10–7a, and the correct design, with a gradual change of thickness, is shown in Fig. 10–7b.



(a) (b)

Fig. 10–7. Blending of different thicknesses.

Very thin sections should be avoided in forgings because it is more difficult to forge them and to remove them from the die without damaging them. Also, quench cracks are likely to occur in thin sections during heat treatment. The minimum thickness recommended for moderate-size drop forgings is $\frac{1}{8}$ in.

Cleaning. The drawing for a drop forging should specify that the forging must be delivered free from scale. Forgings for parts that will be subjected to impact loads, and particularly to repeated loads, must be cleaned by blasting. The action of blasting is similar to that of peening and improves the endurance strength of steel parts. Forgings not subjected to impact loads may be cleaned by pickling.

10–7. Machining considerations. To allow for subsequent machining, the locating points for machining should be carefully selected, sufficient material should be provided for finishing, and drill holes should be spotted.

Locating points. The designer should give careful consideration in selecting the locating points at which a part is held during machining and should indicate them on the drawing. Locating points should be kept away from the parting line because the unavoidable wear of the edges of the dies gradually increases the thickness of the flesh, which is the metal that must be removed—usually by a shear die. The dotted lines in Fig. 10–8 show how die wear occurs at the parting line.

Machining stock. Sufficient material should be allowed for machining at all places where finishing to accurate dimensions is necessary. The allowance for machine finish ranges from $\frac{1}{32}$ in. for small hammer die forgings to $\frac{5}{16}$ in. for large drop forgings. This is not an over-all allowance but indicates the amount of metal to be removed from each machined surface. For press die forgings the allowance for machining can be much smaller, the maximum allowance being $\frac{1}{16}$ in.

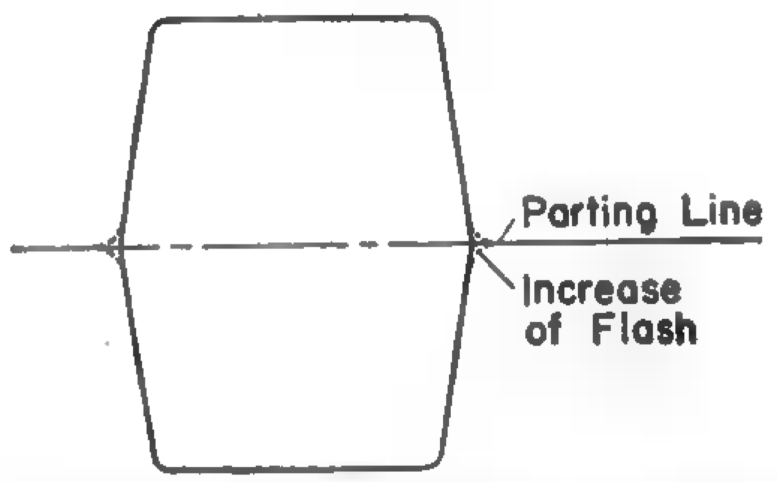


FIG. 10–8. Influence of die wear.

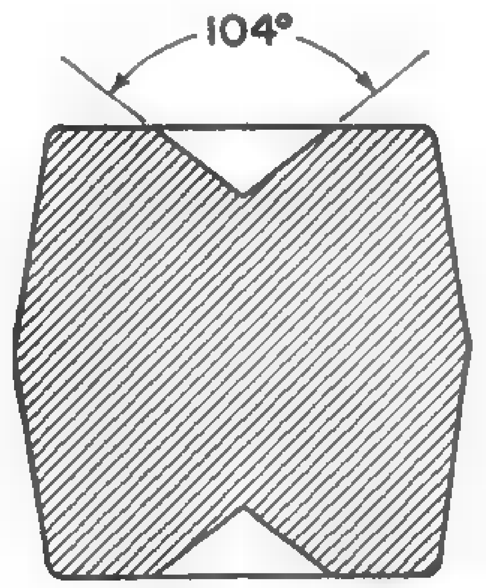


FIG. 10–9. Spotting hole in forging.

Drilling. When large holes parallel to the die motion are to be drilled in forgings, it is economical to specify that the holes be spotted by the forging dies. This reduces the amount of layout work and helps to start the drill at the proper point. Spotting by the die is used only for holes $\frac{1}{2}$ in. in diameter and larger.

Even though drilling is to be done from only one side of the forging, spots are placed on both sides of the piece, as shown in Fig. 10–9. This is done to a certain extent to obtain a more favorable fiber flow and to bring the fiber flow lines closer together. Where accurate center distances between two holes are required, only one of the holes should be die-spotted, the other being drilled from a template.

10–8. Tolerances. The general rule, explained in section 6–3, that a tolerance should never be unnecessarily close must be observed in drop-forging drawings even more strictly than in machining operations. The

Drop Forging Association has set up tolerances for thickness, length and width, draft angle, fillets, and corners. The designer should never specify tolerances closer than "close" figures given in the tables of the DFA.[1]

10–9. Press forging. In press forging the piece is shaped by the application of steady pressure rather than by repeated blows. This procedure requires special machines and is used only for mass production. The dies used are similar to those used for drop forging. However, a much smaller draft may be specified, for both external and internal surfaces. A hole whose depth is less than its diameter can be produced practically without any draft.

Press forging is better than drop forging for pieces with heavy sections because it refines the grain of the entire piece, whereas the effect of hammering does not penetrate so deep below the surface. For the shaping of pieces made up of thin sections drop forging is good.

Press forging is a relatively new process, but its use is increasing rapidly, especially in the production of nonferrous parts. By the use of several steps and retractable split-type dies, it permits the production of such complicated forgings as automobile-engine pistons made of aluminum alloys.

10–10. General conclusion. The points discussed above are only the main points that a designer should know and observe. There are a great number of minor points which are also important and which may be found in special and general literature.[2]

[1] Erik Oberg and Franklin D. Jones, *Machinery's Handbook*, 14th ed. (New York: The Industrial Press, 1953), pp. 1176–77.

[2] *Forging Handbook* (Cleveland: American Society for Metals, 1950); Herbert Chase, *Handbook on Designing for Quantity Production*, 2d ed. (New York: McGraw-Hill Book Company, Inc., 1944); *SAE Handbook;* articles in *Machine Design*, *Product Engineering*, and other periodicals; and Aluminum Company of America, *Designing for Alcoa Forgings* (Pittsburgh: 1950).

PART III: FASTENINGS

CHAPTER 11

Screw Fastenings

11–1. Unified and American screw threads. Screw fastenings are used for holding two or more machine parts together or for adjusting one part with relation to another. In screw fastenings the threads are made in several forms but are always of the triangular type, single-thread.

Screw threads are made right-hand and left-hand. Unless otherwise stated, it is always understood that the thread is right-hand.

After prolonged efforts of special committees, screw-thread standards in the United States, Canada, and Great Britain were unified in 1948. The American standard screw thread is shown in Fig. 11–1. The Unified and American screw threads standard,[1] shown in Fig. 11–2, combines the good features of the old standards of the respective countries. The thread angle of 60°, the depth of thread h, the truncation f, and the number of thread n are identical with the old American National standard, and threads made to the new and old standards are interchangeable.

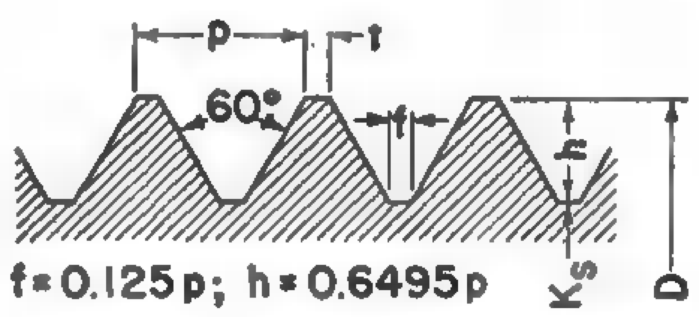


Fig. 11–1. American standard screw thread.

The Unified standard recommends the rounded root contour for the external screw thread, as was used in the British Whitworth standard, but permits a flat root when existing tools are used.

Designations used in formulas for threads are as shown in Fig. 11–1:

p = *pitch*, or the axial distance between two consecutive threads, in inches.
h = depth of the threads, in inches.
n = number of threads per inch of length.
D = *major diameter* of the thread on a screw or nut—the largest diameter of the thread, in inches.
K = *minor diameter* of the thread on a screw (K_s) or nut (K_n)—the smallest diameter of the thread, in inches.
E = *pitch diameter* of an imaginary cylinder whose surface passes through the thread profiles at such points as to make the width of the groove equal to one-half the pitch.

By definition there exists the general relation

$$p = \frac{1}{n} \tag{11–1}$$

[1] ASA B1.1–1949.

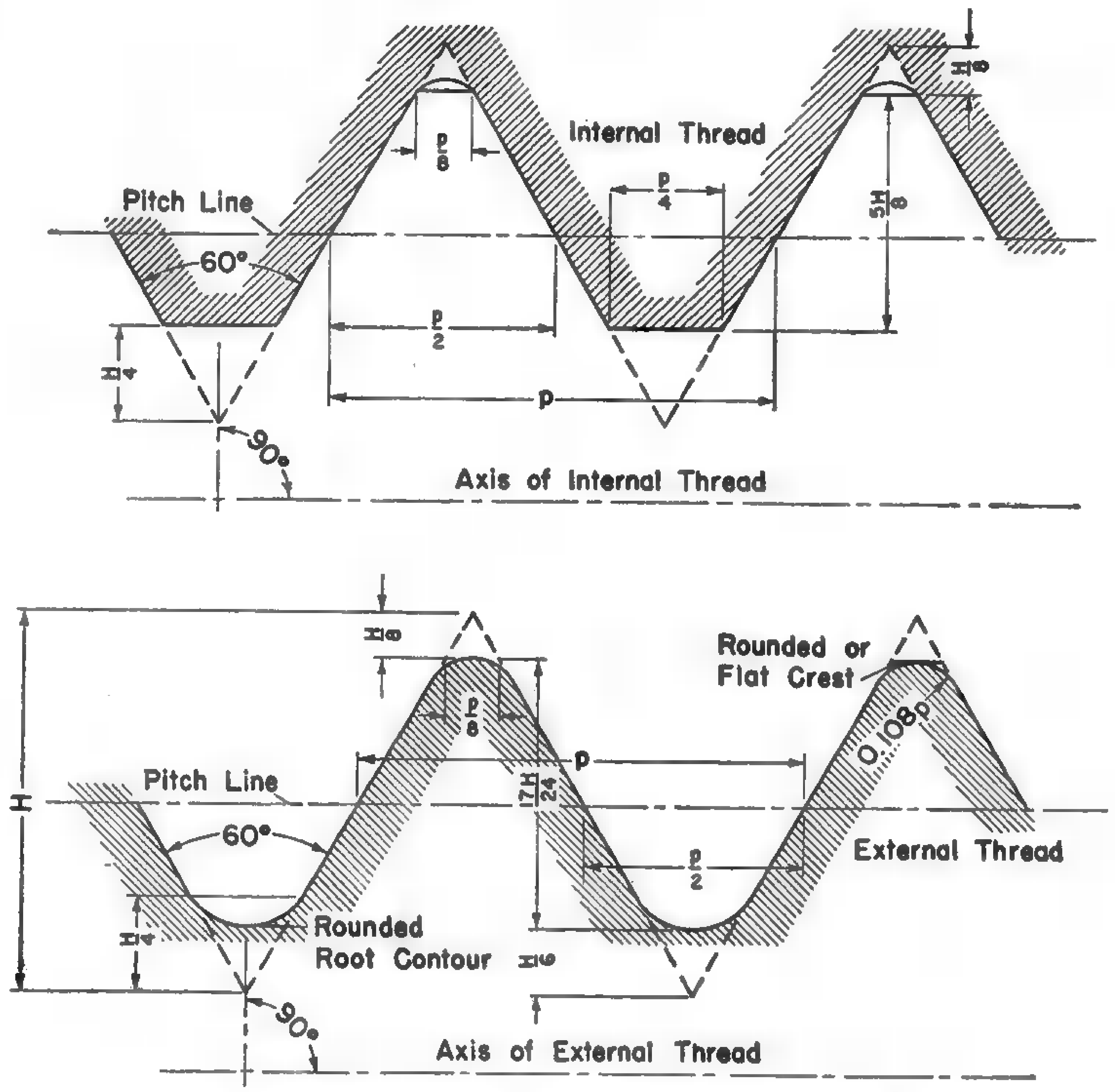


FIG. 11-2. Unified and American screw-thread forms.

The following equations apply to the Unified form of thread:
The basic depth of thread engagement is

$$h_e = 0.5417p = \frac{0.5417}{n} \tag{11-2}$$

The minor diameter of the screw thread is given by the formula

$$K_s = D - \frac{1.22687}{n} \tag{11-3}$$

The minor diameter of the nut thread is given by the formula

$$K_n = D - \frac{1.08253}{n} \tag{11-4}$$

The basic pitch diameter is given by the formula

$$E = D - \frac{0.64952}{n} \tag{11-5}$$

The stress area A_s is the assumed area of a screw and is used for the purpose of computing tensile strength. It is given by the formula

TABLE 11-1

UNIFIED AND AMERICAN COARSE-THREAD SERIES UNC AND NC

Size Number or Size (in.)	Basic Major Diameter D (in.)	Threads per Inch n	Minor-Diameter External Threads K_s (in.)	Stress Area A_s (sq in.)	Size Number or Size (in.)	Basic Major Diameter D (in.)	Threads per Inch n	Minor-Diameter External Threads K_s (in.)	Stress Area A_s (sq in.)
No. 1	0.073	64	0.0538	0.0026	$\frac{3}{4}$	0.7500	10	0.6273	0.3340
No. 2	0.086	56	0.0641	0.0036	$\frac{7}{8}$	0.8750	9	0.7387	0.4612
No. 3	0.099	48	0.0734	0.0048	1	1.0000	[illegible]	0.8466	0.6051
No. 4	0.112	40	0.0813	0.0060					
					$1\frac{1}{8}$	1.1250	7	0.9497	0.7627
No. 5	0.125	40	0.0943	0.0079	$1\frac{1}{4}$	1.2500	7	1,0747	0.9684
No. 6	0.138	32	0.0997	0.0090	$1\frac{3}{8}$	1.3750	6	1.1705	1.1538
No. 8	0.164	32	0.1257	0.0139	$1\frac{1}{2}$	1.5000	6	1.2955	1.4041
No. 10	0.190	24	0.1389	0.0174	$1\frac{3}{4}$	1.7500	5	1.5046	1.8983
No. 12	0.216	24	0.1649	0.0240					
					2	2.0000	$4\frac{1}{2}$	1.7274	2.4971
$\frac{1}{4}$	0.2500	20	0.1887	0.0317	$2\frac{1}{4}$	2.2500	$4\frac{1}{2}$	1.9774	3.2464
$\frac{5}{16}$	0.3125	18	0.2443	0.0522	$2\frac{1}{2}$	2.5000	4	2.1933	3.9976
$\frac{3}{8}$	0.3750	16	0.2983	0.0773	$2\frac{3}{4}$	2.7500	4	2.4433	4.9326
$\frac{7}{16}$	0.4375	14	0.3499	0.1060					
					3	3.0000	4	2.6933	5.9659
$\frac{1}{2}$*	0.5000	13	0.4056	0.1416	$3\frac{1}{4}$	3.2500	4	2,9433	7.0992
$\frac{1}{2}$	0.5000	12	0.3978	0.1374	$3\frac{1}{2}$	3.5000	4	3.1933	8.3268
$\frac{9}{16}$	0.5625	12	0.4603	0.1816	$3\frac{3}{4}$	3.7500	4	3.4433	9.6546
$\frac{5}{8}$	0.6250	11	0.5135	0.2256	4	4.0000	4	3.6933	11.0805

* Not included in the Unified standard.

$$A_s = 0.7854\left(\frac{E_m + K_m}{2}\right)^2 \tag{11-6}$$

This area is somewhat greater than the area corresponding to the minor diameter and takes into account the strengthening effect of the helix of the thread. In computing the stress in a screw in bending or torsion it is safer to base the calculations on the minor diameter.

Two Unified and six American standard thread series are in use.[2]

The *UNC coarse thread series*, Table 11-1, is for general use and for all materials, including materials of lower strength than steel, such as cast iron, bronze, brass, and aluminum.

The *UNF fine thread series*, Table 11-2, is for automotive and aircraft use or for use where maximum strength is required and the nut is also of steel.

The *American extra-fine thread series, NEF,* is not a unified standard. It covers threads from No. 12 to 2 in. and has four to six threads per inch more than the UNF series.

[2] Complete information is given in ASA B1.4-1949; Lionel S. Marks, ed., *Mechanical Engineers' Handbook*, 5th ed. (New York: McGraw-Hill Book Company, Inc., 1951); R. T. Kent, *Mechanical Engineers' Handbook*, 12th ed., Volume 2, *Design and Production*, ed. by Colin Carmichael (New York: John Wiley & Sons, Inc., 1950); and Erik Oberg and Franklin D. Jones, *Machinery's Handbook*, 14th ed. (New York: The Industrial Press, 1949).

TABLE 11-2

UNIFIED AND AMERICAN FINE-THREAD SERIES UNF AND NF

Size Number or Size (in.)	Basic Major Diameter D (in.)	Threads per Inch n	Minor-Diameter External Threads K_s (in.)	Stress Area A_s (sq in.)	Size Number or Size (in.)	Basic Major Diameter D (in.)	Threads per Inch n	Minor-Diameter External Threads K_s (in.)	Stress Area A_s (sq in.)
No. 0*	0.0600	80	0.0447	0.0018	$\frac{3}{8}$	0.3750	24	0.3239	0.0876
No. 1*	0.0730	72	0.0560	0.0027	$\frac{7}{16}$	0.4375	20	0.3762	0.1185
No. 2*	0.0860	64	0.0668	0.0039	$\frac{1}{2}$	0.5000	20	0.4387	0.1597
No. 3*	0.0990	56	0.0771	0.0052					
No. 4*	0.1120	48	0.0864	0.0065	$\frac{9}{16}$	0.5625	18	0.4943	0.2026
					$\frac{5}{8}$	0.6250	18	0.5568	0.2555
No. 5*	0.1250	44	0.0971	0.0082	$\frac{3}{4}$	0.7500	16	0.6733	0.3724
No. 6*	0.1380	40	0.1073	0.0101	$\frac{7}{8}$	0.8750	14	0.7874	0.5088
No. 8*	0.1640	36	0.1299	0.0146					
No. 10*	0.1900	32	0.1517	0.0199	1	1.0000	12	0.8978	0.6624
No. 12*	0.2160	28	0.1722	0.0257	$1\frac{1}{8}$	1.1250	12	1.0228	0.8549
					$1\frac{1}{4}$	1.2500	12	1.1478	1.0721
$\frac{1}{4}$	0.2500	28	0.2062	0.0326	$1\frac{3}{8}$	1.3750	12	1.2728	1.3137
$\frac{5}{16}$	0.3315	24	0.2614	0.0579	$1\frac{1}{2}$	1.5000	12	1.3978	1.5799

* Not included in the Unified standard.

The *American eight-thread series, 8N,* in which $p = \frac{1}{8}$ in., is for threads from 1 in. to 6 in. It is used for fastenings where an initial tension must be set up by elastic deformation of the bolt and of the parts that it holds together. Examples are bolts for high-pressure pipe flanges, and studs for engine cylinder heads.

The *American twelve-thread series, 12N,* in which $p = \frac{1}{12}$ in., is for threads from $\frac{1}{2}$ in. to 6 in. It is used for thin nuts on shafts and sleeves. In diameters larger than $1\frac{1}{2}$ in. it serves as a continuation of the fine thread series, UNF.

The *American sixteen-thread series, 16N,* in which $p = \frac{1}{16}$ in., is for threads from $\frac{3}{4}$ in. to 6 in. It is used chiefly for details where very fine close adjustment is required, such as threaded adjusting collars and retaining nuts.

11–2. Fits. The following three main classes of screw-thread fits are provided by the standards:

Class 1 fit is recommended only for screw-thread work where a substantial clearance between the screw and nut is required for rapid assembly and where shake or play is not objectionable. *Class 1A* tolerances and allowances for external threads and *class 1B* tolerances and allowances for internal threads are established for UNC and UNF thread series.

Class 2 fit represents a high quality of commercial screw-thread work, and *class 2A* and *class 2B* tolerances are established for UNC, UNF, 8N, and 12N thread series.

Class 3 fit represents an exceptionally high grade of commercially threaded product and is recommended only in cases where the high cost of precision tools and continual checking of tools and products is justified. *Class 3A* and *class 3B* tolerances are established for all six standard series.

Screw-thread designation. For brevity on drawings, a designation for a screw thread includes the nominal size, the diameter in inches (or the number, for sizes under $\frac{1}{4}$ in.), the number of threads per inch, the initial letters of the thread series, and the screw-fit class. For example, the designation $\frac{1}{2}$–13 UNC–2A indicates a $\frac{1}{2}$-in. screw with 13 threads per inch which is of the UNC coarse-thread series and which is made to conform to class 2A tolerances and allowances. Unless the designation just described is followed by the letters LH, which stand for left-hand, the screw is understood to be right-hand.

11–3. Pipe threads. *American standard taper pipe threads* are used in the United States for pipes and fittings. The thread is formed with a taper of $\frac{3}{4}$ in. per foot in order to make a tight joint. The threads have slightly flattened roots and crests for a distance L_2, Fig. 11–3, which includes two threads with slightly imperfect crests. Following these threads are three or four that are imperfect because of the leads of the die. The number n of threads per inch, the outside diameter D, and the length L_1 of normal engagement when the part is turned by hand are given in Table 11–3. The pitch p is determined by equation 11–1, and the other dimensions are as follows:

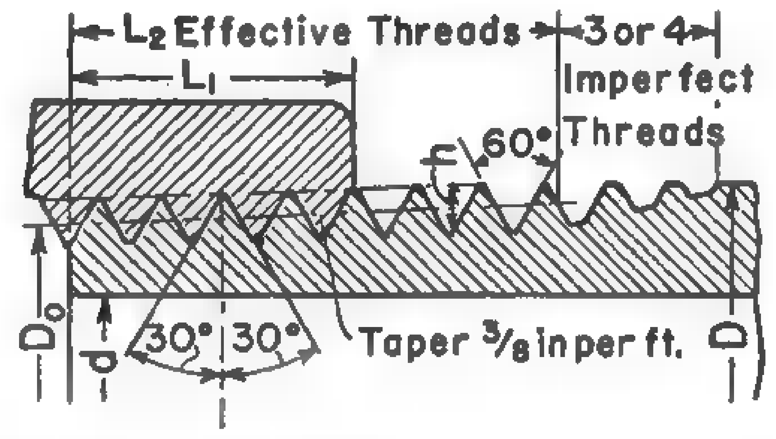


FIG. 11–3. American Standard taper pipe thread.

The depth of thread is

$$h = 0.8p \tag{11-7}$$

The pitch diameter at the end is

$$D_o = D - (0.05D + 1.1)p \tag{11-8}$$

The length of the effective threads is

$$L_2 = (0.8D + 6.8)p \tag{11-9}$$

American standard straight pipe threads are cut with a pitch diameter given by the relation

$$D_1 = D - (0.05D + 1.1)p + \tfrac{1}{16}L_1 \tag{11-10}$$

where the length L_1 is given in Table 11–3.

11–4. Types of screw fastenings. Screw fastenings may be classified as follows: (a) through bolts, (b) stud bolts, (c) tap bolts and cap screws, (d) machine screws, (e) setscrews, (f) special screws, and (g) special details.

TABLE 11–3

ASA Taper Pipe Threads

Pipe Diameter			Number of Threads per Inch	Length of Normal Engagement	Pipe Diameter			Number of Threads per Inch	Length of Normal Engagement
Inside		Actual Outside			Inside		Actual Outside		
Nominal d (in.)	Actual d_1 (in.)	D (in.)	n	L_1 (in.)	Nominal d (in.)	Actual d_1 (in.)	D (in.)	n	L_1 (in.)
$\frac{1}{16}$	0.177	0.313	27	0.160	$2\frac{1}{2}$	2.469	2.875	8	0.682
$\frac{1}{8}$	0.269	0.405	27	0.180	3	3.068	3.500	8	0.766
$\frac{1}{4}$	0.364	0.540	18	0.200	$3\frac{1}{2}$	3.548	4.000	8	0.821
$\frac{3}{8}$	0.493	0.675	18	0.240	4	4.026	4.500	8	0.844
$\frac{1}{2}$	0.622	0.840	14	0.320	5	5.047	5.563	8	0.937
$\frac{3}{4}$	0.824	1.050	14	0.339	6	6.065	6.625	8	0.958
1	1.049	1.315	$11\frac{1}{2}$	0.400	8	7.625	8.625	8	1.063
$1\frac{1}{4}$	1.380	1.660	$11\frac{1}{2}$	0.420	10	9.750	10.75	8	1.210
$1\frac{1}{2}$	1.610	1.900	$11\frac{1}{2}$	0.420	12	11.750	12.75	8	1.360
2	2.067	2.375	$11\frac{1}{2}$	0.436	140 OD	13.868	14.00	8	1.562

Through bolts. A through bolt, also called simply a *bolt*, is a round bar one end of which is threaded and fitted with a nut while the other end is upset to form a head. Nuts are made either hexagonal or square, as shown in Fig. 11–4a and b, which show plain through bolts. The heads also are made either hexagonal or square. Square heads and nuts are used mostly with rough bolts, as in construction work.

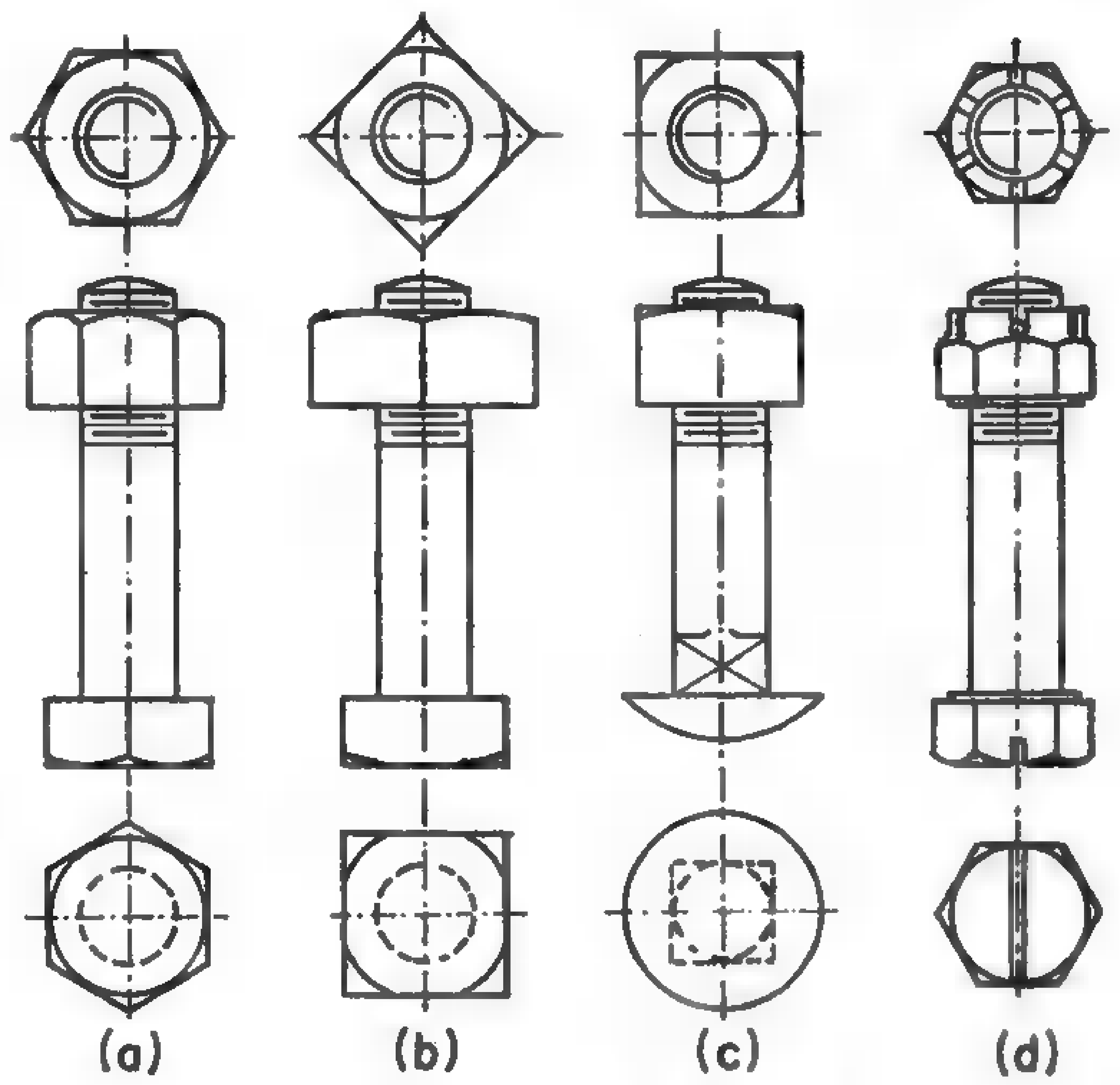


Fig. 11–4. Types of bolts.

Through bolts are the best form of screw fastening because when the nut has been tightened the shank of the bolt is subjected only to tension. If a through bolt is loaded in shear, it carries the load on the shank area.

Through bolts should be used whenever the available space and other conditions permit their use. If conditions are such that a through bolt cannot be inserted, as in a split pulley, a rod threaded from both ends with two nuts may be used, as shown in Fig. 11–5 and Fig. 11–36.

A *machine bolt* has a rough shank, but the head and the nut may be rough or finished, as desired. Commercial machine bolts are made and carried in stock in sizes from $d = \frac{1}{4}$ in. up to $d = 3$ in.

A *coupling bolt* is a machine bolt that is finished all over and is made to be fitted into a reamed hole having the same diameter as the bolt.

A *carriage bolt*, Fig. 11–4c, is used when the head must rest against wood. The part of the shank at the head is square to prevent the bolt from turning when the nut is tightened.

An *automotive bolt* is fitted with UNF threads, is finished all over, and has a somewhat smaller hexagonal head and nut. Such bolts are made in sizes from $\frac{1}{4}$ in. up to $1\frac{1}{2}$ in. and in various lengths up to 6 in. The head is often slotted for a screwdriver. Nuts of the ordinary type and of the castellated type are available. A castellated nut is shown in Fig. 11–4d.

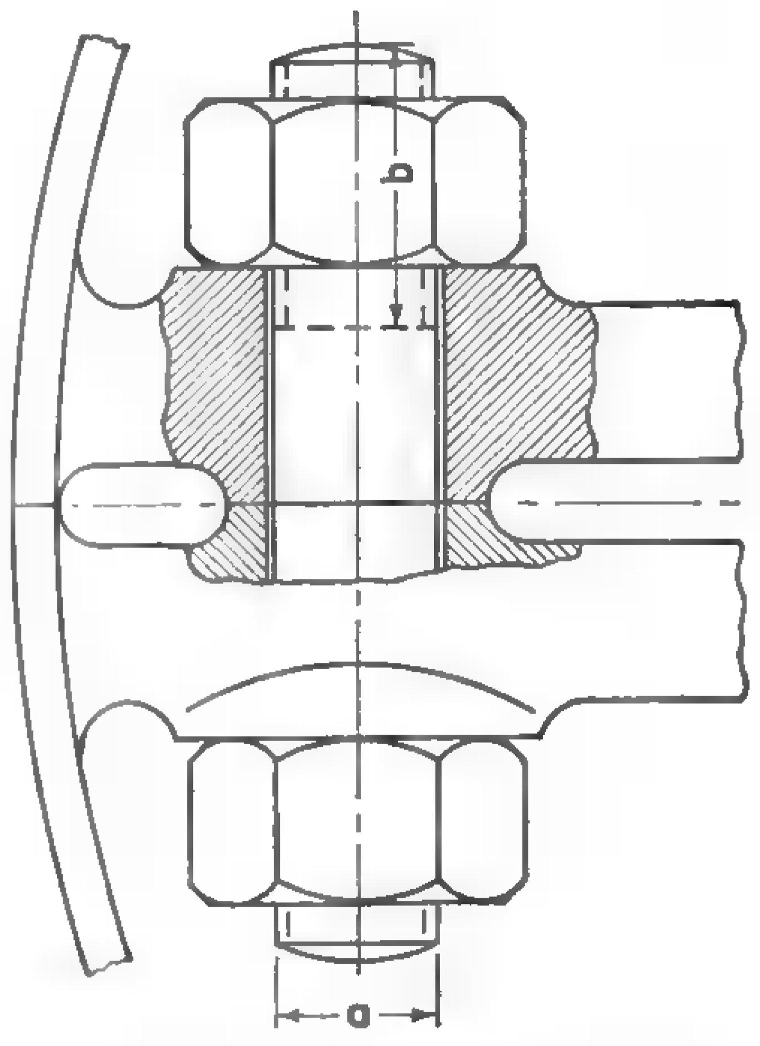


Fig. 11–5. Bolt with two nuts.

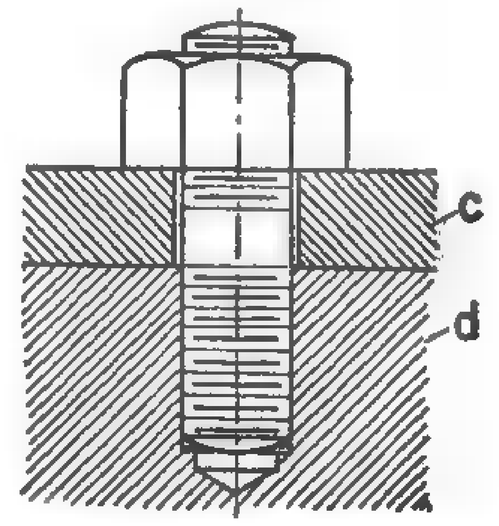


Fig. 11–6. Stud bolt.

Stud bolts. A stud bolt, often called simply a *stud*, is shown in Fig. 11–6. A stud is used when it is desired to fasten two parts c and d together, but when it is not possible or not desirable to drill a hole entirely through the second part d. To prevent unscrewing of the inner end, its thread is made to give a tight fit, while the thread at the nut end is of standard size.

There is no standard for the length of the thread on the end of a stud. However, it is advisable to make the depth of the tapped hole equal to the length of the threaded end, which should be at least equal to the diameter, in steel, and $1\frac{1}{2}$ diameters in cast iron and brass. This prevents stripping of the thread and provides enough frictional resistance against turning when the nut is unscrewed.

A stud is the next-best screw fastening if a through bolt cannot be used. Studs are particularly convenient for positioning the covers of a cylinder head.

Wrenches. Wrenches for tightening nuts and screws are usually made with two ends, the openings in which fit two adjoining sizes. If the handle is straight or inclined at 30°, then the tightening of a hexagonal nut requires a 60° swing of the wrench, as indicated in Fig. 11–7. If the handle is inclined at 15° or 45°, the nut can be tightened even where the swing is limited to 30°. The nut is first turned 30°, as shown in Fig. 11–8a. Then the wrench is taken off the nut, is reversed so that the other face is up, and is put back on the nut as in Fig. 11–8b and turned another 30°.

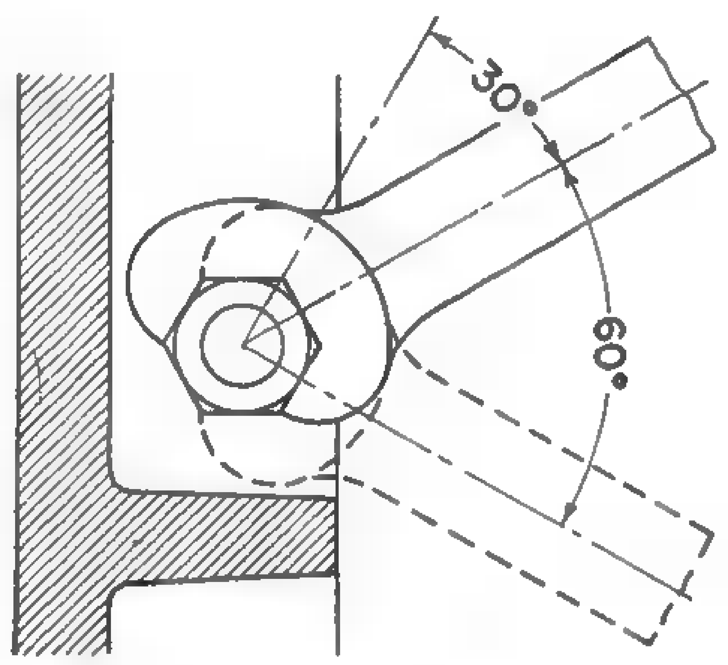


Fig. 11–7. Tightening a nut in a confined space.

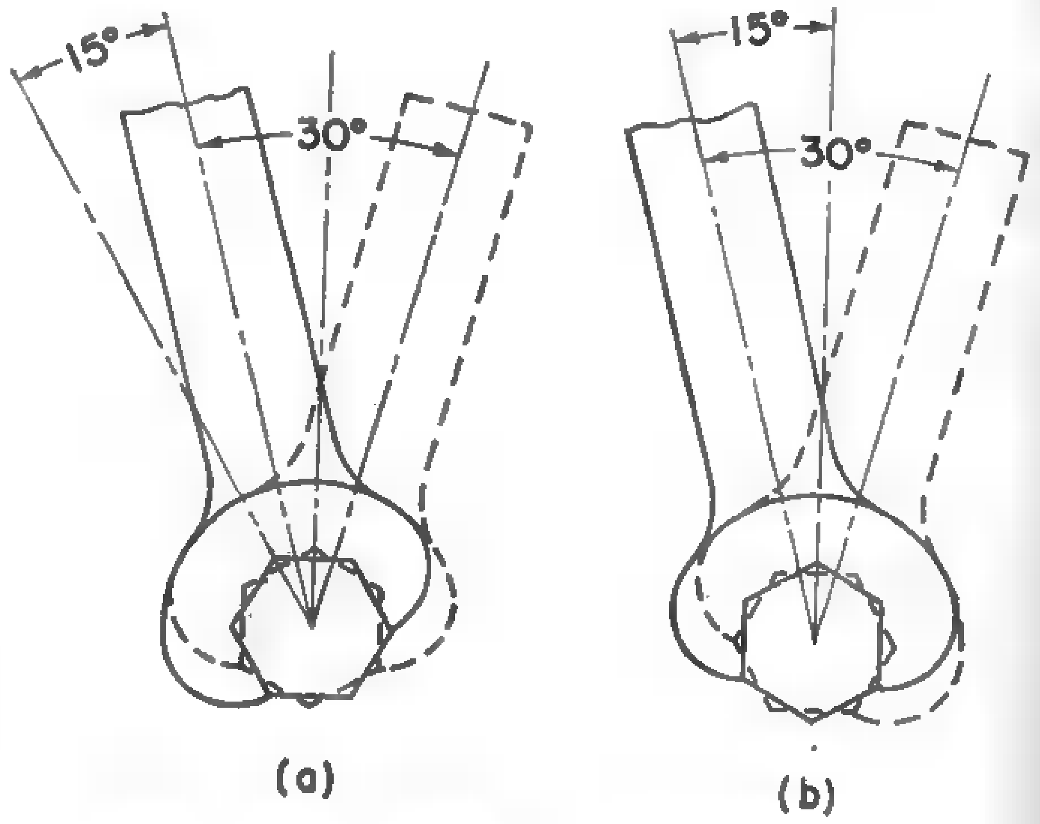


Fig. 11–8. Use of a 15-deg wrench to reduce the swing.

Steel nuts that are to be frequently unscrewed should be casehardened on the outside in order to protect them from wear by the wrench.

In locating bolts and studs in a confined place, as indicated in Fig. 11–7, the designer must remember to allow enough room for the jaws of the wrench and a sufficient angle of swing for the wrench.

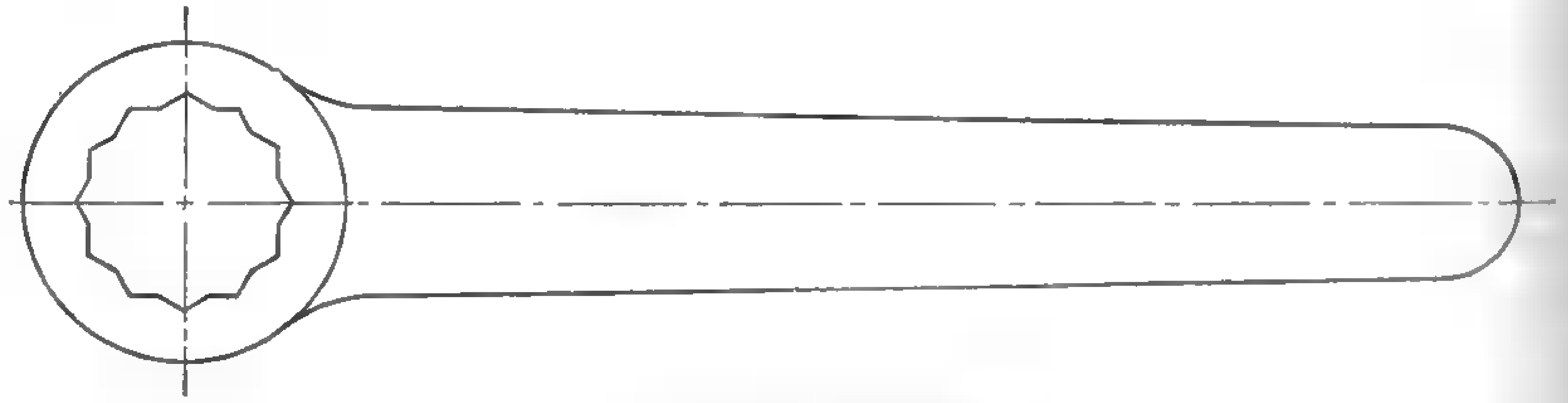


Fig. 11–9. Closed wrench.

By using a *closed wrench* with twelve cut-in corners, like that in Fig. 11–9, the nut can be brought somewhat closer to a wall or rib parallel to which the center line of the bolt or stud lies, and the necessary angle of swing can be reduced considerably. The use of a *socket wrench* helps in the same respect.

A *hook wrench*, Fig. 11–10, is used in places where there is no room for a standard open-end wrench and where a closed or socket wrench also cannot be used, such as for locknuts for ball bearings or stuffing-box glands.

The usual length of a wrench is made equal to $12D$, where D is the nominal diameter of the screw thread.

Tap bolts and cap screws. As shown in Fig. 11–11, a tap bolt is a through bolt without a nut.

Cap screws are made with various heads: hexagonal, square, socket, fillister, round, flat countersunk, and oval countersunk. Commercial cap screws are available in sizes from $\frac{1}{4}$ in. to $1\frac{1}{4}$ in. and up to 6 in. long; cap screws with screwdriver slots are made in sizes from $\frac{1}{8}$ in. up to 1 in. and in lengths up to 5 in. However, cap screws with socket heads like that in Fig. 11–15b are preferred because they can be tightened better.

Tap bolts and cap screws are cheaper than studs and hence are often used instead of them. However, if a tap bolt or cap screw is loaded in shear, it carries the load on the root area, whereas a through bolt carries a shear load on the larger shank area. Long cap screws and tap bolts should not be used, because they twist when they are tightened; and later, when they untwist, this reduces the pressure between the surfaces they hold together.

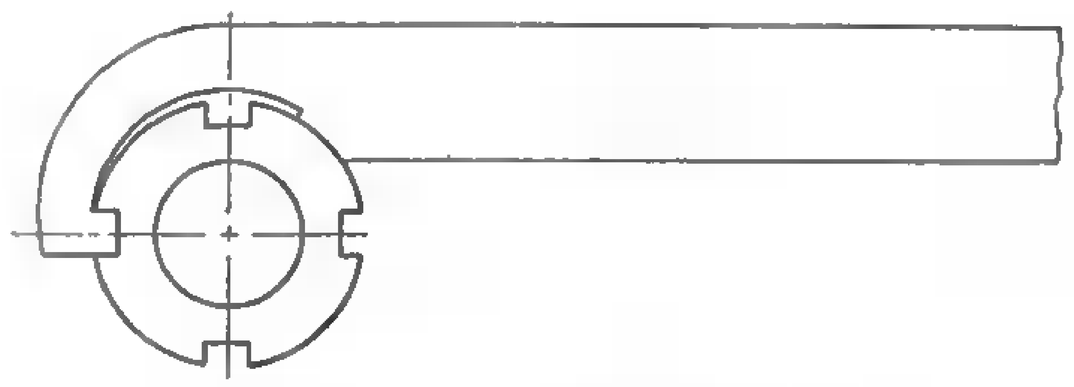


Fig. 11–10. Hook wrench.

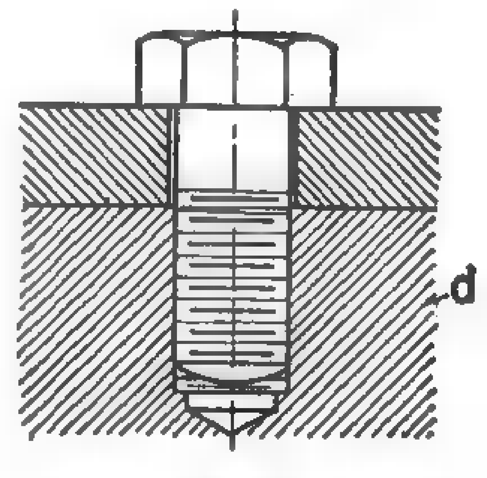


Fig. 11–11. Tap bolt.

The majority of cap screws are made with UNC threads. For screwing into steel, cap screws with UNF threads may be obtained.

Commercial cap screws are made of ordinary machine steel and various alloy steels, some having a tensile strength up to 250,000 psi. Brass and Monel-metal cap screws are also obtainable.

Properly fitted cap screws are good for parts that are seldom removed, but their use should be avoided where they have to be unscrewed frequently, unless the material of the part d, Fig. 11–11, is steel. To secure a good fastening by means of cap screws, the depth of the tapped hole should be not less than one and one-half times the diameter, in steel, and two times the diameter in cast iron.

Machine screws. Small cap screws with various types of heads, most of which have a slot for a screwdriver, are called *machine screws.* They are made with UNC and UNF threads. The sizes are designated by numbers from 0 to 12. The diameter of a No. 0 machine screw is $D = 0.060$ in., and the

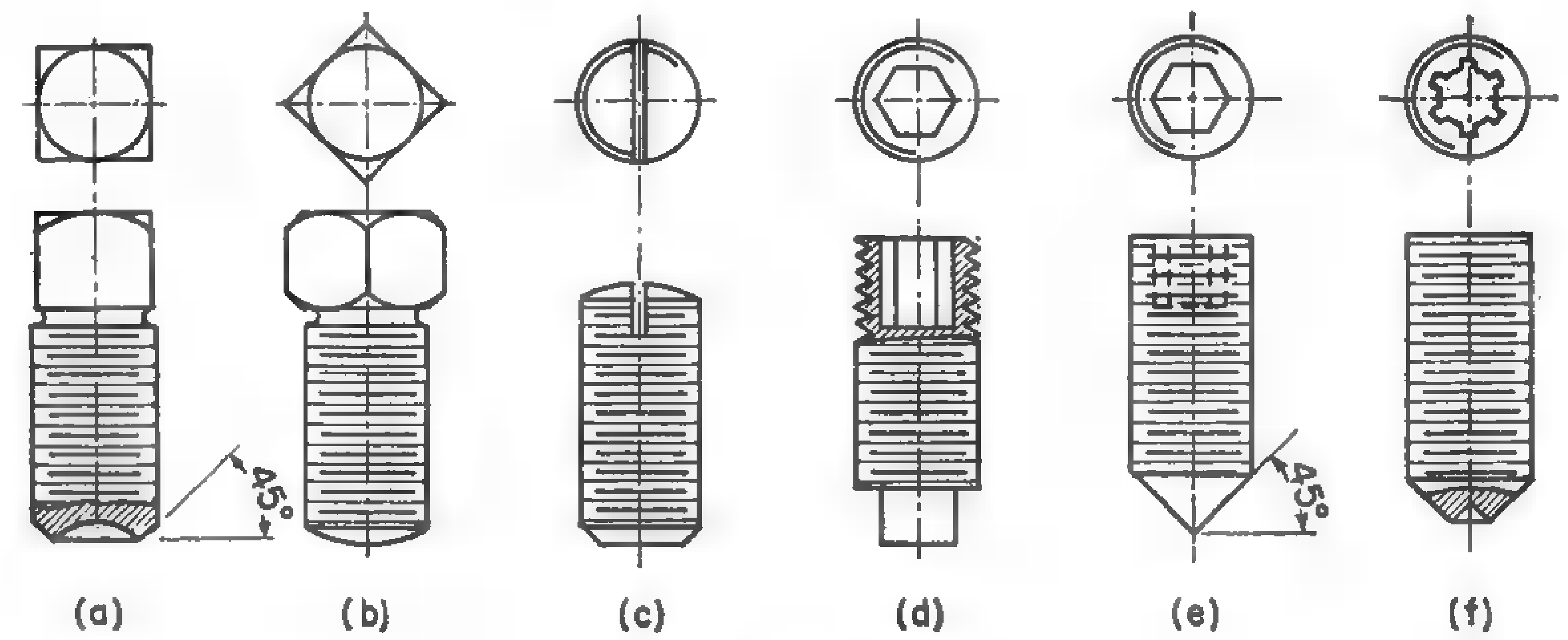


FIG. 11-12. Types of setscrews.

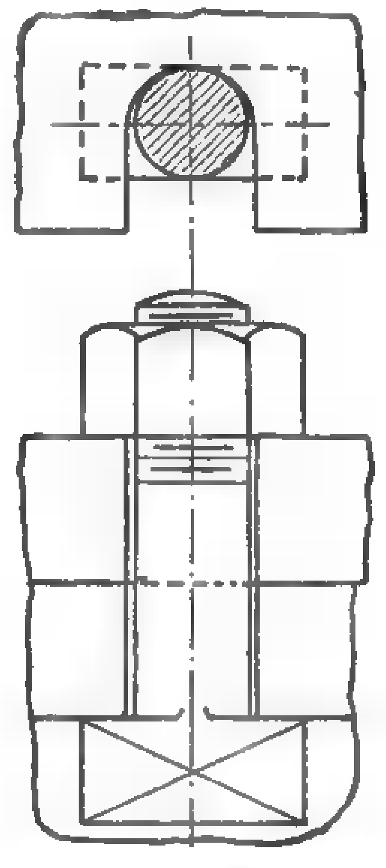


FIG. 11-13. Anchor bolt.

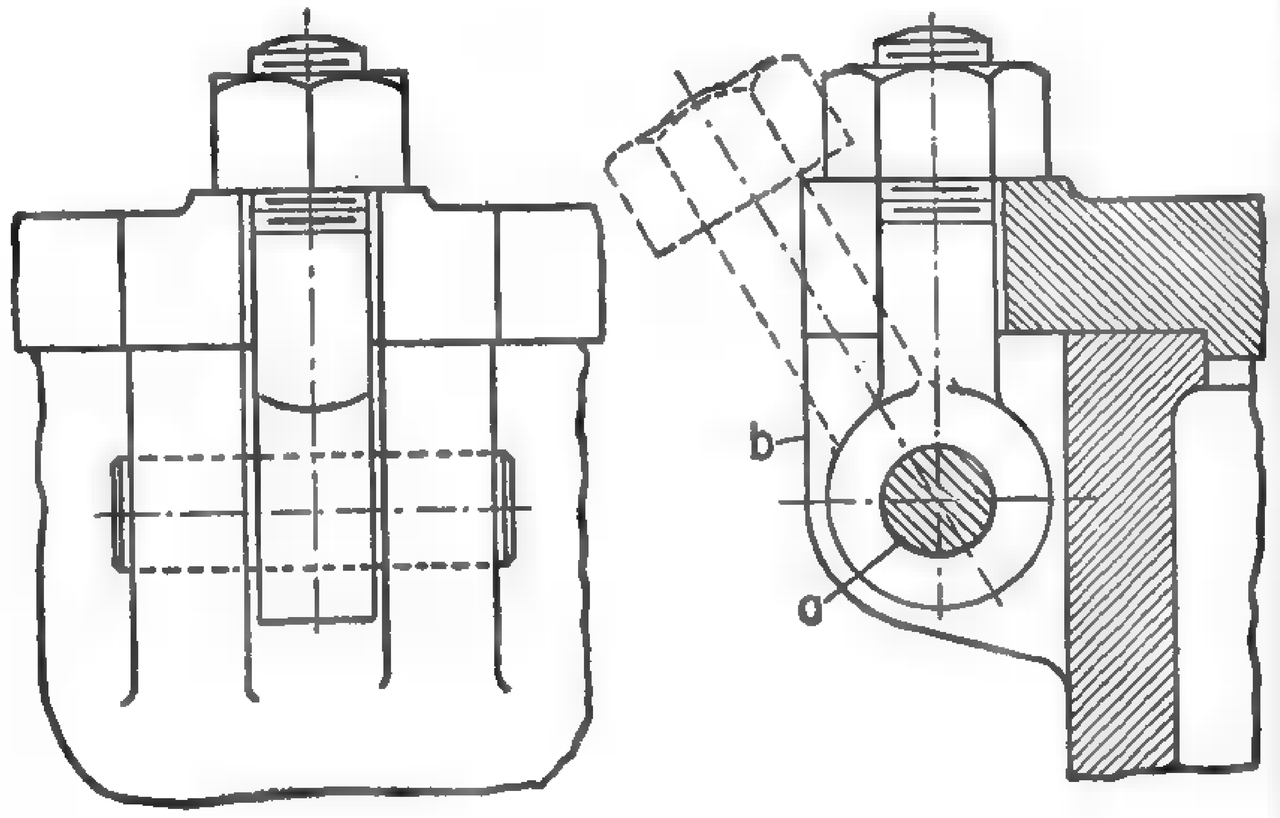


FIG. 11-14. Swinging eyebolt.

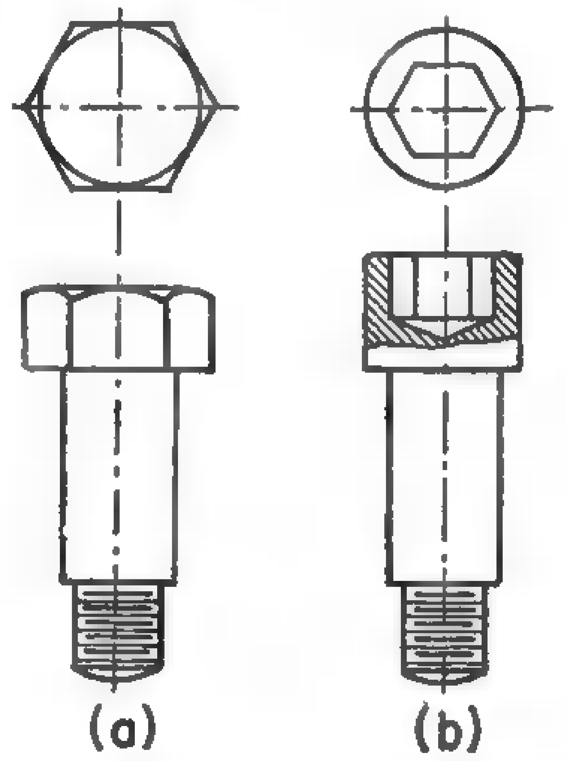


FIG. 11-15. Shoulder screws.

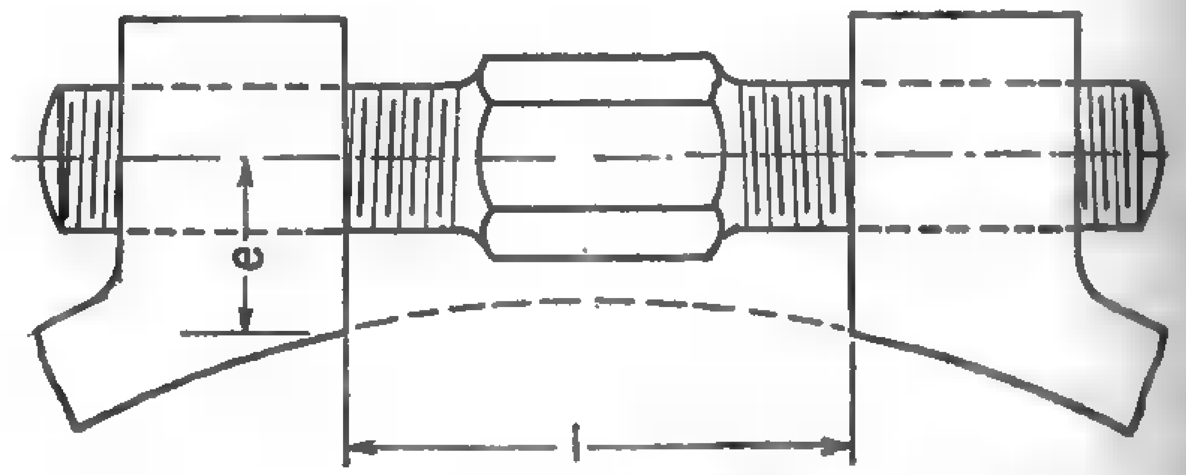


FIG. 11-16. Tie rod.

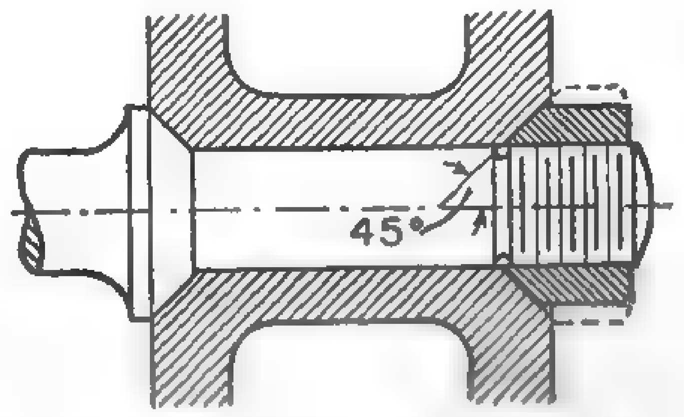


FIG. 11-17. Fastening of piston to rod.

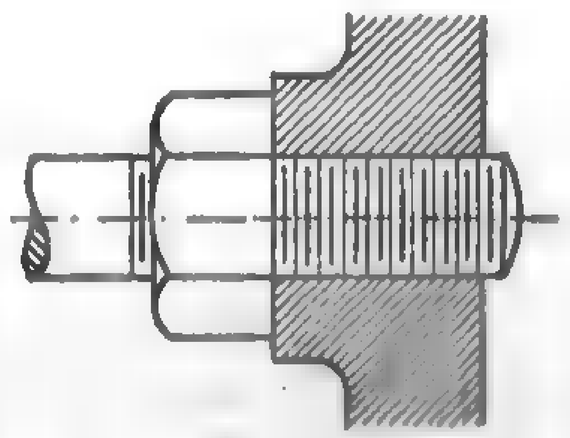


FIG. 11-18. Fastening of piston rod to crosshead.

diameter increases 0.013 in. per number. Thus the diameter of any machine screw can be found from its number by the relation

$$D = 0.060 + 0.013N \tag{11-11}$$

where N is the size number. The diameter of a No. 12 screw is $D = 0.216$ in. For larger sizes regular cap screws are used.

In addition to the number, a machine screw is specified by the number of threads and the length. An example is No. 12–24 × 1 in. M.sc.

Setscrews. A setscrew is a screw whose point presses against a piece and thus holds two parts together by friction. A setscrew may have a square head, Fig. 11–12a or b; it may be headless, with a screwdriver slot, Fig. 11–12c; or it may have a hexagonal or fluted socket, Fig. 11–12d, e, or f. The point may be cup-shaped, Fig. 11–12a or f; cone-shaped, Fig. 11–12e; oval, Fig. 11–12b; flat, Fig. 11–12c; or dog-shaped, Fig. 11–12d. The point is generally hardened.

Special screws. Screw fastenings of special shapes are used freely in various design problems. Only a few typical examples will be given here.

Anchor bolts, or *T-head bolts*, shown in Fig. 11–13, are used with slots instead of holes in the connected parts to facilitate quick removal.

Eyebolts are used for fastening parts requiring frequent removal when misplacing of the bolt must be prevented. The construction and method of application of an eyebolt are shown in Fig. 11–14. The pin *a* is made to give a drive fit in the cast-iron lugs *b* and goes freely through the bolt eye.

Shoulder screws, Fig. 11–15a and b, are chiefly used as fulcrums for rotating or rocking parts, since they prevent binding or clamping of the moving parts.

Tie rods, also called *threaded rods*, are rods with threaded ends which are either screwed into other details or have nuts on both sides. A tie rod may be fitted with a right-hand thread on one end and a left-hand thread on the other, as shown in Fig. 11–16. This makes it possible to adjust the distance l between the connected parts by turning the rod, without disturbing other parts.

Various details are fastened to rods by means of threads. The attachment of a piston to the piston rod by means of a thread and nut is shown in Fig. 11–17 and Fig. 11–33; and the connection between a piston rod and a crosshead by means of a tapped hole and locknut is shown in Fig. 11–18.

Special details. A few special screw devices are shown in Fig. 11–19. The *turnbuckle*, Fig. 11–19a, connects two rods, one of which has a right-hand thread and the other a left-hand thread.

The *clevis*, Fig. 11–19b, is used to fasten a rod to a plate or to a fulcrum pin.

The *wing nut*, Fig. 11–19c, is used where the holding device must be screwed down and unscrewed frequently.

The *cap nut*, Fig. 11–19d, is used to prevent leakage past the threads of studs. A cap nut of the type shown in Fig. 11–19e is sometimes used for the same purpose but oftener for the sake of appearance.

Washers. A thin ring, or washer, of steel is placed under the nut, when the surface of the part is not finished, in order to decrease the friction that occurs when the nut is tightened. A special wedge-shaped washer, Fig. 11–19f, is used to prevent bending of a bolt if the surface under the nut is not normal to the axis of the bolt, as in an I beam.

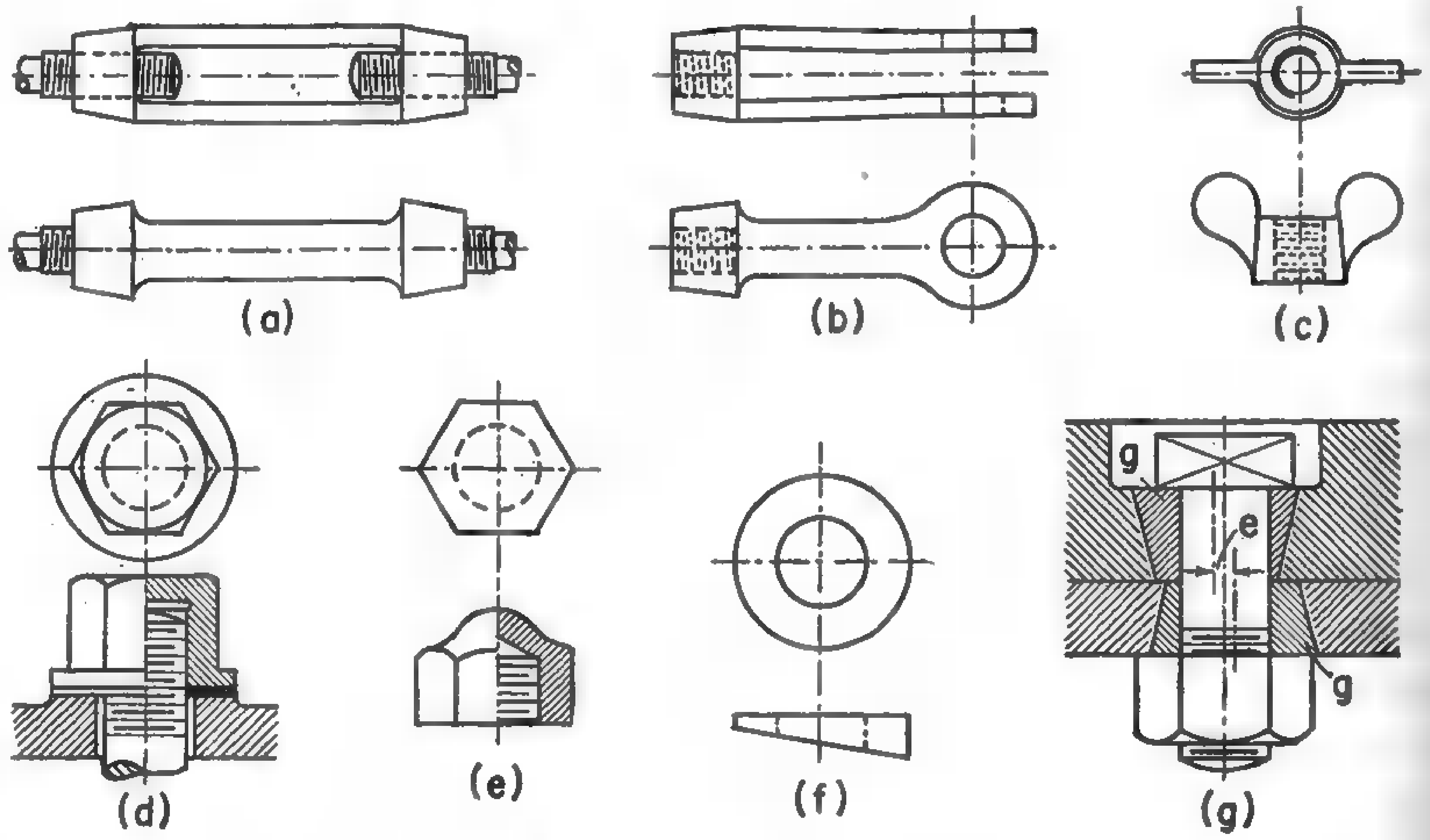


Fig. 11–19. Special screw-fastening details.

Conical washers, as shown in Fig. 11–19g, with eccentrically drilled holes are used to assemble parts in which the center lines of the holes do not coincide. For large sizes this method is cheaper than reaming the holes in place. The washers are made of soft steel or, when there is no impact action or vibration, of a special lead alloy.

11–5. Friction locking devices. Nuts on bolts and studs, when subjected to vibration, tend to work loose. There are many kinds of locking devices. Of the two main groups, those using friction and those based on a positive engagement, the first group will be discussed first.

Lock nuts. One of the most used devices is a *second nut*. When the upper nut is tightened while the lower one is held with a wrench, the bolt stretches and the pressure on the threads of the lower nut is relieved, with the result that the latter acts more like a spacer. For this reason the lower nut can be made shorter. Its height may be only $\frac{1}{2}D$, or half that of a regular nut. However, the average mechanic has a tendency to put the thinner lock nut

on top. To prevent this mistake it is a better practice, if a saving in height is desired, to make both nuts of the same height, which may be about $\frac{3}{4}D$.

The *elastic stop nut*, Fig. 11–20a, uses a fiber collar c which is set in a recess in the nut and grips the thread of the bolt as the nut is screwed home. Elastic stop nuts are made of various materials—steel, brass, and duralumin—in sizes up to 2 in., and have either UNC or UNF threads.

The *spring washer*, or *lock washer*, Fig. 11–20b, tilts the nut slightly so as to produce a pinching action, but its use may dent the lower surface.

A *shakeproof lock washer*, Fig. 11–20c and d, depends on twisted hardened-steel teeth to form a multiple lock of sharp edges which cut into the nut and the machine part. The conical lock washer in Fig. 11–20d is the only locking device for flat, cone-shaped machine-screw heads.

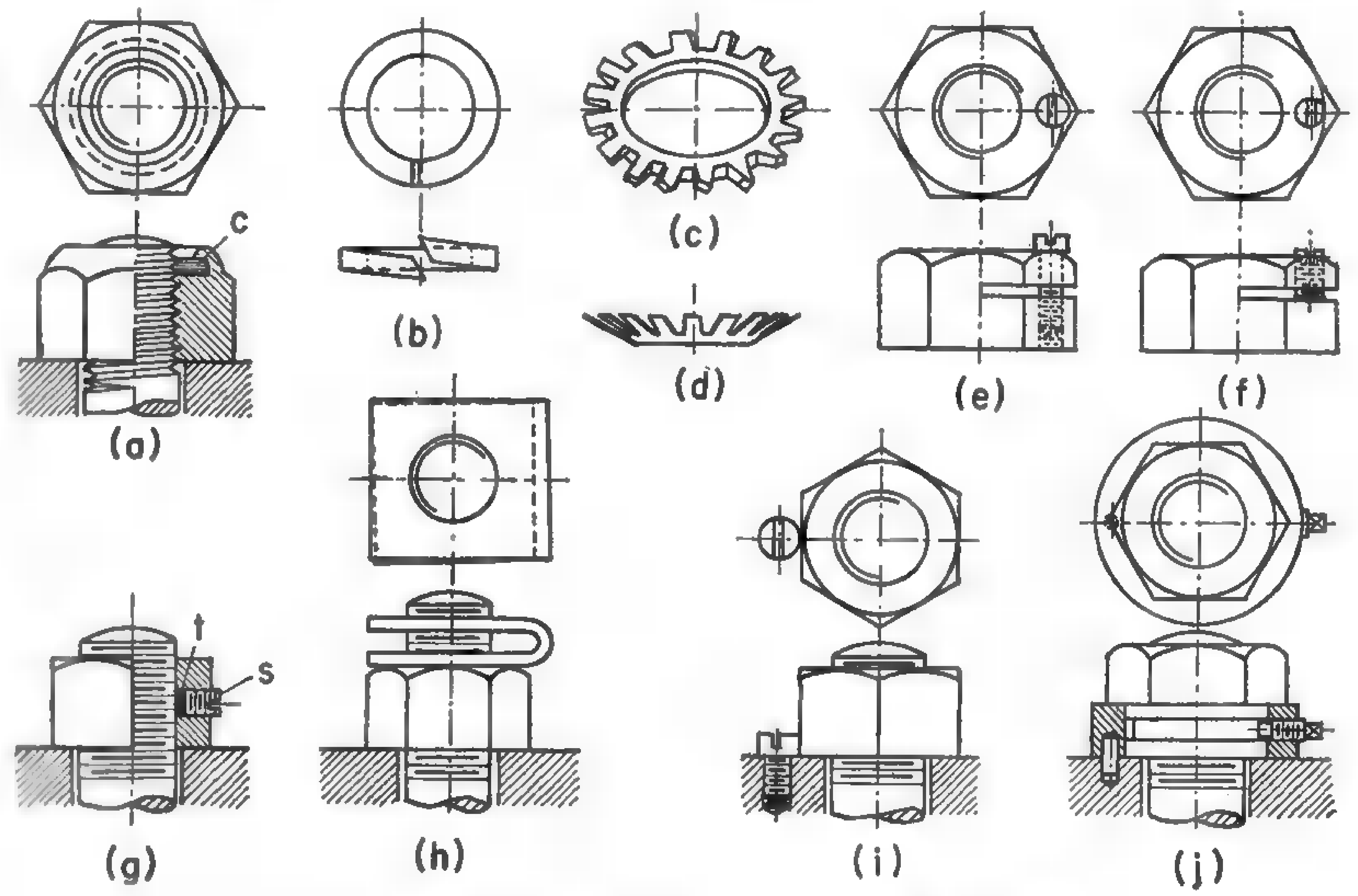


Fig. 11–20. Nut-locking devices based on friction.

A *slotted nut* has a saw-cut on the sides. A small screw pulls the slot together, Fig. 11–20e, or spreads it, Fig. 11–20f, and thus causes the nut to pinch the thread.

The *lock screw*, Fig. 11–20g, is used successfully in a special joint or with a heavy nut. The setscrew s presses against a piece of copper or fiber t to protect the screw thread and to create additional friction. The fine threads of the small setscrew prevent it from unscrewing when the assembly is subjected to vibration which would cause the big nut to unscrew. The *boxer lock*, Fig. 11–20h, is made of a spring steel plate which is tapped and which has one side slightly twisted. Screwed on top of a regular nut, it produces a pinching action which makes it one of the safest locking devices. The

arrangements of lock screws shown in Fig. 11–20i and j have almost positive engagements but can be applied only in special cases.

The wrench-type lock *s*, Fig. 11–21, is also almost positive because the flat-head machine screw *b* will not unscrew under severe vibration.

The *Dardelet thread lock*, Fig. 11–22, consists of a self-locking bolt and nut. The effectiveness of the grip depends on the tightness with which the spiral sloping surfaces of the root of the bolt and the crest of the nut thread jam against each other when the nut is screwed home (Fig. 11–22b). The difficulty of cutting the special thread restricts the use of this otherwise very effective device. Also it is often difficult to apply a sufficient initial tension in the bolt, because the crest of the nut thread may jam against the face of the bolt root before the bolt is stretched sufficiently.

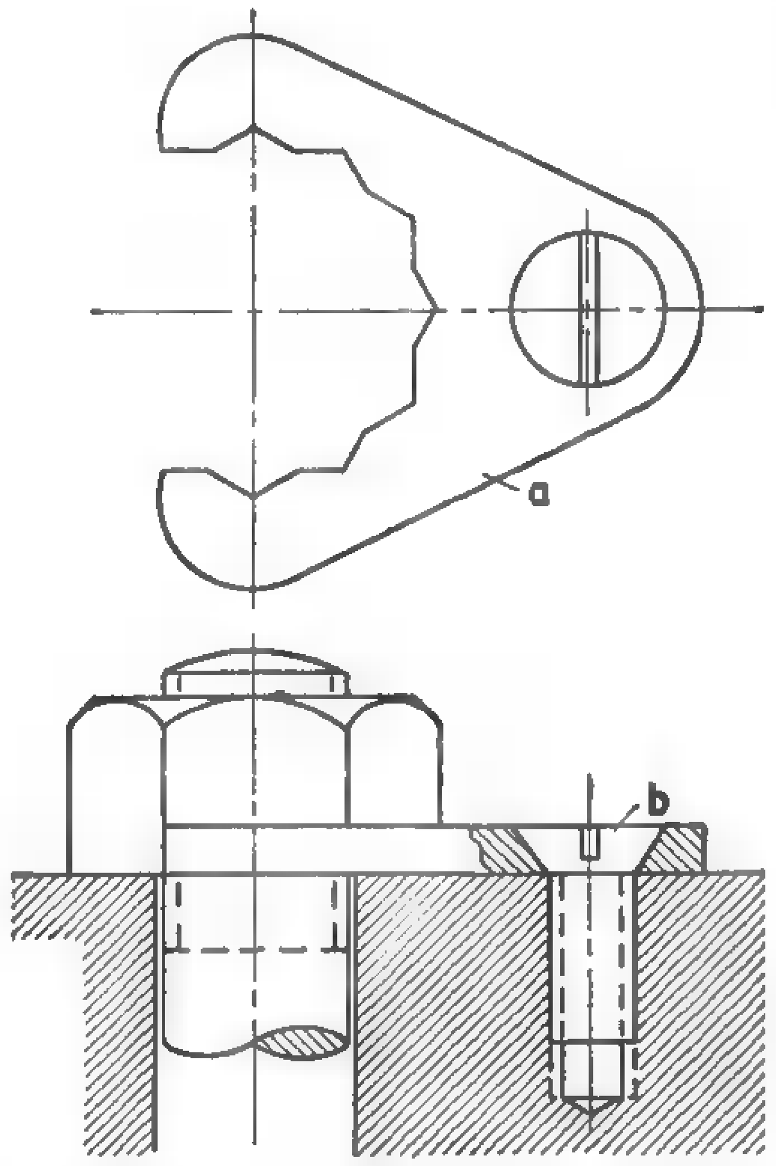


Fig. 11–21. Wrench-type nut lock.

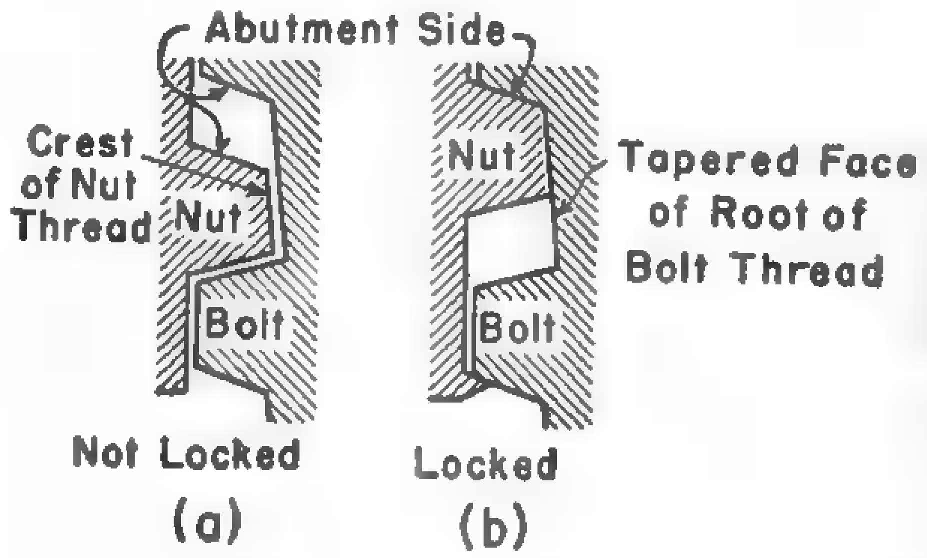


Fig. 11–22. Dardelet thread lock.

A nut having a *conical bottom* with a cone angle of 45° or smaller, Fig. 11–17, is a very dependable lock and is used rather extensively in special screw joints.

11–6. Positive locking devices. The most positive lock is the cotter pin, Fig. 11–23a, which is inserted in a hole drilled through the bolt after the nut has been screwed home. Where the nut is to be removed frequently or pulled up in service, the use of a *castellated nut* (Fig. 11–4d) with a finer UNF thread is a satisfactory solution. Insertion of the cotter pin above the nut makes its removal easier but has the disadvantage that the nut can back up before it is stopped by the pin.

The wiring together of cross-drilled screw heads or tap-bolt heads, or of bolt ends protruding through the nuts, is used when a slight turning of the bolts and nuts is not objectionable.

The *spring wire lock*, Fig. 11–23b, requires a groove to be turned in the nut and six small holes to be drilled through it. The hole in the screw is drilled after the nut is screwed home.

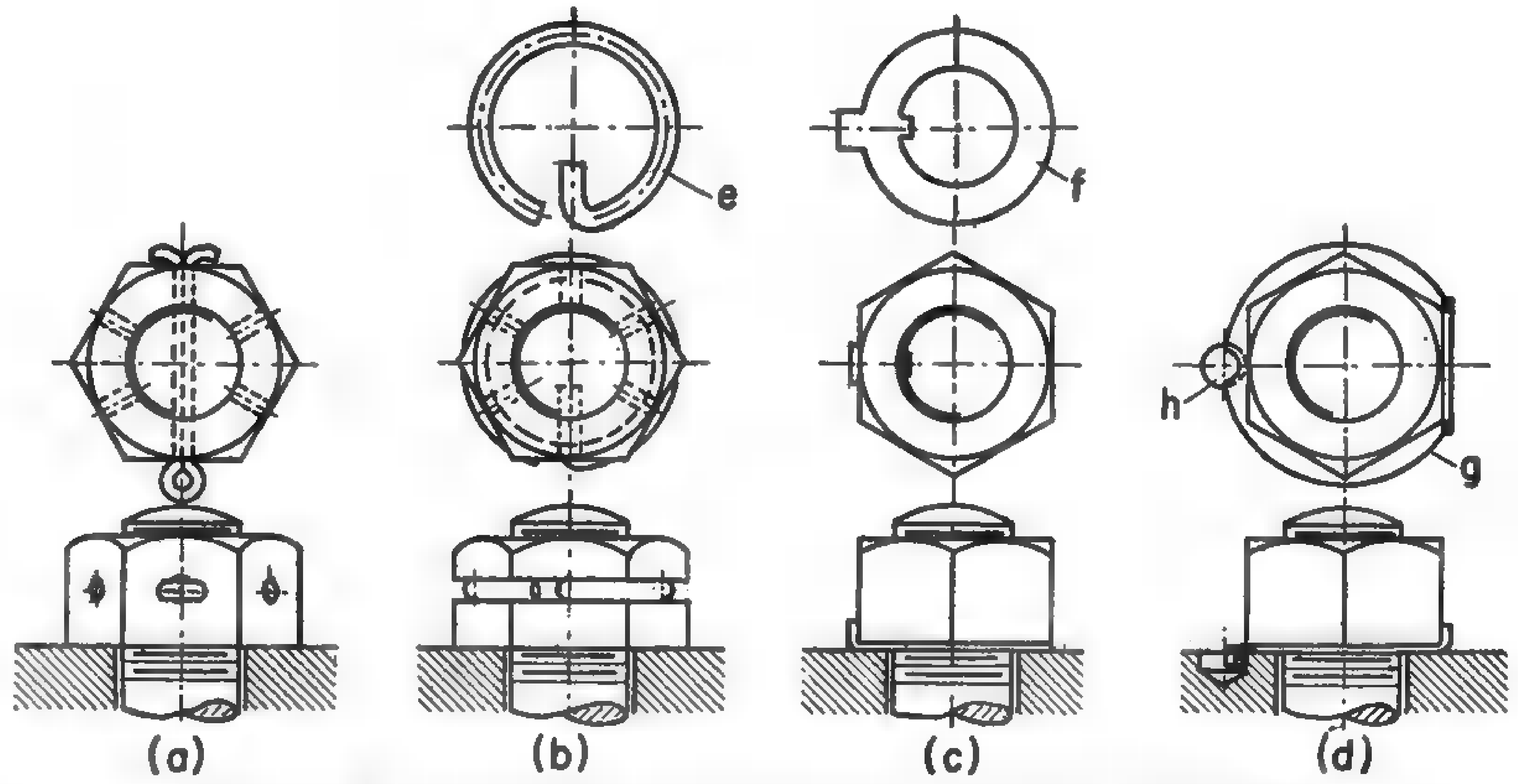


Fig. 11–23. Nut-locking devices with a positive engagement.

The *tongued washer*, Fig. 11–23c, requires the nut to be in a certain position to be really positive and also requires milling of a groove in the bolt end in order to accommodate the inner tongue.

The *upturned washer*, Fig. 11–23d, is used with large nuts, such as those which hold pistons, and is made of soft steel. If the surface under the nut is flat, it is necessary to form (drill) a recess *h* into which part of the washer is hammered. This lock has two advantages: The bolt is not weakened by having holes or grooves cut in it, and the nut may be stopped in any position.

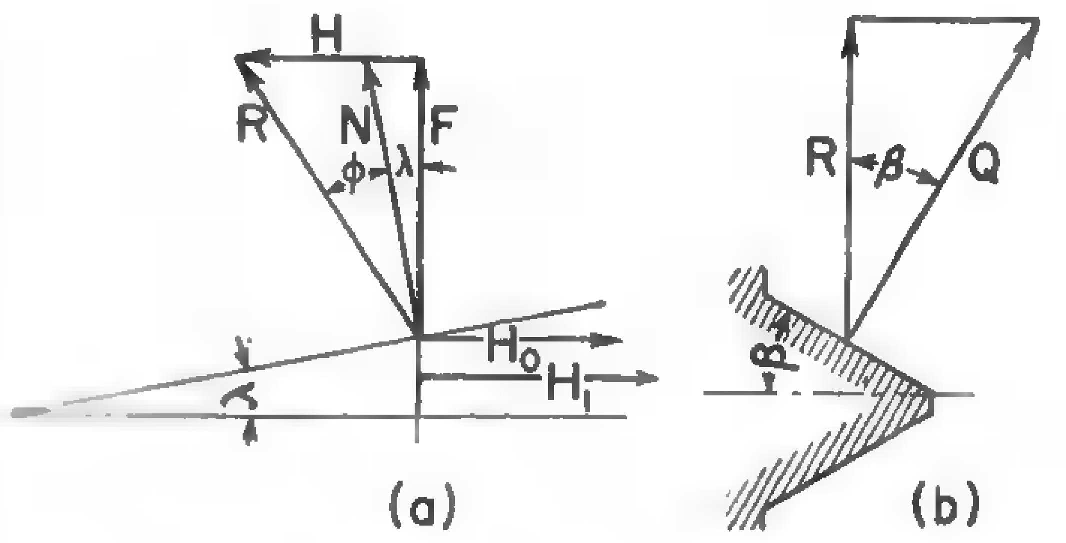


Fig. 11–24. Forces acting on a triangular thread.

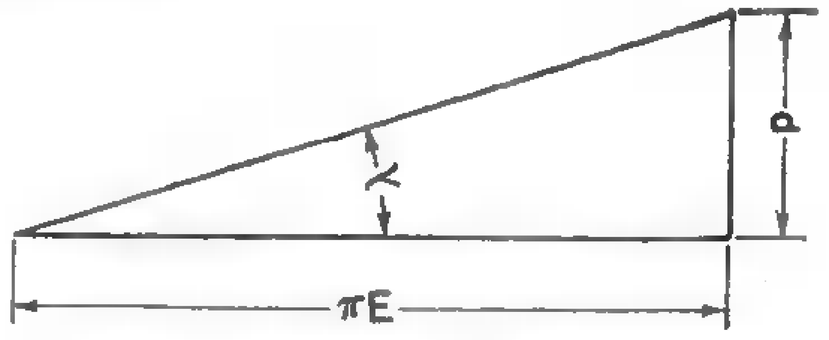


Fig. 11–25. Helix angle of a V thread.

11–7. Efficiency of triangular threads. Friction in a screw thread creates a torsional stress in the screw. The turning moment acts on the screw in a plane normal to the screw axis. The useful force F, Fig. 11–24a, is exerted by the thread in the direction of the axis. If there were no friction, this force would exert a pressure N normal to the surface of the thread.

The angle between the forces F and N is evidently equal to the helix angle λ. In a V thread this angle changes from the outside to the inside of the thread. Its mean value is at the pitch diameter E and may be found from Fig. 11–25, which shows a development of the pitch cylinder. Thus,

$$\tan \lambda = \frac{p}{\pi E} = \frac{1}{\pi E n} \tag{11–12}$$

where, with sufficient accuracy, $E = \frac{1}{2}(D + K)$.

TABLE 11–4

COEFFICIENTS OF FRICTION f FOR SCREWS AND NUTS

Material of Screw	Lubrication	Material of Nut		
		Steel	Brass	Cast Iron
Steel, mild or casehardened, or bronze..........	Dry	0.15–0.25	0.15–0.23	0.15–0.25
	Machinery oil	0.11–0.17	0.10–0.16	0.11–0.17
	Oil with graphite	0.08–0.12	0.04–0.06	0.06–0.09

Because of friction the resultant force R, Fig. 11–24a, will be inclined from the normal by the amount of the angle of friction ϕ, and will have a component H in the plane normal to the screw axis. To do its work, the reaction R must have a component in the direction of F equal to the load but oppositely directed, and the component H must be overcome and balanced by an opposite force H_0. From Fig. 11–24a,

$$H_0 = F \tan (\lambda + \phi) \tag{11–13}$$

If $\tan (\lambda + \phi)$ is expressed in terms of functions of the component angles λ and ϕ, and $\tan \phi$ is replaced by the coefficient of friction f_1, the resulting equation is

$$H_0 = F \frac{\tan \lambda + f_1}{1 - f_1 \tan \lambda} \tag{11–14}$$

In order to take into account the triangularity of the thread, the acting reaction Q, Fig. 11–24b, can be taken with sufficient accuracy as

$$Q = \frac{R}{\cos \beta} \tag{11–15}$$

Since the friction force is directly proportional to the thread pressure, the friction terms in equation 11–14 must be divided by $\cos \beta$. The corrected tangential resistance is

$$H_1 = F \frac{\tan \lambda + f_1 \sec \beta}{1 - f_1 \tan \lambda \sec \beta} \tag{11–16}$$

Values of the coefficient of friction compiled from various sources are given in Table 11–4.[3] The lower values apply to high-grade workmanship; the higher ones apply to average or poor workmanship.

Efficiency. The efficiency of a screw is the ratio of the force that has to be applied for a given output if there is no friction in the threads, to the force actually required in the presence of friction. If there is no friction, $f_1 = 0$ and equation 11–14 becomes

$$H_0' = F \tan \lambda \tag{11–17}$$

[3] C. W. Ham and D. G. Ryan, *An Experimental Investigation of the Friction in Screw Threads*, Bulletin No. 247, University of Illinois Engineering Experiment Station (June, 1932), pp. 9 and 47.

The efficiency e is equal to H_0'/H_1. Substituting values from equations 11–17 and 11–16, and simplifying the result, gives

$$e = \frac{\cos \beta - f_1 \tan \lambda}{\cos \beta + f_1 \cot \lambda} \tag{11–18}$$

Equation 11–18 allows for thread friction only. To find the over-all efficiency when a nut or a screw with a head is being turned, it is necessary to consider the additional friction against the surface on which the nut or head bears. If the outside diameter of the bolt is D, the standard width across the flats of a hexagonal head or nut is $b = 1.5D$, and the diameter at which this friction is applied may be considered to be

$$D' = \tfrac{1}{2}(D + b) = 1.25D \tag{11–19}$$

The tangential friction force at this diameter is Ff_2, where f_2 is the coefficient of friction against the outside surface. Referred to the same mean thread diameter E as the force H_1, the additional friction force is

$$H_2 = Ff_2 \times \frac{1.25D}{E} \tag{11–20}$$

Thus the total friction is $H_1 + H_2$, and the over-all efficiency is

$$e_2 = \frac{H_0'}{H_1 + H_2} \tag{11–21}$$

Substituting for H_0', H_1, and H_2 their values in equations 11–17, 11–16, and 11–20, and simplifying the result, gives

$$e_2 = \frac{\tan \lambda}{\dfrac{\tan \lambda + f_1 \sec \beta}{1 - f_1 \tan \lambda \sec \beta} + 1.25 f_2 \dfrac{D}{E}} \tag{11–22}$$

For the UNC thread the Lewis empirical formula may be used. This is

$$e_2 = \frac{p}{p + D} \tag{11–23}$$

Equation 11–23 is only approximate, but it checks well for $f_1 = 0.10$ and $f_2 = 0.15$.

11–8. Static stresses in screw fastenings. The following stresses must be considered in screw fastenings with a static loading:

a. Initial stresses due to screwing-up forces
b. Stresses due to external forces
c. Stresses due to a combination of forces of types a and b

Stress concentration. In computing the tensile and torsional stresses in the root section of a screw, the stress-concentration effect should be considered. With a ductile material, however, there will be a localized yielding with a

consequent readjustment of the internal stresses. A certain permanent set may occur in the outer fibers at the root. Under a steady load, however, the bolt will not fail if the highest stress is higher than the elastic limit of the material but lower than its ultimate strength.

11–9. Initial stresses. The stresses in a bolt, screw, or stud when it is screwed up tightly are: (a) a tensile stress due to the stretching of the bolt; (b) a torsional stress caused by the frictional resistance of the thread during its tightening; and (c) a bending stress if the surfaces under the head or nut are not perfectly normal to the bolt axis.

Tensile stress. The stretching of the bolt depends on the force F, Fig. 11–24, which can be computed theoretically from the proportions of the thread. The following analysis will explain the procedure. The axial tension F may be treated as a function of the force $H = H_1 + H_2$ which must be applied when screwing down the nut and which may be found from equations 11–16 and 11–20. By taking as two more or less extreme cases a ½-in. screw and a 3-in. screw with UNC threads, assuming as average friction coefficients $f_1 = 0.15$ and $f_2 = 0.20$, and determining the values for the other terms from the corresponding geometrical relations, we find that $H = 0.507F$ for the ½-in. screw and $H = 0.473F$ for the 3-in. screw. The values of the coefficients of friction, Table 11–4, vary much more than these values of H. Therefore an average value of $0.49F$ can be assumed. This gives for the tension in the screw,

$$F = 2.04H \tag{11–24}$$

The tensile stress is then

$$s = \frac{F}{A_s} \tag{11–25}$$

where the stress area A_s is given in Table 11–1.

Torsion. The torsional stress s_s can be computed by the relation

$$s_s = \frac{16T}{\pi K^3} \tag{11–26}$$

In this case the torque is

$$T = \tfrac{1}{2}H_1E \tag{11–27}$$

where the force H_1 can be found by equation 11–16, where E is the pitch diameter and K is the minor diameter of the thread.

The magnitude of the resultant stress can be found by combining s and s_s as shown in section 2–11. For UNC standard screws with sizes from ½ in. to 3 in., and for $f_1 = 0.15$ and $f_2 = 0.20$, the resultant stress is found to be from 28 per cent to 22 per cent greater than the tensile stress alone. However, for a screw with a diameter of ¾ in. or less, the initial stress depends to such an extent upon the judgment and experience of the mechanic that an attempt to calculate it is practically useless.

Repeated tightening. According to accurate measurements, repeated unscrewing and tightening of steel nuts increases the torsional moment that must be applied, because there is a gradual scoring of the threads. After 50 tightenings the torsional resistance is about 100 per cent greater than the resistance at the first tightening, and after 200 tightenings the increase is about 150 per cent.[4]

Initial tension. According to experiments made at Sibley College[5] the load in pounds produced by screwing a threaded member up tight may be estimated as

$$F' = 16{,}000D \tag{11–28}$$

Equation 11–28 was confirmed surprisingly well by German experiments[6] with screws having diameters of ½ in., ¾ in., and 1 in.

Substituting the value of F' from equation 11–28 in equation 11–25 and using the values of D and K from Table 11–1, we get for UNC screws with sizes from ½ in. to 3 in. an average stress

$$s = \frac{28{,}000}{D} \tag{11–29}$$

This expression shows that even 1-in. screws are subjected to an excessively high initial stress. Smaller screws are easily stretched above the elastic limit and even broken. For this reason, bolts that must carry external loads preferably should not be under ¾ in., and sizes under ⅝ in. should not be used.

Equations 11–28 and 11–29 apply to cases where the nuts have metal-to-metal contact. If a bolt fastens two parts with a flexible gasket between them, a smaller effort is applied in order not to crush the gasket, and the initial load may be taken as one-half that given by equation 11–28. Thus

$$F' = 8{,}000D \tag{11–30}$$

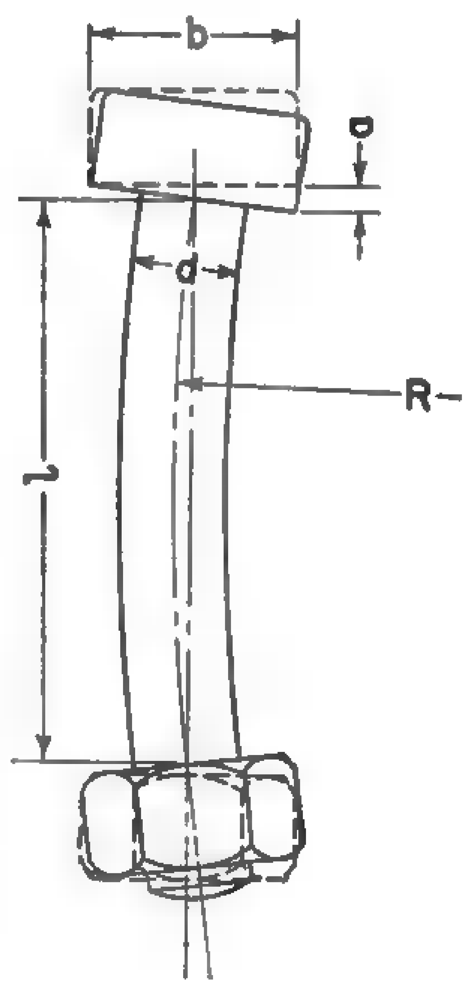


Fig. 11–26. Bending of a bolt.

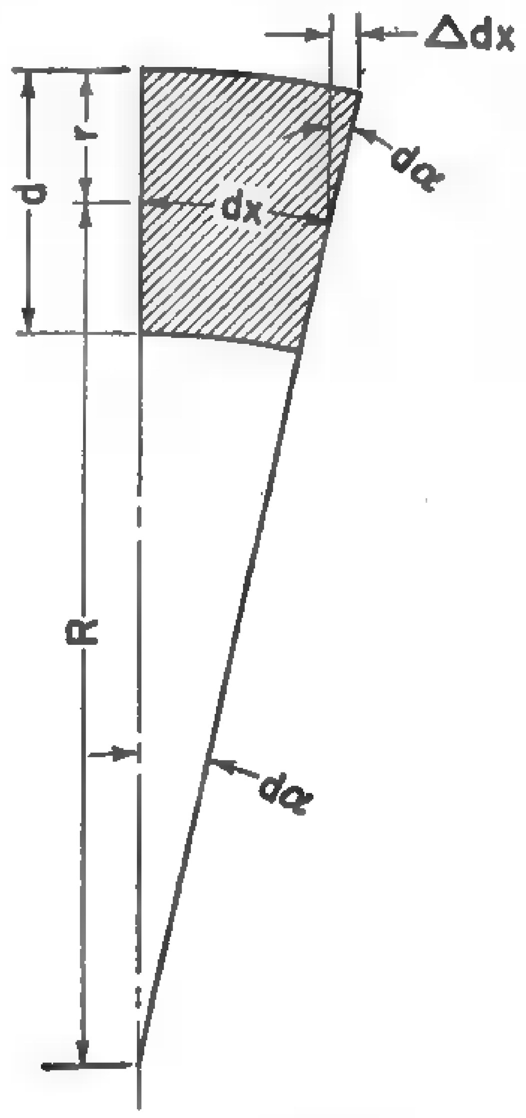


Fig. 11–27. Element of a bolt subjected to bending.

[4] A. Thum, "Vorspannung und Dauerhaltborkeit von Schraubenverbindungen," *Mitteilungen der MPA der T.H. Darmstadt*, Heft 7 (Berlin: VDI-Verlag, 1936), pp. 1–72.

[5] D. S. Kimball and J. H. Barr, *Elements of Machine Design*, 3d ed. (New York: John Wiley & Sons, Inc., 1935), p. 277.

[6] E. Bock, "Das Verhalten der Schraubenverbindung beim Anziehen und Lösen," *Zeitschrift Verein Deutscher Ingenieure*, Vol. 78 (1934), p. 780.

Bending. If the outside surfaces of the parts that are bolted together are not parallel, the bolt will be subjected to bending by a moment M that is constant over the whole free length l, Fig. 11–26. The stress induced by this bending may be found by considering the deformation of an element of the bolt, Fig. 11–27. The unit elongation of the outer fibers is

$$\epsilon = \frac{\Delta\, dx}{dx} = \frac{r\, d\alpha}{R\, d\alpha} = \frac{r}{R} \tag{11–31}$$

Expressing the stress by equation 2–4 and substituting for ϵ its value from equation 11–31 gives

$$s = \epsilon E = \frac{rE}{R} \tag{11–32}$$

From Fig. 11–26,

$$\frac{1}{R} = \frac{2a}{b} \tag{11–33}$$

Substituting the value of R from equation 11–33 in equation 11–32 and noticing that $r = d/2$ and $b = 2d$, as a limit value, we finally get

$$s = \frac{aE}{2l} \tag{11–34}$$

Equation 11–34 shows that for a given material the stress depends only on the ratio a/l.

EXAMPLE 11–1. Determine the bending stress induced in a 1-in. bolt that must hold together two flanges which have a combined thickness of $2\frac{1}{2}$ in. and are so placed that the gap a, Fig. 11–26, is 0.008 in.

By equation 11–34, the stress is

$$s = \frac{0.008 \times 30{,}200{,}000}{2 \times 2.5} = 48{,}300 \text{ psi}$$

In spite of the very small deviation of the surfaces from being parallel, the stress exceeds the elastic limit.

This example shows how important it is to have the outside surfaces true. If they are not machined, then the surfaces around the holes must be *spot-faced.* With structural shapes, wedgelike washers, Fig. 11–19f, must be used.

The sum of the bending stress and the initial stress from tightening the nut can very easily exceed not only the elastic limit but even the ultimate strength of the bolt material. However, because of the ductility of steel the bolt will be bent before breaking. This bending will relieve some of the stress, but it is not permissible in a machine part.

11–10. Stresses due to external forces. Generally the external load applied to a bolt tends to separate the connected machine parts in the direction of the bolt axis, and this action sets up a tensile stress in the bolt. Sometimes when bolts are used to prevent the relative movement of two or more parts, a shear stress is induced in the bolt. When the action of the load is at an angle to the axis, the stress in the bolt becomes tension and shear combined.

Tension. In a bolt subjected to a load F which induces tension, the stress is found by equation 11–25.

Shear. Bolts should preferably not be subjected to shear. When shear stresses cannot be avoided in the design, the bolt shank should be accurately fitted to the holes in the connected parts, at least in the portion near the joint. A better arrangement is to use dowel pins fitted accurately into reamed holes after the bolts have been inserted and tightened.

Eccentric loading on rectangular base. The bolts holding a bracket as in Fig. 11–28 are not efficient. The load F tends to cause the bracket to rotate clockwise about the edge a, thus stretching each bolt to a degree that depends on its distance from the edge a. Since stress is a function of elongation, the loads on the bolts are different. However, for convenience and economy all six bolts are made of the same size.

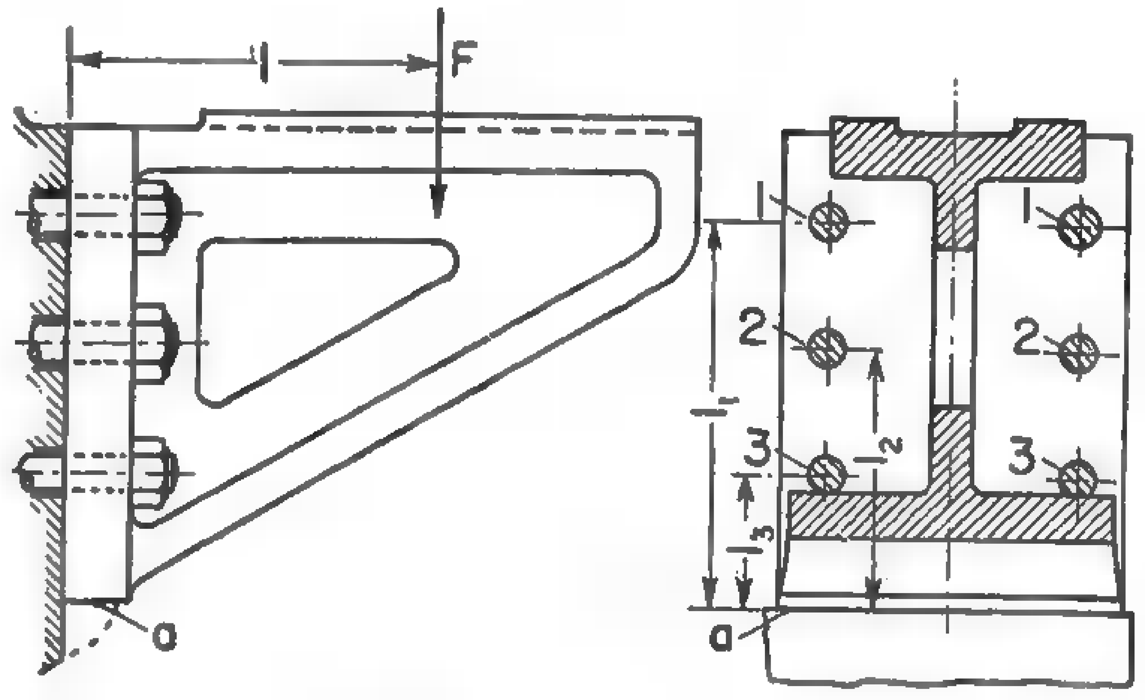


FIG. 11–28. Fastening of a bracket.

The moment Fl of the load F must be balanced by the sum of the moments of the bolt loads about the edge a. If the loads on the bolts are designated by F_1, F_2, and F_3 and their moment arms are designated as in Fig. 11–28,

$$Fl = 2(F_1l_1 + F_2l_2 + F_3l_3) \tag{11–35}$$

Since the stresses induced in the bolts are directly proportional to the elongations produced, and the elongations are proportional to the distances from the edge a, the maximum load comes on the bolts marked *1* and the following relations can be written:

$$F_2 = \frac{F_1 l_2}{l_1} \qquad F_3 = \frac{F_1 l_3}{l_1}$$

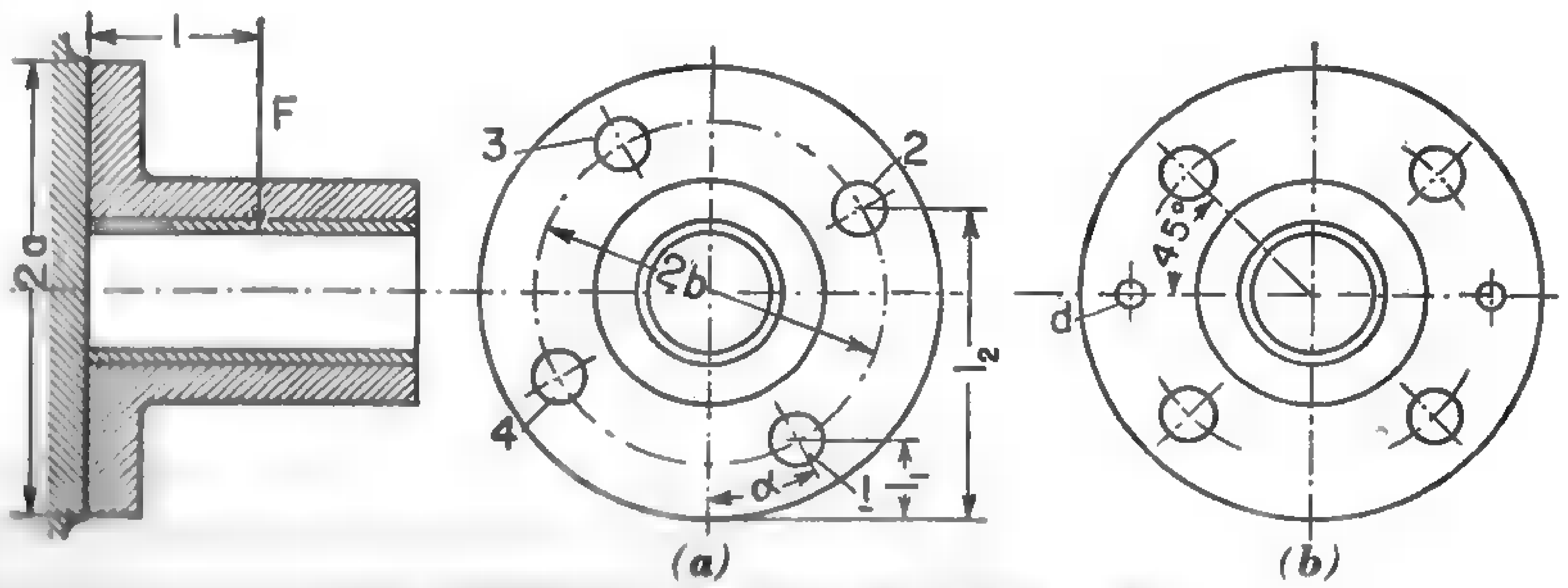


FIG. 11–29. Fastening of a flanged bearing.

Substituting these values of F_2 and F_3 in equation 11–35 and solving the resulting equation for F, gives

$$F_1 = \frac{Fll_1}{2(l_1^2 + l_2^2 + l_3^2)} \tag{11–36}$$

Eccentric loading on circular base. A machine member is frequently made with a circular base fastened by bolts located on a circle. To simplify the discussion a round-flange bearing with four bolts, as shown in Fig. 11–29, will be analyzed. By proceeding in the same way as with a rectangular base, we find the magnitude of the load on bolt *1* to be

$$F_1 = \frac{Fll_1}{l_1^2 + l_2^2 + l_3^2 + l_4^2} \tag{11–37}$$

From Fig. 11–29a,

$$l_1 = a - b \cos \alpha \qquad l_3 = a + b \cos \alpha$$
$$l_2 = a + b \sin \alpha \qquad l_4 = a - b \sin \alpha$$

When these values are substituted in equation 11–37, the result is

$$F_1 = F \frac{l(a - b \cos \alpha)}{4a^2 + 2b^2} \tag{11–38}$$

By repeating the discussion for i bolts, we can write the general expression for the load as

$$F_i = F \frac{2l(a - b \cos \alpha)}{(2a^2 + b^2)i} \tag{11–39}$$

The force F_i evidently has its maximum value when $\cos \alpha$ is a minimum, or when $\cos \alpha = -1$. Then $\alpha = 180°$, and

$$F_{\max} = F \frac{2l(a + b)}{(2a^2 + b^2)i} \tag{11–40}$$

Equation 11–40 gives the absolute maximum value and should be used if the direction of the load F can change with relation to the bolts, as in the case of the base of a pillar crane. If the direction of F is fixed, the maximum load on the bolts can be reduced by locating the bolts so that two of them will be equally stressed, as shown in Fig. 11–29b. In this case the angle α in equation 11–39 becomes $180° - 360°/2i$, and the equation for the maximum load is

$$F'_{\max} = F \frac{2l\left[a + b \cos\left(\frac{180°}{i}\right)\right]}{(2a^2 + b^2)i} \tag{11–41}$$

The bolts should be relieved of shear stresses by using dowel pins d, Fig. 11–29b.

Nuts. The computation of the stresses in bending, shear, and compression that are set up in the threads of a bolt and nut may be omitted if the effective height of a nut or of a tapped hole is made at least equal to the

TABLE 11–5

DIMENSIONS FOR ANGULAR THREADS (FIG. 11–30)

Nut			Tubular Connection	
Material[a]	$\frac{l}{d}$	$\frac{b}{d}$	Material	$\frac{l}{h}$
Steel	0.8–1.0	1.4–1.5	Steel	2.5–3.0
Bronze	1.0–1.2	1.5–1.6	Bronze	2.5–3.0
Cast iron	1.3–1.5	1.6–2.0	Cast iron	2.0–2.5
Aluminum alloy	2.2–2.6	2.0–2.5	Aluminum alloy	2.5–3.0

[a] Material of screw or rod: SAE 1120 screw-stock steel.

values given in Table 11–5. The lower figures refer to first-class workmanship, and the higher ones to average workmanship.

Table 11–5, in connection with Fig. 11–30, gives the dimensions b and l for nuts made of different materials. If a nut on a rod takes only part of the load that the rod can stand, the nut may be treated as a tubular connection and the minimum length l may be determined as a function of the thickness h, as shown in the right-hand column in Table 11–5.

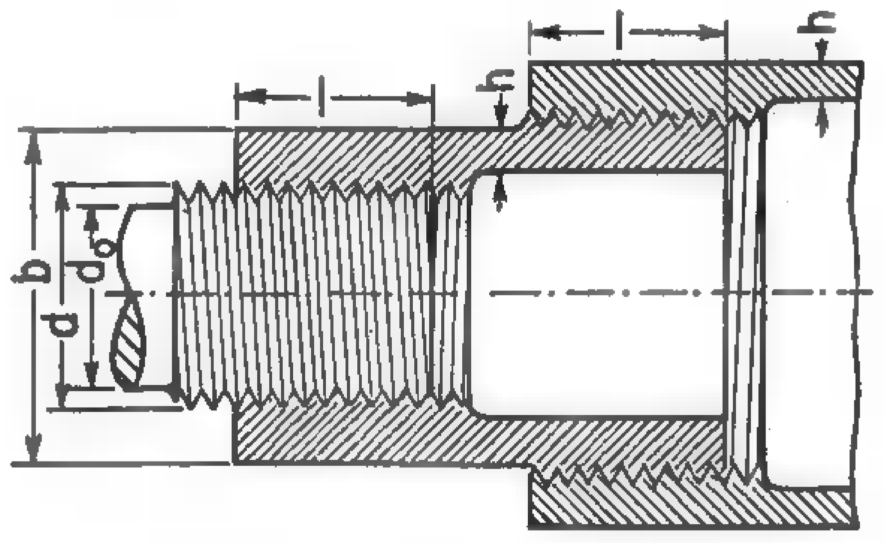


FIG. 11–30. Threaded fastenings.

EXAMPLE 11–2. An angular thread on a 3½-in. steel shaft takes a steady tensile load of 50,000 lb. Determine the minimum length and outside width of a suitable steel nut.

The elastic limit of SAE 1020 steel, according to Table 4–2, is $S_e = 35{,}000$ psi. If a size factor $e_{ss} = 0.75$ and a factor of safety $n = 2$ are used, the design stress is $S_d = 13{,}100$ psi. The corresponding outside diameter b, Fig. 11–30, may be found from the equation

$$50{,}000 = 0.7854(b^2 - 3.5^2) \times 13{,}100$$

which gives $b = 4.13$ in. This corresponds to

$$h = \tfrac{1}{2} \times (4.13 - 3.5) = 0.315 \text{ in.}$$

By the ratio in the right-hand column of Table 11–5,

$$l = 0.315 \times 3 = 0.945 \text{ in.}$$

The dimensions of the nut will probably be $b = 4\frac{1}{4}$ in. and $l = 1$ in.

Where cast iron or aluminum is used, angular threads are permissible only for permanent fastenings because the threads in these materials are easily damaged by repeated unscrewing and tightening. When the bolts are to be screwed and unscrewed repeatedly, a screwed-in steel bushing m, Fig. 11–31a, should be used in the case of cast iron, and for aluminum a cast-in insert n, Fig. 11–31b, of bronze or Monel metal should be used and should be drilled and tapped in place.

11–11. Stresses due to combined loads. The stress resulting in a bolt from the combined action of the initial load put on when the thread is screwed tight and of the external load may be found by the following analysis.

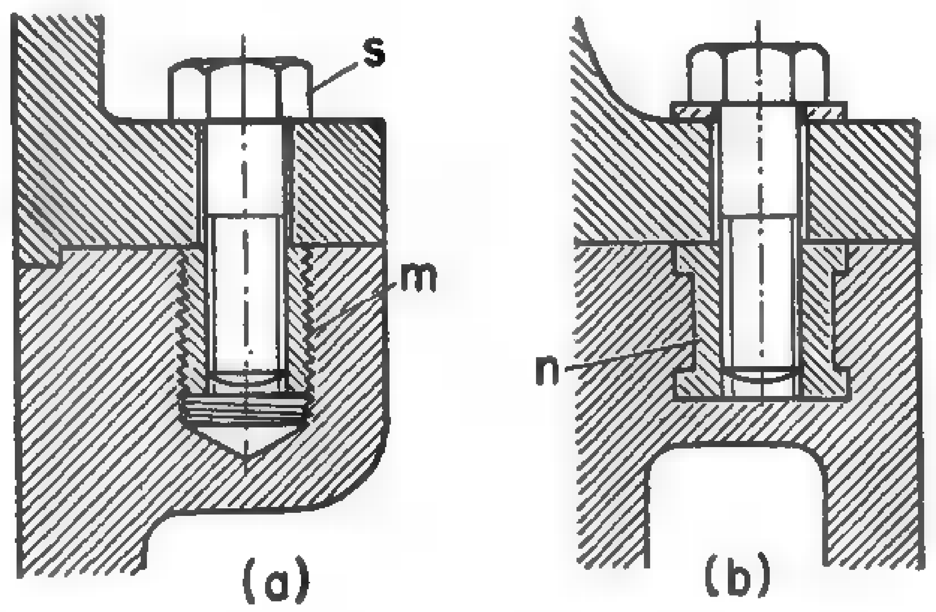


FIG. 11–31. Screw bushings.

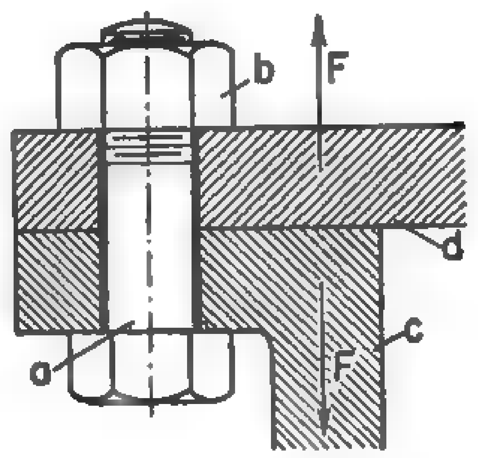


FIG. 11–32. Metal-to-metal joint.

Assume that a flexible gasket is used in a bolted joint like that shown in Fig. 11–32. For one unit, or one bolt with the corresponding parts of the flanges, the following designations will be used:

F' is the initial load due to tightening of the bolt.
F_a is the applied or external load.
F_c is the compression load on the unit area of the gasket.
F is the combined load on the bolt.
e' is the rate of elongation of the bolt, in inches per pound.
c' is the rate of compression of the gasket, in inches per pound.

When the bolt is tightened but before the external load is applied, the bolt stretches and the flanges and the gasket are compressed. However, the modulus of elasticity of the metal of the flanges is so much higher than the modulus of the gasket that the compressive deformation of the flanges can be neglected. The force compressing the gasket will be equal to the force F' stretching the bolt; the initial compression of the gasket will be $F'c'$; and the elongation of the bolt will be $F'e'$. When the load F_a is applied additionally, it will increase the length of the bolt by $(F-F')e'$, and this elongation will permit the gasket thickness to increase by the same amount. The final deformation of the gasket will be

$$F_c c' = F'c' - (F - F')e' \qquad (11\text{–}42)$$

Therefore the compressive load on the gasket is

$$F_c = F' - (F - F')\frac{e'}{c'} \qquad (11\text{–}43)$$

The combined load on the bolt is equal to the sum of the gasket load and the applied external load, or

$$F = F_c + F_a = F' - \frac{(F - F')e'}{c'} + F_a \qquad (11\text{–}44)$$

Solving equation 11–44 for F gives

$$F = F' = \frac{F_a c'}{c' + e'} = F' + CF_a \qquad (11\text{–}45)$$

Equation 11–45 indicates that the load carried by a bolt is equal to the initial tension plus a certain fraction of the external load. The value of the factor C is found as follows: By equation 2–10 the total elongation of the bolt is

$$e = \frac{Fl}{A_b E_b} \qquad (11\text{–}46)$$

Hence the rate of elongation is

$$e' = \frac{e}{F} = \frac{l}{A_b E_b} \qquad (11\text{–}47)$$

In a similar way the rate of compression of the gasket, if its thickness is t in., is

$$c' = \frac{t}{A_g E_g} \qquad (11\text{–}48)$$

Substituting the values of e' and c' from equations 11–47 and 11–48 in equation 11–45, and simplifying the expression for the factor C, gives

$$C = \frac{tA_b E_b}{tA_b E_b + lA_g E_g} \qquad (11\text{–}49)$$

EXAMPLE 11–3. Estimate the value of the factor C in equation 11–45 for a joint with 1-in. bolts and a rawhide gasket $\frac{1}{16}$ in. thick.

Equation 11–49 must be used. In this case $t = 0.063$ in. and $A_b = 0.7854$ sq in.; and from Table 4–2, for SAE 1120 steel, $E_b = 30{,}000{,}000$ psi.

The thickness of each flange may be estimated as 1.25 in. Therefore $l = 1.25 \times 2 + 0.063 = 2.563$ in. The bolt spacing may be estimated as $4d = 4$ in.; the width of the gasket is approximately $2d = 2$ in.; the diameter of the hole punched for the bolt will be about $1\frac{1}{8}$ in., and therefore $A_g = 2 \times 4 - 0.7854 \times 1.125^2 = 7$ sq in.; from Table 4–8, $E_g = 18{,}000$ psi. In equation 11–49,

$$tA_b E_b = 0.063 \times 0.7854 \times 30{,}200{,}000 = 1{,}480{,}000 \text{ in.-lb}$$
$$lA_g E_g = 2.563 \times 7 \times 18{,}000 = 323{,}000 \text{ in.-lb}$$

Hence,

$$C = \frac{1{,}480{,}000}{1{,}480{,}000 + 323{,}000} = 0.82$$

Metal-to-metal joint. For this case the factor C becomes 0, and the external load does not affect the stress in the bolt so long as F_a is less than F'. The load F' may be computed by equation 11–28. For a tight joint, F_a must be smaller than F' by a safe margin, usually at least 20 per cent. If F_a is greater than F', the joint will separate and the stress will be due to the external load alone.

If the joint is to be kept tight, F' must be greater than F_a, and the contact surfaces of the parts c and d, Fig. 11–32, must be true and smooth. Usually they are ground. The presence of a thin, inelastic gasket, such as one made of a copper or lead sheet or a very thin asbestos sheet, does not change the

conditions materially and helps to obtain a tight joint when the surfaces are machine-finished and not ground.

Gasket joint. With a gasket, the bolt carries, in addition to the initial load, a certain fraction of the external load, as shown by equation 11–45. In this case F' may be computed by equation 11–30. Equation 11–49 shows that the additional load depends on the rigidity of the gasket and on the rigidity of the bolt itself. The value of the factor C may be computed by equation 11–49. However, the value of E_g is not always known for certain, and Table 11–6 may serve as a guide.

TABLE 11–6

VALUES OF C IN EQUATION 11–45

Type of Joint	*Factor C*
Soft, elastic gasket with studs	1.00
Soft gasket with through bolts	0.90
Copper-asbestos gasket	0.60
Soft-copper corrugated gasket	0.40
Lead gasket with studs	0.10
Narrow copper ring	0.01
Metal-to-metal joint	0.00

Bolt spacing. To obtain a tight joint when two surfaces are held together by a row of bolts, the distance c between the centers of each two bolts, called the *spacing*, or *pitch*, must not be too great. Bolt spacing is discussed in section 15–6.

11–12. Dynamic stresses in screw fastenings. The stress-concentration factor K' for American Standard threads subjected to impact loads may be taken equal to 2.85 for soft steel,[7] such as SAE 1120 or SAE 1030. For heat-treated SAE 2320 steel, $K' = 3.9$.

EXAMPLE 11–4. Determine the safe useful load for a 1½-in. bolt, 4½ in. long with UNC standard threads and made of SAE 1120 open-hearth steel, for three conditions: (a) static load; (b) sudden change of load from zero to a maximum; (c) impact action resulting from a variable load when the nut is unscrewed one-sixteenth turn.

Static loading. The elastic limit in tension of SAE 1120 steel, from Table 4–2, is 34,000 psi. If the bolt was turned from a 2¼-in. hexagonal bar, the size coefficient, calculated from equation 5–7 and Table 5–1, is

$$e_{sz} = 1 - 0.4 \times (1 - 0.85) \times (2.25 - 0.5) = 0.895$$

The safety factor may be taken as $n = 1.5$. The design stress is then

$$S_d = \frac{34,000 \times 0.895}{1.5} = 20,300 \text{ psi}$$

The stress area, from Table 11–1, is 1.404 sq in., and the maximum safe load is

$$F = 20,300 \times 1.404 = 28,500 \text{ lb}$$

According to equation 11–28, the initial load produced by screwing the nut tight is of the order of

$$F' = 16,000 \times 1.5 = 24,000 \text{ lb}$$

[7] A. Thum and H. Wiegand, "Die Dauerhaltbarkeit von Schraubenverbindungen und Mittel zu ihrer Steigerung," *Z.VDI*, Vol. 77 (1933), pp. 1061–63.

If a separation of the bolted surfaces must be avoided, the useful load should not exceed

$$F_1 = 0.8 \times 24,000 = 19,200 \text{ lb}$$

Sudden load. When the load is applied suddenly, the stress concentration must be taken into account by using a factor $K' = 2.85$. However, this factor does not apply to the static stress created by screwing down the nut. This stress can be estimated as

$$s_1 = \frac{F'}{A} = \frac{24,000}{1.404} = 17,100 \text{ psi}$$

The relation between the stress s_1 and the stress s_2 set up by the outside force may be expressed as

$$s_1 + K's_2 = S_d$$

From this equation

$$s_2 = \frac{(S_d - s_1)}{K'} \tag{11-50}$$

And for this case

$$s_2 = \frac{(20,300 - 17,100)}{2.85} = 1,120 \text{ psi}$$

According to equation 3–21, the stress created by a sudden load is twice as high as that due to the static load. The nominal allowable static stress is therefore lowered to

$$s_2' = \tfrac{1}{2} \times (17,100 + 1,120) = 9,110 \text{ psi}$$

and the external sudden load on the bolt must not exceed

$$F_2 = 9,110 \times 1.404 = 12,830 \text{ lb}$$

or only 45 per cent of the static load of 28,500 lb.

Impact loading. The allowable nominal stress with an impact load can be computed by equation 3–21. The travel of the impact force is given as $\frac{1}{16}p$, where p is the pitch of the screw. From Table 11–1, $n = 6$. From equation 11–1,

$$h = \frac{1}{6 \times 16} = 0.01042 \text{ in.}$$

The unthreaded length of the screw, which is 4½ in., takes the shock. The stresses in equation 3–21 should therefore be referred to the unweakened area, which is

$$A' = 0.7854 \times 1.5^2 = 1.767 \text{ sq in.}$$

The allowable static load, taking into account stress concentration, is

$$F_3 = \frac{F}{K'} = \frac{28,500}{2.85} = 10,000 \text{ lb}$$

and the maximum stress in the unweakened section is

$$s' = \frac{10,000}{1.767} = 5,660 \text{ psi}$$

If these values of h and s' are substituted in equation 3–21, the result is

$$5,660 = s\left[1 + \sqrt{1 + \frac{2 \times 0.01042 \times 30,000,000}{4.5s}}\right]$$

Solving for s gives $s = 214$ psi. Thus the allowable impact load is

$$F_3' = 214 \times 1.767 = 378 \text{ lb}$$

which is only 1.3 per cent of the maximum static load of 28,500 lb.

Repetitive loads. For ductile materials the stress-concentration factor K_r is small and increases with a decrease of ductility. Since for steels the

ductility decreases with an increase of strength, curve d in Fig. 5–8 may be used for all sizes of screws.

EXAMPLE 11–5. Determine the diameter d_1 of the end of the piston rod, Fig. 11–33, of a double-acting steam engine. The cylinder bore is 18 in.; the maximum steam pressure is 170 psi; and the steam is exhausted to the atmosphere.

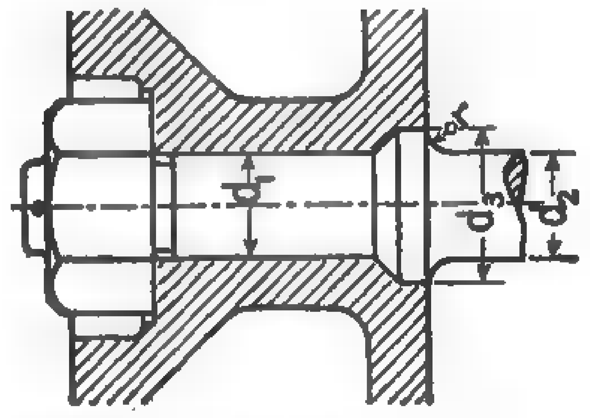


FIG. 11-33. Piston-rod end.

A suitable material is SAE 1045 steel, the endurance diagram of which is given in Fig. 4–5.

The force acting on the full face of the piston is

$$F = 0.7854 \times 18^2 \times 170 = 43{,}300 \text{ lb}$$

On the crank side, because of the presence of the piston rod, the area of the piston face will be about 1.5 per cent smaller than that on the head side, so that $F' = 43{,}300 \times 0.985 = 42{,}650$ lb.

In order to prevent the piston from being lifted from the conical seat, the stress set up by tightening the nut must be about 25 per cent higher than that coming from the steam pressure. This stress corresponds to an additional load on the rod end:

$$F'' = 0.25 \times 42.650 = 10{,}660 \text{ lb}$$

With a static load and with $S_e = 51{,}000$ psi, and with a safety factor of 2, the design stress is $S_d = 25{,}500$ psi. Thus the minimum stress area of the thread is

$$A_s = \frac{42{,}650 + 10{,}660}{25{,}500} = 2.09 \text{ sq in.}$$

The next-larger stress area from Table 11–1 is 2,497 sq in., which corresponds to a major diameter of 2 in. Anticipating the influence of the size factor, assume a thread one size larger, or $2\frac{1}{4}$ in. The rod diameter will be about $d_3 = 3$ in., and the size factor, from Table 5–1, will be $e_{ss} = 0.85$.

The corrected stress area is

$$A_s = \frac{53{,}310}{25{,}500 \times 0.85} = 2.455 \text{ sq in.}$$

which corresponds to a $2\frac{1}{4}$-in. thread.

With a nut tightened as here assumed, the stress in the end section of the rod will not change with the fluctuation of the steam pressure, and static-load conditions will exist. So it is not necessary to take into consideration the stress concentration in the threads.

EXAMPLE 11–6. Find the size of thread necessary for the case of example 11–5 when the nut is tightened only enough to induce a stress equal to one-half that caused by the steam pressure.

In this case the varying loads on the rod are

$$F_1 = 42{,}650 \text{ lb}$$

and

$$F_2 = \tfrac{1}{2} \times 42{,}650 = 21{,}325 \text{ lb}$$

By equation 5–34,

$$\frac{S_a}{S_m} = \frac{42{,}650 - 21{,}325}{42{,}650 + 21{,}325} = 0.333$$

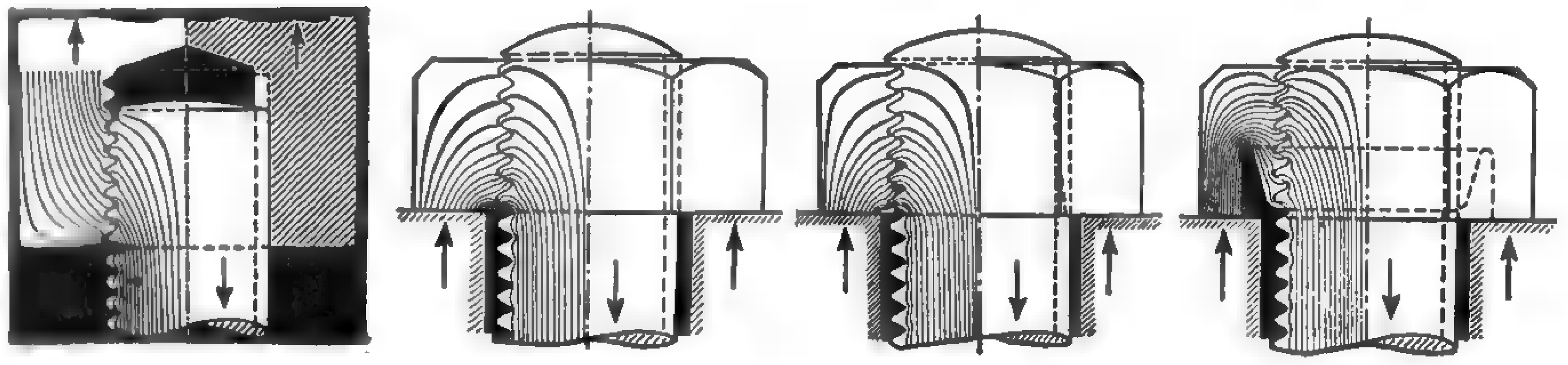
An inclined line drawn in Fig. 4–5, as was done in Fig. 5–10, intersects the upper stress line at the elastic limit and gives $S_a = 12{,}500$ psi. Using the same size factor of 0.85 gives $S_a' = 10{,}620$ psi.

The stress-concentration effect of the threads, taken into account by using curve d, Fig. 5–8, and assuming that $S_u = 92{,}500$ psi, is $e_{sr} = 0.74$. With $n = 2.0$ and $K_r = 1$, since the effect of the threads is taken into account as a surface effect, the required area is

$$A_s = \frac{(42{,}650 - 21{,}325) \times 2}{2 \times 0.74 \times 10{,}620} = 2.71 \text{ sq in.}$$

instead of $A_s = 2.455$ sq in. for a static load, even though the maximum load is reduced from 53,310 lb to 42,650 lb.

Force flow. The endurance limits of screw joints depend to a great extent on the flow of forces in the material. The difference in force flow in various cases is shown in Fig. 11–34. In a stud, Fig. 11–34a, the flow is deflected but keeps the same direction; in a standard bolt and nut, Fig. 11–34b, the force flow is turned 180 deg and the lower threads carry a much greater load than the upper ones. By removing part of the threads with a conical reamer, Fig. 11–34c, the force flow is deflected upward, relieving the stress in the lower threads and increasing it in the upper ones, and thus gives a more uniform stress distribution. The same result is obtained, but to a greater degree, by undercutting the bottom of the nut, Fig. 11–34d, to about one-half its height. If the endurance of an ordinary bolt and nut, Fig. 11–34b, is taken as unity, the conical boring of the nut, Fig. 11–34c, increases the limit about 20 per cent,[8] and the undercutting, Fig. 11–34d, increases it about 30 per cent. The strength of a screwed-in stud end, Fig. 11–34a, is about 95 per cent greater.



(a) (b) (c) (d)

FIG. 11-34. Force flow in different types of bolts and nuts.

Increase of endurance. The chief methods by which the endurance limit of screw stock can be raised, and the danger of failure of a screw through progressive fracture decreased, are as follows:

a. Nitriding of the bolt, which practically eliminates the stress-concentration effect of the threads
b. Creating a compressive stress at the root of the threads by rolling them instead of cutting them,[9] and thus decreasing stress concentration
c. Blending the thread and stock by a large-radius fillet, as shown in Fig. 11–35, and counterboring the hole for a stud so as to remove bending stresses from the last thread
d. Boring the nut conically or undercutting it

[8] *Ibid.*

[9] H. F. Moore and P. E. Henwood, *The Strength of Screw Threads under Repeated Tension*, Bulletin No. 264, University of Illinois Engineering Experimental Station (1934), pp. 12 and 15.

e. Using a lock nut or any other device that prevents the nut from unscrewing and maintains a certain initial tension in the bolt, thereby eliminating rattling and impact effect

A stud may break as a result of progressive fracture at the end of the threads, as in Fig. 11–35. Cutting away the stock diameter to less than the minor thread diameter is helpful if the large-radius fillet extends beyond the first female thread. Counterboring the hole removes the bending stress from the dangerous section.

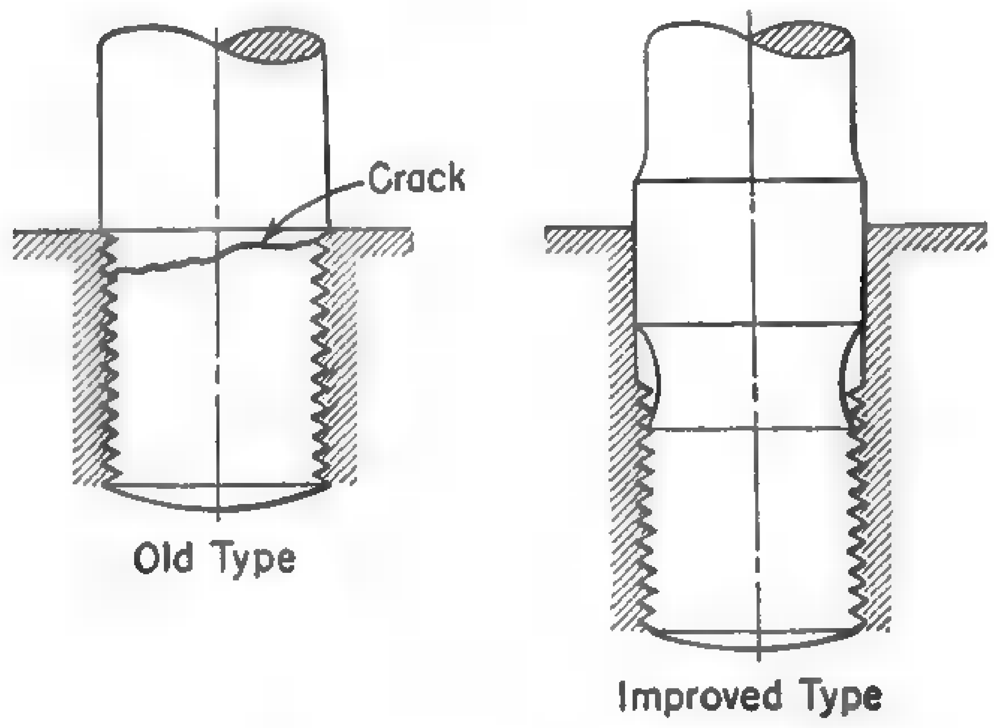


FIG. 11–35. Stress-relieving of studs.

11–13. Design of screw fastenings. Screw fastenings include all types of threaded parts.

Bolts. It is not possible to determine the initial stresses produced in bolts when screwing them up. This is particularly true of sizes up to and including $\frac{3}{4}$ in. To overcome this handicap it is general practice to take the external force alone as the total load on a bolt, and to neglect the initial stress and allow a low working stress instead. Thus Unwin[10] recommends the following formulas for the allowable stresses in bolts of ordinary steel used to make a fluid-tight joint: For rough joints,

$$S_d = 1{,}600(d^2+1) \tag{11–51}$$

and for faced joints,

$$S_d = 2{,}500(1.2d^2+1) \tag{11–52}$$

These values seem to be rather low, but they are fairly close to those given by Carl Bach for the stresses in bolts of $\frac{1}{2}$-in. to 1-in. sizes. Refigured for the same material, the allowable stress for average workmanship (rough joints) given by Bach is 2,100 psi for $\frac{1}{2}$-in. bolts. It increases with the size of the bolt up to 4,600 psi for 1-in. bolts and over. For high-grade workmanship (faced joints) Bach raises the allowable stress for average workmanship by 25 per cent.

Factors of safety. It is customary to compute the allowable design stress by taking the factor of safety n as 2 for low-carbon steels and copper-base alloys, and as 1.5 for high-strength alloy steels.[11] These figures apply to good workmanship and to screws with d not less than 1 in. For inferior workmanship the allowable stresses must be multiplied by 0.85; and for poor workmanship, by 0.7. For screws under 1 in. the design stresses should be

[10] W. C. Unwin and A. N. Mellanby, *The Elements of Machine Design*, rev. ed., Part I (London: Longmans, Green & Company, 1927), pp. 198–204.

[11] C. Höhner, "Richtlinien für Schrauben und Verschraubungen," *Z. VDI*, Vol. 78, (1933), p. 299.

TABLE 11–7

PROPERTIES OF MATERIALS FOR BOLTS AND NUTS

Material	Class	Heat Treatment	Average Values in Tension: Ultimate Strength (psi)	Elastic Limit (psi)	Elongation in 8 in. (%)
Open-hearth steel, SAE 1120, rolled or forged	B	Anneal	62,000	34,000	28
Open-hearth carbon or nickel steel, SAE 2330, rolled	A	.....	80,000	43,000	23
Open-hearth carbon or nickel steel, forged	A	Anneal; oil-tempered	85,000	53,000	25
Open-hearth nickel steel, SAE 3140, forged	High grade	Anneal; oil-tempered	100,000	68,000	21
G-metal: Cu, 88%; Sn, 10%; Zn, 2%; cast	....	.....	40,000	12,000	25
Phosphor bronze, hot-rolled	....	.....	65,000	35,000	23
Monel metal, hot-rolled	....	.....	85,000	35,000	40

lowered. For $\frac{1}{2}$-in. screws, they should be about one-half the values just given.

When a number of bolts work together, especially if conditions may cause a great difference in their tightnesses, still lower design stresses should be used.

Safety stop. Sometimes it may be desirable to make one of the bolts the weakest part of the machine, so that when the machine is overloaded the bolt will break and the machine will stop. In such a case the breaking load of the bolt should be equal to the load that causes the weakest member of the machine connected by this bolt to be stressed close to the elastic limit.

Materials. Table 11–7 shows the main properties of various steels and noncorrosive copper alloys suitable for making bolts, studs, screws, and other threaded parts.

High temperature. Mild steels with ultimate tensile strengths of less than 64,000 psi should not be used for bolts whose temperatures may exceed 570 F.[12] Harder steels can be used for temperatures up to 840 F. Special steels should be used for temperatures above 840 F.

The influence of temperatures above 392 F on the elastic limit of steel may be taken into account by decreasing the elastic limit at room temperature by 1 per cent for every 9 deg above 392 F, up to 932 F.

Design stresses prescribed by the ASME Boiler Construction Code for alloy-steel bolts for temperatures from 700 F to 950 F are given in Table 15–2.

Sizes. Machines should be designed so as to require the smallest possible number of sizes of bolts and screws, in order to reduce the number of drills

[12] *Ibid.*

TABLE 11–8

HOLDING POWER OF CUP-POINT SETSCREWS AT SURFACE (LB)*

Shaft Diameter (in.)	Diameter of Setscrews (in.)								
	$\frac{1}{4}$	$\frac{5}{16}$	$\frac{3}{8}$	$\frac{7}{16}$	$\frac{1}{2}$	$\frac{5}{8}$	$\frac{3}{4}$	$\frac{7}{8}$	1
	Recommended Tightening Torque (lb-in.)								
	65	150	200	300	500	800	1,500	2,000	3,000
1	850	1,500	. . . .	. . . .	. . . .	. . . .	. . . . .	. . . . .	. . . . .
$1\frac{1}{4}$ . . .	985	1,710	1,890	. . . .	. . . .	. . . .	. . . . .	. . . . .	. . . . .
$1\frac{1}{2}$ . . .	1,030	1,920	2,090	2,500	. . . .	. . . .	. . . . .	. . . . .	. . . . .
$1\frac{3}{4}$ . . .	. . . .	2,130	2,290	2,740	3,300	. . . .	. . . . .	. . . . .	. . . . .
2	. . . .	. . . .	. . . .	2.975	3,610	4,000	. . . . .	. . . . .	. . . . .
$2\frac{1}{2}$ . . .	. . . .	. . . .	. . . .	. . . .	4,190	4,600	5,650	. . . . .	. . . . .
3	. . . .	. . . .	. . . .	. . . .	. . . .	5,170	6,330	6,600	. . . . .
4	. . . .	. . . .	. . . .	. . . .	. . . .	. . . .	7,700	7,800	8,300
5	. . . .	. . . .	. . . .	. . . .	. . . .	. . . .	. . . . .	9,000	9,500
6	. . . .	. . . .	. . . .	. . . .	. . . .	. . . .	. . . . .	. . . . .	10,750
Flat surface	1,150	2,240	3,000	3,750	6,150	8,950	12,500	14,000	16,300

* Courtesy of the Allen Manufacturing Company, Hartford, Connecticut.

and taps needed in manufacturing them and the number of wrenches required to service them.

Location of bolts. Screw fastenings are generally subjected to very high stresses. Therefore a good design should avoid any additional stresses that may arise from an inappropriate location of the fastening. Thus the adjusting screw in Fig. 11–16 is subjected to an additional bending stress due to a bending moment equal to the product of the force and the lever arm e. By decreasing the distance e the additional stress can be lowered. When a threaded rod holds together the parts of a split flywheel or pulley rim, as in Fig. 11–36, centrifugal force creates a similar unavoidable bending stress in the rod under the nuts. If the rod fits the holes in the lugs tightly, the bending action will become greater, but it will be confined to the shank in the middle, where the bolt is not weakened by threads. In all such cases it is very difficult to determine the additional load. The only practical way to cope with the situation is to lower the design stress by a certain amount.

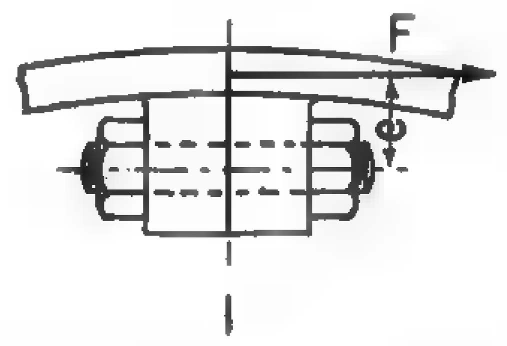


FIG. 11–36. Split-pulley rim.

A case of additional shear stress is illustrated in Fig. 11–28. If the bolts alone are depended upon to resist the downward force F, they must be carefully fitted to insure that each bolt receives its full share of the shearing load. Only through bolts can be used in such a case, because tap bolts or studs cannot be fitted accurately. A better design is to have either dowel pins or a projecting lip a take the shear. In this case the bolts need not fit the holes closely, and tap bolts or studs can be used.

Assembling. The designer of a screw fastening should make sure that all heads and nuts are accessible for tightening, and that room is provided for the insertion of standard wrenches or screwdrivers. When through bolts or threaded rods are used, provision should be made to prevent them from turning when the nuts are tightened.

11–14. Setscrews. Table 11–8 gives the engineering data for cup-point setscrews with hexagonal socket heads made of molybdenum alloy steel having a tensile strength of 250,000 psi. In computing the size and number of setscrews to be used instead of a shaft key, the holding power listed in Table 11–8 must be divided by a safety factor of 1.5 to 2.0. The holding power of a setscrew made of steel of a lower grade is usually given as about one-sixth that shown in Table 11–8. A headless setscrew has a still smaller holding power.

The disadvantage of the cup point is that it raises a burr on the shaft, thus making the removal of the piece more difficult. Filing a flat spot under the point overcomes this difficulty. If a conical point is used, a conical hole should be drilled in the shaft.

CHAPTER 12

Keys, Pins, and Cotters

12–1. Types of keys. The main function of a key is to transmit torque between a shaft and a machine part assembled on it. In most cases keys prevent relative motion, both rotary and axial. In some constructions they allow an axial motion between the shaft and the hub; such keys are called *feather keys*, or *spline keys*. In spite of the tendency to standardize, there are over a dozen key types in use by various manufacturers.

According to various characteristics, keys may be distinguished as straight and tapered; rectangular, dovetailed, chamfered, round, and disk-shaped; radial and tangential; and (according to their use) light-duty and heavy-duty. However, the distinguishing features so overlap that a single, all-embracing classification is impossible. Keys of various types will be briefly described and illustrated, but they will be classified only according to their intended duty.

Light-duty and medium-duty keys. Keys used for light duty and medium duty are shown in Fig. 12–1.

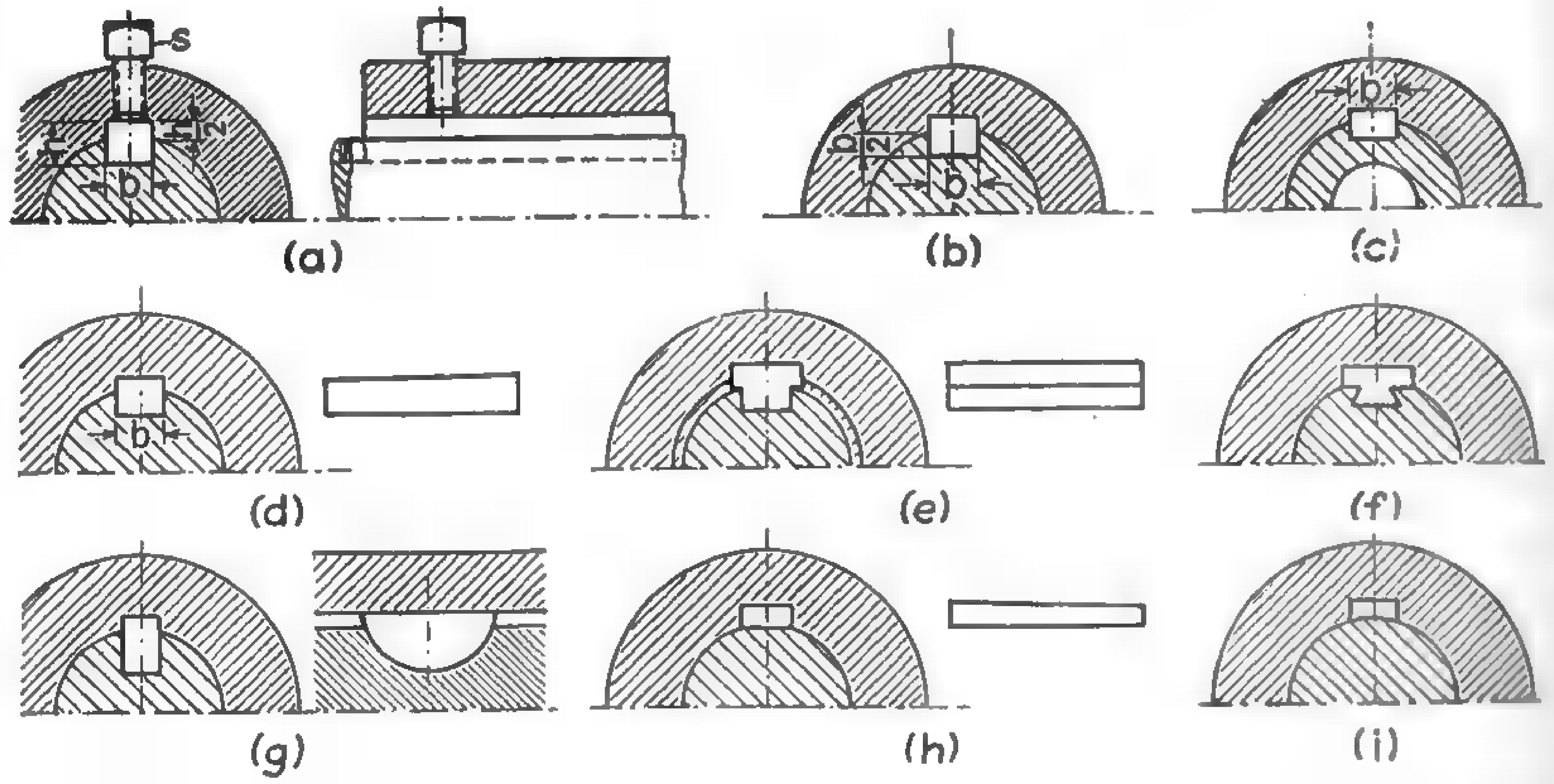


FIG. 12–1. Shaft keys for light and medium duty.

A straight *square key*, Fig. 12–1a, is a standard type used extensively for light duty. A modification of this key is the *rectangular key*, Fig. 12–1b, which has a standard depth in the shaft but is shallow above the shaft for use with a thin hub. Another modification is the key shown in Fig. 12–1c, which is shallow both ways for a thin hub and a hollow shaft or sleeve.

The *standard taper key*, Fig. 12–1d, is made also with a gib head to make the removal of the key easier. A taper key can be used for medium duty or a variable torque.

The *two-width key*, Fig. 12–1e, which fits two sizes of keyseats, is used with a thin bushing and standard-size keyseats in the hub and shaft.

The *dovetailed key*, Fig. 12–1f, is fitted in the shaft to prevent its working loose, and is used as a feather key.

The *Woodruff key*, Fig. 12–1g, is used for small torques to avoid troublesome fitting. The keyseat in the shaft is milled with a special cutter. The keyseat in the hub is usually straight.

The *flat key*, Fig. 12-1h, is used for light duty when it is not desired to cut a keyseat in the shaft. Usually the key has a taper of $\frac{1}{8}$ in. per ft. A straight key can be used but it requires a setscrew to hold it in place, as in the type of key shown in Fig. 12–1a.

The *saddle key*, Fig. 12–1i, depends upon friction alone to transmit the torque. It is used only with very light loads or for temporary service, as in setting eccentrics or cams. Usually it has a taper of $\frac{1}{8}$ in. per ft. If it is made straight, it requires a setscrew.

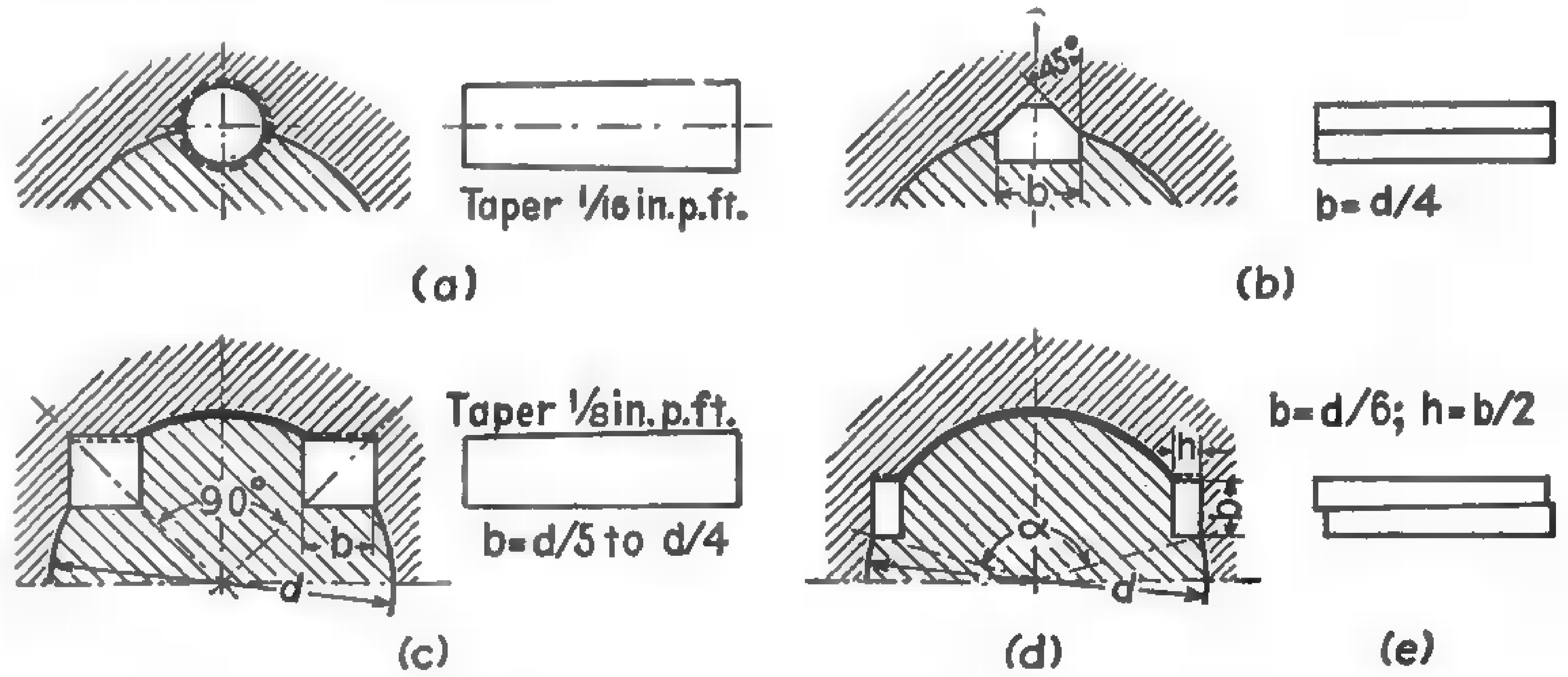


FIG. 12–2. Shaft keys for heavy duty.

Heavy-duty keys. Keys used for heavy duty are shown in Fig. 12–2.

The *round key*, or *pin key*, Fig. 12–2a, also called the *Nordberg key*, was originally used for light and small work. When made tapered, however, it proved satisfactory for heavy duty, because it can easily be fitted accurately.

The *Barth key*, Fig. 12–2b, is a rectangular key with two corners beveled off. This key does not require a tight fit, because the torque itself tends to force the key into its seat and, producing a compressive stress instead of shear, does not tend to turn the key in its seat. Another form of this key is turned 180 deg, so that the beveled sides are in the shaft instead of in the hub. The Barth key is used mostly as a feather key, and it is also used to replace a rectangular feather key that has given trouble.

The *Kennedy key*, Fig. 12–2c, consists of two rectangular taper keys driven from opposite ends of the hub. Diagonals through the keys to the shaft

TABLE 12-1

STANDARD DIMENSIONS OF STRAIGHT KEYS

DIAMETER OF SHAFT D (INCLUSIVE) (IN.)	KEY DIMENSIONS (IN.)			DIAMETER OF SHAFT D (INCLUSIVE) (IN.)	KEY DIMENSIONS (IN.)		
	Width b	Thickness			Width b	Thickness	
		Standard h	Flat h'			Standard h	Flat h'
$\frac{1}{2}$ – $\frac{9}{16}$	$\frac{1}{8}$	$\frac{1}{8}$	$\frac{3}{32}$	$3\frac{3}{8}$–$3\frac{13}{16}$	$\frac{7}{8}$	$\frac{7}{8}$	$\frac{5}{8}$
$\frac{5}{8}$ – $\frac{7}{8}$	$\frac{3}{16}$	$\frac{3}{16}$	$\frac{1}{8}$	$3\frac{7}{8}$–$4\frac{11}{16}$	1	1	$\frac{3}{4}$
$\frac{15}{16}$–$1\frac{1}{4}$	$\frac{1}{4}$	$\frac{1}{4}$	$\frac{3}{16}$	$4\frac{3}{4}$–$5\frac{11}{16}$	$1\frac{1}{4}$	$1\frac{1}{4}$	$\frac{7}{8}$
$1\frac{1}{16}$–$1\frac{3}{4}$	$\frac{3}{8}$	$\frac{3}{8}$	$\frac{1}{4}$	$5\frac{3}{4}$–$6\frac{15}{16}$	$1\frac{1}{2}$	$1\frac{1}{2}$	1
$1\frac{13}{16}$–$2\frac{1}{4}$	$\frac{1}{2}$	$\frac{1}{2}$	$\frac{3}{8}$	7–$9\frac{3}{4}$*	$1\frac{3}{4}$	$1\frac{3}{4}$	1
$2\frac{1}{16}$–$2\frac{13}{16}$	$\frac{5}{8}$	$\frac{5}{8}$	$\frac{7}{16}$	10–$12\frac{1}{2}$*	2	2	1
$2\frac{7}{8}$ –$3\frac{5}{16}$	$\frac{3}{4}$	$\frac{3}{4}$	$\frac{1}{2}$				

* Nonstandard.

center form an angle of 90°. Like the Barth key, this key works in compression; two keys are required to transmit the torque in both directions. It is used for heavy duty, as in rolling mills.

The *Lewis key*, Fig. 12–2d, consists of two tangential taper keys. This key is similar in characteristics to the Kennedy key but is much easier to fit and is used for medium and heavy service. The key is subjected to practically pure compression in the direction of its width b and therefore does not loosen in operation under the heaviest torque and shock action.

The *double-taper tangential key*, Fig. 12–2e, is a modification of the Lewis key and is used for heavy duty in place of the Kennedy key.

12–2. Key data. As already pointed out, keys are made either straight or tapered. A straight key transmits torque by means of shear and compression. It does not disturb the alignment of the keyed parts, but this advantage is offset by the requirement of an accurate fit on the sides. Moreover, it cannot transmit a heavy torque. A tapered key transmits torque chiefly through friction. By taking up any play between the shaft and the hub, it has a tendency to throw the pulleys or gears out of alignment. It can transmit a higher torque, however, and it eliminates any axial motion between the hub and the shaft.

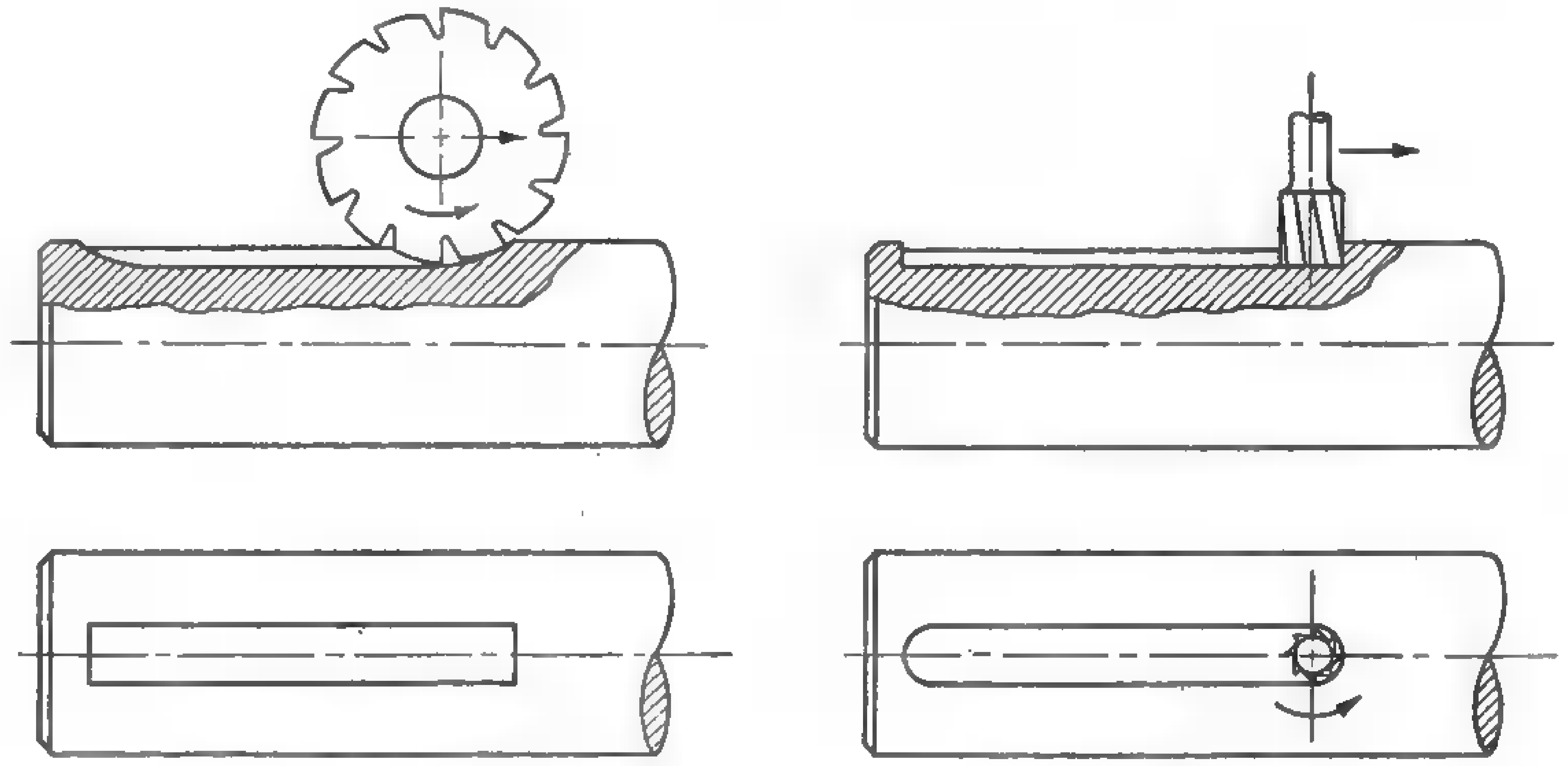
Square keys. In Table 12–1 are shown the dimensions of standard straight keys.[1] No standards have been established for shafts over 6 in; the data in Table 12–1 correspond to average practice. The keys are made square up to $b = 1\frac{1}{2}$ in.; in larger sizes h is often made smaller than b. In all sizes, b is made about equal to $\frac{1}{4}D$, where D is the shaft diameter.

The depths of the keyseats in both the shaft and the hub are made equal to one-half the key thickness, except for the type shown in Fig. 12–1b.

[1] Leaflets AESC B17b and AESC B17d, prepared by the American Society of Mechanical Engineers.

Every straight key is fitted on the sides of the keyseats and has a slight clearance at the top. To prevent any axial motion of the hub a setscrew s, Fig. 12–1a, or some other holding means, should be provided. A setscrew also reduces the wear of the whole connection if the torque varies.

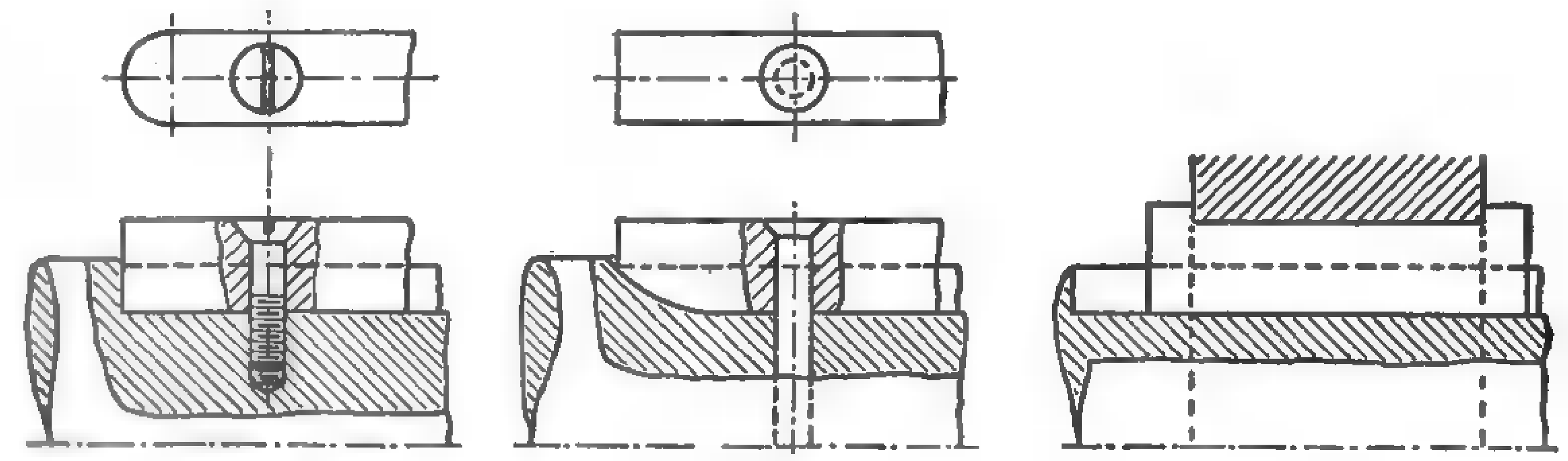
Keys made as in Fig. 12–1b have a thickness $h'' = \frac{1}{2}(h + h')$, where h' is the thickness of a shallow, or flat key, Fig. 12–1c, and is shown in Table 12–1.



(a) (b)

FIG. 12–3. Methods of cutting keyways.

Keyseats. In the hub the keyseat is cut by a special keyseater with a reciprocating motion of the cutter. In the shaft the keyseat is cut by a milling cutter that is either of the disk type, Fig. 12–3a, or of the end type, Fig. 12–3b. The keyseat in Fig. 12–3a is cheaper to make. It is also better, because it does not weaken the shaft by stress concentration as much as the keyseat in Fig. 12–3b, though the latter does hold the key in place axially.



(a) (b) (c)

FIG. 12–4. Methods of fastening feather keys.

Feather keys. A feather key is used when the hub must slide along the shaft. The key is fastened to the shaft by countersunk machine screws, as shown in Fig. 12–4a, or by through-pins riveted over, Fig. 12–4b. If the keyseat extends to the end of the shaft, a key with end lips, Fig. 12–4c, may be used; it slides along the seat with the hub. When the key is fastened to the shaft, the keyseat may be cut either by an end miller, resulting in a keyway

like that shown in Fig. 12–4a, or by a circumferential cutter, as in Fig. 12–4b.

Standard taper key. The thick end of a standard taper key has the dimensions given in Table 12-1 for straight keys. The standard taper is $\frac{1}{8}$ in. per foot of length and starts at a distance b from the end, Fig. 12–5. The keyseat in the shaft has the same depth as that used with a standard square or rectangular key, and only the keyseat in the hub is tapered. The deep end of the keyseat in the hub is cut slightly shallower than the corresponding projection of the key, to allow for fitting. The allowance is about $\frac{1}{64}$ in. for shafts up to 2 in., increasing slightly with the size of the shaft. The key is fitted at the top and bottom and is often relieved on the sides to make fitting easier. However, both theory and practical experience indicate that a taper key holds better when it is fitted accurately on the sides as well as on the top and bottom.

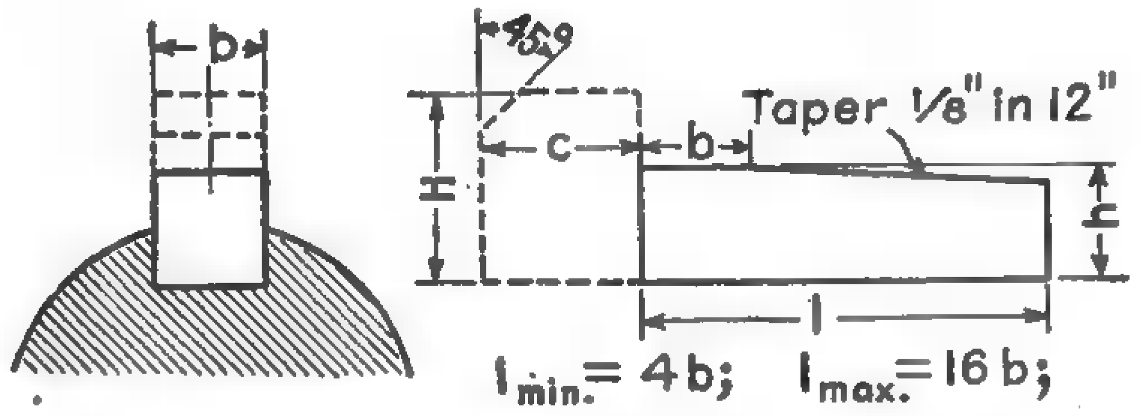


Fig. 12–5. Taper key.

The gib-head dimensions, Fig. 12–5, are approximately $c = 1.2b$ and $H = 1.7b$. The gib head is helpful for removing a key, but it is a dangerous protrusion on a rotating shaft and should be covered by a stationary guard or one fastened to the shaft. At present gib heads are not often used.

Woodruff keys. Woodruff keys, also known as Whitney stock-size keys, are used in shafts up to $2\frac{1}{2}$ in. in diameter. The extra depth of the keyseat, Fig. 12–6, weakens the shaft, but at the same time it precludes all possibility of the key tipping. Another advantage of this key is its tendency to adjust itself to a tapered keyseat in the hub. When a long hub must be secured, the depth of the keyway in the shaft can be decreased by using two or more Woodruff keys having the same thickness but a smaller diameter.

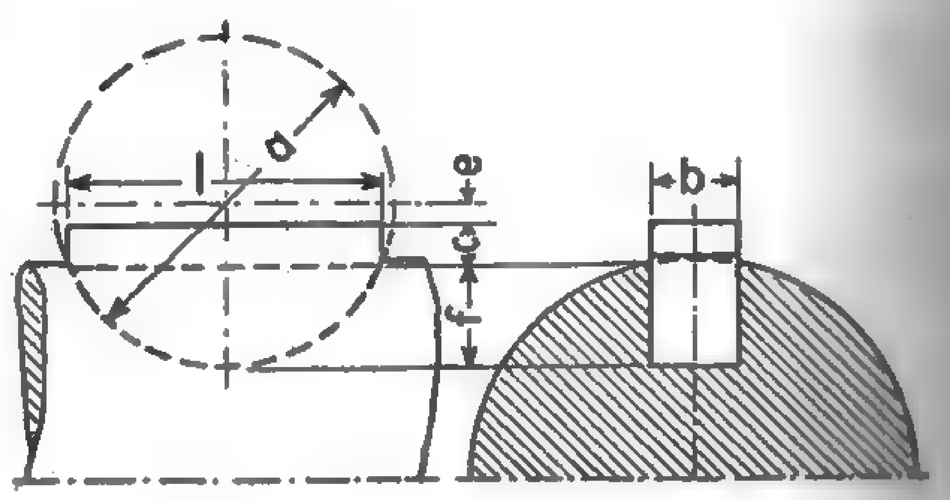


Fig. 12–6. Woodruff key.

The dotted circle in Fig. 12–6 illustrates how two keys are cut from a disk, which itself is cut from round bar stock. Table 12–2 gives dimensions of Woodruff keys of standard sizes. The depth of the keyway in the hub is $c = \frac{1}{2}b$, and the length is practically equal to the stock diameter, or $l = a$. The size of key to be used in a shaft with a certain diameter D is given by the manufacturers in a special table or may be determined by selecting a key having a thickness equal to or slightly greater than $0.17D$.

Other keys. The width b and the height above the shaft of other keys shown in Fig. 12–1 are made equal to the standard dimensions given in Table 12–1, in order to use hubs with standard keyseats.

TABLE 12–2

Dimensions of Standard Woodruff Keys (Fig. 12–6)
(inches)

No.	a	b	e	No.	a	b	e	No.	■	b	■
1....	$\frac{1}{2}$	$\frac{1}{16}$	$\frac{3}{64}$	12...	$\frac{7}{8}$	$\frac{7}{32}$	$\frac{1}{16}$	20...	$1\frac{1}{4}$	$\frac{7}{32}$	$\frac{5}{64}$
2....	$\frac{1}{2}$	$\frac{3}{32}$	$\frac{3}{64}$	A...	$\frac{7}{8}$	$\frac{1}{4}$	$\frac{1}{16}$	21...	$1\frac{1}{4}$	$\frac{1}{4}$	$\frac{5}{64}$
3....	$\frac{1}{2}$	$\frac{1}{8}$	$\frac{3}{64}$	13...	1	$\frac{3}{16}$	$\frac{1}{16}$	D...	$1\frac{1}{4}$	$\frac{5}{16}$	$\frac{5}{64}$
4....	$\frac{5}{8}$	$\frac{3}{32}$	$\frac{1}{16}$	14...	1	$\frac{7}{32}$	$\frac{1}{16}$	E...	$1\frac{1}{4}$	$\frac{3}{8}$	$\frac{5}{64}$
5....	$\frac{5}{8}$	$\frac{1}{8}$	$\frac{1}{16}$	15...	1	$\frac{1}{4}$	$\frac{1}{16}$	22...	$1\frac{3}{8}$	$\frac{1}{4}$	$\frac{3}{32}$
6....	$\frac{5}{8}$	$\frac{5}{32}$	$\frac{1}{16}$	B....	1	$\frac{5}{16}$	$\frac{1}{16}$	23...	$1\frac{3}{8}$	$\frac{5}{16}$	$\frac{3}{32}$
7..	$\frac{3}{4}$	$\frac{1}{8}$	$\frac{1}{16}$	16...	$1\frac{1}{8}$	$\frac{3}{16}$	$\frac{5}{64}$	F....	$1\frac{3}{8}$	$\frac{3}{8}$	$\frac{3}{32}$
8....	$\frac{3}{4}$	$\frac{5}{32}$	$\frac{1}{16}$	17...	$1\frac{1}{8}$	$\frac{7}{32}$	$\frac{5}{64}$	24...	$1\frac{1}{2}$	$\frac{1}{4}$	$\frac{7}{64}$
9....	$\frac{3}{4}$	$\frac{3}{16}$	$\frac{1}{16}$	18...	$1\frac{1}{8}$	$\frac{1}{4}$	$\frac{5}{64}$	25...	$1\frac{1}{2}$	$\frac{5}{16}$	$\frac{7}{64}$
10....	$\frac{7}{8}$	$\frac{5}{32}$	$\frac{1}{16}$	C...	$1\frac{1}{8}$	$\frac{5}{16}$	$\frac{5}{64}$	G...	$1\frac{1}{2}$	$\frac{3}{8}$	$\frac{7}{64}$
11....	$\frac{7}{8}$	$\frac{3}{16}$	$\frac{1}{16}$	19...	$1\frac{1}{4}$	$\frac{3}{16}$	$\frac{5}{64}$				

Round, or pin, keys. Taper-pin keys are fitted halfway into the shaft and hub, as shown in Fig. 12–2a. If a taper pin is used for light duty, it is advisable to use commercial standard taper pins. The taper of these pins is $\frac{1}{4}$ in. per ft, and the corresponding data may be taken from Table 12–4. When such a pin is used as a key, its large diameter d should comply with the relation

$$d = 0.6\sqrt{D} \text{ to } 0.7\sqrt{D} \tag{12–1}$$

where D is the shaft diameter.

Heavy-duty keys. The dimensions of a taper-pin key, as used by the Nordberg Manufacturing Company for heavy duty, are given in Table 12–3. The total taper of the reamer is $\frac{1}{16}$ in. per ft.

The width of a Barth key is given in Table 12–1, and the thickness is halfway between that of a square key and that of a shallow key.

TABLE 12–3

Dimensions of Nordberg Round Keys

Shaft Diameter D (in.)	Reamer (in.)		Shaft Diameter D (in.)	Reamer (in.)	
	Small Diameter a	Flute Length l		Small Diameter a	Flute Length l
2[illegible]-3..........	$\frac{3}{4}$	$4\frac{1}{4}$	7– 9..........	$1\frac{5}{8}$	$6\frac{7}{8}$
[illegible]$\frac{7}{16}$-$3\frac{1}{2}$..........	$\frac{5}{8}$	$4\frac{1}{2}$	10–12..........	2	8
[illegible]-4..........	1	$4\frac{7}{8}$	13–15..........	$2\frac{7}{16}$	$10\frac{1}{4}$
[illegible]-$4\frac{1}{2}$..........	$1\frac{1}{8}$	5	16–18..........	$3\frac{1}{8}$	12
5..........	$1\frac{1}{4}$	$4\frac{5}{8}$	19–21..........	$3\frac{11}{16}$	13
$5\frac{1}{2}$..........	$1\frac{3}{8}$	$4\frac{7}{8}$	22–24..........	$4\frac{1}{4}$	$14\frac{1}{4}$
6..........	$1\frac{1}{2}$	6[illegible]			

A Kennedy key is formed by using standard taper keys chosen according to Table 12–1. To facilitate erection the hub is first bored for a press fit with the shaft and is then rebored, the center being offset about $\frac{1}{64}$ in. to produce the clearance shown in Fig. 12–2c.

A Lewis key, Fig. 12–2d, gives excellent service in heavy work. However, since it is rather expensive to fit, it is not used extensively. The angle α is made equal to 150° or slightly smaller.

The keyseating and fitting are considerably simplified if each key is composed of two tapered keys with the tapers turned in opposite directions so as to make the outside edges parallel, as shown in Fig. 12–2e.

12–3. Strength of keys. In regard to stress analysis and wear, keys of all the various types can be classified in four main groups, as illustrated diagrammatically in Fig. 12–7.

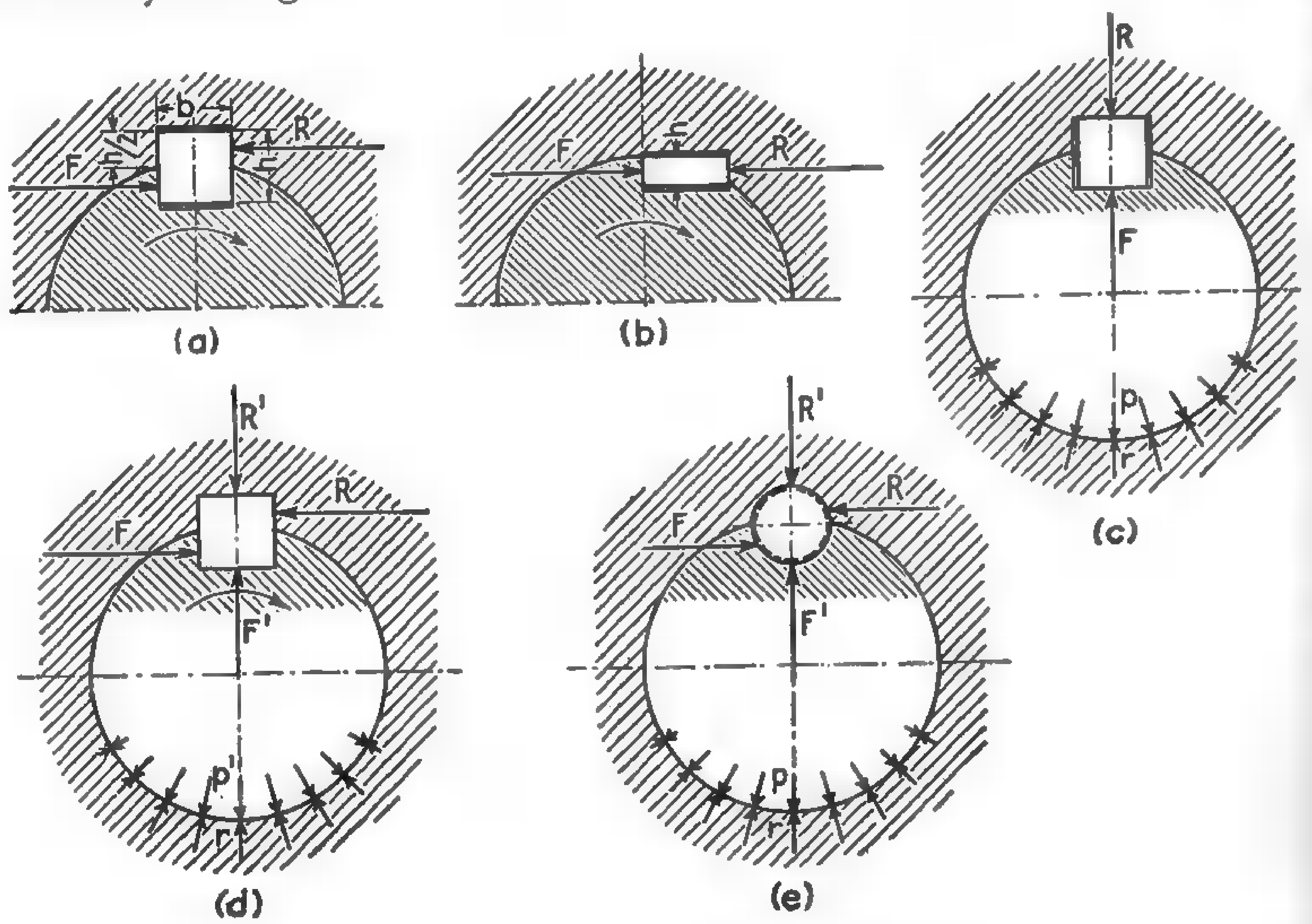


FIG. 12–7. Diagrams of key action.

a) Rectangular fitted keys, Fig. 12–7a, in which the torque is transmitted by means of compressive and shear stresses.

b) Tangential keys, Fig. 12–7b, in which the torque is transmitted by means of compressive stresses alone.

c) Tapered keys, Fig. 12–7c, in which the torque is transmitted by means of friction induced by compressive stresses.

d) Tapered keys fitted on the sides, Fig. 12–7d, in which torque is transmitted by a simultaneous action of compressive and shear stresses and friction. Round tapered pins, Fig. 12–7e, fall in this group.

Rectangular fitted key. Rectangular stock keys of machine steel are made to standard dimensions b and h, and the designer has to find only the length l

of the key necessary to transmit the given torque T. The stresses produced in a fitted key are crushing and shear, and both should be investigated.

Crushing strength. Since a hub is always much more rigid than a shaft, the shaft will be twisted by the torque, whereas the hub will remain practically undistorted. The torque has its maximum value at the point where the shaft enters the hub, and it gradually decreases toward the other end of the hub. The tangential pressure p per unit length of the key, Fig. 12–8, is proportional to the torque. If its maximum value where the shaft enters the hub is p_1, it decreases to p_2 at the end of the key, the length of which is l_2.

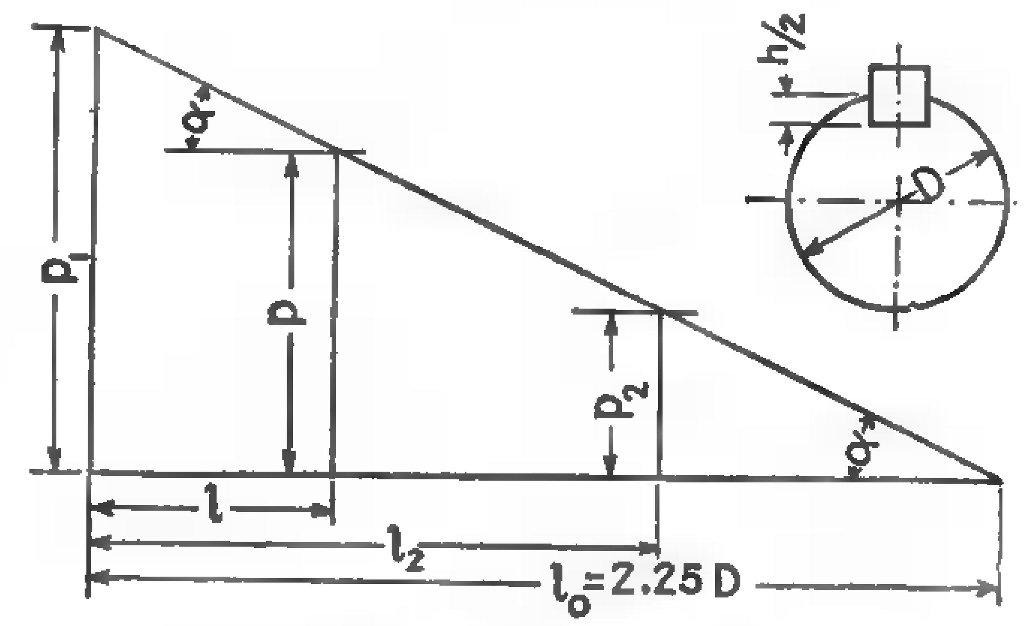


FIG. 12–8. Pressures between key and keyseat.

The tangential pressure p at any intermediate distance l from the hub edge can be computed from the relation

$$p = p_1 - l \tan \alpha \tag{12–2}$$

where $\tan \alpha = (p_1 - p_2)/l_2 = p_1/l_0$.

The torque T transmitted by the key can be determined from the elementary torque formula

$$dT = p \times dl \times \tfrac{1}{2}D \tag{12–3}$$

where D is the shaft diameter. By substituting the value of p from equation 12–2 in equation 12–3 and integrating it from $l = 0$ to $l = l_2$, the torque can be expressed by the equation

$$T = \tfrac{1}{2}p_1 D l_2 - D l_2^2 \tan \alpha \tag{12–4}$$

Since the pressure per unit length of the key is equal to the product of the bearing stress s_b and the area $0.5h$ of the key side 1 in. long, it follows that $p_1 = 0.5 s_{b1} h$. According to practical experience, a length of key over $2.25D$ is useless. If the tangential pressure p_2 is considered zero when $l_2 = l_0 = 2.25D$,

$$\tan \alpha = \frac{p_1}{l_0} = \frac{s_{b1} h}{4.5D} \tag{12–5}$$

In general the torque transmitted by the key is

$$T = \tfrac{1}{4}s_{b1} h D l_2 - \tfrac{1}{18}s_{b1} h l_2^2 \tag{12–6}$$

where s_{b1} is the nominal stress at the dangerous point.

The significant stress, according to equation 5–12, is the nominal stress times K', where the stress-concentration factor K' may be determined by equation 5–9. The theoretical stress factor K due to a concentrated load

may be taken as 4, as found for keys fitted at the sides.[2] For impact action, q may be taken as 0.4. For a well-fitted key the impact is reduced to a sudden load and q may be taken as 0.2, which gives a stress-concentration factor $K'=1.6$. For a steady load, $K'=1$. The safety factor n may be taken as 1.5 for a steady torque, and it should be increased up to 2.5 for a strongly fluctuating torque.

To determine the length l_2 of a square key required to transmit a certain torque by applying equation 12–6, the design stress S_d must be used instead of the stress s_{b1}. If solving equation 12–6 gives a negative quantity under the square root, this condition will indicate that one key is not enough to transmit the required torque T. If solving equation 12–6 gives a length l_2 less than D, a practical rule is to make l_2 equal to D.

Strength in shear. The resistance of a key to shear, when the flexibility of the shaft is taken into consideration, may be represented by the same diagram as that shown in Fig. 12–8, with the maximum unit resistance of $p_1 = s_{s1}b$, where s_{s1} is the maximum shear stress at the end of the key. The torque can also be determined by an equation similar to equation 12–6. If it is assumed that the maximum useful length of a key is $2.25D$,

$$\tan \alpha = \frac{p_1}{l_0} = \frac{s_{s1}b}{2.25D} \tag{12–7}$$

The torque becomes

$$T = \tfrac{1}{2}s_{s1}bDl_2 - \tfrac{1}{9}s_{s1}bl_2^2 \tag{12–8}$$

where s_{s1} is the nominal shear stress at the dangerous point. From equation 12–8,

$$s_{s1} = \frac{T}{l_2b(0.5D - 0.11l_2)} \tag{12–9}$$

The safety factor n may be taken as 1.5 to 2.5, the exact value depending on the character of the torque, but stress concentration may be neglected because of the low value of K'.

EXAMPLE 12–1. Find the dimensions of a square fitted key for a $3\frac{7}{16}$-in. steel shaft to transmit 95 hp at 200 rpm; the torque fluctuates and the key is well-fitted.

The torsional moment, or torque, is

$$T = \frac{63{,}030 \times 95}{200} = 29{,}900 \text{ lb-in.}$$

From Table 12–1 the width of the key is $b = \frac{7}{8}$ in., and its height is $h = \frac{7}{8}$ in. For SAE 1010 steel the elastic limit in compression is $S_e = 31{,}000$ psi, and from equation 5–14 $S_b = 62{,}000$ psi. The size coefficient according to equation 5–8 is

$$e_{sz} = 1 - 0.4 \times (1 - 0.84) \times (0.875 - 0.5) = 0.976$$

[2] A. G. Soiokian and G. B. Karelitz, "Photoelastic Study of Shearing Stresses in Keyways," *Transactions of the American Society of Mechanical Engineers*, Vol. 54 (1932), APM 54-10, p. 112.

Because the torque fluctuates, a safety factor n of 2.5 should be used. Hence the allowable bearing stress is

$$S_d = \frac{62{,}000 \times 0.976}{2.5} = 24{,}200 \text{ psi}$$

With a well-fitted key, $K'=1.6$, whence

$$S_d' = \frac{24{,}200}{1.6} = 15{,}100 \text{ psi}$$

Substituting this value for s_{b1} and the other data in equation 12–6 gives

$$29{,}900 = \tfrac{1}{4} \times 15{,}100 \times 0.875 \times 3.437 \times l_2 - \tfrac{1}{18} \times 15{,}100 \times 0.875 l_2^2$$

Solving this equation for l_2 gives two roots, 3.37 and 12.09. The first root is the answer; the second one is extraneous because it contradicts the condition that $l_2 \leqq 2.25D$. Therefore the proper key length is $3\frac{1}{2}$ in.

Check for shear: The maximum shear stress, by equation 12–9, is

$$s_{s1} = \frac{29{,}900}{3.5 \times 0.875 \times (0.5 \times 3.437 - 0.11 \times 3.5)} = 7{,}470 \text{ psi}$$

Since the elastic limit in shear is $S_{es}' = 20{,}000 \times 0.976 = 19{,}540$, the safety factor is

$$n = \frac{S_{es}'}{S_{s1}} = \frac{19{,}540}{7{,}470} = 2.62$$

This result indicates that the crushing stress, with $n = 2.5$, is more dangerous.

12–4. Taper key. The relation involving the circumferential force F_t, Fig. 12–9, and the pressure F between the shaft and the hub is

$$F_t = f_1F \tag{12–10}$$

where f_1 is the coefficient of friction between the shaft and the hub and can be taken as 0.25. But

$$F = blp \tag{12–11}$$

where p is the pressure, or compressive stress, in the key. Therefore the allowable pressure is only one-half that for a straight key. When the key is driven home, p may attain a high value.

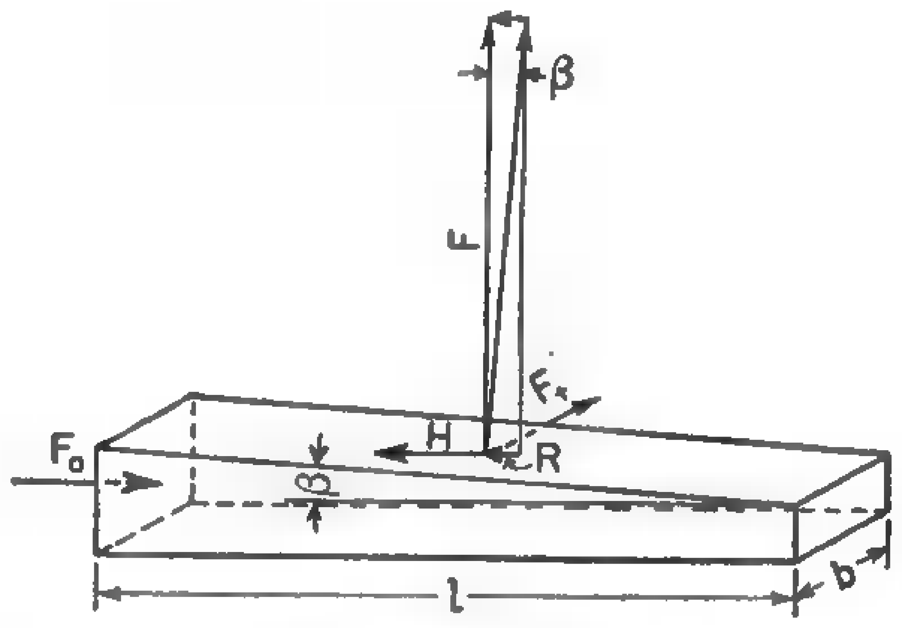


FIG. 12–9. Diagram of forces acting on a taper key.

Also, the relation between the torque T and the force F_t is

$$T = \tfrac{1}{2}F_tD \tag{12–12}$$

Hence

$$T = \tfrac{1}{2}f_1blpD \tag{12–13}$$

The necessary key length is then

$$l = \frac{2T}{f_1bpD} \tag{12–14}$$

The axial effort F_a, Fig. 12–9, necessary to drive the key home is

$$F_a = H + R = 2Ff_1 + F \tan \beta \tag{12–15}$$

It is a good practice to have the key greased. The coefficient of friction f_2 may then be taken as 0.10 on both the top and bottom surfaces of the key. If the taper is $\frac{1}{8}$ in. per ft, $\tan \beta = 0.0104$. From equation 12–11,

$$F_a = 0.21pbl \tag{12–16}$$

EXAMPLE 12–2. Find the dimensions of a tapered key for the same conditions as those in example 12–1.

From Table 12–1, the width is $b = \frac{7}{8}$ in. With a drive fit, a safety factor n of 1.5 is sufficient. The allowable pressure is

$$p = \frac{31{,}000 \times 0.976}{1.5} = 20{,}000 \text{ psi}$$

From equation 12–14, the required length is

$$l = \frac{2 \times 29{,}900}{0.25 \times 0.875 \times 20{,}000 \times 3.437} = 3.99 \text{ in., or 4 in.}$$

This is slightly greater than the length of a fitted straight key.

The effort required to drive the key home, by equation 12–16, is

$$F_a = 0.21 \times 20{,}000 \times 0.875 \times 4.0 = 14{,}700 \text{ lb}$$

Effect of elasticity. With a tapered key, as in the case of a fitted key, the shaft has a tendency to twist in the hub, especially if the torque fluctuates. The torque distribution is approximately the same as with a fitted key. As a result, where the torque is a maximum the shaft will move slightly in the hub. This motion produces a gradual wear of the surfaces pressed together. The wear extends the motion farther into the hub, and so on until the shaft becomes loose. A good fit on the sides limits the torsional motion of the shaft and lengthens the life of this key connection. If the torque fluctuates, however, the key will ultimately loosen. Therefore a tapered rectangular key is not satisfactory for a heavy and fluctuating torque; keys of other special types should be used.

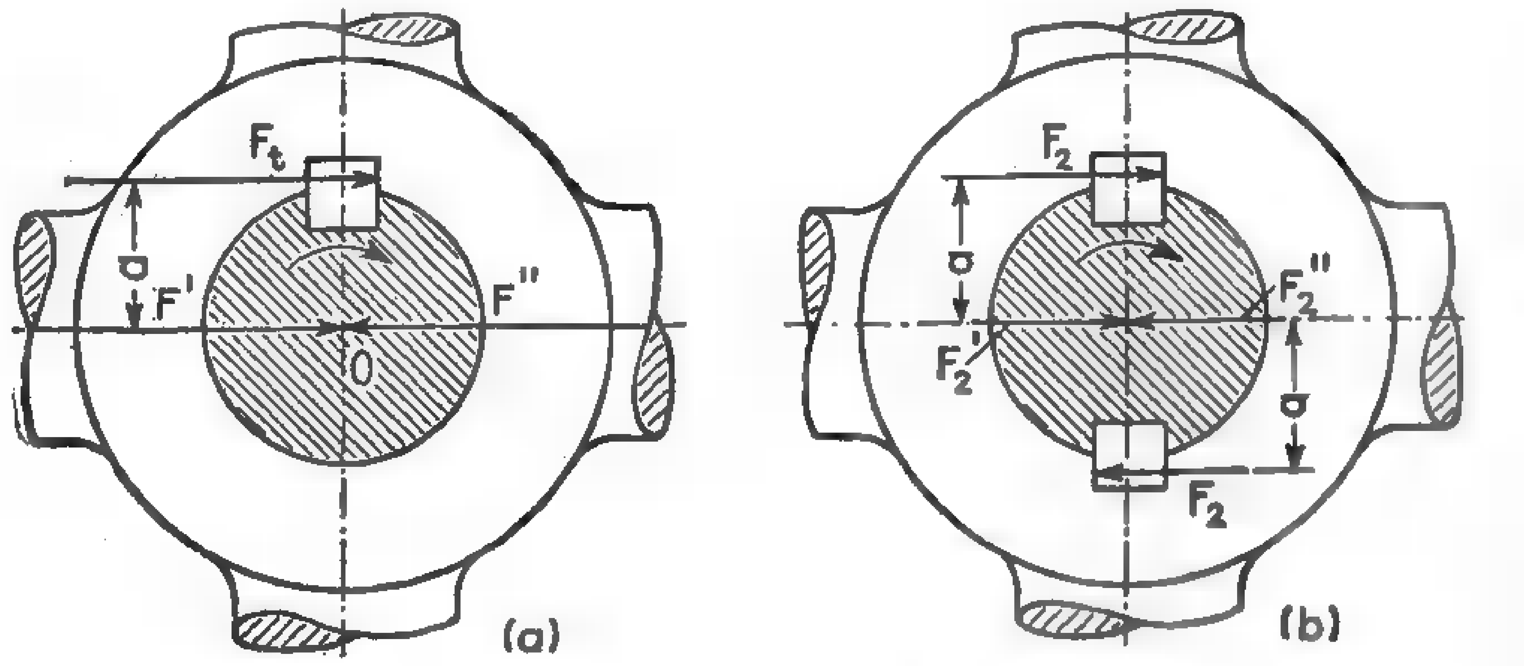


FIG. 12–10. Action of feather keys.

12–5. Friction of feather keys. When a hub connected to a shaft by a feather key and subjected to a torque T is moved along the shaft, it presents a resistance which can be found from Fig. 12–10a. The torque T produces a circumferential force, which is

$$F_t = \frac{T}{a} \tag{12–17}$$

Two opposite forces $F' = F'' = F_t$ applied in the center plane do not change the existing equilibrium but give a couple formed by the force F_t and F'' and a single force F'. The couple, whose moment is $F_t a = T$, tends to rotate the hub about the center O. The force F' presses the hub against the shaft. When the hub is shifted lengthwise, a resistance on the key and on the shaft must be overcome. This resistance is

$$R = fF_t + f_2F' \tag{12–18}$$

Since f and f_2 are approximately equal,

$$R = 2fF_t \tag{12–19}$$

If two feather keys are used, as in Fig. 12–10b,

$$T = F_2a + F_2a = 2F_2a \tag{12–20}$$

A comparison of equations 12–17 and 12–20 shows that

$$F_2 = \tfrac{1}{2}F_t \tag{12–21}$$

Two opposite forces $F_2' = F_2'' = F_2$ applied in the center plane give two couples, both tending to rotate the hub clockwise. When the hub is being shifted, the only resistance to be overcome is that on the keys:

$$R_2 = 2fF_2 \tag{12–22}$$

Substituting values of F_2 from equation 12–21 and F_t from equation 12–19 results in

$$R_2 = \tfrac{1}{2}R \tag{12–23}$$

The addition of the second key halves the shifting resistance by eliminating the eccentric application of forces. Naturally, a hub with two feather keys requires very accurate fitting in order to give the advantage disclosed by the preceding analysis.

12–6. Parallel-side splines. The development of the automobile required a connection between circular shafts and hubs that would be strong, light, and suitable for mass production. This demand led to the development of the integral spline shaft. The first standard spline connections were developed and adopted by the SAE in 1914. These splines consist of multiple integral keys with parallel sides milled on the outside surface of the shaft. The hub is bored to the small diameter of the shaft and has keyseats broached in it to receive the projecting ribs on the shaft, as shown in Fig. 12–11. Evidently this construction can be used only with hubs made of a material soft enough to be broached. The SAE has standardized four types of spline fittings, namely, the four-, six-, ten-, and sixteen-spline types. All dimensions are given as functions of the shaft diameter D, Fig. 12–11. The bore

of the hub is $d = D - 2h$. The height h of the spline for each type depends on the operating conditions. Fit A, which is recommended for a permanent fit, has the smallest height h. For fit B, which is recommended where the hub is to slide when not under load, the specified height is about 50 per cent greater than that for fit A. For fit C, which is recommended where the hub is to slide when under load, the specified height is about 100 per cent greater than that for fit A.

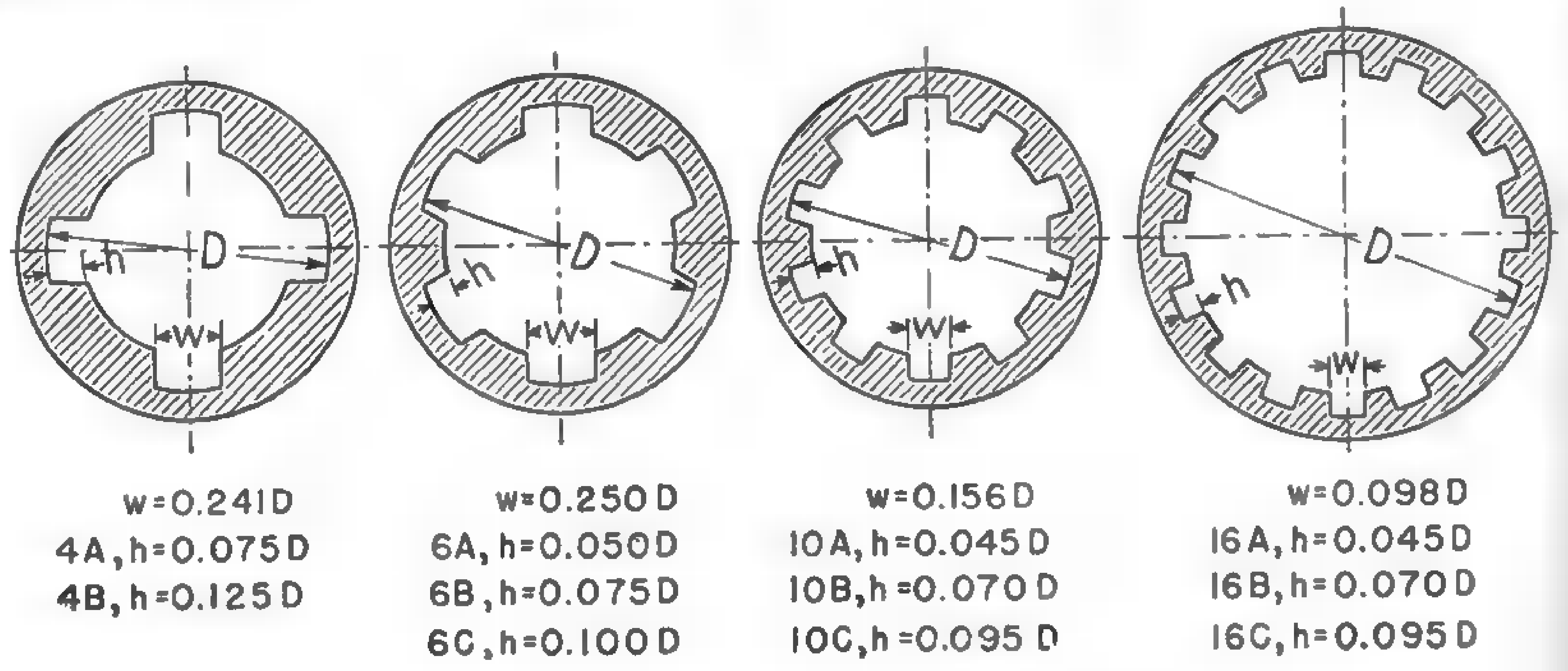


FIG. 12–11. SAE standard parallel-side splines.

The torque which an integral multispline shaft can transmit safely by pressure between the sides of the spline can be expressed as

$$T = \tfrac{1}{2}phli(D - h) \qquad (12\text{–}24)$$

where p is the allowable pressure, in pounds per square inch;
i is the number of splines;
l is the engaged length of each spline, in inches.

The length l required for a given torque T and a selected pressure p may also be found by equation 12–24.

The pressure p must be much lower than the elastic limit S_e in compression, to prevent wear of the sliding surfaces. Since p must be considered a bearing pressure, the very low pressure of 1,000 psi is recommended[3] for p for fit C. The value 2,000 psi may be used for fit B; and 3,000 psi is suitable for fit A.

From the standpoint of strength, tests have shown that the spline grooves weaken the shaft more in torsion than in bending, as was to be expected in view of the nature of the discontinuity. The elastic strength in torsion of a shaft with spline grooves is approximately equal to the strength of a straight shaft the diameter of which is $D' = D - 2h$. In bending the elastic strength is equal to the strength of a shaft for which $D' = D - h$.

Stress concentration. The main drawback of the integral spline connection with straight sides is the forming of cracks in the fillets a (Fig. 12–12) of the

[3] *SAE Handbook* (New York: Society of Automotive Engineers, 1953), pp. 605–7.

splined shaft if the torque fluctuates. According to the SAE standards the fillet radius should not exceed 0.015 in. This radius is very small, especially for the larger shafts, and results in a high form-stress factor K, Fig. 3–31. In ductile materials the factor of sensitivity q is low, giving a moderate stress-concentration factor K'. In brittle materials, such as tempered steel, q is high, ranging up to 0.6, and the factor K' is high.

12–7. Involute splines. Lately there has been an increasing tendency to use involute stub splines instead of straight-side splines, both for automotive and general machinery applications. These splines are produced by the methods and tools used for cutting involute gears. A standard pressure angle of 30° is used. Involute splines are considerably stronger than straight-side splines, and no stress concentrations or cracks, such as those shown in Fig. 12–12, ever occur.

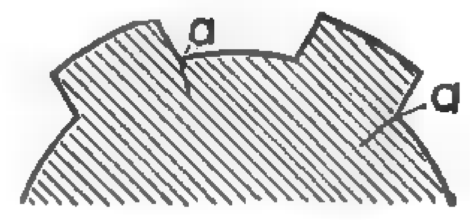


FIG. 12–12. Failure of a spline shaft.

The SAE has established fifteen standard pitches, with diametral pitches from $\frac{1}{2}$ to $\frac{48}{96}$ and with 6 to 50 teeth. The first figure in the pitch designation is the diametral pitch p_d, which determines the pitch diameter of a spline for a given number of teeth, and the second figure designates the diametral pitch p_d', which determines the size of the addendum a and dedendum d. For a flat root, a and d are equal and

$$a = d = \frac{1}{p_d'} \qquad (12\text{–}25)$$

All data for manufacturing and checking the accuracy of the obtained splines are given in elaborate tables in the SAE Handbook.[4]

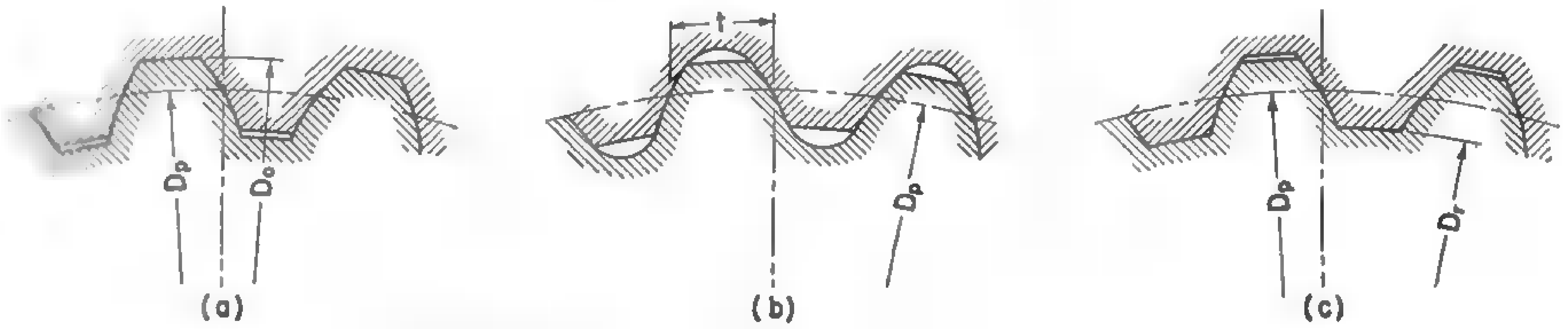


FIG. 12–13. Ways of fitting involute splines.

Fits. The SAE standards establish three methods of fitting spline members together. In one method the fit is controlled by varying the major, or outside, diameter D_0 of the external spline, Fig. 12–13a. In a second method the fit is controlled by varying the tooth thickness t. This fit is used for splines of the full-fillet type, Fig. 12–13b. In the third method the fit is controlled by varying the minor, or root, diameter D_r of the internal splines, Fig. 12–13c.

For each of these three methods there are three classes of fits—sliding, close, and press fits. Sliding fits must have clearance at all points; close fits

[4] *Ibid.*, pp. 521–79; also ASA B5.15–1950 (New York: American Standards Association, 1950).

must be close at one point (dimension) of the tooth profile; and press fits must have an interference at only one point (dimension) of the tooth profile.

The SAE Handbook gives the limits for clearances and interferences for the three classes of fits. These limits vary with the diametral pitch.

The involute splines are made either with a so-called *flat root*, Fig. 12–13a and c, or with a *full-fillet root*, Fig. 12–13b.

The advantages of involute splines with a 30° pressure angle are:

a) The teeth have the maximum strength through the minor diameter, where it is needed.

b) Involute splines are self-centering and therefore tend to equalize the bearing and shear stresses among all teeth.

c) The tooth surface is smooth, being obtained by the generating action of the hobbing operation.

Length of spline. The stresses created in the spline by the torque transmitted by the shaft are shear in the teeth, and bearing stress at the contacts between the teeth.

The area resisting shear is that of the pitch cylinder with the diameter D_p, which is the same for both the external and internal splines. This area is

$$A_s = \frac{\pi D_p L}{2} \tag{12-26}$$

where L is the length of the spline. If the design stress in shear is designated as S_{ds}, the torque capacity of the teeth in shear is

$$T_s = \left(\frac{\pi D_p L}{2}\right)\left(\frac{D_p}{2}\right) S_{ds} = 0.7854 D_p^2 L S_{ds} \tag{12-27}$$

The minimum height of contact on one tooth is

$$h = \frac{0.8}{p_d} = \frac{0.8 D_p}{i} \tag{12-28}$$

The corresponding area of contact of all i teeth is

$$A = \left(\frac{0.8 D_p}{i}\right) L i = 0.8 D_p L \tag{12-29}$$

The torque capacity of the spline in bearing stress, with $S_b = 2S_{dc}$, is

$$T_b = (0.8 D_p L)\left(\frac{D_p}{2}\right) 2S_{dc} = 0.8 D_p^2 L S_{dc} \tag{12-30}$$

For steel, the ratio of the elastic limit in shear to the elastic limit in compression is about 0.6. If the safety factor is the same, $S_{ds} = 0.6 S_d$ and dividing equation 12–27 by equation 12–30 gives $T_s = 0.59 T_b$. Therefore the design of splines is critical in shear.

The necessary length of a spline may be obtained by equating the torque capacity of the spline to that of the shaft. The effective shaft diameter may be considered equal to the pitch diameter D_p of the spline. Then

$$T = \frac{\pi D_p^3 S_{ds}}{16} \tag{12-31}$$

Experience shows that because of inaccuracies in spacing and tooth form, only about 25 per cent of the teeth are in actual contact. Therefore, introducing the factor 0.25 in the second member of equation 12–27, equating the resulting expression to the second member of equation 12–31, and solving for L, we get $L = D_p$. This value is in accordance with standard practice. It is also sufficient to obtain stability of the part with the internal spline.

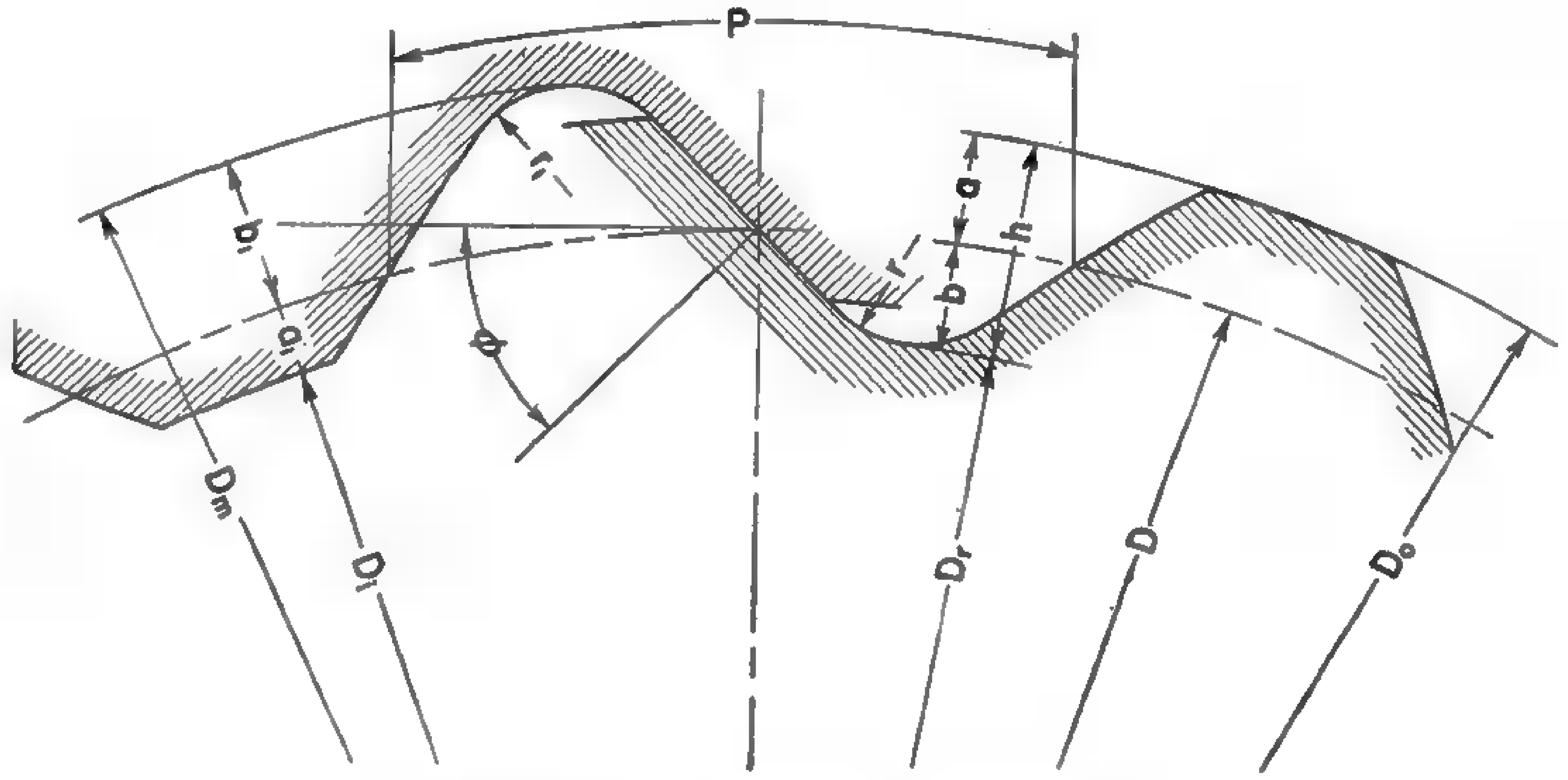


FIG. 12–14. Involute serration.

Involute serrations. In addition to the splines previously described, the Society of Automotive Engineers and the American Standards Association have adopted standards for splines with much finer pitches and a pressure angle of 45°. These splines have been given the name *involute serrations.* The shape of an involute serration is shown in Fig. 12–14. The pitches range from 10/20 to 128/256. The three coarser pitches may use 6 to 100 teeth; the six finer pitches use from 6 to 50 teeth. The pitch diameters range from 0.10 to 10.0 in.

Involute serrations are used for permanent connections of shafts with parts mounted on them. They are made with three classes of fits. A class A fit is loose; a class B fit is close; and a class C fit is a press fit. Involute serrations are fitted on the sides of the teeth. All values necessary for manufacturing and checking the accuracy are given in tables.[5]

12–8. Pins. Geometrically, pins may be divided into cylindrical pins, called *straight pins*, and conical pins, or *taper pins*. Dynamically, pins may be classified as those used only to locate the relative position of two parts when there is little or no force acting upon the pin, and those that fasten two or more parts together and are subjected to considerable stresses, which are mostly shear stresses but sometimes bending stresses. Locating pins are

[5] *SAE Handbook*, pp. 580–604; also ASA B5.26–1950.

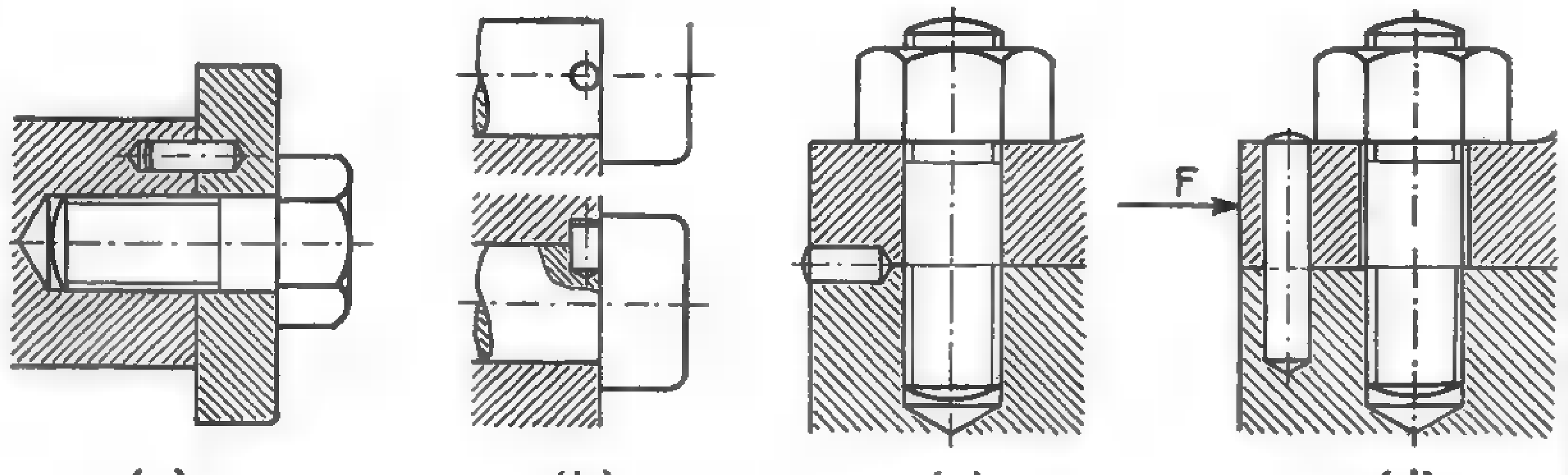


FIG. 12–15. Dowel pins.

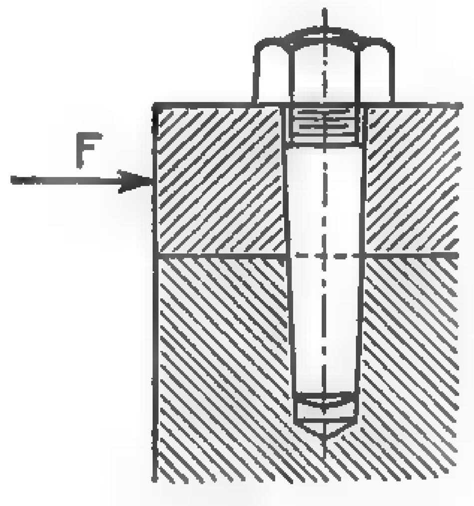


FIG. 12–16. Taper dowel.

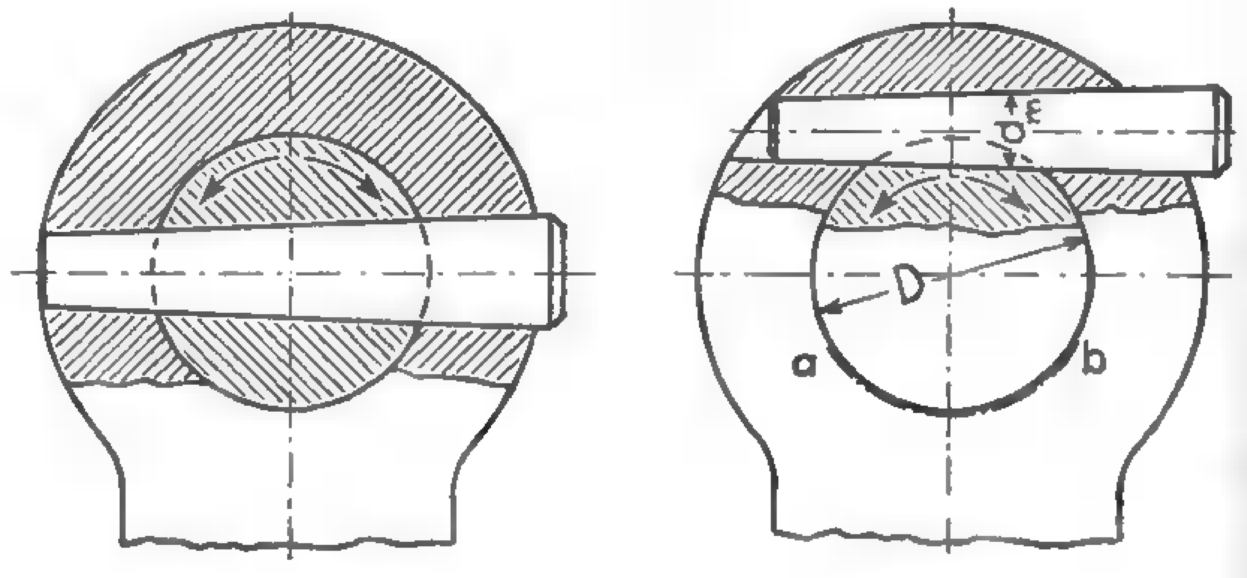


FIG. 12–17. Taper pins.

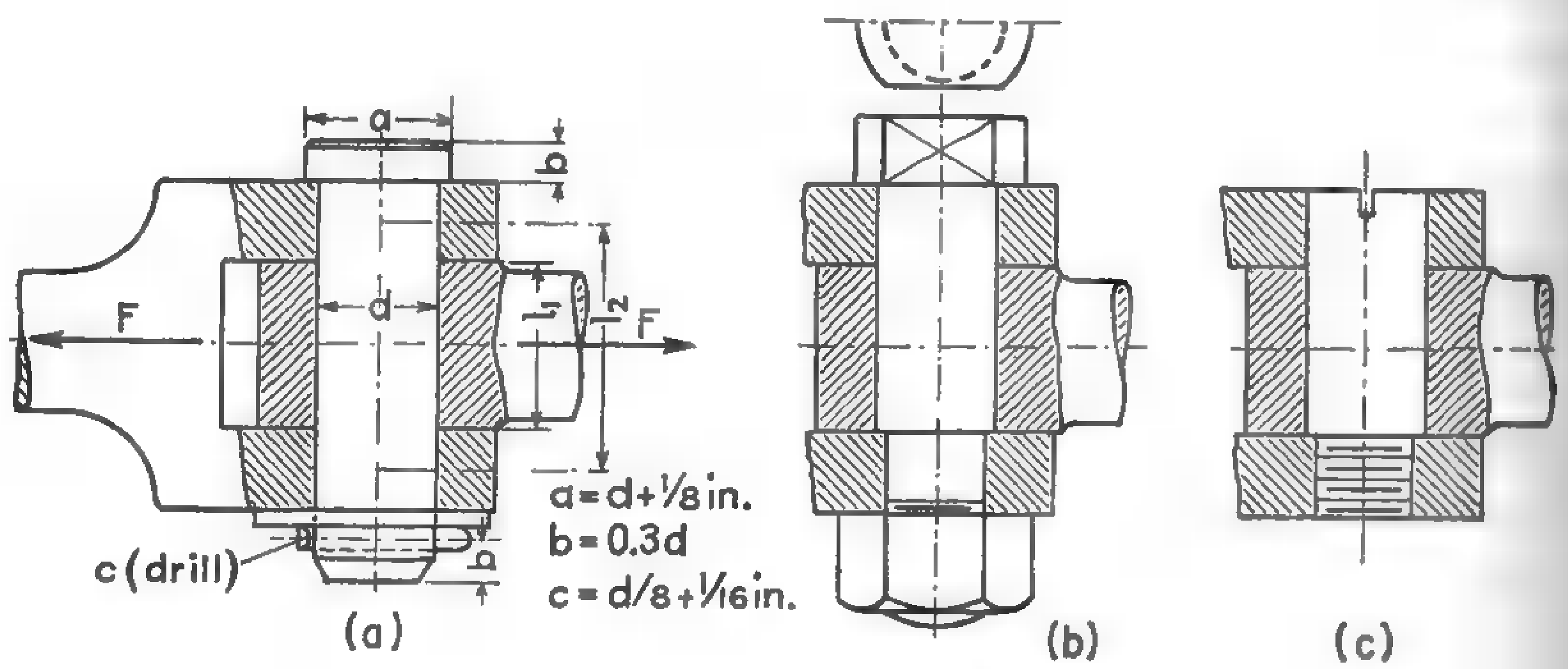


FIG. 12–18. Knuckle-joint pins.

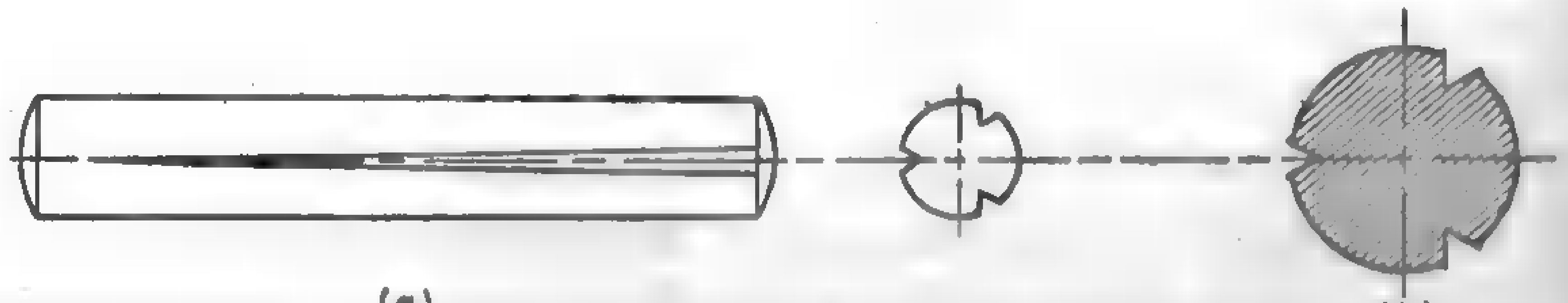


FIG. 12–19. Grooved pin.

called *dowel pins*, or simply *dowels*. A connecting pin, like a dowel, may be used either as a permanent connection or as a fulcrum for a movable joint.

The various uses of dowel pins are illustrated in Fig. 12–15. To insure accuracy of assembly the dowel is sometimes made tapered, Fig. 12–15d.

Connecting pins are made tapered. When a transverse force F acts on the flange, as shown in Fig. 12–15d, the dowel pin becomes a connecting pin which must be properly dimensioned to resist the force F. To remove a taper pin when there is no through hole, its large end may be threaded, Fig. 12–16, and a nut may be used as a puller. The round key, Fig. 12–2a, is another example of a connecting pin. To secure a part on a shaft in regard to both rotation and axial movement, the pin is inserted as shown in Fig. 12–17a. However, if the torque transmitted to the hub is not heavy and it is desired to avoid unnecessary weakening of the shaft, the pin may be inserted as shown in Fig. 12–17b. All connecting pins must be fitted well, using standard pins and reamers, and they must be driven home to prevent them from working loose.

Several types of pins used as fulcrums in knuckle joints are shown in Fig. 12–18. Shoulder screws, Fig. 11–15, are used for the same purpose. In a knuckle joint with all holes of the same diameter, like that shown in Fig. 12–18a, a simple cap screw with a nut and lock nut may be used.

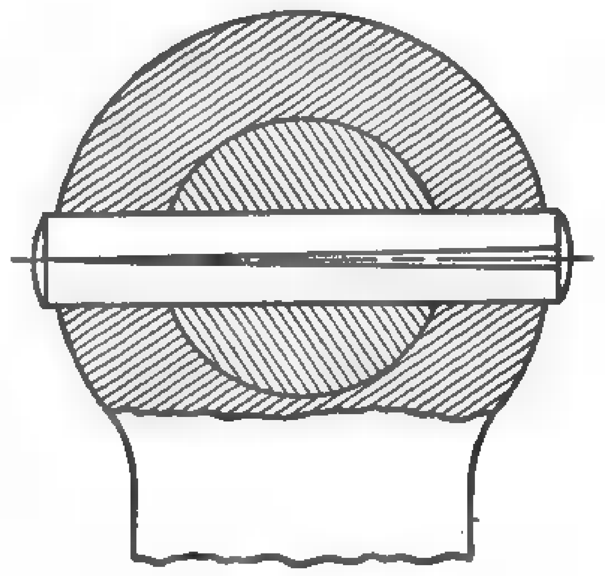


FIG. 12–20. Grooved pin as a key.

Grooved pins are now finding wide application because they do not require reaming of the drilled holes. These pins, as shown in Fig. 12–19, have three tapered rolled grooves with protruding edges. The edges deform elastically and prevent loosening of the pin under the action of variable loading and even under severe vibration. These pins are used in place of straight or conical pins in the manner shown in Fig. 12–20. They can be re-used up to twenty-five times.

Roll pins are also a recent innovation, designed by the Elastic Stop Nut Corporation to replace various dowel, pivot, and tapered pins. The roll pin is made of SAE 1045 steel or type 420 stainless steel, and is formed by rolling a plate into the shape of a cylinder with an axial gap. Both ends of the pin are chamfered, which permits it to be driven into a hole with a diameter slightly smaller than that of the pin. The spring action of the compressed pin holds it securely against any vibration or shock.

At present, roll pins are made in thirteen diameters to fit holes with diameters ranging from $\frac{3}{32}$ in. to $\frac{1}{2}$ in., and in a wide range of stock lengths. The use of a roll pin is illustrated in Fig. 12–21. Figure 12–21a shows how easily the pin, with its chamfered ends, can be driven into a predrilled hole; Fig. 12–21b shows the compression of the pin as it is driven; Fig. 12–21c shows

TABLE 12–4

MORSE STANDARD TAPER PINS

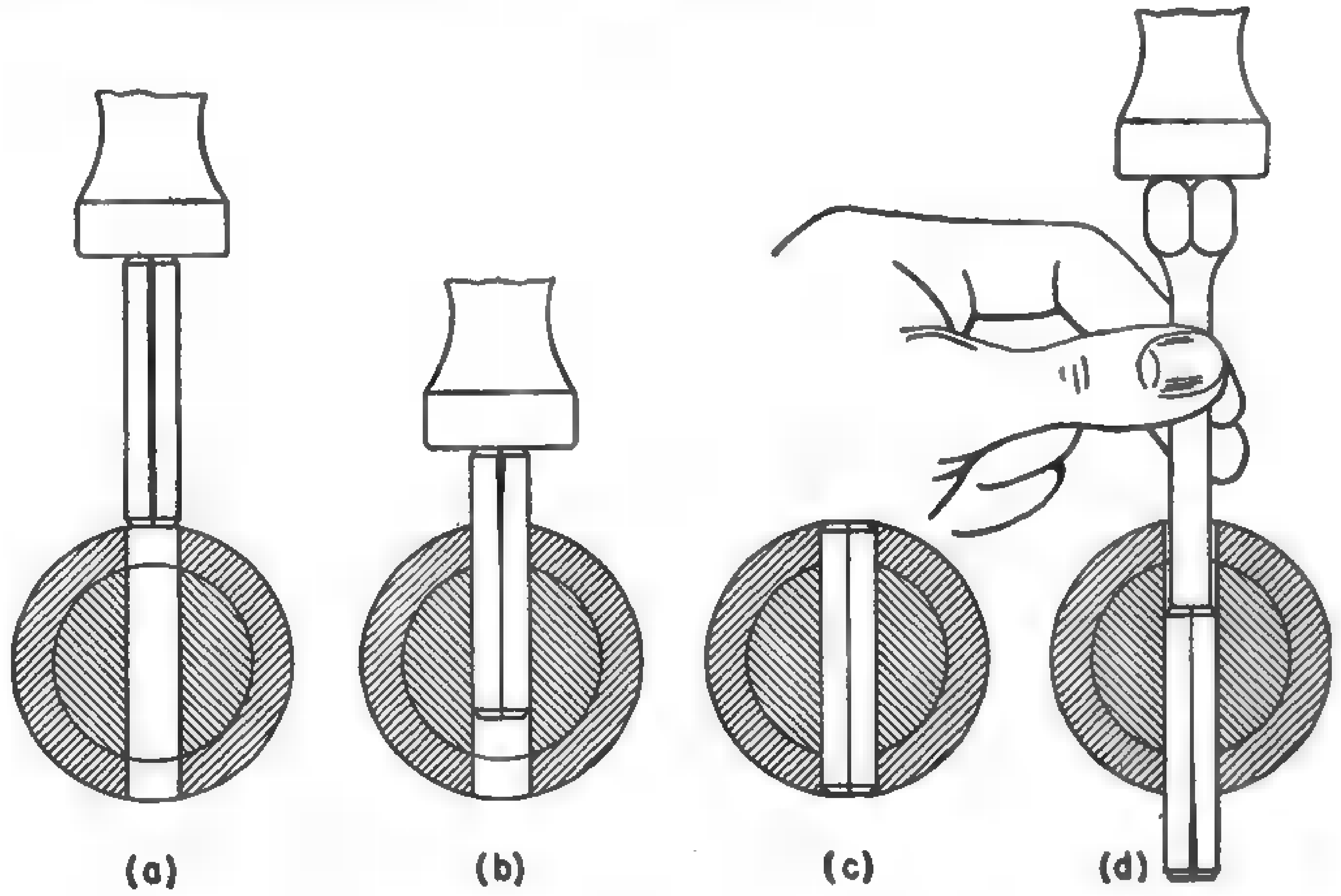
Size Number	Large Diameter d_1 (in.)	Lengths Available l (in.)	Size Number	Large Diameter d_1 (in.)	Lengths Available l (in.)
0	0.156	$\frac{1}{2}$–3	6	0.341	$\frac{3}{4}$–5
1	0.172	$\frac{1}{2}$–3	7	0.409	1 –5
2	0.193	$\frac{3}{4}$–$3\frac{1}{2}$	8	0.492	$1\frac{1}{4}$–5
3	0.219	$\frac{3}{4}$–$3\frac{1}{2}$	9	0.591	$1\frac{1}{2}$–6
4	0.250	$\frac{3}{4}$–4	10	0.706	$1\frac{1}{2}$–6
5	0.289	$\frac{3}{4}$–4			

how it locks permanently in place; and Fig. 12–21d shows how easy it is to remove the pin when necessary.

Taper pins are made in standard sizes in accordance with the Morse dimensions given in Table 12–4. The stock lengths l increase by $\frac{1}{4}$ in. The taper is $\frac{1}{4}$ in. per ft, or 0.0208 in. per in. Thus the diameter d_s at the small end can be found by the equation

$$d_s = d_1 - 0.0208l \tag{12–32}$$

where d_1 is the large diameter in Table 12–4.



(a) (b) (c) (d)

FIG. 12–21. Joint with a roll pin.

Taper pins are made of ordinary machine steel, or of 0.10 carbon steel. The diameter of a taper pin is selected so that the force acting upon the pin does not induce an excessive shear stress. A pin may work in single shear, like the one shown in Fig. 12–16, or in double shear, like the one shown in Fig. 12–17a. A tangential pin, shown in Fig. 12–17b, produces friction between the shaft and the hub on the arc *ab*. To insure sufficient pressure

the hole is first reamed to a taper with the hub on the shaft, and the hub must then be taken off and the hole in it reamed slightly larger. In a pin of this type, satisfactory operation depends on the skill of the mechanic to such an extent that any theoretical calculations are futile.

The mean diameter of the pin is made

$$d_m = 0.20D \text{ to } 0.25D \tag{12–33}$$

where D is the shaft diameter.

A *knuckle pin* should be designed to have sufficient bearing area, sufficient strength in bending, and sufficient strength in double shear.

With the designations of Fig. 12–18, the bearing pressure is

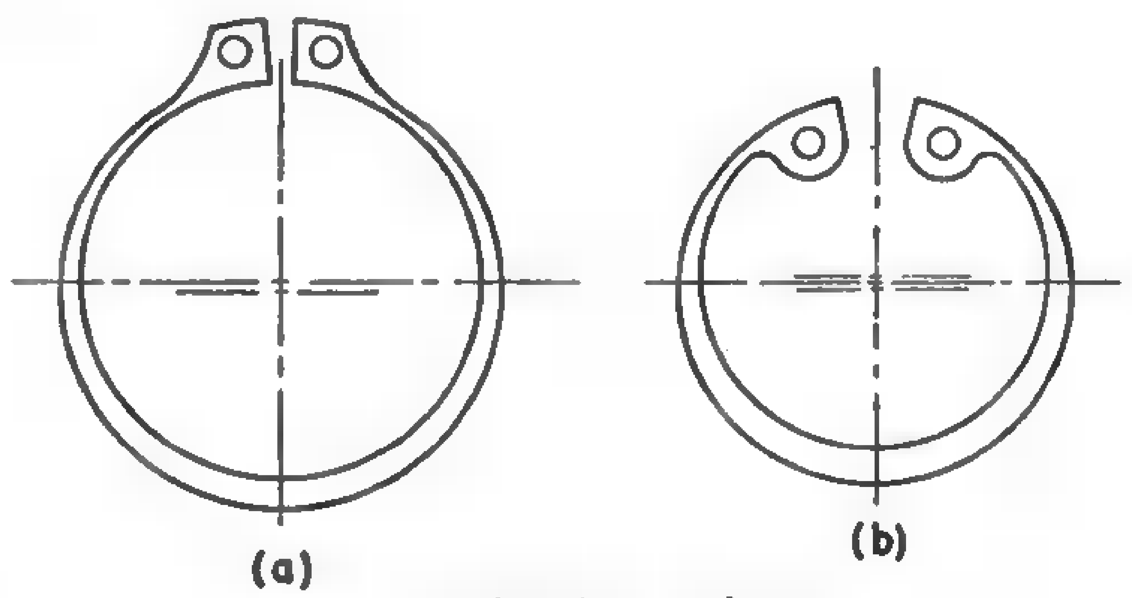
$$p = \frac{F}{dl_1} \tag{12–34}$$

For ordinary machine steel, p should not exceed 3,000 psi if the rocking motion is small and if a small amount of wear is not objectionable. Otherwise, p should not be more than 2,000 psi.

In computing the bending stress, one should regard the load as applied at the center of the eye, and the points of support should be taken at the center of each fork shank.

The designer should determine the required diameter d for all three stress conditions. The largest of the three values for d will be the correct size to use.

Snap rings are used to prevent axial motion of two concentric parts. They are of two types: external, Fig. 12–22a, and internal, Fig. 12–22b. The rings can be easily slipped in place, and the holes in the ends serve to permit their removal. They are made of high-carbon heat-treated steel in sizes from $\frac{1}{2}$ in. to $8\frac{3}{8}$ in. in diameter, and in thicknesses from 0.042 to 0.125 in. Figure 12–23 illustrates the use of an external snap ring *e* for locating the inner race of a ball bearing, and the use of an internal ring *i* to act as a stop in the housing. The grooves for snap rings must be made with a slight clearance for ease of insertion, and the rings cannot be inserted while the parts are under axial load.[6]



(a) (b)

FIG. 12–22. Snap rings.

12–9. Cotter joints. A cotter is a cross key used for fastening a rod to some part having a socket, the rod being subjected to tension or compression

[6] P. F. Rossman, "Designing Snap Ring Fastenings," *Machine Design*, Vol. 13 (May, 1941), pp. 49–51, 106.

in the direction of its axis. A few typical cotter joints are shown in Fig. 12–24. In Fig. 12–24a a hub is connected to a rod; in Fig. 12–24b a rod with a socket is connected to another rod with a collar; in Fig. 12–24d two rods are connected by means of a double cotter so as to obtain parallel edges; and in Fig. 12–24e two halves of a casting are connected by a tie rod with cotters.

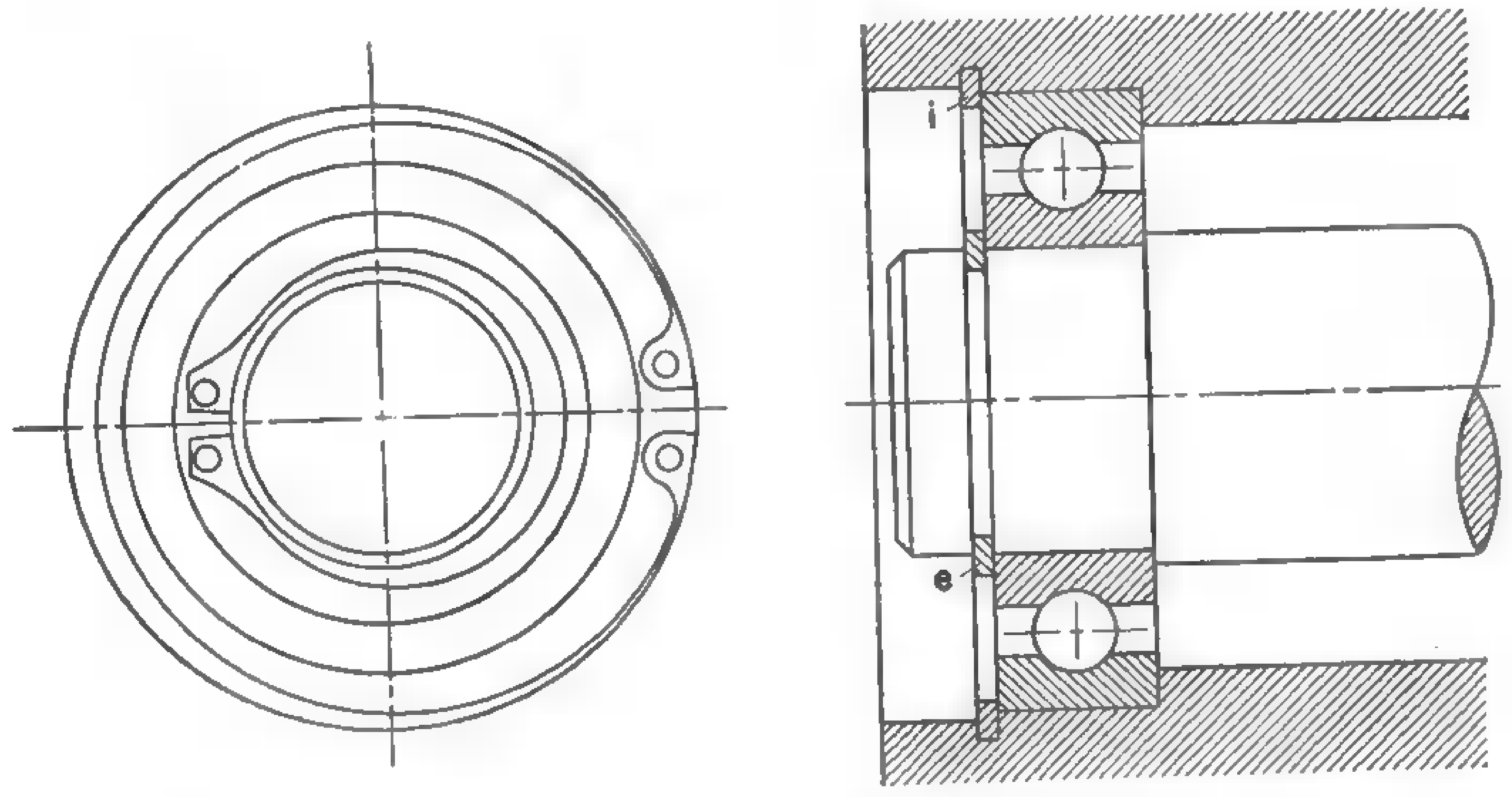


Fig. 12–23. Ball bearing installation with snap rings.

Analysis of a cotter joint. An analysis of stresses in a cotter joint, which can serve as a guide in the determination of proportions for such joints in general, will be made with reference to the type of joint shown in Fig. 12–24b. Because of the elasticity of materials, a cotter joint—like a screw fastening—can work satisfactorily only if the cotter is previously tightened to such an extent that the initial force F set up by the wedge action is greater than

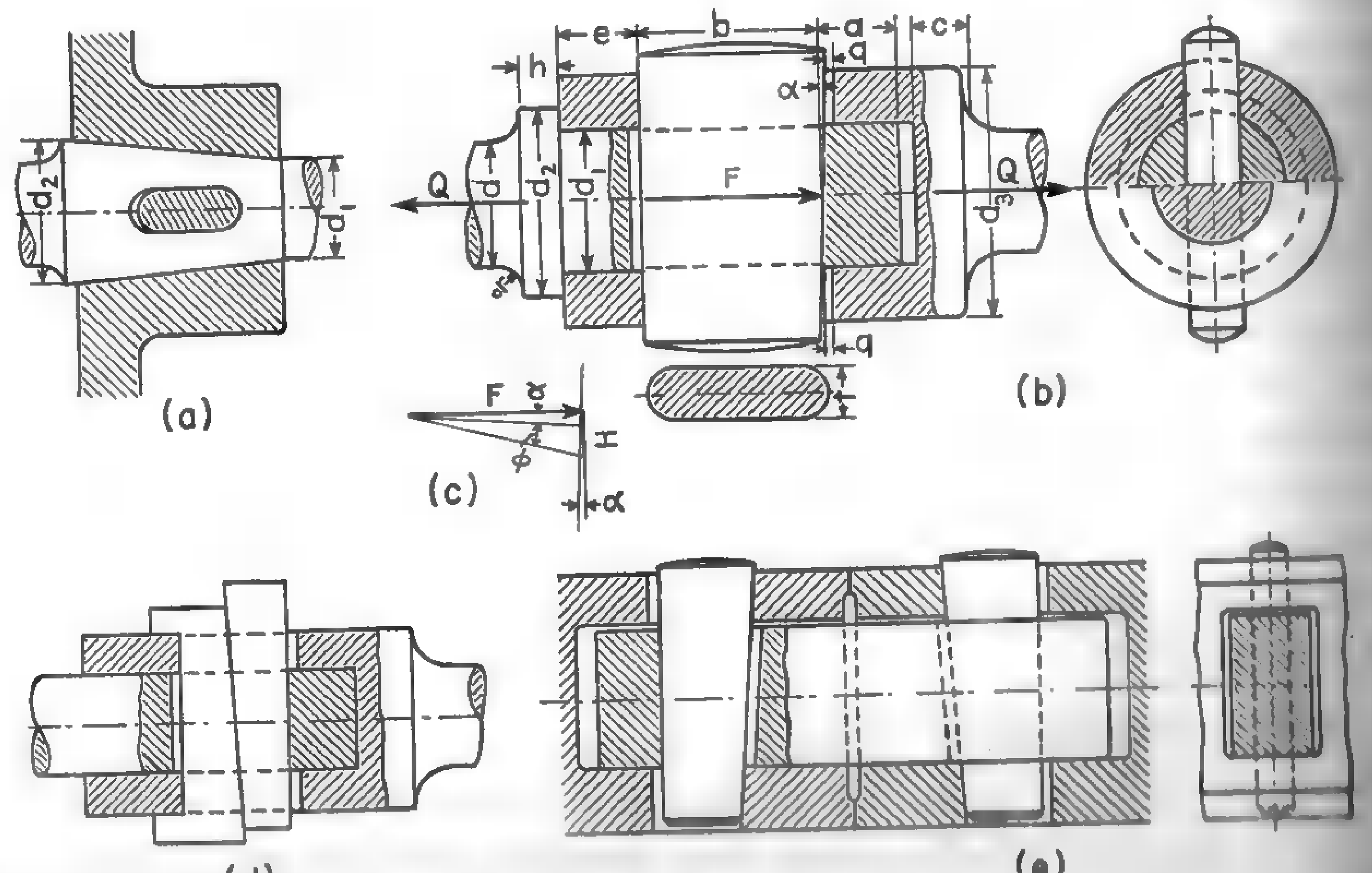


Fig. 12–24. Types of cotter joints.

the external load Q applied to the rod. The force F should be approximately $1.25Q$. The force H, Fig. 12–24c, necessary to produce F can evidently be found from the relation

$$H = F \tan(\alpha + \phi) \tag{12–35}$$

where α is the angle of the cotter slope and ϕ is the angle of friction.

The joint may fail if the stress induced by the force F or the load Q, whichever is acting at a certain place, exceeds the safe value in any one of twelve places.

The load Q affects the following places:

a) *Rod in tension.* The stress concentration due to the change from d to d_2 with a fillet r must be taken into account.

b) *Rod-socket connection.* The rod can separate from the socket by shearing if the dimension c is not long enough.

c) *Socket end.* The socket end may fail in double shear in the dimension e.

The force F set up by the initial tightening of the cotter affects the following places, whether the load Q acts or not:

d) *Rod across the slot.* The rod may fail in tension at this section. Stress concentration at the ends of the slot in the rod exists but may be neglected in view of constant load conditions.

e) *Socket across the slot.* This section is stressed in tension, and stress concentration at the end of the slot may again be neglected.

f) *Rod end.* The end of the rod may fail in double shear through the length a.

g) *Collar.* The collar may fail by shear if its height h is too small.

h) *Cotter.* The cotter may fail in double shear.

i) *Cotter.* The cotter may also fail in bending. For the sake of safety it may be assumed that the load is concentrated at the center line and the points of support are located at the center of each side of the socket.

Finally, *crushing* may occur at certain surfaces of contact because of excessive compressive or bearing stresses if the corresponding dimensions are too small. The following areas of contact must be checked:

j) *Collar and socket.*

k) *Socket and cotter.*

l) *Rod and cotter.*

Design. Taper on the cotter should be not more than $\frac{1}{2}$ in. per ft, or 1:24, in order to prevent loosening of the joint through load fluctuation. The taper may be increased up to $1\frac{1}{2}$ in. per ft, if some locking device, such as a setscrew, is applied to the cotter. If the joint is subjected to vibration the use of a setscrew is advisable even with a smaller taper.

Cotter dimensions which may help to start the design may be taken as follows:

$$t = 0.4d \tag{12–36}$$

and

$$b = 4t = 1.6d \tag{12-37}$$

Sufficient clearances q must be provided for take-up.

In order to decrease the stress concentration in the slots of the rod and of the socket, the cross section of the cotter should not be rectangular, but should be semicircular on the short sides, as shown in Fig. 12–24a and b,

Fillets at the junctions of the collar and the socket with the rods must be large enough to avoid dangerous localized stresses.

Bearing stresses. To avoid large and clumsy proportions, rather high bearing stresses S_c must be allowed. An examination of successful practice shows that actual bearing stresses computed from the initial force F are as high as 20,000 psi for machine steel (about SAE 1030). A conical rod end, Fig. 12–24a, should be preferred to one with a collar, Fig. 12–24b, since it can be fitted more easily and is not as likely to loosen. The bearing stress must be computed from the projected ring area of the cone. Thus the same dimensions d_1 and d_2 are needed regardless, whether a cone or a collar is used.

CHAPTER 13

Press, Shrink, and Friction Joints

13–1. General explanations. A *press joint,* also called a *force joint,* is obtained by forcing a shaft into a hole that is slightly smaller than the shaft. This is possible because of the elasticity of the materials. The tendency of the materials to return to their original dimensions produces the grip that holds the hub and shaft together.

A *shrink joint* differs from a press joint chiefly in the method of assembling it. The hub is heated to expand its bore in order to slip it on the shaft. When the hub cools down to the temperature of the shaft, the grip is produced in the same way as in the force joint. The shrink joint is also used to connect machine parts by means of special rings, anchors, and tie rods.

In a *friction joint* the holding grip is produced either by the conical shape of the shaft end and the hub bore and by the pull of a nut, or by a slotted hub whose bore is smaller than the shaft and which is spread by a wedge when the joint is being assembled.

Comparison of joints. The assembling of a shaft and a hub by means of a press joint is simpler than with a shrink joint, especially if a hydraulic press of sufficient capacity is available. Shrink joints are used mainly in places where it is difficult or impossible to assemble a press joint, as in the case of rings or anchors. However, tests have shown[1] that shrink joints assembled with the same interference as press joints give more than three times the holding power against both torsion and axial pull. This superior effectiveness is due to the absence of abrasion between the surfaces of the shaft and the hub during assembly.

Press joints and shrink joints are permanent connections, while friction joints can be dismantled. Any one of these joints can be used when machine parts must be connected more securely than can be accomplished with a key or screw joint, especially when they are subjected to shock or vibration.

Allowances and tolerances. In Table 13–1 are given the allowances and tolerances recommended for press and shrink joints over the full range of fits.[2] In general, *allowance* is defined as the minimum clearance space between mating parts. A negative allowance, as in a press fit, is called an *interference.* *Tolerance* is defined as the permissible amount of variation in the size of a part.

[1] J. J. Wilmore, "Shrink and Force Fits," *American Machinist,* Vol. 22 (February 16, 1899), p. 126.

[2] *Tentative American Standard Tolerances, Allowances and Gages for Metal Fits* (New York: American Standards Association, 1925).

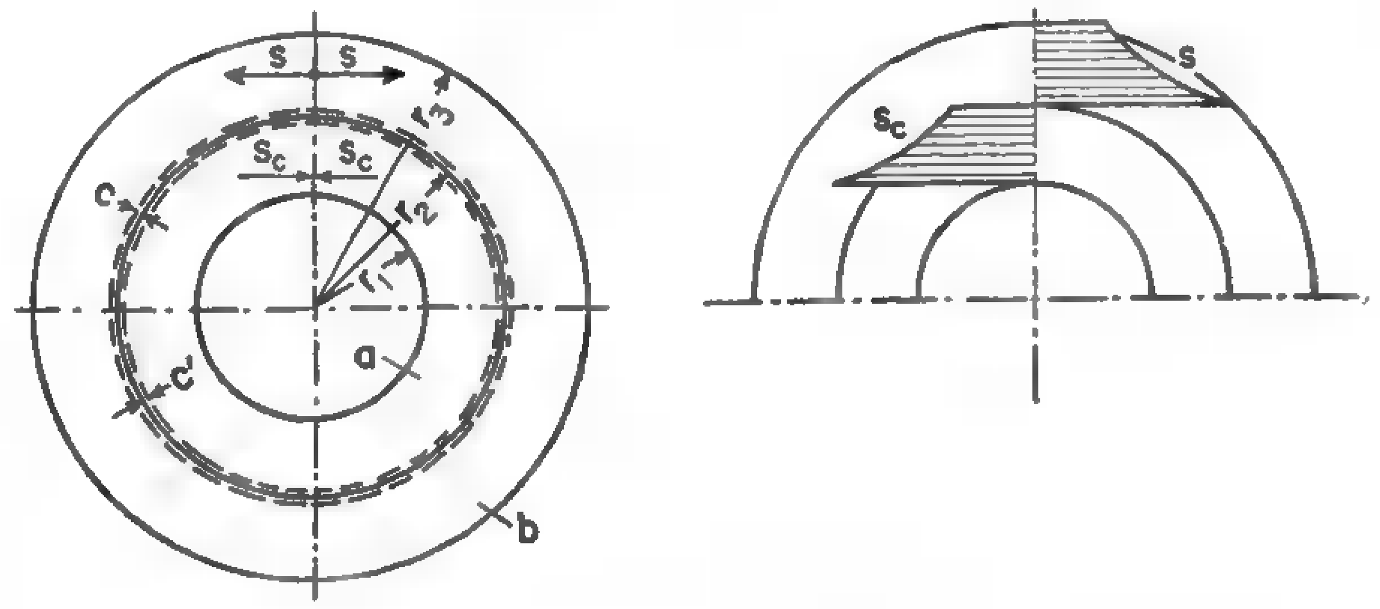
TABLE 13–1

RECOMMENDED ALLOWANCES AND TOLERANCES FOR VARIOUS FITS

Class	Type of Fit	Method of Assembly	Diametral Allowance (in.)	Selected Total Interference (in.)	Tolerance in Hole or Shaft (in.)
1	Loose	Interchangeable	$0.0025\sqrt[3]{d^2}$	. . .	$0.0025\sqrt[3]{d}$
2	Free	Interchangeable	$0.0014\sqrt[3]{d^2}$	. . .	$0.0013\sqrt[3]{d}$
3	Medium	Interchangeable	$0.0009\sqrt[3]{d^2}$	. . .	$0.0008\sqrt[3]{d}$
4	Snug	Interchangeable	0.0000	. . .	$0.0006\sqrt[3]{d}$
5	Wringing	Selective	. . .	0.0000	$0.0006\sqrt[3]{d}$
6	Tight	Selective	. . .	$0.00025d$	$0.0006\sqrt[3]{d}$
7	Medium force	Selective	. . .	$0.0005d$	$0.0006\sqrt[3]{d}$
8	Heavy force	Selective	. . .	$0.001d$	$0.0006\sqrt[3]{d}$

All hole tolerances are positive. The shaft tolerances are negative for the working fits (classes 1 to 4) and are positive for the force fits (classes 5 to 8). The hole is the base, or nominal, size. Table 13–1 is not intended to cover every kind of work; individual conditions may make some deviations from the standard advisable. However, Table 13–1 will apply to the majority of cases occurring in practice.

13–2. Press fits. Class 7 fits, Table 13–1, are the tightest recommended for holes in cast iron since they stress cast iron to its elastic limit. Class 8 fits are used for holes in steel, whose elastic limit is considerably higher.



(a) (b)

FIG. 13–1. Hub shrunk on hollow shaft.

Stress due to a force fit. If e is the elongation or contraction of any radius r, the elongation or contraction per unit of length is e/r, in both the radial and circumferential directions. If s is the stress that accompanies this strain, then from equation 2–4

$$e = \frac{sr}{E} \tag{13–1}$$

If r_2, Fig. 13–1a, is the radius of the contact surface of the hollow shaft a pressed into the hub b, then before assembling, the outer radius of the shaft

was (r_2+c) and the inner radius of the hub was (r_2-c'). The shaft a is in the condition of a thick solid cylinder with open ends subjected to an external pressure, which induces a compressive stress in it. The hub b is in the condition of an open-end thick hollow cylinder subjected to an internal pressure, which induces a tensile stress in the tangential direction. Because of the elasticity of metals, the highest compressive stress s_c will be at the inner surface of the shaft and the highest tensile stress s will be in the inner fibers of the hub.

The radial interference before the parts are assembled, designated by i, is obviously

$$i = (r_2+c) - (r_2-c') = c+c' \tag{13–2}$$

The common radius r_2 can be used both for the original outside shaft radius and for the original inside hub radius, without too great inaccuracy. From equation 13–1, remembering that the compressive stress is negative,

$$i = \left(\frac{s_c}{E_s} + \frac{s}{E_h}\right) r_2 \tag{13–3}$$

where E_s and E_h are the moduli of elasticity of the shaft and hub, respectively.

The relation between the stresses s_c and s, the radii of the shaft and hub, and the pressure between them may be found by applying equations 2–70 and 2–71. If equation 2–70 is applied to the surface of the hollow shaft with the designations of Fig. 13–1, where $p_i = 0$, $p_o = p_2$, $r_i = r_1$, and $r_o = r_2$, and also $s_r = s_c$, the result is

$$s_c = -p_2 \tag{13–4}$$

The tangential stress at the inner surface of the hub may be found from equation 2–71, substituting $p_i = p_2$, $p_o = 0$, $r = r_i = r_2$, and $r_o = r_3$. Thus

$$s = \frac{p_2(r_3^2 + r_2^2)}{r_3^2 - r_2^2} \tag{13–5}$$

The variation of the tangential stresses in the shaft and hub is shown diagrammatically in Fig. 13-1b.

If the ratio of the absolute values of the stresses is designated by n,

$$\frac{s_c}{s} = -n \tag{13–6}$$

Substituting the values of the stresses from equations 13–4 and 13–5 gives

$$n = \frac{r_3^2 - r_2^2}{r_3^2 + r_2^2} \tag{13–7}$$

From equations 13–3 and 13–6,

$$s = \frac{i}{\left(\frac{n}{E_s} + \frac{1}{E_h}\right) r_2} \tag{13–8}$$

and

$$s_c = \frac{-in}{\left(\frac{n}{E_s} + \frac{1}{E_h}\right) r_2} \tag{13–9}$$

EXAMPLE 13–1. A steel crank is to be pressed upon the end of a hollow steel shaft with an outside diameter of 9 in. and an inside diameter of 3 in.; the outside diameter of the crank hub is 16 in. Using a recommended fit, determine the maximum stress in the hub and the pressure between the hub and shaft. Also find the corresponding stresses at the outer and inner surfaces of the shaft.

With steel on steel, class 8 fit in Table 13–1 may be used. The interference referred to the radius is $i = 0.001 \times \frac{9}{2} = 0.0045$ in.

The modulus of elasticity is the same for both the shaft and the hub: $E_s = E_h = 30{,}000{,}000$ psi.

From equation 13–7, $n = 0.519$.

From equation 13–8, the maximum tangential tensile stress in the hub is

$$s = \frac{0.0045 \times 30{,}000{,}000}{(0.519+1) \times 4.5} = 19{,}750 \text{ psi}$$

From equation 13–6, the compressive stress on the outside surface of the shaft is

$$s_c = 19{,}750 \times -0.519 = -10{,}250 \text{ psi}$$

From equation 13–4, the pressure at the contact surface is

$$p_2 = 10{,}250 \text{ psi}$$

The stress at the inner surface of the shaft can be found from equation 2–71. Since $p_i = 0$ and $r = r_1 = 1.5$ in.,

$$s_c \frac{-4.5^2 \times 10{,}250 - 4.5^2 \times 10{,}250}{4.5^2 - 1.5^2} = -23{,}060 \text{ psi}$$

The fact that the maximum stresses in the hub and the shaft, both of which are made of steel, have approximately the same values shows that the relations r_3/r_2, r_1/r_2, and i were selected properly. If these stresses should differ considerably, the value of i should be slightly altered.

This discussion assumes that the materials are elastic and that Hooke's law applies. For cast iron, Hooke's law applies only approximately. For the combination of a cast-iron hub on a steel shaft, there is a semirational formula based on Lamé's theory and the records of several manufacturers.[3] This formula, slightly modified to conform to the designations of this text, gives the stress s at the inner surface of the cast-iron hub. It is

$$s = \frac{Ei}{r_2 + 0.14 r_3} \tag{13–10}$$

The following simple formula for a steel hub on a steel shaft, also modified, gives the stress at the inner surface of the hub:

$$s = Ei \frac{r_3^2 + r_2^2}{2 r_3^2 r_2} \tag{13–11}$$

Equation 13–11 may be used for preliminary design, but the final figures should be checked by proceeding as has been explained.

It should be remembered that the maximum stress in the hub should always be well below the elastic limit. Otherwise a permanent set will occur,

[3] A. L. Jenkins, "Formulas for Forced and Shrink Fits," *American Machinist*, Vol. 42 (March 4, 1915), pp. 377–84.

with the result that the pressure between the shaft and the hub will be lost and the joint will become loose.[4]

Forcing pressures. The force required to assemble a force-fit joint depends to a great extent on the material, the finish of the surfaces, and the lubricant applied. Although experimental data relating to this feature are rather incomplete, they indicate that the pressure required, in addition to the influences mentioned, is directly proportional to the interference i, the shaft diameter d, and the hub length l.

If the radial pressure p_2 is computed from equation 13–4 or equation 13–5 for a given value of i, then the axial force, in tons, theoretically necessary to press the shaft into the hub is

$$F = \frac{\pi d l f p_2}{2{,}000} \tag{13–12}$$

where f is the friction coefficient, which may vary from 0.085 to 0.125 for unlubricated surfaces but with special lubricants can be lowered to about 0.05.[5]

The value of F, in tons, for a steel shaft in a cast-iron hub may be computed by the formula[6]

$$F = 6{,}000 \frac{(r_3 + 0.3 r_2) li}{r_3 + 6.33 r_2} \tag{13–13}$$

And for a steel shaft in a steel hub,

$$F = 4{,}120 \frac{(r_3^2 - r_2^2) li}{r_3^2} \tag{13–14}$$

EXAMPLE 13–2. Find the force that a press must produce to assemble the shaft and hub discussed in example 13–1, assuming that the hub length is 15 in.

By equation 13–12,

$$F = \frac{\pi \times 9 \times 15 \times 0.10 \times 10{,}250}{2{,}000} = 217 \text{ tons}$$

By equation 13–14,

$$F = 4{,}120 \times \frac{(8^2 - 4.5^2) \times 15 \times 0.0045}{8^2} = 190 \text{ tons}$$

The agreement between the two results is satisfactory.

Transmission of torque. Experiments by the Westinghouse Electric Corporation indicate that the elastic torque of a press joint does not depend on the length of the joint, if the latter is longer than half of the diameter. The term *elastic torque* means the torque at which slip begins at one end of the joint because of twisting of the shaft. The *ultimate torque* is the torque at which the joint slips throughout its full length. A press fit should be designed to prevent elastic torque, especially if an alternating torque is applied. In

[4] "Design of Press and Shrink Fit Assemblies," *Journal of Applied Mechanics*, Vol. 5, No. 1 (March, 1938), p. A-32; Joseph Marin, "Designing Shrink Fit Assemblies," *Machine Design*, Vol. 14 (June and July, 1942), pp. 68–73 and pp. 72–75.

[5] H. L. Guy, "Factors Affecting the Grip in Force, Shrink, and Extension Fits," *Mechanical Engineering*, Vol. 56 (1934), p. 235.

[6] Jenkins, *loc. cit.*

the presence of an elastic torque, wear and abrasion take place and cause a final loosening of the joint. A sideways-fitted key inserted in a press fit prevents the starting of wear and abrasion. The ultimate torque may be used as a basis of the design only in the case of a steady torque application.

The ultimate torque T of a press fit or a shrink fit may be found by the relation

$$T = \tfrac{1}{2}\pi d^2 l f p_2 \tag{13-15}$$

where f may be taken as 0.10 for press fits and 0.125 for shink fits.[7]

The torque that can be safely transmitted by a press fit or a shrink fit may be found by dividing the ultimate torque from equation 13–15 by a safety factor n which may be between 1.5 and 2.5, the value depending on the uniformity of the transmitted torque.

Disk with hub. All calculations may be made by assuming that the disk and the hub are two separate pieces placed side by side.

Stress concentration. As pointed out in section 12–3, a shaft that transmits a torque always twists. As pointed out in section 5–10, when a hub is pressed or shrunk on a shaft, the radial compressive stress (designated p_2 in equations 13–4 and 13–5) increases near the end where the shaft protrudes from the hub, as shown in Fig. 5–15. This stress concentration may increase p_2 to a value above the elastic limit, and the twisting of the overstressed shaft entering the rigid hub will gradually loosen it, regardless of the accuracy with which it was assembled. The lengthening of the fit will increase the time for which the fit will hold. The rate at which the shaft will work loose can be materially decreased by making the end of the hub less rigid, as shown in Fig. 5–28b. A hyperbolic shape, as shown in Fig. 13–2, is still better. The ribs a support the rim without increasing the torsional rigidity of the hub. Naturally, if the torque is transmitted to both shaft ends the hub must be made symmetrical.

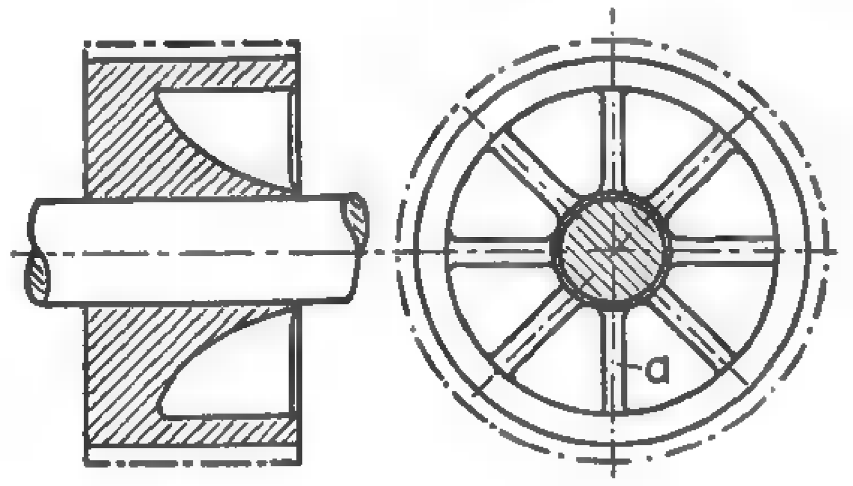


FIG. 13-2. Gear with a flexible hub.

13–3. Shrink joints. Since the holding power of a shrink joint is considerably greater than that of a press joint with the same interference, there is no reason to use a greater interference in a shrink joint. However, a rather common practice is to make the interference in a shrink joint almost twice as large, even though this is often detrimental to the connection because a large interference may produce stresses above the elastic limit.

Types. Besides being used instead of press joints to connect shafts to hubs, shrink joints are used to connect machine parts by means of rings and anchors.

[7] *Design Work Sheets* (New York: Product Engineering, 1932), p. 99.

Figure 13–3a shows one of several hoops used to connect heavy frames; Fig. 13–3b shows a ring shrunk on the hub end of a split pulley or flywheel; and Fig. 13–3c shows one of the four anchors used to connect two halves of a split flywheel. Another example of a shrink connection is the heating of a long tie rod having nuts on each end, with a blowtorch, before tightening the nuts. Finally, shrink joints are used to fasten rims of a certain material on wheels of another material, as in the case of gears and rail-car wheels. Circular hoops should be preferred to oval hoops because of the greater ease of machining circular hoops and the greater accuracy in determining the actual interference and the resulting stresses.

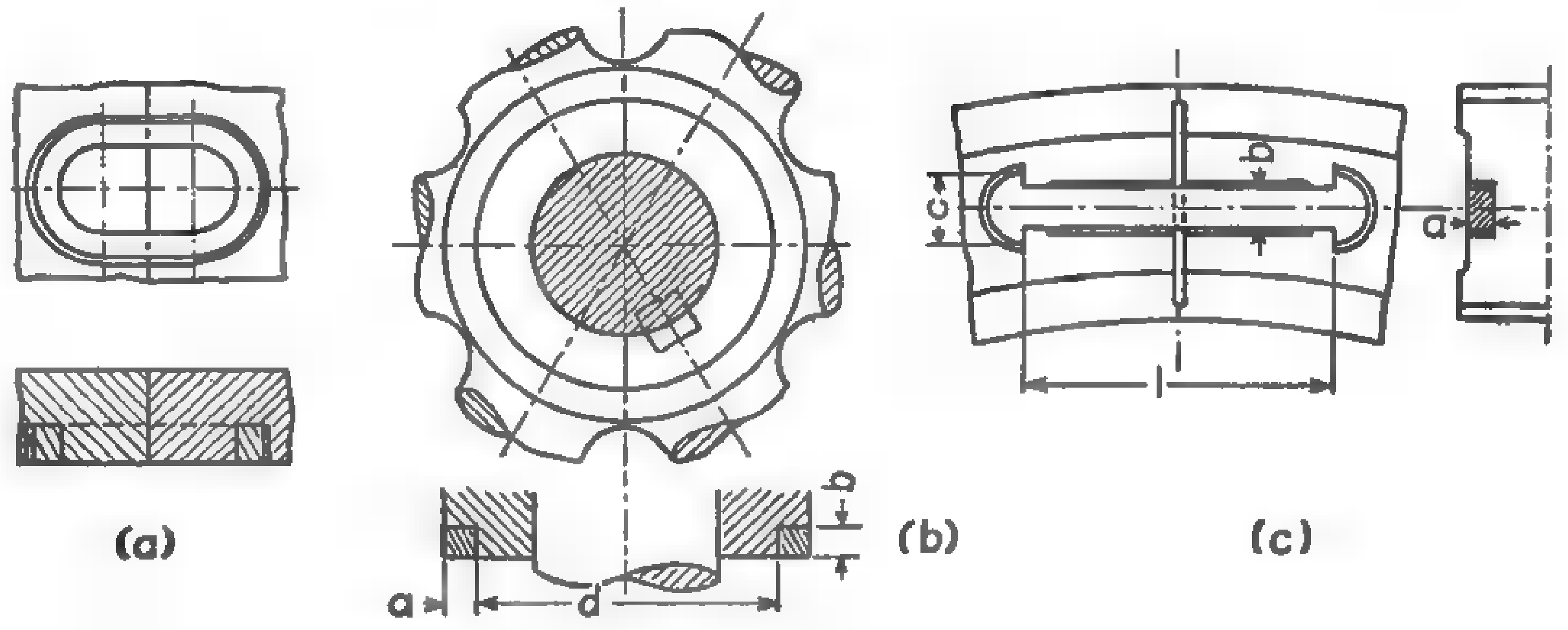


FIG. 13-3. Types of shrink joints.

Stresses. The stresses in hubs shrunk on shafts are exactly the same as those that occur in the case of press joints. If a hoop which is shrunk on to hold two parts together is radially thin compared to its diameter, the assumption can be made without appreciable error that the stress is uniform throughout the cross section of the hoop. The contraction in the part on which the hoop is placed is so small in such a case as to be negligible, and the stress in the hoop may be considered as due to stretching it over an incompressible body. This assumption is in favor of strength. The method of computation can be best illustrated by the following example.

EXAMPLE 13–3. Two circular steel rings are to be shrunk on the ends of the hub of a split flywheel, Fig. 13–3b. The outside diameter of the hub is $d = 16.020$ in., and the centrifugal force that must be resisted by the hoops is 180,000 lb. Determine the inside diameter of the hoops and their cross section, provided the maximum tensile stress does not exceed 30,000 psi.

If it is assumed that $E = 30{,}000{,}000$ psi and that the hub is incompressible, equation 13–3 gives the magnitude of the radial interference as

$$i = \frac{sr_2}{E} = \frac{30{,}000 \times 16.020}{30{,}000{,}000 \times 2} = 0.008 \text{ in.}$$

The inside diameter of the rings must be

$$d_o = 16.020 - 2i = 16.020 - 2 \times 0.008 = 16.004 \text{ in.}$$

The interference per inch of diameter is $2 \times 0.008/16 = 0.001$ in., which corresponds to a class 8 fit; and the tolerance q, according to Table 13–1, is

$$q = 0.0006\sqrt[3]{d} = 0.0006\sqrt[3]{16} = 0.0015 \text{ in.}$$

The initial tension set up by the shrinkage must be greater than the centrifugal force in order that the hub halves shall be held together. If a safety margin of 50 per cent is assumed, the force that each ring must resist is

$$F = \tfrac{1}{2} \times 180{,}000 \times 1.5 = 135{,}000 \text{ lb}$$

Since this force is taken up by two cross sections $a \times b$,

$$ab = \frac{135{,}000}{30{,}000 \times 2} = 2.25 \text{ sq in.}$$

The section dimensions may be $a = b = 1.5$ in.

Anchors. The necessary linear interference i for shrink anchors may be computed by the equation

$$i = \frac{S_d l}{E} \tag{13-16}$$

where l is the effective length, Fig. 13-3c, and the stress S_d may be taken with a safety factor n of 1.25.

The force that is exerted by such an anchor is

$$F = abS_d \tag{13-17}$$

where it is convenient to make the ratio b/a between 2 and 3 and to select the dimension a as the thickness of an available stock plate in order to avoid unnecessary machining.

Shrink-on temperatures. The temperature t_2 to which a piece ready to be shrunk on must be heated to go on the larger part having a temperature t_1 depends on the radial interference i and may be computed from the equation

$$a'd(t_2 - t_1) \geqq 2i \tag{13-18}$$

where a' is the coefficient of linear expansion and d is the diameter or length of the shrink link.

Example 13-4. Find the temperature to which the hoops of example 13-3 must be heated for assembling.

The coefficient of linear expansion for steel, from Table 5-3, is $a' = 0.065/10{,}000$. From equation 13-18, the temperature should be

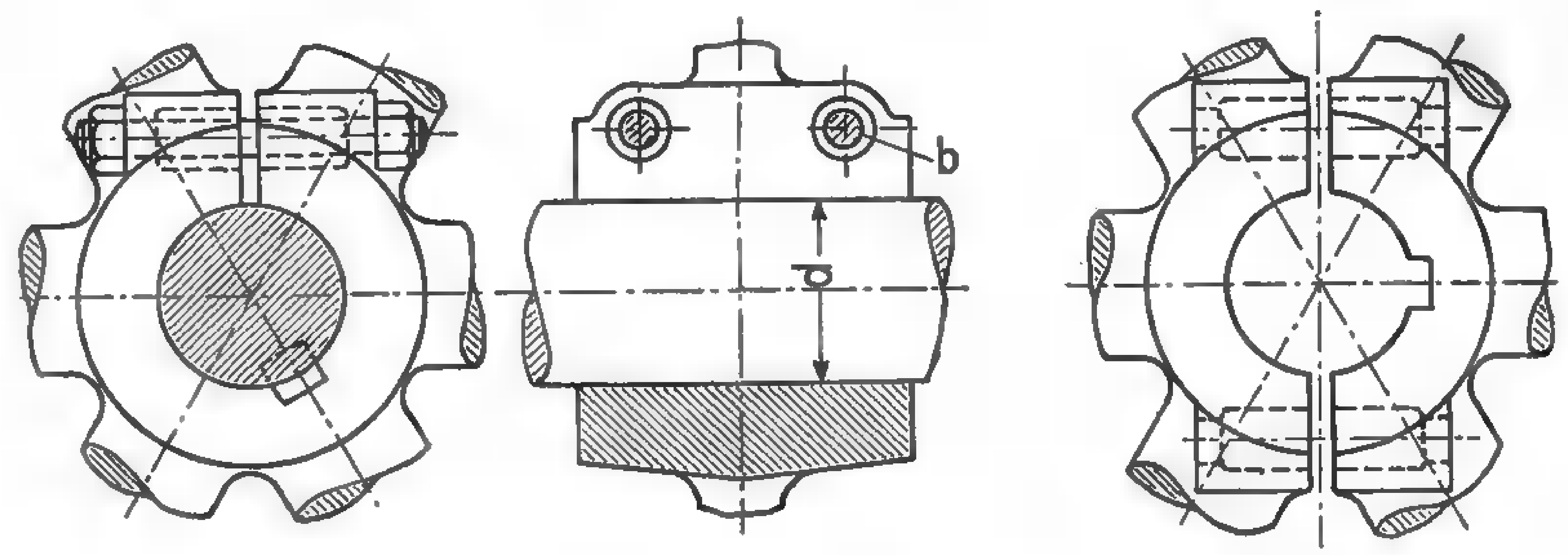
$$t_2 \geqq \frac{2 \times 0.008 \times 10{,}000}{0.065 \times 16} + t_1 \geqq 154 + t_1$$

If we assume a safety margin of about 20 per cent and a room temperature of $t_1 = 70$ F, the temperature to which the hoops must be heated will be

$$t_2 = 154 \times 1.2 + 70 = 255 \text{ F}$$

Shrink fits are also assembled by using dry ice to cool the piece that goes into a hole, such as a pin, to a temperature t_3, thus creating the required temperature difference $(t_1 - t_3)$.

13-4. Friction joints. The connection of a shaft to another rotating part by means of a friction joint is attained either by a cylindrical fit or by a conical fit.



(a) (b)

Fig. 13-4. Split-pulley hubs.

For a cylindrical friction joint a class 6 fit is used. The split hub, Fig. 13-4a, slides on the shaft freely when a steel wedge is driven into the slot. After the wedge has been removed, an additional grip on the shaft is obtained by tightening the bolts b. The diameter of the bolts could be figured from the torque transmitted by the pulley, but usually it is made equal to $\frac{1}{6}d$. A square, side-fitted key may be used. If a taper key is used, it should be placed so as not to act against the bolts. In a large pulley or flywheel the hub may be split into two pieces, as shown in Fig. 13-4b. In this case the initial gripping force is set up by the rim tension through the arms.

In a conical fit the force of friction is created by the pull of a nut, as shown in Fig. 13-5. The shank is turned down to the minor-diameter size in order to increase the elasticity of the fastening and thus to insure more constant pressure and friction between the conical surfaces. Still better results may be obtained by inserting a spring-type washer instead of the solid washer a.

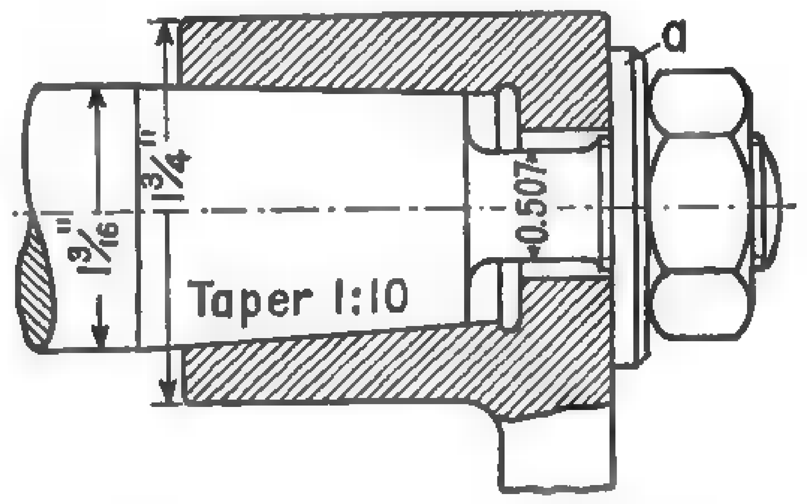


Fig. 13-5. Hand crank.

The static coefficient of friction f between the conical surfaces may be taken from Table 13-2.[8]

TABLE 13-2

Coefficient of Friction between Conical Surfaces

Metals in Contact	Condition of Surfaces	Coefficient f
Any	Greased with tallow	$\leqq 0.10$
Any	Oil-lubricated	≈ 0.15
Steel on cast iron	Dry	0.16
Steel on steel	Dry	0.22
Steel on cast iron or on steel	Shrink joint, "cold-welded"	≈ 0.33

[8] J. Bach, "Kegelreibungsverbindungen," *Zeitschrift Verein Deutscher Ingenieure*, Vol. 79 (1935), pp. 1570-71.

The diametrical taper is made from $1\frac{1}{2}$ in. per ft, or 1:8, down to $\frac{3}{4}$ in. per ft, or 1:16, and even smaller.

The torque transmission in a conical fit is often secured by a key that may be either rectangular or round. A round key is driven into a hole drilled half-and-half in the connected parts.

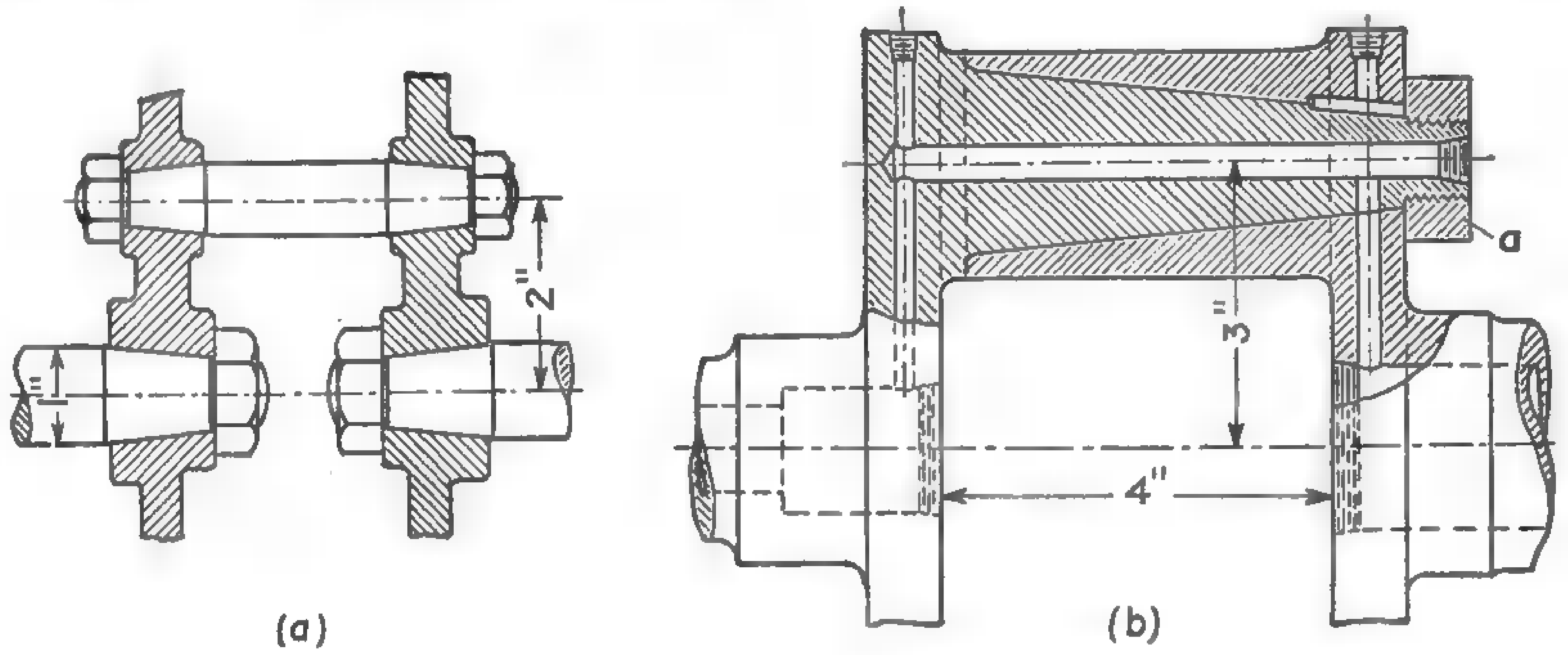


FIG. 13-6. Small built-up crankshafts.

Figure 13–6a shows a built-up crankshaft of a motorcycle engine, and Fig. 13–6b shows the center part of a two-piece crankshaft of a radial airplane engine. In the latter the two halves are connected by a lapped-in conical fit with a taper of 1:8 and a round-pin key. The propeller hub is also fastened with a conical fit. The taper of this hub is given by the SAE standards. The hub is secured additionally by either a rectangular key or an integral multiple spline.

The proper size of the pulling nut in a conical fit may be determined by an analysis similar to that applied to taper keys.

CHAPTER 14

Springs

14–1. General considerations. Springs are used to connect two parts or bodies by a flexible joint. Their functions are:

a. To cushion, absorb, or control energy due either to shock, as in car springs or railway buffers, or to vibration, as in spring-supports and vibration dampers.
b. To control motion—by maintaining contact between two elements, such as a cam and its follower; by restoring a machine part to its normal position when the disturbing force is removed, as in a governor or valve; or by producing the necessary pressure in a friction device, as in a brake or clutch.
c. To store energy, as in clockworks or starters.
d. To measure forces, as in spring balances, gages, or engine indicators.

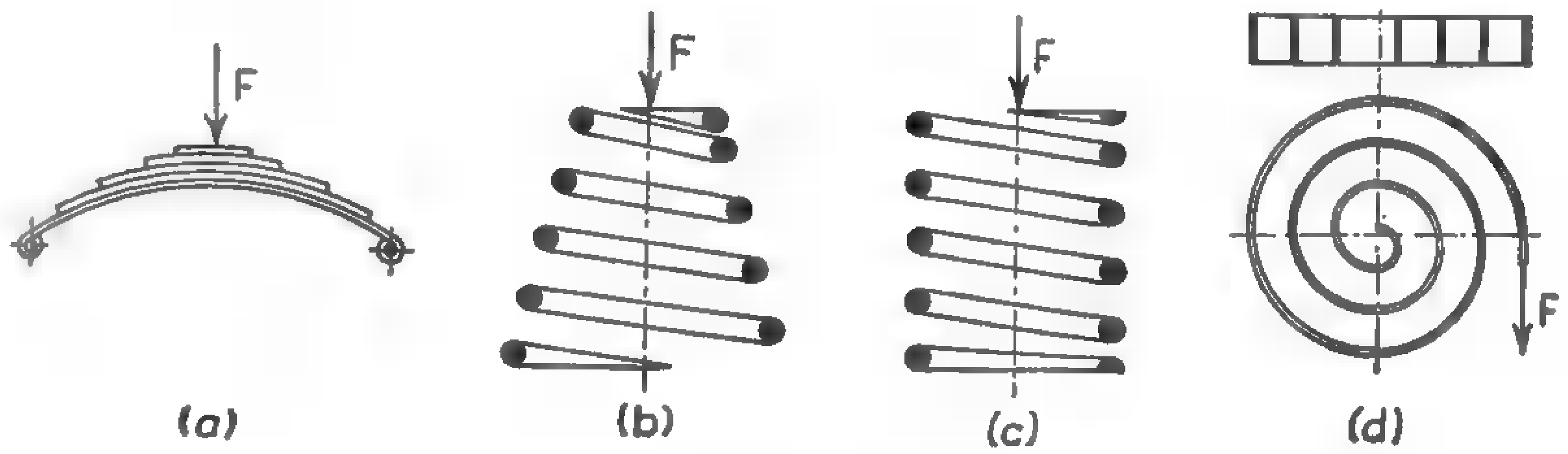


FIG. 14–1. Shapes of springs.

Classification. Springs can be classified according to shape, as indicated in Fig. 14–1. In Fig. 14–1a is shown a *plate spring*, or *leaf spring;* and in Fig. 14–1b is shown a *helical conical spring*. If the radius of the coils of a helical spring is constant, as in Fig. 14–1c, it becomes a *cylindrical spring*. If the angle of the helix is zero, as in Fig. 14–1d, it is a *spiral spring*.

Special springs for large loads and deflections are made in the form of a series of flat-tapered or dish-shaped disks.[1]

In leaf, spiral, and disk springs the stress induced by the load F is *bending*. In a helical spring the main stress induced by an axial load F is *torsion*. If a helical spring works in torsion similarly to a spiral spring, the main stress is *bending*.

[1] F. W. Brecht and A. M. Wahl, "The Radially Tapered Disk Spring," *Transactions of the American Society of Mechanical Engineers*, Vol. 52 (1930), APM-52-4, p. 45.

14–2. Leaf springs. A *flat spring* may have the form of a cantilever beam, as in Fig. 14–2a, b, or c; or it may have the form of a simple beam, as in Fig. 14–2d, e, or f, which can be considered as a double cantilever beam. Some beams shown in Fig. 14–2 are of constant cross section, and others are of uniform strength obtained by keeping either a constant thickness h or a constant width b. The main stresses are tension on one side of the neutral axis and compression on the other. The additional transverse shear may be neglected as far as strength is concerned.

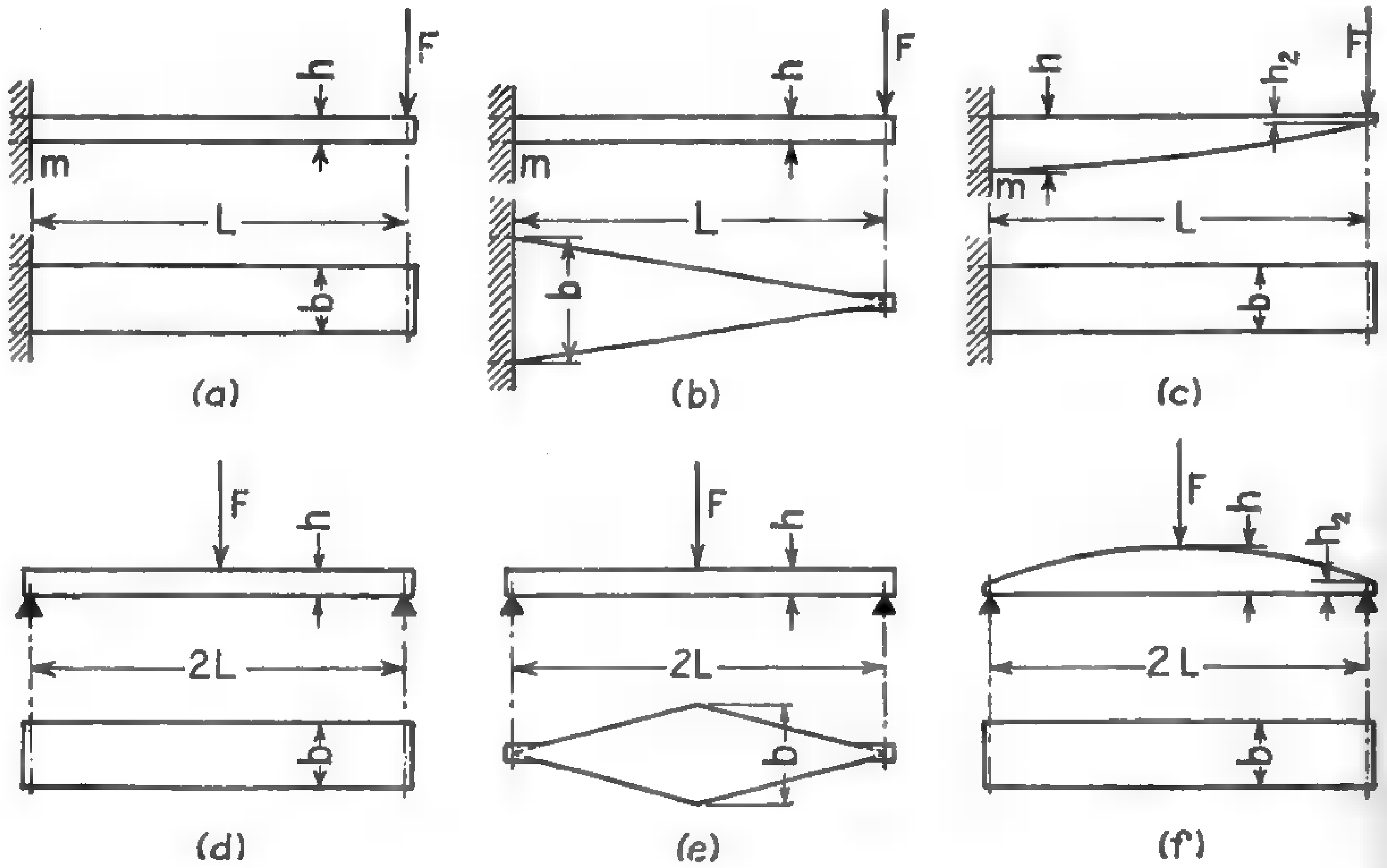


FIG. 14–2. Beams with a rectangular section.

The maximum stress in each of the six cases in Fig. 14–2 is $s = M/Z$, and it may be expressed by the one general equation

$$s = \frac{c_1 Fl}{bh^2} \tag{14-1}$$

Similarly, the maximum deflection is

$$y = \frac{c_2 Fl^3}{Ebh^3} \tag{14-2}$$

The constants c_1 and c_2 can be taken from Table 14–1, which also contains data on the resilience for the various types of springs.

By dividing equation 14–1 by equation 14–2, the terms F and b are eliminated and the following convenient relation for the design of a spring is obtained:

$$h = \frac{c_2 s l^2}{c_1 E y} \tag{14-3}$$

where l and y are given; s is the allowable stress which must be selected for the material; and E is the modulus of elasticity. After that, the width for a given load F can be found from equation 14–1. Thus,

TABLE 14–1

CONSTANTS IN BEAM EQUATIONS

Constant	Cantilever Beam (Fig. 14–2)			Simple Beam (Fig. 14–2)		
	a	*b*	*c*	*d*	*e*	*f*
c_1—for the stress	6	6	6	3	3	3
c_2—for the deflection	4	6	8	2	3	4
Unit resilience, in.-lb per cu in.	$\frac{s^2}{18E}$	$\frac{s^2}{6E}$	$\frac{s^2}{6E}$	$\frac{s^2}{18E}$	$\frac{s^2}{6E}$	$\frac{s^2}{6E}$

$$b = \frac{c_1 Fl}{sh^2} \tag{14-4}$$

In Fig. 14–2 it is shown that beams of uniform strength permit a considerable saving in material. At the same time they have a greater deflection, as shown in Table 14–1. This means that their resilience and capacity for absorbing impact energy are also greater in the same proportion, as was shown in section 3–4. The shear stress, which can be neglected in the section with the maximum width b, becomes important toward the end of the beam. Therefore the beam cannot come to a point, but it must have either a certain minimum height h_2 or a certain minimum width b_2, to resist this shear.

EXAMPLE 14–1. Determine the thickness and width of a flat steel spring to carry a load of 750 lb with a deflection of about $1\frac{1}{4}$ in. The spring must be supported at the ends, the distance between the supports being 26 in., and is loaded at the center. Allow a maximum stress of 50,000 psi and make the spring with a constant thickness and varying width.

According to Table 4–2 the steel used must have a modulus of elasticity of 31,000,000 psi. The conditions correspond to those in Fig. 14–2e. From Table 14–1, the constants are $c_1 = 3$ and $c_2 = 3$. According to Fig. 14–2e, $l = \frac{26}{2} = 13$ in.

By equation 14–3,

$$h = \frac{3 \times 50{,}000 \times 13^2}{3 \times 31{,}000{,}000 \times 1.25} = 0.218, \text{ or } \tfrac{7}{32} \text{ in.}$$

By equation 14–4,

$$b = \frac{3 \times 750 \times 13}{50{,}000 \times 0.219^2} = 12.22 \text{ in.}$$

14–3. Laminated springs. In order to decrease the width b, if it becomes too large, as in example 14–1, the lozenge-shaped plate can be assumed to be cut into narrow strips, as indicated in Fig. 14–3, and assembled with a clamp c, as in Fig. 14–3b. Evidently

$$b' = \frac{b}{i} \tag{14-5}$$

where i is the number of strips or leaves.

This laminated spring is a beam of approximately uniform strength. The unit stress and the maximum deflection, if friction between the leaves is neglected, will be the same as the values for the original plate.

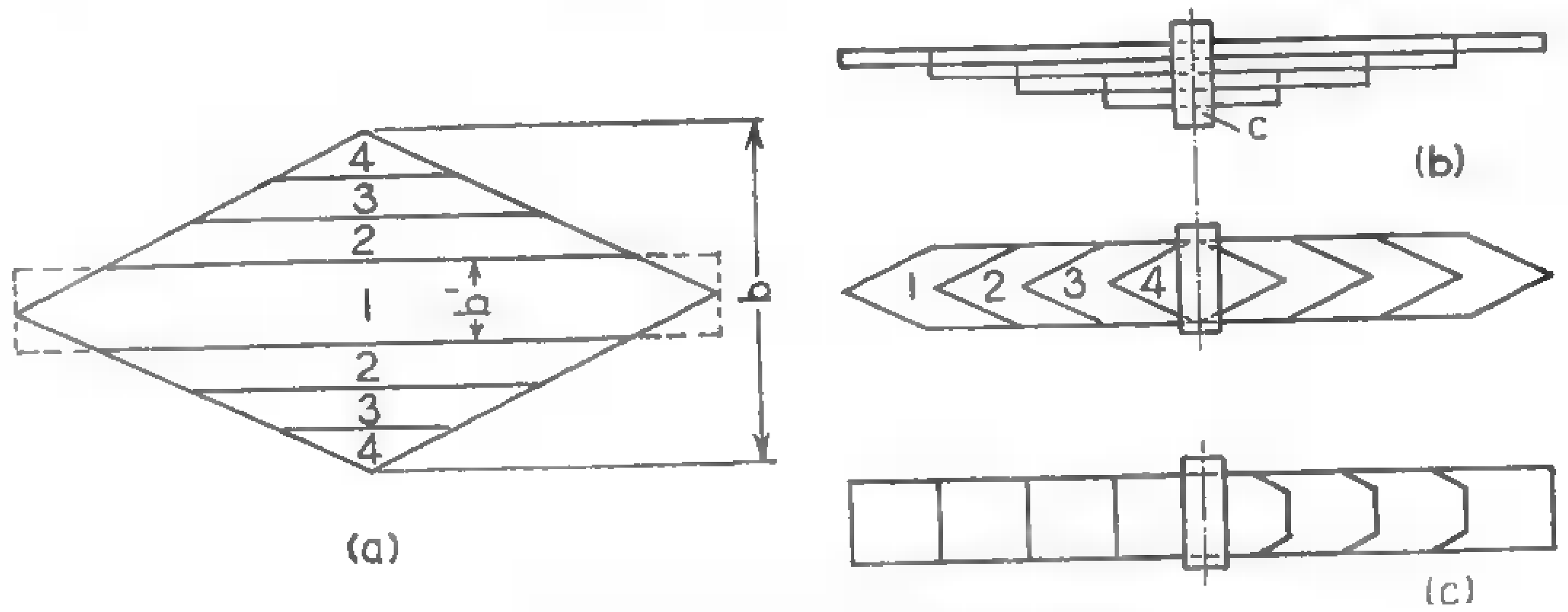


FIG. 14-3. Laminated leaf springs.

Equations 14-1 to 14-4 apply to laminated springs with the substitution of ib' for b. Thus the load, from equation 14-1, is

$$F = \frac{sib'h^2}{c_1 l} \tag{14-6}$$

and the maximum deflection is

$$y = \frac{c_2 F l^3}{Eib'h^3} \tag{14-7}$$

In an actual laminated spring the full-length leaves must have square ends by which they are fastened to the supports. Sometimes the shorter, or *graduate*, leaves are simply cut square at the ends, as shown in Fig. 14-3c at the left. To make the transition from one leaf to the next gradual, it is better to taper the ends in width or in thickness, or in both dimensions, as shown on the right side in Fig. 14-3c. This change does not affect the stress found by equation 14-6, but it may influence the deflection found by equation 14-7.

The resilience of a laminated spring is equal to that of an equivalent lozenge-shaped spring.

EXAMPLE 14-2. Change the spring of example 14-1 to a laminated leaf spring and find: (a) the stress which will be induced if the load comes down with a shock, deflecting the spring 3 in.; (b) the magnitude of the impact energy which the spring will absorb in this case; (c) the height from which the load must drop; and (d) the corresponding impact force.

a) If the spring is made of 6 leaves, the width b' is $12.22/6 = 2.037$ in.

The stress under the increased deflection can be found by noticing that according to equation 14-3 the stress is proportional to the deflection. Hence,

$$s = \frac{50,000\times 3}{1.25} = 120,000 \text{ psi}$$

b) The impact is equal to the resilience of the spring, which, according to Table 14-1, is

$$U = V\times\frac{s^2}{6E} = \frac{26\times 12.22\times 0.219}{2}\times\frac{120,000^2}{6\times 31,000,000} = 2,700 \text{ in.-lb}$$

c) Since the total impact energy $K_i = W(h+e)$, where e is the deflection, or 3 in., the height of drop must be

$$h = \frac{2,700}{750} - 3 = 0.60 \text{ in.}$$

Checking h by equation 3-18 gives

$$h = \left[\left(\frac{120,000}{50,000} - 1\right)^2 - 1\right]\frac{1.25}{2} = 0.6 \text{ in.}$$

d) The force of the impact is found by equation 3-23

$$F = \frac{2U}{e} = \frac{2\times 2,700}{3} = 1,800 \text{ lb}$$

This force F will act upon the end supports of the spring.

Other factors. This analysis did not take into consideration several factors which influence the design of actual springs. First comes the shape of the leaves. The most common type of leaf spring is the *semielliptic*, Fig. 14-4. The unloaded spring is curved, or *cambered*, the magnitude of the camber being such that the spring is approximately straight under the full static load. Under the maximum dynamic load the camber will be negative.

The long leaf a fastened to the supports is called the main leaf or *master leaf*. Its ends are bent to form an eye c. The center bolt d holds the spring leaves together. The hole drilled for this bolt weakens the spring. Fortunately, the pressure exerted by the U clips e, which hold the spring to the seat, materially reduces the bending stresses in the center part with the bolt. However, the bending action is not reduced enough to allow the omission of this part from the length l of the spring in computations relating to deflections and stresses. Also, the U clips e create a stress concentration at the edge of the spring seat. A soft pad placed between the leaf and the seat will reduce the stress concentration. The *rebound clips* f serve to distribute to the shorter leaves some of the load of the rebound which otherwise would be taken by the master leaf alone.

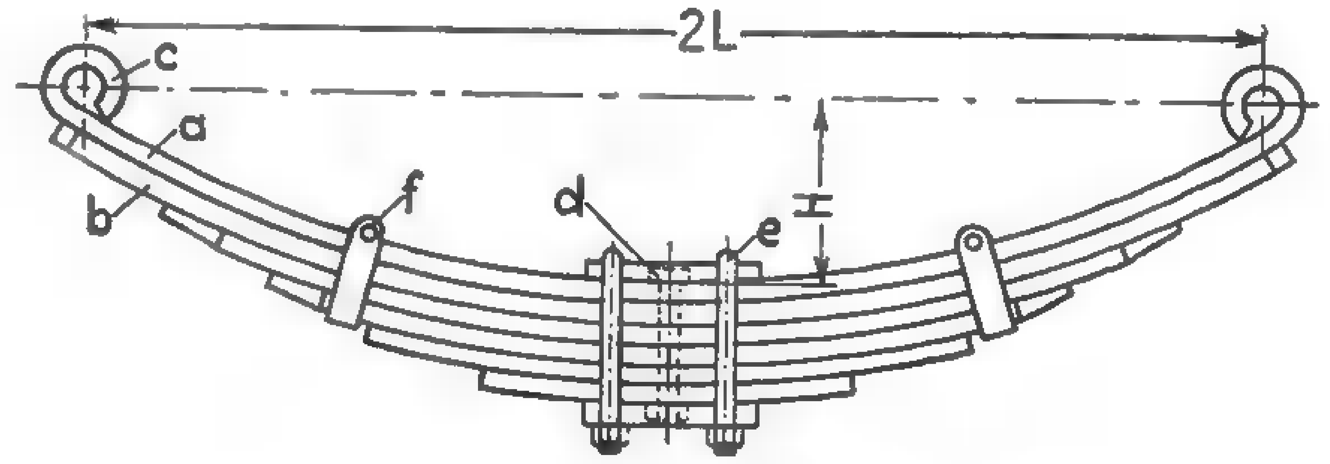


FIG. 14-4. Laminated semielliptic spring.

Full-length leaves. Laminated springs for heavy loads usually have under the master leaf one or more additional full-length leaves, as b in Fig. 14-4. With such an arrangement the spring ceases to be of uniform strength. However, the relation between the load F and the stress s remains the same, being

given by equation 14–6. The deflection can be calculated by introducing a correction factor k_4. Thus,

$$y=\frac{c_2Fl^3k_4}{Eib'h^3} \tag{14–8}$$

This factor k_4 may be determined by the formula[2]

$$k_4=\frac{1-4r+2r^2(1.5-\log_e r)}{(1-r)^3} \tag{14–9}$$

where $r=i'/i$, with i the number of full-length blunt-ended leaves and i the total number of leaves.

Support influence. The ends of the master leaf of a laminated spring are fastened to the supports by means of hinges. This arrangement produces longitudinal loads and additional stresses. To these may be added stresses from transverse forces and from twisting. One method of taking care of this increase of stresses in the master leaf is to make the master leaf of a stronger material than the rest of the spring.

Another way is to reduce the combined stress in the master leaf by making it thinner than the rest,[3] as can be seen from equation 14–3. The simplest way to reduce the stress is to make the radius of curvature of the master leaf a, Fig. 14–4, greater than that of the next one. When the spring is assembled the master leaf is given an initial bend by the center bolt and is thus given an initial stress in a direction opposite to that which the load will create. Therefore, when the load is applied, the stress in the master leaf is first relieved of the initial stress and is then only stressed in the opposite direction. The other leaves receive initial stresses in the same direction as those coming from the load. Hence, the stress from the vertical load in the master leaf will always be lower than that in the rest of the leaves. Such prestressing caused by a difference in the radii of curvature is termed *nipping*. In automobile springs all leaves are nipped to a certain extent. This helps to keep them in contact on the rebound, and also keeps out dirt.[4]

14–4. Leaf-spring design. Carbon steel SAE 1095 that is properly heat-treated is commonly used for laminated springs. Silicon-manganese steels, SAE 9250 and SAE 9260, the latter having a slightly higher carbon content and higher mechanical properties, are both standardized by the Society of Automotive Engineers for leaf springs. Chrome-vanadium steel has a still greater strength, as well as greater toughness and resistance to failure through repeated stresses.

[2] J. B. Peddle, "Chart for Full and Semielliptic Leaf Springs," *American Machinist*, Vol. 38 (April 17, 1913), p. 645; also F. A. Halsey, *Handbook for Machine Designers*, 2d ed. (New York: McGraw-Hill Book Company, Inc., 1916), p. 201.

[3] F. Franz, "Remedy for Spring Breakage," *Automotive Industries*, Vol. 45 (1921), p. 924.

[4] P. M. Heldt, *Motor Vehicles and Tractors* (Nyack, N. Y.: published by the author, 1929), pp. 468 ff.

TABLE 14–2

WIRE AND SHEET-METAL GAGES

Gage No.	Steel Wire, Washburn & Moen (Roebling) Gage d (in.)	Nonferrous Wire, American (Brown & Sharpe) Gage d (in.)	Sheet Steel, Birmingham Wire Gage h (in.)	Gage No.	Steel Wire, Washburn & Moen (Roebling) Gage d (in.)	Nonferrous Wire, American (Brown & Sharpe) Gage d (in.)	Sheet Steel, Birmingham Wire Gage h (in.)
0000......	0.3938	0.460	0.454	8.......	0.1620	0.128	0.165
000......	0.3625	0.410	0.425	9.......	0.1483	0.114	0.148
00......	0.3310	0.365	0.380	10.......	0.1350	0.102	0.134
0......	0.3065	0.325	0.340	11.......	0.1205	0.091	0.120
1......	0.2830	0.289	0.300	12.......	0.1055	0.081	0.109
2......	0.2625	0.258	0.284	13.......	0.0915	0.072	0.095
3......	0.2437	0.229	0.259	14.......	0.0800	0.064	0.083
4......	0.2253	0.204	0.238	15.......	0.0720	0.057	0.072
5...	0.2070	0.182	0.220	16.......	0.0625	0.051	0.065
6......	0.1920	0.162	0.203	17.......	0.0540	0.045	0.058
7......	0.1770	0.144	0.180	18.......	0.0475	0.040	0.049

Sizes. Spring plates are made in accordance with the Birmingham wire gage, Table 14–2, and also in thicknesses varying in thirty-secondths of an inch from $\frac{3}{16}$ to $\frac{1}{2}$ in. The sizes recommended by the SAE Handbook are: BWG Nos. 4, 3, and 2, and $\frac{5}{16}$, $\frac{3}{8}$, and $\frac{7}{16}$ in.

The width of the strips varies in quarters of an inch from $1\frac{3}{4}$ to 4 in.

Limit stresses. All springs must be heat-treated. The mechanical properties of spring materials that are properly tempered are given in Table 14–3 for materials about $\frac{1}{2}$ in. thick. The influence of the thickness t is considerable in spring leaves and can be taken into account by multiplying the values in Table 14–3 by a size coefficient which is computed by the formula[5]

$$e_{sz}=0.8+\frac{0.1}{h} \tag{14–10}$$

Safety factor. For static loads, as in governor springs, a safety factor n of 1.5 is sufficient. If the calculations are based on elastic limits, in railroad service the values of n can range from 2 to 2.25; in automobile design n is taken as 2.25 to 2.5 for rear springs and from 3.25 to 3.5 for front springs. If the design takes into account the shock action and is based on endurance values, a factor of safety of 1.5 or even 1.25 is sufficient.

Design procedure. So many variable factors enter into the design calculations of a leaf spring that no definite rules can be given and cut-and-try methods have to be applied.

[5] Derived from data given in D. S. Kimball and J. H. Barr, *Elements of Machine Design*, 3d ed. (New York: John Wiley & Sons, Inc., 1935), p. 215

TABLE 14-3

PROPERTIES OF SPRING MATERIALS

Material	Ultimate Strength in Tension S_u (kpsi)	Elastic Limits: Tension S_l (kpsi)	Elastic Limits: Shear S_s (kpsi)	Endurance Limits: Bending S_{en} (kpsi)	Endurance Limits: Torsion S_{en} (kpsi)	Moduli of Elasticity: Direct E (psi)	Moduli of Elasticity: Transverse G (psi)
Music wire............	290	250	150	135	80	30,000,000	11,300,000
Cr-V steel, SAE 6150...	240	190	110	90	55	30,500,000	12,000,000
Si-Mn steel SAE 9250..	220	160	95	80	48	30,000,000	11,400,000
Carbon steel, SAE 1095.	200	140	85	70	42	29,700,000	11,400,000
Carbon steel, SAE 1050.	120	75	45	57	34	29,700,000	11,200,000
Monel metal, cold-rolled	120	70	42	50	30	25,500,000	9,600,000
Phosphor bronze, SAE 77 hard and SAE 81..	98	50	30	46	20	15,000,000	6,200,000
Brass wire, SAE 80, wire grade A............	95	47	28	20	12	14,500,000	5,500,000
Brass, SAE 70, hard-drawn..............	90	45	27	15	10	14,500,000	5,500,000
Duraluminum, C 17 ST, SAE 26............	65	34	20	13.5	8	10,000,000	3,900,000

The first step is to decide what material will be used and to assume its approximate thickness h. With the data of Table 14-3 and equation 14-10 the limit stress S_l can be determined. Having selected a suitable safety factor n, the design stress $S_d = S_l/n$ is determined.

The value of ib' can be found from equation 14-6 by substituting the selected value of S_d for S, using the given load F, and using a length l that

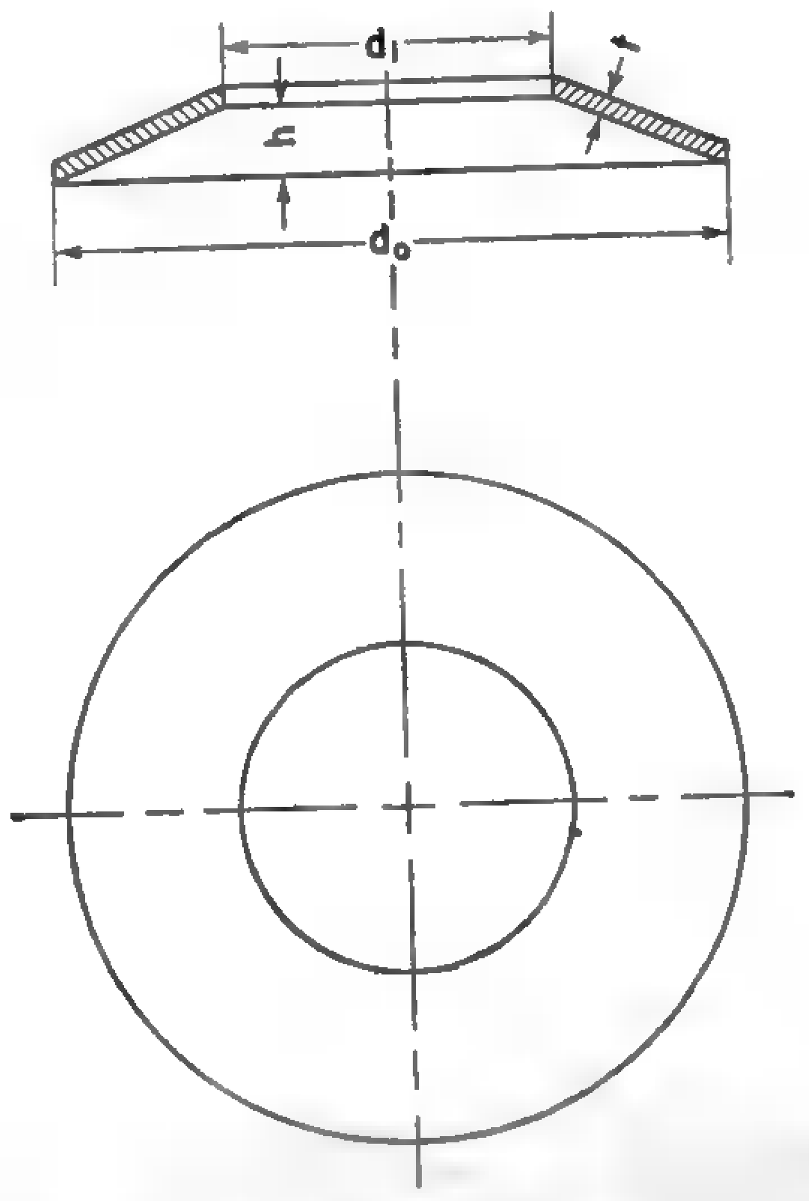


FIG. 14-5. Disk spring.

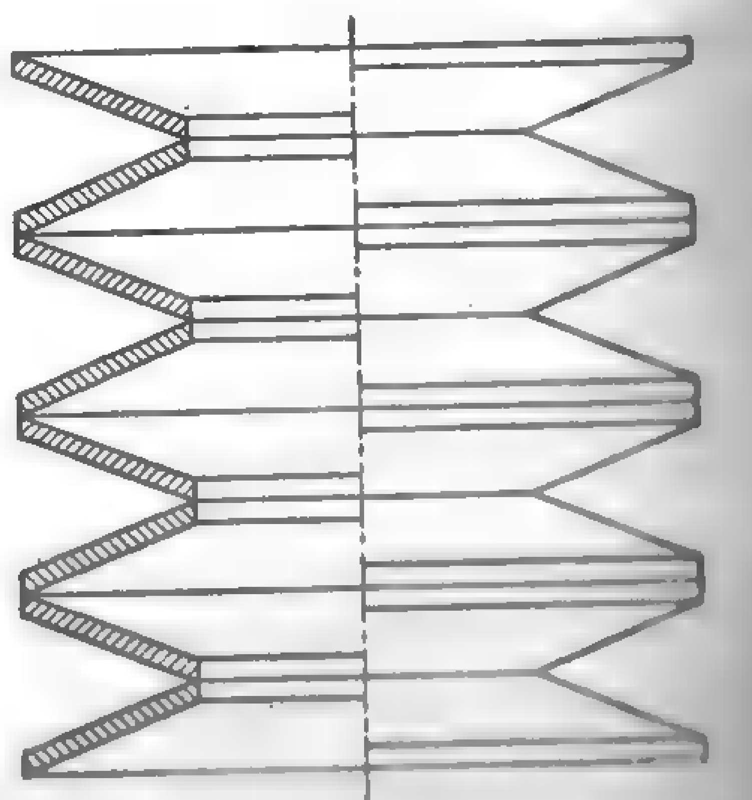


FIG. 14-6. Nest of disk springs.

is given or is determined by other considerations. The number of leaves i usually should be from 5 to 10, but in railroad cars it may be increased up to 14. If this does not give a strip width b' within the recommended limits $1\frac{3}{4}$ and 4 in., the thickness h must be changed correspondingly and the whole procedure repeated.

In a particular case other factors, such as nipping and variable leaf thickness, must be taken into consideration in finding both the maximum stress and the deflection.

Test check. Because of the involved calculations and the influence of friction, the actual performance of a laminated spring may differ somewhat from that indicated by the design figures. Before the final adoption of the design for quantity production, the spring performance should be checked under expected working conditions.

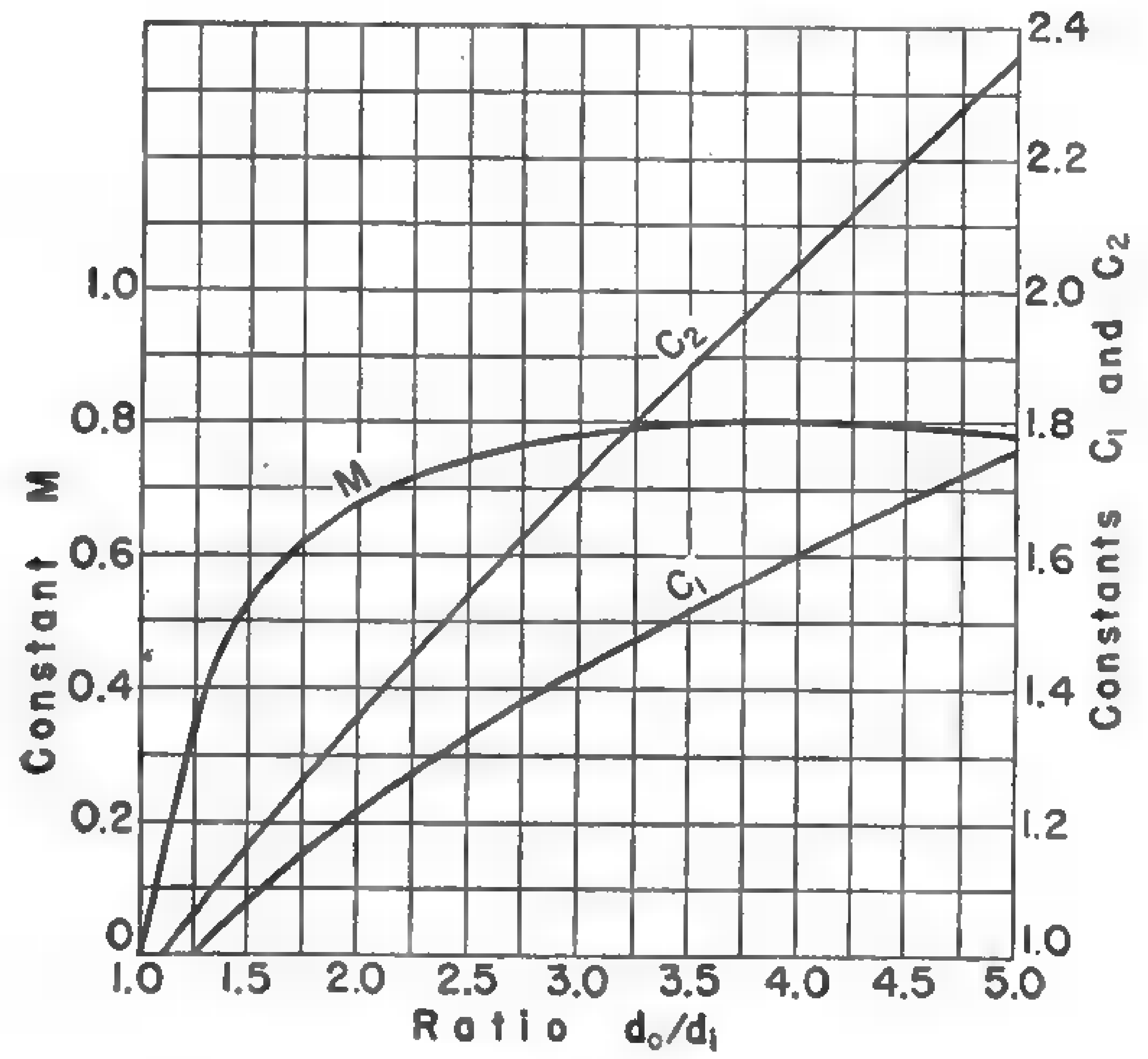


FIG. 14-7. Diagram of disk constants.

14-5. Disk springs. Disk springs, also called *Belleville springs*, are used where high-capacity compression springs must fit into small spaces. Each spring consists of several annular disks that are dished to a conical shape, as in Fig. 14-5. They are stacked up one on top of another, as in Fig. 14-6, in order to increase the deflection.

The load is applied uniformly around the edge, and the relation between the load F and the axial deflection y of each disk is given by the equation

$$F = \frac{4Ey}{(1-\mu^2)Md_o^2}\left[\left(h-y\right)\left(h-\frac{y}{2}\right)t + t^3\right] \tag{14-11}$$

where μ is Poisson's ratio; M is a constant which depends on the ratio d_o/d_i, as indicated in Fig. 14-7; and d_o, d_i, h, and t are spring dimensions.

The maximum stresses induced at the edges are given by the equation[6]

$$s = \frac{4Ey}{(1-\mu^2)d_o^2}\left[C_1\left(h-\frac{y}{2}\right) \pm C_2 t\right] \tag{14-12}$$

[6] J. O. Almen and A. Lazzlo, "Disk Springs Facilitate Compactness," *Machine Design*, Vol. 8 (June, 1936), p. 80; W. W. Boyd, "Deflection and Capacity of Belleville Springs," *Product Engineering*, Vol. 3 (September, 1932), p. 361, and Vol. 4 (February, 1933), p. 63; and Brecht and Wahl, *loc. cit.*

where the plus sign is used for the stress at the inner edge, the minus sign is used for the stress at the outer edge, and C_1 and C_2 are constants depending on the ratio d_o/d_i, as indicated in Fig. 14–7.

Experience with springs of this type shows that under static conditions the stresses computed by equation 14–12 may reach 220,000 psi for steel having an elastic limit in tension of 120,000 psi. Endurance tests show that the maximum stresses computed by equation 14–12 may reach 180,000 psi. Actual stresses are unknown. The endurance life of a disk spring is considerably increased if the corners of the disk edges are rounded off. The ratio d_o/d_i should be between 1.5 and 5.

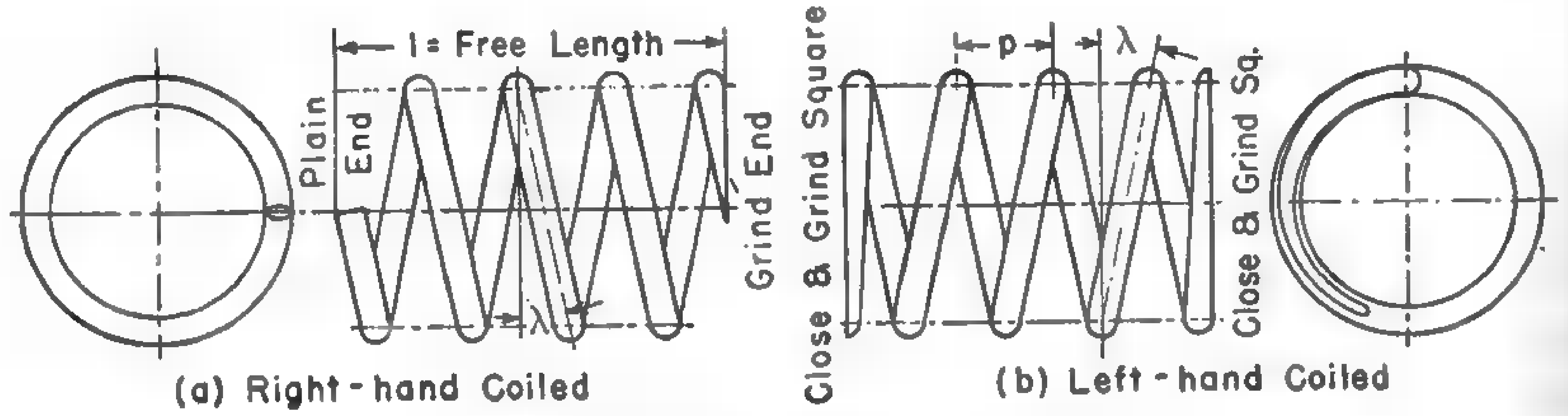


FIG. 14–8. Compression springs.

14–6. Helical springs. Helical springs are used to take up forces which tend to shorten, lengthen, or twist them. However, most springs work in compression. The advantages of compression springs are that they are cheaper to make and that they continue to function to a certain extent even if a coil breaks. In Fig. 14–8 are shown two compression springs that differ in the type of coiling and in the method of finishing the ends. The best end finish, which gives a uniform compression without additional distortion, is by bending the ends, closing them, and grinding them square to the axis. In such a spring practically a whole coil on each end is inactive, and this fact must be taken into consideration in figuring the spring deflection. Helical springs are usually made of round wire.

14–7. Cylindrical compression springs. Compression springs may be made of round wire, or wire with a rectangular cross section. The first part of this discussion will deal with round-wire springs.

Stress analysis. A quadrant of a coil of a round-wire spring, Fig. 14–9, will be considered as a free body. The load F acting along the axis of the spring, which has a mean diameter, or pitch diameter, D, produces a torsional moment determined by the formula

$$T = \frac{1}{2}FD \tag{14–13}$$

The same torsional moment acts upon all sections of the loaded spring. The internal resisting moment, by equations 2–13 and 2–14, is

$$s_s Z = \frac{1}{16} s_s \pi d^3 \tag{14–14}$$

where d is the wire diameter, in inches. Equating the external moment to the internal one and solving for s_s results in

$$s_s = \frac{8FD}{\pi d^3} \tag{14–15}$$

In addition to the shear stress s_s, there will be a compressive stress due to the component F' acting along the coil, and also a direct shear stress $s_{s2} = F/A$, where $A = 0.7854d^2$. Unless the helix angle or pitch angle λ is large, the compressive stress is negligible. The direct shear stress s_{s2} is usually neglected. However, if the spring diameter d is comparatively small, s_{s2} may become too great to be neglected.

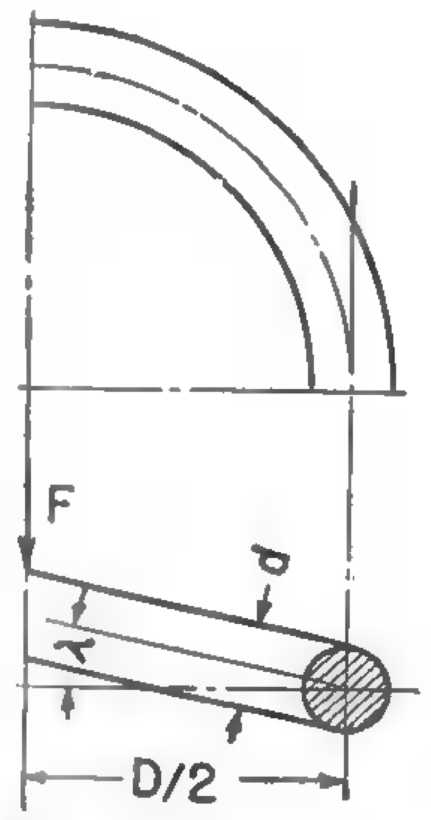


FIG. 14–9. A quadrant of a coil.

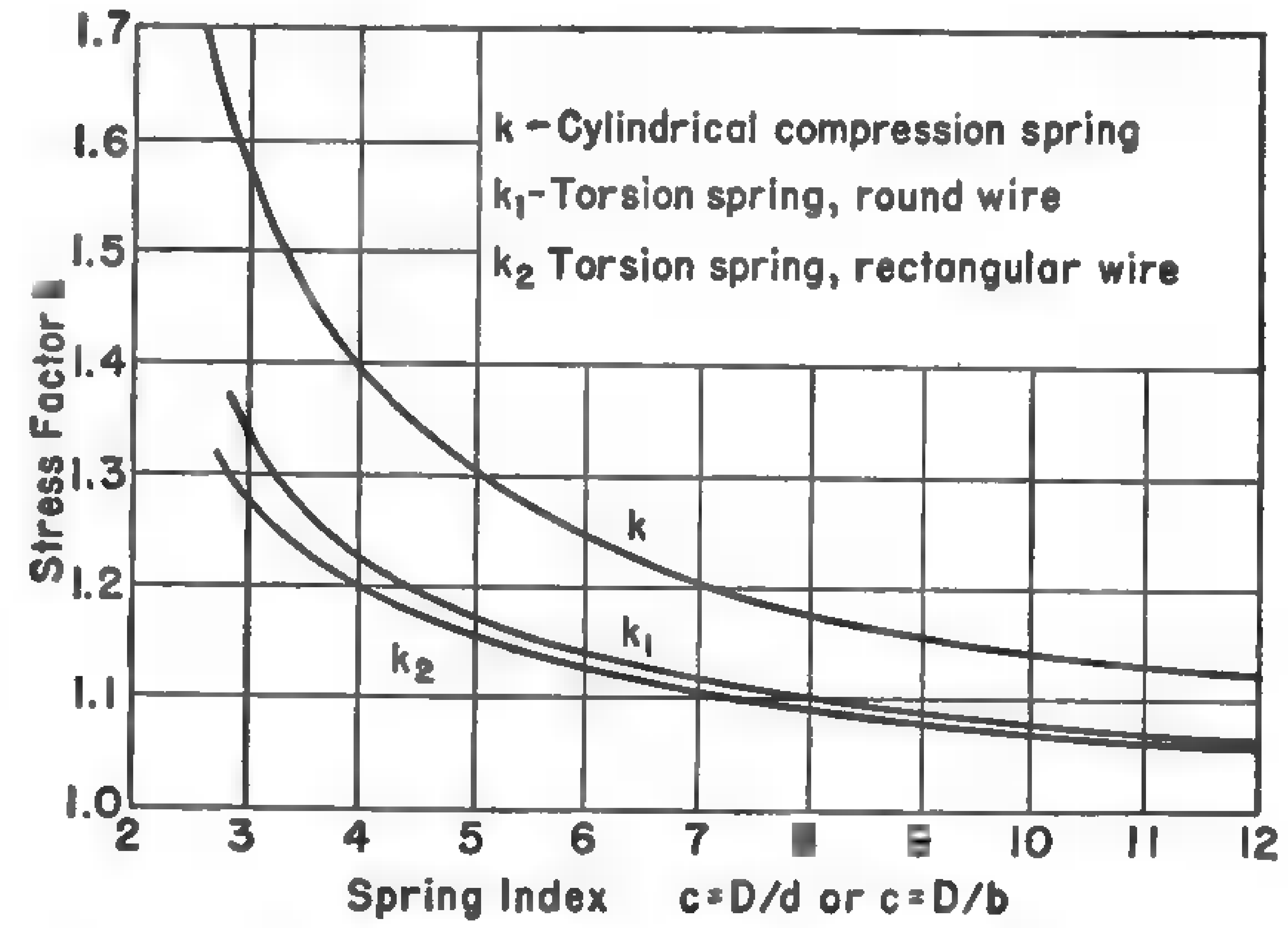


FIG. 14–10. Stress factors for helical springs.

By a theoretical analysis, which has been confirmed by actual tests, a more accurate formula for the stress has been established.[7] This formula is

$$s_s = \frac{8FDk}{\pi d^3} \tag{14–16}$$

where the stress factor k is a function of the diameter ratio $c = D/d$, which is called the *spring index*. For closely coiled springs,

$$k = \frac{4c-1}{4c-4} + \frac{0.615}{c} \tag{14–17}$$

For the sake of convenience a curve for finding k is given in Fig. 14–10. This curve shows that the stress factor increases very rapidly as the spring index c decreases. It is advisable to have c at least 3.

[7] A. M. Wahl, "Stresses in Heavily Coiled Helical Springs," *Trans. ASME*, Vol. 51 (1929), APM-51-17, p. 186, and Vol. 52 (1930), APM-52-18, p. 217; A. M. Wahl, "Helical Compression and Tension Springs," *Journal of Applied Mechanics*, Vol. 2 (March, 1935), PA-35.

Deflection. The angular deflection θ, found by using the general equation 2–16 and substituting for s_s its value from equation 14–15, is

$$\theta = \frac{16FDl}{\pi d^4 G} \tag{14–18}$$

The length l of the bar is approximately equal to πDi, where i is the number of active coils, which is greater by one-half coil than the number of free coils.[8] Since the axial deflection y of the whole spring is equal to the angular deflection times the mean radius of the coil,

$$y = \frac{8FD^3 i}{d^4 G} \tag{14–19}$$

Values of y calculated by equation 14–19 check well with those measured in actual springs.

Substituting for F its expression from equation 14–16 gives

$$y = \frac{\pi i s_s D^2}{kdG} \tag{14–20}$$

This equation allows finding the maximum deflection permissible for a selected stress limit. Solving equation 14–20 for s_s gives

$$s_s = \frac{kydG}{\pi i D^2} \tag{14–21}$$

This equation is convenient for checking the stress in a spring in operation.

Spring scale. A useful characteristic of a spring is the force F_o necessary to compress it 1 in. This force, which is termed the *spring scale,* is

$$F_o = \frac{F}{y} \tag{14–22}$$

Combining equations 14–22 and 14–19 gives

$$F_o = \frac{d^4 G}{8iD^3} \tag{14–23}$$

From equation 14–22, the deflection is

$$y = \frac{F}{F_o} \tag{14–24}$$

By equation 14–19 the deflection is directly proportional to the load. If the force F_1 exerted by a spring under certain conditions is given, and the force F_2 exerted with an additional compression y' of the spring is also given, then

$$F_o = \frac{F_2 - F_1}{y'} \tag{14–25}$$

[8] R. F. Fogt, "Number of Active Coils in Helical Springs," *Trans. ASME,* Vol. 56 (1934), RP-56-4, pp. 469–72.

Conversely, the total deflection y_2, by equation 14–24, is

$$y_2 = \frac{F_2}{F_o} = \frac{y' F_2}{F_2 - F_1} \tag{14–26}$$

This relation is illustrated by Fig. 14–11.

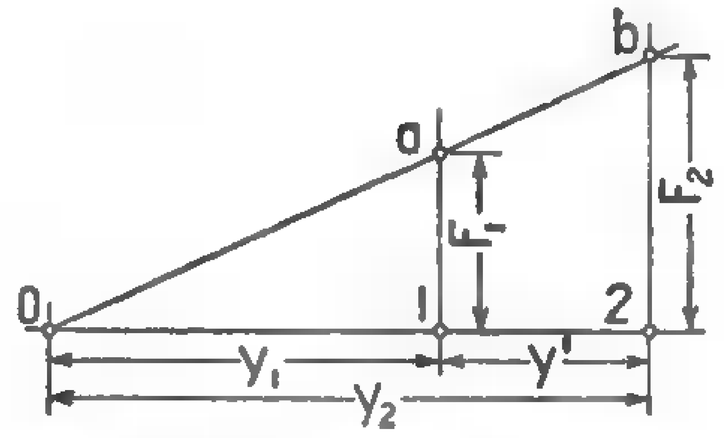


FIG. 14–11. Relation between loads and deflection in a helical spring.

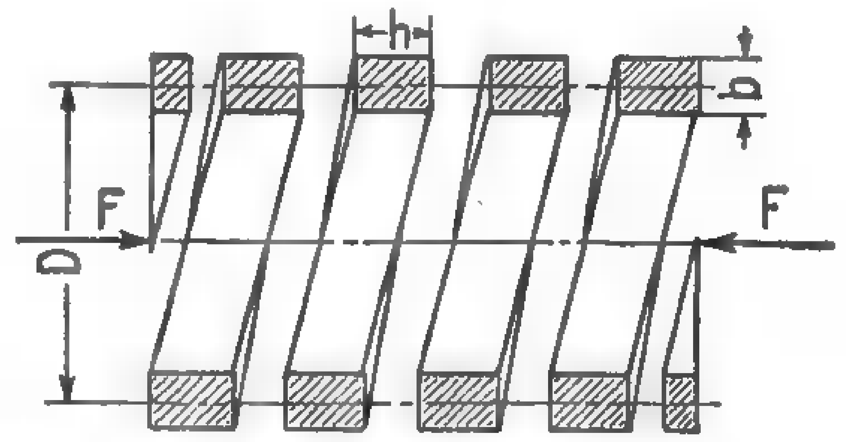


FIG. 14–12. Spring with rectangular section.

Resilience. The resilience U of a spring is equal to the energy absorbed. By the general equation 3–23, in which the value of y is given by equation 14–19,

$$U = \frac{Fy}{2} = \frac{4F^2 D^3 i}{d^4 G} \tag{14–27}$$

Substituting for F its value from equation 14–16 gives

$$U = \frac{\pi^2 d^2 D i s_s^2}{16k^2 G} \tag{14–28}$$

Since the volume V of the spring is $\pi Di \times \frac{1}{4}\pi d^2$,

$$U = \frac{V s_s^2}{4k^2 G} \tag{14–29}$$

This equation could be written directly by taking the expression for resilience from Table 3–3 and substituting the corrected stress s_s/k for the elastic limit S_s. Equation 14–29 shows that, in order to have a greater resilience and a better utilization of the material, k should be low. In turn, the spring index c should be large.

Finally, another equation for U may be obtained by substituting in equation 14–28 the value of s_s from equation 14–21. The result is

$$U = \frac{y^2 d^4 G}{16iD^3} \tag{14–30}$$

This equation is very convenient for checking the resilience of a given spring directly from its dimensions and deflection.

Rectangular-section springs. The equation for the stress produced in the spring shown in Fig. 14–12 can be derived by proceeding as just explained

for equation 14–15. Taking into consideration the additional direct shear stress and the influence of the curvature of the coils, there results the equation

$$s_s' = \frac{kFD(1.5h+0.9b)}{b^2h^2} \tag{14–31}$$

where k is determined by equation 14–17 or Fig. 14–10 with $c=D/b$.

The deflection may be found from the equation

$$y = \frac{2.83iFD^3(b^2+h^2)}{b^3h^3G} \tag{14–32}$$

For design purposes it is convenient to let $b/h=m$. Then

$$s_s' = \frac{kFD(1.5+0.9m)}{m^2h^3} \tag{14–33}$$

and

$$y = \frac{2.83iFD^3(1+m^2)}{m^3h^4G} \tag{14–34}$$

For a spring made of square wire, $m=1$. In this case

$$s_s' = \frac{2.4kFD}{h^3} \tag{14–35}$$

and

$$y = \frac{5.66iFD^3}{h^4G} \tag{14–36}$$

As may be found by using equation 3–29, the resilience of a spring with a rectangular cross section, generally speaking, is smaller than that of a round-wire spring. However, it increases as the ratio b/h decreases.

The main advantage of a rectangular-section spring, as compared with a round-section spring, is the possibility of obtaining a stronger spring within given space limitations.

14-8. Design data. The material most commonly used for cylindrical compression springs is carbon steel with a carbon content from 0.50 to 1.0 per cent. Alloy steels are used to increase the life of springs subjected to high stresses and repeated loads. Spring brass and phosphor bronze are used when corrosion must be prevented. Monel metal is used for springs exposed to corrosion and also to high temperatures.

Sizes. The diameter of steel wire is specified according to the Washburn and Moen (Roebling) wire gage, Table 14–2. However, sizes that are multiples of $\frac{1}{32}$ in. can usually be obtained; and sizes $\frac{3}{8}$ in. and larger come in multiples of $\frac{1}{32}$ in. up to $\frac{5}{8}$ in., and in multiples of $\frac{1}{16}$ in. for larger diameters.

Nonferrous round wire is specified according to the American Wire Gage (Brown and Sharpe).

Square bars of SAE 1095 steel can be obtained in even $\frac{1}{16}$-in. sizes. Rectangular-section bars are made only to order.

Safe stresses. Mechanical properties of all spring materials are given in Table 14–3 for $\frac{1}{2}$-in. sections. For other sections having a different diameter d the values for elastic and endurance limits must be multiplied by a size factor e_{sz}. For steel e_{sz} can be determined by the relation

$$e_{sz} = 0.86 + \frac{0.07}{d} \tag{14–37}$$

If the diameter of the wire is less than 0.09 in., this value should be used for d. For rectangular sections the value of b or h, whichever is smaller, is used instead of d in equation 14–37.

For Monel-metal wire the influence of the diameter is much less and can be taken into account by applying the equation[9]

$$e_{sz} = 0.986 + \frac{0.0043}{d} \tag{14–38}$$

For springs in infrequent or intermittent service the calculations may be based on elastic limits in shear with a safety factor from 1.5 to 2, the value depending on the service. For continuous service and rapid regular applications of the load, spring manufacturers advise lowering of the stress by 20 to 50 per cent, which means using a safety factor from 1.8 to 2.5. However, in the case of repeated loading the correct method of designing the spring is to use the endurance diagram. With such a procedure a safety factor n of 1.5 is again sufficient.

Properly done, *shot peening* increases the endurance life of helical springs more than 200 per cent and permits the use of a nominal safety factor n of 1.25. Shot peening must be specified on the drawing.

14-9. Design procedure. As in the design of leaf springs, the design of a coil spring involves a cut-and-try method, and the results should be checked by actual testing of the spring. Furthermore, a slight variation in the spring scale found by equation 14–22 must be expected in springs that seem to have identical dimensions.

The data usually given are the load on the spring, or the force which it must exert, and the desired force increase with a certain deflection. Often certain space limitations are given, such as the minimum inside diameter or the maximum outside diameter and the length of the spring under working conditions.

The values to be found usually are: the pitch diameter D, the size of the wire d, the number of coils i, and the free length l. From these basic values the spring scale F_o can be found.

[9] *Inco*, Vol. 11, No. 1 (1931), p. 21. [Published by the International Nickel Company.]

The usual design procedure is as follows: First the pitch diameter D is selected to conform to given space limitations or to other conditions. The material and safety factor n are selected next, and the approximate value for the safe stress S_d is then determined. With these data the wire diameter d can be found from equation 14–15. Thus,

$$d=\sqrt[3]{\frac{8FD}{\pi S_d}} \tag{14–39}$$

Based on this preliminary value of d, the size factor e_{sz} is found by equation 14–37 or equation 14–38. This value can be used to obtain a more accurate value for S_d. Also, the stress factor k is determined from Fig. 14–10 or by equation 14–17. Now the more accurate value of d is found from equation 14–16. The resulting relation is

$$d=\sqrt[3]{\frac{8kFD}{\pi e_{sz} S_d}} \tag{14–40}$$

If there are no space limitations, it is convenient to set $D=cd$ and to select a suitable value for c which will determine the value of k. In this case equation 14–16, when solved for d, gives

$$d=\sqrt{\frac{8Fck}{\pi S_d}} \tag{14–41}$$

Final dimensions. The necessary minimum diameter d having been determined, the actual d is taken as the nearest size from Table 14–2. A larger size decreases the working stress and gives a slightly stiffer spring, while a smaller size increases the stress and gives a softer spring.

High-grade steel wire, particularly the kind used for internal-combustion engine valve springs, may be obtained from $\frac{1}{8}$ in. to $\frac{1}{4}$ in. in diameter in increments of about 0.003 in. The exact sizes should be obtained from the wire manufacturers.

With d determined and the total deflection y given, the number of active coils is found from equation 14–19. Thus,

$$i=\frac{yd^4G}{8FD^3} \tag{14–42}$$

If y is not known, it must be determined by equation 14–24 or equation 14–26 as the case may be.

If i becomes too small, the spring is too soft. According to equation 14–42 the pitch diameter D should be decreased. This change will also slightly decrease d, and the calculations must be repeated. If i is too great, D must be increased.

When i is finally determined, the minimum free length l_o of the spring can be found. If the ends are bent before grinding,

$$l_o \geq (i+2)d+y \tag{14–43}$$

To this should be added a certain length, a in., to prevent the spring from being compressed solid with the application of the maximum load.

Buckling. A helical compression spring having a great length in proportion to its pitch diameter D may buckle at a comparatively low load. Such a spring is a very flexible column, and the critical axial load F_{cr} that can cause buckling may be found by the formula[10]

$$F_{cr}=F_oK_l l_o \tag{14–44}$$

where F_o is the spring scale, in pounds per inch;
K_l is a factor depending on the ratio l_o/D, as shown in Fig. 14–13;
l_o is the free length of the spring, in inches.

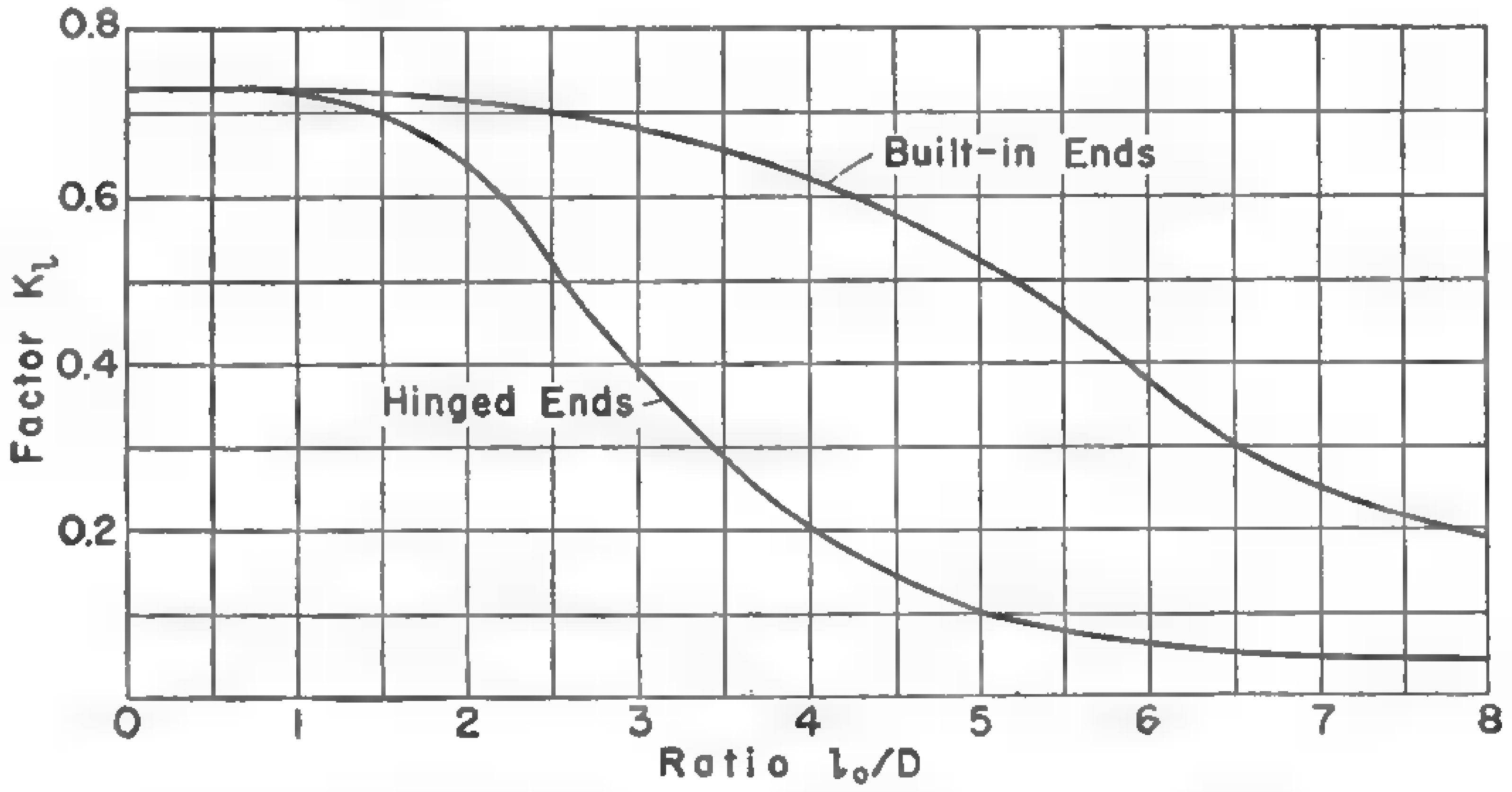


FIG. 14–13. Buckling factor for helical compression springs.

In using Fig. 14–13, plain ends as in Fig. 14–8a may be considered hinged ends, and closed and ground square ends as in Fig. 14–8b may be considered built-in ends.

EXAMPLE 14–3. Design a valve spring for an automobile motor. When closed, the spring should produce a force of about 10 lb; the spring must fit over the valve bushing, which has an outside diameter of $\frac{3}{4}$ in., and must go inside a space $1\frac{3}{8}$ in. in diameter; the valve lift is $\frac{1}{4}$ in. In designing the spring, take into account the repeated stresses only indirectly, by the usual simplified method.

Si-Mn steel, SAE 9250, may be selected as a suitable material. As a small-size wire, and hence a large e_{sz}, must be anticipated, a preliminary value of $S_d=0.51S_s$ may be assumed. Since S_s in Table 14–3 is 95,000 psi, $S_d=0.5\times 95{,}000=47{,}500$ psi.

The load with the valve open may be assumed as

$$F_2=1.2F_1=1.2\times 10=12 \text{ lb}$$

The pitch diameter can be assumed as a mean value between the two space limitations. Thus

$$D=\tfrac{1}{2}(0.75+1.375)=1.063 \text{ in.}$$

By equation 14–39,

$$d=\sqrt[3]{\frac{8\times 12\times 1.063}{\pi\times 47{,}500}}=\sqrt[3]{0.000682}=0.088 \text{ in.}$$

[10] A. M. Wahl, "When Helical Springs Buckle," *Machine Design*, Vol. 15 (May, 1943), p. 96.

By equation 14–37,

$$e_{sz} = 0.86 + \frac{0.07}{0.088} = 1.65$$

The spring index $c = 1.063/0.088 = 12.1$; and by Fig. 14–10 the stress factor $k = 1.12$. The safety factor for continuous service and rapid stress change should be taken as $n = 2 \times 1.5 = 3.0$. Then $S_d = 95{,}000/3 = 31{,}700$ psi. By equation 14–40,

$$d = \sqrt[3]{\frac{8 \times 1.12 \times 12 \times 1.063}{\pi \times 1.65 \times 31{,}700}} = 0.0885 \text{ in.}$$

The nearest W & M gage size in Table 14–2 is No. 13 with $d = 0.092$ in. The total deflection, by equation 14–26, is

$$y_2 = \frac{0.25 \times 12}{12 - 10} = 1.5 \text{ in.}$$

With these data the number of active coils, by equation 14–42, is

$$i = \frac{1.5 \times 0.092^4 \times 11{,}400{,}000}{8 \times 12 \times 1.063^3} = 10.5$$

If this number of coils seems to be too great, a slightly larger pitch diameter D should be selected without ignoring the space limitations, and all calculations should be repeated.

The free length, by equation 14–43, is

$$l_o \geqq (10.5 + 2) \times 0.092 + 1.5 = 1.15 + 1.5 = 2.65 \text{ in.}$$

Probably a free length of $2\frac{3}{4}$ in., fully compressed to 1.25 in., would be used.

Impact loading. Depending on the limitations imposed on the design, the dimensions of a spring needed to sustain impact loading are based on either equation 14–28 or equation 14–30. In either case it is best to select a value of $c = D/d$ and a number of coils i, and to determine the wire diameter by equating the spring resilience to the impact energy that the spring must absorb. If only the strength of the spring must be considered, equation 14–28 should be used. If a certain deflection y is prescribed, equation 14–30 should be used. However, in the latter case, after the wire diameter d is found, the stress must be checked by equation 14–21 or equation 14–28. If this stress exceeds the permissible design value S_d, the spring resilience must be increased by increasing either the number of coils i or the pitch diameter D, or by increasing both. It should be noted that in this case equations 14–16 and 14–19 cannot be used unless a fictitious force F is found which will produce the same deflection and stress as when the impact energy is absorbed by the spring.

EXAMPLE 14–4. Design the springs for a mechanical sieve. The weight of the dropping frame and the material being sifted is 200 lb; the height of drop is $\frac{1}{2}$ in.; no stops except the springs themselves are used; eight springs are taking the full impact, and each time the frame is lifted by the mechanism they open up to their free length; the load may be assumed as distributed evenly among all eight springs.

The energy which each spring must absorb is

$$K_i = \frac{200 \times (0.5 + y)}{8} \text{ in.-lb}$$

where y is the deflection of the spring. From preliminary calculations it may be assumed that $y = 1.875$ in., and the estimated energy will be

$$K_i = \frac{200 \times (0.5 + 1.875)}{8} = 59.4 \text{ in.-lb}$$

Select $c = D/d = 10$. Then $D = 10d$; and from Fig. 14–10, $k = 1.14$. Next select the material from Table 14–3, choosing SAE 9250. For this material, $G = 11{,}400{,}000$ psi and the endurance limit in torsion is $S_{en} = 48{,}000$ psi. Also, take the safety factor n as 1.7 because the repeated stress fluctuation is not accompanied by a reversal of the stress. If a wire diameter d of 0.25 in. is assumed, the size factor found by equation 14–37 is

$$e_{sz} = 0.86 + \frac{0.07}{0.25} = 1.14$$

The design stress is then

$$S_d = \frac{48{,}000 \times 1.14}{1.7} = 32{,}200 \text{ psi}$$

Try using $i = 10$ free coils. Substituting these values in equation 14–28, and solving it for d, results in

$$d = \sqrt[3]{\frac{59.4 \times 16 \times 11{,}400{,}000 \times 1.14^2}{9.87 \times 10 \times 10 \times 32{,}200^2}} = 0.239 \text{ in.}$$

The nearest larger gage is No. 3 with $d = 0.2437$ in. Therefore, in accordance with the preceding assumption,

$$D = 10 \times 0.2437 = 2.437 \text{ in.}$$

The increase of d from 0.239 to 0.2437 in. will lower the stress from 32,200 to 31,200 psi. The corresponding actual deflection y, found by equation 14–20, is

$$y = \frac{\pi \times 10 \times 31{,}200 \times 2.437^2}{1.14 \times 0.2437 \times 11{,}400{,}000} = 1.825 \text{ in.}$$

Therefore, the energy to be absorbed will be a little smaller than the value assumed. It is

$$K_i = \frac{200 \times (0.5 + 1.825)}{8} = 58.1 \text{ in.-lb}$$

A recheck of U by equation 14–30 gives

$$U = \frac{1.825^2 \times 0.2437^4 \times 11{,}400{,}000}{16 \times 10 \times 2437^3} = 58.0 \text{ in.-lb}$$

This is as close as can be obtained with a slide rule.

The free length l_o of the springs is found by assuming the total number of coils as $10 + 2 = 12$ and allowing a clearance of about $\frac{1}{16}$ in. between the coils when they are fully compressed. The result is

$$l_o = 12 \times 0.2437 + 11 \times 0.0625 + 1.825 = 5.437 \text{ or } 5\tfrac{7}{16} \text{ in.}$$

The scale F_o of the spring can be found from equation 14–19 by substituting 1 in. for y. Thus,

$$F_o = \frac{0.2437^4 \times 11{,}400{,}000}{8 \times 2.437^3 \times 10} = 34.7 \text{ lb per in.}$$

The actual safety factor can be found by equation 5–19, in which

$$U = \frac{\pi^2 \times 0.2437^2 \times 2.437 \times 10 \times (48{,}000 \times 1.14)^2}{16 \times 1.14^2 \times 11{,}400{,}000} = 180 \text{ in.-lb}$$

Then

$$n = \sqrt{\frac{180}{58.1}} = 1.76$$

Repeated loading. In the case of repeated loading, encountered particularly in valve springs of internal-combustion engines or plunger return springs of oil engines, the calculations should be based on the endurance

diagram, the stress amplitude S_a, from equation 5–38, and the mean stress S_m, from equation 5–39. The corresponding torque amplitude T_a, based on equation 14–13, is

$$T_a = \tfrac{1}{2}(T_1 - T_2) = \tfrac{1}{2}(F_1 - F_2) \times \tfrac{1}{2}D = \tfrac{1}{4}(F_1 - F_2)D \tag{14–45}$$

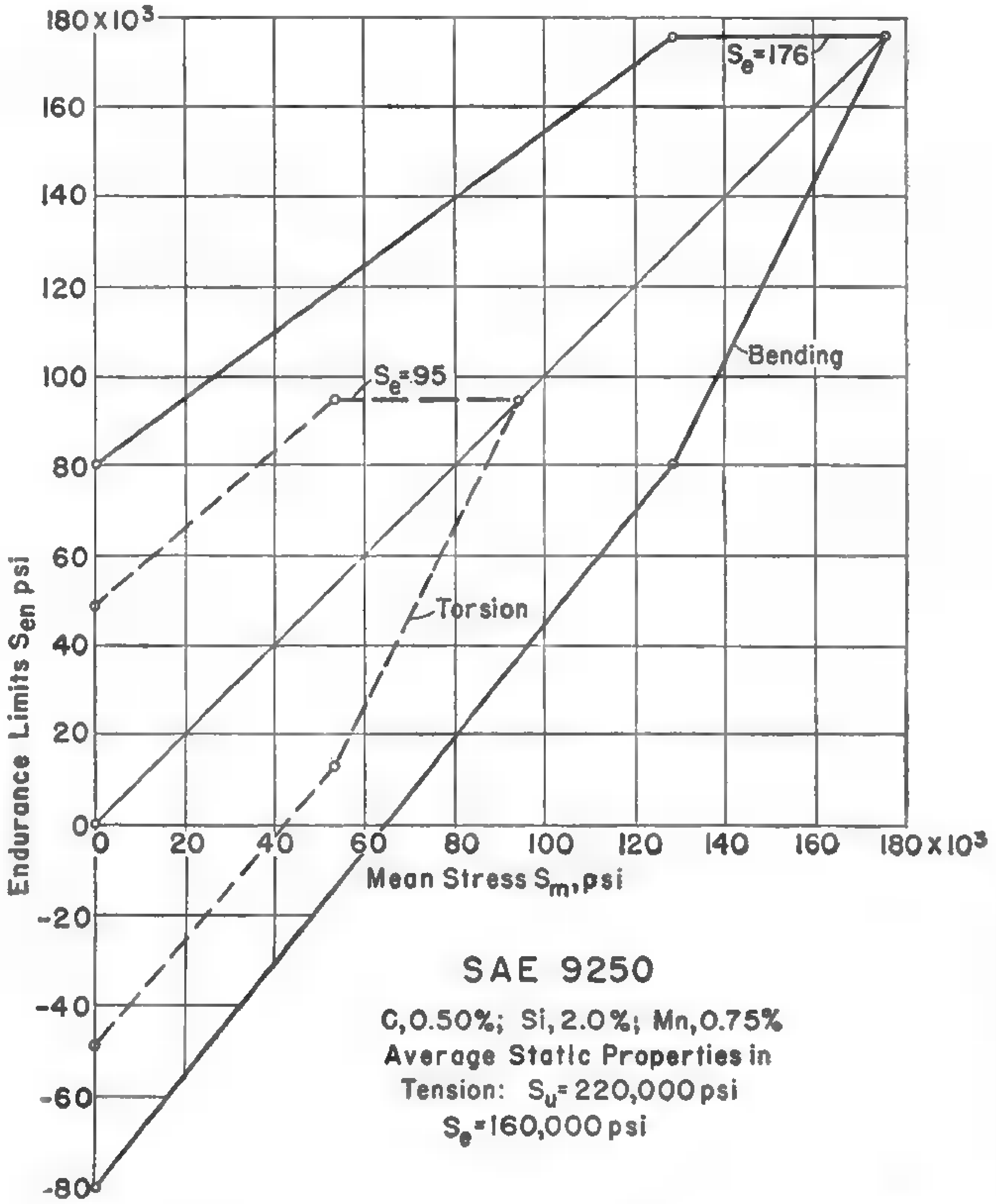


FIG. 14–14. Endurance diagram of SAE 9250 spring steel.

Substituting this value of T_a in equation 5–38, and k instead of K_r, gives

$$S_a = nk\frac{(F_1 - F_2)D}{4Z_o e_{sz} e'_{sr}} \tag{14–46}$$

Similarly, the mean torque is

$$T_m = \tfrac{1}{2}(T_1 + T_2) = \tfrac{1}{4}(F_1 + F_2)D \tag{14–47}$$

and the mean stress, found by applying equation 5–39, is

$$S_m = nk\frac{(F_1 + F_2)D}{4Z_o e_{ss}} \tag{14–48}$$

For a round bar, $Z_o = \frac{1}{16}\pi d^3$. Therefore the necessary wire diameter d, determined from equation 14–46, is

$$d = \sqrt[3]{4nk\frac{(F_1 - F_2)D}{\pi e_{sz} e'_{sr} S_a}} \tag{14–49}$$

In equations 14–46, 14–48, and 14–49 the stress factor k determined from equation 14–17 must be used for the stress-concentration factor K_r. The surface coefficient e_{sr}' should be determined by equation 5–23 and Fig. 5–8. Curve b will correspond to a usual clean surface, and curve e will apply for a spring with scale as is often found after quenching. The removal of this scale and the addition of a smooth finish allows an increase of the stress and therefore the use of a smaller diameter d.

EXAMPLE 14–5. Design the springs of example 14–4, making a more accurate allowance for the danger of failure through repeated stresses.

Since among the endurance diagrams given in Chapter 4 there is no diagram for SAE 9250 steel, such a diagram must be laid out by the method of interpolation, primarily by using data from Table 14–3. Since this diagram may be useful also for leaf-spring design, the two types of loading—torsion (shear) and bending—were considered in laying out Fig. 14–14. The general shape of Fig. 4–11 was followed to a certain extent since its relative data come fairly close to the data of Si-Mn steel in Table 14–3.

By using the graphic method illustrated by Fig. 5–10, where the force $F_1 = 0$ and $F_2 = F_o y = 34.7 \times 1.885 = 65.5$ lb, the stress amplitude was found to be $S_a = 40{,}000$ psi. The various factors in equation 14–49 can at first be assumed to be based on the value of $d = 0.2437$ found by the static method. Thus, with the spring index $c = 10$, the stress factor from Fig. 14–10 is $k = 1.14$. Since the size factor remains the same, $e_{ss} = 1.14$. The surface factor, taken from curve b in Fig. 5–8, is $e_{sr} = 0.88$. By equation 5–23,

$$e_{sr}' = 0.425 + 0.575 \times 0.88 = 0.931$$

A safety factor of 2.50 is sufficient. With these data, equation 14–49 gives

$$d = \sqrt[3]{\frac{4 \times 2.50 \times 1.14 \times 65.5 \times 2.437}{\pi \times 1.14 \times 0.931 \times 40{,}000}} = \sqrt[3]{0.0136} = 0.238 \text{ in.}$$

This corresponds to the same No. 3 wire with $d = 0.2437$ in.

Thus, the accurate method gave the same size of wire but a greater assurance that the spring would not break in operation.

Rectangular section. In designing a spring with a rectangular cross section, the procedure remains the same but the ratio m must be assumed. The smaller the value of m, the stiffer the spring will be, according to equation 14–34. Usually m is made from 1 down to about 0.1, although m is occasionally made greater than 1.

14–10. Concentric springs. Concentric springs are used for one of the following three purposes:

a. To obtain a greater spring force in a given space
b. To insure the operation of a mechanism in the event that one spring breaks
c. To obtain a spring force which does not increase in a direct relation to the deflection, but increases faster

In cases a and b two or more concentric springs have the same free lengths and are compressed equally. In case c the springs are made of different

lengths, as l_1 and l_2 in Fig. 14–15. The shorter spring begins to act only after the longer spring is compressed a certain amount.

To prevent any tendency to bind, the adjacent coils are wound in opposite directions. In the design of concentric springs, if the same material is used, the springs should be designed with approximately the same safety factor. In order to have approximately the same stress factor k, the same spring index c is desirable. This requirement in turn calls for a smaller wire diameter d with the decrease of the pitch diameter D. Because the size factors are different, the design stresses will be different.

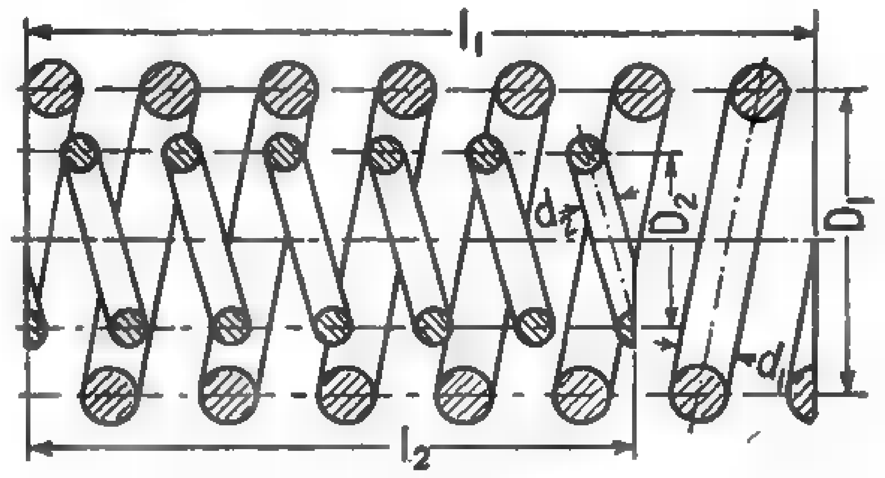


FIG. 14–15. Concentric springs.

The approximate relation between the sizes of two concentric springs wound from round wire of the same material can be obtained by solving equation 14–40 for S_d, substituting for e_{sz} a simpler though less accurate expression $e_{sz} = (0.5/d)^{0.15}$, and canceling k. This gives the relation

$$\left(\frac{d_1}{d_2}\right)^{2.85} = \frac{F_1 D_1}{F_2 D_2} \tag{14–50}$$

The diameters D_1 and D_2 are selected from geometrical considerations. Also, F_2 is expressed in terms of F_1 by the relation $F_2 = mF_1$, where $m \leq 1$. This gives $F_1 = F/(1+m)$, where F is the total maximum spring load. The wire diameter d_1 of the outer spring is found in the usual way, and the wire diameter d_2 is then found by equation 14–50.

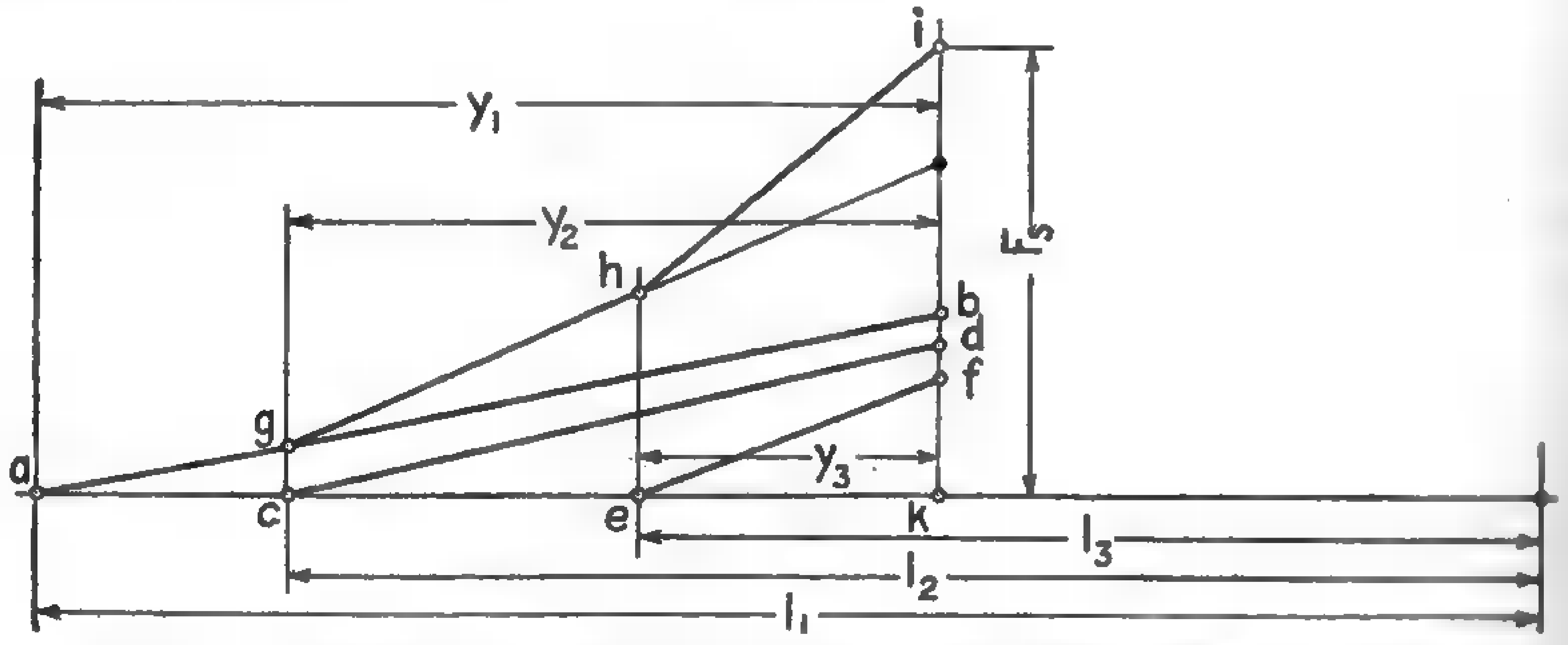


FIG. 14–16. Multiple spring with a variable force-deflection ratio.

Variable force-deflection ratio. Where the force-deflection ratio is variable, the springs have different free lengths and different deflections. In Fig. 14–16 is shown a case involving three springs. Their free lengths are l_1, l_2, and l_3; their maximum deflections are y_1, y_2, and y_3, respectively; the inclined lines a–b, c–d, and e–f represent the forces of the springs; and the curve $aghi$ represents the gradual increase of the spring force of the combination up to $F_s = i$–k.

Such multiple springs are used in governors of variable-speed engines to take care of the variable centrifugal force, which is proportional to the square of the speed.

14–11. Vibration of cylindrical springs. If the natural frequency of a spring is a small multiple of the frequency of the outside force which acts upon it, the spring will have a tendency to vibrate with its own natural frequency. Under these conditions the middle coils will begin to surge back and forth. This surging changes the spring force and interferes with the proper operation of the spring. Furthermore, surging of the coils increases the maximum deflection between adjoining coils and therefore raises the stress above that figured by equation 14–21. This excessive stress may cause a failure of the spring through progressive fracture. The farther the natural frequency of the spring is above the frequency of the external force, the smaller is the danger of surge interference. Therefore, it is desirable to use springs with a high natural frequency.

The natural frequency of the spring may be found from equation 5–43. In determining the values of F and W, two cases must be considered: first, when one end of a spring is at rest; and second, when both ends of the spring are fixed. In the first case, the amplitude of the coils moving back and forth gradually decreases toward the coil which is at rest. The mass in motion may be considered as one-half the whole spring mass, or $W/2g$. Therefore, from equation 5–43, the frequency is

$$f = \frac{1}{T} = \frac{1}{2\pi}\sqrt{\frac{2F_o g}{W}} = 4.42\sqrt{\frac{F_o}{W}} \tag{14–51}$$

where F_o is the scale of the spring, in pounds per inch.

With both ends fixed, the maximum vibration or surge will take place at the middle coil. The force required to move the middle coil the same distance y will be four times as large as in the former case, because the coils on one side must be compressed while those on the other side are stretched and because the same deflection y is produced in one-half the coils. The active weight is again about one-half the spring weight. Thus,

$$f = 8.84\sqrt{\frac{F_o}{W}} \tag{14–52}$$

Some authors[11] recommend formulas which give values about 10 per cent higher. Other authors[12] change the coefficient in equation 14–52 to 9.83, the results then being 11 per cent higher.

[11] C. H. Kent, "Don't Overlook Surge in Designing Springs," *Machine Design*, Vol. 7 (October, 1935), p. 38; A. Swan, "Valve Springs," *Automobile Engineer*, Vol. 16 (1926), pp. 218, 290.

[12] H. R. Ricardo, *The High-Speed Internal Combustion Engine*, 3d ed. (New York: Interscience Publishers, 1941), p. 256; R. T. Kent, *Mechanical Engineers' Handbook*, 12th ed., Vol. II, *Design and Production*, ed. by Colin Carmichael (New York: John Wiley & Sons, Inc., 1950), p. 10-21.

EXAMPLE 14–6. Determine the dangerous engine speeds for a valve spring of a four-stroke engine. The outside diameter of the spring is 3 in., the wire diameter is $\frac{1}{4}$ in., and the spring has eight coils; with the valve closed the spring is compressed $2\frac{1}{2}$ in. and has a force of 145 lb.

The scale of the spring is

$$F_o = 145/2.5 = 58 \text{ lb per in.}$$

The weight of the spring is

$$W = 0.7854 \times (0.25)^2 \times \pi \times (3 - 0.25) \times 8 \times 0.28 = 0.95 \text{ lb}$$

By equation 14–51, the frequency is

$$f = 4.42\sqrt{\frac{58}{0.95}} = 34.5 \text{ vibr per sec, or 2,070 vibr per min}$$

Since the camshaft runs at half of the speed of the engine, the most dangerous engine speed would be $2{,}070 \times 2 = 4{,}140$ rpm. Other dangerous speeds would be $4{,}140/2 = 2{,}070$ rpm, $4{,}140/3 = 1{,}380$ rpm, and $4{,}140/4 = 1{,}035$ rpm. Lower engine speeds are not likely to set up surge action, because of the damping effect of the spring material between the impulses. The surge frequency of the middle coil will be twice as high, or still less dangerous.

Surge elimination. The computation of the stress caused by surging is rather involved and does not give reliable results because of the uncertainty of the damping effect of the spring material. Insofar as the design is concerned, the best policy is to have the natural frequency considerably above the number of impulses of the disturbing force. To find the frequency, substitute in equation 14–51 the value of F_o given by equation 14–22, and replace W by $\frac{1}{4}\pi^2 Did^2 w$, where i is the number of active coils, and w is the specific weight of the spring material, in pounds per cubic inch. Then, reducing the terms gives

$$f = \frac{d}{iD^2}\sqrt{\frac{G}{w}} \tag{14–53}$$

Examination of equation 14–53 in connection with equation 14–21 shows that it is not easy to raise f without increasing the stress. Tests show that the spring deflection due to surge depends on the initial spring load. This load permits reduction of the ill effects of surging but requires a careful study of the spring behavior under operating conditions.[13] Means for adjusting the initial load by changing the working length of a spring should be provided in the design if the theoretical analysis indicates a possibility of spring surge.

14–12. Extension springs. The equations for compression springs apply as well to extension springs. A compression spring is usually wound with the coils touching, as in Fig. 14–17, and often with some initial tension. Extensions springs are used in machinery considerably less than compression springs. The main reasons for this are:

[13] Ricardo, *op. cit.*, p. 233.

a) They are more expensive to manufacture.
b) They require a more elaborate fastening of the ends.
c) They are more likely to be stressed beyond the elastic limit.
d) In case of failure, the kinematic constraint is lost.

In Fig. 14–17 are shown two types of spring ends—a hook or eye a bent from the spring wire itself, and a cast-iron or steel plug b which is screwed into the spring and has a tapped hole for a threaded connecting detail.

To overcome the last two disadvantages listed, the arrangement shown in Fig. 14–18 may be used instead of an extension spring.

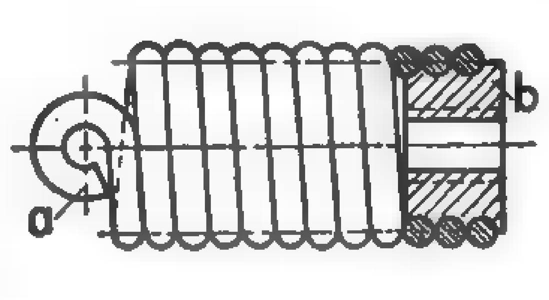


FIG. 14–17. Tension spring.

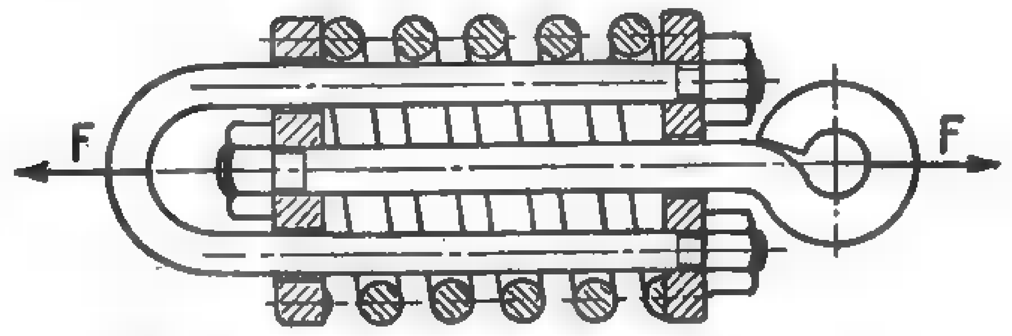


FIG. 14–18. Compression spring for tension purposes.

14–13. Conical springs. Conical helical springs usually work in compression and are made of round or rectangular stock. A conical spring is used either where space limitations prohibit the use of a cylindrical spring or where a single spring with a variable stiffness is desired.

From Fig. 14–19 and equation 14–16 it can be seen that the largest coil with the diameter D_2 is subject to the highest stress, and that the stress gradually decreases toward the end with the smallest diameter D_1. Naturally, in designing the spring the highest stress must be taken into account. For a round stock, this stress will be found by equation 14–16. For a square stock, it will be found by equation 14–31 or equation 14–33, with the substitution of D_2 for D.

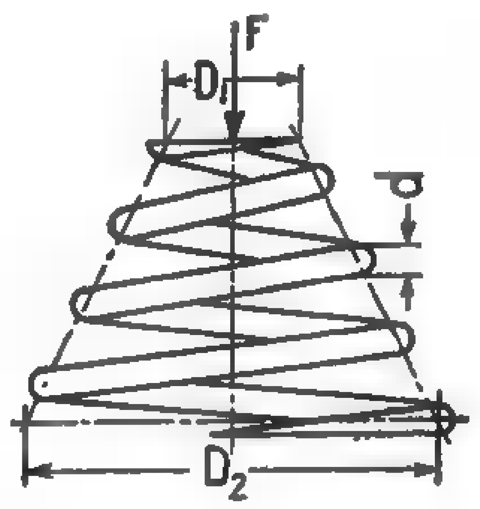


FIG. 14–19. Conical spring.

If the spring is so designed that with an increase of the load the coils having the larger diameters will gradually compress solidly, the smallest coil must be designed for the maximum load.

The axial deflection y for i coils of round stock may be computed by the relation[14]

$$y = \frac{2iF(D_2^3 + D_2^2 D_1 + D_2 D_1^2 + D_1^3)}{d^4 G} \tag{14–54}$$

By substituting for F its value from equation 14–16, the deflection can be expressed in terms of the maximum stress. Thus,

$$y = \frac{\pi i s_s (D_2^3 + D_2^2 D_1 + D_2 D_1^2 + D_1^3)}{4dD_2 kG} \tag{14–55}$$

[14] O. A. Leutwiler, *Elements of Machine Design* (New York: McGraw-Hill Book Company, Inc., 1917), p. 141.

The axial deflection of a conical spring made of rectangular stock with a radial thickness b and an axial dimension h, as in Fig. 14–12, is

$$y = \frac{0.71iF(b^2+h^2)(D_2^3+D_2^2D_1+D_2D_1^2+D_1^3)}{b^3h^3G} \tag{14-56}$$

14–14. Torsion springs. The torsional moment T on a spring produces a bending stress in the wire, the bending moment on the wire being numerically equal to T. In addition to this stress, there is a direct tensile or compressive stress due to the force F that is tangential to the coil. Therefore, the maximum stress is

$$s = \frac{T}{Z} + \frac{F}{A} \tag{14-57}$$

where Z is the section modulus of the wire and $F = 2T/D$. By reason of the curvature of the coil the actual bending stress will be greater than that given by equation 14–57. The correction can be made as described for compression springs by introducing a factor k', giving the equation

$$s = \frac{k'T}{Z} + \frac{2T}{DA} \tag{14-58}$$

The deflection, measured by the distance traveled by a point on the pitch diameter of the end coil to which the pull is applied, is approximately

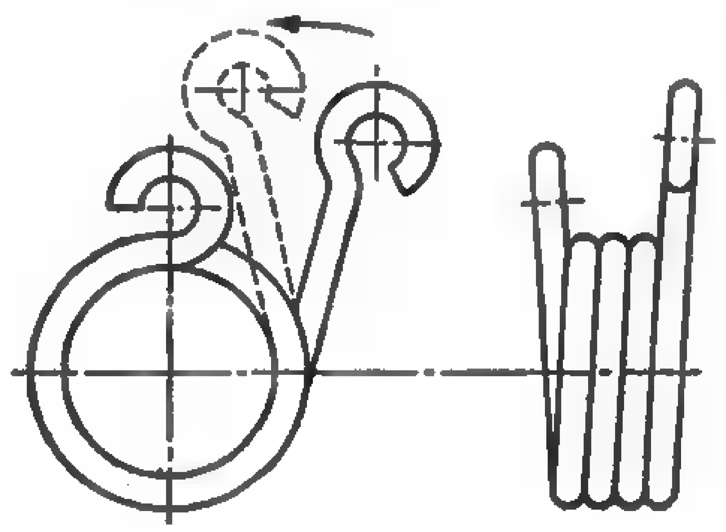


FIG. 14–20. Torsion spring.

$$y = \frac{TLD}{2EI} \tag{14-59}$$

where L is the length of the coil part of the spring and is equal to $i\pi D$, and I is the moment of inertia of the wire.

Round-wire spring. In a round-wire spring, Fig. 14–20, $Z = \frac{1}{32}\pi d^3$; $A = \frac{1}{4}\pi d^2$; and $k' = k_1$ is a function of the spring index $c = D/d$ and may be found from curve k_1 in Fig. 14–10. Therefore,

$$s = \frac{8T(4k_1D+d)}{\pi d^3 D} \tag{14-60}$$

Rectangular-wire spring. For a rectangular section, as in Fig. 14–12, $Z = \frac{1}{6}hb^2$, $A = bh$, and $k' = k_2$. The value of k_2 can be found from Fig. 14–10, where $c = D/b$. The stress is then

$$s = \frac{6k_2T}{b^2h} + \frac{2T}{Dbh} \tag{14-61}$$

Additional data, such as resilience for the springs discussed and formulas for other special types of springs, can be found in handbooks.[15]

[15] Lionel S. Marks, ed., *Mechanical Engineers' Handbook*, 5th ed. (New York: McGraw-Hill Book Company, Inc., 1951), pp. 455 ff; R. T. Kent, *op. cit.*, pp. 10-21 ff.

PART IV: DETAILS FOR HANDLING FLUIDS

CHAPTER 15

Cylinders, Heads, and Cover Plates

15–1. Thin-wall cylinders. Cylindrical vessels may be divided into two general classes: (a) those having thin walls, such as boiler shells, pressure tanks, and pipes; and (b) those having comparatively thick walls.

The analysis of stresses induced in the walls of a thin-wall cylinder by an internal pressure will be made by assuming that the stresses are distributed uniformly over the cross section of the cylinder and by neglecting the restraining action of the heads at the ends of the cylinder. In a cylinder having a diameter d, with the ends closed by heads, the internal pressure p against the heads produces an axial force $\frac{1}{4}\pi d^2 p$. If h is the thickness of the walls, the area of any circumferential cross section is πdh. Hence, the axial force on the ends causes in any circumferential section a longitudinal tensile stress s_1 found by the equation

$$s_1 = 0.25\frac{pd}{h} \tag{15–1}$$

The magnitude of the hoop stress s_2 is found by taking a ring section of the cylinder 1 in. long. If it is imagined that this ring is cut in two by an axial plane, the force acting on each half ring is pd and it induces a reaction $2hs_2$. Equating these forces gives

$$s_2 = 0.5\frac{pd}{h} \tag{15–2}$$

Thus the simple longitudinal stress is one-half the hoop stress. When the biaxial loading is taken into account by equation 2–44, the resulting longitudinal stress becomes

$$s'_1 = 0.25\frac{pd}{h} - 0.5\frac{\mu pd}{h} \tag{15–3}$$

In the same way, the resulting hoop stress is found to be

$$s'_2 = 0.5\frac{pd}{h} - 0.25\frac{\mu pd}{h} \tag{15–4}$$

For steel, for which $\mu = 0.303$, the governing stress found by equation 15–4 is

$$s_2 = 0.424\frac{pd}{h} \tag{15–5}$$

Since the difference is not very large and it is on the safe side to do so, the simpler equation 15–2 is commonly used in practice rather than equa-

tion 15–4. Therefore the necessary thickness may be determined by the equation

$$h = \frac{0.5pd}{s_2} \tag{15–6}$$

15–2. Thick-wall cylinders. In a cylinder with thick walls, the actual hoop stress varies along the wall thickness. It has the highest magnitude at the inner surface of the cylinder and gradually decreases toward the outer surface, the distribution being similar to that of the stress in the outer ring of Fig. 13–1b. The magnitude of the stress depends on the cylinder-end condition; that is, on whether the cylinder is closed or open at the ends.

At the same time, the design procedure depends on whether the material is ductile or brittle. This characteristic indicates what theory of failure should be applied for the given case, as explained in section 5–2, and hence what equation must be used.

Cylinder with closed ends. If the cylinder is closed at the ends, the hoop stress induced by the inner pressure governs the design. For a cylinder of *ductile* material having an inside diameter d_1 and an outside diameter d_2 and subjected to an inner pressure p, the governing stress may be computed by Clavarino's formula, which is based on the maximum-strain-energy theory. Thus,

$$s = \frac{p[(1-2\mu)d_1^2 + (1+\mu)d_2^2]}{d_2^2 - d_1^2} \tag{15–7}$$

Substituting the wall thickness h for $(d_2 - d_1)/2$ in equation 15–7 and solving the resulting equation for h, results in

$$h = \frac{d_1}{2}\left[\sqrt{\frac{s + (1-2\mu)p}{s - (1+\mu)p}} - 1\right] \tag{15–8}$$

where Poisson's ratio μ may be taken from Table 2–1.

For a *brittle* material the governing stress should be based on the maximum-normal-stress theory. From equation 2–71,

$$s = \frac{p(d_2^2 + d_1^2)}{d_2^2 - d_1^2} \tag{15–9}$$

The necessary wall thickness h may then be computed from the equation

$$h = \frac{d_1}{2}\left(\sqrt{\frac{s+p}{s-p}} - 1\right) \tag{15–10}$$

Cylinder with open ends. In a cylinder with open ends the pressure seal is obtained by a separate piston. In a cylinder of this type the longitudinal stress is zero.

For a *ductile* material the stress may be determined in accordance with the distortion-energy theory. The equation is

$$s = \frac{p\sqrt{d_1^4 + d_2^4/\mu}}{d_2^2 - d_1^2} \tag{15–11}$$

The necessary wall thickness h is then

$$h = \frac{d_1}{2}\left[\sqrt{\frac{s^2 + p\sqrt{4s^2 - p^2/\mu}}{s^2 - p^2/\mu}} - 1\right] \tag{15–12}$$

Equation 15–12 may be made more convenient for design purposes by dividing the numerator and the denominator under the radical sign by p^2 and designating the ratio s/p by a. Then

$$h = \frac{d_1}{2}\left[\sqrt{\frac{a^2 + \sqrt{4a^2 - 1/\mu}}{a^2 - 1/\mu}} - 1\right] \tag{15–13}$$

For a *brittle* material the maximum-normal-stress theory should be applied again. In this case there is no difference between open ends and closed ends, and equations 15–9 and 15–10 should be used.

It should be noted that for a ductile material and the same values of d_1, p, and s, the thickness h found by equation 15–8 is smaller than the thickness found by equation 15–12 or equation 15–13.

In determining the necessary wall thickness, the value of the design stress S_d is used for s in all of the foregoing equations.

Relative thicknesses of walls. To determine whether the simpler formulas given for thin-wall cylinders may be used or the more accurate formulas given for thick-wall cylinders must be used, it is necessary to ascertain when the latter will give larger values. On the basis of a mean value 0.3 for μ, it will be found that for closed ends and ductile materials, when $p \leqq S_d/6$, the thin-wall formula can be used; when $p \geqq S_d/6$, the thick-wall formula must be used. For open ends the thick-wall formulas give larger values.

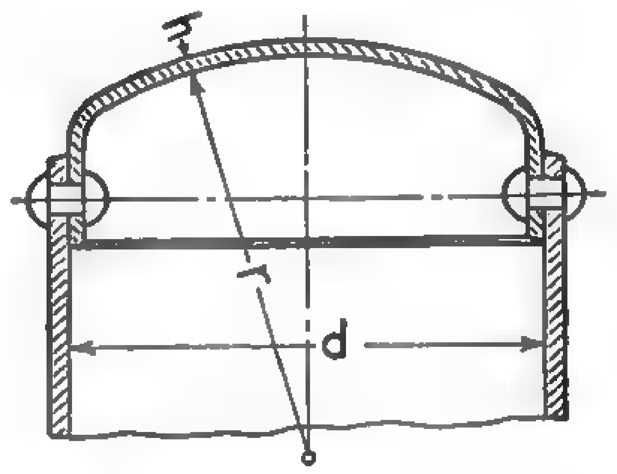


Fig. 15–1. Dished boiler head.

15–3. Circular heads, covers, and plates. The conditions of stress distribution in cylinder heads and covers are rather involved, and more or less empirical formulas must be used.

Dished head. The thickness of a dished head (Fig. 15–1) that is riveted or welded to a cylindrical shell, should be[1]

$$h = \frac{2.08pr}{S_e} \tag{15–14}$$

where S_e is the elastic limit of the material from Table 4–2. The radius r should not be greater than the diameter d of the shell. If there is a manhole in the head, the coefficient 2.08 should be increased to at least 2.4.

[1] *ASME Boiler and Pressure Vessel Code*, 1952 Edition, Section I, *Power Boilers* (New York: American Society of Mechanical Engineers, 1952).

TABLE 15–1

COEFFICIENTS FOR DETERMINING HEAD THICKNESS

Type of Head in Fig. 15–2	Coefficient c	Remarks
A or A′......	0.162	Plate rigidly riveted or bolted to the shell flange.
B...........	0.162	Integral flat head; $d \leqq 24$ in.; $h \geqq 0.05d$.
C...........	0.30	Flanged plate attached by a lap joint; $r \geqq 3h$.
D or E......	0.25	Plate butt-welded or forged integral; $r \geqq 3h_f$.
F...........	0.50	Plate fusion-welded with fillet weld; throat $h_l \geqq 1.25h_s$.
G or H......	$0.30+K$	Bolts tend to dish the plate; K is found by the relation $K = 1.4Wh_G/Hd$, where W = total bolt load in pounds; H = total pressure on area bounded by the outside diameter of the gasket, in pounds; and h_G and d are as shown in Fig. 15–2.

Flat heads. The minimum required thickness h of an unstayed flat head or cover plate should be determined by the relation[2]

$$h = d\sqrt{\frac{cp}{S_d}} \tag{15–15}$$

where d is the diameter, or shortest span, as indicated in Fig. 15–2, in inches;
c is an empirical coefficient, given in Table 15–1;
p is the maximum inside pressure, in pounds per square inch;
S_d is the allowable design stress in pounds per square inch, which may be taken from Table 15–2.

Plate uniformly loaded. The thickness h of a plate with a diameter d supported at the circumference and subjected to a pressure p distributed uniformly over the total area may be calculated by the equation

$$h = k_1 d\sqrt{\frac{p}{S_d}} \tag{15–16}$$

TABLE 15–2

DESIGN STRESSES FOR BOLTED FLANGED HEADS

Maximum Temperature (Deg F)	Minimum of Specified Range of Tensile Strength of Flange Material at Room Temperature (psi)					Allow Bolt Stress (psi)
	45,000	50,000	55,000	60,000	70,000	
700............	10,650	11,850	13,050	14,200	16,600	14,200
750............	9,450	10,500	11,550	12,600	14,700	12,600
800............	8,100	9,000	9,900	10,800	12,600	10,800
850............	6,750	7,500	8,250	9,000	10,500	9,000
900............	5,400	6,000	6,600	7,200	8,400	7,200
950............	4,050	4,500	4,950	5,400	6,300	5,400

[2] "Revisions and Addenda to the Boiler Construction Code," *Mechanical Engineering*, Vol. 56 (1934), p. 310.

TABLE 15–3

COEFFICIENTS IN FORMULAS FOR COVER PLATES

Material of Cover Plate	Method of Holding Edges	Circular Plate		Rectangular Plate		Elliptical Plate
		k_1	k_2	k_3	k_4	k_5
Cast iron........	Supported, free	0.54	0.038	0.75	1.73	1.5
	Fixed	0.44	0.010	0.62	1.4; 1.6*	1.2
Mild steel........	Supported, free	0.42	....	0.60	1.38	1.2
	Fixed	0.35	....	0.49	1.12; 1.28	0.9

* With gasket.

The maximum deflection is given in this case by the equation

$$y = k_2 d^4 \frac{p}{Eh^3} \tag{15–17}$$

The coefficients k_1 and k_2 depend on the method of holding the edges and the material of the plate and are given in Table 15–3.

Plate loaded centrally. The thickness h of a flat cast-iron plate supported freely at the circumference with a diameter d and subjected to a load F distributed uniformly over an area $\frac{1}{4}\pi d_o^2$ may be calculated by the equation

$$h = 1.2\sqrt{\left(1 - \frac{0.67d_o}{d}\right)\frac{F}{S_d}} \tag{15–18}$$

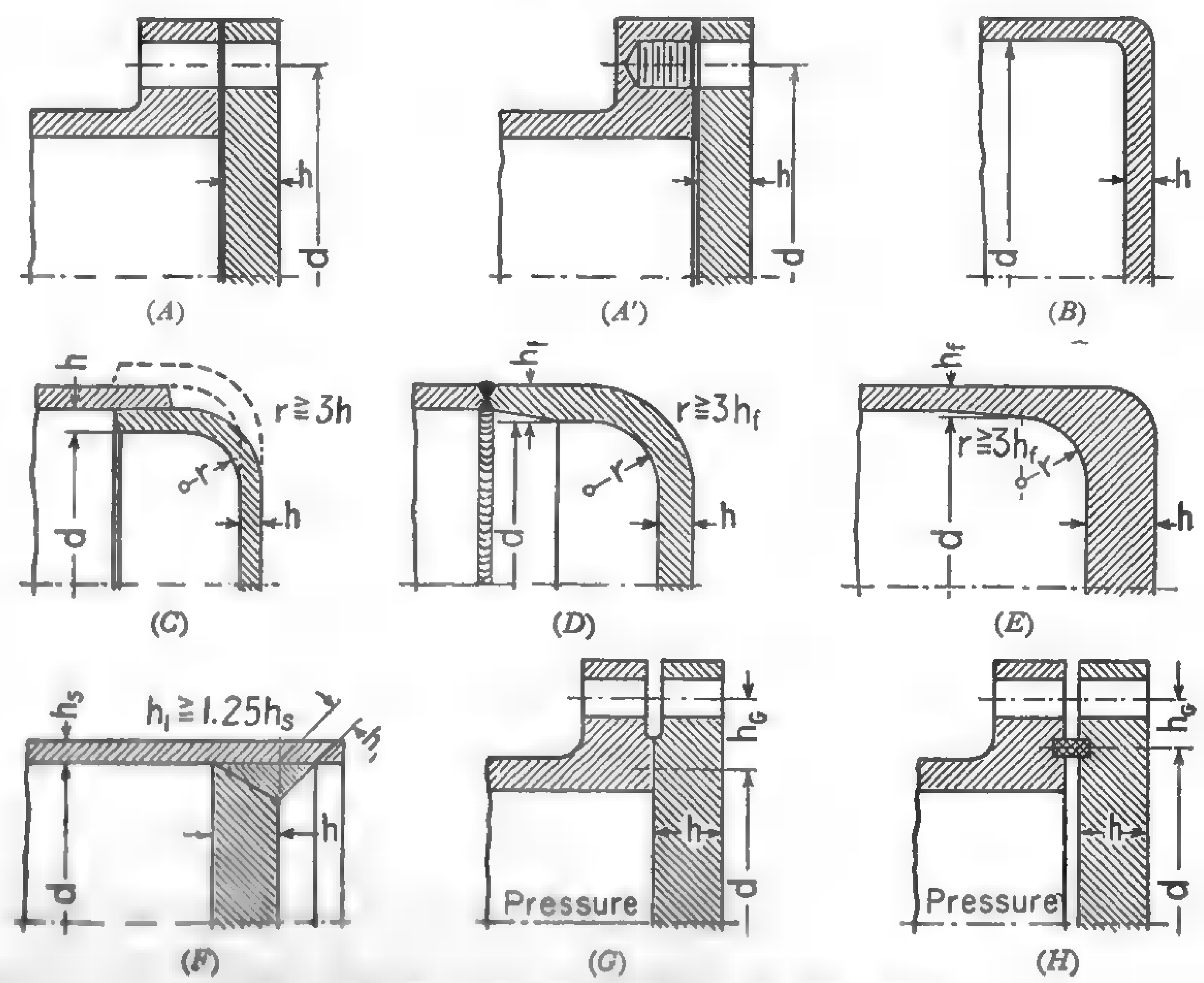


FIG. 15–2. Types of cylinder heads and covers.

The deflection is given in this case by the equation

$$y=\frac{0.12d^2F}{Eh^3} \tag{15-19}$$

If the plate with the above given type of loading is *fixed* rigidly around the circumference, its thickness may be calculated by Grashof's formula, which is

$$h=0.65\sqrt{\frac{F}{S_d}\log_e\left(\frac{d}{d_o}\right)} \tag{15-20}$$

The deflection for this case may be found by equation 15-19, with the numerical coefficient 0.12 changed to 0.055.

EXAMPLE 15-1. Determine the thickness of a cover of type A, Fig. 15-2, made of nickel cast iron, grade II, for a cylinder having an inside diameter of 12 in. and carrying a pressure of 275 psi.

From Table 4-1, the elastic limit in tension for grade II nickel cast iron is 17,000 psi. If we first assume a safety factor n of 2 and a size coefficient e_{sz} of 0.95, the allowable stress is

$$S_d=\frac{17{,}000\times 0.95}{2}=8{,}000 \text{ psi}$$

By equation 15-15, in which c is taken as 0.162 and d is estimated to be $14\frac{1}{2}$ in.,

$$h=14.5\sqrt{\frac{0.162\times 275}{8{,}000}}=14.5\times 0.0746=1.08, \text{ or } 1\tfrac{1}{8} \text{ in.}$$

This is the thickness required at the edge; the center thickness may be made 1 in. The edge of the plate should be checked in bending. The moment per inch of circumference, at the inner diameter $d_o=12$ in., is

$$M=\frac{0.7854\times 12^2\times 275\times 1.25}{\pi\times 12}=1{,}032 \text{ lb-in.}$$

The section modulus for a width of 1 in. is

$$Z=\tfrac{1}{6}\times 1\times 1.125^2=0.211 \text{ in.}^3$$

The stress is

$$s=\frac{1{,}032}{0.211}=4{,}890 \text{ psi}$$

This is below the allowable stress of 8,000 psi.

ASME design data. The Applied Mechanics Division of the American Society of Mechanical Engineers has worked out, for various types of loading of circular plates, simple formulas that are convenient for slide-rule use and involve numerical factors which may be taken from curves.[3]

15-4. Rectangular plates. In deriving a formula for the strength of a rectangular plate it is assumed that the critical section passes through the center of the plate. A square cast-iron plate fixed at the edges and subjected to a uniformly distributed load or a load concentrated at the center fails

[3] A. M. Wahl and Stewart Way, "Stress and Deflection of Circular Plates," *Journal of Applied Mechanics*, Vol. 3 (March, 1936), p. A-28.

as shown in Fig. 15-3. It fractures first along the diagonal lines from c to d, e, f, and g and then breaks along the fixed edges. When loaded uniformly and clamped between rigid edges, a thin plate may fail by shearing off along the edges between d, e, f, and g. A square plate freely supported at the edges fails by breaking along the diagonal lines only.

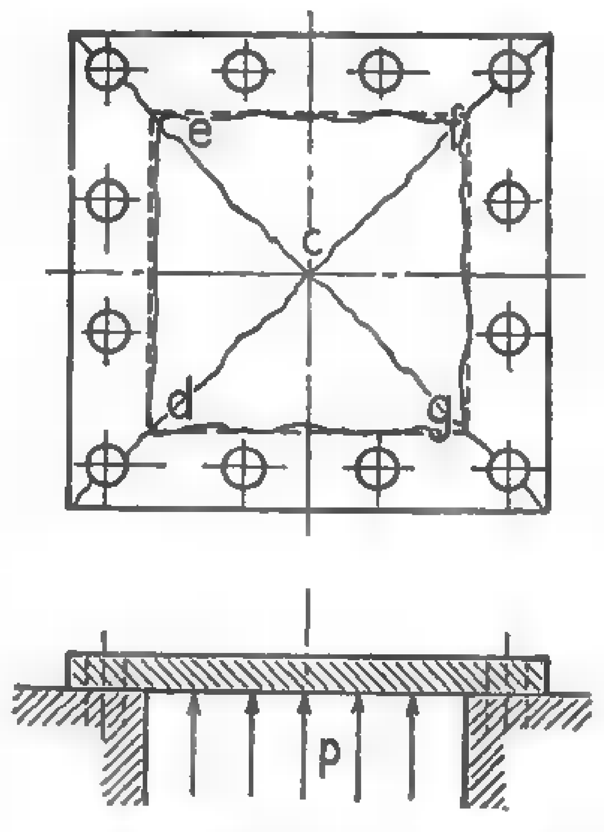


FIG. 15-3. Failure of a square plate.

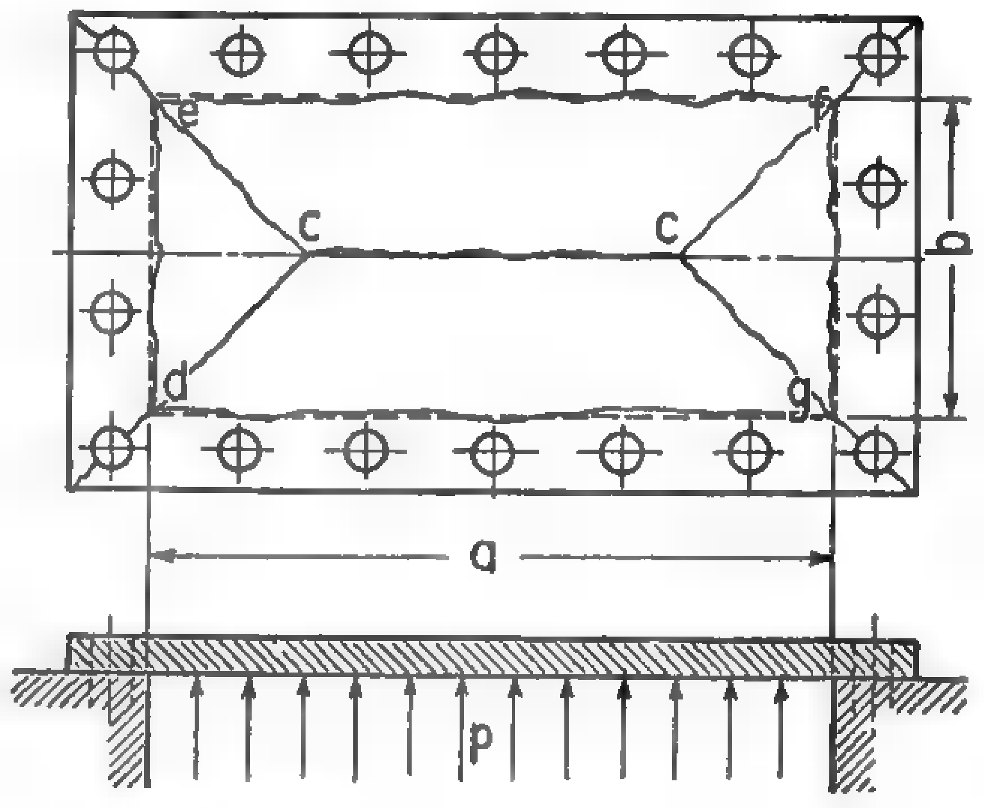


FIG. 15-4. Failure of a rectangular plate.

The probable manner of failure of a rectangular plate is shown in Fig. 15-4. A plate freely supported along the edges will fail by breaking along the center line c–c and the diagonals to d, e, f, and g. A plate held rigidly at the edges presents a greater resistance to failure than does a plate freely supported. This is evident if the plates are considered as beams, in which case the bending moments depend on the method of fastening, as shown in Table 2-4. This table shows also that in a plate with rigidly held edges the sections at the edges are stressed at least as heavily as, if not more heavily than, that at the center.

Exact formulas for determining the strengths of rectangular plates have not been established yet. The existing formulas are based on certain assumptions which are not fully corroborated by tests. For design purposes these formulas are used with empirical coefficients and give only approximate results. Therefore sufficiently large safety factors must be used.

Uniform load. According to Grashof and Bach, the thickness h of a rectangular plate subjected to a pressure p, as in Fig. 15-4, can be found from the equation

$$h=abk_3\sqrt{\frac{p}{S_d(a^2+b^2)}} \tag{15-21}$$

where a is the length of the plate, in inches;
b is its breadth, in inches;
k_3 is a coefficient.

Values of k_3 are given in Table 15–3. However, k_3 depends also on the conditions of the contact surface of the plate, the material of the gasket, and the initial force required to make a tight joint. The values in Table 15–3 are the low-limit values for the most favorable conditions.

Concentrated load. If a rectangular plate similar to that already discussed carries a load F at the intersection of the diagonals, the relation between the thickness h and the stress S_d is determined by the equation

$$h = k_4 \sqrt{\frac{abF}{S_d(a^2+b^2)}} \qquad (15\text{–}22)$$

where the values of k_4 are given in Table 15–3. If the edges are not fixed rigidly, but only held more or less securely, as with a gasket, k_4 should be taken as 1.6 for cast iron and 1.28 for steel.

ASME design data. Simple formulas that are convenient for slide-rule use and involve numerical factors determined by curves have been worked out by the American Society of Mechanical Engineers.[4]

15–5. Other plates. The relation between the thickness h and the stress for a uniformly loaded elliptical plate can be computed by equation 15–21, in which the value of k_5 taken from Table 15–3 is substituted for k_3 and in which the dimensions a and b are the major and minor axes, respectively.

Data for computing the strength and deflection of plates of other shapes for various types of loading may be found in handbooks.

15–6. Bolted joints. Flanges of vessels under internal pressure are subjected to bending, as shown to an exaggerated degree in Fig. 15–5a. When a joint is closed by means of through bolts, the arm of the bending moment is comparatively large. The stress reaches its maximum at the corner where the flange joins the wall. Here the stress is still increased by a stress concentration, the magnitude of which can be estimated from Fig. 3–24 or Fig. 3–25. It is desirable to use a large fillet radius r. In order to keep l as small as possible, it is advantageous to spot-face the bolt head or nut.

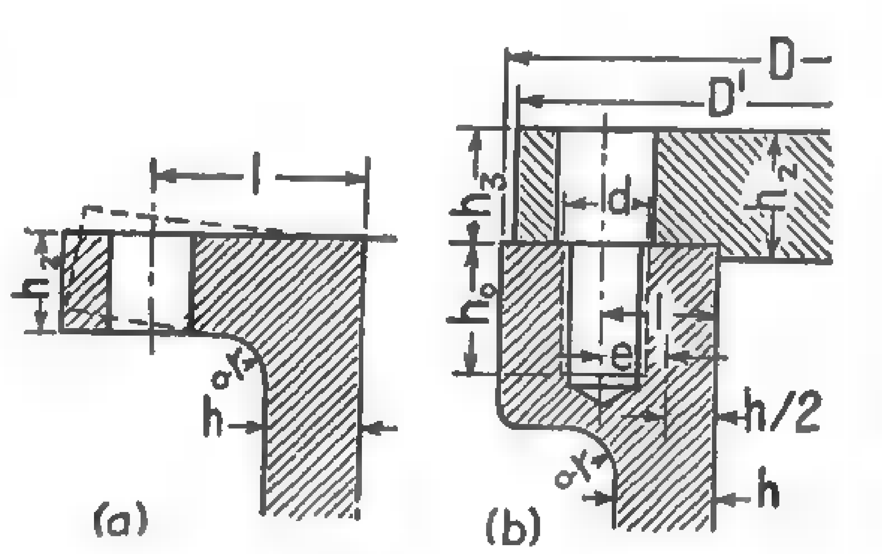


FIG. 15-5. Flanges on vessels.

The flange thickness h_2, Fig. 15–5a, according to common practice, is made equal to $1.25d$ to $1.5d$, where d is the bolt diameter; but it should not be less than $1.1h$ to $1.25h$. After h_2 has been thus selected, the thickness should be checked in bending by considering the flange or the cover edge of thickness h_3, Fig. 15–5b, as a cantilever beam.

[4] I. A. Wojtaszak, "Stress and Deflection of Rectangular Plates," *JAM*, Vol. 3 (June, 1936), p. A-71.

Studs. The use of studs, as in Fig. 15–5b, has many advantages for heavy pressures. It decreases the bending stress at the flange root since the moment arm e can be made very small. The distance l can be made equal to $1.25d$ to $1.5d$. The depth h_o of the tapped hole is made equal to $1.25d$ in a steel casting, $1.5d$ to $1.75d$ in cast iron, and $1.75d$ to $2d$ in aluminum. If the flange is not turned on the outside, it is advisable to make the cover diameter D' $\frac{1}{8}$ or $\frac{1}{4}$ in. smaller than the flange diameter D. Rough castings never match, and if the diameters are supposed to be equal, the joint looks worse than when the two pieces are purposely made of slightly different sizes.

Bolt spacing. To insure a tight joint the distance c between two bolts, often called the *pitch*, must not be greater than a certain limit, which depends on the rigidity of the flange or cover and the inner pressure p. On the other hand, this distance should be made not smaller than $3d$ in order to leave room for the wrench when tightening the nuts. The United States Navy has standards for the pitch c for water and steam joints which may serve as a guide in the design. According to these standards, $c = 7d$ for low pressures from 0 to 50 psi. The pitch c gradually decreases with the increase of pressure, and $c = 3.5d$ for pressures from 175 to 200 psi.

When bolts are spaced equally on a circle, it is convenient for laying out the holes in the shop to have the number of bolts a multiple of 2 or 3, such as 4, 6, 8, or 12.

The proper size of bolts to use was discussed in sections 11–9, 11–11, and 11–13.

EXAMPLE 15–2. Find the number and size of stud bolts to hold the cover of example 15–1 with a metal-to-metal joint.

Since h should be between $1.25d$ and $1.5d$, the bolt diameter d should be approximately equal to $0.67h$ to $0.8h$, or $\frac{3}{4}$ in. to 1 in. Their number may be determined by the least distance between two bolts for insuring a tight joint. The cover not being very rigid, for $p = 275$ psi, according to the United States Navy Standards, $c = 3.5d$. If 1-in. bolts are used, $c = 3.5$ in. and the diameter D of the bolt circle can be made equal to $12 + 2 \times 1 = 14$ in. Thus, the required number of bolts is

$$i = \frac{\pi \times 14}{3.5} = 12.5$$

The stress in the bolts must be checked. Use SAE 1120 steel with a safety factor n of 2, and take the size coefficient as

$$e_{sz} = 1 - 0.4 \times (1 - 0.85) \times (1 - 0.5) = 0.97$$

The safe stress is then

$$S_d = \frac{34{,}000 \times 0.97}{2} = 16{,}500 \text{ psi}$$

The safe load on one bolt, based on the stress area in Table 11–1, is

$$F' = 0.605 \times 16{,}500 = 9{,}980 \text{ lb}$$

However, according to equation 11–28, the initial load from tightening the nuts is $F_o = 16{,}000 \times 1 = 16{,}000$ lb. Thus the actual safety factor n' of a tightened 1-in. bolt is only $2 \times 9{,}980/16{,}000 = 1.25$.

According to section 11–11, for a metal-to-metal contact the required number of bolts based on the consideration of strength alone is

$$i=\frac{0.7854\times 14^2\times 275}{9{,}980}=\frac{42{,}400}{9{,}980}=4.25$$

For comparison the allowable stress given by Unwin's formula (equation 11–52) may be used, although this formula applies to a thin gasket rather than to a metal-to-metal joint. By this formula,

$$S_d=2{,}500\times(1.2\times 1^2+1)=5{,}500 \text{ psi}$$

The safe load on one bolt is

$$F'=0.605\times 5{,}500=3{,}330 \text{ lb}$$

and the number of bolts should be

$$i=\frac{42{,}400}{3{,}330}=12.8$$

According to Bach (section 11–13) the allowable stress for this case is $4{,}600\times 1.25=5{,}750$ psi and

$$i=\frac{42{,}400}{0.605\times 5{,}750}=12.2$$

Since Unwin's and Bach's stresses are very low, twelve 1-in. bolts may be used safely.

EXAMPLE 15–3. Find the number and size of stud bolts to hold the cover of example 15–1, using a gasket joint.

For a joint with a flexible gasket the load on the bolts, according to section 11–11, is equal to the sum of the initial load F_o and the fluid pressure. Evidently, the size of the bolts must be increased if the same convenient number $i=12$ is to be used. Increasing the major diameter to $d=1\frac{1}{8}$ in. gives, by Unwin's formula (equation 11–52),

$$S_d=2{,}500\times(1.2\times 1.125^2+1)=6{,}300 \text{ psi}$$

The safe load on one bolt is

$$F'=0.763\times 6{,}300=4{,}800 \text{ lb}$$

The diameter of the bolt circle must be increased to $12+1.125\times 2=14.25$ in., and the number of bolts should be

$$i=\frac{0.7854\times 14.25^2\times 275}{4{,}800}=\frac{43{,}800}{4{,}800}=9.1$$

However, 12 will be used.

The safety factor can now be found as follows: By equation 11–28, with a factor of 0.[illegible] for a gasket joint, the initial load is approximately $8{,}000\times 1.125=9{,}000$ lb. The load from the fluid pressure is $F=43{,}800/12=3{,}650$ lb, and the stress in each bolt is

$$s=\frac{(9{,}000+3{,}650)}{0.694}=18{,}250 \text{ psi}$$

The size coefficient for $1\frac{1}{8}$-in. steel is 0.96, and the safety factor is

$$n=\frac{34{,}000\times 0.96}{18{,}250}=1.79$$

CHAPTER 16

Packings and Seals

16–1. Gaskets. If there is no relative motion between the parts forming a joint, the joint can be made tight by means of a gasket. When a gasket is used, the surfaces in contact need not be as accurately finished as in the case of a metal-to-metal joint. The material of the gasket depends on the pressure, temperature, and chemical properties of the fluid against which the joint must be sealed. *Rubber* is a good material for cold water and cold gases under moderate pressures. *Paper* and *cork-composition* sheets are used to seal oil, gasoline, or water. *Asbestos* gaskets can be used for water, steam, or hot gases without regard to the temperature. For pressures over 100 psi it is better to use special gasket materials made of asbestos fiber with the addition of binders for increased tensile strength. For high temperatures and pressures, such as are encountered in automobile engines, special copper-asbestos gaskets are made in which a thin copper sheet gives the necessary strength and prevents blowouts. For high and sudden pressures, gaskets are confined in a ring space, as in Fig. 16–1. The width b is made so small that the gasket is compressed above the elastic limit. For ordinary temperatures the gasket g may be made of *lead;* for higher temperatures it is a ring of soft copper.

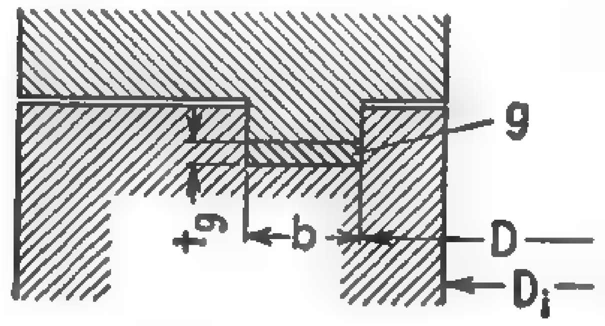


FIG. 16–1. Sealing of high pressures.

Thickness. A fibrous gasket should be thin in order to prevent a blowout. Gaskets are made with thicknesses from 0.010 in. up to $\frac{1}{16}$ in. and not over $\frac{1}{8}$ in., the value depending on the surface. Asbestos gaskets are made $\frac{1}{32}$ to $\frac{1}{16}$ in. thick. The thickness t_g of a copper gasket, Fig. 16–1, need not be over $\frac{1}{64}$ in., provided the surfaces are finished. However, since the danger of a blowout is eliminated, t_g can be made considerably greater.

Ground joints without any gaskets are used for very high pressures. Both the cylinder and the head must be very rigid, so as not to be distorted by tightening of the bolts or by the inside pressure.

16–2. Packings. To seal the joint between two parts when one of them is in motion with respect to the other, *stuffing boxes* are used. In most instances the sealing is produced by a pressure exerted against the moving part. This pressure may be produced either externally or by the fluid itself.

Packings for reciprocating motion. The most commonly used soft-packing stuffing box is shown in Fig. 16–2. The packing material is hemp or cotton

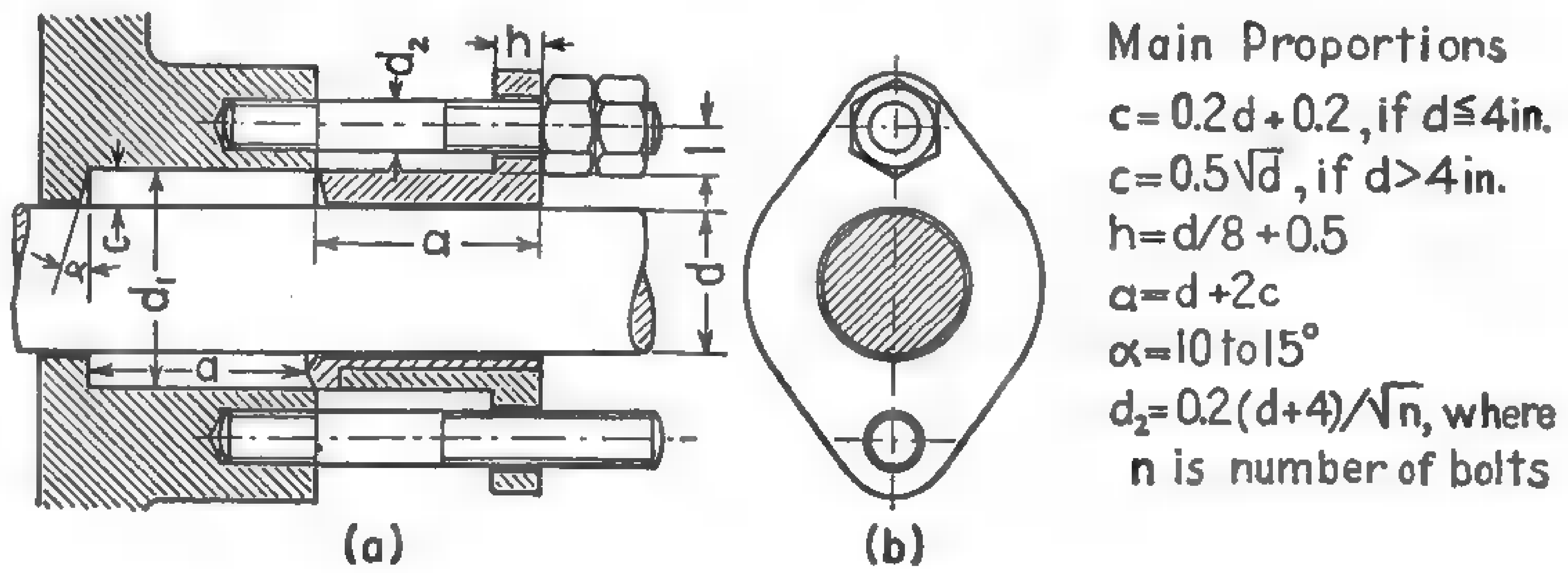


FIG. 16–2. Stuffing box with bolted gland.

rope soaked in grease, for cold water or air, and graphited asbestos rope, for steam or hot water. The bottom of the box and the end of the gland are usually conical, to produce a radial pressure on the rod. However, a flat bottom and a double-cone end of the gland, as shown in the lower half of

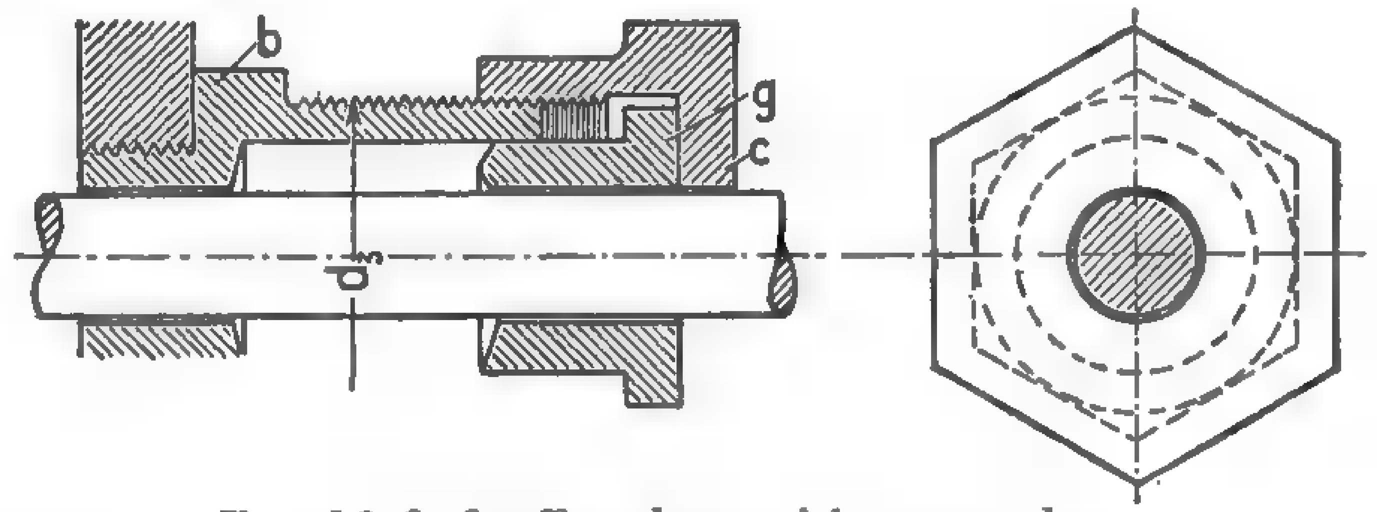


FIG. 16–3. Stuffing box with screwed cap.

Fig. 16–2 and the upper half of Fig. 16–3, give a seal at the box wall and at the rod without creating excessive friction.[1] The gland flange, which is elliptic for two stud bolts, is made round for three bolts and square for four bolts.

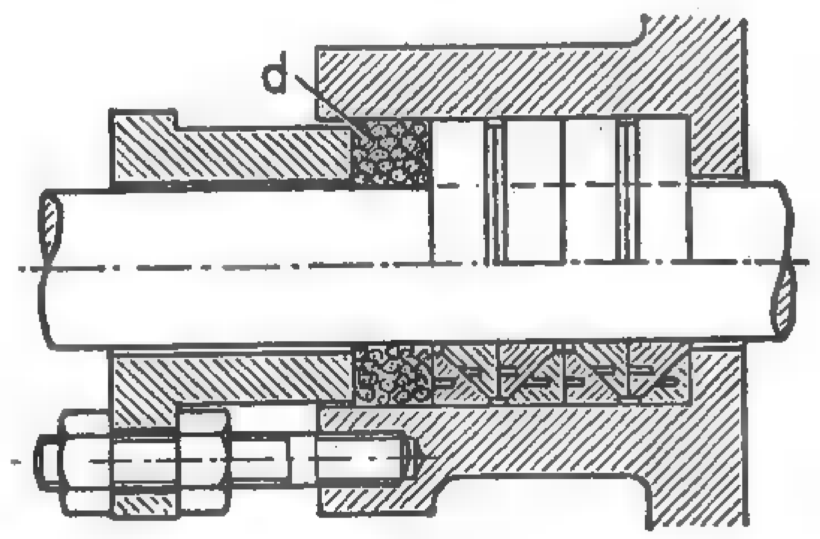


FIG. 16–4. Stuffing box with metallic packing.

A screwed-on cap, Fig. 16–3, is used for smaller rod diameters. In the lower half is shown another type of gland end.

In Fig. 16–4 is shown a stuffing box with metal packing rings. The rings are split into two or more parts to obtain the radial pressure from the 45° cones. They are made of babbitt or brass, or in larger sizes of soft cast iron. A layer of soft packing p is necessary to give the packing a certain flexibility. This packing can be used only with a perfectly cylindrical rod where there is no side play.

A flexible metal packing for larger rod diameters is shown in Fig. 16–5. Each ring is split into three parts which are pressed against the rod by garter

[1] E. D. Waters, D. B. Wesstrom, and Frank S. G. Williams, "Design of Bolted Flanged Connections," *Mechanical Engineering*, Vol. 56 (1934), p. 736; F. A. Halsey, *Handbook for Machine Designers*, 2d ed. (New York: McGraw-Hill Book Company, Inc., 1916), p. 435.

coil springs s, and pins a prevent the gaps of a pair of rings from aligning. The split rings are made of cast iron. A copper-asbestos gasket g prevents any leakage along the wall of the box. This packing is built in units or sections, two of which are shown in Fig. 16–5. For higher pressures the number of sections is increased up to ten or more.

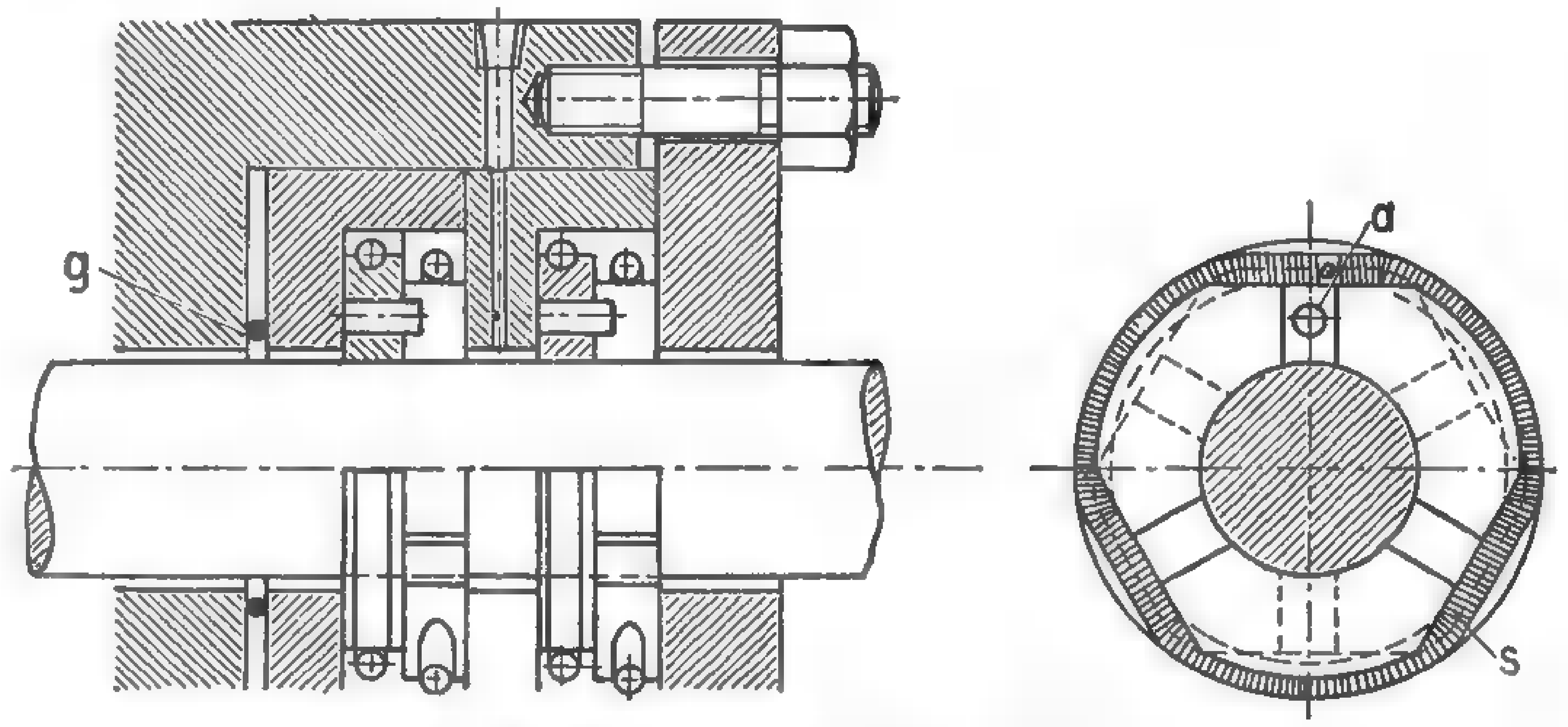


FIG. 16–5. Flexible metal packing.

In Fig. 16–6 are shown packings in which the fluid pressure helps to keep the joint tight. In Fig. 16–6a is shown a U-shaped leather collar used either alone or with a soft packing ring f. In Fig. 16–6b a U-shaped leather collar l is used without any gland; to facilitate its insertion it may be cut diagonally without impairing its sealing effect. In Fig. 16–6c there is a flanged collar l, and the sealing along the wall is obtained by the pressure of the gland g. Finally, in Fig. 16–6d is shown a so-called *chevron packing*—a

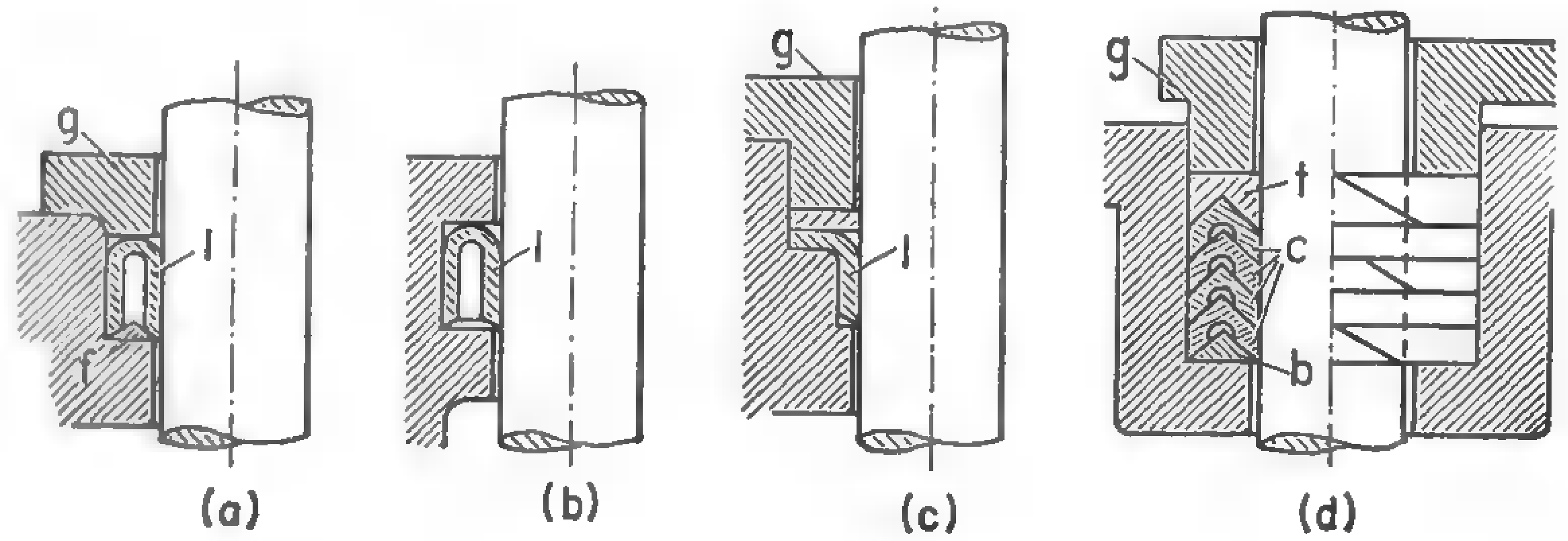


FIG. 16–6. Self-sealing packings.

series of V-shaped collars c molded of a cotton duck and rubber compound for water, and of an asbestos cloth and rubber cement for high temperatures; the collars c are split at an angle, as are also the top and bottom collars t and b, which are molded of the same composition. These packings act automatically, the pressure with which they are held against the sliding rod and their sealing effect varying with the fluid pressure itself.

In Fig. 16–7 are shown methods of sealing the water jacket in cylinders of internal-combustion engines. In Fig. 16–7a each of the two rubber rings

is made of a round rubber band with d greater than a. When confined in the space having a cross-sectional area $a \times b$ they do not quite fill it, allowing a slight expansion of the liner l in the direction indicated by the arrow. In Fig. 16–7b the rubber ring slides along with the end of the liner. The diameter d of the ring is made equal to $\frac{1}{4}$ in. for small cylinders and is gradually increased up to $\frac{3}{8}$ or $\frac{7}{16}$ in. for the largest sizes; the depth c may be made equal to $0.8d$.

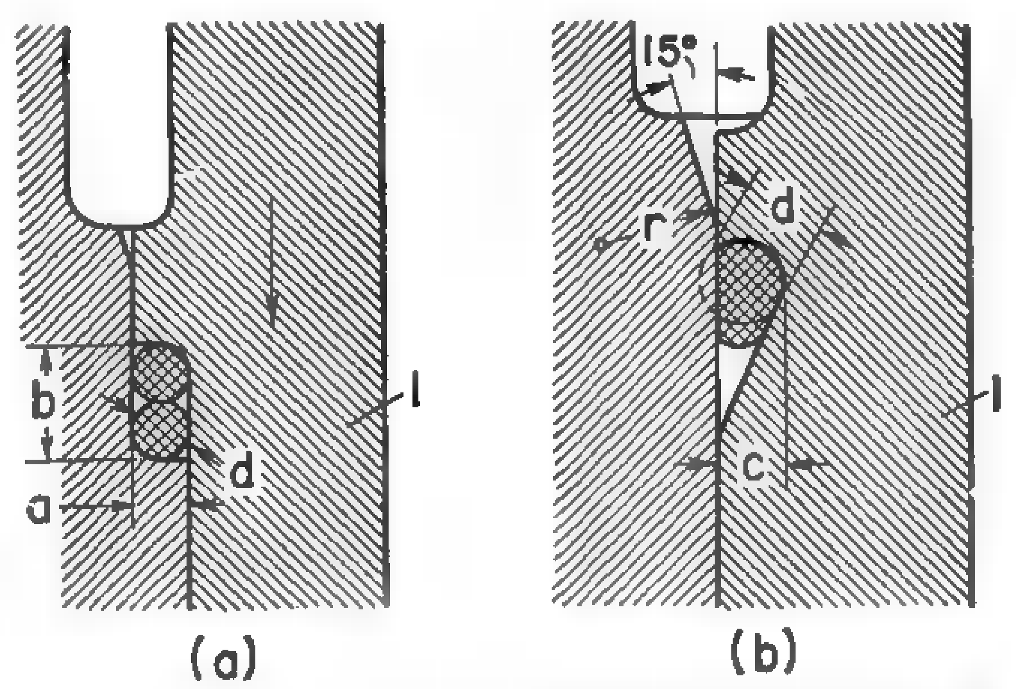


FIG. 16–7. Joints with rubber packing rings.

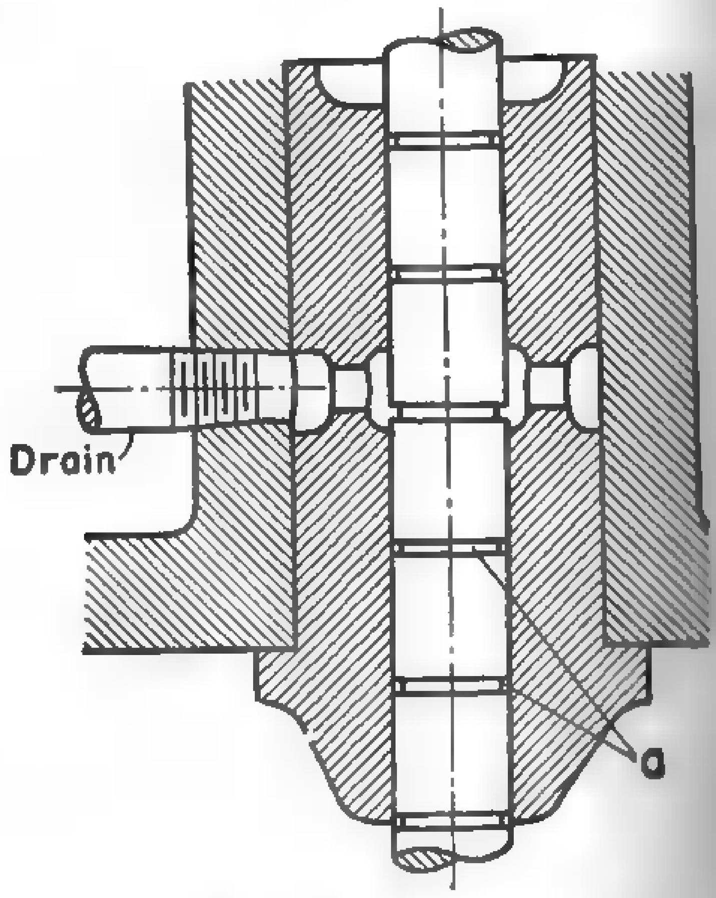


FIG. 16–8. Labyrinth packing.

A packingless leakproof joint can be obtained by using a very close fit. The advantage of such a joint is the absence of friction, except that produced in the film of the fluid which is sealed. The joint is made by grinding and then lapping the plunger in the bushing. In order to have a leakproof joint the clearance must be of the order of 0.0001 in. This requires precision and expensive work. In Fig. 16–8 is shown a packingless stem and guide of a steam-engine poppet valve. The circular grooves a increase the resistance to leakage of steam or gas; hence the name *labyrinth packing* is applied. With liquids such grooves are useless and only decrease the effective length of the seal. Packingless fuel-oil pump plungers of oil engines give good service for pressures up to 4,000 psi and more, if made of very hard material such as heat-treated Nitralloy. Because of the exceedingly small clearance, special care must be exercised to avoid distortion through uneven clamping in assembling.

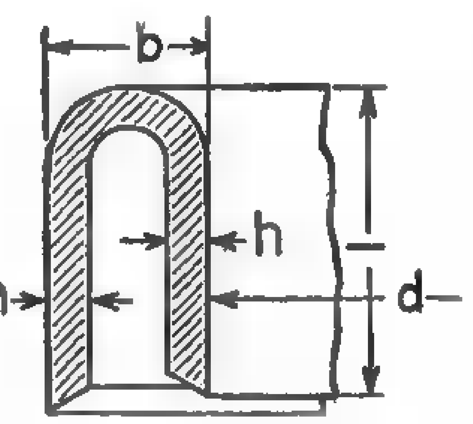


FIG. 16–9. U collar.

Packing for rotary motion. Many of the stuffing boxes previously described, such as those shown in Figs. 16–2 to 16–5, can also be used for rotary or rocking motion. Packings along the lines of Fig. 16–6 wear out too fast when used for rotary motion.

16–3. Design remarks. The procedure in designing a packing depends on the type of packing.

Elastic packings. The friction force F_r exerted by a soft packing upon the reciprocating rod may be computed from the relation

$$F_r = kdp \tag{16–1}$$

where k is an empirical coefficient;
d is the diameter of the sliding member, in inches;
p is the fluid pressure to be sealed, in pounds per square inch. If p is less than 50 psi, use $p = 50$ psi.

When the nuts holding the gland are tightened only enough to prevent leakage, the coefficient k may be taken equal to 0.2[2]

Gland. The flange thickness h, found from data of Fig. 16–2, should be checked by considering the flange as a centilever beam having a length l and carrying a load equal to the working strength of the bolt.

The gland is made of bronze in small sizes, and of cast iron with a bronze bushing for larger sizes.

To reduce friction the gland should be bored slightly larger, by about $0.03d$, than the diameter d of the sliding member. The outside diameter of the gland should be smaller than the bore of the box by the same amount.

Bolts. There is no rule governing the selection of the number of bolts, n. A survey of existing machinery shows that two bolts are used in small boxes, with d up to approximately $2\frac{1}{2}$ or 3 in.; three bolts are used up to 4 in.; and four bolts are used on all larger sizes. Six bolts are seldom found, except on sizes over 10 in.

The bolt diameter d_2 is found by equating the working strength of the bolts to the pressure p exerted by the fluid upon the gland and the frictional force F_r. With the notations of Fig. 16–2,

$$0.7854 d_o^2 n S_d = 0.7854(d_1^2 - d^2)p + F_r \tag{16–2}$$

where d_o is the minor bolt diameter. The allowable stress S_d can be taken as about 10,000 and should not be over 12,000 psi, because the bolts are subject to an additional load due to friction between the packing and the reciprocating rod, and also because the bolts may be tightened under load.

For reciprocating motion the stud bolts should be made long enough to take two nuts when the gland barely enters into the packing space.

To prevent uneven tightening of the bolts and cocking of the gland in large boxes, the nuts are sometimes equipped with gear-shaped collars which are engaged by a gear concentric with the box. By turning this gear all nuts are tightened equally.

Box body. In a box with a threaded cap, as in Fig. 16–3, the root diameter of the threads must be equal to or greater than the minimum outside diameter of the box. Thus

$$d_3 \geq d + 2c + 2h \tag{16–3}$$

[2] Halsey, *op. cit.*, p. 244.

where c is the gland thickness and h is the wall thickness determined to withstand the fluid pressure p.

The thread used is either straight pipe thread or UNF standard thread, Table 11-2. The UNC standard thread is too coarse.

Lubrication of the moving member should be provided. In small and low-speed boxes lubrication may be by gravity. With a metal packing, Fig. 16-5, pressure lubrication should be used.

Self-sealing packings. The thickness h, Fig. 16-9, of a U-shaped collar for great pressures can be determined by combining data of Houghton,[3] Welch, and Jenkins.[4] The approximate thickness should be

$$h = 0.12d^{0.2} \tag{16-4}$$

The width b is made as small as possible, about equal to $4h$, and the depth l can be made from $1.2b$ to $1.8b$.

Speeds. Leather collars should be used only for moderate speeds, not over 200 fpm.[5] With high speeds the wear is excessive and a soft packing with a gland should be used.

Friction. The friction resistance F_r can be computed from the relation[6]

$$F_r = F_o + fAp \tag{16-5}$$

where F_o is the friction of the stuffing box when there is no fluid pressure, in pounds;
f is the coefficient of friction;
A is the area of the leather collar in contact with the sliding member, in square inches;
p is the pressure of the fluid, in pounds per square inch.

The coefficient f may be taken as 0.01 for rubber and soft lubricated leather;[7] for hard leather, f is many times as great, ranging up to 0.15.

Lubrication. Leather packings must have provision for lubrication by pressure oil, to reduce friction and to prolong the life of the packing.

Rotary motion friction. The tangential friction force F_r for rotary motion can be determined from equation 16-1, as for reciprocating motion. The torsional resistance therefore becomes

$$T = \frac{F_r d}{2} = \frac{kd^2p}{2} \tag{16-6}$$

[3] *Hydraulic Engineering* (Philadelphia: E. F. Houghton & Company, 1926), p. 35.
[4] H. J. Spooner, *Machine Design*, 6th ed. (London: Longmans, Green & Company, 1930), p. 66, and Halsey, *op. cit.*, p. 242.
[5] Hütte, *Des Ingenieurs Taschenbuch*, 26th ed., Vol. II (Berlin: Wilhelm Ernst and Sohn, 1931), p. 66.
[6] *Ibid.*, p. 69.
[7] K. Kutzbach, "Versuche über Stopfbüchsen mit hohen Flüssigkeitsdruck," *Zeitschrift Verein Deutscher Ingenieure*, Vol. 80 (1936), p. 609.

TABLE 16-1

ABSOLUTE VISCOSITIES Z

Fluid	Temperature (deg F)	Absolute Viscosity (centipoises)	Temperature	Absolute Viscosity
Steam	68	0.0097	500	0.018
Air	68	0.018	200	0.022
Water	32	1.79	100	0.69
Water	68	1.0	160	0.40
Gasoline	68	0.6	180	0.3
Kerosene	68	2.7	180	1.3
Fuel oil, 30° Baumé	68	5	180	1.6
Fuel oil, 24° Baumé	68	40	180	4
Spindle oil	68	20-35	180	3-4
Machine oil	68	200-500	210	5.5-16
Castor oil	68	1,000	110	200

16-4. Packingless seals. Leakage of the fluid past a rod, as in Fig. 16-8, can be computed with fair accuracy by the formula[8]

$$V = 1.79(100c)^3(p_1 - p_2)\frac{d}{lZ} \tag{16-7}$$

where V is the discharge, in cubic inches per second;
c is the radial clearance between the rod and the bushing, in inches;
p_1, p_2 are pressures on each end of the joint, in pounds per square inch;
d is the rod diameter, in inches;
l is the length of the joint, in inches;
Z is the absolute viscosity of the fluid, in centipoises.

Table 16-1 gives the viscosities of fluids commonly encountered in machines. Viscosities at other temperatures may be determined by interpolation.

Lubrication. When a packingless joint is used for sealing gas, provision should be made for lubrication, preferably under pressure.

16-5. Comparison of packings. Simple stuffing boxes with a soft packing are inexpensive but can be used only for low pressures, not over 150 psi.

Self-sealing leather collars have small friction and long life and can be used up to the highest pressures, but they are suitable only for cold water or oil.

Metallic packings are more expensive and take up more space, but they have small friction and long life and can be made for any pressure that may occur, and for any fluid.

[8] A. M. Rothrick and E. T. Marsh, *Effect of Viscosity on Fuel Leakage between Lapped Plungers and Sleeves on the Discharge from a Pump Injection*, Report No. 477, National Advisory Committee for Aeronautics (1934), p. 9.

PART V: HOISTING MACHINERY

CHAPTER 17

Chains and Wire Ropes

17–1. Chains. Chains are of two main classes, known as coil chains and stud-link chains.

Coil chains. The type of chain used on hoists, cranes, and dredges is shown in Fig. 17–1 and is known as a coil chain. Such a chain is designated by the diameter d of the link stock, but no rigid standards exist for the link length l or the width b. These dimensions as well as the breaking load F_u or proof load F may be obtained from catalogs of chain manufacturers.[1] The usual sizes run from $\frac{1}{4}$ in. to $2\frac{1}{2}$ in.

The material used for crane chains is basic open-hearth steel for the smaller sizes and high-grade wrought iron for the larger sizes.

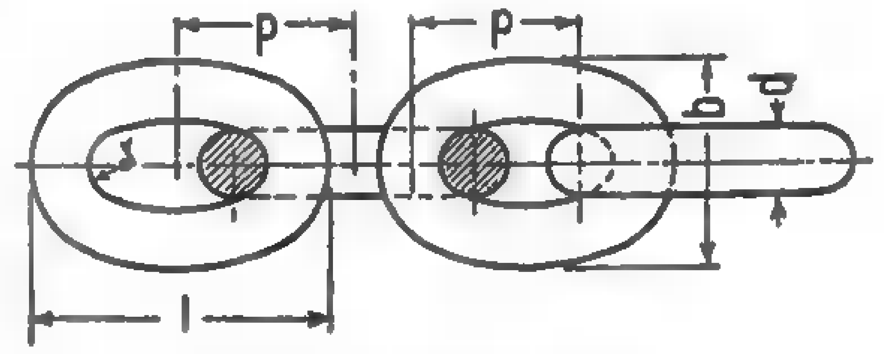


FIG. 17–1. Coil chain.

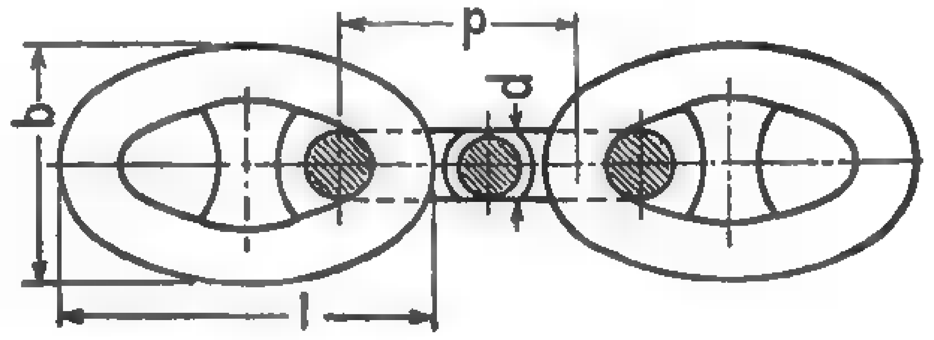


FIG. 17–2. Stud-link chain.

Load capacity. Chains are tested with a proof load equal to approximately one-half the breaking load, or practically equal to the load at the elastic limit. The safe load is often taken as one-half the proof load. This practice corresponds to using a safety factor n of 2. However, since chains are likely to be subjected to shock action, it is better to take n as 3 for intermittent machine operation, and as 3.5 for continuous machine operation.

Stud-link chains. A stud-link chain is shown in Fig. 17–2. Such chains are used for heavy hoisting and in marine work for anchors and moorings. Within the elastic limit a stud-link chain will carry a load of 20 to 25 per cent greater than that which is safe for open-link chains.

An additional advantage of stud-link chains is that they do not kink or tangle as readily as open-link chains.

17–2. Chain drums and sheaves. A drum used for winding up a chain should be provided with machined helical grooves. Two forms of such grooves are used, the more common form being shown in Fig. 17–3a.

[1] Lionel S. Marks, ed., *Mechanical Engineers' Handbook*, 5th ed. (New York: McGraw-Hill Book Company, Inc., 1951), p. 931.

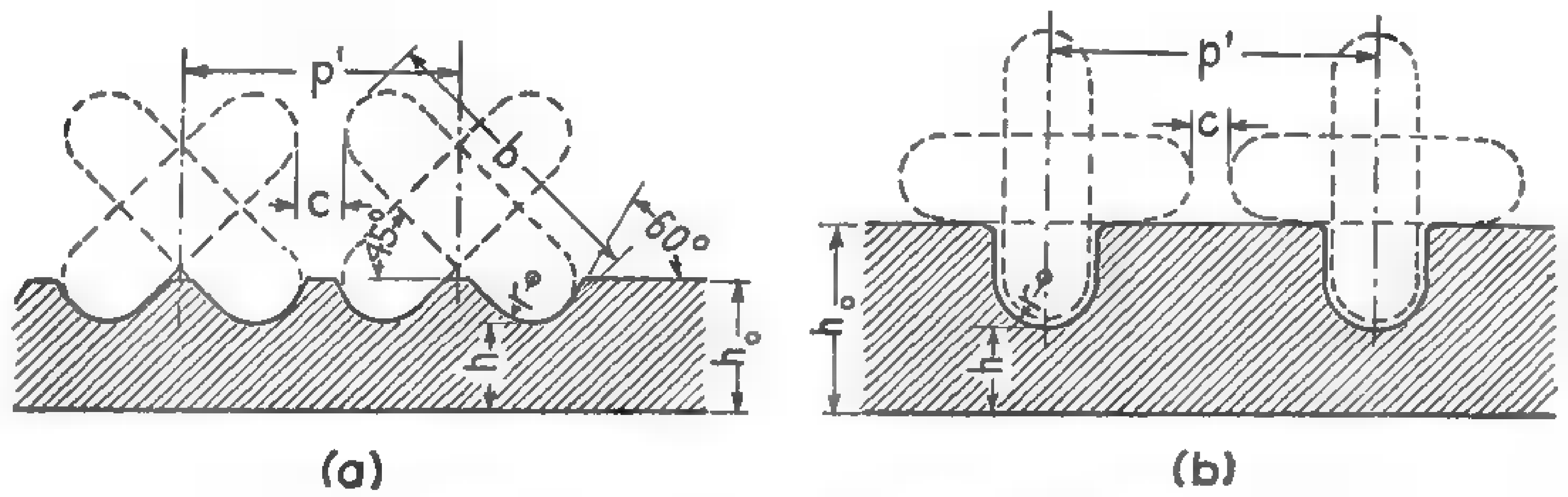

(a) (b)

FIG. 17–3. Grooves in chain drums.

The grooves are cut so as to leave a clearance c of about $\frac{3}{16}$ in. From the geometry of the sketch, the pitch p is

$$p' = 0.707b + 0.293d + \tfrac{3}{16} \tag{17–1}$$

where all values are expressed in inches.

The radius of the groove should be

$$r = 0.5d + \tfrac{1}{16} \tag{17–2}$$

Deeper grooves, like those in Fig. 17–3b, require a heavier drum wall and also, with the same clearance c, a greater drum length to take the same length of chain, since their pitch is greater. This pitch is

$$p' = b + \tfrac{3}{16} \tag{17–3}$$

The diameter D of the drum depends on the size of the chain, its speed, and its desired life. For short-link chains the drum diameter should be

$$D \geqq 20d \tag{17–4}$$

and it should preferably be about $30d$. If a chain is wound on a relatively small drum, the bending of the links is increased and the life of the chain is reduced.

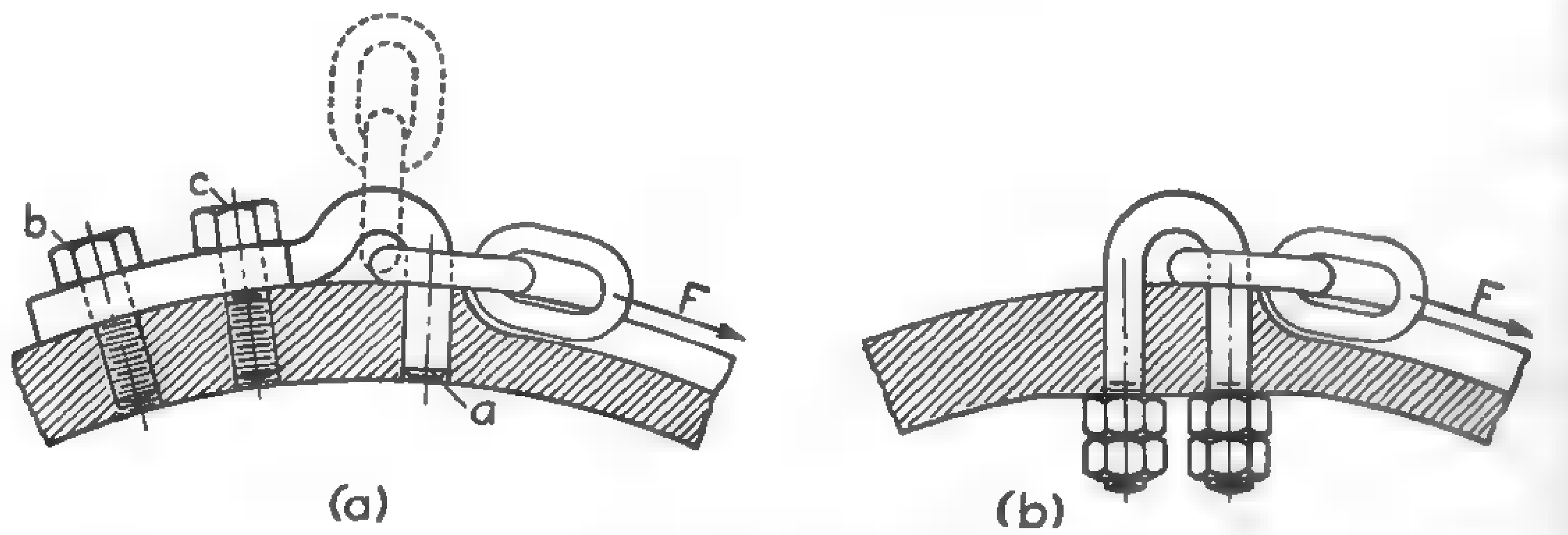

(a) (b)

FIG. 17–4. Chain anchors.

The length of the drum should always be such that the required chain length will go on the drum in a single layer. In order to reduce the load coming upon the chain anchor, one or two coils of the chain should remain on the drum when the load is in its lowest position.

Anchors. In Fig. 17–4a is shown an anchor type which is easily assembled but has the disadvantage of subjecting the tongue a to bending. In case the

chain should assume the position shown by the dotted lines, it imposes a much greater load on the cap screw c than on the screw b. The design shown in Fig. 17–4b is cheaper to make and gives a good load distribution, but it may not be as convenient for assembling.

Design of drums. A crane drum may be made with bushings and may revolve on a stationary shaft, or the drum may be fastened to a revolving shaft.

An exact stress analysis for a hoist drum is a rather difficult problem, but an approximate procedure consisting of the following steps may be applied:

a) The minimum thickness of the metal, as h in Fig. 17–3, is determined from considerations relative to casting the drum and machining the grooves.

b) The bending stress s_1 is determined by considering the drum as a hollow round beam loaded at the middle and supported at the ends. The stress s_1 should not exceed 3,000 psi for cast iron[2] and 5,000 psi for cast steel.

c) The tangential compressive stress s_2 due to the tension in the chain when it is wound on the drum may be determined by the relation

$$s_2 = \frac{F}{p'h} \tag{17–5}$$

where p' is the pitch of the grooves, or the distance between the centers of two adjacent grooves. The stress s_2 should not exceed 15,000 psi for ordinary cast iron, 17,000 psi for the best cast iron, or 12,000 to 16,500 psi for cast steel.

d) The shear stress due to the torsional moment created by the load may be calculated. Usually, however, this stress is negligibly small.

By considering the wall thickness equal to h and neglecting the metal above the groove, a strong yet not excessively heavy drum is obtained.

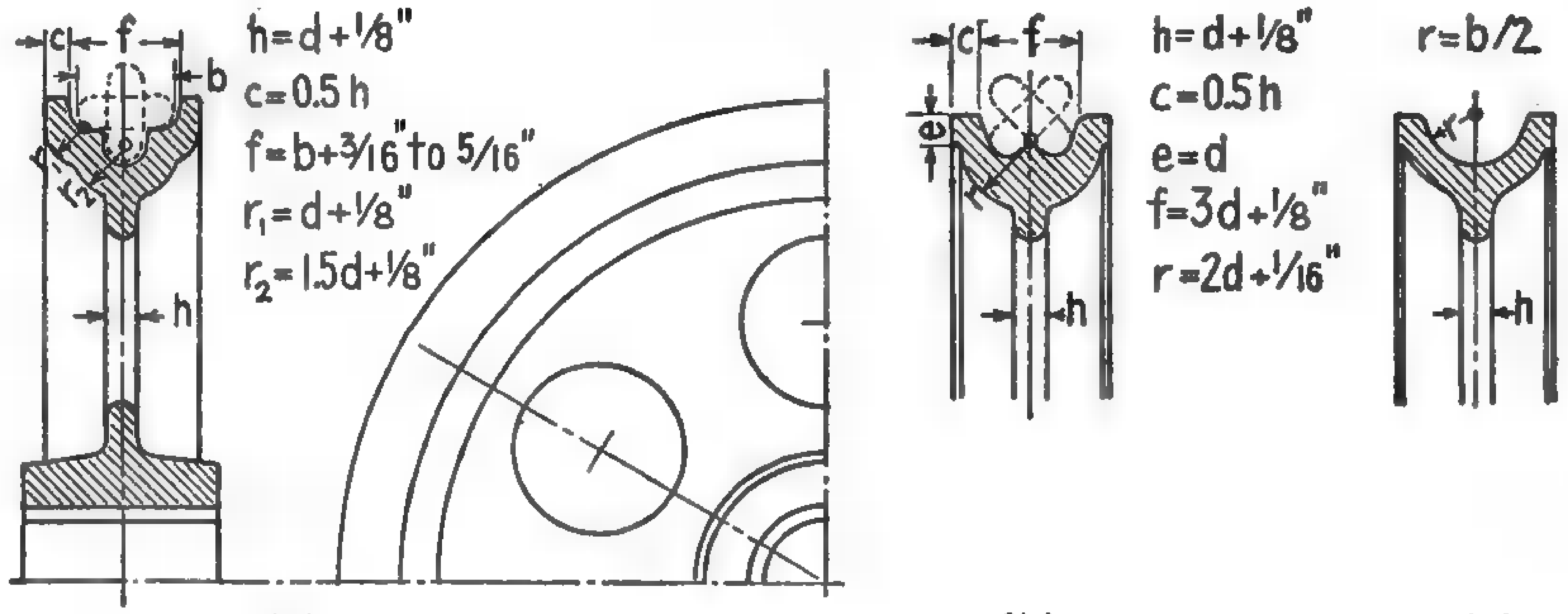
Sheaves. Chain sheaves are of two types: plain sheaves which only guide the chain in changing its directions, and sheaves with pockets which serve to pull the chain.

In Fig. 17–5a and b are shown sheave designs with grooves similar to those used on drums. The center webs may be made plain, but round holes are helpful for handling the sheave and for decreasing the weight. In Fig. 17–5c is shown a simpler but satisfactory groove. The sheave diameter D should not be less than $20d$.

Pocket sheaves are used in place of drums, particularly for anchor chains. A sheave of this type has a rim similar to that in Fig. 17–5a, but there are cross ribs to catch the horizontal links, the vertical links going into the central groove.

17–3. Wire rope. Wire ropes are built up of strands made of wires twisted together. Ordinarily the strands are twisted into rope in the opposite

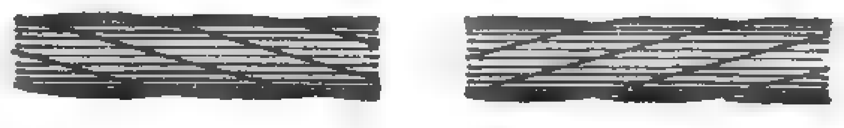
[2] Erik Oberg and Franklin D. Jones, *Machinery's Handbook*, 14th ed. (New York: The Industrial Press), p. 445.



(a) (b) (c)

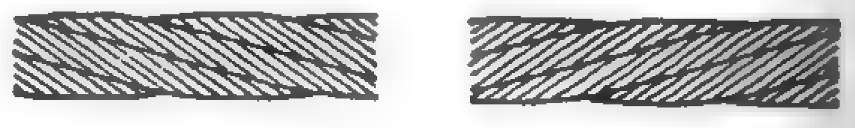
FIG. 17-5. Chain sheaves.

direction to the twist of wires in the strand, as shown in Fig. 17-6. The rope is then known as *regular lay rope.* When wires and strands are twisted in the same direction, as in Fig. 17-7, the rope is known as *lang lay rope.* Tests show that lang lay rope has a life several times as long as regular lay rope.[3] However, regular lay rope is more generally used than lang lay rope because is has less tendency to twist and spin. Ropes with strands twisted in the right direction, as in Fig. 17-6a or Fig. 17-7a, are known as *right lay ropes*; those twisted in the left direction, as in Fig. 17-6b, or Fig. 17-7b, are *left lay ropes.*



(a) (b)

FIG. 17-6. Regular-lay rope.



(a) (b)

FIG. 17-7. Lang-lay rope.

Standard wire ropes are made of six strands with a hemp core saturated with a lubricant. The number of wires in a strand may be 7, 19, or 37. The rope is designated accordingly as 6×7, 6×19, or 6×37. *Extra-flexible hoisting rope* is made of 8 strands of 19 wires, or 8×19 construction. Hoisting ropes differ also in the metal of the wires, as indicated in Table 17-1. The breaking strength of wire ropes as found by tests is about 83 per cent of the combined strength of all wires. This reduction is due to the difficulty of getting a perfect grip on the rope. As a result, not all the wires carry their full share of the load; and since the inner wires in a strand are shorter than the outer wires, they are more easily overloaded. Table 17-1 gives the approximate breaking strength of standard 6×19 hoisting ropes, and the minimum sheave diameters that are recommended.

High-grade steel increases the strength of the rope but reduces its flexibility. When using one of the stronger ropes, it is therefore good practice

[3] [Symposium,] "Drantseilforschung," *Zeitschrift Verein Deutscher Ingenieure,* Vol. 75 (1931), p. 1485.

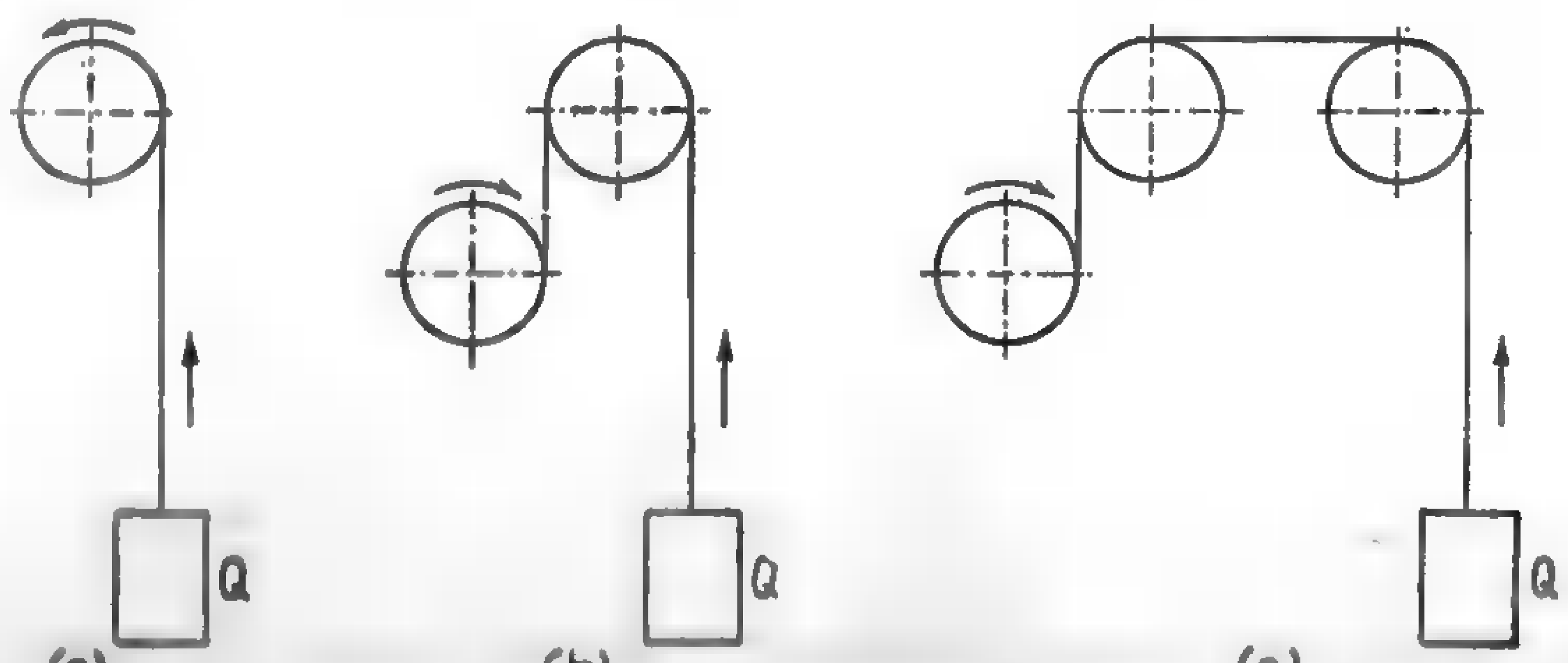
TABLE 17-1

STANDARD 6×19 HOISTING ROPES

Rope Diameter (in.)	Approximate Weight (lb per ft)	Minimum Drum or Sheave Diameter Recommended		Approximate Strength, Tons of 2,000 Lb			
		Iron Rope (in.)	Mild Plow Steel Rope (in.)	Wrought Iron	Mild Plow Steel	Plow Steel	Improved Plow Steel
1/4	0.10	18	12	1.1	2.07	2.39	2.74
5/16	0.16	24	15	1.6	3.22	3.71	4.26
3/8	0.23	28	18	2.5	4.62	5.31	6.10
7/16	0.31	32	22	3.2	6.25	7.19	8.27
1/2	0.40	36	24	4.2	8.13	9.35	10.7
9/16	0.51	42	28	5.3	10.2	11.8	13.5
5/8	0.63	46	30	6.4	12.6	14.5	16.7
3/4	0.90	54	36	9.1	18.0	20.7	23.8
7/8	1.23	64	42	12.4	24.3	28.0	32.2
1	1.60	72	48	16.0	31.6	36.4	41.8
1 1/8	2.03	..	54	...	39.8	45.7	52.6
1 1/4	2.50	..	60	...	48.8	56.2	64.6
1 3/8	3.03	..	66	...	58.8	67.5	77.7
1 1/2	3.60	..	72	...	69.6	80.0	92.0
1 5/8	4.23	..	78	...	81.2	93.4	107.0
1 3/4	4.90	..	84	...	93.6	108.0	124.0
1 7/8	5.63	..	90	...	107	123	141
2	6.40	..	96	...	121	139	160
2 1/8	7.23	..	102	...	...	156	179
2 1/4	8.10	..	108	...	...	174	200
2 1/2	10.00	..	120	...	...	212	244
2 3/4	12.10	..	132	...	...	254	292

to make the sheave or drum diameter equal to that required by a rope of mild plow steel of the same strength.

The strength of extra-flexible 8×19 or special flexible 6×37 hoisting rope is about 10 to 12 per cent lower than that of standard 6×19 rope of the same material. Also, the more-flexible ropes are made of thinner wire and are more subject to outside abrasion.



(a) (b) (c)

FIG. 17-8. Schemes of rope direction.

TABLE 17–2

HOISTING-ROPE DATA

Type of Service	Working Factors n_o		Rope Speeds (fpm)	
	Minimum	Maximum	Normal	Maximum
Cranes				
Hand-operated	3	5	100	....
Motor-operated	4	7	200	....
Hot-ladle cranes, slings	8	...	200	....
Derricks, haulage rope	3	...	....	2,000
Small electric and air hoists	7	...	200	....
Mine hoists (according to shaft depths)				
Up to 500 ft	8	10	1,200	1,600
500 to 1,000 ft	8	10	1,600	2,000
1,000 to 2,000 ft	7	9	2,000	2,500
2,000 to 3,000 ft	6	8	2,500	3,000
Over 3,000 ft	5	8	3,000	3,000
Miscellaneous hoisting	5	...	200	....
Passenger elevators	8	12	250–300	1,250

The life of rope depends to a great extent on the number of bendings it undergoes at every operation. A bending over a guide sheave, as in Fig. 17–8b, is equivalent to two bendings, as it is a reversed bending; that is, it requires bending the rope first in one direction and then in the opposite direction. However, it makes no difference whether the angle of contact with the sheave is 90°, 180°, or 360°. Both the stress and the life of the rope will remain the same in each case. Thus, if all conditions are equal and the life of a rope in Fig. 17–8a is taken as unity, the life of the rope in Fig. 17–8b will be about one-fourth as long, and that in Fig. 17–8c will be only one-sixth or one-seventh as long. A design that eliminates reversed and unnecessary bendings will lengthen the life of the rope. The life of the rope decreases also with the speed.

Working loads. The working load of a rope is based not on the elastic limit but on its breaking strength. Therefore, to avoid confusion with the safety factor the ratio of the breaking strength to the *effective load* (defined in section 17–4) will be designated as the *working factor* n_o. Average practice is to take n_o as 6. However, n_o may be taken either lower or higher, as shown in Table 17–2, the actual value depending on the service. For transportation of persons, state laws require a minimum value of 8 for n_o. In mine hoists lower values of n_o are used for hoisting materials from considerable depths. For hauling persons, n_o is increased to 8 by decreasing the maximum useful load. For passenger service the maximum rope speed usually should not exceed 1,000 fpm. Only for very tall buildings and great depths should it be increased up to 1,250 fpm.

For wrought-iron ropes it is difficult to obtain a value of n_o greater than 5, and this restriction automatically limits their use.

Friction and efficiency. When a rope is wound on a sheave, the friction between the fibers offers resistance to bending, which must be overcome by the pull applied to the running-off side of the rope.

In hoisting practice, for average conditions, a friction loss of 5 per cent for each bending, or a rope efficiency e_r of 0.95, is considered a safe value; and a friction loss of 3 per cent for each bending, or a rope efficiency e_r of 0.97, is used for very flexible ropes with extra-large pulleys.

17–4. Stresses in hoisting rope. The following main stresses are created in a hoisting rope under load:

a. Direct stress due to the load hoisted and the weight of the rope
b. Stress due to the bending of the rope about the sheave
c. Stress during starting
d. Stress due to change of rope speed, including stops

Bending stress. The commonly used formula proposed by Reuleaux for the bending stress in wire is

$$s=\frac{E_r d_w}{D} \qquad (17\text{–}6)$$

where E_r is the modulus of elasticity of the rope as a whole, in pounds per square inch;
d_w is the diameter of the wire, in inches;
D is the sheave diameter, in inches.

According to experiments, E_r can be taken equal to 11,000,000 psi for wrought-iron ropes and to 12,000,000 psi for steel ropes.[4] The modulus of elasticity E_r is less than one-half that of the wire material because the rope is composed of twisted wires, which act as helical springs.

The wire diameter d_w may be taken as in Table 17–3, where d is the nominal diameter of the rope.

TABLE 17–3

WIRE DIAMETER OF HOISTING ROPE

Rope	d_w	*Rope*	d_w
6×7	0.106d	6×37	0.045d
6×19	0.063d	8×19	0 050d

If s is the bending stress in each wire, the load on the whole rope due to bending can be taken as

$$Q_b=0.7854 d_w^2 \times n \times s \qquad (17\text{–}7)$$

where n is the total number of wires in the rope section.

Stress during starting. The general case of starting is when the rope has a slack h which must be taken out before the rope is taut and starts to exert

[4] J. F. Howe, "Determination of Stresses in Wire Rope as Applied to Modern Engineering Problems," *Transactions of the American Society of Mechanical Engineers*, Vol. 40 (1918), p. 1043.

a pull on the load. The stress due to this slack depends on the acceleration a which is necessary in order to impart to the drum and rope the rope velocity v_s, at the instant when the rope is taut. If this acceleration is considered constant, which is a natural assumption, then

$$v_s = \sqrt{2ah} \tag{17–8}$$

where v_s is in feet per second, a is in feet per second per second, and h is in feet.

Taking up the slack is an impact action, and the corresponding stress may be found by using equation 3–20. However, this equation was derived for a weight falling under the influence of the force of gravity with an acceleration g. Since the acceleration of the hoist rope is a, the distance h in equation 3–20 must be multiplied by the ratio of the accelerations. The stress due to starting and slack may therefore be determined by the relation

$$s' = s\left(1 + \sqrt{1 + \frac{2ahE_r}{slg}}\right) \tag{17–9}$$

Equation 17–9 shows that the length of rope l decreases the impact stress. If the comparatively small resistance of the hemp core is disregarded, the static stress s may be found from the equation

$$s = \frac{Q + W_r}{0.7854 d_w^2 n} \tag{17–10}$$

where Q is the load and W_r is the weight of the rope.

If loads are substituted for stresses, the starting load in the rope near the drum becomes

$$Q_{st} = (Q + W_r)\left(1 + \sqrt{1 + \frac{2ahE_r}{slg}}\right) \tag{17–11}$$

This equation, in conjunction with equation 17–8, gives

$$Q_{st} = (Q + W_r)\left(1 + \sqrt{1 + \frac{v_s^2 E_r}{slg}}\right) \tag{17–12}$$

If there is no slack, $h = 0$, $v_s = 0$, and

$$Q_{st} = 2(Q + W_r) \tag{17–13}$$

Acceleration. Stress due to acceleration is directly proportional to the acceleration a given to the load. The magnitude of the corresponding load is equal to the mass multiplied by the acceleration:

$$Q_a = (Q + W_r)\frac{a}{g} \tag{17–14}$$

Usually it is not the acceleration a but the time t necessary to attain a velocity v that is given. Then, if v is in feet per minute and t is in seconds, a may be found from the relation

$$a = \frac{v}{60t} \tag{17–15}$$

Change in speed. The additional load created by a change of speed may be found by equation 17–12, and the acceleration may be found by equation 17–15 by using for v the change of velocity $(v_2 - v_1)$.

Sudden stopping of the hoist drum when lowering the load produces a stress several times as great as the static stress because of the kinetic energy of the moving masses. This kinetic energy is absorbed by the rope, and the resulting stress may be computed by equating the kinetic energy to the resilience of the rope and solving for this stress. If during stopping the load moves down a certain distance, the corresponding change of potential energy must be added to the kinetic energy, and it is necessary to add also the work of stretching the rope during stopping, which may be computed from the impact stress.

Effective load. The sum of the useful load Q, the weight W_r of the rope, and the load Q_b equivalent to bending is called the *effective load.* During starting, the starting load Q_{st} takes the place of $Q + W_r$; and during acceleration of the load the effective load is increased by Q_a.

EXAMPLE 17–1. Determine the size of wire rope necessary for a mine hoist carrying a load Q of 8.4 tons to be lifted from a depth of 750 ft. A rope speed of 1,600 fpm must be attained in 10 sec.

The simplest method is to assume a certain size of rope from preliminary calculations and to find the actual working factor n_o by taking all stresses into consideration. The breaking strength of the rope should be about six times the maximum load. At starting the stress is doubled. Also, to this stress is added the stress due to bending, which with the minimum permissible drum diameter is about equal to the static stress. Therefore a nominal working factor of about 15 may be assumed. The minimum breaking strength must then be $8.4 \times 15 = 126$ tons.

The commonly used type is the 6×19 rope. Since the cost of various ropes for a given strength is about the same, it is desirable to use a high-quality steel giving a smaller size and a smaller weight for the same strength. Thus, according to Table 17–1, a $1\frac{7}{8}$-in. plow-steel rope seems to be the proper choice.

The diameter of the drum may be assumed from Table 17–1 as suitable for the corresponding $2\frac{1}{8}$ in. rope of mild plow steel, or 102 in.

The wire diameter, by Table 17–3, is

$$d_w = 0.063 \times 1\tfrac{7}{8} = 0.118 \text{ in.}$$

The bending stress, according to equation 17–6, will be

$$s = 12{,}000{,}000 \times \frac{0.118}{102} = 13{,}900 \text{ psi}$$

The load equivalent to bending, by equation 17–7, is

$$Q_b = 0.7854 \times 0.118^2 \times 6 \times 19 \times 13{,}900 = 17{,}300 \text{ lb, or } 8.65 \text{ tons}$$

The weight of the rope, from Table 17–1, is

$$W_r = 5.63 \times 750 = 4{,}220 \text{ lb, or } 2.1 \text{ tons}$$

The starting load without slack, from equation 17–13, is

$$Q_{st} = 2 \times (8.4 + 2.1) = 21.0 \text{ tons}$$

The acceleration, found by equation 17–15, is

$$a=\frac{1{,}600}{60\times 10}=2.67 \text{ fpsps}$$

The corresponding additional load, by equation 17–14, is

$$Q_a=\frac{(8.4+2.1)\times 2.67}{32.2}=0.9 \text{ ton}$$

Thus the effective load during starting is

$$Q_{max}=Q_b+Q_{st}=8.65+21.0=29.65 \text{ tons}$$

The working factor during starting is

$$n_o'=\frac{127}{29.65}=4.27$$

During the first 10 sec after starting,

$$n_o=\frac{127}{8.65+8.4+2.1+0.9}=6.3$$

During the uniform lifting or lowering of the load,

$$n_o=\frac{127}{8.65+8.4+2.1}=6.6$$

This shows that the $1\frac{7}{8}$-in. rope is satisfactory. However, if a higher working factor n_o is desired, the drum diameter must be increased and all calculations must be repeated.

EXAMPLE 17–2. Determine the influence of a rope slack of only 1 ft on the hoist rope discussed in example 17–1.

The acceleration of the rope while it is made taut is $a=2.67$ fpsps, as found in example 17–1. By equation 17–8, the rope velocity, when the rope becomes taut, is

$$v_s=\sqrt{2\times 2.67\times 1}=2.31 \text{ fps, or } 138.6 \text{ fpm}$$

The static stress in the rope, by equation 17–10, is

$$s=\frac{(8.4+2.1)\times 2{,}000}{0.7854\times 0.118^2\times 6\times 19}=16{,}850 \text{ psi}$$

By equation 17–12, the starting load is

$$Q_{st}=(8.4+2.1)\times\left(1+\sqrt{1+\frac{12{,}000{,}000\times 2.31^2}{16{,}850\times 750\times 32.2}}\right)=10.5\times(1+\sqrt{1+0.157})=21.9 \text{ tons}$$

Thus the slack increases the load by 4.5 per cent. If the length l of the rope is smaller, the influence of slack will be greater. However, in this case the amount of slack and the velocity v_s at the moment when the rope is made taut will also be smaller.

17–5. Bearing stress. As a result of recent research work on failure of wire ropes due to repeated bending, a new design criterion has been offered.[5] This criterion is based on the bearing pressure exerted on the wire of a rope and is a dimensionless ratio B which can be calculated from the relation

$$B=\frac{2Q_c}{S_u dD} \tag{17–16}$$

where Q_c is the total load in tension on the wire, in pounds;
S_u is the ultimate tensile strength of the wire rope, in pounds per square inch;

[5] D. C. Drucker and H. Tachau, "A New Design Criterion for Wire Rope," *Trans. ASME*, Vol. 66 (1945), p. A-43.

d is the rope diameter, in inches;
D is the sheave or drum pitch diameter, in inches;

In Fig. 17–9 are given average curves showing the relation between the bearing-pressure ratio B and the number of bends causing failure for various constructions of rope. All curves have similar shape and are of the same order of magnitude. From Fig. 17–9 it appears that a value of B below 0.0011 will insure long life for a stiff 6×12 rope; below 0.0012, for a standard 6×19 rope; below 0.0015, for extra-flexible 6×37 hoisting rope; and below 0.0012, for 6×24 and 8×19 ropes.

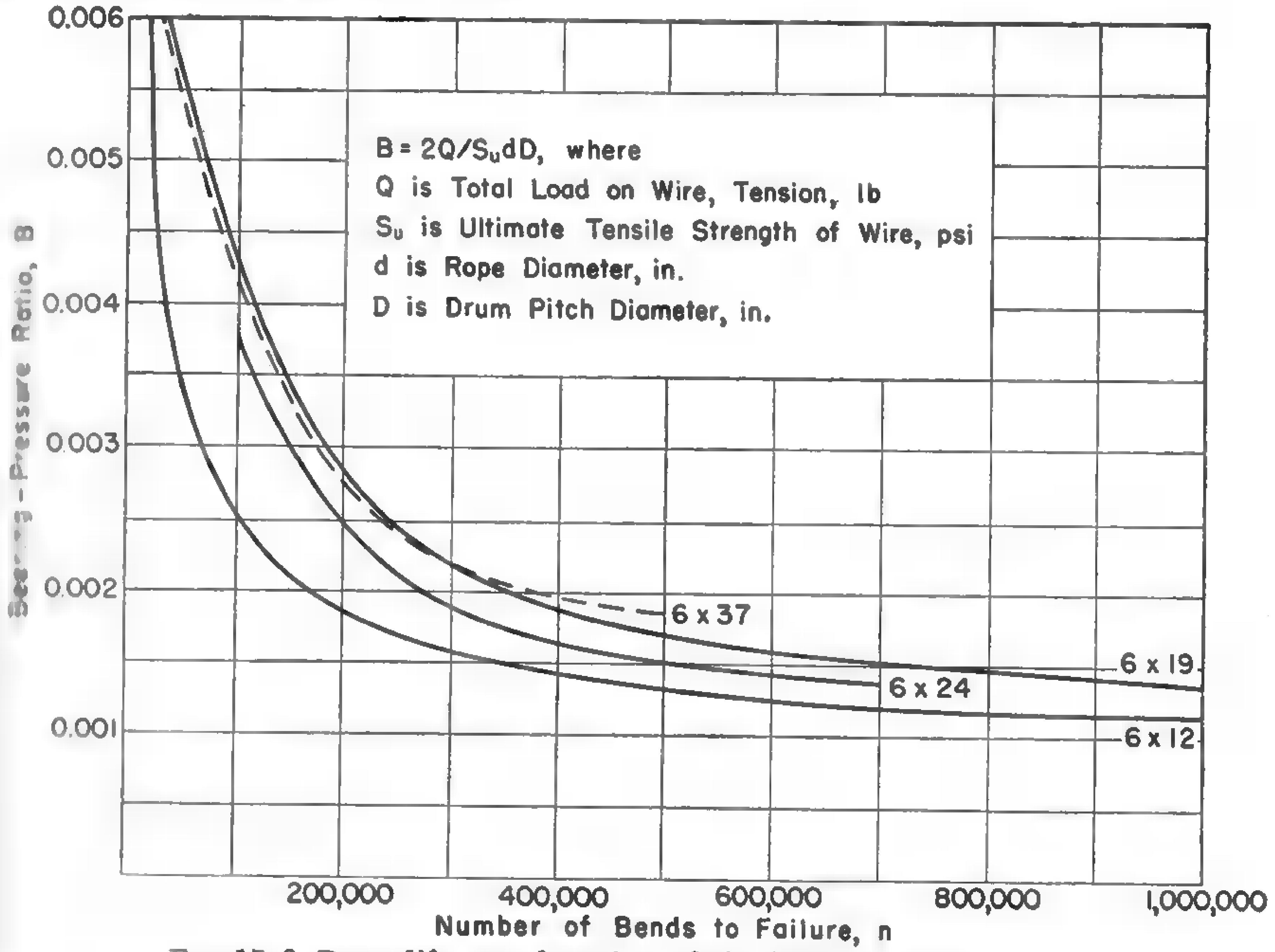


FIG. 17–9. Rope life as a function of the bearing-pressure ratio.

There are not enough practical data to discard the conventional design of a rope that takes into account bending stresses by equation 17–6, and instead to base the selection of the rope size and the drum diameter exclusively on equation 17–16. However, it seems appropriate to check d and D by this equation.

EXAMPLE 17–3. Check by the bearing-pressure ratio the rope size and the drum diameter found in example 17–1.

The total load in equation 17–16 is the sum of the useful load, or 8.4 tons, and the weight of the rope, or 2.1 tons. Thus,

$$Q_c=(8.4+2.1)\times 2{,}000=21{,}000 \text{ lb}$$

The ultimate strength S_u may be found from the breaking strength of 127 tons. It is

$$S_u=\frac{127\times 2{,}000}{0.7854\times 0.118^2\times 6\times 19}=203{,}000 \text{ psi}$$

By equation 17–16, the bearing-pressure ratio is

$$B = \frac{2 \times 21{,}000}{(203{,}000 \times 1.875 \times 102)} = 0.00108$$

This value is slightly lower than the high-limit value of 0.0012 based on Fig. 17–9.

According to equation 17–16, the value of B may be increased by reducing d and D. Taking for both d and D the next-smaller values, or $d = 1\frac{3}{4}$ in. and $D = 96$ in., changes the weight of the rope to $4.90 \times 750/2{,}000 = 1.84$ tons and gives the total load as

$$Q_c = (8.4 + 1.84) \times 2{,}000 = 20{,}480 \text{ lb}$$

Therefore,

$$B = \frac{2 \times 20{,}480}{203{,}000 \times 1.75 \times 96} = 0.001196$$

This is very close to the limit value of 0.0012.

On the other hand, the working factors obtained by checking by the conventional method for bending and acceleration loads are

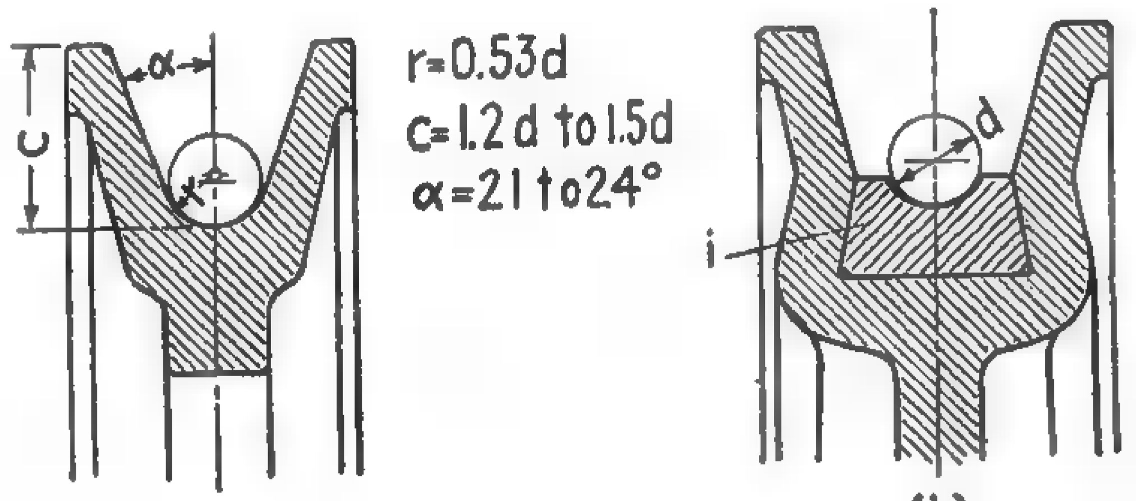
$$n_o = \frac{112}{27.7} = 4.03$$

and

$$n_o = \frac{112}{17.4} = 6.43$$

These factors are very little less than those obtained in example 17–1 with the heavier rope and bigger drum diameter.

17–6. Rope sheaves and drums. Table 17–1 gives the minimum diameters of sheaves and drums recommended by rope manufacturers. As a general rule the sheave diameter D should not be much less than $1{,}000\, d_w$, where d_w is the diameter of the heaviest wire in the rope. However, D is often made equal to $500d_w$, or even to $300d_w$, naturally with a material decrease of the life of the rope. Drum diameters can be made about 10 per cent smaller than sheave diameters with the same length of rope life.



(a) (b)

FIG. 17–10. Rope sheave rims.

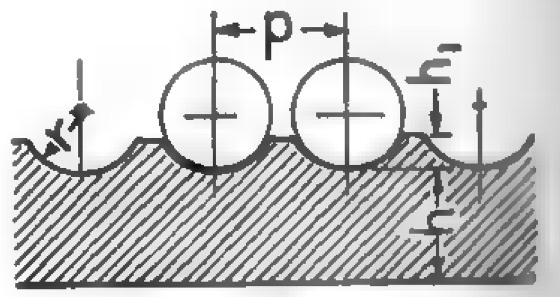


FIG. 17–11. Rope drum grooving.

Sheaves for rope are made similar to those for chains (see Fig. 17–5), the main difference being in the rim groove, as shown in Fig. 17–10. For light and medium service the sheaves are made of cast iron, but for heavy crane service they are often made of steel castings. To prevent wear of the individual wires, the grooves are finished smooth. The life of the rope can be increased materially by use of an insert i, Fig. 17–10b, at the bottom. This may be made of leather, hard rubber, wood, or an artificial Bakelite-like material with a fibrous insert such as used for noiseless gears. The radius r

of the bottom of the groove should be made slightly larger than the radius of the rope so as to prevent wedging of the rope in the groove. A good relation is

$$r = 0.53d \qquad (17\text{–}17)$$

A still-larger radius does not give sufficient bearing area and decreases the life of the rope.

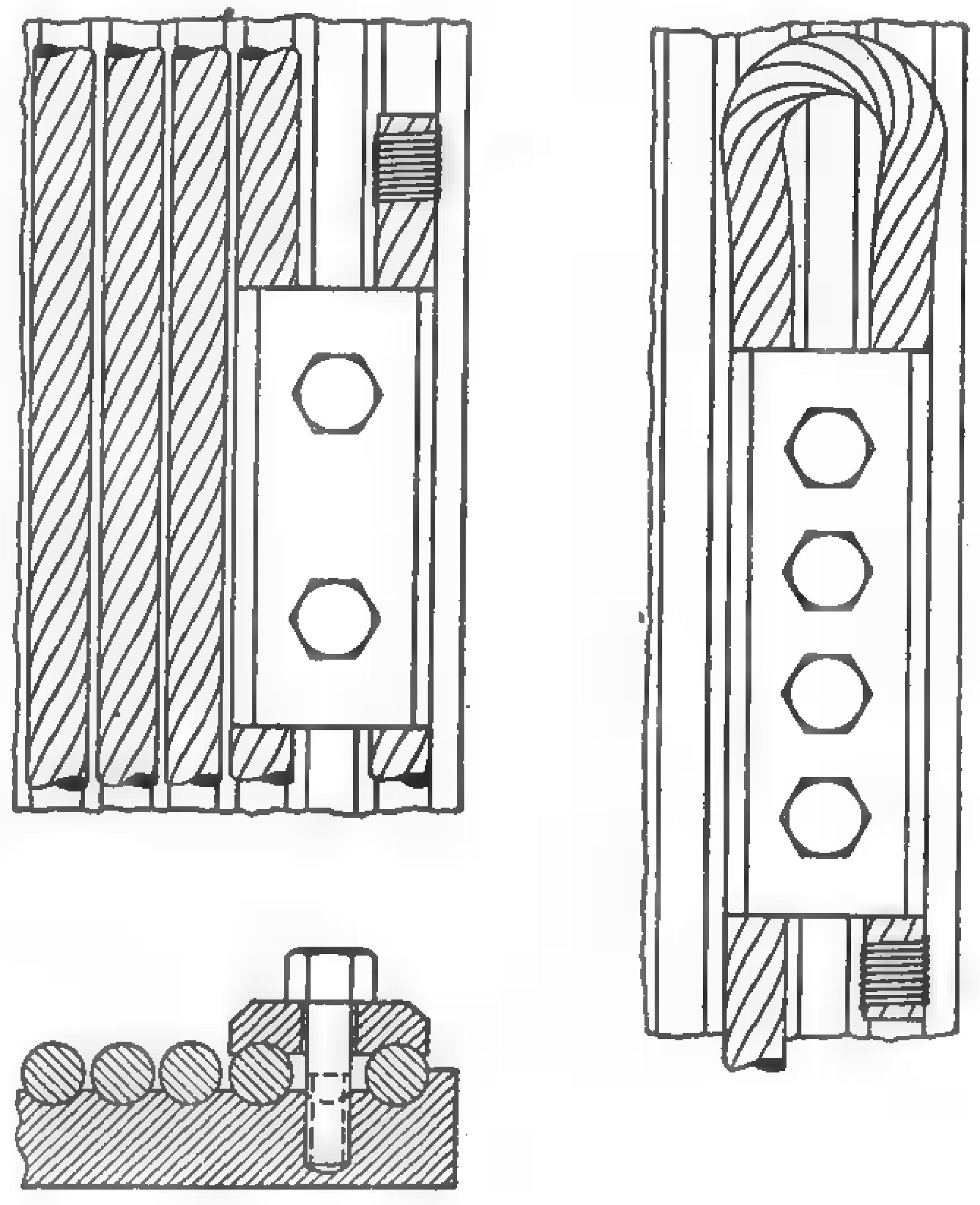
Small drums in hand hoists are made plain. A hoist operated by a motor or an engine has a drum with helical grooves, as shown in Fig. 17–11. The pitch p of the grooves must be made slightly larger than the rope diameter, to avoid friction and wear between the coils. A satisfactory relation is

$$p = 1.15d \qquad (17\text{–}18)$$

The radius of curvature of a groove may be made the same as that for a sheave, or $r = 0.53d$. The height h_1 of the groove ribs may be found by the relation

$$h_1 = 0.25d \qquad (17\text{–}19)$$

The drum thickness h is determined by the procedure outlined for a chain drum. When the bending stress s_1 is computed, the total weight of the rope is added to the useful load, although actually one-half the rope weight is evenly distributed over one side of the drum. A practical rule is to make h equal to d and then to check h for strength.



(a) (b)

FIG. 17–12. Rope anchors.

Rope anchors. The rope may be anchored to the drum with a steel clamp (shown in Fig. 17–12a) that catches a whole turn of the rope. If only the end of the rope is anchored, as in Fig. 17–12b, a longer clamp with four tap bolts must be used in order to take the full load applied to the rope when necessary.

CHAPTER 18

Brakes

18–1. General considerations. The function of a brake is to regulate the speed of a mechanism by transforming, through friction, the energy of a moving body into heat, and dissipating it. Only those brakes will be discussed here which are applied to rotating sheaves, drums, or wheels.

Energy equations. In general the energy absorbed by a brake is equal to the sum of the energy given up by the live load and the energy of all moving parts that are being retarded. To deduce an expression for the tangential force F_t which must be applied to the brake sheave in order to decrease the velocity of a load Q, the case of a hoisting drum lowering a load will be considered. The following additional designations will be used:

v_1, v_2 = speed of the live load before and after the brake is applied, respectively, in feet per second;

ω_1, ω_2 = angular velocity of the rotating parts, in radians per second;

n_1, n_2 = speed of the brake sheave, in revolutions per minute;

t = duration of the brake application, in seconds;

D = diameter of the brake sheave, in inches.

The change of the speed of the live load from v_1 to v_2 represents a decrease of kinetic energy by the amount

$$E_k=\frac{Q(v_1^2-v_2^2)}{2g} \tag{18-1}$$

This must be absorbed by the brake. During the same time t the brake must absorb the change of the potential energy, which is equal to the load times the mean velocity $\frac{1}{2}(v_1+v_2)$ times t. Thus,

$$E_p=\tfrac{1}{2}Q(v_1+v_2)t \tag{18-2}$$

Finally, the brake must also absorb the change of the kinetic energy of all rotating parts, such as the hoist drum and various gears and sheaves. This is

$$E_r=\Sigma\frac{Wk_o^2(\omega_1^2-\omega_2^2)}{2g} \tag{18-3}$$

where W and k_o designate the weight and the radius of gyration, respectively, of each of these parts.

The work to be done by the tangential force F_t at the brake sheave surface, in t sec, is

$$W_k=\frac{F_t\pi D(n_1+n_2)t}{12\times 2\times 60} \tag{18-4}$$

Equating this work to the sum $(E_k+E_p+E_r)$, and solving for F_t, gives

$$F_t=\frac{458.5(E_k+E_p+E_r)}{D(n_1+n_2)t} \tag{18-5}$$

The magnitude of F_t depends on the final velocities v_2 and n_2 and on the braking time t. It attains a maximum value if $v_2=0$ and $n_2=0$, when the load is stopped.

The torque which the brake must absorb is

$$T=\tfrac{1}{2}F_tD \tag{18-6}$$

Heat dissipation. The energy E absorbed by the brake and transformed into heat must be dissipated to the surrounding air in order to prevent an excessive temperature rise of the brake. The temperature rise depends on the amount of energy which the brake is required to absorb per unit of time and on the weight of the heated parts, chiefly of the sheave rim. The highest permissible temperature t_2 depends on the material of the friction surfaces. For leather, fiber, and wood facing, t_2 should not exceed 150 to 160 F to prevent charring. For asbestos and metal surfaces that are slightly lubricated, t_2 should not exceed 200 to 220 F, to prevent burning of the oil film. In automobile brakes with asbestos block lining, t_2 goes up to 400 F, and even to 500 F.

Instead of computing the temperature rise, it is more practical to establish the relation between the energy to be absorbed and the factors influencing its absorption and dissipation. The chief factors are the size and character of the friction surface and of the surface dissipating the heat. The energy absorbed by a brake per second can be computed by the equation

$$E=fpA_fv \tag{18-7}$$

where f is the friction coefficient;

p is the specific pressure, in pounds per square inch;

A_f is the contact area of the friction surfaces, in square inches;

v is the relative velocity of the friction surfaces, in feet per second.

This energy must not be greater than the capacity of the brake to dissipate the heat of friction. This capacity can be considered as the product of the area A_d that dissipates the heat and a factor k which is a function of the service conditions. Thus

$$fpA_fv\leq kA_d \tag{18-8}$$

Values of k taken from actual brake performance are:[1]

20 for continuous operation, lowering brakes, wood on cast iron

40 for intermittent operation, stopping brakes, wood on cast iron

60 for good heat dissipation, metal on cast iron in an oil bath

[1] Hütte, *op. cit.*, p. 159.

40 for continuous operation, woven asbestos on steel
60 for intermittent operation, woven asbestos on steel

The magnitudes of f and the limiting values for p in equation 18–8 are given in Table 18–1.

TABLE 18–1

FRICTION COEFFICIENTS AND ALLOWABLE PRESSURES

Materials in Contact	Friction Coefficient f			Allowable Pressure (psi)
	Dry	Greasy	Lubricated	
Cast iron on cast iron	0.2 –0.15	0.10–0.06	0.10–0.05	150–250
Bronze on cast iron	....	0.10–0.05	0.10–0.05	80–120
Steel on cast iron	0.30–0.20	0.12–0.07	0.10–0.06	120–200
Wood on cast iron	0.25–0.20	0.12–0.08	....	60–90
Fiber on metal	....	0.20–0.10	....	10–30
Cork on metal	0.35	0.30–0.25	0.25–0.22	8–15
Leather on metal	0.5–0.3	0.20–0.15	0.15–0.12	10–30
Wire asbestos on metal	0.5–0.35	0.30–0.25	0.25–0.20	40–80
Asbestos blocks on metal	0.48–0.40	0.30–0.25	....	40–160
Asbestos on metal, short action	....	....	0.25–0.20	200–300
Metal on cast iron, short action	....	....	0.10–0.05	200–300

The coefficient of friction depends on the nature of the friction surfaces, the specific pressure, and the rubbing velocity. The results of tests for friction between steel wheels and cast-iron blocks,[2] for rubbing velocities v from 0 to 88 fps, may be expressed by the equation

$$f=\frac{0.6}{\sqrt[3]{v}} \tag{18–9}$$

The influences of velocity and specific pressure on the coefficients of friction between steel drums and different brake linings in automobiles are shown in Fig. 18–1.[3]

Finally, it may be readily seen that the tangential force F_t in equation 18–6 may be computed by the relation

$$F_t=fpA_f \tag{18–10}$$

From equation 18–8,

$$A_d \geq \frac{F_t v}{k} \tag{18–11}$$

Rating of brakes. The natural and frequently used way of rating a brake is by the torque T (in pound-feet) which it can absorb. Substituting the value for F_t from equation 18–10 in equation 18–6 gives

$$T=\tfrac{1}{2}fpA_fD \tag{18–12}$$

[2] Lionel S. Marks, ed., *Mechanical Engineers' Handbook*, 5th ed. (New York: McGraw-Hill Book Company, Inc., 1951), p. 221.

[3] A. Vallance and V. L. Doughtie, *Design of Machine Members*, 2d. ed. (New York: McGraw-Hill Book Company, Inc., 1943), p. 216.

Another way of rating, also often used, is in terms of horsepower. The horsepower P of a brake is usually considered to be equal to the horsepower of the motor used for hoisting the load. Actually the load may be lowered at a higher rate of speed than it is raised. This increases the energy or horsepower which the brake has to absorb, but the safety margin in the rating of the brake must anticipate such an overload.

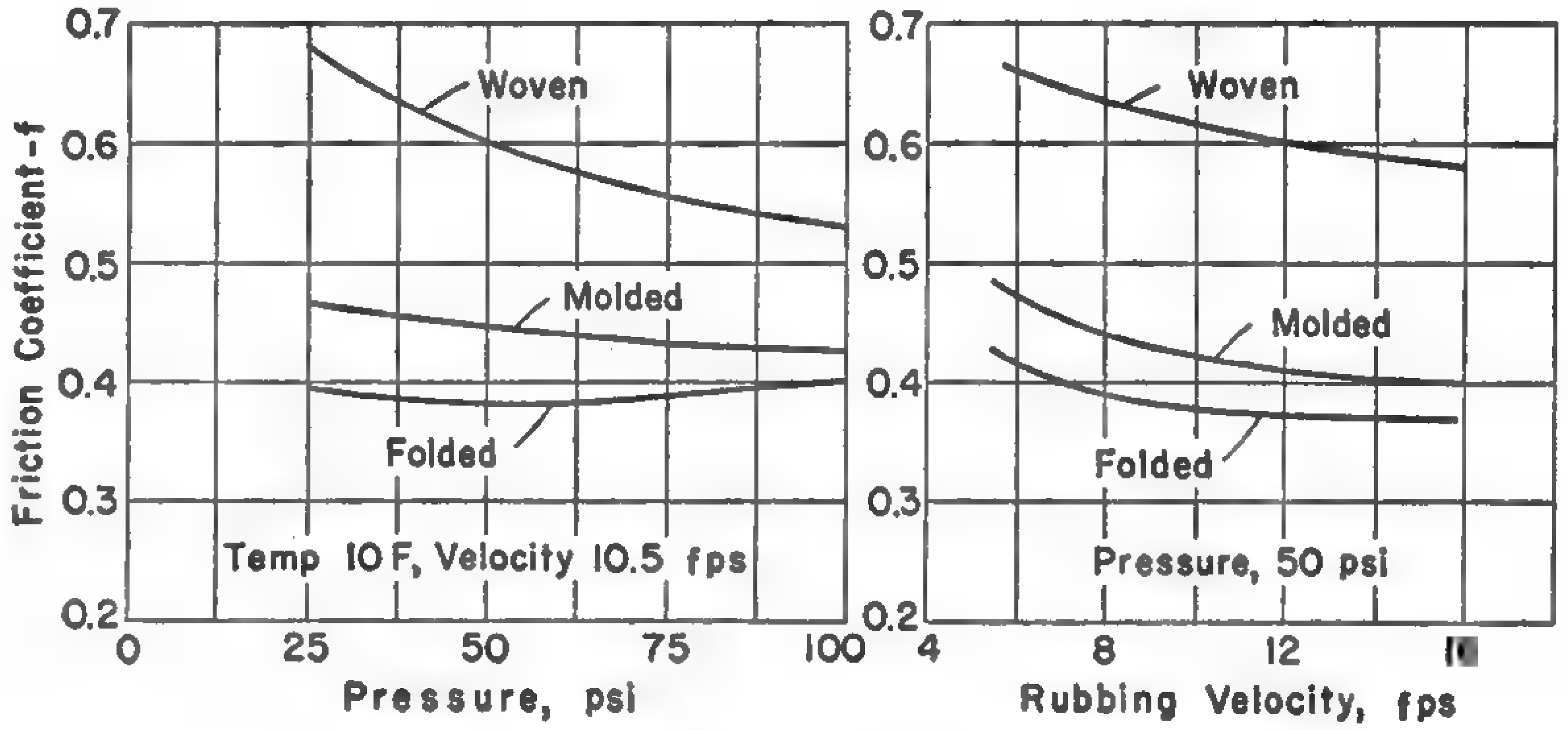


FIG. 18–1. Friction coefficients of brake linings.

Classification. Brakes may be divided into two groups, according to the direction of the acting force: (a) *radial brakes* and (b) *axial brakes.*

Radial brakes, in turn, may be subdivided into *external brakes* and *internal brakes.* Another basis of classification may be the shape of the friction detail; thus there may be distinguished *block brakes* and *band brakes.*

Axial brakes may be subdivided into *cone brakes* and *disk brakes.*

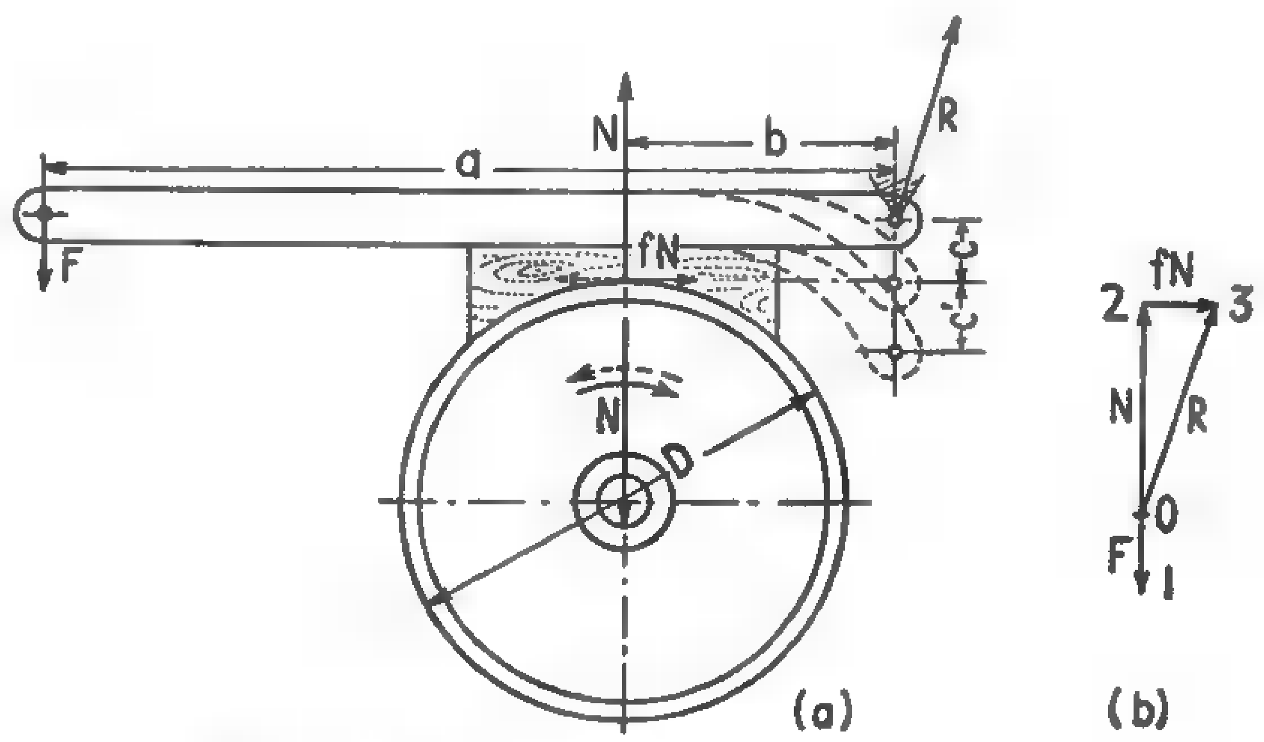


FIG. 18–2. Single-lever block brake.

18–2. Block brakes. When the brake is applied by operating a single lever, the operating lever with the friction block can be considered as a free body in equilibrium under the action of the applied force F, Fig. 18–2a, the normal reaction N between the sheave and block, the friction force fN between them, and the pin reaction R, the last being unknown in magnitude

and direction. For a body in equilibrium the moments with respect to any point must balance. Therefore, if moments are taken with respect to the axis of the fulcrum pin, and clockwise rotation is considered,

$$Fa+fNc=Nb \tag{18–13}$$

For counterclockwise rotation,

$$Fa=Nb+fNc \tag{18–14}$$

From these two equations the force needed to operate the brake, in general, is

$$F=\frac{(Nb\pm fNc)}{a} \tag{18–15}$$

But

$$fN=F_t \tag{18–16}$$

where F_t is given by equation 18–5. Hence equation 18–15 may be written as follows:

$$F=F_t\frac{b\pm fc}{fa} \tag{18–17}$$

The magnitude of F depends on the direction of rotation. It is smaller for clockwise rotation. In fact, it may be zero if $b=fc$; and it may be negative if $b<fc$. The magnitude of the friction coefficient f is rather uncertain and varies with the condition of the surfaces and with the velocity and specific pressure. Therefore, to avoid grabbing of the brake, b must be kept sufficiently larger than fc. On the other hand, a negative value of F means that the sheave can turn only if the lever is slightly lifted. This gives an automatic brake used for lowering loads. To be certain that the automatic brake will not begin to slip as the result of a change of f, the lever ratio b/c must be smaller than the smallest value of f for the existing conditions.

If $c=0$ in Fig. 18–2a, F does not depend on the direction of rotation. This is an advantage if the brake is operated in both directions. If, on the other hand, the fulcrum point is lowered by an additional amount c', the influence of the direction of rotation is reversed.

The magnitude of the force F determines the dimensions of the operating lever. To determine the size of the fulcrum pin and its bearings, the magnitude of the force R coming upon it must be known. This force is easily found graphically, as shown in Fig. 18–2b. It is the geometrical resultant 03 of the forces F, N, and fN drawn to scale, with their respective directions of action.

Although it is simple and reliable, the single-lever block brake is not used much, because the normal force N exerts a heavy pressure on the shaft bearings and produces bending of the shaft. This is particularly objectionable in large brakes.

Double-block brakes, with two brake blocks located diametrically opposite each other, are used to overcome the drawbacks of single-lever brakes. A

double-block brake used on cranes is shown in Fig. 18–3. The brake is set by a spring s which pulls the upper ends of the brake levers together. When a force F is applied to the bell crank b, which has its middle fulcrum i on the end of the brake lever, the spring s is compressed and the brake is released. This type of brake is often used on electric cranes, and the effort F is produced by an electromagnet or solenoid. When the current is off, there is no pull F on the bell crank, and the brake is set automatically, thus preventing the load from moving down.

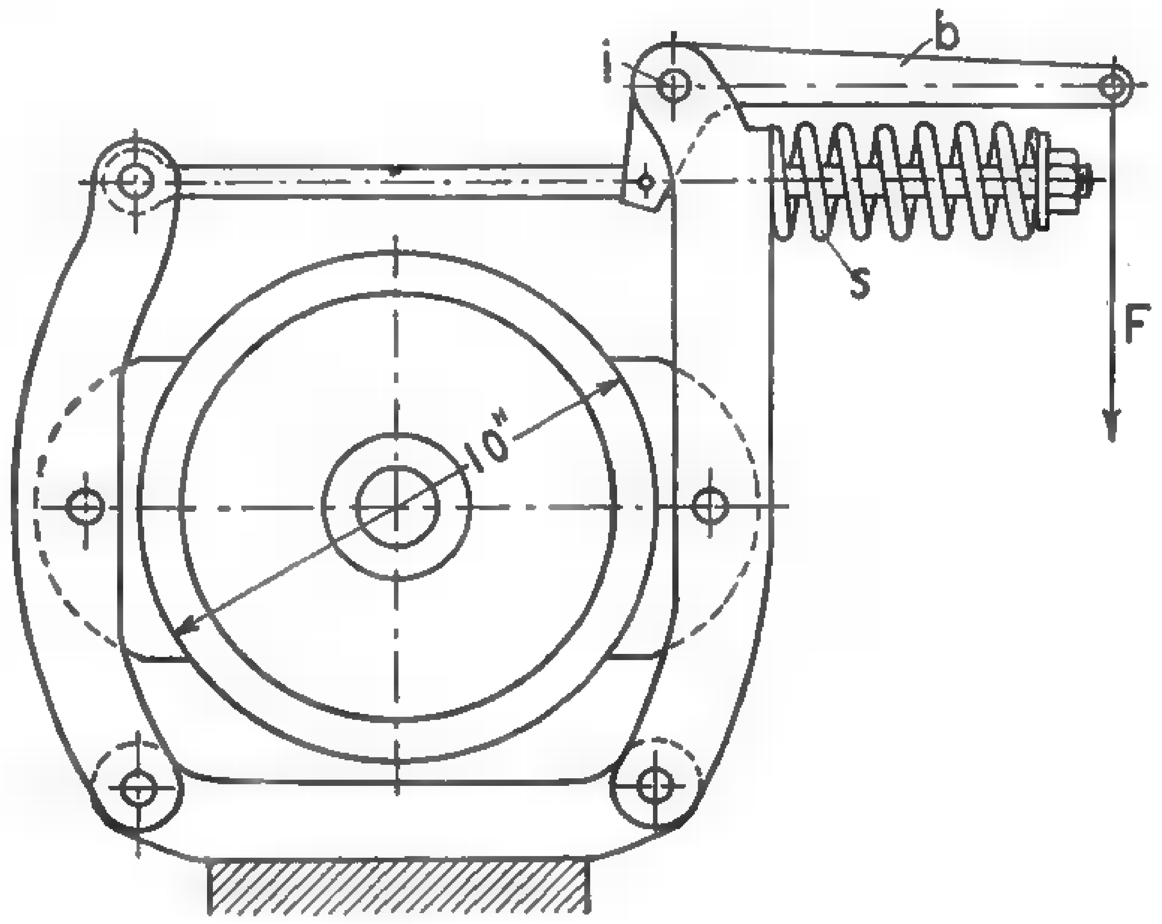


FIG. 18–3. Double-block crane brake.

Design procedure. It is first necessary to select a sheave diameter D in accordance with the given load, the given rotative speed, and other data. The second step is to determine the tangential effort F_t by equation 18–5 and to compute the torque capacity T by equation 18–6. The next step is to decide what materials to use on the friction surfaces, to select the corresponding friction coefficient from Table 18–1, and to find the normal force N from equation 18–16.

After this the necessary area A_{sf} of the brake shoes is found from the relation

$$A_{sf}=\frac{N}{p} \tag{18–18}$$

where A_{sf} is the projected area normal to the direction of N, in square inches. In regard to p, there is another limiting factor—the capacity of the brake to dissipate the generated heat of friction. According to experience, for wooden or molded asbestos blocks, if v is the rubbing velocity, in feet per second, the following limits should be assumed: $pv\leqq 500$ for continuous operations, as in lowering a load, and $pv\leqq 900$ for intermittent operations.

EXAMPLE 18–1. Determine (a) the capacity and (b) the main dimensions of a double-block brake for the following conditions: The brake sheave is mounted on the drum shaft; the hoist with its load weighs 6,000 lb and moves downward with a velocity of 240 fpm; the pitch diameter of the hoist drum is $D=48$ in.; the hoist must be stopped in a distance of 10 ft; the kinetic energy of the drum may be neglected.

a) The shaft speed is

$$n=\frac{240\times 12}{\pi\times 48}=19.11 \text{ rpm}$$

Select a brake-sheave diameter D of 50 in. The time t for stopping in 10 ft, with $v_1=240/60=4$ fps and $v_2=0$, is

$$t=\frac{10}{0.5\times(4+0)}=5 \text{ sec}$$

The tangential effort, by equation 18–5, is, after substitution and simplification,

$$F_t=\frac{(6{,}000\times 4+6{,}000\times 5\times 32.2)\times 48}{32.2\times 50\times 5\times 1}=5{,}900 \text{ lb}$$

The torque capacity, by equation 18–6, is

$$T=\frac{1}{2}\times 5{,}900\times\frac{50}{12}=12{,}300 \text{ lb-ft}$$

and the power P is, therefore,

$$P=\frac{12{,}300\times 19.11}{5{,}252}=44.7 \text{ hp}$$

b) A suitable brake-shoe material is wood. A cast-iron brake sheave will be used, and from Table 18–1 a safe value for the coefficient of friction is $f=0.20$.

The normal force, by equation 18–16, is

$$N=\frac{5{,}900}{0.2}=29{,}500 \text{ lb}$$

For continuous operation, with $v=4$ fps and $pv \leqq 500$,

$$p \leqq \frac{500}{4} \leqq 125 \text{ psi}$$

According to Table 18–1 the maximum value of p can be 90 psi; use 80 psi.

The brake-shoe area, by equation 18–18, is

$$A_{sf} \geqq \frac{29{,}500}{80}=368.5 \text{ sq in.}$$

If the projected length L of each shoe is taken $=\frac{1}{2}D=25$ in., and there are two shoes, the width B of the shoes should be

$$B=\frac{368.5}{2\times 25}=7.37, \text{ or } 7\tfrac{1}{2} \text{ in.}$$

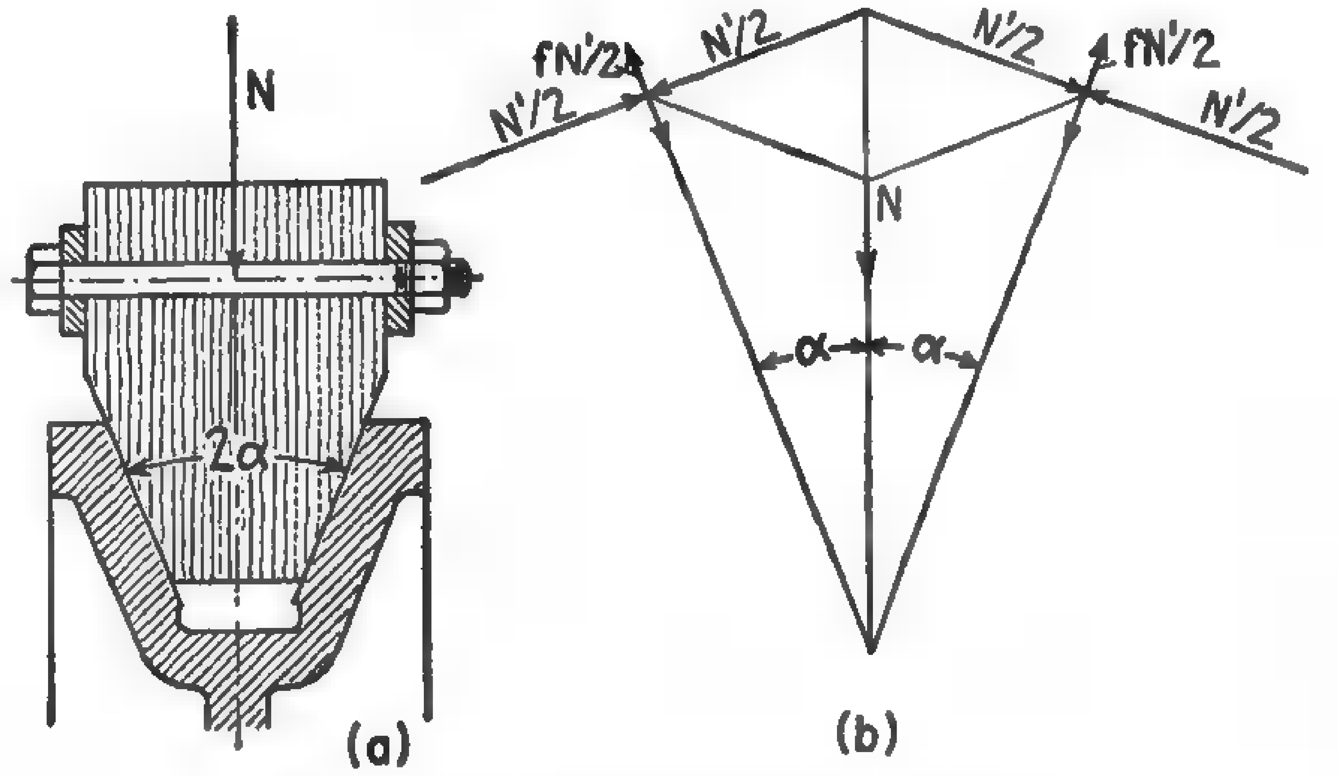


FIG. 18–4. Grooved brake sheave.

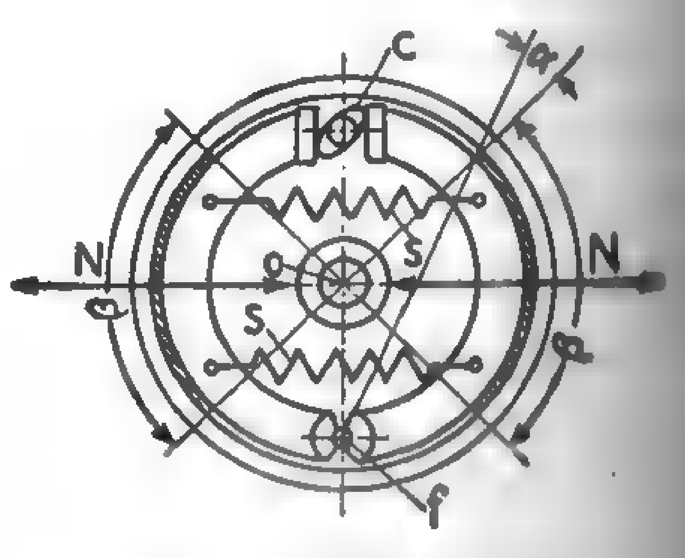


FIG. 18–5. Internal block brake.

Grooved sheaves. Grooved sheaves, as in Fig. 18–4a, are used to increase the force F_t without increasing the normal force N. Summing up the forces along the vertical axis in Fig. 18–4b results in

$$N=N'(\sin\alpha+f\cos\alpha) \tag{18–19}$$

For the case of a brake operating in both directions, apply equation 18–15, in which $c=0$, the value of N is taken from equation 18–19, and N' is then eliminated by using the relation $fN'=F_t$. The result is

$$F=\frac{F_t b(\sin\alpha+f\cos\alpha)}{fa} \tag{18–20}$$

The expression $f/(\sin\alpha+f\cos\alpha)$ is called the *apparent coefficient of friction.*

The groove half-angle α should not be made smaller than 20°, to prevent grabbing and locking, if the friction coefficient should increase. On the other hand, it is seldom made greater than 30°, as then the advantage of the grooves is decreased.

Internal block brakes. Internal block brakes are very compact and are particularly suitable if rotation occurs in both directions. In Fig. 18–5 is shown a widely used automobile brake arrangement with metal-asbestos lining. The brake is applied by turning the elliptical cam c clockwise. When the cam is turned back, the springs s pull the blocks away from the sheave surface, releasing the brake. In order to prevent grabbing and locking of the brake, the smallest angle α, formed by the lines going through the end of the lining and the centers o and f, must be greater than the angle of friction ϕ. For $f=0.5$, the maximum center angle of the lining is $\beta=72°$; and for $f=0.4$, the angle β may be 90°.

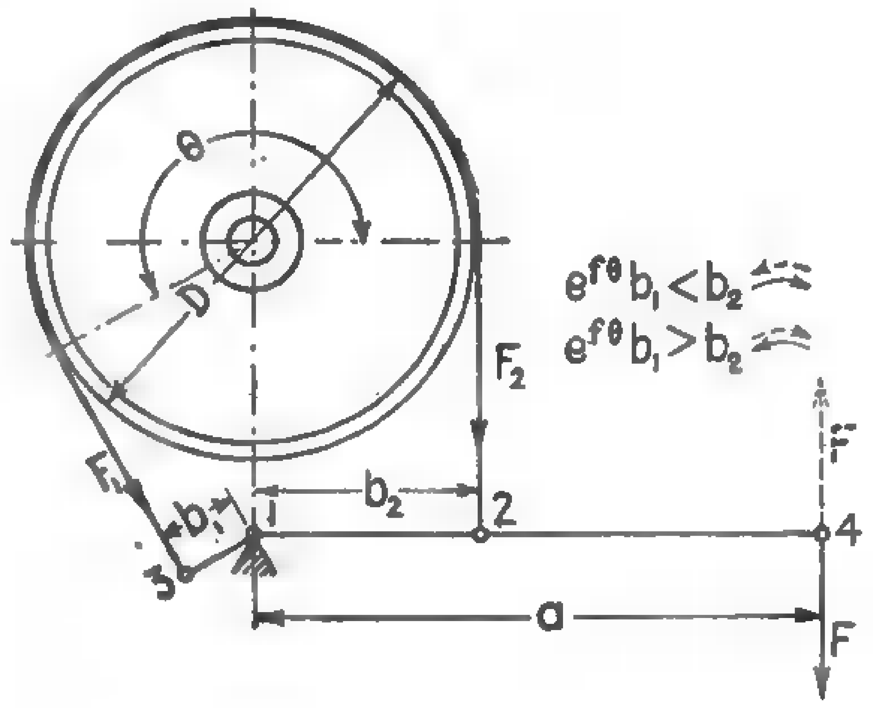


FIG. 18–6. Differential brake.

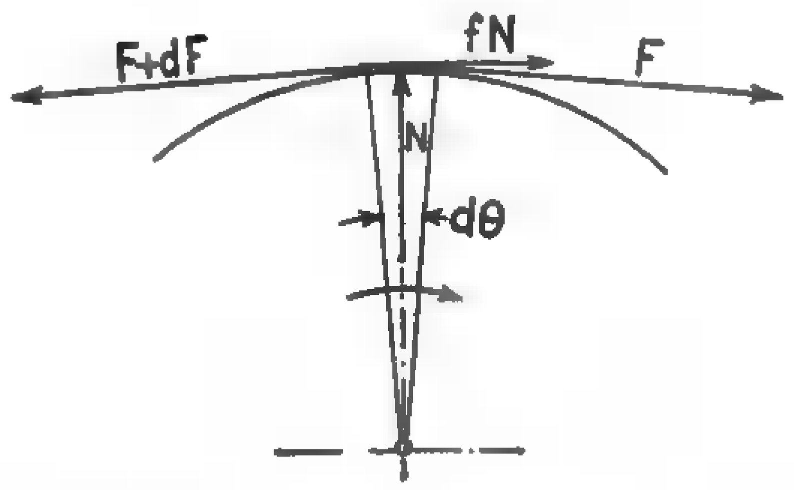


FIG. 18–7. Band tension.

18–3. Band brakes. A general arrangement of band brakes is shown in Fig. 18–6. Before a relation is deduced between the tangential force F_t and the pull F at the lever end, it should be noted that owing to friction between the sheave and the band the tensions F_1 and F_2 in the two band ends are different. Such brakes are called *differential brakes.*

Differential brakes. The desired relation for differential brakes may be found by using Fig. 18–7. An infinitesimal length of the band is subtended by the angle $d\theta$. The tension at one end is F, and at the other end is $(F+dF)$; each of these tensions makes an angle $(\frac{1}{2}\pi-\frac{1}{2}d\theta)$ with the vertical center

line. The pressure between the band and sheave rim is designated by N, and with a friction coefficient f the friction force is fN.

This piece of band is held in equilibrium by the four forces F, $(F+dF)$, N, and fN. The summation of the horizontal and vertical components, respectively, gives the following equations:

$$-dF \cos\left(\tfrac{1}{2}d\theta\right) + fN = 0 \tag{18–21}$$

and

$$-(2F+dF) \sin\left(\tfrac{1}{2}d\theta\right) + N = 0 \tag{18–22}$$

Eliminating N, and substituting $\frac{1}{2}d\theta$ for $\sin(\frac{1}{2}d\theta)$ and 1 for $\cos(\frac{1}{2}d\theta)$, results in

$$fF\,d\theta - dF = 0 \tag{18–23}$$

Separating the variables gives

$$\int_{F_2}^{F_1} \frac{dF}{F} = f \int_0^{\theta} d\theta \tag{18–24}$$

Integrating gives the ratio of the tight tension to the loose tension. Thus,

$$\frac{F_1}{F_2} = e^{f\theta} \tag{18–25}$$

The net tension is evidently equal to the tangential braking force, or

$$F_1 - F_2 = F_t \tag{18–26}$$

Eliminating F_2 from equations 18–25 and 18–26 gives

$$F_1 = \frac{F_t e^{f\theta}}{e^{f\theta} - 1} \tag{18–27}$$

From equations 18–25 and 18–27,

$$F_2 = \frac{F_t}{e^{f\theta} - 1} \tag{18–28}$$

This equation shows that for a required F_t the magnitudes of F_1 and F_2 decrease with an increase of the friction coeficient f, and particularly with an increase of the angle of contact θ. Thus, for an average value of $f = 0.3$, the following results are obtained: For $\theta = \pi$, $F_1/F_t = 1.64$ and $F_2/F_t = 0.64$; for $\theta = 1.5\pi$, $F_1/F_t = 1.32$ and $F_2/F_t = 0.32$; and for $\theta = 3.5\pi$, $F_1/F_t = 1.04$ and $F_2/F_t = 0.04$.

Considering the operating lever as a free body and taking moments about the fulcrum l, and assuming clockwise rotation in Fig. 18–6, there results

$$Fa + F_1 b_1 = F_2 b_2 \tag{18–29}$$

Substituting the values for F_1 and F_2 from equations 18–27 and 18–28, and solving for F, gives

$$F = \frac{F_t(b_2 - e^{f\theta} b_1)}{(e^{f\theta} - 1)a} \tag{18–30}$$

The conditions represented in Fig. 18–6 require that $b_2 > e^{f\theta} b_1$, or

$$\frac{b_2}{b_1} > e^{f\theta} \tag{18–31}$$

If $b_2/b_1 = e^{f\theta}$, then F is zero and the brake becomes self-locking. This is undesirable and even dangerous because of the possible fluctuation of the friction coefficient f.

If $b_2/b_1 < e^{f\theta}$, the pull F becomes negative, the brake is applied automatically, and a pull must be applied in the opposite direction, as shown by dotted lines in Fig. 18–6, in order to allow the sheave to turn and thus to lower the load.

If the direction of rotation is reversed, or is counterclockwise, the greater tension F_1 will act at the right end of the band, and the smaller tension F_2 will act at the left end. A similar analysis gives

$$F = \frac{F_t(e^{f\theta} b_2 - b_1)}{(e^{f\theta} - 1)a} \tag{18–32}$$

If b_2 does not differ much from b_1, the influence of $e^{f\theta}$, according to equations 18–30 and 18–32, is small. The main factor determining the magnitude of F for a given F_t is the average ratio of the lever arms, or the ratio $(b_1 + b_2)/2a$.

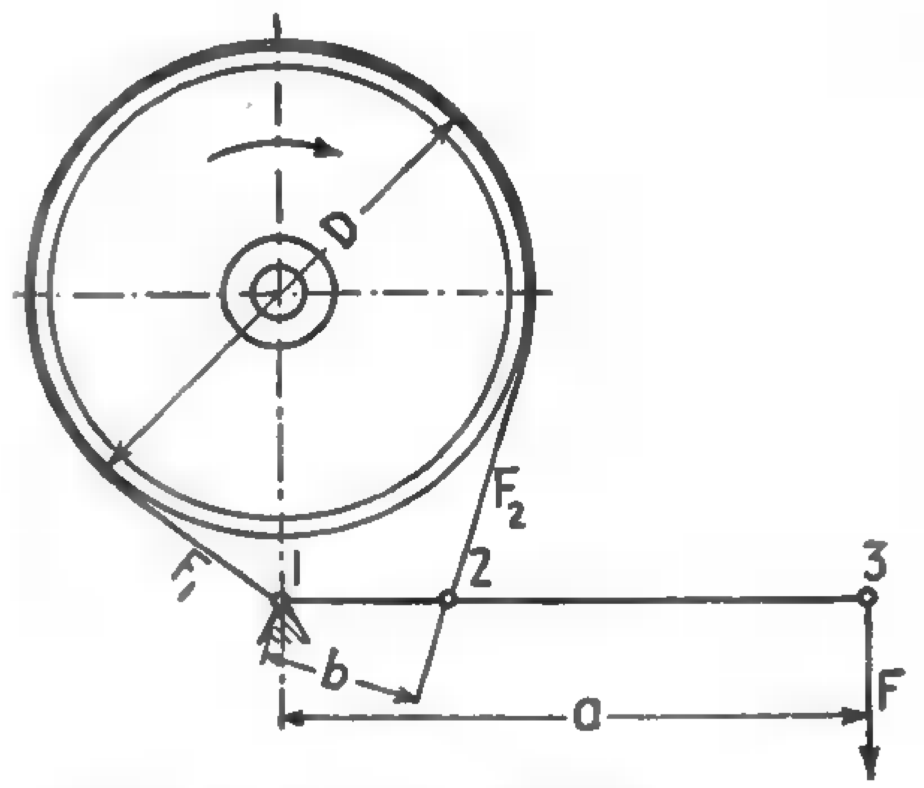


FIG. 18–8. Simple band brake.

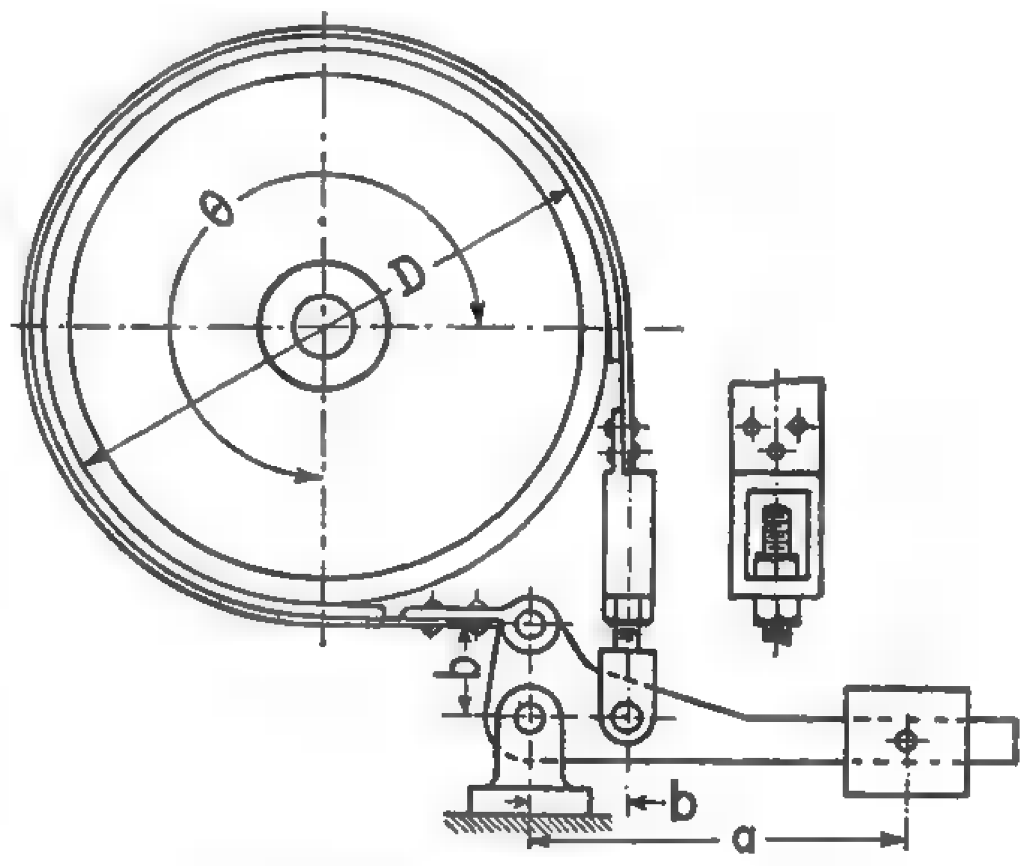


FIG. 18–9. Brake for rotation in both directions.

Simple band brake. A simple band brake is a special case of the differential brake, in which one of the band ends is fastened to the fixed fulcrum. One of these arrangements is shown in Fig. 18–8. The expression for F is obtained from equation 18–30 or equation 18–32, the proper one depending on the direction of rotation of the sheave. In this case, $b_1 = 0$ and $b_2 = b$. Thus, for clockwise rotation, equation 18–32 gives

$$F = \frac{F_t b}{(e^{f\theta} - 1)a} \tag{18–33}$$

Equation 18–33 shows a certain advantage of a simple band brake over a differential brake. By making the angle of contact θ large, the necessary pull F can be made small without increasing unduly the lever arm a. With this consideration in view, brakes are built in which the band makes several complete turns around the sheave.

For counterclockwise rotation, equation 18–32 gives

$$F = \frac{F_t e^{f\theta} b}{(e^{f\theta}-1)a} \tag{18–34}$$

In order not to increase F unnecessarily, the rotation should be clockwise. This means that the band end with the higher tension must be fastened to the fixed fulcrum.

Band brake for rotation in both directions. For hoist brakes in which the rotation is reversed, as in cranes, elevators, and mine hoists, it is desirable to have the same pull F, regardless of the direction of rotation. A study of Fig. 18–6 shows that this can be accomplished if both the moment of the tension F_1 and the moment of the tension F_2 act in the same direction and in the opposite direction to the moment of the pull F. To obtain this result the overhanging lever end *3–1* must be turned to another position and the lever arms b_1 and b_2 must be equal. A brake with this arrangement is shown in Fig. 18–9. By applying the same method of analysis, or by using either equation 18–30 or equation 18–32, with proper substitutions, the required magnitude of the pull is determined to be

$$F = \frac{F_t b(e^{f\theta}+1)}{(e^{f\theta}-1)a} \tag{18–35}$$

With this arrangement the required pull for rotation in one direction is heavier than that required by other brakes, but a considerably lighter pull is sufficient when rotation occurs in the opposite direction.

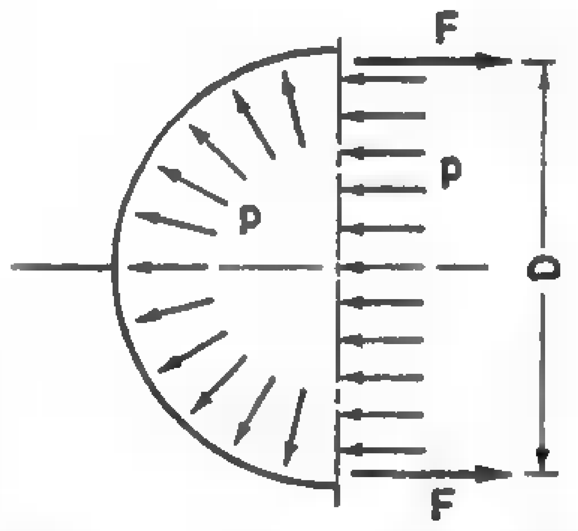


Fig. 18–10. Forces acting on a brake band.

Pressure on band. The magnitude of the pressure p between the band and the brake sheave may be found by considering Fig. 18–10. The sum of the horizontal components of the pressures is equal to the product pD. Thus, the sum of the forces applied to the band ends, or $2F$, is equal to pDw, where w is the band width. For a rotating sheave, $2F = F_1 + F_2$ and the average pressure is

$$p = \frac{F_1 + F_2}{Dw} \tag{18–36}$$

Because of stretching of the band, the pressure around the sheave is not constant. It is highest at the tight end of the band, where it is

$$p_1 = \frac{2F_1}{Dw} \tag{18–37}$$

It decreases gradually toward the other end to the minimum value of

$$p_2 = \frac{2F_2}{Dw} \tag{18–38}$$

Design remarks. The best material for brake drums and sheaves is cast iron. Particularly suitable is hard-wearing martensitic cast iron having a nickel content of 5 to 6 per cent and a Brinell hardness of 400 to 450. The proper material for brake bands is mild steel.

A practical rule is to determine the band thickness h by the relation

$$h = 0.005D \tag{18–39}$$

After h has been selected, the width w is determined from considerations of strength by applying the equation

$$wh = \frac{F_1}{S_d} \tag{18–40}$$

In selecting S_d, a safety factor n of $2 \times 2 = 4$ should be used because of the possible sudden application of a brake band. The width w must be checked by equation 18–37.

The brake-band ends are fastened to the lugs either by welding or by riveting with $\frac{3}{8}$-in. rivets.

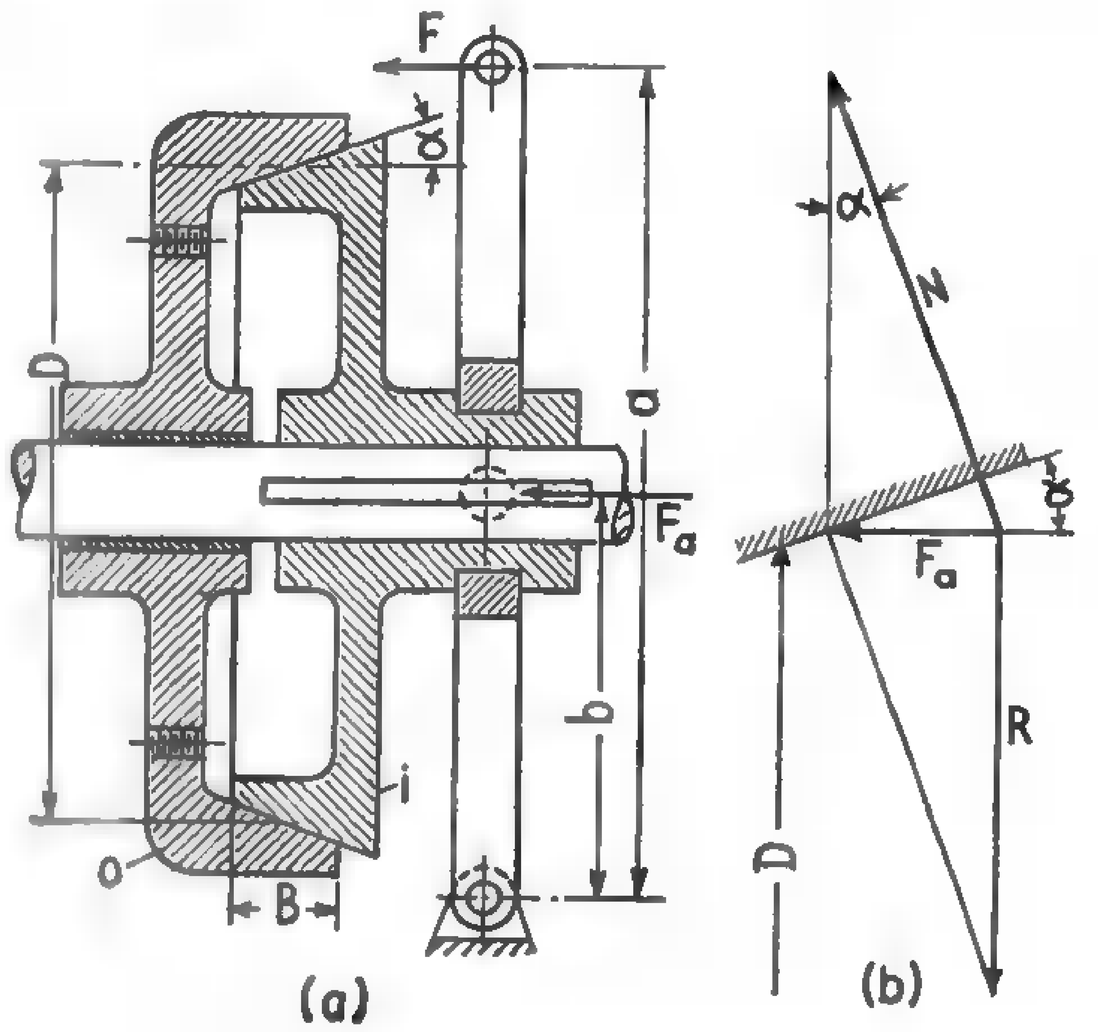


Fig. 18–11. Cone brake.

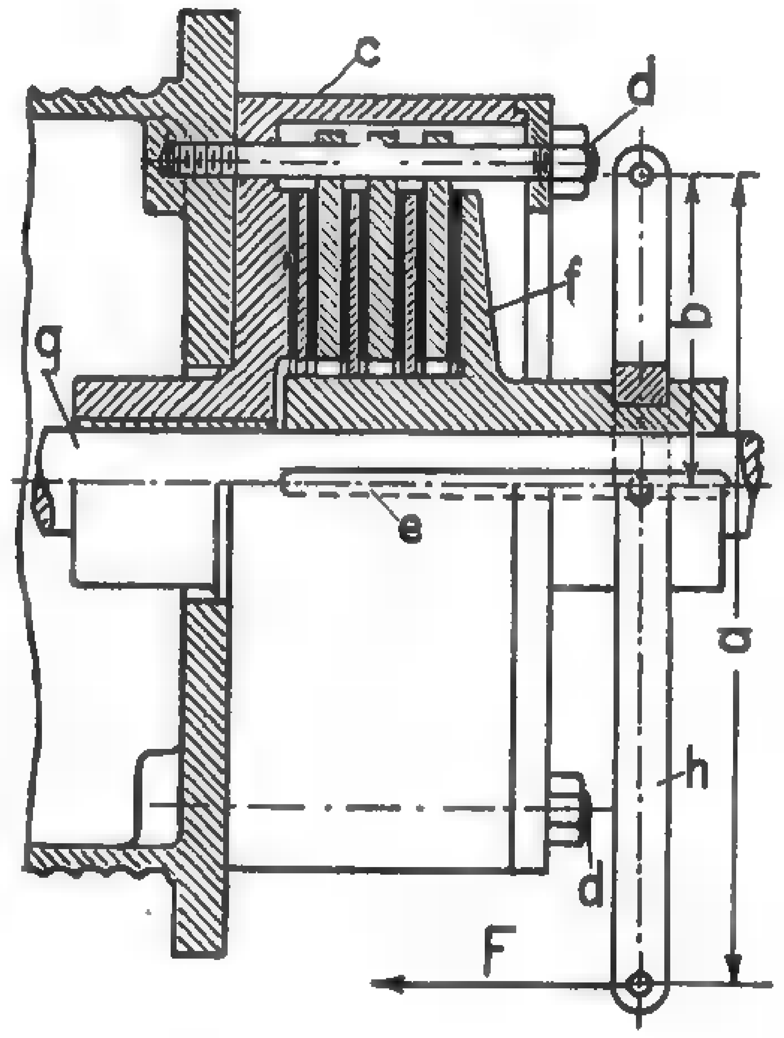


Fig. 18–12. Multidisk brake.

18–4. Cone brakes. A semidiagrammatic drawing of a cone brake is shown in Fig. 18–11a. The outer cone *o* may form a part of the hoist drum or be attached to it, while the inner cone *i* is splined to a shaft which can rotate in only one direction, being prevented from running in the opposite direction by a ratchet and pawl.

Force analysis. The magnitude of the force F at the end of the operating lever may be computed as follows: The axial force F_a applied at the cone

surface can be resolved, as shown in Fig. 18–11b, into a normal force N and a radial force R. The normal force is

$$N = \frac{F_a}{\sin \alpha} \tag{18–41}$$

The radial force is

$$R = \frac{F_a}{\tan \alpha} \tag{18–42}$$

In a conical surface the radial forces balance each other. The tangential force, or braking force, F_t, is equal to the normal force multiplied by the friction coefficient, or

$$F_t = fN = \frac{fF_a}{\sin \alpha} \tag{18–43}$$

The braking torque is then

$$T = \frac{fF_aD}{2 \sin \alpha} \tag{18–44}$$

where D is the mean diameter of the cone.

Owing to the leverage,

$$F_a = \frac{Fa}{b} \tag{18–45}$$

The relation between the operating force F and the braking force F_t may be obtained by combining equations 18–43 and 18–45.

$$F = \frac{F_tb \sin \alpha}{fa} \tag{18–46}$$

The area A of the contact surfaces can be determined, with the designations of Fig. 18–11, by the relation

$$A = \frac{\pi DB}{\cos \alpha} \tag{18–47}$$

The average pressure between the contact surfaces is

$$p = \frac{N}{A} = \frac{F_a}{\pi DB \tan \alpha} \tag{18–48}$$

Design remarks. The female cone is usually made of cast iron. The inner cone is also of cast iron, but it is often lined with wood or asbestos blocks in order to increase f. The angle α is made from 10 to 18 deg.

The axial width B, Fig. 18–11, is made from $0.12D$ to $0.22D$, and both D and B are so selected that the pressure p found by equation 18–48 does not exceed the value given in Table 18–1.

18–5. Disk brakes. A disk brake is a special form of a cone brake with the angle α equal to 90°. A disk brake usually has more than two surfaces in contact. A multidisk brake is shown diagrammatically in Fig. 18–12. The housing c is fastened to the hoist drum and runs freely with it on the shaft g. The flange f rotates with the shaft, but it can slide axially on the feather key e. The shaft is driven by a motor and can rotate in only one direction, being prevented from rotating in the other direction by a ratchet mechanism. Between the inner faces of the housing c and the flange f are assembled a number of friction disks. The larger disks are of cast iron and rotate with the housing c, having holes through which bolts d pass. The smaller disks are of steel, and each has in the center a square hole which fits over the square hub of the flange f. When the load is hoisted, this flange is locked to the housing and to the hoist drum by the pull F on the lever, and it acts as a clutch. When the load must be lowered, the motor is stopped. The shaft cannot run backward, and the operator, by decreasing the pull F, allows the drum to run backward.

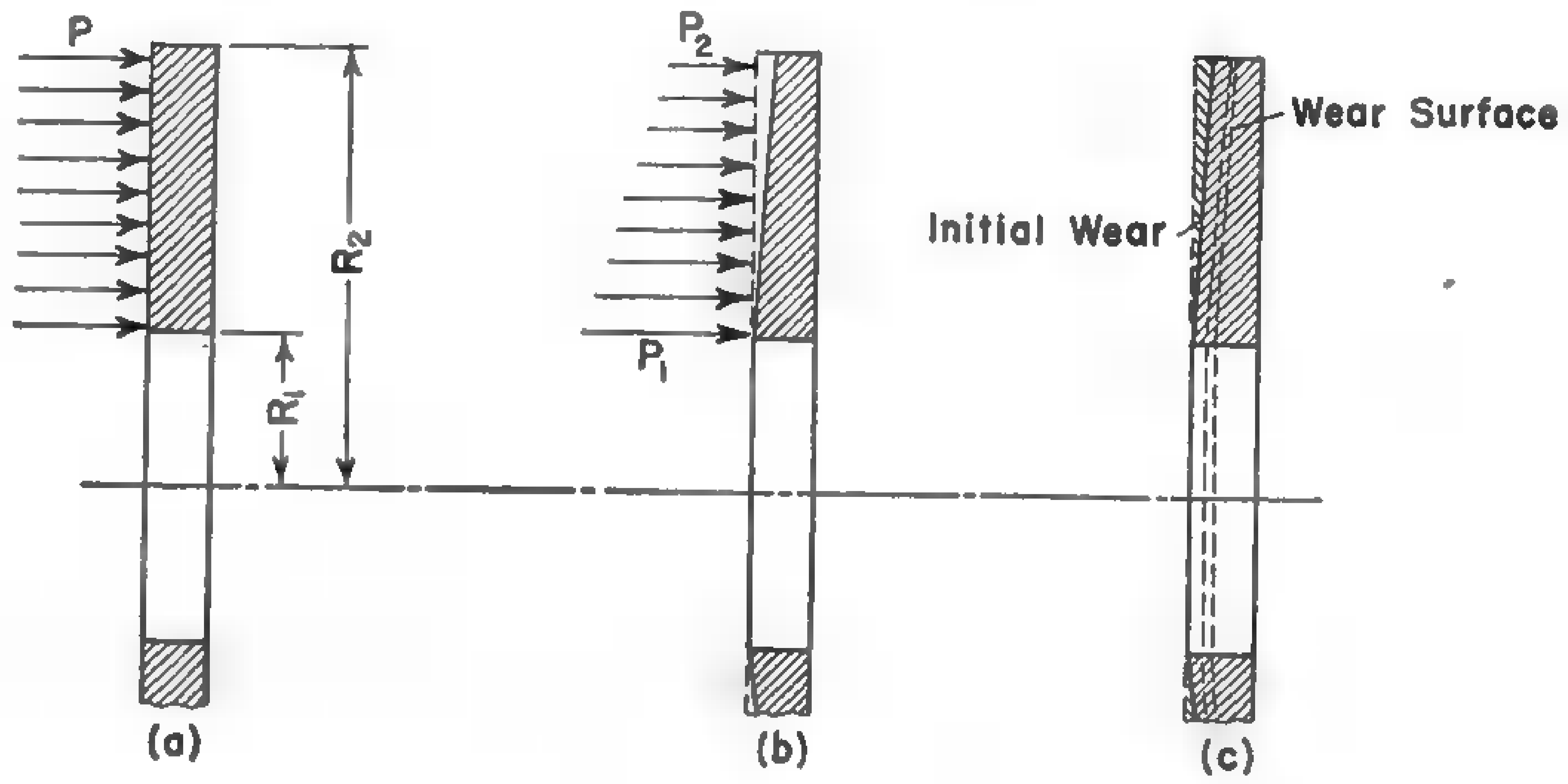


FIG. 18–13. Disk pressure and wear.

Pressure distribution. A reasonable assumption is that the pressure p between the disks is distributed uniformly, as indicated in Fig. 18–13a. This is true with new plates. However, after a certain length of time in operation, the plates become thinner toward the outside diameter, as shown exaggerated in Fig. 18–13b. Such a condition can be explained if it is assumed that the wear is proportional to the work of friction. This work at an element of area is proportional to the product of the pressure and the velocity of rubbing. Therefore, the initial wear of a plate increases from the inside radius R_1 toward the outside radius R_2. After the plate is worn in, further wear is uniform, as indicated in Fig. 18–13c. This indicates that the product pR is constant. Hence, in general,

$$pR = p_1R_1 = p_2R_2 \tag{18–49}$$

Force analysis. The torsional resistance of one pair of disks may be found by integrating the elemental torque which is equal to the product of the tangential force dF_t and the radius of friction R. In this case,

$$dF_t = fp\, dA \tag{18–50}$$

where, from equation 18–49,

$$p = \frac{p_1R_1}{R} \tag{18–51}$$

and the elemental ring area is

$$dA = 2\pi R\, dR \tag{18-52}$$

Thus the torque expression becomes

$$T = \int_{R_1}^{R_2} fp\, dA\, R = \int_{R_1}^{R_2} \frac{fp_1R_1}{R} \times 2\pi R\, dR \times R = 2\pi fp_1R_1 \int_{R_1}^{R_2} R\, dR = \pi fp_1R_1(R_2^2 - R_1^2)$$

If $\frac{1}{2}D_1$ and $\frac{1}{2}D_2$ are substituted for R_1 and R_2, the torque for i pairs of surfaces is

$$T = \frac{\pi ifp_1D_1(D_2^2 - D_1^2)}{8} \tag{18-53}$$

The axial force F_a transmitted to the disks from the flange f, Fig. 18–12, may be obtained from the relation

$$F_a = \int_{R_1}^{R_2} p\, dA \tag{18-54}$$

Substituting the values from equations 18–51 and 18–52 in equation 18–54 and integrating, there results

$$F_a = \tfrac{1}{2}\pi p_1D_1(D_2 - D_1) \tag{18-55}$$

Because of the leverage, the pull F (Fig. 18–12) on the operating lever is

$$F = \frac{F_ab}{a} \tag{18-56}$$

Design procedure. First, the outside diameter D_2 is selected in accordance with load and speed conditions, and the corresponding inside diameter D_1 is estimated. Next, the mean tangential effort F_t is determined by equation 18–5, and the torque requirement T is found by equation 18–6. The value of the mean diameter D may be taken as

$$D = 0.5(D_2 + D_1) \tag{18-57}$$

However, a more accurate value may be obtained through integration of the expression for elemental torque. This value is

$$D = \frac{2(D_2^3 - D_1^3)}{3(D_2^2 - D_1^2)} \tag{18-58}$$

The remaining steps in the design are to decide what friction materials will be used, to select the corresponding values of f and p_1, and to determine from equation 18–53 the number i of pairs of friction surfaces. If the number i becomes too small, say under 4, the diameter D_2 was selected too large; on the other hand, if the number i becomes too large, the diameter D_2 was selected too small. In either case, a new value of D_2 should be selected and the calculations repeated.

In order to obtain smooth operation and to have little wear, it is advisable to take for f and p_1 the lower limits for the respective materials that are given in Table 18–1.

The lever pull F also can be decreased by increasing the number of plates. However, more than seven pairs of surfaces are seldom used because of poor heat dissipation.

18–6. Comparison of brakes. Three factors can be used as a basis for comparison of brakes: (a) the ratio of the effort F to the tangential braking force F_t; (b) the motion of the operating lever necessary to apply a brake or to release it; and (c) the heat dissipation.

Of the great variety of types, only the six more-typical brake arrangements, listed in Table 18–2, will be discussed.

TABLE 18–2

COMPARISON OF HOIST BRAKES

BRAKE CHARACTERISTICS	BLOCK BRAKES		BAND BRAKES		AXIAL BRAKES	
	Double Block	V-Grooved Sheave	Simple	Both Directions of Rotation	Cone	Multidisk
Force ratio $\frac{F}{F_t}$	$\frac{b}{fa}$	$\frac{b \sin\alpha}{fa}$	$\frac{b}{a(e^{f\theta}-1)}$	$\frac{b(e^{f\theta}+1)}{a(e^{f\theta}-1)}$	$\frac{b \sin\alpha}{fa}$	$\frac{b}{nfa}$
Average numerical value.	0.667	0.282	0.0323	0.165	0.161	0.097
Relative value	20.6	8.7	1	5.1	5.0	3.0
Travel at lever end......	$\frac{ha}{b}$	$\frac{ha}{b \sin\alpha}$	$\frac{ha\theta}{2\pi b}$	$\frac{ha\theta}{4\pi b}$	$\frac{ha}{b \sin\alpha}$	$\frac{ih'a}{b}$
Average travel, in.	0.313	0.740	2.943	1.471	1.292	0.219
Maximum capacity, hp..	2,000	25	300	100	50	120

Force ratio. For the sake of convenience the expressions for the ratio F/F_t are shown in Table 18–2. In computing the numerical values for F/F_t the coefficient of friction f was assumed to be 0.3. In compliance with actual conditions, the leverage a/b was assumed as 5 : 1, with the exception of the band brakes, in which it was taken as 10 : 1, as actually made. For the multidisk brake, seven pairs of surfaces were assumed.

The line of relative values shows that the simple band brake requires the smallest effort, and the multidisk brake comes next. The block brake requires the largest effort. Since this brake has certain advantages for heavy loads, it is used quite extensively; but the ratio F/F_t is made more favorable by additional leverage, as indicated in Fig. 18–3.

Lever-end travel. In Table 18–2 the expressions for the travel of the lever end are given as functions of the normal distance h between the sheave and the stationary braking surface to prevent dragging. In computing the numerical values a minimum magnitude of $\frac{1}{16}$ in. was assumed for h. For

the disk brake the total spreading ih' was taken as $\frac{1}{16}$ in. Naturally, the small travel of the ordinary block brake will be increased in the same proportion as the force ratio is decreased if an additional lever system is used.

Heat dissipation. The heat is dissipated best with axial conical brakes, and block brakes with cast-iron shoes come next. Band brakes, especially those with asbestos lining, have a smaller radiating area and therefore, other conditions being equal, run hotter. Nevertheless, because of their powerful action they are used on hoists and are also used very extensively on automotive vehicles. Multidisk brakes are the poorest and therefore are not used for heavy loads.

CHAPTER 19

Screws for Power Transmission

19–1. Power threads. Screws are used to produce uniform, slow, and powerful motion, such as is required in presses, jacks, lathes, valves, and other machinery. The efficiency of V threads is very low, partly because of the large angle of the profile. By making this angle small or zero, threads with a higher efficiency are obtained. Those employed for power screws are the square, buttress, Acme, and round-groove threads.

Square threads. The square thread, Fig. 19–1, has the highest efficiency and therefore is extensively used in spite of its higher manufacturing cost. There is no national standard for square threads. However, Sellers' standard is used quite widely. Its threads are shallower than those of a real square thread. The proportions are shown in Fig. 19–1, and the number of threads is given in Table 19–1.

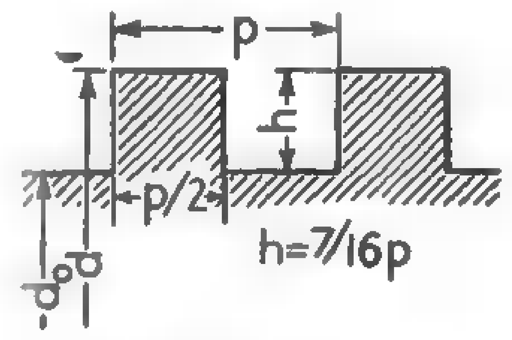


FIG. 19–1. Square thread.

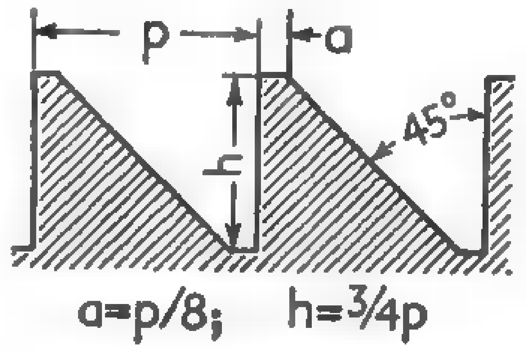


FIG. 19–2. Buttress thread.

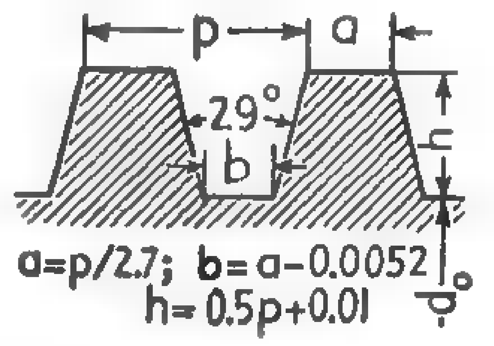


FIG. 19–3. Acme thread.

Buttress threads. The buttress thread, or trapezoidal thread, Fig. 19–2, is used for the transmission of power in one direction only. The efficiency of this thread is the same as that of the square thread, while its cost of manufacturing is slightly smaller. There are no standard proportions for this thread, but those commonly used are given in Fig. 19–2. The number of threads is made greater than that given in Table 19–1. For sizes up to and including $1\frac{3}{4}$ in., the increase is 1; for larger sizes it is $\frac{3}{4}$.

Acme thread. The Acme thread, Fig. 19–3, is used chiefly for lead screws and similar service where lost motion is objectionable. Such lost motion, caused by wear, can be eliminated by means of a nut split lengthwise. The efficiency of the Acme thread is slightly lower than that of a square thread, but its cost of production is smaller since it can be cut by dies. The number of threads per inch is commonly made the same as that of Sellers' thread, Table 19–1, and all proportions can be computed from the data of Fig. 19–3.

Multiple threads. Power screws with two or three parallel threads are employed to increase the travel of the nut per revolution. The mechanical

TABLE 19–1

SELLERS' SQUARE THREADS

Diameter d (in.)	Number of Threads per Inch n	Diameter d (in.)	Number of Threads per Inch n	Diameter d (in.)	Number of Threads per Inch n	Diameter d (in.)	Number of Threads per Inch n	Diameter d (in.)	Number of Threads per Inch n
$\frac{1}{4}$......	10	$\frac{9}{16}$....	6	1.....	4	$1\frac{3}{4}$....	$2\frac{1}{2}$	3.....	$1\frac{3}{4}$
$\frac{5}{16}$.....	9	$\frac{5}{8}$.....	$5\frac{1}{2}$	$1\frac{1}{8}$....	$3\frac{1}{2}$	2.....	$2\frac{1}{4}$	$3\frac{1}{4}$....	$1\frac{3}{4}$
$\frac{3}{8}$......	8	$\frac{11}{16}$....	5	$1\frac{1}{4}$....	$3\frac{1}{2}$	$2\frac{1}{4}$....	$2\frac{1}{4}$	$3\frac{1}{2}$....	$1\frac{5}{8}$
$\frac{7}{16}$.....	7	$\frac{3}{4}$.....	5	$1\frac{3}{8}$....	3	$2\frac{1}{2}$....	2	$3\frac{3}{4}$....	$1\frac{1}{2}$
$\frac{1}{2}$......	$6\frac{1}{2}$	$\frac{7}{8}$.....	$4\frac{1}{2}$	$1\frac{1}{2}$....	3	$2\frac{3}{4}$....	2	4.....	$1\frac{1}{2}$

advantage of a multiple thread is correspondingly smaller, but the efficiency is higher because of the increase of the helix angle. The *pitch* of a screw is the distance p between consecutive threads, whether it is a single-thread or multiple-thread screw. The distance l which the nut of a multiple-thread screw advances for one revolution of the screw is called the *lead*. In a single-thread screw the pitch and the lead are equal. The helix angle at the mean diameter is called the *lead angle*. The relation between l, p, and the number of parallel threads i evidently is

$$l = pi \tag{19–1}$$

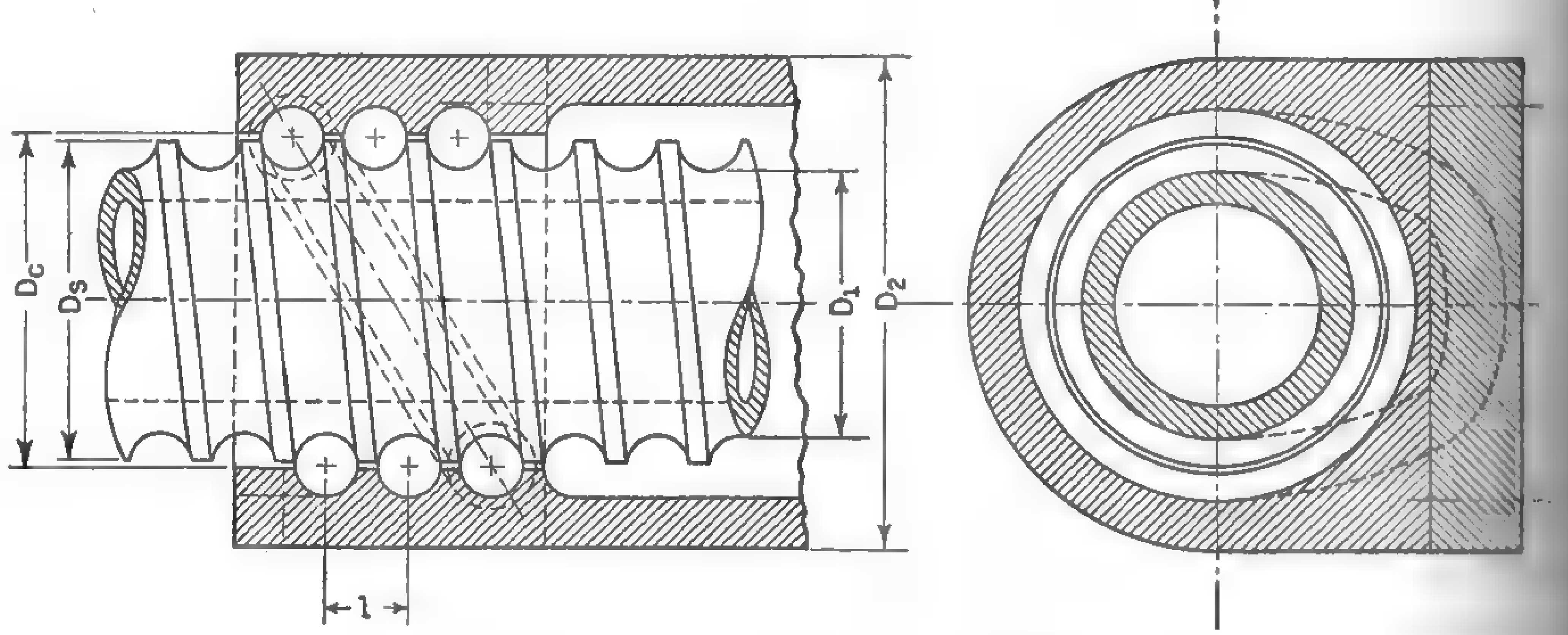


FIG. 19–4. Screw and nut with balls.

Round threads with balls. In order to reduce friction in the threads and thus to increase the efficiency of power screws, a special round thread with steel balls inserted between the screw and the nut has been developed. The thread grooves have approximately semicircular cross sections. The nut contains $1\frac{1}{2}$ or $2\frac{1}{2}$ rows of bearing balls and a return groove, as shown in Fig. 19–4.

The efficiency of such a screw is of the order of 90 per cent and higher. So far these screws are used chiefly for automobile steering gears and power

actuators. However, there are many more places where they may be used to advantage.[1]

19–2. Force analysis. If the notations and the reasoning in section 11–7 are used, the relation for the tangential force H_1, Fig. 19–5a, which must be applied to a square thread to exert an axial force F, is

$$H_1 = F \tan (\lambda + \phi) \tag{19–2}$$

Expressing $\tan (\lambda + \phi)$ by functions of the component angles λ and ϕ, and using the designation $f_1 = \tan \phi$, gives

$$H_1 = \frac{F(\tan \lambda + f_1)}{1 - f_1 \tan \lambda} \tag{19–3}$$

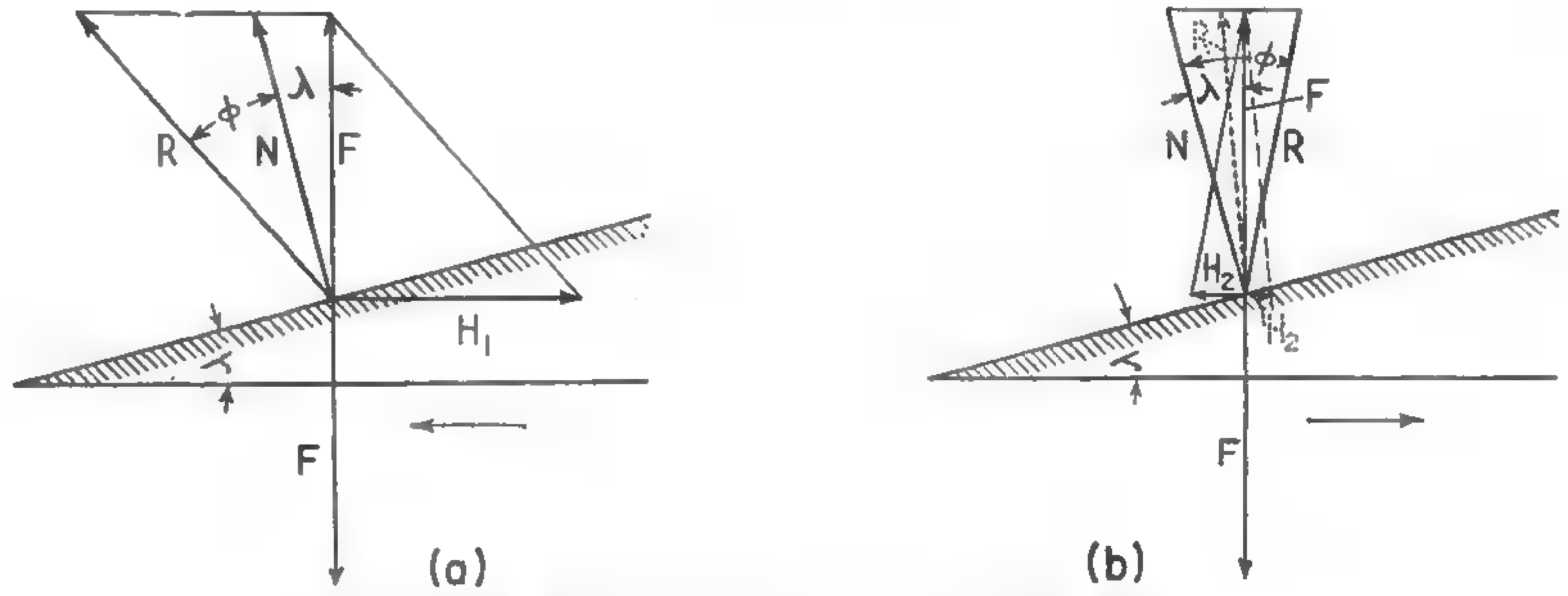


FIG. 19–5. Forces acting on a square thread.

If the motion is reversed, as indicated in Fig. 19–5b, the screw is used for lowering the load F. The friction changes the normal force N to the resultant force R, which is inclined in the direction of the screw motion. In this case

$$H_2 = F \tan (\phi - \lambda) \tag{19–4}$$

or

$$H_2 = \frac{F(f_1 - \tan \lambda)}{1 + f_1 \tan \lambda} \tag{19–5}$$

If $\phi < \lambda$, H_2 becomes negative, as shown by the dotted lines in Fig. 19–5b, no force is required to lower the load F, and the load will begin to turn the screw. The screw is not self-locking.

If d_m designates the mean diameter of the screw, the torque necessary to overcome the load F is

$$T = \tfrac{1}{2} F d_m \tan (\lambda + \phi) \tag{19–6}$$

In the absence of friction, the torque would be

$$T_o = \tfrac{1}{2} F d_m \tan \lambda \tag{19–7}$$

Hence the efficiency is

$$e = \frac{T_o}{T} = \frac{\tan \lambda}{\tan (\lambda + \phi)} \tag{19–8}$$

[1] Design data and procedure for screws with ball nuts are given in V. L. Maleev, *Machine Design*, rev. ed. (Scranton: International Textbook Company, 1946), pp. 248 ff.

Another equation for efficiency may be obtained by using equation 11–18, with $\beta=0$. Thus

$$e=\frac{\tan\lambda(1-f_1\tan\lambda)}{\tan\lambda+f_1} \tag{19–9}$$

The torque and the efficiency can also be expressed in terms of the screw dimensions by using the relation

$$\tan\lambda=\frac{l}{\pi d_m} \tag{19–10}$$

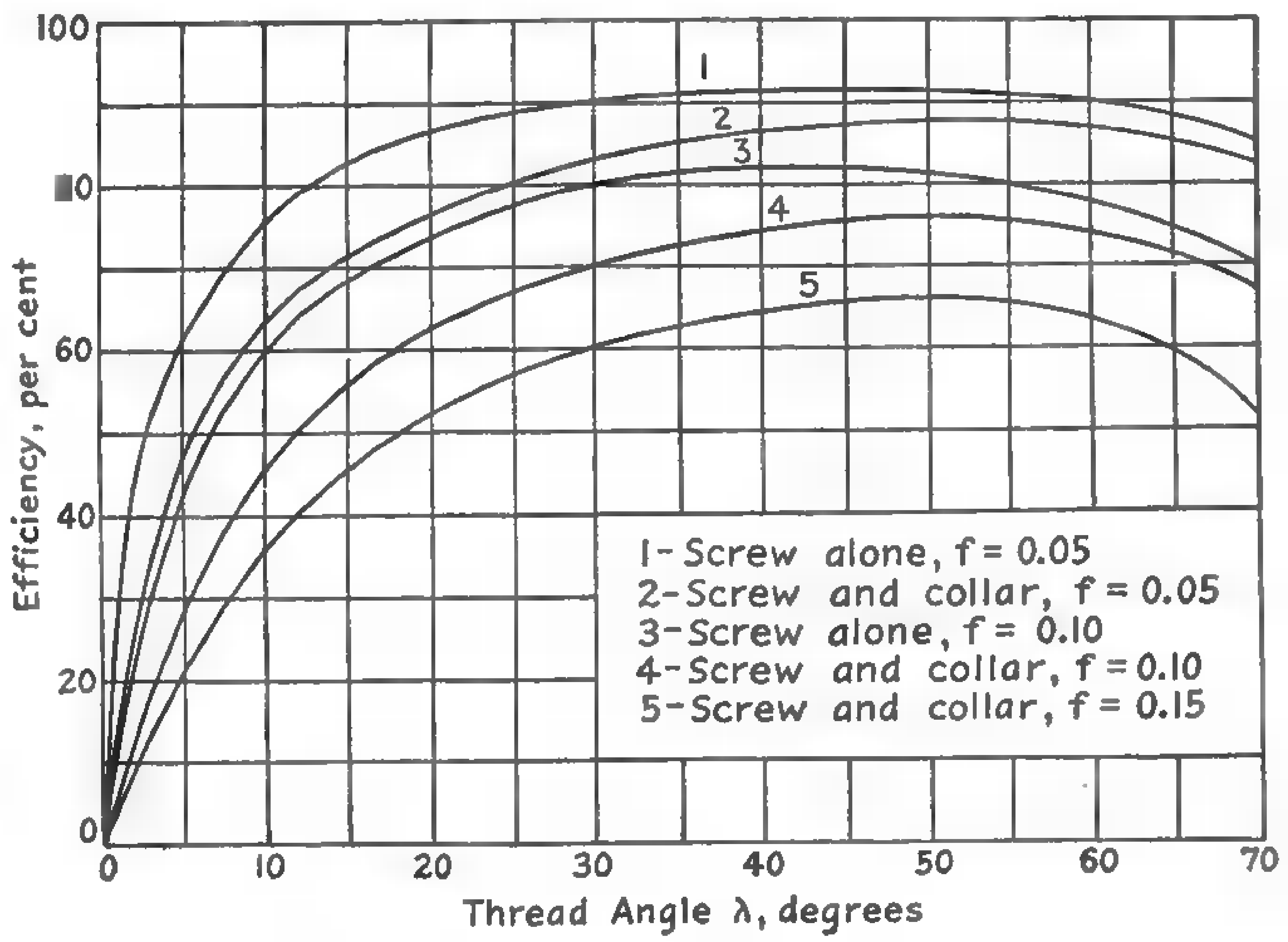


FIG. 19–6. Efficiencies of squares screws.

In the preceding equations only the friction in the threads is taken into account. The pivot friction, such as that between the cap and the screw in a screw jack or in a thrust collar of a lead screw, can be determined by an analysis similar to that used in deriving equation 11–22 for V-shaped screws. The influence of the helix angle λ and friction on the efficiency of a screw is shown in Fig. 19–6.[2] The efficiency increases very rapidly as λ increases up to 15° to 20°; it increases more slowly until it reaches a maximum for values of λ between 40° and 50°; it then begins to decrease. In single-thread screws λ varies from 4° to 7° and therefore their efficiency is below 50 per cent. In multiple-thread screws a higher efficiency can be obtained but the screws lose the self-locking characteristic.

Since an Acme thread is a V thread with a small angle, $\beta=14\frac{1}{2}°$, the tangential force may be determined by equation 11–16, and the efficiency may be computed by equations 11–18 and 11–22.

The *coefficient of friction* may be taken from Table 11–4. For the sake of lowering both the friction and the wear, it is advisable to use different materials for the screw and the nut. Since the nut is usually smaller and less expensive than the screw, the nut is made of softer material, such as brass or bronze. More recent investigations show that the materials of the screw and nut have less influence on the coefficient of friction than does workmanship.[3] For an average-quality material and workmanship, and heavy-oil lubrication, the coefficient of friction in motion may be taken as 0.125. For inferior workmanship it is safer to take the coefficient as 0.15. The starting friction is about 35 per cent higher.

[2] D. S. Kimball and J. H. Barr, *Elements of Machine Design*, 3d ed. (New York: John Wiley & Sons, Inc., 1935), p. 290.

19–3. Bearing pressure. In order that the threads of a screw may transmit the required work without excessive wear, the unit pressure upon the surfaces in contact must not exceed certain values. These values depend on the materials of the screw and nut and on the relative speed between the rubbing surfaces. These limit values, upon which the design is based, are determined under the assumption that the load is distributed uniformly among all threads. This assumption is not correct, as a simple analysis will show.[4]

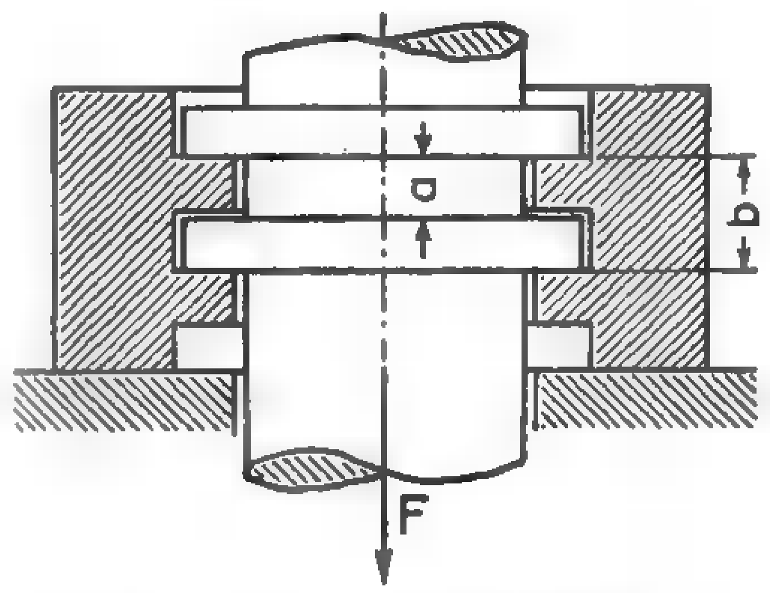


FIG. 19–7. Load distribution in a screw and nut.

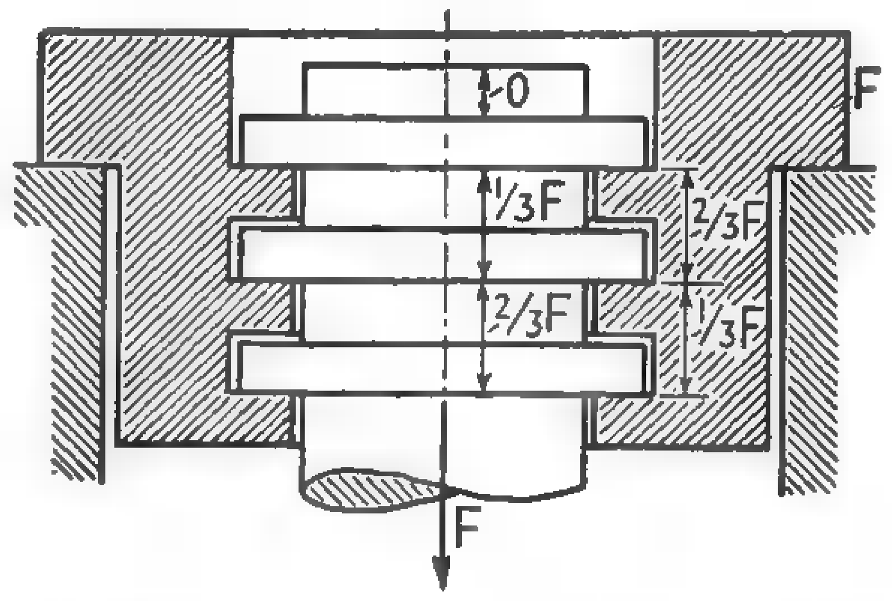


FIG. 19–8. Screw and nut loaded in tension.

In Fig. 19–7 two threads of a screw working in tension are replaced by collars, and the corresponding part of a nut is replaced by a sleeve with two inner collars. The figure shows that if each pair of collars carries one-half the load, part *a* of the screw is in tension and part *b* of the nut is in compression. Since *a* becomes longer and *b* becomes shorter, the upper collar, or the nut, will move away from the corresponding screw collar. This being impossible under our assumption of load distribution, it follows that the lower pair of collars carries all the load, and that the upper pair only stays in contact, without carrying any load.

The explanation of the necessity of using more than one thread to carry a certain load is in the elastic and plastic deformation of the threads which

[3] C. W. Ham and D. G. Ryan, *An Experimental Investigation of the Friction in Screw Threads*, Bulletin No. 247, University of Illinois Engineering Experiment Station (June, 1932).

[4] W. Trinks, "Things That Are Commonly Wrong in Textbooks on Machine Design," *The Journal of Engineering Education*, Vol. 23 (March, 1933), p. 523.

transmit part of the load from the lower thread to the next threads. Nevertheless the load distribution is far from uniform.

In Fig. 19–7 the screw is in tension and the nut is in compression. The results will be the same if the situation is reversed, the screw being stressed in compression and the nut in tension. If both the screw and the nut work in tension, or if both work in compression, the results will be different. Such an arrangement is shown in Fig. 19–8. Let it be assumed that each pair of collars carries one-third the total load F. Then the force distribution in the screw and in the nut is indicated by the values 0, $\frac{1}{3}F$, $\frac{2}{3}F$, and F. It is evident that the upper part of the nut between the upper and middle collars stretches more than does the corresponding part of the screw, while the lower part of the nut stretches less than does the corresponding part of the screw. Consequently the upper and lower pairs of collars remain in contact while the middle pair becomes disengaged, because the upper part of the nut stretches more than the upper part of the screw. In an actual screw all threads will be in contact, but the inner threads will carry a smaller load than the top and bottom threads.

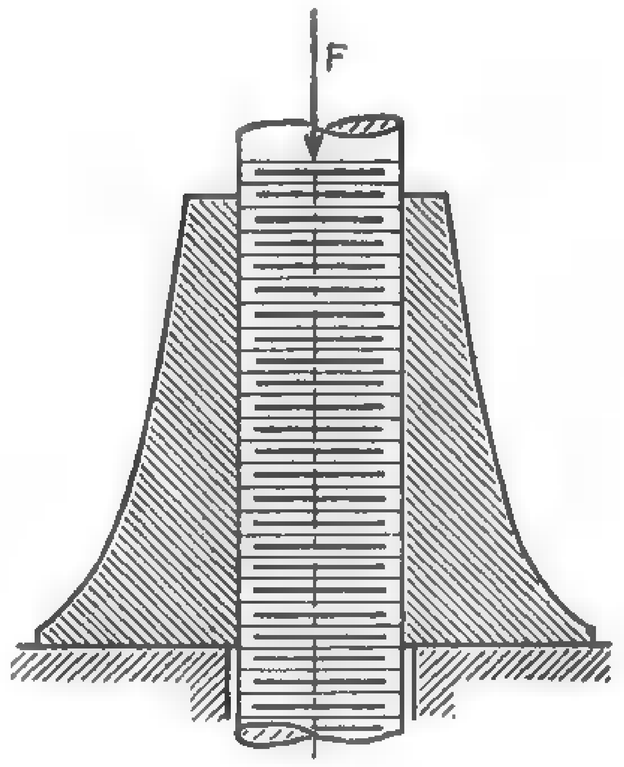


Fig. 19–9. Nut with uniform pressure on threads.

This analysis shows that the bearing pressure is not uniform on all threads which are in contact, and that the pressure is more uniformly distributed if the screw and nut are both subjected to the same stress, either both being in tension or both being in compression. These theoretical conclusions are supported by tests with undercut nuts, Fig. 11–34d. Ordinary nuts which transmit motion under heavy pressure are subject to great wear because of unequal pressure. By using a variable-section nut, Fig. 19–9, the equation for which is a parabola, a uniform pressure and reduced wear can be obtained.[5]

19–4. Differential and compound screws. In some cases a very slow advance of the screw is desired, whereas in other cases a very fast movement is required. With a single-thread screw, slow motion can be obtained by using a small pitch. This, however, results in a weak thread. A fast motion is obtained by using a multiple-thread screw, but this means expensive machining and loss of the self-locking feature.

Differential screws. To a certain extent the difficulties just mentioned can be overcome by using the arrangement shown in Fig. 19–10a. The screw has two threads of the same hand but of different pitches. As a result each revolution of the screw causes the nuts m and n to move toward or away from each other a distance equal to the difference of the pitches. If one

[5] S. Timoshenko and J. M. Lessels, *Applied Elasticity* (East Pittsburgh: Westinghouse Technical Night School Press, 1925).

nut and two screws are used, and the screws are prevented from turning, the arrangement in Fig. 19–10b results. In a third modification, shown in Fig. 19–10c, the hub of the handwheel a has a coarse-pitch outside thread and a finer-pitch inside thread; and each revolution of the handwheel a moves the screw b a distance equal to the difference of the pitches.

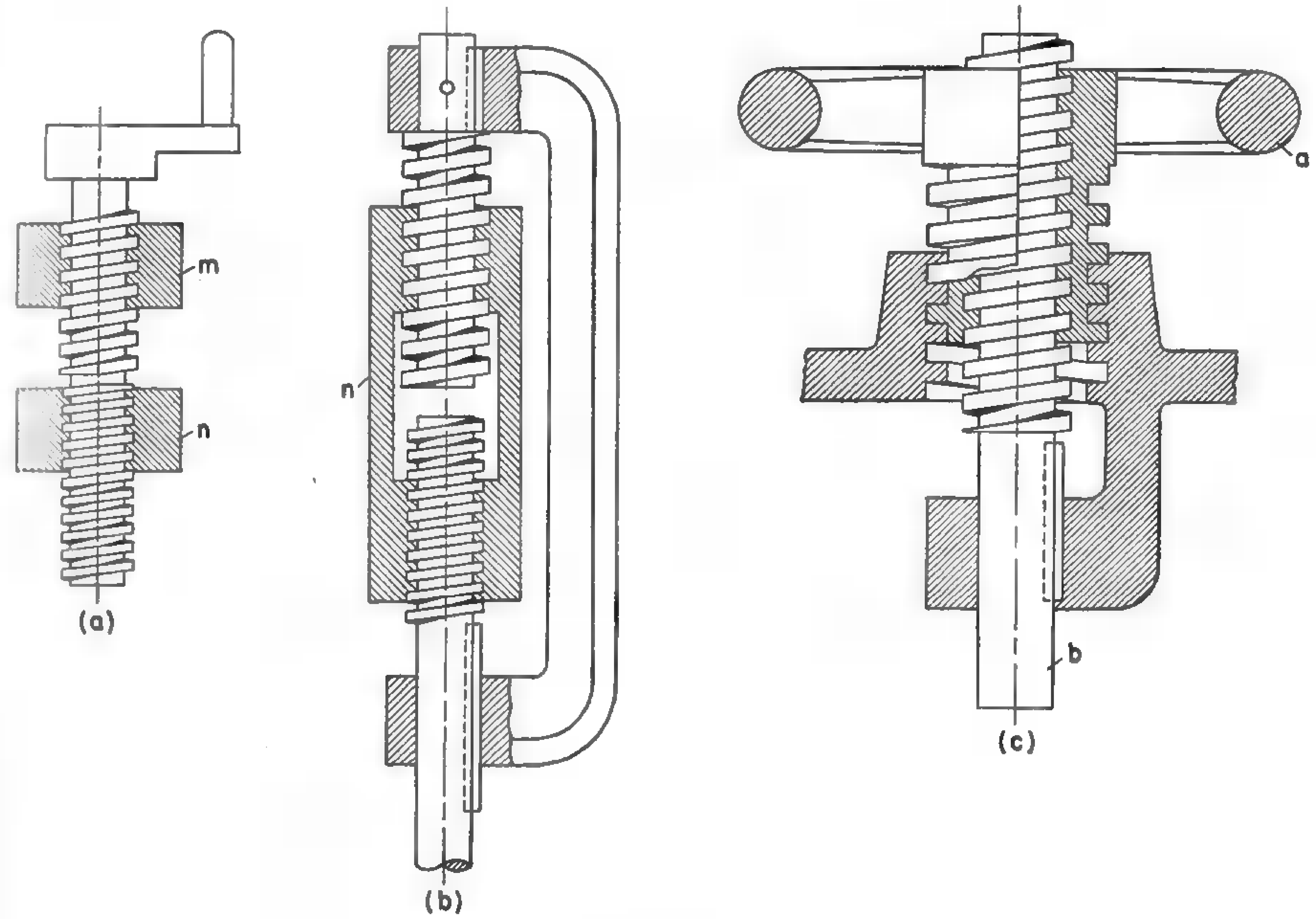


Fig. 19–10. Differential screws.

If the number of threads of the coarser pitch is n_1 and the number of threads of the finer pitch is n_2, the motion h produced by a differential screw is

$$h = p_1 - p_2 = \frac{1}{n_1} - \frac{1}{n_2} \tag{19–11}$$

Differential screws and nuts are cut in a lathe. There are no standards for pitches, but Table 19–1 may be used as a guide. The numbers of threads can be determined by using the desired motion h as a basis and assuming the difference $(n_2 - n_1)$ as $\frac{1}{8}$ for a very coarse pitch and gradually increasing this difference up to 1 for a fine pitch. In general the increments between consecutive pitches in Table 19–1 should be used.

Example 19–1. Determine the size of differential screw (Fig. 19–10b) needed to exert a force of 12,000 lb and to produce a motion of about 0.025 in. for each revolution of the nut.

The minimum minor diameter d_o may be found by assuming a low design stress S_d of 8,000 psi in order to take care of any possible column action. The necessary cross-sectional area is

$$A = \frac{12{,}000}{8{,}000} = 1.5 \text{ sq in.}$$

and

$$d_o = \sqrt{\frac{1.5}{0.7854}} = 1.380 \text{ in.}$$

The approximate number of threads may be found by using equation 19–11 and assuming, in accordance with Table 19–1, that $n_2 - n_1 = \frac{1}{4}$, or $n_2 = n_1 + 0.25$. Thus

$$0.025 = \frac{1}{n_1} - \frac{1}{n_1 + 0.25}$$

Solving this equation gives $n_1 = 3.04$. Using the nearest values of $n_1 = 3$ and $n_2 = 3\frac{1}{4}$, and checking, gives

$$h = \frac{1}{3} - \frac{1}{3.25} = 0.333 - 0.308 = 0.025 \text{ in.}$$

The major screw diameter may be determined now by referring to Fig. 19–1. With $d_o = 1.380$ in.,

$$d = 1{,}380 + 2 \times \tfrac{7}{16} \times 0.333 = 1.380 + 0.291 = 1.671 \text{ in.}$$

However, using the nearest stock size gives $d = 1\frac{3}{4}$ in. and

$$d_o = 1.750 - 0.291 = 1.459 \text{ in.}$$

The minimum number of threads i in engagement may be found by assuming a bronze nut and a permissible bearing pressure $p = 2{,}500$ psi from Table 19–2. Then

$$i = \frac{12{,}000}{2{,}500 \times 0.7854 \times (1.750^2 - 1.459^2)} = 6.53$$

Compound screws. If the threads in Fig. 19–10a, Fig. 19–10b, or Fig. 19-10c are of different hands, the motion of the nut or screw, as the case may be, is equal to the sum of the pitches for every revolution of the actuating part. Such screws are called *compound screws*. With compound screws the pitches usually are made equal.

Efficiency. The expression for the efficiency of a differential screw may be found by noticing that when the coarse thread is raising the load the finer thread lowers it. Each torque is computed for the tangential force found by taking the sum of H_1 from equation 19–3, for the coarser thread, and H_2 from equation 19–5, for the finer thread. The numerator in the expression for the efficiency is the sum of these torques with $f = 0$. If the respective helix angles are designated by λ_1 and λ_2, and the mean thread diameters by d_1 and d_2, the efficiency is

$$e = \frac{d_1 \tan \lambda_1 - d_2 \tan \lambda_2}{\dfrac{(\tan \lambda_1 + f)d_1}{1 + f \tan \lambda_1} + \dfrac{(f - \tan \lambda_2)d_2}{1 + f \tan \lambda_2}} \tag{19–12}$$

When equation 19–12 is applied with some typical values of λ_1, λ_2, d_1, d_2, and f, it will be found that the efficiency of differential screws is very low, ranging from 2.5 to 8.8 per cent at the most. Tests confirm this conclusion.[6]

[6] R. T. Kent, *Mechanical Engineers' Handbook*, 12th ed., Vol. 2, *Design and Production*, ed. by Colin Carmichael (New York: John Wiley & Sons, Inc., 1950), p. 7-11.

The efficiency of a compound screw is found in a similar way, but only the tangential force H_1 found by equation 19–3 is used in determining the torque for all terms. Thus

$$e = \frac{d_1 \tan \lambda_1 + d_2 \tan \lambda_2}{\dfrac{(\tan \lambda_1 + f)d_1}{1 - f \tan \lambda_1} + \dfrac{(\tan \lambda_2 + f)d_2}{1 - f \tan \lambda_2}} \tag{19–13}$$

Numerically the efficiency of a compound screw is of the same order as that of a single screw. In fact, if both screws have the same diameter and pitch, equation 19–13 can be simplified to the form of equation 19–9.

19–5. Design considerations. Screws used for the transmission of power are subjected to the following stresses: tension or compression in the central section; shear in the threads, due to the axial load; shear due to the external torque; and bearing pressure.

Tension or compression. The service of the screw and the method of mounting it determine the kind of stress resulting from the direct load. If the stress is tension, its nominal magnitude is equal to the load divided by the area at the root of the thread. The absence of fillets in the threads invokes a localized stress, with a form stress factor K of 4.5 to 5. For ductile materials and static load the factor of sensitivity q may be assumed to be about 0.15, giving a stress-concentration factor K' of 1.5 to 1.6. The safety factor n may be taken as 1.5 to 2.

If the load produces a compressive stress, the stress concentration, although present, is not so dangerous and may be neglected. If the length of the screw exceeds six to eight times the root diameter, the screw must be treated as a short column.

Shear. The shear stress in the threads, which is due to the axial load, is usually not dangerous, and the number of threads resisting the shear action is determined from consideration of wear rather than strength. Nevertheless a check of the average stress should be made. Stress concentration may be neglected. The threads of the screw may shear at the root diameter; the threads of the nut may shear at the major diameter, which is equal to the outside diameter of the screw. Since the screw and the nut are of different materials, the stresses and safety factors for both should be computed.

Torsional shear is induced in the screw by the external turning moment T, which can be found by equation 19–6. The stress is found by equation 2–14, in which D is the minor diameter of the screw.

Bearing pressure. As shown in section 19–3, the pressure between the threads of a screw and nut is not uniform. However, such an assumption simplifies the design and is always made. The inaccuracy of this assumption is taken into account by using low values for the bearing pressure p_b. The necessary number of threads i' is then determined from the equation

$$F = 0.7854(d^2 - d_0^2)p_b i' \tag{19–14}$$

TABLE 19–2

SAFE BEARING PRESSURES IN POWER SCREWS

SERVICE	MATERIAL		SAFE BEARING PRESSURE p (PSI)	REMARKS
	Screw	Nut		
Hand press.....	Steel	Bronze	2,500–3,500	Low speed, well-lubricated
Screw jack......	Steel	Cast iron	1,800–2,500	Low speed, not over 8 fpm
		Bronze	1,600–2,500	Low speed, not over 10 fpm
Hoisting screw..	Steel	Cast iron	600–1,000	Medium speed, 20 to 40 fpm
		Bronze	800–1,400	Medium speed, 20 to 40 fpm
Lead screw.....	Steel	Bronze	150–240	High speed, 50 fpm and over

where d is the outside diameter of the screw and d_2 is the inside diameter of the nut. The allowable values of p_b determined from actual service are given in Table 19–2.

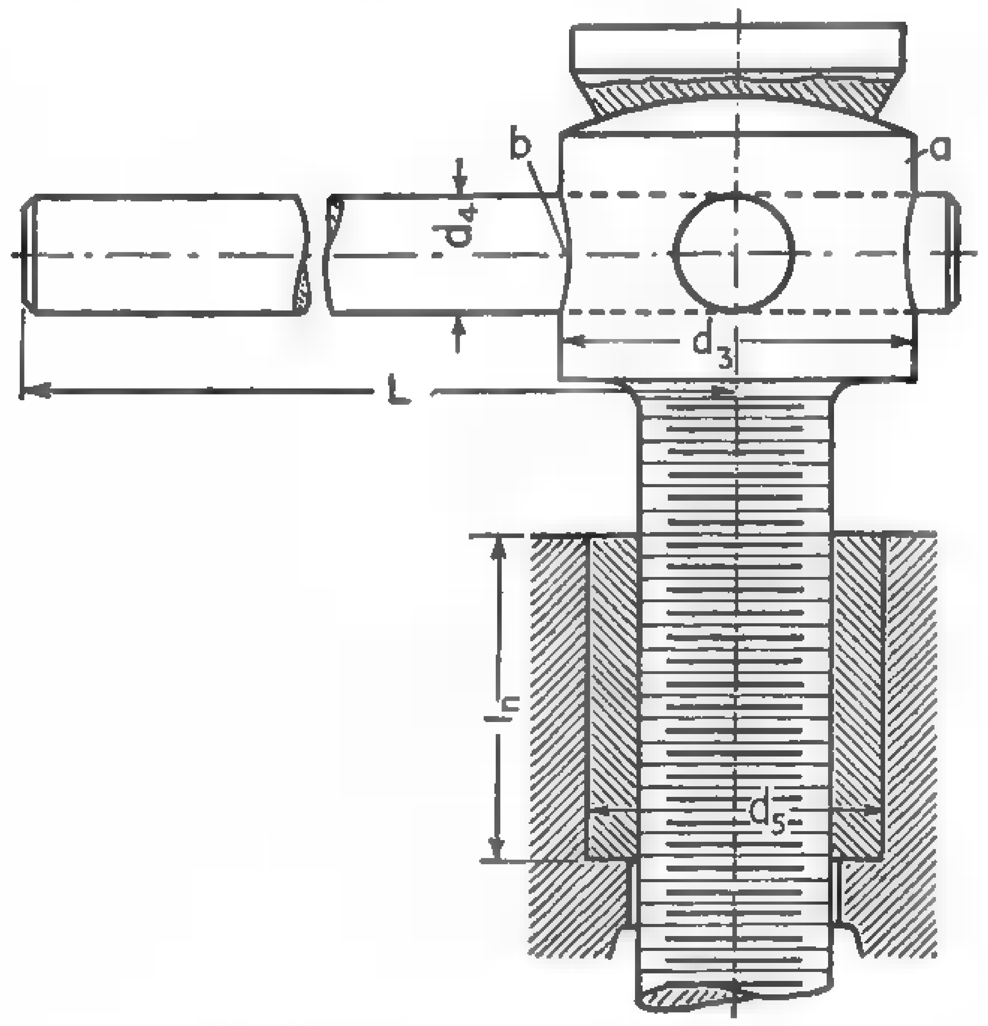


FIG. 19–11. Screw jack.

EXAMPLE 19–2. Determine the main dimensions of the screw jack in Fig. 19–11 for a load of 17.5 tons and a lift of 14 in.

The screw can be made of SAE 1030 steel. The following values are taken from Table 4–2: In tension, $S_e = 42{,}000$ psi; in compression, $S_{ec} = 42{,}000$ psi; in shear $S_{es} = 26{,}000$ psi. The nut can be made of phosphor bronze. Table 4–3 gives: $S_e = 18{,}000$ psi, $S_{ec} = 16{,}000$ psi, and $S_{es} = 15{,}000$ psi.

For preliminary computations a comparatively high safety factor n of 2.25 may be assumed to take care of the size influence.

The minimum root diameter of the screw for simple compression is

$$d_o = \sqrt{\frac{17.5 \times 2000 \times 2.25}{42{,}000 \times 0.7854}} = 1.55 \text{ in.}$$

For the Sellers' square thread, from Table 19–1, the proper size seems to be $d = 2\frac{1}{2}$ in., with 2 threads per inch. By Fig. 19–1 the root diameter is

$$d_o = d - \tfrac{7}{8}p = 2.50 - \frac{0.875}{2} = 2.062 \text{ in.}$$

The length of the protruding screw should be taken about one diameter, or $2\frac{1}{2}$ in., longer than the lift, to allow for the head part a, Fig. 19–11. Thus $l = 14 + 2\frac{1}{2} = 16\frac{1}{2}$ in. Since $l/d_o = 16.50/2.062 = 8.02$, the diameter must be determined by a short-column formula. If Ritter's formula (equation 2–37) is used, the diameter d_o may be found by substituting $0.7854d_o^2$ for A and $0.25d_o$ for k, and solving for d_o. However, it is quicker to assume the root diameter d_o and to check the stress s.

For $d = 2\frac{1}{2}$, the value of $d_o = 2{,}062$ may be tried first. Then

$$\frac{l}{k} = \frac{l}{0.25d_o} = \frac{16.50}{0.25 \times 2.062} = 32.0$$

In Ritter's formula, by Fig. 2–17, $n = 0.25$; from Table 4–2, $E = 30{,}200{,}000$ psi. The size influence can be taken into account by use of equation 5–8 and Table 5–1. In this case

$$e_{ss} = 1 - 0.4 \times (1 - 0.82) \times (2.50 - 0.5) = 0.86$$

Then, $S_e' = 42{,}000 \times 0.86 = 36{,}100$ psi, and the design stress, with a safety factor $n = 2$, is

$$S_d = \frac{36{,}100}{2} = 18{,}050 \text{ psi}$$

From equation 2–37, with $A = 0.7854 \times 2.062^2 = 3.335$ sq in., the stress becomes

$$s = \frac{35{,}000}{3.335} \times \left(1 + \frac{32^2 \times 36{,}100}{\pi^2 \times 0.25 \times 30{,}200{,}000}\right) = 10{,}500 \times (1 + 0.499) = 15{,}780 \text{ psi}$$

Therefore a $2\frac{1}{2}$-in. screw is satisfactory. To make sure, the maximum normal stress must be checked later.

The number of threads i' necessary to obtain proper bearing pressure is found from equation 19–14. If a clearance of $\frac{1}{64}$ in. is allowed between the root diameter of the screw and the inner diameter of the nut, or $d_2 = d_o + 0.016 = 2.062 + 0.016 = 2.078$ in., and p_b is taken as 2,000 psi from Table 19–2,

$$i' = \frac{F}{0.7854(d^2 - d_2^2)p_b} = \frac{35{,}000}{0.7854 \times (2.50^2 - 2.078^2) \times 2{,}000} = 11.5$$

With 2 threads per in., or $p = 0.5$ in., the length of the nut is found to be

$$l_n = 11.5 \times 0.5 = 5.75, \text{ or } 5\tfrac{3}{4} \text{ in.}$$

Next, the average angle of helix, referred to the mean diameter, may be found. Thus,

$$d_m = \tfrac{1}{2}(d + d_o) = \tfrac{1}{2} \times (2.50 + 2.062) = 2.281 \text{ in.}$$

and

$$\tan \lambda = \frac{1}{\pi d_m n} = \frac{1}{\pi \times 2.281 \times 2} = 0.0698$$

Hence $\lambda = 3° \, 59'$. To find the efficiency of the screw by equation 19–8, with $f_1 = 0.125$ from Table 11–4, the angle of friction must be found. If $\tan \phi = f = 0.125$, then $\phi = 7° \, 8'$ and $\lambda + \phi = 3° \, 59' + 7° \, 8' = 11° \, 7'$; and

$$e = \frac{0.0698}{\tan 11° \, 7'} = \frac{0.0698}{0.197} = 0.354$$

The pivot friction between the top of the screw and the cap can be taken into account by using a relation similar to equation 11–20. Thus,

$$H_2 = \frac{Ff_2 d_m'}{d_m}$$

The corresponding torque becomes

$$T_c = \tfrac{1}{2}d_m H_2 = \tfrac{1}{2}Ff_2 d_m' \tag{19–15}$$

and the equation for the over-all efficiency, which is similar to equation 11–22, becomes

$$e_3 = \frac{\tan \lambda}{\tan(\lambda + \phi) + f_2 d_m'/d_m} \tag{19–16}$$

where the friction coefficient may be taken as 0.10 and where the mean diameter of the cap d_m' may be taken approximately as $0.75d$. With $f_2 d_m'/d_m = 0.10 \times 0.75 \times 2.50/2.281 = 0.082$, equation 19–16 gives

$$e_3 = \frac{0.0698}{0.197 + 0.082} = 0.250$$

In a square thread the total height of the threads is $\frac{1}{2}l_n$, and the shear stress in the threads of the screw can be found from the equation

$$s_s = \frac{2F}{\pi d_o l_n} = \frac{2 \times 35{,}000}{\pi \times 2.062 \times 5.75} = 1{,}880 \text{ psi}$$

This is very small. In the threads of the nut the stress is still smaller.

The torsional moment transmitted by the screw is, by equation 19–6,

$$T = \tfrac{1}{2} \times 35{,}000 \times 2.281 \times 0.197 = 7{,}870 \text{ lb-in.}$$

If the reinforcing action of the threads is neglected, the torsional stress is

$$s_s' = 7{,}870 \times \frac{16}{\pi \times 2.062^3} = 4{,}570 \text{ psi}$$

The corresponding safety factor is

$$n = \frac{26{,}000 \times 0.86}{4{,}570} = 4.88$$

which is satisfactory.

Now it is advisable to check the magnitude of the maximum normal stress. The direct normal stress was found to be 15,780 psi, and the shear stress is $S_s = 4{,}570$ psi. By equation 2–48, in which $\mu = 0.3$, the maximum normal stress is

$$s'' = \tfrac{1}{2} \times (1 - 0.3) \times 15{,}780 + (1 + 0.3)\sqrt{\tfrac{1}{4} \times 15{,}780^2 + 4{,}570^2} = 17{,}350 \text{ psi}$$

Then $n' = 36{,}000/17{,}350 = 2.07$, which is very satisfactory for hand-operated machinery.

The diameter d_3 (Fig. 19–11) at the top of the screw should be made approximately equal to $1.75d$. Thus

$$d_3 = 2.50 \times 1.75 = 4.37, \text{ or } 4\tfrac{3}{8} \text{ in.}$$

The length L of the turning bar can be found by equating the torque T plus the torsional resistance T_c at the cap, to the turning moment LF_t'. By equation 19–15,

$$T_c = \tfrac{1}{2} \times 35{,}000 \times 0.10 \times 0.75 \times 2.50 = 3{,}280 \text{ lb-in.}$$

If it is assumed that four men will act, and $F_t' = 300$ lb,

$$L = \frac{T + T_c}{F_t'} = \frac{7{,}870 + 3{,}280}{300} = 37.2 \text{ in.}$$

To give a grip for the hands, the length will be made about 44 or 46 in.

Finally, the diameter d_4 of the turning bar should be determined. When the external bending moment is equated to the moment of the stress in the bar at point b, the result is

$$F_t(L - 0.5d_3) = ZS_d$$

where $Z = \frac{1}{32}\pi d_4^3$. With a safety factor $n = 1.5$, for SAE 1020, $S_d = 22{,}600$ psi and

$$d_4 = \sqrt[3]{\frac{300 \times (37.2 - 2.19) \times 32}{22{,}600\pi}} = 1.68, \text{ or } 1\tfrac{3}{4} \text{ in.}$$

PART VI: PARTS TRANSMITTING ROTARY MOTION

CHAPTER 20

Shafts

20-1. Introductory considerations. A *shaft* is a rotating member which transmits power. An *axle* is a machine part which is loaded chiefly in bending and carries such rotating parts as wheels and gears. An axle may be stationary, or it may rotate. Short shafts and axles in machines are called *spindles*. However, the term shaft is often applied to all machine parts here mentioned, irrespective of the type of loading or size.

A *headshaft*, or *stubshaft*, is directly connected to an engine or motor. A *line shaft*, often called a *transmission shaft*, is a comparatively long shaft which receives its motion from a motor and transmits it to various machines. A *countershaft*, or *jackshaft*, is a short shaft placed between a prime mover and a line shaft or a driven machine.

Materials. Shafts are made of Bessemer steel, open-hearth steel, and alloy steels—and some copper alloys, when resistance to corrosion is desired.

Ordinary shafts are made of Bessemer or open-hearth steel with a carbon content of 0.15 to 0.40 per cent. Such steel is commonly called medium, or machinery, steel. Steel shafting is sometimes finished by cold rolling or cold drawing. This produces a somewhat stronger bar than hot rolling. However, cold-finished shafts have several disadvantages: The diameter tolerances are not very close; the shaft does not come precisely straight; and the fibers at and near the outer surface are under stress which, when partially released through keyseating, causes a distortion of the shaft. The straightening of a distorted or twisted shaft is a difficult and expensive operation.

Most commercial shafting, after being hot-rolled, is turned and polished. The best is known as T&G, or *turned-and-ground*, shafting. It is made of open-hearth mild steel that is similar to SAE 1015 but has a slightly higher content of manganese, phosphorus, and sulfur. On special order, turned-and-ground bars are furnished with a higher carbon content and of stainless steel.

When greater strength is required, as in high-speed machinery, shafts are made of alloy steels, the most common being nickel, nickel-chromium, and chrome-vanadium steels. Alloy-steel shafts are always heat-treated. For spline shafts subjected to severe shocks, SAE 3245 steel that is oil-quenched from 1500 F and tempered to 800 F is particularly suitable.

Shafts for special purposes, especially those having integral connecting flanges, are forged, as are most hollow shafts.

Commercial sizes. Formerly shafting was made of bars rolled to even sizes by $\frac{1}{4}$-in. increments. The bars were reduced $\frac{1}{16}$ in. in finishing, and the commercial sizes of shafts were established $\frac{1}{16}$ in. under the nominal sizes. At present both turned-and-ground and cold-rolled shafting up to 2 in. can be obtained in $\frac{1}{16}$-in. increments; shafting from 2 in. to 5 in. comes in increments of $\frac{1}{4}$ in. in nominal sizes, and also $\frac{1}{16}$ in. under the nominal sizes. In larger sizes, up to 8 in., shafting comes in increments of $\frac{1}{2}$ in.; it is usually in nominal sizes but also may be obtained $\frac{1}{16}$ in. undersize. All standard bearings, set collars, and other shaft attachments are bored $\frac{1}{16}$ in. undersize. T & G bars are ground 0.001 in. under the nominal size up to $2\frac{7}{16}$ in., and 0.002 in. under in larger sizes, with a tolerance of -0.001 in. in all sizes.

Standard lengths are 20 and 24 ft. However, most manufacturers carry shorter lengths in stock, from 10 ft up, in 2-ft increments. Lengths over 26 ft can be obtained at extra cost.

20–2. Shafts with steady loading. The loads to which shafts may be subjected cause simple torsion, simple bending, combined torsion and bending, or combined torsion and compression with or without bending.

Strength in simple torsion. For a given torsional moment T, in pound-inches, and for an allowable shear stress S_d, the shaft diameter is found from equation 2–13. Thus,

$$D=\sqrt[3]{\frac{16T}{\pi S_d}}=1.72\sqrt[3]{\frac{T}{S_d}} \tag{20–1}$$

Substituting in equation 20–1 the value for T from equation 2–17 gives

$$D=\sqrt[3]{\frac{321{,}000P}{nS_d}}=68.5\sqrt[3]{\frac{P}{nS_d}} \tag{20–2}$$

Rigidity. In cases where the torsional deflection must be taken into consideration, the angle θ of torsion can be determined from equation 2–15. Multiplying by $180/\pi$ to change from radians to degrees, and collecting the numerical factors, results in

$$\alpha=\frac{584lT}{D^4G} \tag{20–3}$$

In drive shafts of machine tools the angle α should be very small, not over 0.08 deg per foot of shaft length. In camshafts of internal-combustion engines the total angle should not exceed 0.5 deg, irrespective of the length. In most shafts, it is good practice to limit the torsional angle to 1 deg in a length of $L=20D$.

The shaft diameter corresponding to a prescribed deflection α can be computed by solving equation 20–3 for D.

Strength in simple bending. For a given bending moment M and an allowable stress S_d, the shaft diameter may be determined from equation 2–22.

Safety factor. For stationary shafts the safety factor n may be from 1.5 to 2. In a rotating shaft the bending stress varies from a maximum tension to a maximum compression, and n should be taken 50 per cent higher, if referred to the elastic limit S_e.

Stiffness. In many cases the stiffness of a shaft is not less important than its strength, and therefore the transverse deflections must be limited. Excessive transverse deflections may create uneven wear of the bearings if the bearings are not self-adjusting. These deflections depend on the distribution of loads acting upon the shaft as well as on the method of supporting it. For slow or moderate speeds they can be calculated in a given case from the general equation 2–23 or, for shafts of constant diameter, by formulas used for beams and listed in Table 2–4.

Bending moments. In calculating the bending moment coming on a shaft it is customary to measure each moment arm to the middle of the bearing. It is assumed that the clearance between the shaft and the bearing permits the shaft to deflect up to the middle of the bearing. This is a reasonable assumption and, besides, gives results which are on the safe side.

When a machine part with a hub is forced or shrunk upon a shaft, the stress-concentration effect of the hub may cause the shaft to fail near the edge of the hub. This possibility must be taken into account in determining the maximum stress. It is better to reduce stress concentration by using a tapered hub, as shown in Fig. 5–30b or Fig. 13–2.

Combined torsion and bending. A revolving shaft transmitting power and carrying pulleys, gears, sprockets, or sheaves is subjected to simultaneous torsion and bending. Its design must be based on a significant stress which is a resultant of the stresses in torsion and bending.

The design of steel shafting usually is based on the maximum-shear theory of failure.[1] From equation 2–13, with $Z_o=\frac{1}{16}\pi D^3$, the relation between the equivalent torsional moment and the combined stress is

$$T_e=\tfrac{1}{16}s_s'\pi D^3 \tag{20–4}$$

The combined shear stress in this case is

$$s_s'=\sqrt{(\tfrac{1}{2}s)^2+s_s^2}$$

where s is the normal stress found by equation 2–22, and s' is the simple shear stress found from equation 2–12. When proper substitutions are made in equation 20–4, the result is

$$T_e=\sqrt{M^2+T^2} \tag{20–5}$$

The necessary shaft diameter is found from equation 20–4 by using for s_s' the design stress $S_d=S_e'/n$, where S_e' is the elastic limit in shear, and using for T_e the value found by applying equation 20–5.

[1] *Code for the Design of Transmission Shafting*, Engineering and Industrial Standards (New York: American Society of Mechanical Engineers, 1927).

Torsion and compression. Occasionally a shaft may carry an axial load. This case can be handled by applying the general principles indicated in section 2–11.

Effect of keyways. A keyway lowers both the strength and the rigidity of a shaft. For a shaft made of mild steel and having a keyway of approximately standard proportions the lowering of the strength based on static tests[2] in torsion can be taken into account by introducing a factor similar to a stress-concentration factor, namely,

$$K'' = 1 + \frac{0.2b + 1.1h}{D} \tag{20–6}$$

where b and h are the width and depth of the keyway, respectively, and D is the shaft diameter. If there is a keyway, the design stress S_d must be divided by K''. For a standard keyway, with $b = \frac{1}{4}D$ and $h = \frac{1}{8}D$, $K'' = 1.2$.

According to these tests the length of the keyway does not affect the strength of the shaft. Also, bending applied simultaneously with torsion does not affect the strength additionally, so far as the key is concerned.

Rigidity. The same tests[3] indicate that the increase of the angle of twist α, equation 20–3, may be expressed by a coefficient. Its value is

$$K_1 = 1 + \frac{0.4b + 0.7h}{D} \tag{20–7}$$

Naturally, this increase applies only to the keyseated length of the shaft.

20–3. Determination of moments. A shaft is very seldom of uniform strength. A small shaft is usually made with a constant diameter. In the case of a large shaft, the diameter is varied in steps, and the shaft approaches uniform strength only at certain points where the diameter is changed. In either case the determination of the bending moments, and particularly the maximum value, is of great importance. For the sake of simplicity it is advisable to find the moments separately in the horizontal and vertical planes and then to find the resultant moments.

Generally speaking, the torsional moment also varies at different shaft sections, and its maximum value may not occur at the section at which the bending moment is greatest. In this case, it may be advisable to determine the equivalent moment for two sections—the section with the maximum bending moment and the section with the maximum torque. If the shaft diameter is to be constant, the larger value is used for the design. Two methods of determination of moments may be applied—the analytical and the graphical.

[2] H. F. Moore, *The Effect of Keyways on the Strength of Shafts*, Bulletin No. 42, University of Illinois Engineering Experiment Station (1909).
[3] *Ibid.*

No attempt will be made here to discuss the principles of mechanics involved. However, it seems advisable to refresh the memory of the designer in regard to the proper procedure by means of an illustrative example.

EXAMPLE 20–1. A 48-in. belt pulley B, Fig. 20–1, receives 100 hp from a horizontal drive, and it runs at 225 rpm. Of this power, a spur gear G with a pitch diameter of $15\frac{1}{2}$ in. delivers 60 hp to a gear located to its right, and lower, the line connecting their centers forming an angle of 55° with the horizontal line; and the balance of the power is delivered by an 8.3-in. bevel gear H, the normal tooth pressure being vertical and upward. The pulley and the gears are keyed to the shaft, which is supported by bearings C and D. The belt pulley weighs 400 lb. Determine all bending moments and torsional moments necessary for designing the shaft.

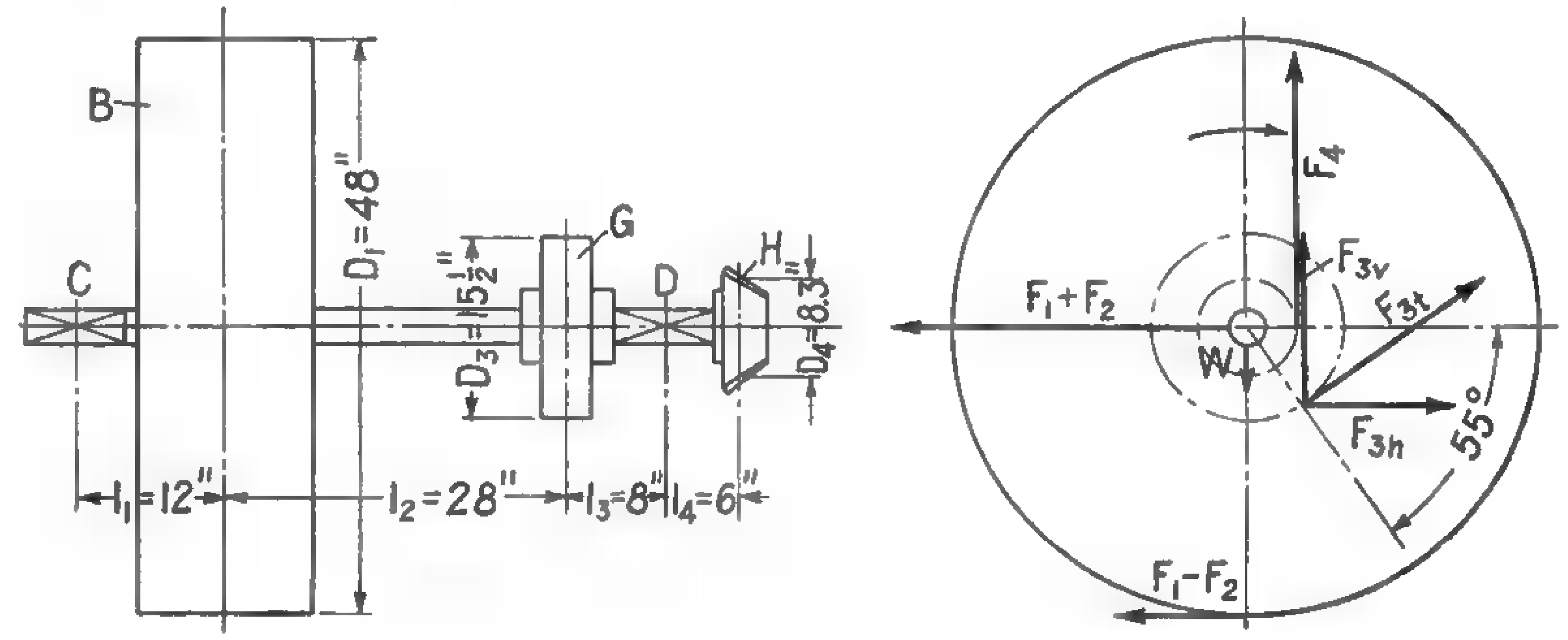


FIG. 20–1. Shaft loaded in bending and torsion.

The procedures for determining the forces acting upon the shaft are explained in chapters 27, 30, and 31. Only the bare calculations will be given here.

The net belt tension is

$$F_1 - F_2 = \frac{33{,}000 \times 12 \times P}{\pi D n} = \frac{33{,}000 \times 12 \times 100}{\pi \times 48 \times 225} = 1{,}167 \text{ lb}$$

The pull of the belt acting upon the bearings, in this case horizontally, is $F_1 + F_2$. Approximately, it is equal to $3(F_1 - F_2)$, or

$$F_1 + F_2 = 1{,}167 \times 3 = 3{,}500 \text{ lb}$$

Similarly, the tangential component of the tooth pressure in the spur gear is

$$F_{3t} = \frac{33{,}000 \times 60 \times 12}{\pi \times 15.5 \times 225} = 2{,}165 \text{ lb}$$

The normal tooth load, with a tooth-pressure angle β of approximately 15°, is

$$F_3 = \frac{2{,}165}{\cos \beta} = \frac{2{,}165}{0.966} = 2{,}240 \text{ lb}$$

This load must be resolved into a vertical component and a horizontal component. The horizontal component acting upon the shaft is

$$F_{3h} = F_3 \sin(\alpha - \beta) = 2{,}240 \sin 40° = 2{,}240 \times 0.643 = 1{,}440 \text{ lb}$$

and the vertical component acting upon the shaft is

$$F_{3v} = F_3 \cos(\alpha - \beta) = 2{,}240 \times 0.766 = 1{,}716 \text{ lb}$$

The tangential component of the tooth load in the bevel gear is

$$F_{4t} = \frac{33{,}000 \times 40 \times 12}{\pi \times 8.3 \times 225} = 2{,}705 \text{ lb}$$

The normal tooth pressure, which, as stated, is acting upward, is

$$F_4 = \frac{F_{4t}}{\cos\beta} = \frac{2,705}{0.966} = 2,800 \text{ lb}$$

In this case there is no horizontal component, and the vertical component acting upon the shaft is $F_{4v} = F_4 = 2,800$ lb.

The horizontal reaction in the left bearing C is

$$R_{1h} = \frac{3,500 \times (28+8)}{12+28+8} - \frac{1,440 \times 8}{12+28+8} = 2,625 - 240 = 2,385 \text{ lb}$$

The horizontal reaction in the right bearing D is

$$R_{2h} = \frac{3,500 \times 12}{48} - \frac{1,400 \times 40}{48} = 875 - 1,200 = -325 \text{ lb}$$

These values may be checked as follows:

$$R_{1h} + R_{2h} = 2,385 - 325 = 2,060 \text{ lb}$$

and

$$(F_1 + F_2) + F_{3h} = 3,500 - 1,440 = 2,060 \text{ lb}$$

The vertical reactions are

$$R_{1v} = \frac{400 \times 36}{48} - \frac{1,716 \times 8}{48} + \frac{2,800 \times 6}{48} = 300 - 286 + 350 = 364 \text{ lb}$$

$$R_{2v} = \frac{400 \times 12}{48} - \frac{1,716 \times 40}{48} - \frac{2,800 \times (6+48)}{48} = 100 - 1,430 - 3,150 = -4,480 \text{ lb}$$

As a check, $364 - 4,480 = -4,116$ lb, and $400 - 1,716 - 2,800 = -4,116$ lb.

The bending moment in the horizontal plane at the middle of the pulley hub is

$$M_{bh} = R_{1h} \times l_1 = 2,385 \times 12 = 28,600 \text{ lb-in.}$$

and that at the middle of the spur-gear hub is

$$M_{gh} = R_{2h} \times l_3 = -325 \times 8 = -2,600 \text{ lb-in.}$$

The bending moments in the vertical plane are

$$M_{bv} = R_{1v} \times l_1 = 364 \times 12 = 4,368 \text{ lb-in.}$$
$$M_{gv} = R_{2v} \times l_3 - F_4(+l_3 l_4) = -4,480 \times 8 - (-2,800)(8+6) = 3,360 \text{ lb-in.}$$

When several forces act in the same plane, as in the case of the reaction on bearing D and a force outside the support, it is well to remember that the resulting bending moment is obtained by superimposing the simple moments. The force on the overhung bevel gear produces a bending moment at the bearing D, which is

$$M_{dv} = F_4 \times l_4 = 2,800 \times 6 = 16,800 \text{ lb-in.}$$

The axial thrust F_a on the bevel gear causes a bending moment in the horizontal plane. This thrust, from other data, is $F_a = 1,425$ lb, and the moment is

$$M_{dh} = F_a \times 0.5D_4 = 1,425 \times 0.5 \times 8.3 = 5,920 \text{ lb-in.}$$

The resultant moments at these three dangerous points are as follows:

$$M_b = \sqrt{28,600^2 + 4,368^2} = 28,900 \text{ lb-in.}$$

$$M_g = \sqrt{2,600^2 + 3,300^2} = 5,080 \text{ lb-in.}$$

$$M_d = \sqrt{5,920^2 + 16,800^2} = 17,800 \text{ lb-in.}$$

Thus the maximum bending moment is at the pulley and is equal to 28,900 lb-in. The torsional moment in the left end, up to the pulley hub, is $T = 0$.

Between the pulley and the spur gear the torque is

$$T = \tfrac{1}{2}(F_1 - F_2)D_1 = 0.5 \times 1,167 \times 48 = 28,000 \text{ lb-in.}$$

Between the spur gear and the bevel gear the torque is

$$T' = T - \tfrac{1}{2}F_3D_3 = 28,000 - 0.5 \times 2,240 \times 15 = 11,200 \text{ lb-in.}$$

The dangerous section is at the pulley hub. To compute the equivalent torsional moment, apply equation 20–5 and take into account the stress concentration at the keyway by introducing the factor $K'' = 1.2$. Then

$$T_e = \sqrt{(28,900)^2 + (1.2 \times 28,000)^2} = 44,300 \text{ lb-in.}$$

20–4. Shear diagram. The procedure for finding a maximum bending moment is considerably simplified if the point at which it acts is known. This point can be easily found by means of a shear diagram, which also gives a graphical check of the values of the bearing reactions. The shear diagram is constructed by taking for the abscissas, distances along the shaft; and for the ordinates, all forces acting in the plane investigated, including the bearing reactions with their respective directions.

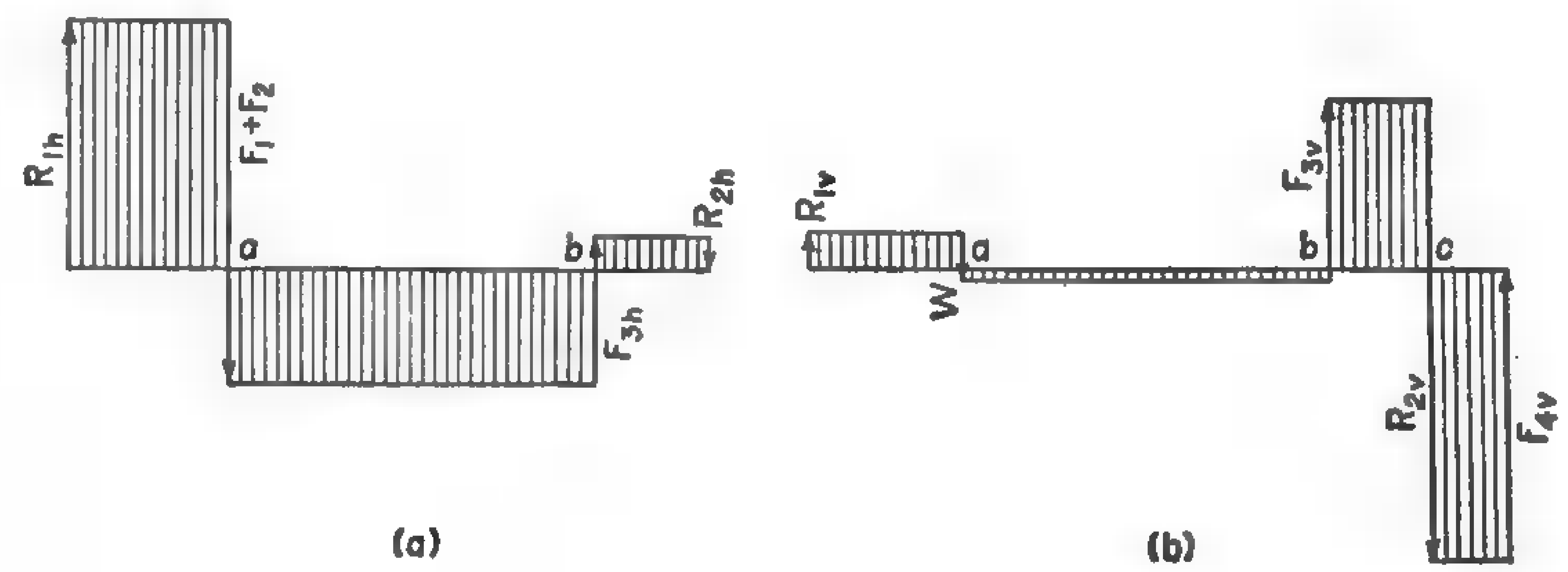


Fig. 20–2. Shear diagrams for shaft.

In Fig. 20–2a is shown the shear diagram for the forces in the horizontal plane in example 20–1. Point a, where the shear diagram intersects the axis the first time, is at the shaft section where the bending moment reaches the maximum value; point b indicates the position of the minimum bending moment. The shear diagram for the forces in the vertical plane is shown in Fig. 20–2b. In this case, points a and c indicate two sections of maximum bending moment, and point b indicates the position of a minimum bending moment.

If the reactions are computed correctly, the shear diagram, with the axis, must form a continuous contour. Also, if the construction of the shear diagram is correct, the sum of the diagram areas above the axis must be equal to that of the areas below it. However, to find the point of the maximum bending moment, the shear diagram does not have to be drawn accurately.

20–5. Shafts with impact loading. Shafts are subjected to bending impact as well as to torsional impact, and often a shaft is subjected to both simultaneously. Generally the impact has no initial velocity and thus is equivalent to a sudden load, which, as has been shown, creates a stress twice as high as that due to a steady load. For design purposes it is convenient to

consider a sudden application of the load and to multiply by 2 the steady bending moment M, or the torque T, in the design equations.

Heavy shocks, particularly those caused when the direction of rotation of the shaft is reversed under load, may increase the stress up to three times that due to a steady load; and allowance for such shocks can be made by multiplying by 3 the moments M and T in the design equations.

It is a general rule that the resistance to impact is measured by the resilience of a part and that the resilience reaches its maximum when all sections of the part are stressed equally and as high as possible, consistent with strength. This rule applies in full to shafts. If a shaft is made with a constant diameter, as many shafts are, the diameter must be determined by the maximum stress due to impact. Heavy shafts, especially short ones, should be made to approach uniform strength and should not be more rigid than is necessary for proper operation.

The designer must keep in mind the size coefficient e_{sz} (equation 5–8), the stress-concentration factor K' (equation 5–9), and the remarks in section 5–6 in regard to the index of sensitivity q.

20–6. Repetitive loading. Any rotating shaft carrying a transverse load is subjected to repeated stresses.

Bending. The bending stress in any outside fiber of such a shaft changes at every revolution from a maximum tension to a maximum compression. In order to avoid failure through progressive fracture, the stress must be kept below the endurance limit.

The shaft diameter necessary to resist bending is computed from equation 2–22. If the shaft has no discontinuity at the design section, the design stress to be used should be determined in the following manner: Take the endurance limit S_{en} in bending from the endurance diagram for the shaft material, and multiply this limit by the size factor e_{sz} (equation 5–8) and by the surface factor e_{sr} (Fig. 5–8), and then divide the result by the safety factor n. If the shaft has a discontinuity, the stress-concentration factor K_r (equation 5–41) and the index of sensitivity q (Table 5–2) must be used instead of the surface factor e_{sr}.

Torsion. The shaft diameter necessary to resist torsion must be computed from equation 20–4. To determine the design stress in this case, use the endurance limit S_{en} in torsion instead of the elastic limit S_e; and introduce all other factors mentioned in the preceding paragraph, using the surface factor e_{sr} from Fig. 5–8 in equation 5–23.

Torsion seldom varies from a maximum value in one direction to the same value in the opposite direction. Usually, the torque fluctuates back and forth from a maximum value T_1 to a smaller one T_2. In this case the actual stress amplitude S_a may be found from the corresponding endurance

diagram by using equation 5–38. The design amplitude is found by dividing S_a by the safety factor n, which in this case may be taken as 1.25 to 1.5.

Simultaneous torsion and bending. In this case the amplitudes of the bending and torsional moments should be combined by applying equation 20–5. The shaft diameter is computed by equation 20–4, in which the equivalent moment is replaced by the moment amplitude and in which the stress is replaced by the design stress amplitude. The design stress amplitude is computed by using an equation similar to equation 5–26 and by introducing the stress-concentration factor K_r. Thus,

$$S_{da} = \frac{e_{sz}e_{sr}'S_a}{nK_r} \tag{20–8}$$

where e_{sr}' is computed by equation 5–23 and S_a is found from the endurance diagram for torsion.

Discontinuities which must be taken into account in the design—or, if possible, avoided—are rough surfaces, abrupt changes of the diameter with an insufficient fillet, press and shrink fits with high pressures, and keyways. Numerical data for the evaluation of the corresponding stress-concentration factors are given in sections 3–7, 3–8, and 5–8 to 5–10.

EXAMPLE 20–2. Determine the diameter of the shaft for the data of example 20–1. Assume that the torque constantly fluctuates between a maximum value and zero. The work delivered is as indicated in example 20–1.

If the average, or mean, torque is designated by T_m, with $T_2 = 0$, the maximum torque is $T_1 = 2T_m$. From example 20–1, $T_m = 28{,}000$ lb-in. The torque amplitude, according to its definition, is $T_a = \frac{1}{2}(2T_m) = 28{,}000$ lb-in.

Since the bending moment is produced by the same forces which produce the torque, it is evident that the bending moment will fluctuate in the same proportion as the torque. However, in a revolving shaft the amplitude will be measured from the maximum value in one direction to the same value in the opposite direction. Thus it can be assumed that $M_1 = 2M$ and that $M_2 = -2M$. The maximum bending moment was found in example 20–1 to be $M_{gh} = 28{,}600$ lb-in. Hence $M_1 = 57{,}200$ lb-in. Also, $M_a = \frac{1}{2} \times [57{,}200 - (-57{,}200)] = 57{,}200$ lb-in.; and $M_m = 0$.

The amplitude of the equivalent torsional moment, by equation 20–5, is

$$T_{ae} = \sqrt{57{,}200^2 + 28{,}000^2} = 63{,}600 \text{ lb-in.}$$

From Fig. 4–3, which is the nearest to SAE 1020, $S_a = 16{,}000$ psi. The size coefficient may be taken from Table 5–1 as $e_{sz} = 0.84$. The surface coefficient, from Fig. 5–8, is $e_{sr} = 0.92$; and by equation 5–3,

$$e_{sr}' = 0.425 + 0.575 \times 0.92 = 0.954$$

The keyway factor $K_r = 1.14$, and the safety factor n can be taken as low as 1.5 because all other adverse influences are taken into account separately. Thus, by equation 20–8, the design stress amplitude is

$$S_{da} = \frac{0.84 \times 0.954 \times 16{,}000}{1.5 \times 1.14} = 7{,}500 \text{ psi}$$

The minimum shaft diameter, from equation 20–1, is

$$D = \sqrt[3]{\frac{16 \times 63{,}600}{\pi \times 7{,}500}} = \sqrt[3]{43.2} = 3.51 \text{ in.}$$

Conclusion. Many machines, including various prime movers, cannot perform their work without shafts. Whether a shaft is large or small in diameter and length, it is important and indispensable for the functioning of the whole mechanism. Since shafts on the average are the most stressed machine parts, no effort should be spared in designing them properly. For a revolving shaft, the corresponding endurance diagram should be used. Transmission shafts are probably the only shafts which are sufficiently standardized to permit them to be designed directly by formulas.

20–7. Design of transmission shafting. Transmission shafts are used so often that the American Society of Mechanical Engineers has worked out a special procedure for their design.[4] The general formula for the outside diameter D, in inches, given in the ASME Code, is

$$D = \sqrt[3]{\frac{16\sqrt{[K_mM + \frac{1}{8}aFD(1+K^2)]^2 + (K_tT)^2}}{\pi S_d\,(1-K^4)}} \qquad (20\text{–}9)$$

The notations not used before, or needing explanation, are as follows:

S_d is the design stress, taken as the allowable *shear* stress, in pounds per square inch;

K_m is the numerical combined shock and endurance factor to be applied to the computed bending moment;

K_t is the numerical combined shock and endurance factor to be applied to the computed torsional moment;

$K = D_1/D$ is the ratio of the inside diameter to the outside diameter of a hollow shaft;

a is the ratio of the maximum intensity of stress to the average intensity, resulting from the axial loading only;

F is the axial tensile or compressive load, in pounds.

Values of K_m and K_t are given in Table 20–1. In most cases equation 20–9 will be simplified by the elimination of factors that do not apply. Thus, in the absence of an axial force, $F = 0$ and the corresponding term disappears; and for pure torsion, $M = 0$.

The presence of the unknown diameter D in the second term in brackets under the radical sign in equation 20–9 compels the use of the cut-and-try method. First D must be assumed and a value for D computed by equation 20–9. The approximate value of D thus found is then inserted under the radical sign, and a corrected value for D is computed.

Allowable stresses for simple torsion. According to the ASME Code, for commercial-steel shafting, the allowable shear stress for simple torsion is $S_s = S_d = 8{,}000$ psi. For steel shafting purchased under definite physical specifications $S_d = 0.3S_e$, where S_e is the elastic limit in tension; but S_d should not be higher than 0.18 S_u, where S_u is the ultimate tensile strength.

[4] *Code for the Design of Transmission Shafting.*

TABLE 20–1

SHOCK AND ENDURANCE FACTORS

Nature of Loading	K_m	K_t
Stationary shafts:		
Gradually applied load	1.0	1.0
Suddenly applied load	1.5–2.0	1.5–2.0
Rotating shafts:		
Steady or gradually applied loads	1.5	1.0
Suddenly applied loads, minor shocks only	1.5–2.0	1.0–1.5
Suddenly applied loads, heavy shocks	2.0–3.0	1.5–3.0

Combined torsion and bending. The allowable stresses for combined torsion and bending are the same as those for simple torsion.

Simple bending. For commercial shafting the allowable stress in simple bending is twice that for simple torsion, or $S_d = 16{,}000$ psi. For shafting with definite specifications $S_d = 0.6S_e$, but it should not be higher than $0.36S_u$.

Keyways. In shafts with keyways the allowable stresses are 75 per cent of the values just given.

Axial loading. The maximum stress s resulting from an axial force F is

$$s = a\frac{F}{A} \qquad (20\text{–}10)$$

where A is the cross-sectional area of the shaft, in square inches, and a is the column-action factor. For short columns, or when $l/k < 115$, a is found by the relation

$$a = \frac{1}{1 - 0.0044\dfrac{l}{k}} \qquad (20\text{–}11)$$

For long columns, or when $l/k \geqq 115$, Euler's formula must be used. From equation 2–38,

$$a = \frac{S_e}{\pi^2 nE}\left(\frac{l}{k}\right)^2 \qquad (20\text{–}12)$$

The ASME Code gives $n = 1$ for hinged ends, and $n = 2.25$ for fixed ends. For both ends pinned, guided, and partly restrained, as they are in bearings, $n = 1.6$.

Hollow shafts. The terms $(1+K^2)$ and $(1-K^4)$ in equation 20–9 take into consideration the weakening of the shaft by making it hollow with an inside diameter $D_1 = KD$. For solid shafts, $K = 0$.

Graphical solutions. The ASME Code gives two charts which may save time in computing the shaft diameters. However, these charts are of real value only when a great number of shafts must be designed.

Transverse shear. In a shaft subjected to bending on a short span by a heavy transverse load F, the maximum shear stress occurs in the fibers in the neutral plane, and its magnitude may be computed for a solid circular section from the expression in Table 2–3 for case b. Thus,

$$s_s = \frac{16F}{3\pi D^2} = \frac{1.69F}{D^2} \tag{20–13}$$

For a hollow circular shaft, equation 2–18 gives

$$s_s = \frac{16F(D^3 - D_1^3)}{3\pi(D^4 - D_1^4)(D - D_1)} = \frac{1.69F(1-K^3)}{D^2(1-K^4)(1-K)} \tag{20–14}$$

Example 20–3. Using the recommendations of the ASME Code, find the shaft diameter for the conditions of example 20–1.

For a solid shaft without an axial load, equation 20–9 becomes

$$D = \sqrt{\frac{16}{\pi S_d}\sqrt{(K_m M)^2 + (K_t T)^2}}$$

From example 20–1, the maximum bending moment is $M = 28{,}900$ lb-in., and the maximum torque is $T = 28{,}000$ lb-in.

For commercial steel shafting the nominal stress $S_d = 8{,}000$ psi. Because of the presence of a keyway the design stress should be lowered to $S_d = 8{,}000 \times 0.75 = 6{,}000$ psi. From Table 20–1, for a revolving shaft with a steady load, $K_m = 1.5$ and $K_t = 1$. Thus

$$D = \sqrt{\frac{16}{6{,}000\pi}\sqrt{(1.5 \times 28{,}900)^2 + 28{,}000^2}} = \sqrt{\frac{16 \times 51{,}600}{6{,}000\pi}} = 3.525 \text{ in.}$$

Example 20–4. Determine the diameter of a shaft, using data of example 20–1 but assuming that the torque fluctuates from a maximum value to zero. The work delivered is as indicated in example 20–1.

The procedure is similar to that in example 20–3. However, the load should be considered as applied suddenly but without heavy shocks, and the higher values from Table 20–1 should therefore be used. For $K_m = 2.0$ and $K_t = 1.5$,

$$D = \sqrt{\frac{16}{6{,}000\pi}\sqrt{(2 \times 28{,}900)^2 + (1.5 \times 28{,}000)^2}} = 3.929 \text{ in.}$$

By the more accurate method of endurance diagrams, example 20–2, the size was found to be 3.51 in., or somewhat smaller. However, with the more accurate method the low safety factor n of 1.5 could be used. With a safety factor n of 2, such as used in the Code, the size would become $d = 3.51\sqrt[3]{2/1.5} = 3.89$ in., which is practically the same as the value found in example 20–4.

20–8. Practical design considerations. In addition to the information given at the end of section 20–1, it may be stated that standard bearings, set collars, clutches, and pulleys, and other parts carried in stock, are bored $\frac{1}{16}$ in. undersize in $\frac{1}{4}$-in. increments up to $5\frac{15}{16}$ in. Therefore transmission shafts should be made from T & G shafting turned with diameters in $\frac{1}{4}$-in. increments and $\frac{1}{16}$ in. under the even sizes, beginning with $\frac{15}{16}$ in. and running up to $5\frac{15}{16}$ in. Some manufacturers of power equipment carry in stock shafts that are $\frac{1}{16}$ in. under the even size, and parts with diameters from $6\frac{7}{16}$ in. up to $7\frac{15}{16}$ in., in $\frac{1}{2}$-in. increments.

TABLE 20–2

Data for Shaft Calculations

Type of Shaft Loading	Coefficient C in Equation 20–15	Allowable Stress S_d (psi)
Shafts heavily loaded, subjected to shock or reversed under full load	5	2,500
Line shafts and countershafts loaded in bending but not reversed...	6.5	4,000
Line shafts bare or with pulleys close to the bearings............	9.5	6,400

Example 20–5. Determine the actual sizes of transmission shafting that would be used in examples 20–3 and 20–4.

The computed value of d, or 3.525 in., will require shafting with a nominal size of $3\frac{11}{16}$ in., for which $d = 3.685$ in.

The computed value of d, or 3.92 in., will require shafting with a nominal size of $3\frac{15}{16}$ in., for which $d = 3.935$ in.

Simplified design procedure. In practice the magnitude of the bending moment is seldom known exactly. Therefore it is customary to determine the shaft diameter as a function of the torsional moment alone by using equation 20–1 or equation 20–2 with a correspondingly low allowable shear stress S_d to allow for bending. Average values of S_d recommended in catalogs of manufacturers of power-transmission machinery are given in the right-hand column of Table 20–2. Normally S_d may be taken as 4,000 psi.

Rigidity in torsion. Usual practice is to limit the angle of torsion to 1 deg in a length L of $20D$. For shafts over 20 ft long it is recommended that the angle not exceed 0.1 deg per ft, or even 0.075 deg per ft, of shaft length.[5]

Stiffness. The transverse deflection of line shafts and countershafts is usually limited to 0.01 in. per foot of length. Sometimes a greater stiffness may be desired.

Instead of computing the deflection, which in the case of higher speeds is increased by the centrifugal whirl, the maximum distance L between the bearings, in feet, may be computed by the empirical formula[6]

$$L = \frac{1{,}500}{n + 1{,}500} C D^{3/4} \tag{20–15}$$

where C may be taken from Table 20–2, and where n is the speed in revolutions per minute.

Load factor. The rotation of most machines is not absolutely uniform. Only steam turbines and gas turbines have a real uniform rotation, owing to the large flywheel effect of their rotors and their high speeds. The speed

[5] Hütte, *op. cit.*

[6] Pinkney, *Machinery* (May, 1915); also Lionel S. Marks, ed., *Mechanical Engineers' Handbook*, 5th ed. (New York: McGraw-Hill Book Company, Inc., 1951), p. 930.

TABLE 20–3

LOAD FACTORS FOR VARIOUS MACHINES

Driver	Driven Machinery	Factor k_l
Steam turbine	Electric generator, steady load; turbine blower	1.00
	Electric generator, uneven load; centrifugal pump	1.25
	Induced-draft fan; line shaft, gear drive	1.50
	Rolling mill, gear drive	2.00
Electric motor	Turbine blower; metalworking machinery	1.25
	Centrifugal pump; woodworking machinery	1.50
	Line shaft; ship propeller; double-acting pump	1.75
	Triplex single-acting pump; elevator; crane	1.75
	Compressor, air or ammonia	1.75
	Rolling mill; rubber mill	2.50
Steam engine	Values for electric-motor drive multiplied by 1.2 to 1.5	
Gas and oil engines	Values for electric-motor drive multiplied by 1.3 to 1.6, the factor depending on the coefficient of steadiness of the flywheel.	

of electric motors fluctuates a little because of the intermittent impulses given to their rotors. Prime movers with reciprocating pistons have still less uniformity of rotation. At the same time, the torque required to operate machinery also varies. In addition the rotating parts of the driven machines have masses and flywheel effects. Therefore a machine part such as a coupling, clutch, or shaft that transmits power from a prime mover to a driven machine is subjected to a fluctuating torque the maximum value of which may exceed the average torque by as much as 100 per cent or more.

The ratio of the maximum torque to the average, or nominal, torque is called the *load factor* and is designated as k_l. Some typical values of the load factor are given in Table 20–3.

If, because of lack of more accurate data, a shaft is designed by the simplified procedure, the transmitted torque T or horsepower P should be multiplied by the load factor k_l, data for which are shown in Table 20–3.

EXAMPLE 20–6. Find the diameter of a transmission shaft connected to a six-cylinder, 100-hp oil engine through a belt drive. The shaft is driving woodworking and metalworking machinery and runs at 225 rpm. Assume that the maximum load is equal to the rated engine power.

The required diameter may be found by applying equation 20–2 and using $S_d = 4{,}000$ psi from Table 20–2. The maximum horsepower is found by using Table 20–3. For a load consisting of metalworking and woodworking machinery the average value of k_l, or 1,375, would be used for an electric-motor drive. A six-cylinder oil engine has a high coefficient of steadiness, and a factor of 1.3 would be sufficient. Since the shaft is not directly connected to the engine but is driven through a belt, which absorbs a considerable amount of speed variation, the factor may be lowered to about 1.2. The final load factor is therefore $k_l = 1.375 \times 1.2 = 1.65$. Thus, by equation 20–2,

$$D = 68.5\sqrt[3]{\frac{100 \times 1.65}{225 \times 4{,}000}} = 68.5\sqrt[3]{0.000184} = 3.89, \text{ or } 3\tfrac{15}{16} \text{ in.}$$

CHAPTER 21

Couplings and Positive Clutches

21–1. General considerations. A *coupling* is used to connect two shafts permanently; it is disconnected only for repairs or to make a change in the installation. A *clutch* permits easy and quick connection and disconnection of two shafts. A clutch is also used instead of a key to connect the shaft with a revolving part, such as a pulley or a gear.

Couplings are of two main types, *rigid* and *flexible*.

Clutch constructions are based on the *positive-action* and *friction* principles.

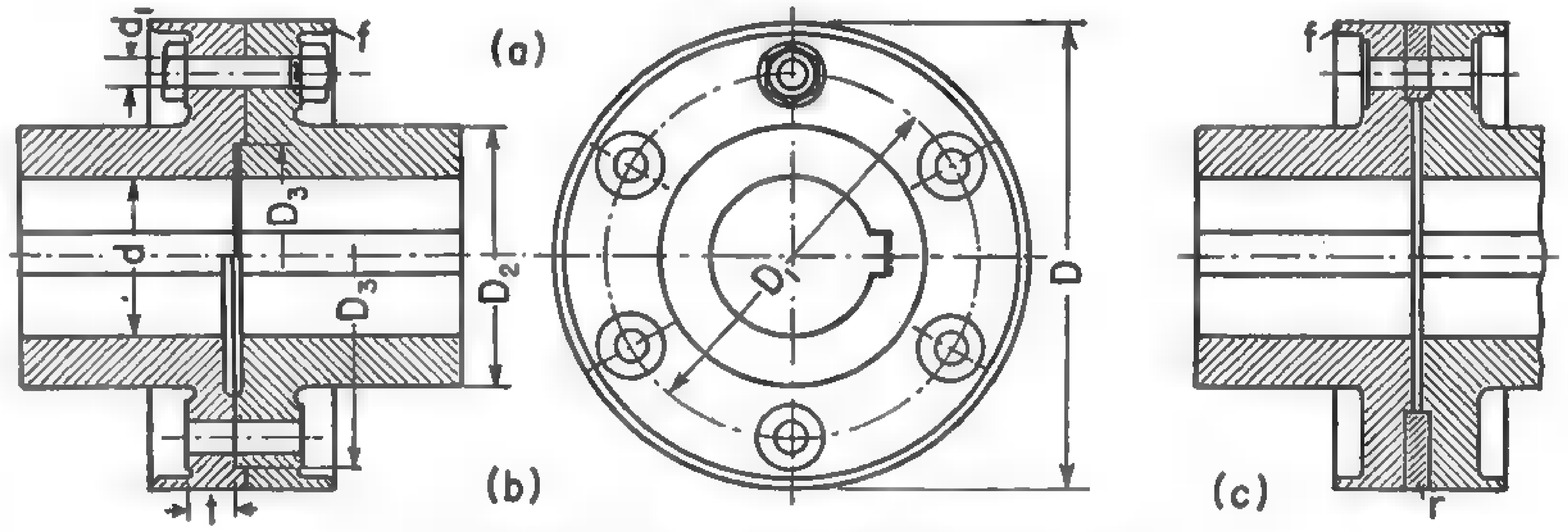


FIG. 21–1. Types of flange couplings.

21–2. Flange coupling. The flange coupling, Fig. 21–1, consists of two cast-iron flanges, keyed to the shaft ends and bolted together. The best practice is to press or shrink the flanges on the shafts, in which case straight keys should be used. If the flanges are put on only with a push fit, taper keys must be used. After the connection has been assembled, it is advisable to true up the faces of the flanges in a lathe to make sure that they are normal to the axis of rotation.

Register. A register is necessary to insure correct alignment of the two shafts. It is usually made by turning a projection on one flange and boring a slightly deeper female register in the other flange. The register is made either with a small diameter D_3, as in Fig. 21–1a, or with a large diameter, as in Fig. 21–1b, in which case the inner part of the face may be relieved or left rough. Another method is to make the faces of both flanges flat and to obtain a register by protruding one shaft end about $\frac{1}{4}$ in. beyond the face and having the shaft end in the mating flange slightly shorter, about $\frac{5}{16}$ in. inside the face.

A third type of register uses a separate ring r, Fig. 21–1c. The ring is usually split into two halves. This construction permits an easy disconnection of the shafts when one of them must not rotate.

Steady torque that always acts in the same direction may be transmitted by friction between the flange faces; and holes slightly larger than the bolt shank may be drilled. A better practice is to ream the holes to fit the bolts when assembling the coupling, and to let the bolts work in shear. In either case the surfaces under the bolt heads and nuts must be normal to the bolt axis. Either the surfaces may be spot-faced, as in the upper half of Fig. 21–1c, in which case slightly raised bosses, Fig. 21–1a and 21–1b, are advisable; or the flanges are finished all over, as in the lower half of Fig. 21–1c.

Couplings transmitting a large and not very steady torque, such as in boat propeller shafts, are made with tapered headless bolts fitted into reamed holes. A taper of 1:20 or 1:24, referred to the diameter, is used.

The projecting bolt heads and nuts must be covered by a safety flange f, Fig. 21–1, which also increases the rigidity of the flanges.

Dimensions. The torque acting on a flange coupling is transmitted through the bolts. The size of the bolts depends on the number of bolts n and on the bolt-circle diameter D_1. There is no direct relation between these two factors and the torque. Both n and D_1 must be selected more or less arbitrarily; the bolt size and the other dimensions can then be computed.

So far the dimensions of flange couplings are not standardized. However, the dimensions of couplings built by different manufacturers do not vary much. Since the torque capacity of a coupling is limited by the torque capacity of the shaft, it is logical, in establishing empirical expressions for the main dimensions, to present them as functions of the shaft diameter d.

The commonly used approximate number of bolts is

$$n = 0.5d + 3 \qquad (21\text{–}1)$$

The preliminary value for the bolt diameter d_1 may be determined by the empirical formula

$$d_1 = \frac{0.5d}{\sqrt{n}} \qquad (21\text{–}2)$$

The average value of the diameter of the bolt circle, as found in stock couplings, is

$$D_1 = 2(d + 1) \qquad (21\text{–}3)$$

A suitable value for the hub diameter D_2 is the same as that used for belt pulleys, or

$$D_2 = 1.5d + 1 \qquad (21\text{–}4)$$

Since the bolts should be located halfway between the hub and the outside of the flange, the outside diameter D should be taken as

$$D = 2.5d + 3 \qquad (21\text{–}5)$$

This is in good agreement with data for stock couplings in catalogues.

The hub length l usually is established by the relation

$$l = 1.25d + 0.75 \qquad (21\text{–}6)$$

Design procedure. The first step in design is to determine the main dimensions n, D_1, D_2, D, and l from equations 21–1 to 21–6. In deciding on the number of bolts n, the nearest larger even number must be used, except for shafts under $1\frac{1}{2}$ in., which are made with three bolts.

The next step is to determine the tangential force F_t necessary to transmit the required torque T, referred to the selected bolt-circle diameter D_1. After this the diameter d_1 determined from equation 21–2 should be checked to make certain that a sufficient friction force F_t is produced. Here it is a good practice to assume that only one-half the total number of bolts is properly tightened. If the bolts are fitted, they should be checked for shear and crushing. Shear of bolts may occur between the flange faces. If the bolts are not fitted into reamed holes, it is advisable to consider again that only half the total number of bolts are doing the work.

Crushing of bolts may take place if the projected bearing surface td_1, Fig. 21–1, is not sufficient. According to equation 5–14 the allowable stress S_d for steel in bearing can be taken about twice as great as in ordinary compression.

The flange thickness t is found by considering that the flange may shear at the hub. As a general rule, t should be slightly greater than the bolt diameter d_1.

The shaft keys must be checked in shear and crushing. If necessary, two keys located 90 deg apart are used in each hub.

Finally, it is necessary to determine from practical consideration those dimensions that cannot be computed by stress analysis, such as the type, diameter, and depth of the register, and the thickness and height of the safety sleeve to cover the nuts and bolt heads.

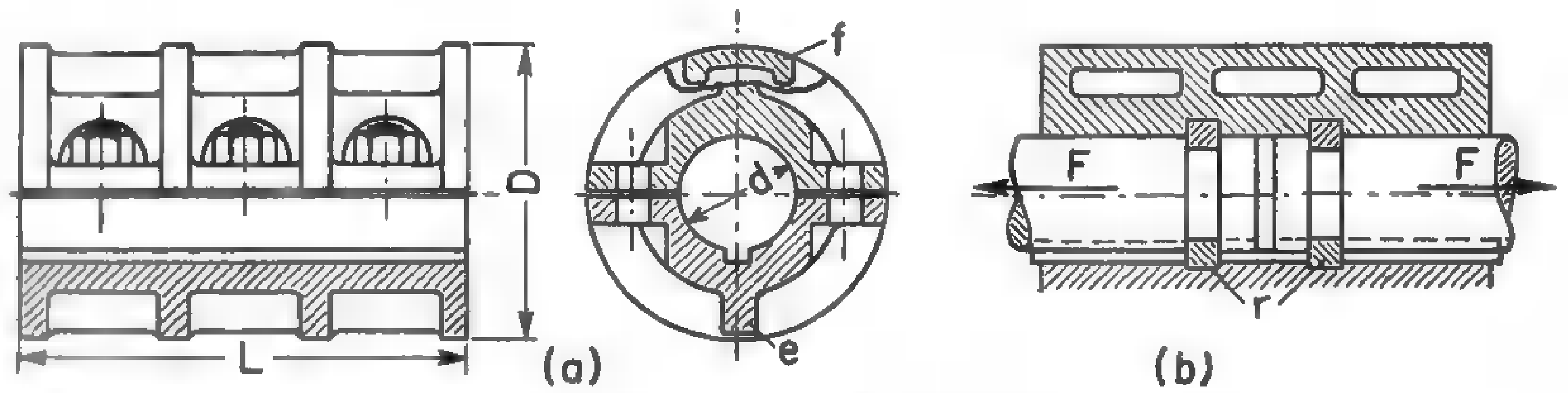


FIG. 21–2. Types of ribbed clamp couplings.

21–3. Clamp coupling. The coupling shown in Fig. 21–2a is made in two parts which are bolted together with through bolts. During the boring operation the two halves are separated by a thin shim. A one-piece square key assists in transmitting the torque. For diameters up to $3\frac{15}{16}$ in., six bolts are used; for larger sizes, up to $6\frac{15}{16}$ in., eight bolts are used. For a $1\frac{3}{16}$-in. shaft, the length L should be about $4.5d$, and the outside diameter D should be about $3.5d$; with an increase in size these dimensions are made relatively smaller, being reduced to $L = 3.2d$ and $D = 2.7d$ for a $6\frac{15}{16}$-in. shaft.

In the smaller sizes the coupling is made of cast iron; in larger sizes, of cast steel. In sizes of $4\frac{7}{16}$ in. and larger, some manufacturers make the couplings of cast iron with reinforcement f, Fig. 21–2a, instead of the axial rib e.

Parts of a shaft subjected to an axial force tending to pull them apart must be equipped with split rings r, Fig. 21–2b, ground on the sides to fit accurately into the grooves.

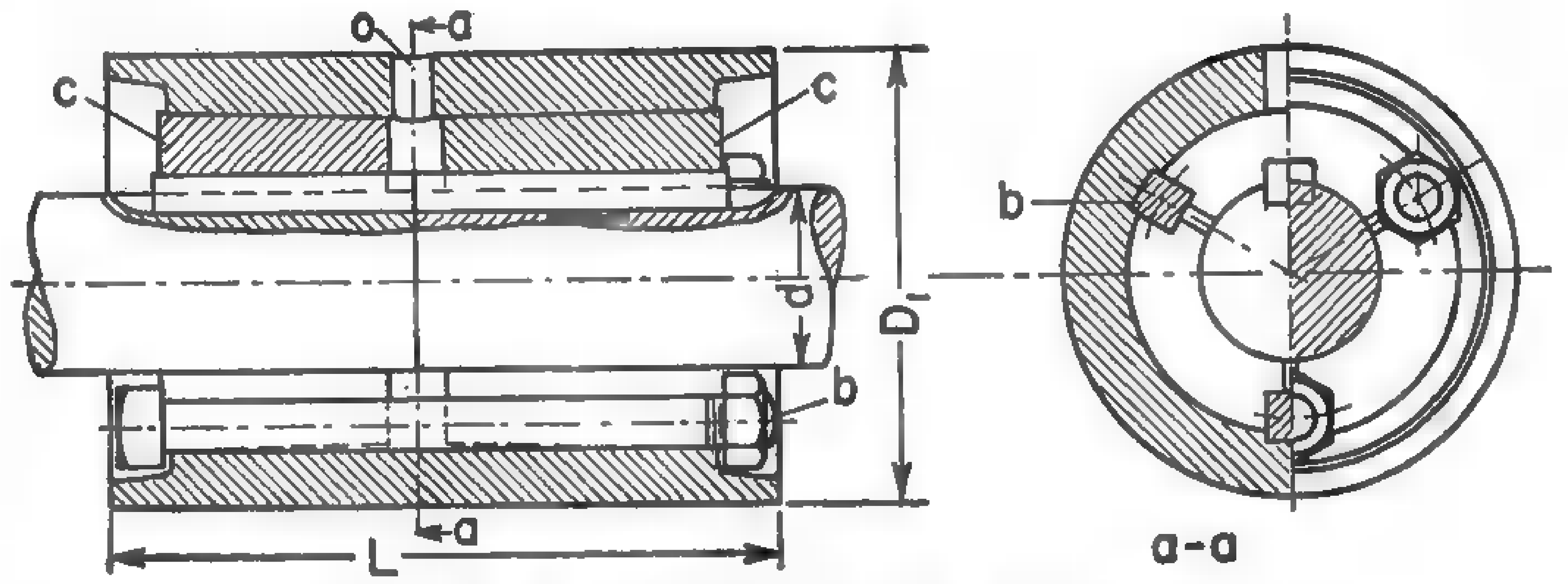


Fig. 21–3. Sellers cone-vise coupling.

Compression couplings are based on the pressure which can be created by wedge action produced by a hollow cone. In Fig. 21–3 the cast-iron split cones c are pulled together by three bolts b. Square keys make the coupling still more positive. The square shape of the shanks of the bolts b facilitates the tightening of the nuts. The inspection opening o permits the mechanic to see whether the cones are pulled together.

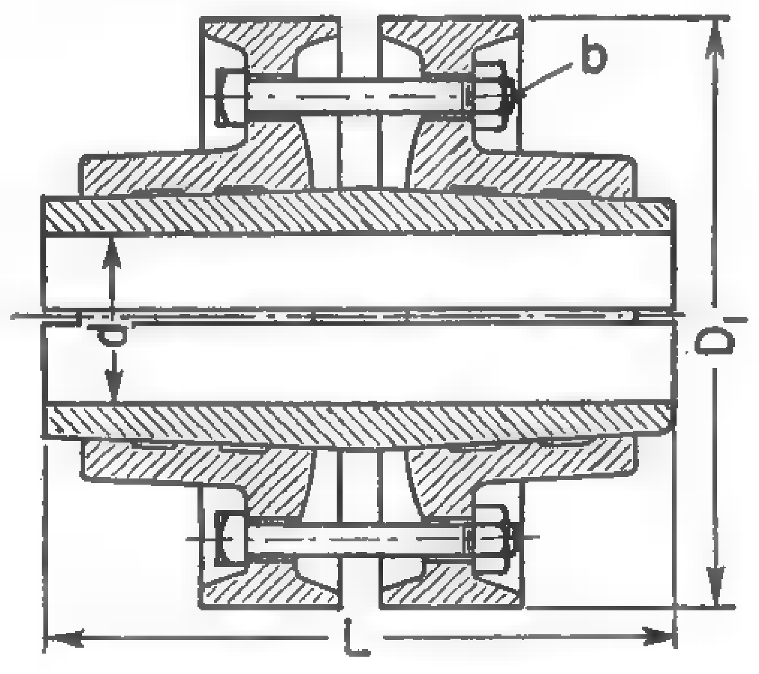


Fig 21–4. Flange compression coupling.

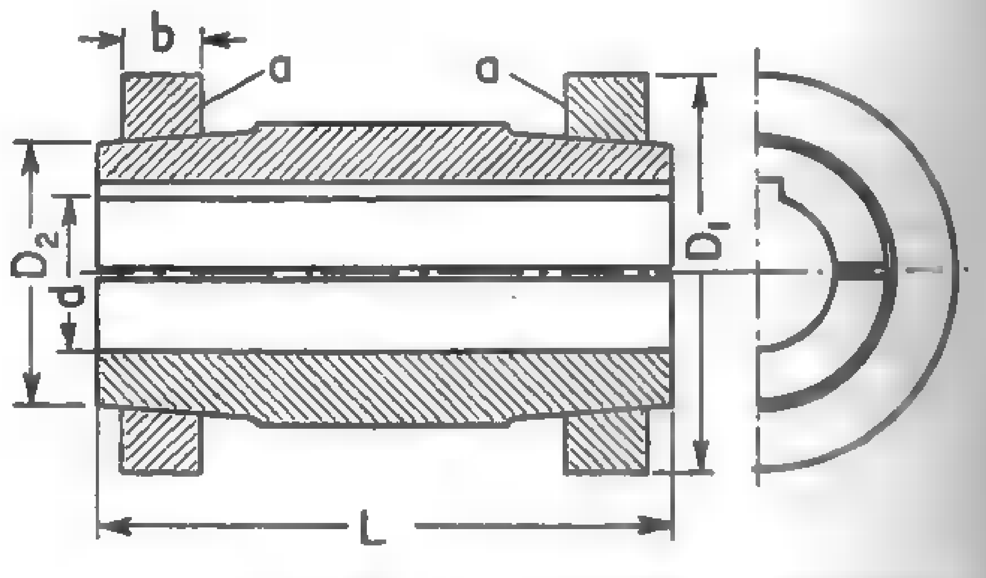


Fig. 21–5. Ring compression coupling.

The Sellers coupling is used for shafts with diameters from $1\frac{7}{16}$ in. up to $8\frac{15}{16}$ in. The length L is made about $4d$; the outside diameter D_1 is made $2.6d$ for values of d up to $4\frac{7}{16}$ in. and is made relatively smaller, down to about $D_1 = 2.4d$, for the large sizes. The taper is made $\frac{3}{8}$ in. per foot, three times as much as in standard taper keys. Other dimensions may be found from usual strength considerations.

In Fig. 21–4 is shown a coupling in which the split double cone is stationary and the two outside cones are pulled together by bolts b. In sizes up to $d = 2\frac{3}{16}$ in., four bolts are used; in larger sizes, up to $3\frac{15}{16}$ in., it is better to use six bolts. The outside dimensions D_1 and L are approximately the same as those for the flange coupling, Fig. 21–1.

Finally, in Fig. 21–5 is shown a compression coupling in which the wedge action is obtained by driving on, usually by hammer blows, two steel rings a. This type, in addition to being rather inexpensive, has the advantage that it can be used for outdoor service where the coupling is likely to become rusted. This coupling is made for shafts up to $5\frac{15}{16}$ in.

In compression couplings, $L = 4d$. For the coupling in Fig. 21–5, $D_1 = 2.5d$, $D_2 = 1.9d$, and $b = 0.5d$. For the coupling in Fig. 21–4, the outside diameter and the bolt-circle diameter are made the same as those for a plain flange coupling.

21–4. Flexible couplings. The purpose of a flexible coupling is to allow for imperfect alignment of two joining shafts, or to absorb impact from the fluctuation of torque or of angular speed. In practice, perfect alignment and constant torques and speeds are very seldom obtained. If two shafts are even slightly misaligned and are connected by a rigid coupling, they are subjected to continuous reversal of bending stresses, which eventually leads to a failure through progressive fracture. In addition, there is always excessive friction and wear of the bearings. Therefore, in most cases of joining shafts which transmit power, it is necessary or at least desirable to use a flexible coupling of one type or another.

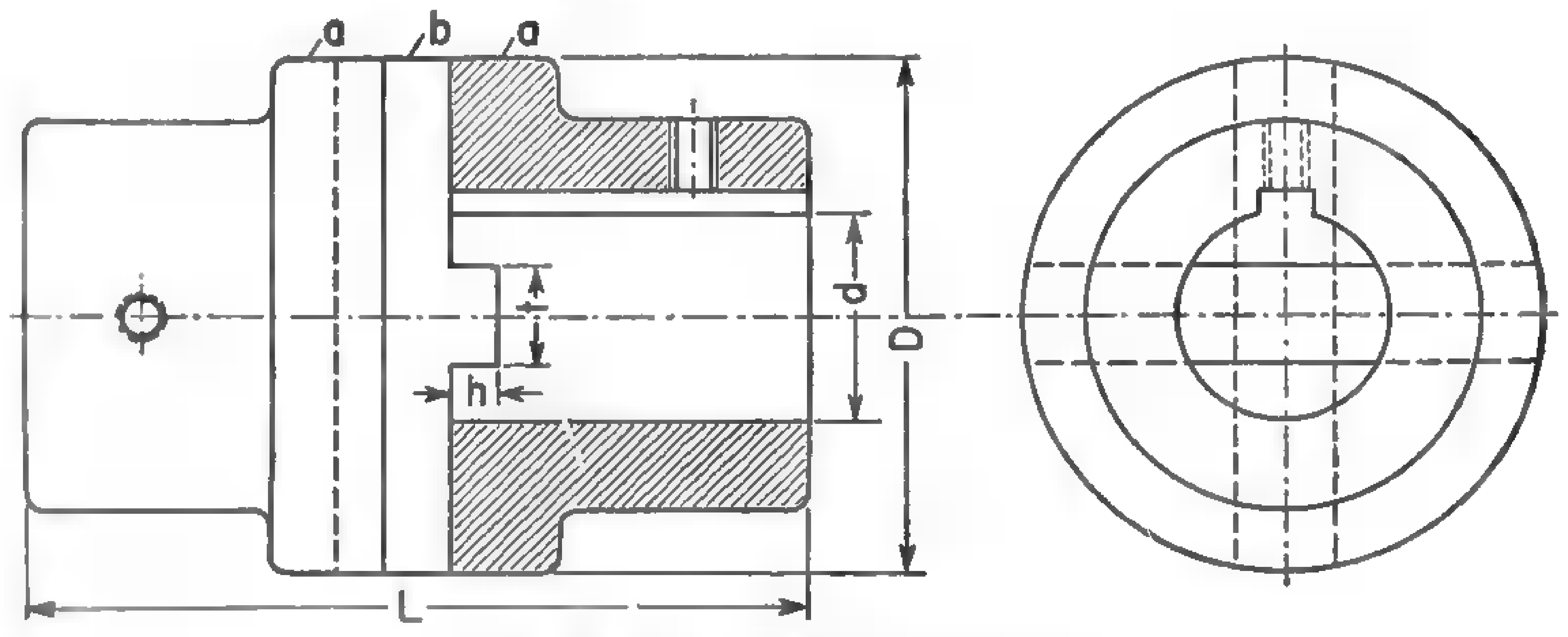


Fig. 21–6. Oldham flexible coupling.

From the standpoint of design, all flexible couplings may be divided into two classes: couplings whose flexibility is obtained kinematically by the use of rigid members in which constraint is absent in certain directions; and couplings with incorporated flexible members. The first type counteracts misalignment only, while the second type anticipates both misalignment and impact. A few of the more typical constructions will be briefly discussed.

21-5. Couplings with kinematic flexibility. Several types of couplings that provide flexibility kinematically will be described here.

Oldham coupling. In Fig. 21-6 is shown the Oldham coupling. The tongues of the centerpiece b, located at right angles to each other, are fitted to grooves in each hub a. In the position shown, the left-hand tongue can slide up and down while the right-hand tongue can slide to and fro. The combined action produces a flexible connection which is especially adapted to shafts which are parallel but not collinear. This connection also takes care of slight angular misalignment. The Oldham coupling is intended for low speeds.

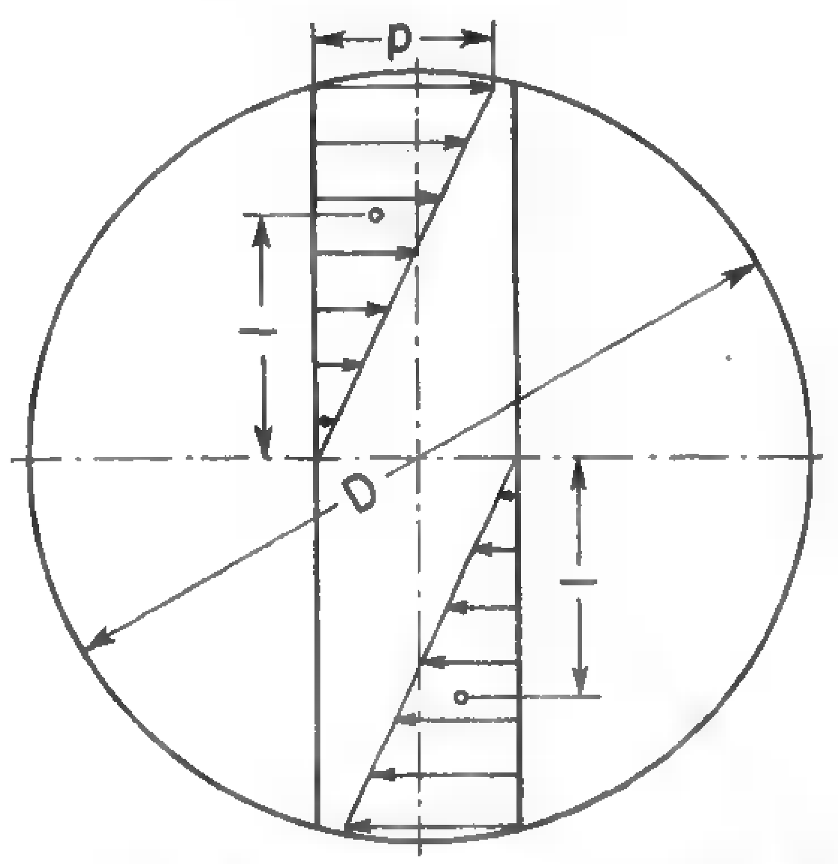


Fig. 21-7. Pressure distribution in an Oldham coupling.

The design of the Oldham coupling must be based on the allowable pressure between the faces of the grooves and the tongues. Because of the elasticity of the material, the pressure distribution for perfectly flat surfaces can be assumed to change on each side, as shown in Fig. 21-7, from a maximum value p at the periphery of the coupling to 0 at the center line. The total pressure on each side then is $F = \frac{1}{4}pDh$, where h is the axial dimension of the contact area. The distance to the pressure-area centroid from the center line is $l = \frac{2}{3} \times \frac{1}{2}D = \frac{1}{3}D$. Thus the torque transmitted by both sides of the tongue is

$$T = 2Fl = \tfrac{1}{6}pD^2h \tag{21-7}$$

The horsepower which can be transmitted at n rpm is

$$P = \frac{Tn}{63{,}030} = \frac{pD^2hn}{378{,}180} \tag{21-8}$$

The allowable pressure p must not exceed 1,200 psi. For a considerable misalignment, meaning an appreciable sliding travel, the maximum pressure should not exceed 1,000 psi. **The outside diameter D is made equal to** about $3d$ to $4d$, and the dimension t of the tongue is made about $0.45d$. Other dimensions may be scaled from Fig. 21-6.

American flexible coupling. In Fig. 21-8 is shown the American flexible coupling, which operates on the same principle as the Oldham coupling. It consists of two identical hubs f, Fig. 21-9a, turned at right angles to one another, and a floating centerpiece b between them. The floating member b is a square cast-iron block with screwed-on bearing strips c, made of Bakelite reinforced with canvas and impregnated with graphite. A hole in the center provides clearance for the shaft ends. The piece b is hollow, and the

cavity is filled with lubricant, which reaches the surface of the bearing strips c through porous reeds in the block and felt pads in the strips.

The coupling may be used with an angular misalignment up to 1 deg and a parallel shaft eccentricity e, Fig. 21-8, not greater than $0.003d$. Its design can be based on equations 21-7 and 21-8 by substituting $a\sqrt{2}$ for D.

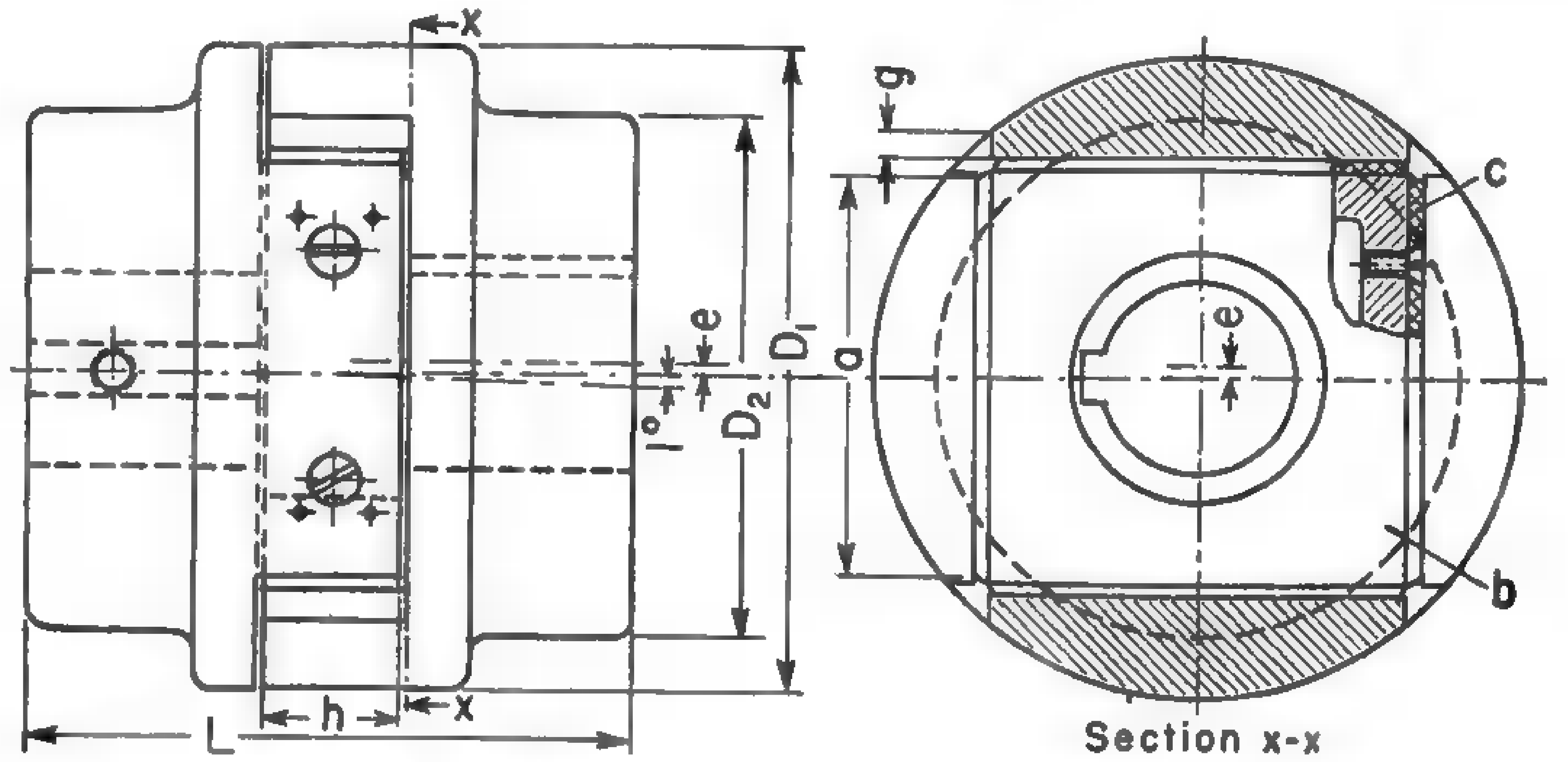


Fig. 21-8. Assembled American flexible coupling.

The allowable pressure is $p = 1{,}000$ psi for light service, and it may be increased to 1,500 psi for heavy-duty service. The width h of the block is made equal to $0.3a$. With this value, equation 21-7 becomes

$$T = 0.1pa^3 \tag{21-9}$$

The design procedure for a given torque T is as follows: The first step is to select the pressure p and then to determine the side a from equation 21-9. Next, the outside diameter D_1 may be determined graphically by

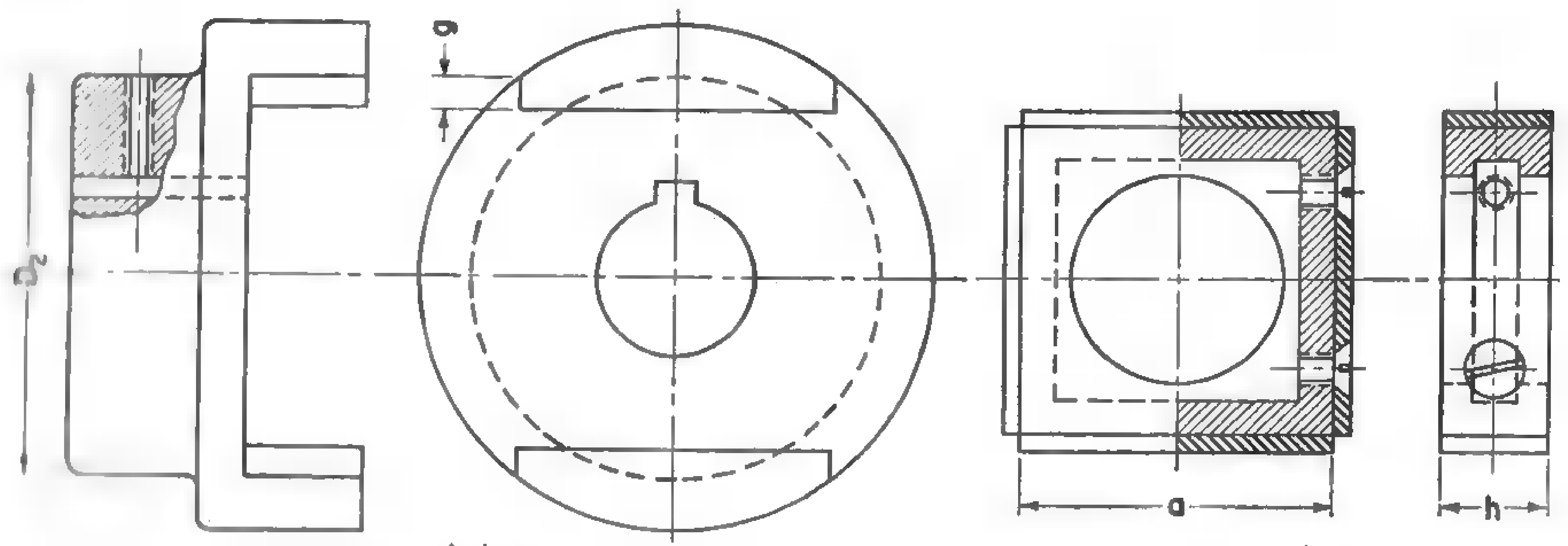


Fig. 21-9. Parts of American flexible coupling.

allowing from $\frac{1}{8}$ in. to $\frac{3}{16}$ in. for the Bakelite strips c and slightly more for the thickness g of the edges. The hub diameter D_2 may be computed by using equation 21-4, and the length L is made about $3.5d$.

Fast's self-aligning coupling. The coupling known as Fast's self-aligning coupling consists of two hubs a, Fig. 21-10, with gear teeth. These hubs are

enclosed in a casing composed of two parts, *c* and *d*, bolted together by bolts *e*. Each part of the casing contains an integral gear, the teeth *f* of which mesh all around with the teeth on the hub. The clearance between the engaging teeth takes care of misalignment; *g* is a lubricating-oil hole closed with a pipe plug.

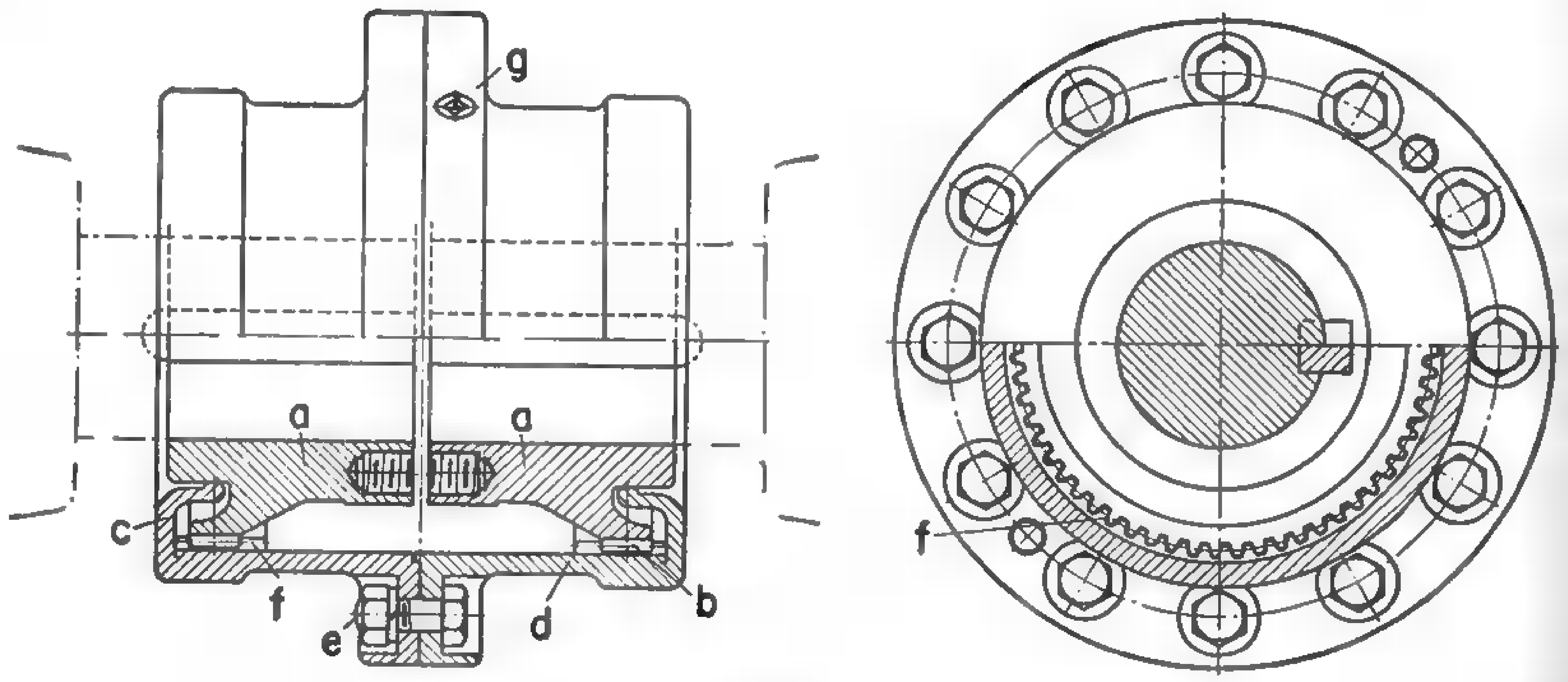


FIG. 21–10. Fast's self-aligning coupling.

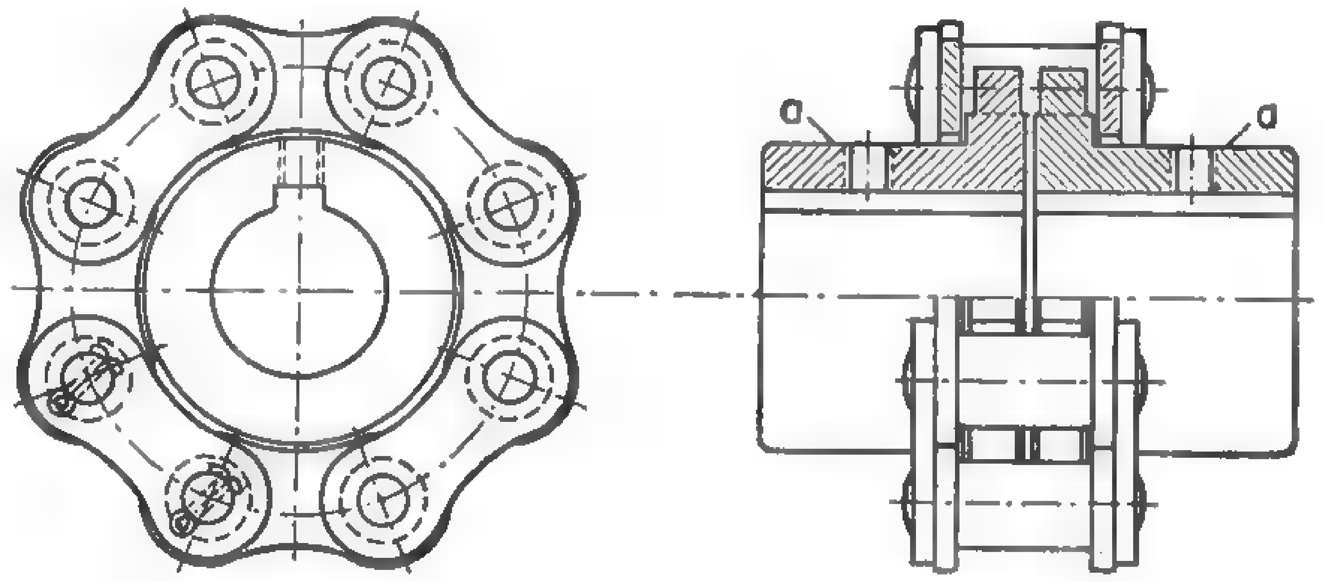


FIG. 21–11. Clark chain coupling.

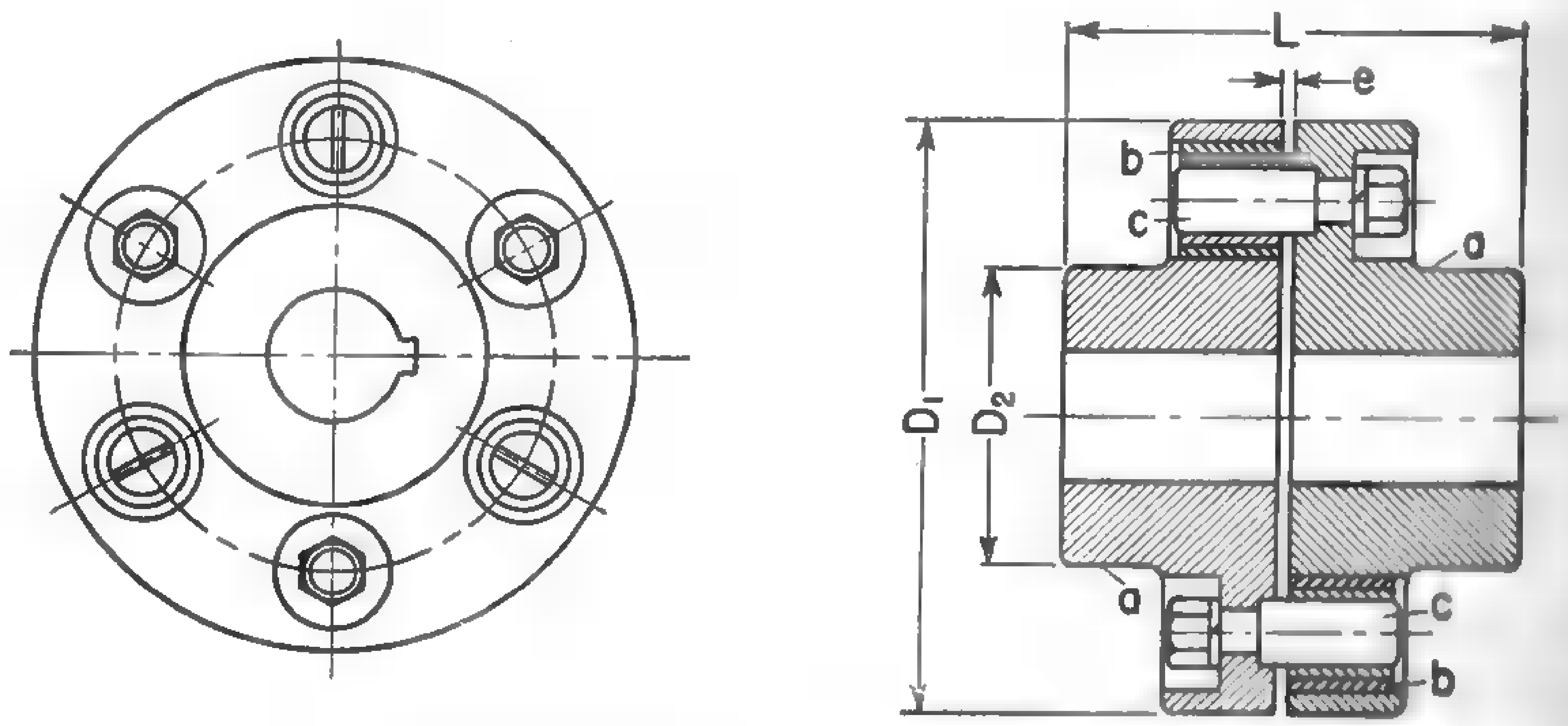


FIG. 21–12. Ajax flexible coupling.

Clark chain coupling. The Clark chain coupling consists of two sprockets *a*, Fig. 21–11, connected by a roller chain. Some manufacturers use a double-row chain. Couplings employing silent chains are also used.

21–6. Flexible couplings with resilient members. The *Ajax flexible coupling* consists of two cast-iron flanged hubs *a*, Fig. 21–12, with rubber bushings *b* and steel pins *c*. The rubber bushings are cemented into the holes and have thin bronze bushings cemented inside them. This process eliminates the wear of the rubber bushings due to sliding between them and the pins. The bronze bushings are impregnated with graphite and thus are self-lubricating. Standard couplings are made with 6 to 12 pins, the number depending on the load conditions.

Heavy-duty couplings are built with 16 pins, eight in each flange, the rubber bushings and the pins alternating in the flanges.

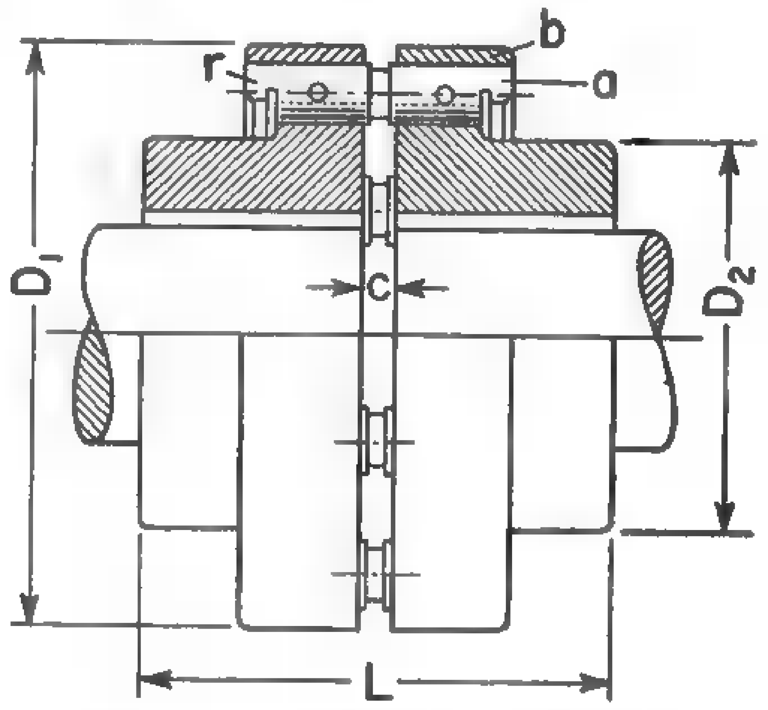


FIG. 21–13. Francke coupling.

The torque capacity of rubber-bushing couplings can be based on a specific pressure of $p = 300$ psi on the projected area of the bronze bushing.

The *Francke coupling*, Fig. 21–13, uses laminated steel pins *a*, which are relatively flexible. One end of each pin is fastened to the flange by a spring-retaining ring *r*. The other end of each pin can slide in a bronze bushing peened in the flange *b*. Thus the coupling can care for some endwise motion, as well as for angular misalignment.

The *Westinghouse-Nuttall coupling*, Fig. 21–14, uses helical springs *c* as connecting members between the two cast-steel halves *a* and *b*. These springs are compressed between casehardened spring seats *d*. The latter rest against the twin arms of the spider *b*, and transmit the torque which

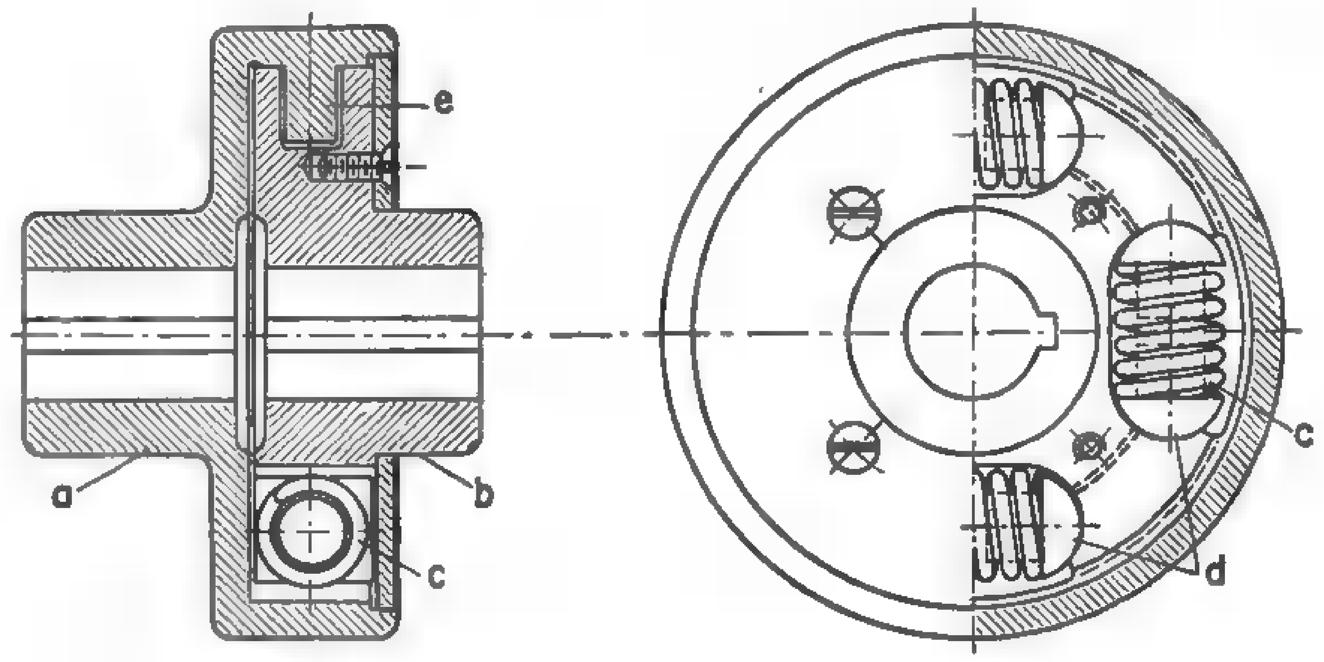


FIG. 21–14. Westinghouse-Nuttall flexible coupling.

comes from hub *a* and fingers *e*. A longitudinal clearance between the two halves of the coupling permits a certain end play of the connected shafts. Because of the small clearances, the coupling is not suitable for taking care of misalignment. However, it is used to absorb impact, and particularly to remove the critical speed from the operating range of shafts driven by diesel engines.

The *Falk flexible coupling*, Fig. 21–15, consists of two flanged steel hubs *a* and *b*, and a ring *d* which is made of a tempered spring-steel strip and which forms a complete cylindrical grid. In Fig. 21–15 is shown a coupling combined with a safety shear pin *e*, the right-hand hub being divided into the hub proper *b* and a ring-shaped flange *c*. The peripheries of the flanges have radial fingers formed by milling. The grooves between the fingers widen inwardly toward each other, and the spring grid *d*, through which the torque is transmitted, is inserted into them. When the torque is applied, the spring is bent along the arcs of the fingers. The curvature of the arcs is such that with an increase of the torque the active spring length gradually decreases from l to l', Fig. 21–15b, making the spring stiffer. The steel cover *f* encloses the coupling tightly and holds the lubricant.

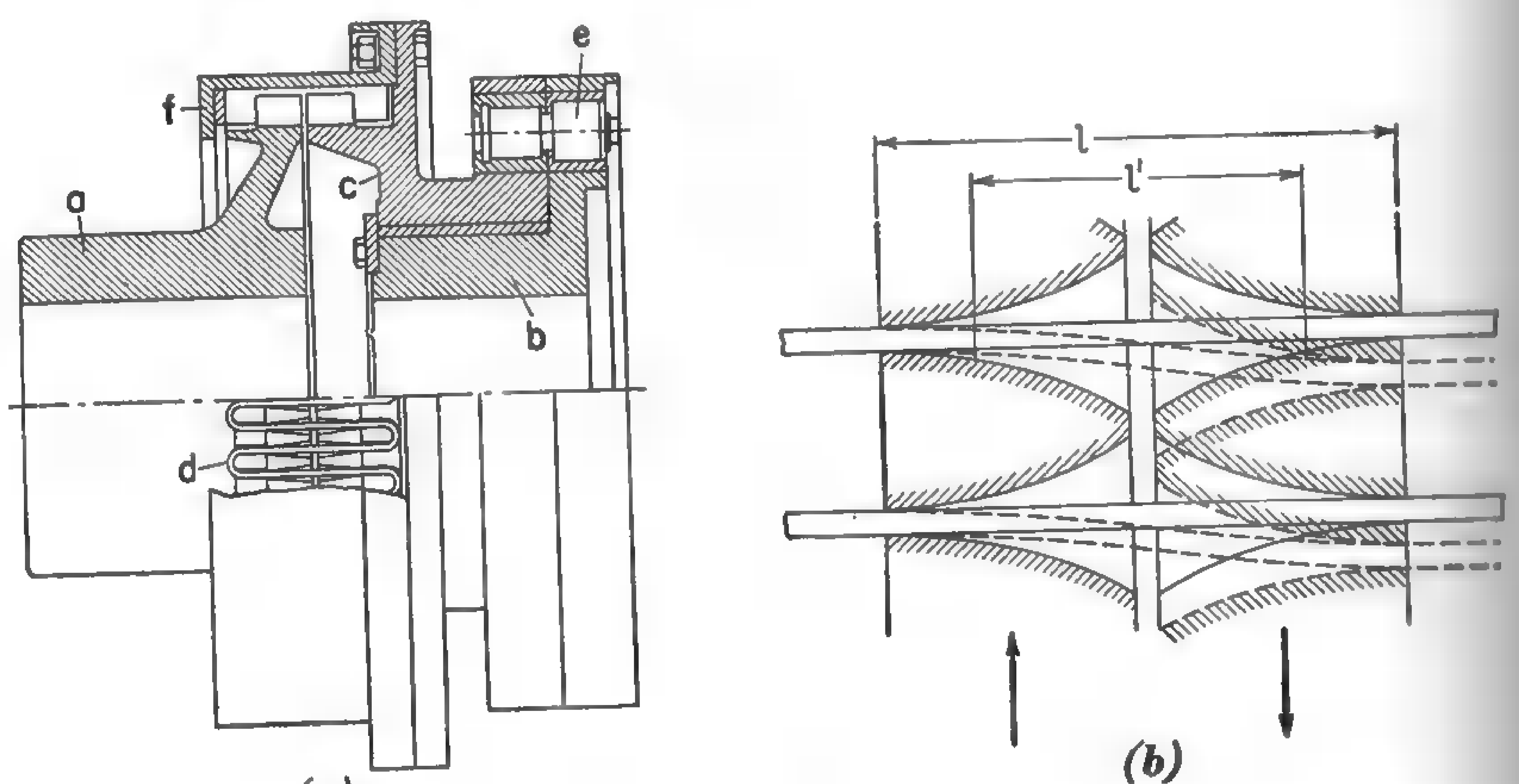


Fig. 21–15. Falk flexible coupling.

This coupling can withstand heavy shocks, as well as all kinds of misalignment.

Load factor. The torque capacities of flexible couplings given in catalogues, or computed by using the pressures or stresses indicated in the foregoing explanations, refer to uniform rotation. In most cases the rotation is not quite uniform and the couplings are subjected to impact. This condition must be taken into account by dividing the nominal coupling capacity by a load factor k_l, Table 20–3.

Speed. The torque capacity of a coupling can be considered to be practically independent of the rotative speed of the shafts.

21–7. Universal joints. The object of a universal joint is to transmit the torque between two shafts whose center lines form a rather large angle, as 5 to 15 deg, or even 30 deg.

In Fig. 21–16 is shown a universal joint used for speeds not over 100 rpm and for shafts up to $4\frac{15}{16}$ in. The hubs *a* and the inner yoke *b* are made of cast iron, and the pins are made of steel.

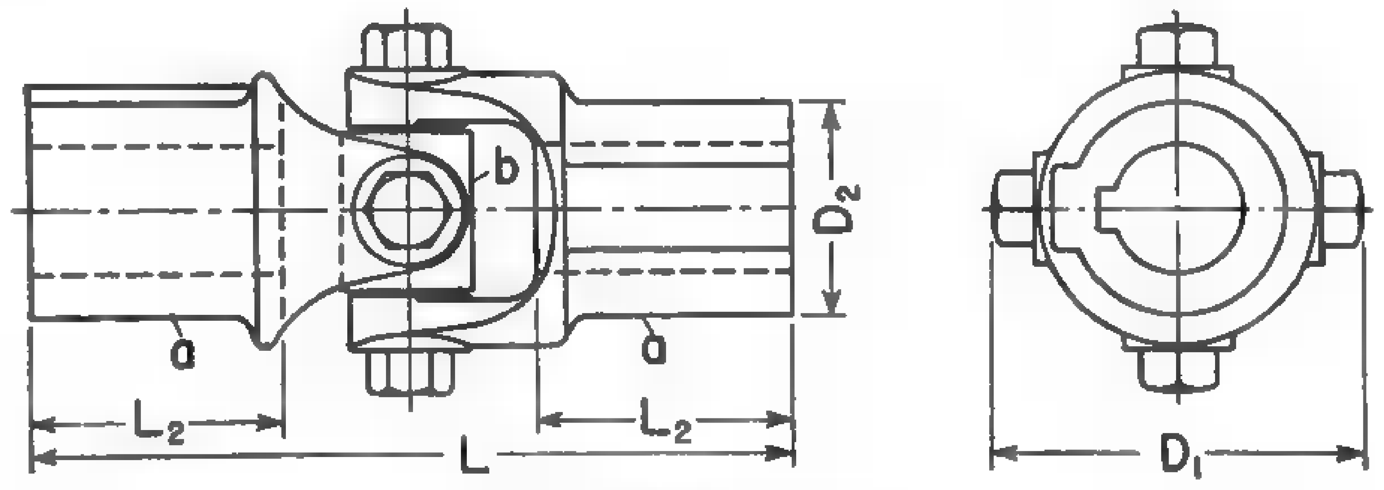


Fig. 21–16. Universal joint.

The curves in Fig. 21–17 show the relation, found by actual tests, between the efficiency and the operating angle in a set of two universal joints. Both joints were at the same angle. Curve *a* was obtained with yokes on the intermediate shaft in the same plane; curve *b* was obtained with yokes at 90 deg.

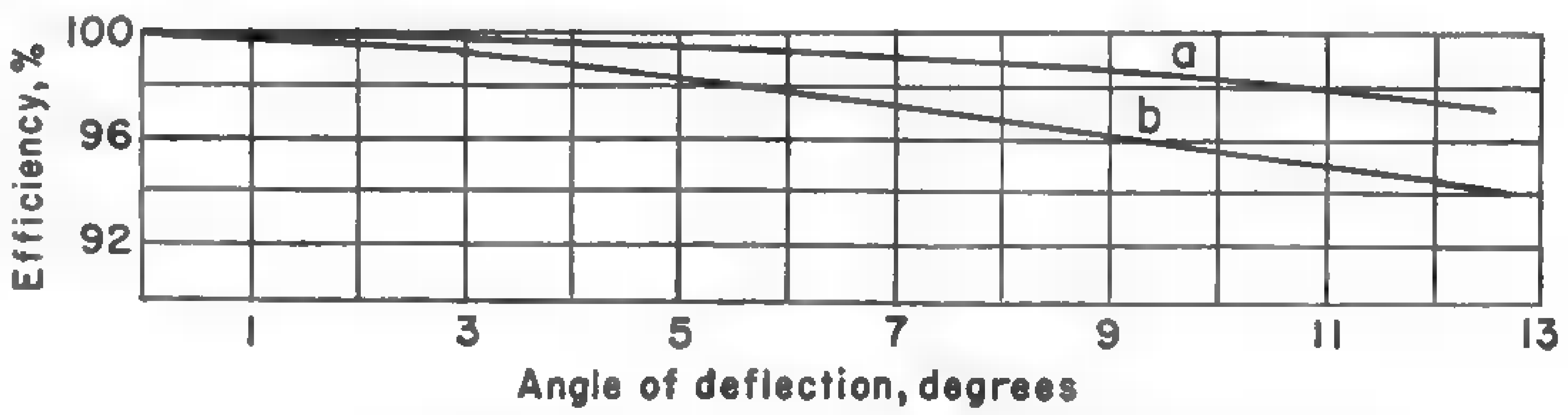


Fig. 21–17. Efficiencies of a double universal joint at different shaft angles.

In Fig. 21–18 is shown a universal joint which at the same time is a flexible coupling and consists of a rubberized canvas disk *c* to which are fastened, straddling each other, the fingers of the tubular shaft ends *a* and *b*. This joint allows an angle up to 4 or 5 deg. If a greater angle is necessary, the ends must have only two fingers instead of three, the coupling becoming a regular universal joint.

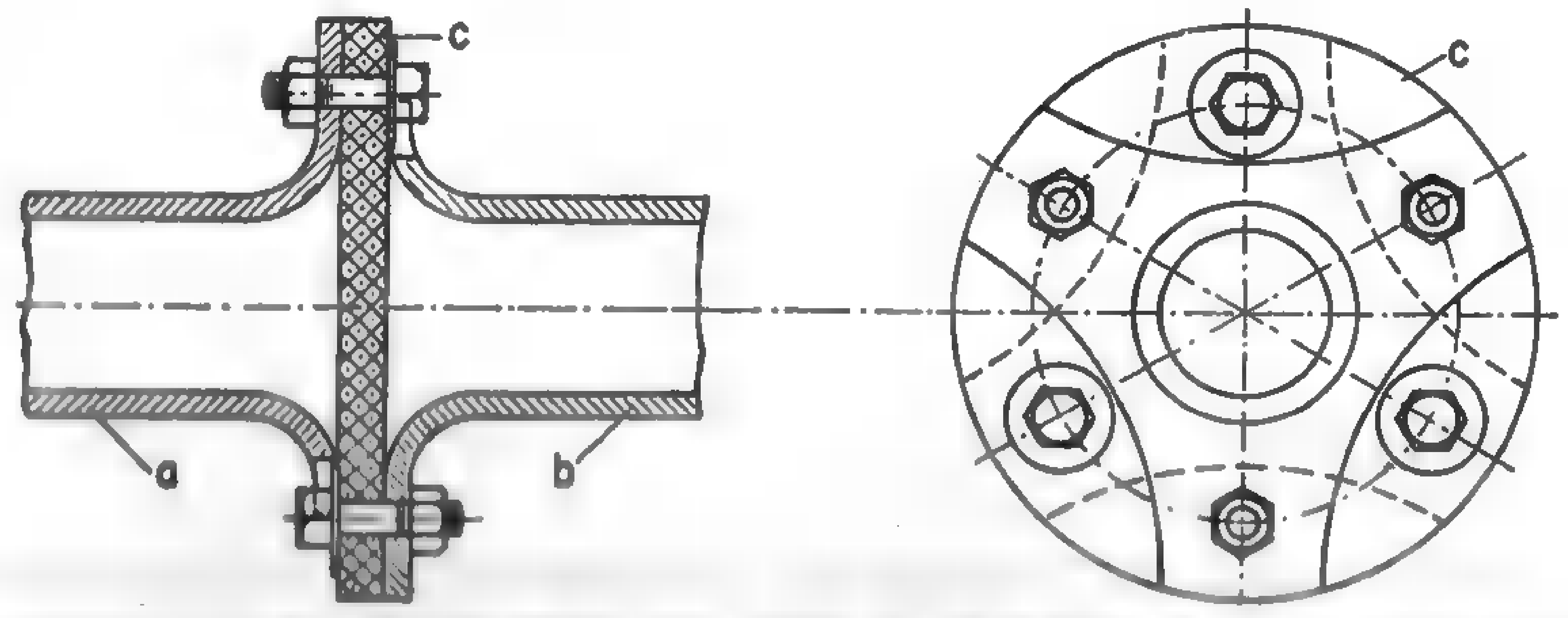


Fig. 21–18. Flexible coupling with a rubber disk.

21-8. Slip coupling. The object of a slip coupling is to permit relative rotation, or slip, between the driving shaft and the driven shaft. A slip coupling is a safety device that prevents damage to rotating parts because of overloading. The slip coupling is adjusted so that it will begin to slip if the transmitted torque exceeds a predetermined value. Usually the slip begins if the load exceeds by 10 to 20 per cent the maximum load for which the shafts and other parts are designed.

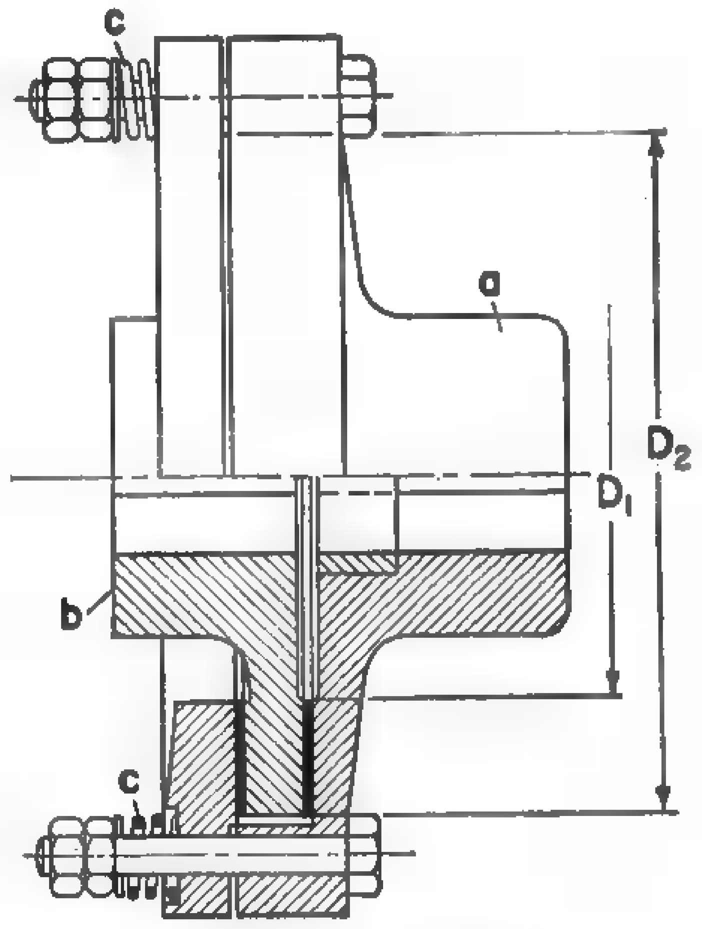


FIG. 21-19. Slip coupling.

A typical slip coupling is shown in Fig. 21-19. The hub a is keyed to the driving shaft; the driven hub b has a flange faced on both sides with friction material; a number of small but heavy springs c are loaded by screwing down the nuts, and thus create the necessary pressure on the friction surfaces.

Force analysis. Slip in such a coupling occurs infrequently, and no measurable wear takes place. Therefore it can be assumed that the unit pressure p between the disks is uniform and that the axial force exerted by the springs, with the designation of Fig. 21-19, is

$$F_a = 0.7854(D_2^2 - D_1^2)p \tag{21-10}$$

With two pairs of friction surfaces, the tangential force is equal to

$$F_t = 2F_a f \tag{21-11}$$

Also, the radius of its application, with sufficient accuracy, is

$$0.5D_m = \frac{D_2 + D_1}{2 \times 2} \tag{21-12}$$

Substituting these values in the general expression of torque, such as equation 18-12, gives

$$T = 0.3927(D_2^2 - D_1^2)(D_2 + D_1)pf \tag{21-13}$$

Design. The values for p and f may be taken from Table 18-1. The diameters D_2 and D_1 may be selected so that, approximately, $D_2/D_1 = 1.6$.

The number of springs i ranges from 6, for small couplings and for a shaft diameter $d = 2\frac{3}{16}$ in., up to 16 for $d = 6\frac{7}{16}$ in.

21-9. Positive clutches. A jaw clutch is the commonest type of positive clutch. Jaw clutches are made either with square jaws, as shown in Fig. 21-20, for driving in both directions, or with spiral jaws, as in Fig. 21-21, for driving in one direction only.

Jaw clutches. When a clutch is used to connect a belt pulley, sprocket, or gear to a rotating shaft, the hub of the driven member has jaws to mesh with those on the sliding half of the clutch. The hub is held in the axial directions by two set collars and usually has a bronze bushing.

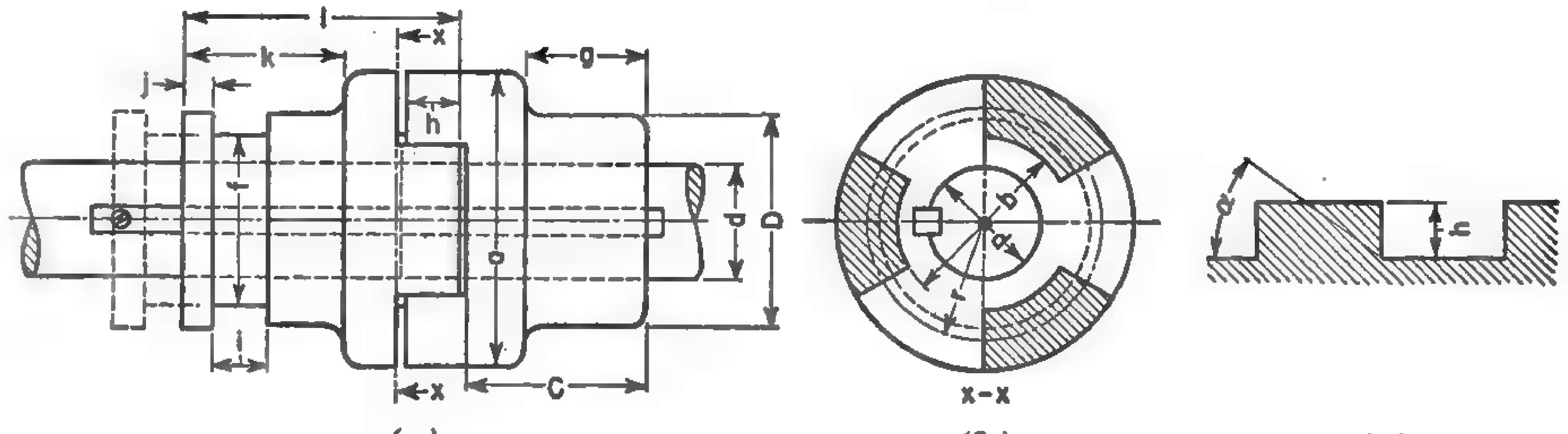


FIG. 21-20. Square-jaw clutch.

Jaw-clutch coupling. When a clutch is used for connecting two shafts, as in Fig. 21-20 or Fig. 21-21, it is called a *clutch coupling*. In a clutch coupling, one half of the clutch, usually the driver, is keyed fast to one shaft, while the other half, the follower, is attached to the other shaft by means of a feather key or spline which permits it to slide along the shaft when the shifting lever is operated. Approximately, the main proportions of a cast-iron jaw clutch, Fig. 21-20, are as follows:

$a = 2.2d + 1.0$	$g = d + 0.2$	$j = 0.2d + 0.15$
$c = 1.2d + 1.2$	$h = 0.3d + 0.5$	$k = 1.2d + 0.8$
$f = 1.4d + 0.3$	$i = 0.4d + 0.25$	$l = 1.7d + 2.3$

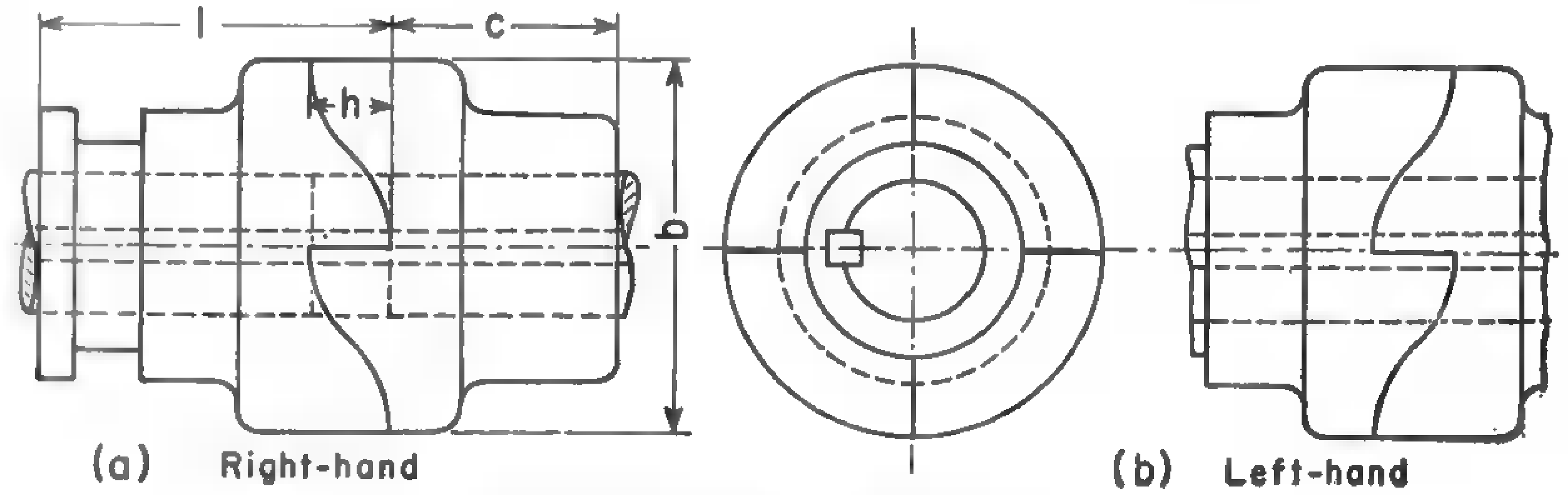


FIG. 21-21. Spiral-jaw clutch.

The counterbore b is made larger than the shaft diameter to clear the feather keys.

Some of the dimensions of the coupling can and should be checked by considering stresses. Thus, it is advisable to check the jaws in shearing; the sides of the jaws in crushing; and the feather key in shear and crushing.

Tests show that cast-iron blocks loaded in compression fail by shearing along a plane making an angle α of about 35 deg with the direction of pressure. In Fig. 21-20c is shown a development at the mean diameter of the jaws. The area in shear is $0.5(a - b)h/\sin\alpha$; and the component of

the tangential force that will act in the plane of shear is evidently $F_t/\cos\alpha$. It should be assumed, for the sake of safety, that only one-half the total number of jaws i is in actual contact. For $\sin\alpha/\cos\alpha = \tan\alpha = 0.7$, the shear stress is

$$s_s = \frac{F_t \sin\alpha}{0.5\cos\alpha(a-b)h\times 0.5i} = \frac{2.8F_t}{(a-b)hi} \tag{21-14}$$

A safety factor n of 2 should be used for the allowable shear stress.

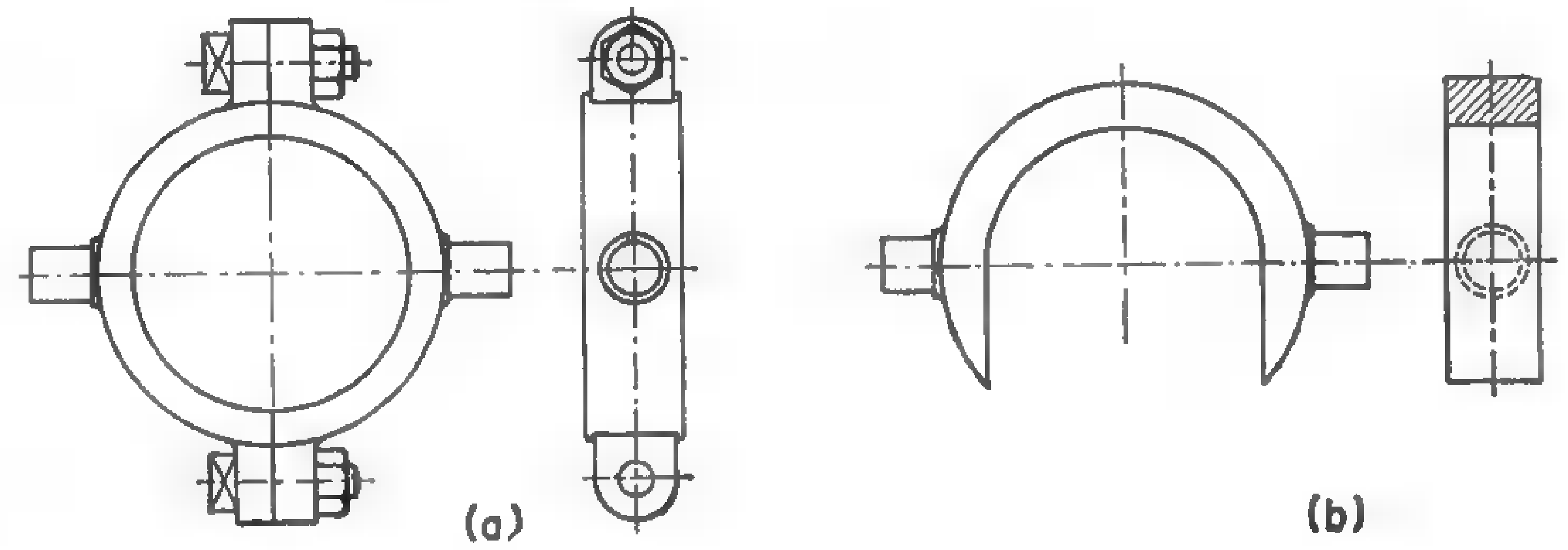


Fig. 21–22. Clutch-shifting collars.

The allowable bearing pressure between cast-iron jaws should be taken as $p = 3{,}000$ psi for small clutches, but this value may be increased, as the size increases, up to $p = 6{,}000$ psi for a $3\frac{15}{16}$-in. shaft.

The number of square jaws is usually two or three, but more are sometimes used for large shafts. If it is necessary to preserve a relative angular position of the shafts, one of the jaws is made slightly smaller or larger than the others.

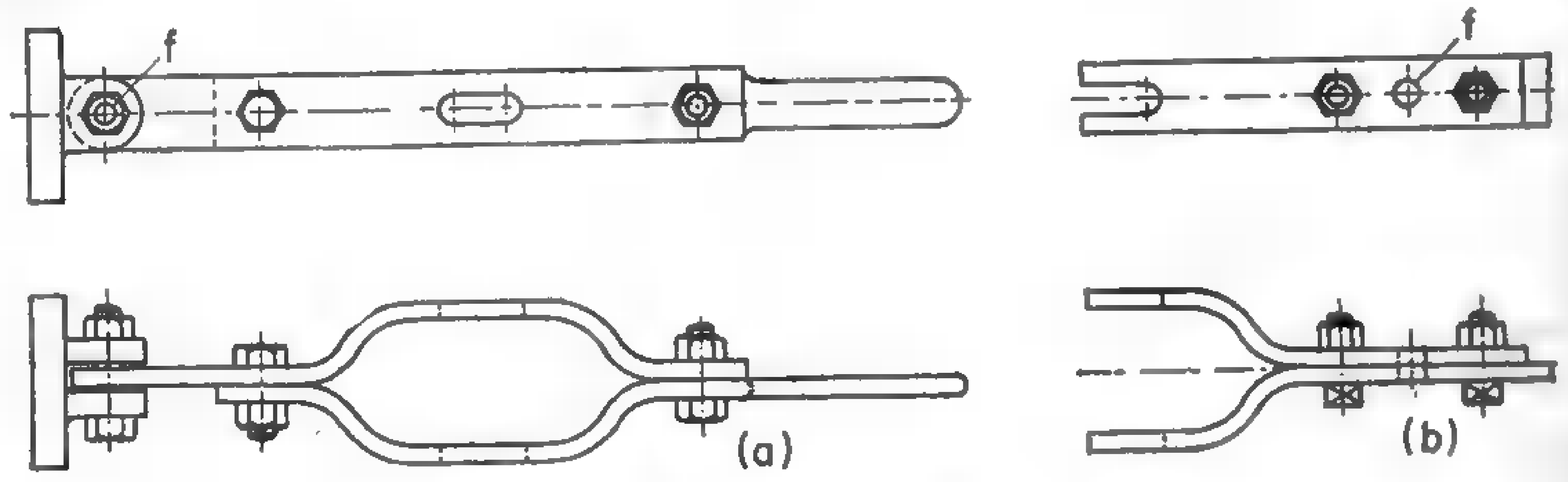


Fig. 21–23. Shifting levers.

Speed limitations. Jaw clutches and couplings can be engaged only at low speeds, below 60 rpm. However, when engaged, the shaft can run at much higher speeds. The spiral jaws are engaged more easily and are used when engagement is frequent. They are also very convenient for presses, punches, and shears in which they are automatically thrown out of engagement when the direction of rotation of the shaft is reversed after completion of the working cycle.

Clutches with cast jaws are used for shafts not over $3\frac{15}{16}$ in. in diameter. Clutches with finished square jaws are used for shafts up to 10 in. in diameter.

Shifting mechanisms. Collars, also called *bands,* are made of brass. Their usual shape is shown in Fig. 21–22a. In Fig. 21–22b is shown a horseshoe collar which can be used only on a horizontal shaft with a shifting lever in the vertical plane.

Shifting levers are made of flat iron with the fulcrum f at the end, as in Fig. 21–23a, or fork-shaped with the fulcrum f between the fork and handle, as in Fig. 21–23b.

CHAPTER 22

Friction Clutches

22-1. General considerations. The object of a friction clutch is to connect a stationary machine part to a rotating part, to bring it up to speed, and to transmit the required power with a minimum of slippage. In certain cases a friction clutch serves as a safety device by slipping when the torque transmitted through it exceeds a safe value, thus preventing the breakage of parts in the transmission train.

Friction clutches are based on the same principles as brakes, and their construction is also similar to that of a brake. In fact, every time a brake is locked, it becomes a friction clutch. Equations and experimental data given in Chapter 18 can be used for friction clutches either directly or with certain modifications explained at various points in the following discussion.

Fluid clutches. Fluid clutches, also called *hydraulic clutches*, or rather *hydraulic couplings*, are coming into extensive use for connecting internal combustion engines (or electric motors) to driven machinery. Their main advantage is that they absorb impact loads imposed by the driven machine, thus eliminating excessive stresses and increasing the service life of the whole equipment. Another advantage is that a hydraulic coupling cuts a long shaft into two shorter lengths, each of which has a higher natural frequency of vibration. This reduces the danger of torsional vibration.

However, a fluid coupling is essentially a piece of hydraulic machinery, and its design is therefore outside the scope of this book.

Design requirements. The following considerations must be observed in the design of a friction clutch:

a. Selection of a type suitable for given operating conditions
b. Selection of suitable materials forming the contact surfaces
c. A sufficient torque capacity
d. Engagement and acceleration without shock
e. Quick disengagement without drag
f. Provision for holding the contact surfaces together by the clutch itself
g. Low weight in order to keep down the inertia, especially in high-speed service
h. Balancing of all moving parts, especially in high-speed service
i. Provision for taking up wear of the contact surfaces
j. Accessibility and provisions for facilitating repairs
k. In an industrial clutch, protection of projecting parts

l. In a clutch frequently operated, provision for carrying away the heat generated at the contact surfaces

Classification. Friction clutches may be divided into two main groups, according to the direction of the acting force: (1) *axial clutches* and (2) *radial clutches*, or *rim clutches*. A friction clutch used to connect two shafts is often termed a clutch coupling, or friction coupling.

An axial clutch is one on which the contact pressure is applied in a direction parallel to the axis of rotation. Axial clutches, in turn, can be subdivided into (a) cone clutches, (b) disk clutches, and (c) combined cone and disk clutches. *Magnetic clutches* are usually of the disk type.

In a rim clutch the contact pressure is applied upon a rim in a radial direction. These clutches, like brakes, may be subdivided into band clutches and block clutches, or into external, internal, and combined internal-and-external clutches.

Only a few examples of the more representative clutch types will be discussed. These examples and the field of their application in accordance with their torque capacity and speed may serve as a guide for the analysis and design of other arrangements.

22-2. Selection of type. The factors which must be taken into consideration in deciding what type of clutch is to be used are torque, rotative speed, available space, and frequency of operation.

Torque. To transmit a heavy torque or a fluctuating torque a clutch must have sufficient gripping power. This condition is fulfilled in general by multidisk clutches and for low-speed service by cone and rim clutches of large diameter. The use of special materials for the contact surfaces may increase the torque capacity of a clutch.

Speed. Light, compact, and internally balanced clutches, such as the double-cone and multidisk types, are best suited for high rotative speeds.

Space limitations. Multidisk, twin-cone, and double-cone clutches are more compact than other types.

Frequency of operation. Clutches which are in frequent or continuous operation should have a small travel, a simple engaging mechanism, and large cooling areas to dissipate the heat generated during the engaging of the clutch under load. In this case single-disk clutches with metal contact surfaces, and cone clutches, are the most suitable.

22-3. Materials for contact surfaces. A material suitable for use as a friction surface must meet the following conditions:

a) It must have a high coefficient of friction.
b) It must not be affected by moisture and oil.
c) It must resist wear.
d) It must be capable of resisting high temperatures caused by slippage.

Materials in common use are wood, asbestos, cork, leather, and various metals.

Wood. Maple, elm, and pine, in lighter service, are used with cast iron in many clutches. All woods have a high coefficient of friction (see Table 18–1). Maple and elm resist wear fairly well, but neither complies with condition b or condition d.

Asbestos. Asbestos fabric and molded blocks have the same coefficient of friction and, according to Table 18–1, comply well with condition a. They also comply well with conditions b, c, and d. In addition, their friction coefficient is less affected by oil or grease than is that of any other material.

Asbestos-metallic blocks are molded under a very high pressure into any desired shape.

Cork. Cork is used in the shape of round plugs or inserts in connection with some other material. The surface covered by cork inserts varies from 10 to 40 per cent of the total friction area. Because of its higher coefficient of friction, cork increases the torque capacity of the clutch. In general, the cork inserts project slightly over the harder surface and are operative chiefly in engaging the clutch. After full engagement, the harder material—metal or wood—forms the surface in contact. In a lubricated clutch, cork inserts also help to keep the friction surface lubricated.

Leather. Most small cone clutches are faced with leather. Oak-tanned leather and so-called chrome leather seem to be equally serviceable. Before the leather facing is fastened to the cone, it should be soaked in castor oil or neat's-foot oil or boiled in tallow. The excess oil or grease is removed by passing the facing between rolls.

Metals. If one of the friction surfaces is faced with a fibrous material, the other surface must be of cast iron, cast steel, or rolled steel. In some clutches, both contact faces are made of metal. Cast iron bears against cast iron, cast steel, or bronze; or hard saw steel bears against steel or bronze. In order to obtain smooth operation in a clutch with metal-to-metal surfaces, these surfaces must be lubricated.

Other requirements for good clutch design will be discussed simultaneously with the analysis of different clutch types.

The *load factors* must be used as given in Table 20–3 for shafts.

22–4. Cone clutches. In Fig. 22–1a is shown schematically a cone cut-out clutch in which the outer cone a is the driving member and the inner cone c can be moved axially.

In Fig. 22–2 is shown an industrial clutch with cast-iron contact surfaces. The cone c is keyed to the shaft, while the pulley p with the cone cup rotates on the cone hub and carries with it the levers l. When the thimble t is forced under the rollers r, by means of the sliding ring s, the levers l bring the cone

surfaces into contact. Heavy springs at c, not shown in the illustration, throw the surfaces apart when the thimble is shifted to the right. The wear is taken up by adjusting the collar g held by a locknut h. If the clutch is operated frequently, the cone should be lined with wood or asbestos blocks.

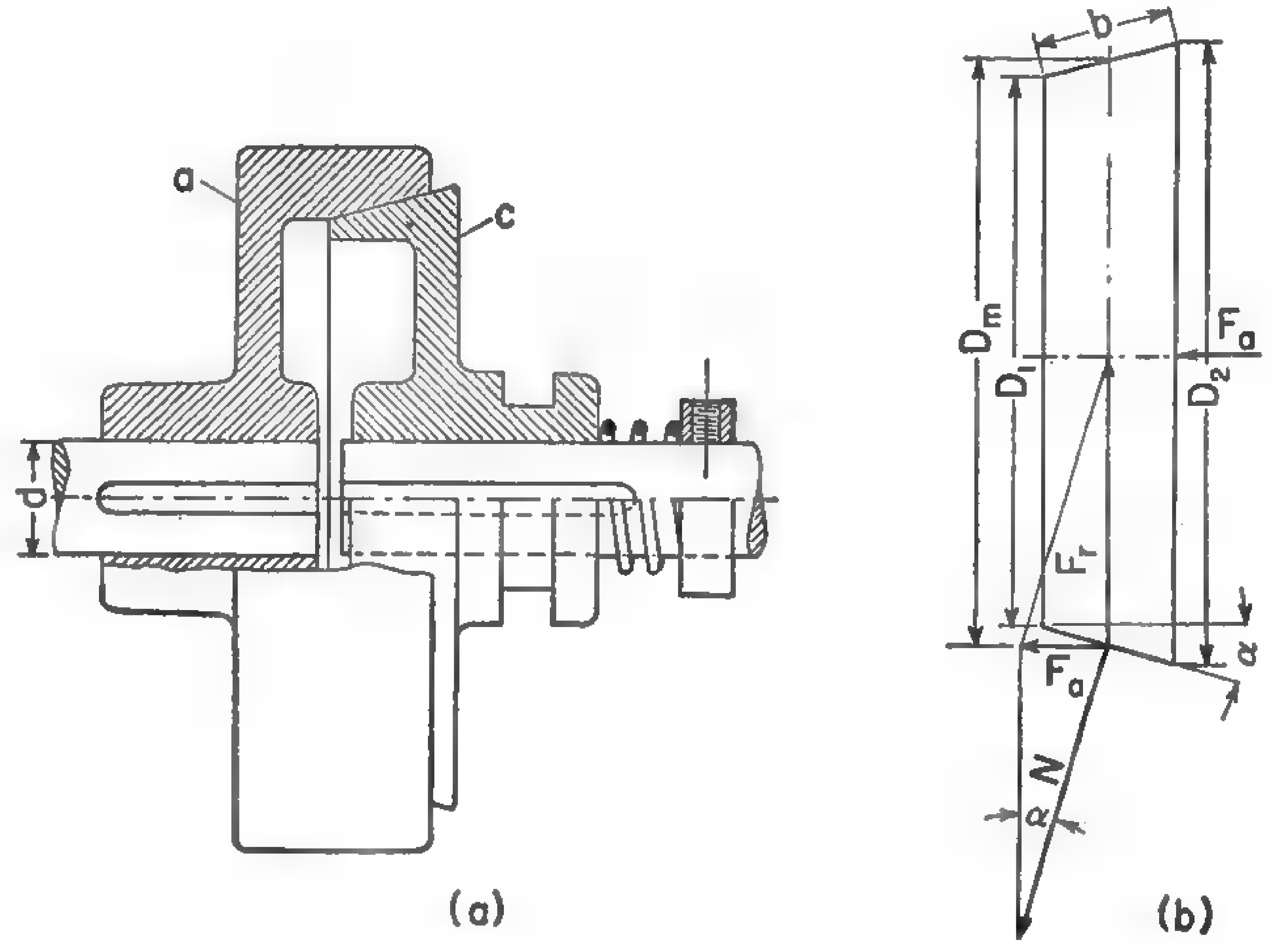


FIG. 22–1. Cone-clutch coupling.

A double-cone clutch having cones faced with asbestos lining is shown in Fig. 22–3. Because of the symmetrical arrangement of the cones, their angle can be large enough (35° to 45°) to eliminate sticking and drag. The cast-iron friction ring f is held to the housing h by a feather key i; the right cone c slides on the feather key k. When the cones are pulled together by the lever mechanism, the lever l is pushed slightly past the vertical or deadline position, and there is no need for an outside force for holding the contact surfaces together. The ring r with the fulcrum forks serves to take up the wear.

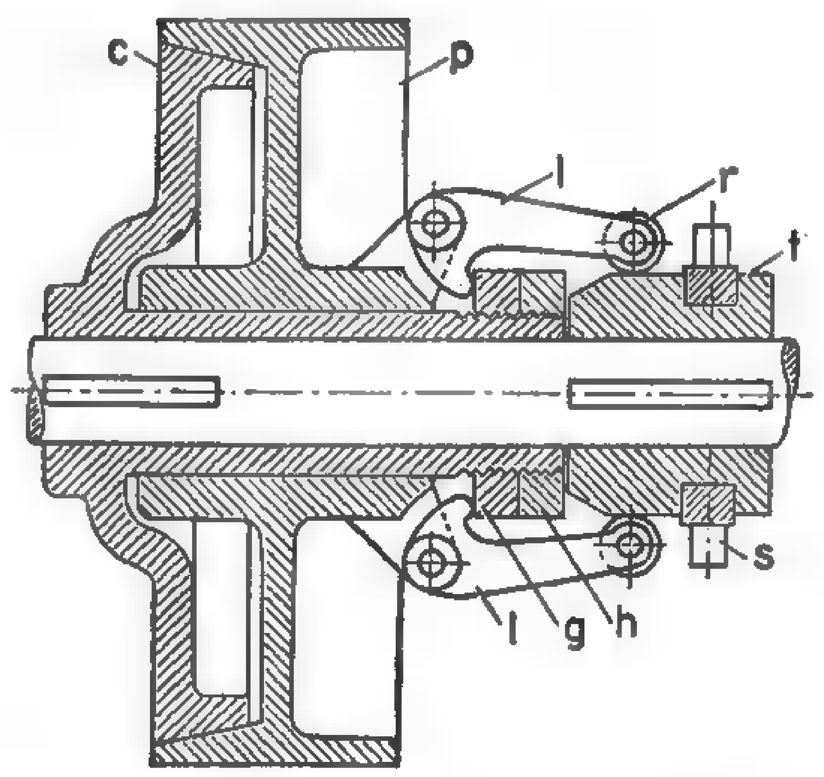


FIG. 22–2. Industrial cone clutch.

Force analysis. The forces acting upon the inner cone in Fig. 22–1a are shown in Fig. 22–1b. In finding the expression for the torque T which the clutch can transmit when a certain axial force F_a is applied, the following designations also will be used: p for the unit normal pressure at the contact surface, and f for the coefficient of friction.

The torque which the clutch will transmit is equal to the torsional moment of the frictional resistance between the inner and outer cones. For an analysis, the male cone c may be considered as a free body. It is

acted upon by the axial force F_a, the normal force N created by F_a and distributed around the cone, and the tangential force F_t due to friction. From Fig. 22–1b,

$$F_a = N \sin \alpha \tag{22–1}$$

The friction force F_t is equal to fN and

$$F_t = \frac{fF_a}{\sin \alpha} \tag{22–2}$$

The torque transmitted through friction is

$$T = \frac{F_t D_m}{2} = \frac{fF_a D_m}{2 \sin \alpha} \tag{22–3}$$

where the mean diameter is approximately $D_m = \frac{1}{2}(D_1 + D_2)$.

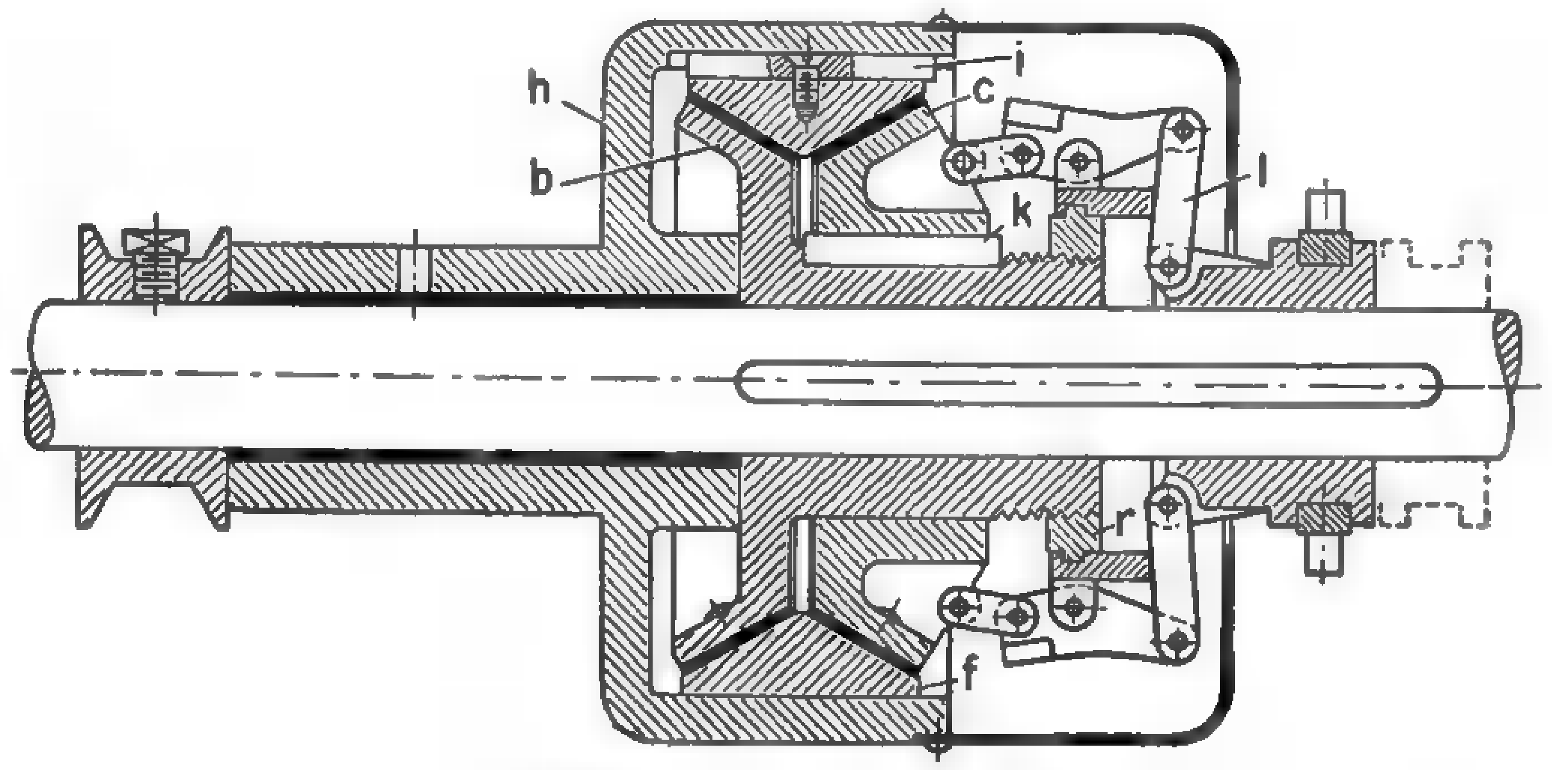


FIG. 22–3. Double-cone clutch.

The horsepower P that a cone clutch will transmit is found by substituting the value of T from equation 22–3 in the general equation 2–17 and taking into account the load factor k_l, Table 20–3. This gives

$$P = \frac{fF_a D_m n}{126{,}060 \sin \alpha \; k_l} \tag{22–4}$$

Since the normal force is also equal to the average normal pressure p multiplied by the total area in contact,

$$N = \pi D_m b p = \frac{F_a}{\sin \alpha} \tag{22–5}$$

Combining equations 22–4 and 22–5 results in

$$P = \frac{fpD_m^2 bn}{40{,}120 k_l} \tag{22–6}$$

The relation between the axial force F_a and the clutch dimensions, as found from equation 22–5, is

$$F_a = \pi D_m b p \sin \alpha \tag{22–7}$$

The axial force F_a exerted by the spring, as determined by equation 22–1 or equation 22–7, is sufficient to transmit the torque. For engaging the clutch the force must be greater in order to overcome friction when cone c is pressed into cone a. By reasoning similar to that used in deriving equation 18–19, it will be found that the force F_a' necessary to engage the clutch is

$$F_a' = N(\sin \alpha + f \cos \alpha) \tag{22–8}$$

or, with equation 22–5,

$$F_a' = \pi D_m b p(\sin \alpha + f \cos \alpha) \tag{22–9}$$

The actual force required to engage a clutch is slightly greater than F_a' because of friction in the joints. If a spring is used to engage a clutch, it must be able to exert a still greater force in order to have a certain reserve strength.

Design data. The friction coefficient may be taken as 0.15 for cast iron on cast iron, 0.2 for a surface faced with leather or wood, 0.3 for a surface faced with asbestos, and 0.25 for cone surfaces with cork inserts.

Slip. In normal operation all friction clutches have a slip of about 2 per cent.

Cone-face angle. The SAE rules recommend, for cone clutches faced with leather or asbestos or having cork inserts, a standard angle α of 12.5°. For industrial clutches faced with wood, the angle α may range from 15° to 25°.

Mean diameter. If the ratio D_m/b is designated by q, and if this substitution is made in equation 22–6 and the equation is solved for D_m, the result is

$$D_m = 34.2 \sqrt[3]{\frac{Pk_l q}{fpn}} \tag{22–10}$$

The permissible pressure p in equation 22–10 must be selected by using Table 18–1 as a guide. To prevent excessive wear of the contact surfaces it is recommended that p be taken near the lower limit, or even slightly below that limit. The value of q varies in existing designs from 4.5 to 8. The greater q is made, the smaller will be the difference in the peripheral speeds at the outer and inner diameters of the cone, and hence the less the wear will be. On the other hand, the greater q is made, the larger D_m and the clutch in general will be.

After D_m is found from equation 22–10, it may be checked by comparison with the shaft diameter d. In commercial clutches D_m varies from $5d$ to $10d$. Another check is made by computing the peripheral velocity v. For high-speed, leather-faced clutches v should be between 2,000 and 5,000 fpm. For clutches not balanced inherently this speed must be considerably lower; for clutches with metal-to-metal contact surfaces the speed limits are 300 to 1,000 rpm.

Shaft sleeves. The hub of the clutch half in which the shaft turns when the clutch is not engaged and which does not move axially must have a sleeve, as shown in Fig. 22–3, which can be replaced when it is worn. The type of sleeve depends on the speed. Cast-iron, wick-oiled sleeves can be used for speeds up to 450 rpm, and bronze-bushed sleeves can be used up to 800 rpm. Sleeves for higher speeds should have ball bearings, one on each end of the sleeve.

EXAMPLE 22–1. (a) Determine the main dimensions of a cone clutch similar to that in Fig. 22–1. It is to be faced with leather and is to transmit 40 hp at 750 rpm from an electric motor to an air compressor. (b) Also find the axial force that must be produced by the spring.

a) The shaft size needed to transmit the given torque is, by equation 20–2, with $S_d = 6{,}400$ psi and $k_t = 1.75$ (Table 20–3),

$$d = \sqrt[3]{\frac{321{,}000 \times 40 \times 1.75}{750 \times 6{,}400}} = \sqrt[3]{4.67} = 1.673 \text{ in.}$$

The next-larger standard size is $1\frac{11}{16}$ in. However, for the sake of rigidity it is advisable to take a still larger size and to use $d = 1\frac{15}{16}$ in.

The mean cone diameter D_m may be determined by selecting the following average values: $q = 6$, $f = 0.2$, and $p = 15$ psi. Then, by equation 22–10,

$$D_m = 34.2\sqrt[3]{\frac{40 \times 1.75 \times 6}{0.2 \times 15 \times 750}} = 34.2 \times \sqrt[3]{0.186} = 19.5 \text{ in.}$$

A check shows that $D_m/d = 19.5/1.9375 = 10$, which is within the usual limits. The peripheral velocity is

$$v = \frac{\pi \times 19.5 \times 750}{12} = 3{,}830 \text{ fpm}$$

This is also satisfactory.

The trial value of the face width may be taken as

$$b = \frac{19.5}{6} = 3.25 \text{ in.}$$

The small cone diameter, with $\alpha = 12.5°$, is then

$$D_1 = D_m = b \sin \alpha = 19.5 - 3.25 \times 0.2164 = 18.8, \text{ or } 18\tfrac{3}{4} \text{ in.}$$

The outside cone diameter is

$$D_2 = 19.5 + 3.25 \times 0.2164 = 20.2, \text{ or } 20\tfrac{1}{4} \text{ in.}$$

The corresponding face width is

$$b = \frac{D_2 - D_1}{2 \sin \alpha} = \frac{20.25 - 18.75}{2 \times 0.2164} = 3.464 \text{ in.}$$

b) The minimum axial force F_a found from equation 22–7 is

$$F_a = \pi \times 19.5 \times 3.464 \times 15 \times 0.2164 = 688 \text{ lb}$$

22–5. Disk clutches. In Fig. 22–4 is shown a heavy-duty friction coupling that is engaged. In order to disengage the clutch, the collar *c* is moved to the right and the steel levers *l* turn about the fulcrum *a* and push the bolts *b* to the left. The wear of the asbestos lining is taken up by the nuts on the bolts *b*.

A single-disk dry-plate automobile clutch coupling, commonly called a clutch, is shown in Fig. 22–5. The clutch disk *d* consists of a thin steel plate faced on both sides with asbestos lining. The clutch is kept engaged by springs *s* and is released by pushing toward the flywheel the bearing *b*, which

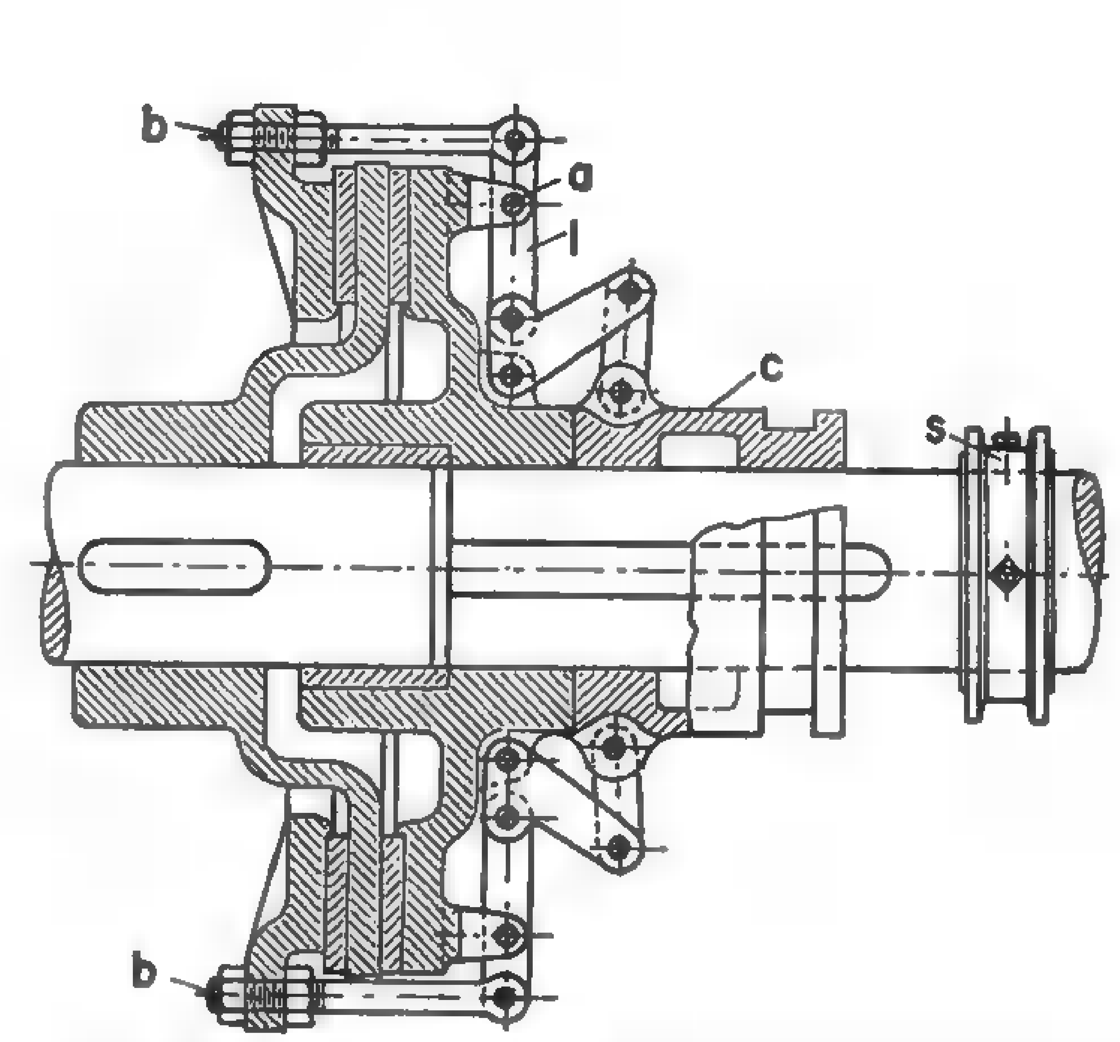


FIG. 22–4. Heavy-duty disk clutch coupling.

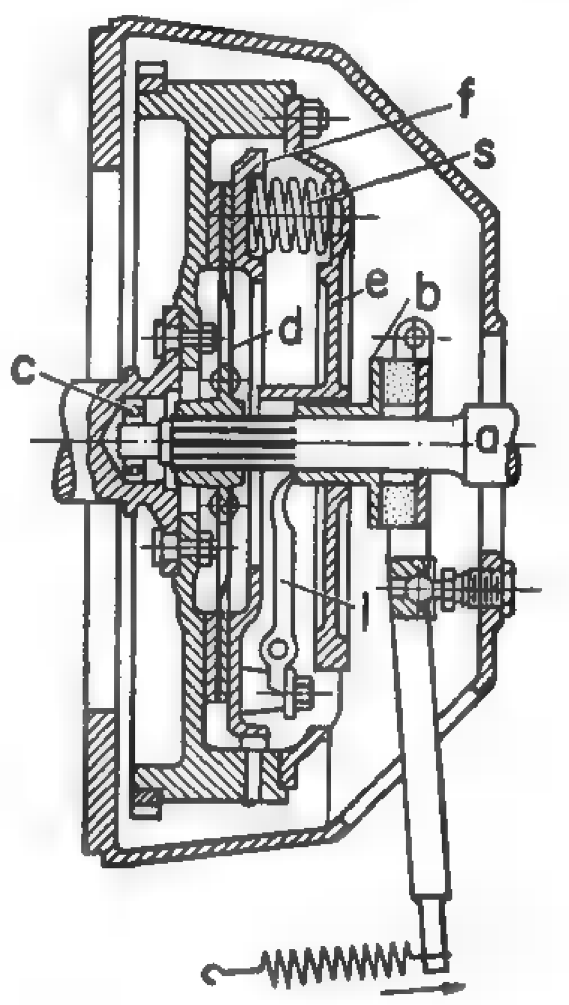


FIG. 22–5. Automobile clutch coupling.

is in contact with the inner ends of the release levers *l*. These levers are pivoted on pins fastened to the clutch cover *e*. The outer fork-shaped ends of levers *l* engage lugs on the pressure plate *f* and pull this plate away from the disk *d*, compressing the springs *s*.

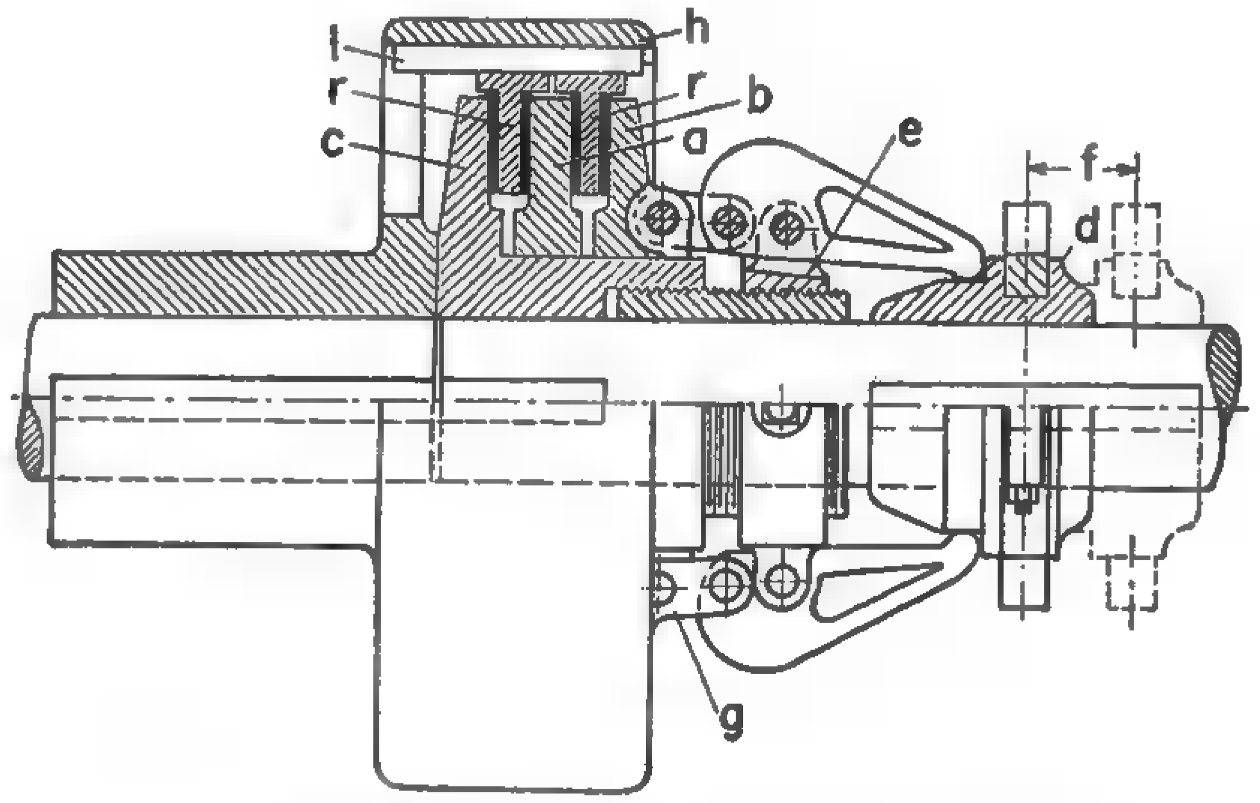


FIG. 22–6. Multidisk clutch coupling.

A multidisk industrial clutch coupling is shown in Fig. 22–6. As a result of the use of four pairs of contact surfaces, this coupling is compact and is suitable for comparatively high speeds. The asbestos-lined rings *r* can slide on the feather key *l* and can rotate with the clutch housing *h*, which is keyed to the left shaft. The inner disk *a* and the right disk *b* slide on a feather key

fastened to the hub of the left disk c, which itself is keyed to the end of the right shaft. When the clutch collar d is shifted to the left in order to engage the clutch, the link g must be pushed past a position parallel to the shaft axis, thus locking the mechanism. The threaded yoke collar e serves to take up wear.

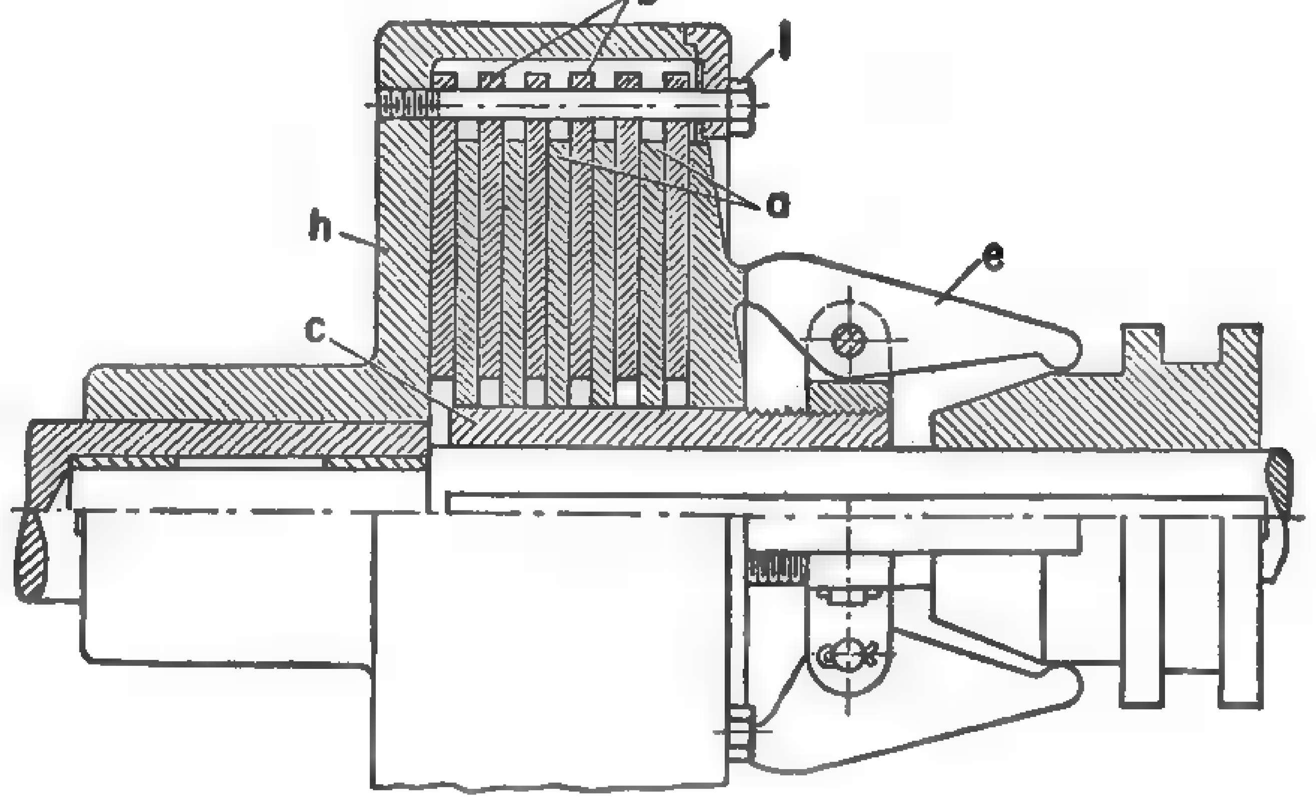


FIG. 22–7. Multidisk marine-type clutch coupling.

A multidisk clutch coupling often used for driving boats by internal combustion engines is shown in Fig. 22–7. It is similar in construction to that in Fig. 22–6, but both the driving disks a and the driven disks b are comparatively thin cast-iron plates without any facing, and they run in an oil bath. The driving disks a have square holes fitted to the square-shaped hub c. The driven plates b are connected to the housing h by pins l, instead of by a key. For smooth operation, the clutch is partially filled with heavy lubricating oil.

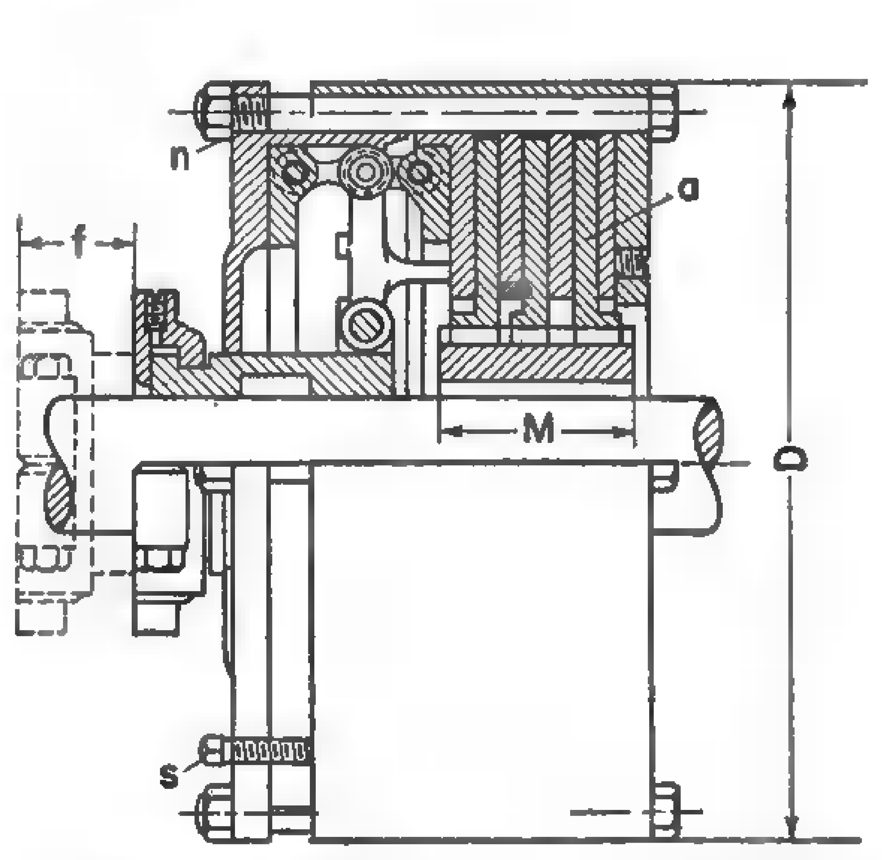


FIG. 22–8. Multidisk industrial clutch.

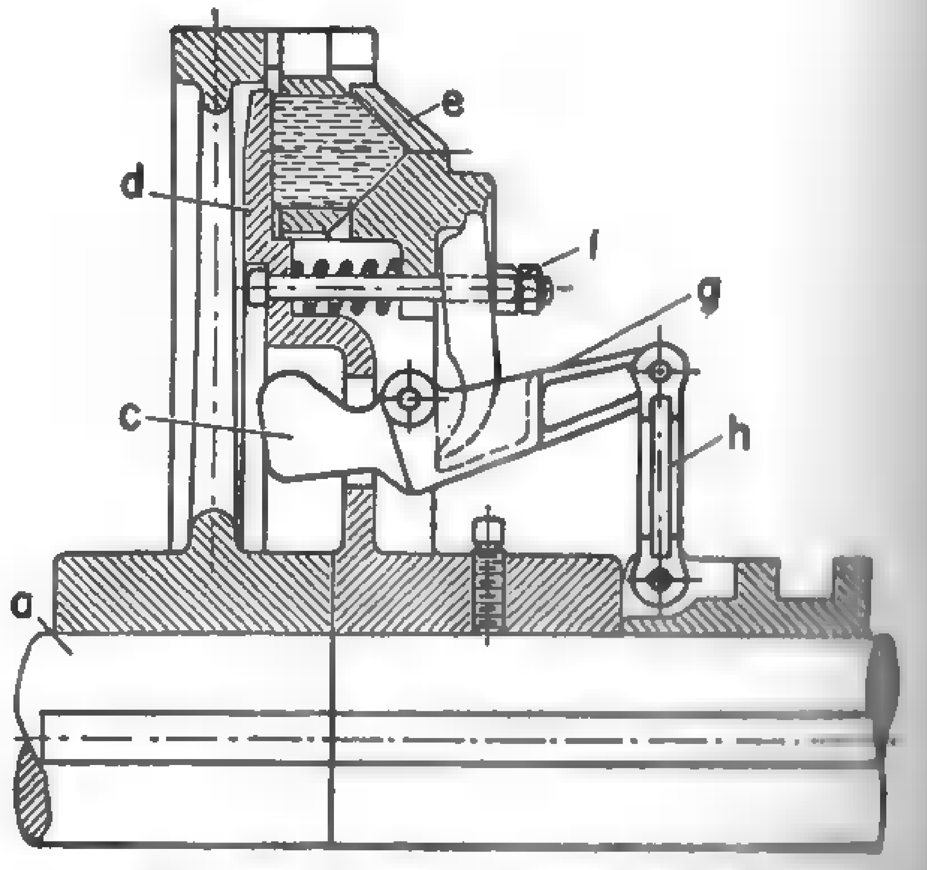


FIG. 22–9. Cone-disk clutch.

A multidisk industrial clutch that is suitable for high speeds is shown in Fig. 22–8. Its construction is similar to the marine clutch in Fig. 22–7, but it is fully enclosed and is actuated by a very powerful double-toggle

mechanism, shown schematically in Fig. 22–17. The smaller disks a, Fig. 22–8, are connected to the hub by a feather key. The necessary adjustment for taking up wear is made by turning nuts n and locking them by screws s. The plates are lubricated to obtain smooth engagement and to reduce wear.

Finally, in Fig. 22–9 is shown an industrial clutch with hardwood or asbestos blocks. The surface of the block in contact with the flange d is flat, while the other end, in contact with the ring e, has the form of a double cone in order to increase friction. The clutch is engaged by a double-toggle mechanism. Counterweights c are put on to balance part of the centrifugal force acting on the lever g. The springs between the parts d and e help to disengage the clutch and to prevent excessive wear of the blocks when the clutch is disengaged. Bolts i are provided to take up wear.

Basic data. Equation 18–53, which was derived for the torque in a brake with i pairs of friction surfaces, can be used without any changes for the torque on a disk clutch. The horsepower that the clutch can transmit may be obtained by substituting this value for T in the general equation 2–17 and introducing the load factor k_l. After the constants are collected, the result is

$$P = \frac{ifp_1 nD_1(D_2^2 - D_1^2)}{160{,}480k_l} \tag{22-11}$$

Similarly, the axial force F_a necessary to create this torque may be found by equation 18–55. Thus,

$$F_a = \tfrac{1}{2}\pi p_1 D_1(D_2 - D_1) \tag{22-12}$$

Design. Both the procedure outlined and special remarks given in section 18–5 apply in full to disk clutches. The values for f and p may be taken from Table 18–1.

Speed. Industrial clutches are designed for a torque at 100 rpm. When the clutch is used at a higher speed, it can be considered that the torque is not changed if the clutch is so designed that the centrifugal force does not affect the engaging mechanism and the pressure on the friction surface. Otherwise its rating should be divided by a speed factor k_s when the clutch is used at a higher speed of n rpm. Some designers determine the speed factor by the relation

$$k_s = 0.9 + 0.001n \tag{22-13}$$

In actual practice, however, it is better to determine k_s from actual tests.

EXAMPLE 22–2. A certain clutch is designed to transmit 65 hp at 100 rpm. Determine its expected minimum capacity at 600 rpm.

By equation 22–13,

$$k_s = 0.9 + 0.001 \times 600 = 1.5$$

Therefore the clutch rating at 600 rpm should be

$$P = \frac{65 \times 600}{100 \times 1.5} = 230 \text{ hp}$$

22–6. Rim clutches. There are a number of different types of rim clutches which vary in the method of gripping the rim and in the shape of the rim. Rim clutches can therefore be classified as block clutches, band clutches, slip-ring clutches, and roller clutches.

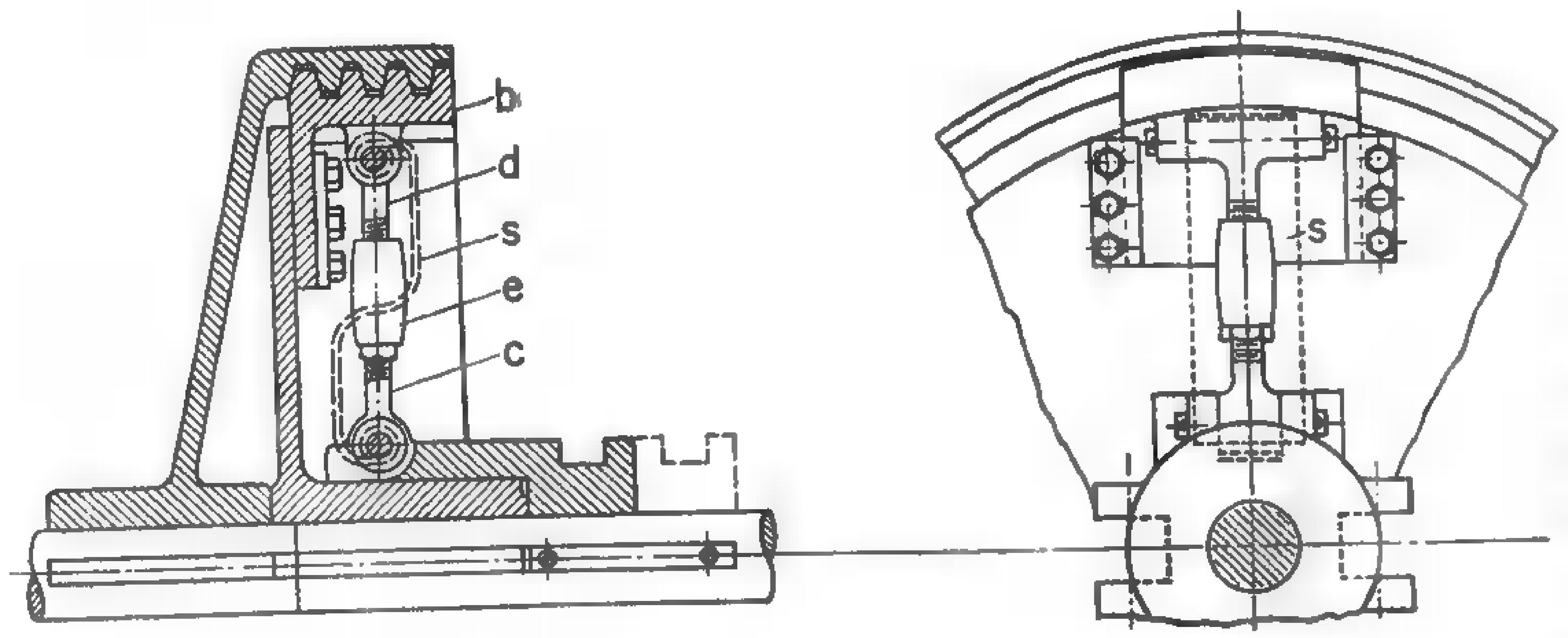


Fig. 22–10. Leblanc block clutch coupling.

In Fig. 22–10 is shown a typical block clutch with a grooved rim surface to increase the torque. The clutch has four, or in larger sizes six, cast-iron blocks b which are pressed against the rim either by adjustable push rods made up of parts c, d, and e, or by flat S-shaped springs s shown in dotted lines. Centrifugal force helps to keep the clutch engaged. Similar clutches are made with flat rims and blocks that are lined with leather or asbestos fabric.

A rim clutch with a double grip, internal and external, is shown in Fig. 22–11. The double-lever engaging mechanism in conjunction with the double row of wooden blocks gives this clutch a high torque capacity.

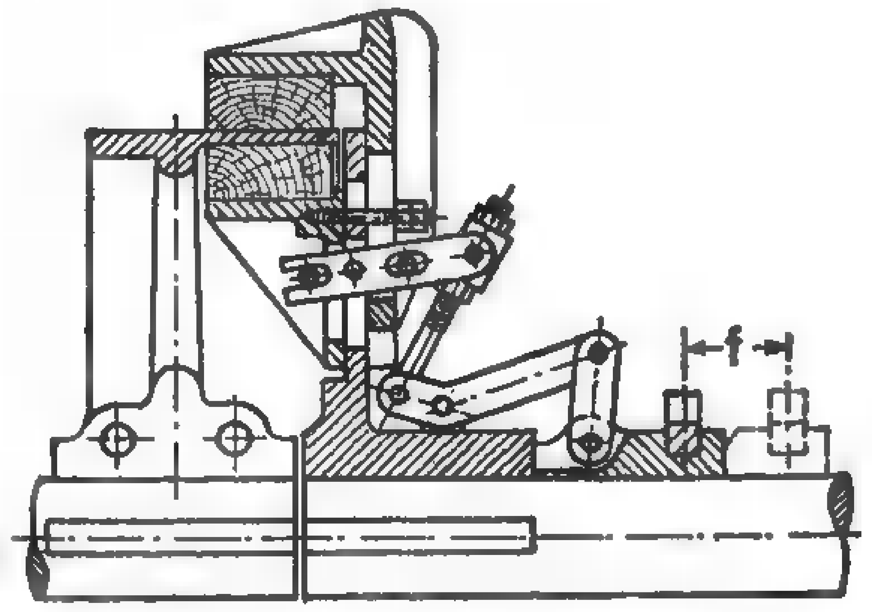


Fig. 22–11. Hill rim clutch coupling.

Force analysis. There are no experimental data regarding the actual distribution of the contact pressure on a rim clutch. For each block, or shoe, of a clutch with a grooved rim, the angle β, Fig. 22–12, is relatively small. Therefore the simplest assumption is justified and the normal pressure p may be considered uniform over the whole contact area of each block. When the clutch is being engaged, the equation for equilibrium of forces along the vertical axis is

$$N = N'(\sin\alpha + f\cos\alpha) \tag{22–14}$$

After the block is pressed on firmly,

$$N = N'\sin\alpha \tag{22–15}$$

Since $F_t = fN'$ and $N' = 2\beta Dbp$, the torque which the clutch can transmit is

$$T = \tfrac{1}{2}i_1 i_2 F_t D = i_1 i_2 f\beta D^2 bp \tag{22–16}$$

where i_1 is the number of grooves in the rim, i_2 is the number of shoes, b is the inclined face, Fig. 22–12, and β is expressed in radians. If another assumption is used for the pressure distribution, such as uniform wear, the torque equation will be changed, but numerically the difference is smaller than the uncertainty in regard to the friction coefficient.

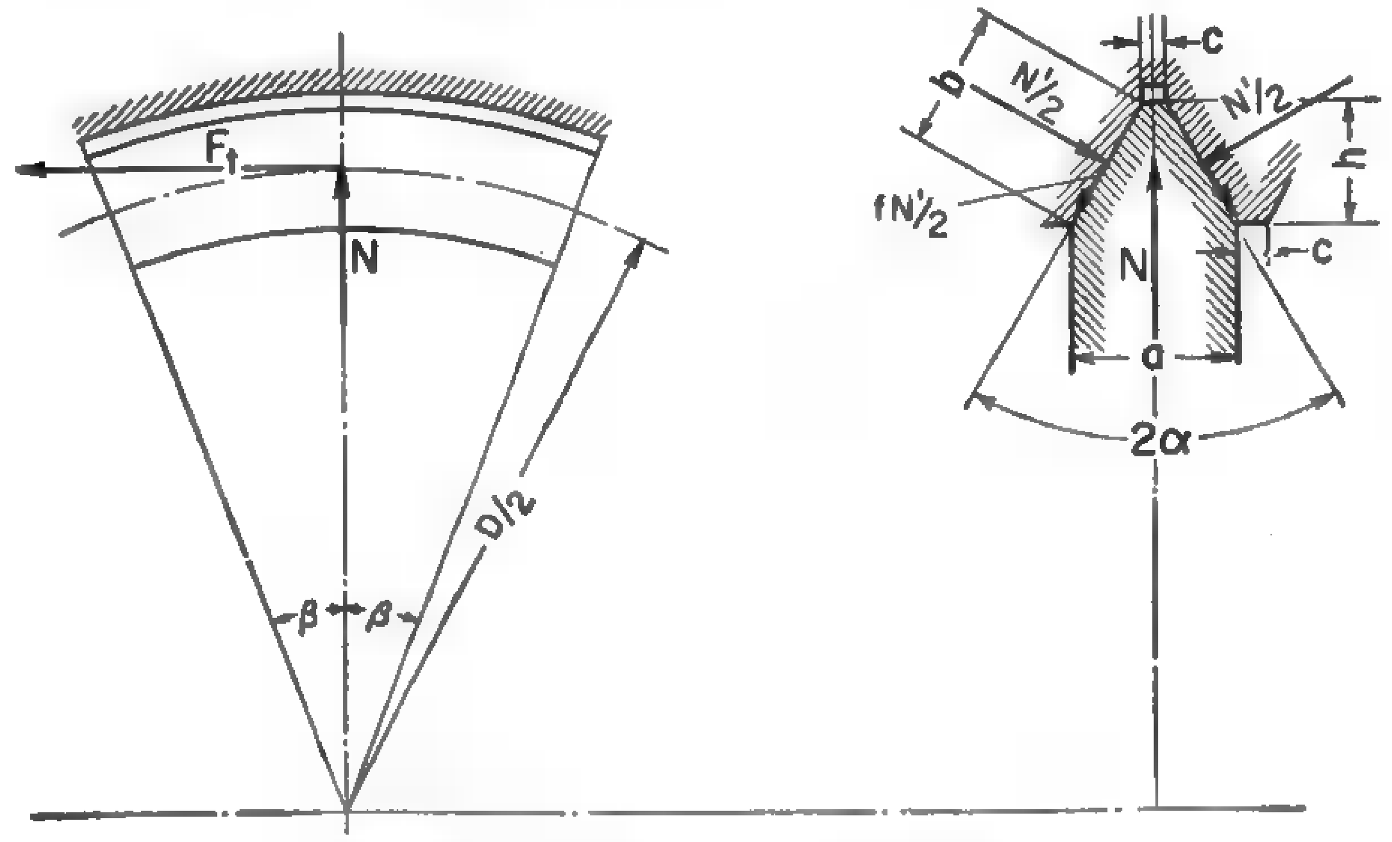


Fig. 22–12. Grooved rim clutch.

If the clutch has a flat rim, $i_1 = 1$ and the number of sides b is only one-half that of a grooved rim. Thus equation 22–16 becomes

$$T = \tfrac{1}{2}if\beta D^2 bp \tag{22–17}$$

Design data. In order to prevent grabbing, the groove angle α should not be made smaller than 15° with cast-iron blocks, or smaller than 20° with wood or asbestos blocks. An angle α greater than 25° decreases the torque too much. The width b of the inclined face, in inches, may be determined by the equation

$$b = 0.01D + 0.25 \tag{22–18}$$

The number of grooves varies from three for small clutches up to six or seven for large ones. The total arc of contact, $2i_2\beta$, should be about 180°, or one-half the circumference. The space c can be made between $0.15h$ and $0.25h$.

Example 22–3. Determine the main dimensions and the force of the S-shaped springs for a clutch similar to that in Fig. 22–10 to transmit 800 hp at 220 rpm from an electric motor to a mine hoist.

The torque to be transmitted is, by equation 2–17,

$$T = \frac{63{,}030 \times 800}{220} = 229{,}200 \text{ lb-in.}$$

According to Table 20–3, a load factor of 1.75 should be used. The design torque is then $T = 401{,}200$ lb-in.

If the peripheral velocity is assumed as 1,300 fpm at 100 rpm, or 2,860 fpm at 220 rpm, the mean diameter D_m may be selected as

$$D_m = \frac{2{,}860 \times 12}{\pi \times 220} = 49.7, \text{ or } 49\tfrac{3}{4} \text{ in.}$$

For cast-iron shoes on a cast-iron rim the angle α may be taken as 15°. From Table 18–1, safe values would be $f = 0.15$ and $p = 150$ psi. The groove face, by equation 22–18, is

$$b = 0.01 \times 49.75 + 0.25 = 0.748 \text{ in.}$$

With these values the arc of contact of each shoe, by equation 22–16, is

$$2\beta = \frac{401{,}200 \times 2}{0.15 \times 49.75^2 \times 0.748 \times 150 \times i_1 \times i_2} = \frac{19.3}{i_1 i_2}$$

If seven grooves are made in the rim, $i_1 = 7$, and if six shoes are used, or $i_2 = 6$, then the arc required is $2\beta = 0.460$ radians, or 26.3°. This is satisfactory, as the total arc is $26.3 \times 6 = 157.8°$, which is somewhat less than 180°.

The depth of the grooves is

$$h = b \cos \alpha = 0.748 \times 0.966 = 0.723, \text{ or } \tfrac{3}{4} \text{ in.}$$

The inside rim diameter is then $D' = 49\tfrac{3}{4} - \tfrac{3}{4} = 49$ in. The distance between the grooves is

$$c = 0.25h = 0.25 \times 0.75 = 0.1875, \text{ or } \tfrac{3}{16} \text{ in.}$$

The width of each groove is

$$a = 2h \tan \alpha + c = 2 \times 0.75 \times 0.268 + 0.187 = 0.589, \text{ or } \tfrac{19}{32} \text{ in.}$$

and the width of the rim is

$$B \geq 7 \times (\tfrac{19}{32} + \tfrac{3}{16}) + \tfrac{3}{16} = 5\tfrac{11}{16} \text{ in.}$$

To find N, or the pressure which each spring s must exert, it is necessary first to compute the magnitude of N', which is

$$N' = 2 i_1 \beta \frac{Dbp}{i_2} = 7 \times 0.460 \times \frac{49.75 \times 0.748 \times 150}{6} = 3{,}000 \text{ lb}$$

Now, by equation 22–15,

$$N = 3{,}000 \times 0.259 = 777 \text{ lb}$$

Band clutches. The force analysis and design of band clutches can be conducted along the lines of band brakes. These clutches are rather bulky, and are not balanced; they can be used only for low speeds and at present are seldom used.

22–7. Expansion clutches. There exist a great variety of expansion-ring, or slip-ring, clutches. Some have cast-iron rings, and some have heavy steel bands lined with asbestos fabric. They differ chiefly in the mechanism for expanding the ring.

The cast-iron split ring a in Fig. 22–13 is fastened to the hub b that is keyed to the shaft c and is fitted into the outer shell s, which runs loose. When the nosepiece n is pushed toward the flange, the action of a pair of levers l with rollers r on one end expands the half rings and thus engages the shell s to which is fastened a gear or a pulley.

Moment of friction. There are no data available in regard to the actual distribution of the pressure exerted by the ring of an expansion clutch upon the clutch shell. If the average pressure is designated by p, the tor-

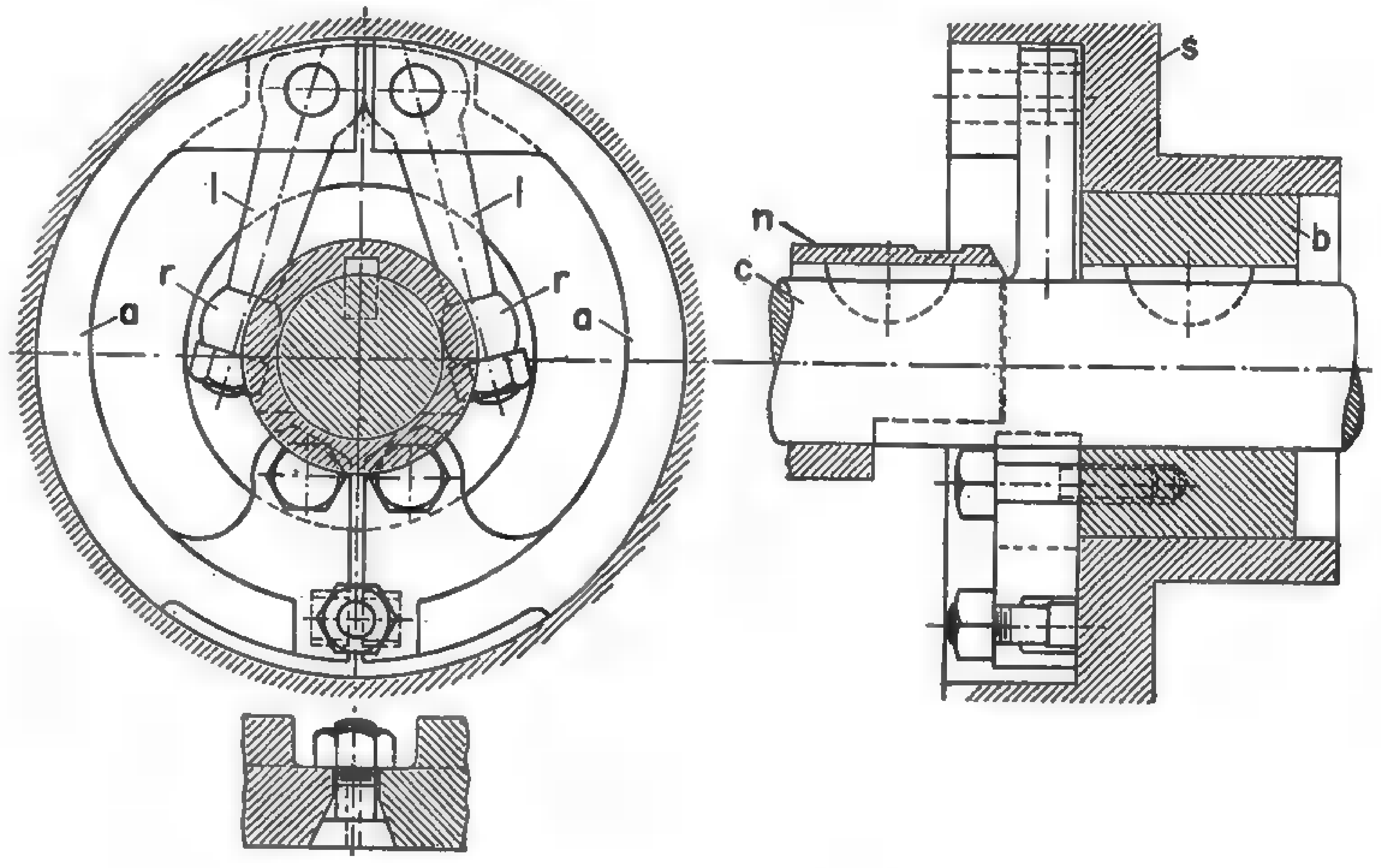


FIG. 22–13. Machine-tool expansion clutch.

sional moment of friction acting upon the elementary area $\frac{1}{2}bD\,d\theta$ in Fig. 22–14 is

$$dT = \tfrac{1}{4} f p b D^2 d\theta \qquad (22\text{–}19)$$

The angle of contact to the ring may be assumed to be equal to 2π. Assuming the coefficient of friction f to be constant, and integrating equation 22–19, gives

$$T = \tfrac{1}{2} f \pi p b D^2 \qquad (22\text{–}20)$$

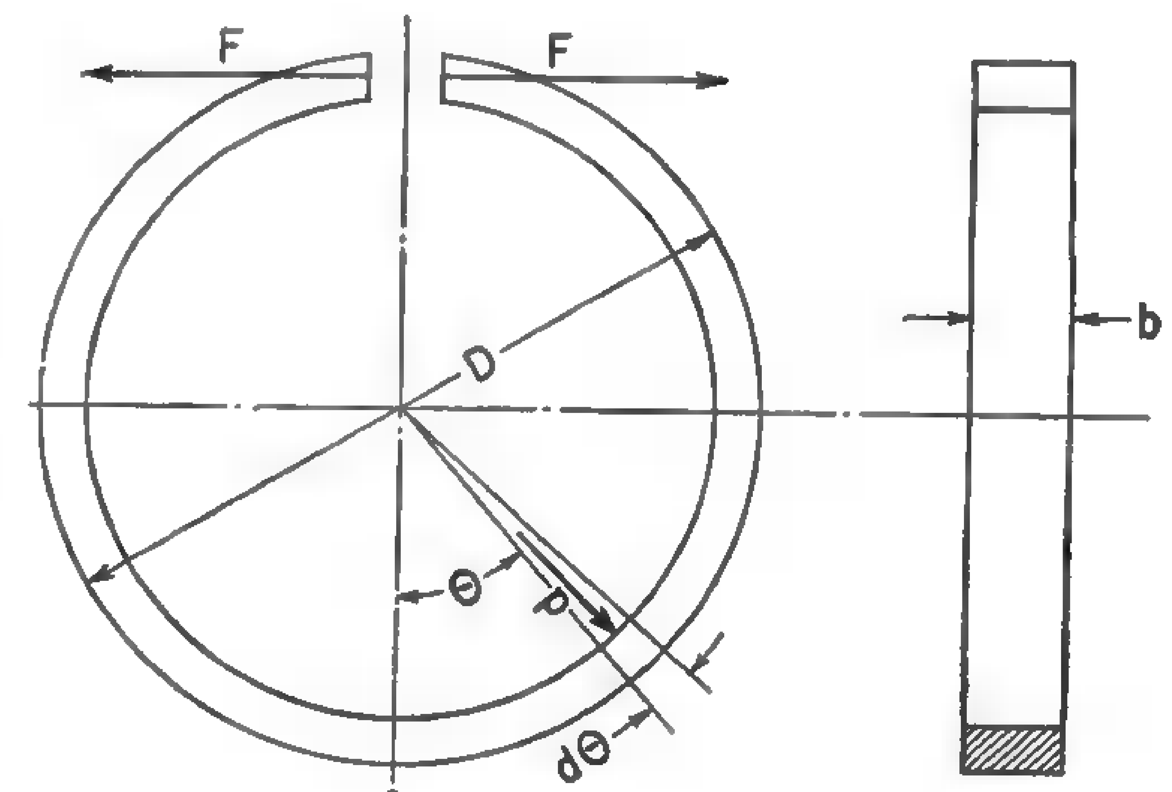


FIG. 22–14. Clutch expansion ring.

Power capacity. The horsepower P transmitted by the clutch at n revolutions per minute may be found by substituting the value of T from equation 22–20 in equation 2–17 and introducing the load factor k_l. Thus,

$$P = \frac{f p b n D^2}{40{,}120 k_l} \qquad (22\text{–}21)$$

Force to spread the ring. The outside diameter of the split ring is made very little smaller than the inside diameter of the shell. Even with a comparatively heavy ring, the force necessary to spread the ring so as to bring it into contact with the shell is negligibly small as compared with the force F, Fig. 22–14, which must be exerted by the operating mechanism to produce the necessary pressure p against the shell.

The force acting upon an elementary length $\frac{1}{2}Dd\theta$ of the ring is $\frac{1}{2}pbDd\theta$, and the moment of this force about the section at c, Fig. 22–14, is

$$dM = \tfrac{1}{4}pbD^2 \sin\theta d\theta \tag{22–22}$$

Integrating from 0 to π gives, for the total bending moment upon the ring,

$$M = \tfrac{1}{2}pbD^2 \tag{22–23}$$

Since this moment must equal that due to the force F,

$$FD = \tfrac{1}{2}pbD^2$$

or

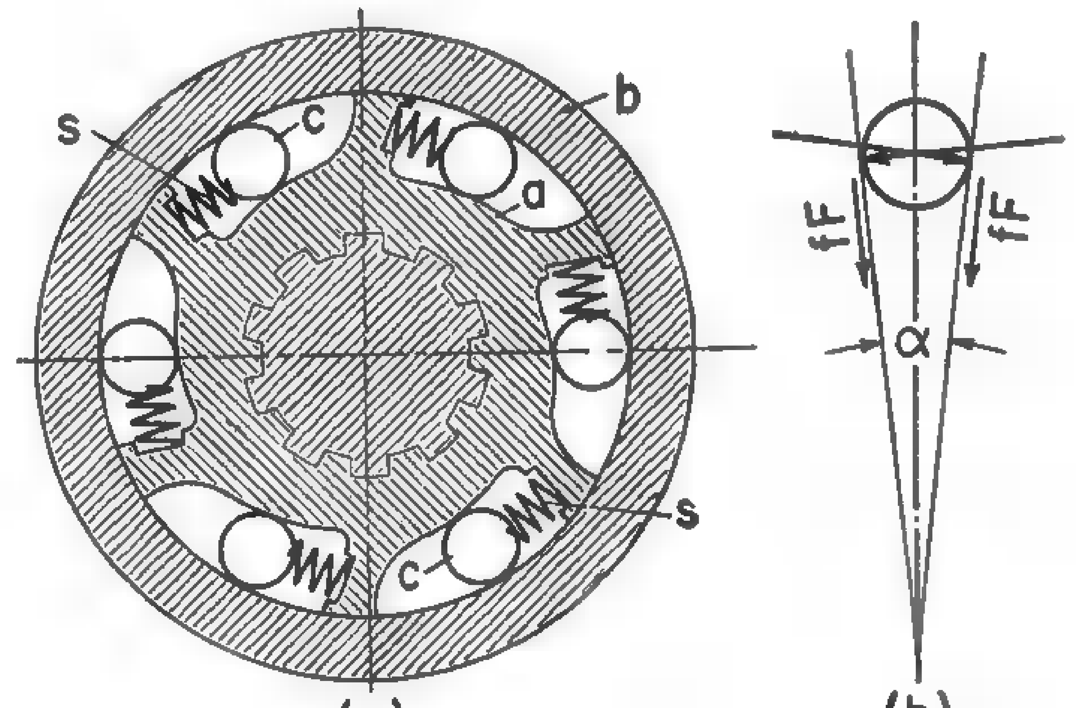
$$F = 0.5pbD \tag{22–24}$$

When equations 22–20 and 22–24 are combined, the result is

$$F = \frac{T}{f\pi D} \tag{22–25}$$

Design data. The outside diameter of a split cast-iron ring is usually made $\frac{1}{64}$ to $\frac{1}{32}$ in. smaller than the inner diameter of the shell. If an asbestos-lined ring of flat steel is bent, the difference is made $\frac{1}{32}$ in. for a small clutch and up to $\frac{1}{4}$ in. for a large one.

The width b may be determined from equation 22–24 by using for p values given in Table 18–1. The coefficient of friction f may be assumed as 0.125 for cast iron on cast iron and as 0.25 for asbestos-lined rings on cast iron.



(a) (b)

FIG. 22–15. Roller clutch.

22–8. Roller clutches. In Fig. 22–15 is represented a roller clutch which is used as a free-wheeling clutch in power transmission. This clutch is in fact a friction ratchet. The cam a is keyed to the drive shaft and consists of several recesses which form inclined planes. The rollers c, impelled up these inclined planes by friction between them and the shell b, wedge themselves between a and b. This action causes the shell b, which is keyed to the driven shaft, to rotate with the drive shaft. The rollers are held in place by a cage. In some industrial clutches the cage is used to engage and disengage the clutch while the drive shaft is running. In automobile use the clutch is engaged automatically when the rotary speed of the driving shaft is greater than that of the driven one, and it is disengaged when the speed relation is reversed.

The condition for the operation of the clutch is that the angle α, Fig. 22–15b, between the tangents to the curves of the cam and the shell at points of roller contact, must be smaller than twice the angle of friction ϕ; that is, $\alpha < 2\phi$, where $\tan\phi = f$. In order that the rollers will not stick and the clutch may be disengaged readily, the difference between 2ϕ and α must be small.

Design data. The friction coefficient f between hardened and polished steel surfaces may vary from 0.03 to 0.05, its value depending on the material and lubrication. For $\tan\phi = 0.03$, the angle $\phi = 1°\,43'$; thus, α must be less than $3°\,26'$.

The force F crushing the roller is evidently

$$F = \frac{F_t}{\tan\alpha} \tag{22–26}$$

where F_t is the tangential force necessary to transmit the torque at a pitch diameter D. This torque is $\frac{1}{2}F_tD$.

When three rollers are used, it is safe to assume that the torque is distributed evenly among all three rollers. In practice a greater number of rollers are used, and extreme accuracy in workmanship is required in order that all rollers may carry the torque evenly.

The allowable load upon a roller depends on the roller diameter d. For i rollers it may be computed by the equation[1]

$$F \leqq iS_bk'ld \tag{22–27}$$

where k' is the coefficient of the flattening of the roller, l is the length of the roller, and S_b is the allowable crushing stress. For high-grade hardened chrome-steel, such as SAE 5185, S_b may be taken as high as 150,000 psi. Numerically

$$k' = \frac{4.64S_b}{E} \tag{22–28}$$

The roller diameter d is made from $0.1D$ to $0.25D$.

The outer shell not only must be strong enough to withstand the tendency of the force F to split it but also must be rigid enough not to show any distortion.

EXAMPLE 22–4. **Determine the main dimensions of a roller clutch, Fig. 22–15, to transmit the full torque capacity of a $1\frac{1}{4}$-in. shaft.**

For steel for which the elastic limit in shear is $S_e = 26{,}000$ psi, and for a sudden load for which the safety factor is $n = 4$, the torque found from equation 20–1, without considering the size factor, is

$$T = \frac{\pi \times 1.25^3 \times 6{,}500}{16} = 2{,}490 \text{ lb-in.}$$

[1] Hütte, *Des Ingenieurs Taschenbuch*, 26th ed., Vol. II (Berlin: Wilhelm Ernst & Son, 1931), p. 131.

If the cam diameter is taken as $D = 2D_o$, or 2.5 in., the tangential force is

$$F_t = \frac{2{,}490 \times 2}{2.5} = 2{,}000 \text{ lb}$$

By equation 22–26, in which the cam angle α is assumed to be 3° 20′, or $\tan \alpha = 0.0582$, the crushing force F is

$$F = \frac{2{,}000}{0.0582} = 34{,}400 \text{ lb}$$

The coefficient k' is, by equation 22–28,

$$k' = \frac{4.64 \times 150{,}000}{30{,}000{,}000} = 0.0232$$

The diameter d of the rollers may be $0.20D = 0.20 \times 2.5 = 0.5$ in. If the clearance between each pair of rollers is approximately equal to the roller diameter, the number of rollers i is found from the relation

$$i = \frac{\pi(D+d)}{2d} = \frac{\pi(2.5+0.5)}{2 \times 0.5} = 9.4 \text{ (use 9)}$$

The roller length, from equation 22–27, is

$$l = \frac{34{,}400}{9 \times 150{,}000 \times 0.0232 \times 0.5} = 2.20 \text{ in., or } 2\tfrac{1}{4} \text{ in.}$$

22-9. Clutch linkages. Clutch-operating linkages are used to obtain a relatively large force F_p necessary to press the friction elements against one another, by applying a moderate external force F_e. The mechanical advantage F_p/F_e is obtained by different combinations of links and levers. However, all these arrangements are based on three simple principles: toggle mechanism, leverage, and inclined plane.

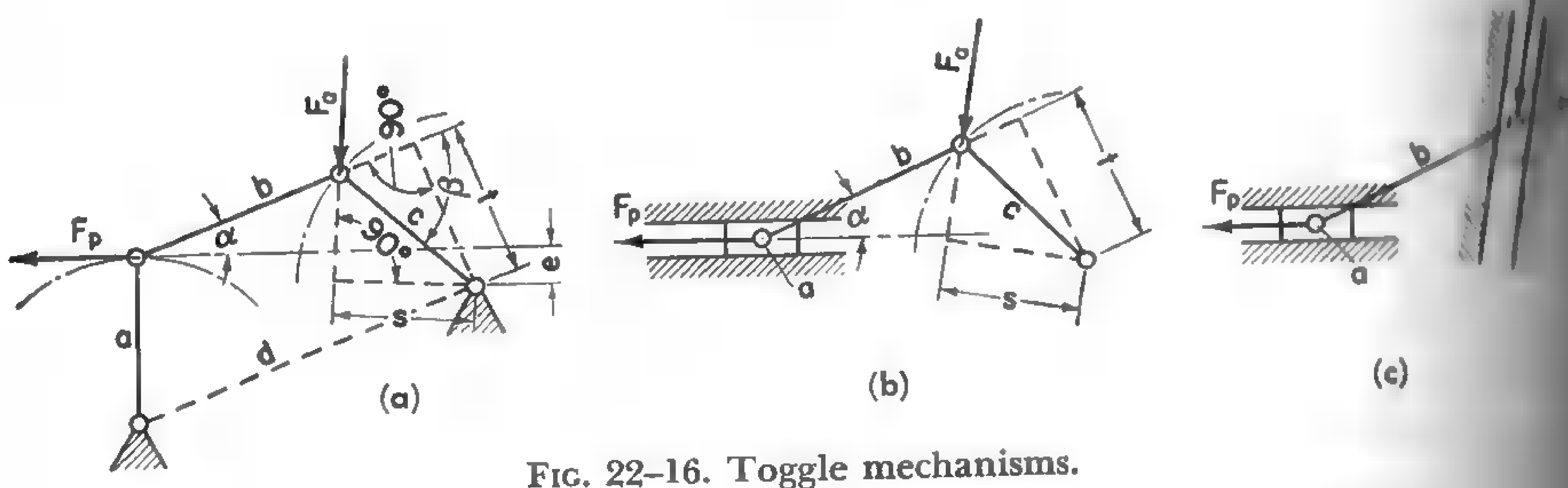


FIG. 22-16. Toggle mechanisms.

Toggle mechanism. In Fig. 22–16a is shown a typical four-bar linkage with two cranks a and c, one connecting rod b, and a stationary link d. The relation between the produced force F_p and the acting force F_a is

$$F_p = \frac{F_a s \cos \alpha}{t} \tag{22–29}$$

A modification of the mechanism is obtained by substituting a slide a, Fig. 22–16b, which is the equivalent of an infinitely long crank. The relation between F_p and F_a is again expressed by equation 22–29. If $\beta = 0$, the distance t theoretically must be 0, and $F_p = \infty$. Actually, because of friction and elasticity of the links, F_p merely becomes very large. When the link b is pushed further down, the angle β is negative and the force F_p will become slightly smaller than its maximum value. However, the mechanism becomes locked, which is very desirable when a clutch is engaged. Usually, $e = 0$; and $\beta = 0$ when $\alpha = 0$.

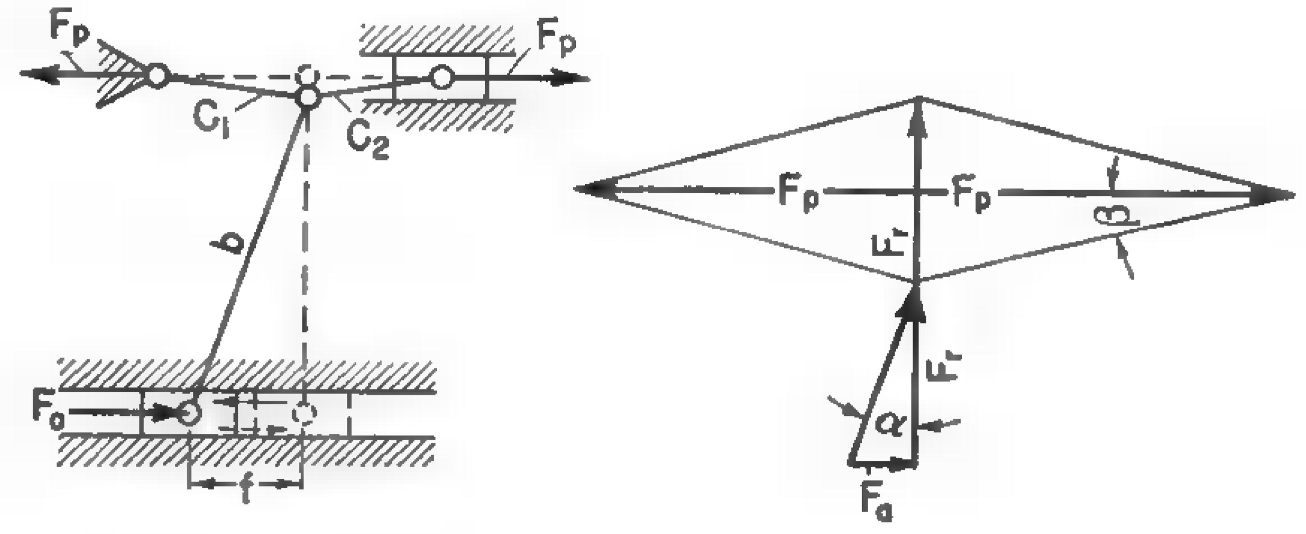


FIG. 22-17. Toggle mechanism.

If both cranks are changed to slides, the arrangement in Fig. 22–16c is obtained. For this case

$$F_p = \frac{F_a}{\tan \alpha} \tag{22–30}$$

In Fig. 22–17a is shown a combination of two toggle joints with a common slide a and crank b. The mechanical advantage gained by such an arrangement is illustrated in Fig. 22–17b, from which it is easy to obtain the relation

$$F_p = \frac{F_a}{2 \tan \alpha \tan \beta} \tag{22–31}$$

The toggle joint is a very powerful arrangement which gives a mechanical advantage up to 20, and even more.

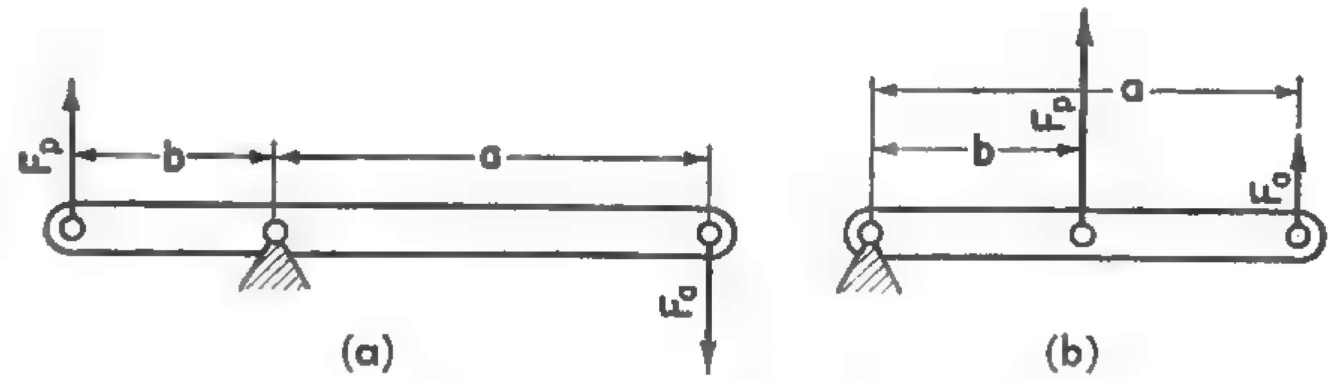


FIG. 22-18. Lever mechanisms.

Lever mechanisms. Two types of lever mechanisms used in clutch linkages are shown in Figs. 22–18a and b. For both cases the force relation is

$$F_p = F_a \frac{a}{b} \tag{22–32}$$

In a clutch linkage the ratio a/b varies from 3 to about 5. A bell-crank lever, as shown in Fig. 22–2 or Fig. 22–4, is equivalent to a straight lever as far as the mechanical advantage is concerned.

Inclined plane. Actually, a clutch uses a conical surface. The analysis is the same as that given in section 11-7 for a helical thread. With the designations of Fig. 22-19,

$$F_p = \frac{F_a}{\tan(\lambda + \phi)} \tag{22-33}$$

The angle λ usually is made between 15° and 20°. The friction angle ϕ corresponds to about 8.5°. It can be decreased to about 4° by using a roller on the lever, as shown in Fig. 22-2. In such a case λ may be increased up to 25° in order to shorten the axial travel of the engagement collar.

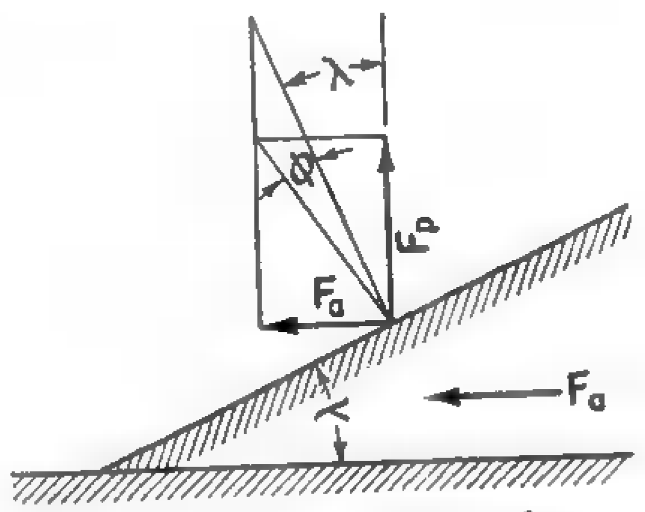


FIG. 22-19. Inclined-plane action.

The inclined-plane arrangement gives a mechanical advantage of 1.5 to about 2.7.

Combination linkage. In most cases, in order to increase the mechanical advantage, several mechanisms are used in series. The mechanical advantage of such a combination linkage is equal to the product of the mechanical advantages of the individual mechanisms.

Thus the combination in Fig. 22-2 consists of an inclined plane and a lever; that in Fig. 22-3 consists of a toggle joint, a lever, and another toggle joint; and that in Fig. 22-6 consists of an inclined plane, a lever, and a toggle joint. The clutch of Fig. 22-8 uses a double toggle mechanism, as illustrated by Fig. 22-17; and the clutch in Fig. 22-10 uses a single toggle joint, as illustrated by Fig. 22-16c.

CHAPTER 23

Bearings with Sliding Contact

23-1. General considerations. A bearing is a machine part which supports a moving part and confines its motion. That part of a shaft which rotates in a bearing is called a *journal*. Bearings in which one rubbing surface slides over another are called *plain bearings* and may be divided into two classes: those with a continuous rotary motion and those with an intermittent motion. To the first class belong journal bearings, which carry a load acting at right angles to the shaft axis, and thrust bearings, which take a load acting in the direction of the shaft axis. To the second class belong bearings of parts having a rocking motion, as wrist pins, or a linear reciprocating motion, as crossheads. A crosshead may be considered a rocking part with an infinitely large radius of the bearing surface.

Bearings with a continuous rotary motion form the great majority of all bearings. They are also the only ones in which an oil film pressure sufficient to support the journal can be created by the journal itself. Bearings with an intermittent motion must depend for proper operation either on an outside source for obtaining the necessary oil pressure or on an abundant oil supply and a low specific bearing pressure.

The failure of a bearing with a sliding contact, or the need of replacement, may be due to excessive wear of the bearing surfaces, overheating, or cracking of the bearing metal.

Excessive wear. Wear is caused by metal-to-metal contact. Wear cannot be entirely eliminated, but it can be appreciably reduced by providing sufficient bearing area and adequate lubrication.

Overheating. Overheating is primarily caused by metal-to-metal contact because of an excessive load or improper lubrication. Unless overheating is stopped in time, it may cause either seizing of the journal or melting of the bearing surface. Seizing will occur if the journal runs in a bearing of hard metal, such as a copper alloy. The bearing surface will melt if the bearing is lined with a metal, such as babbitt, which melts at a low temperature. Lubrication decreases the danger of overheating. However, overheating may occur even with proper lubrication if the heat dissipation of the bearing is not adequate.

Cracking. The bearing metal may crack if it is subjected to heavy shock loads, such as are taking place in the running gear of internal-combustion engines when the compressive stresses in the bearing metal exceed its en-

durance limit. The remedies are to lower the specific bearing pressure or to use a bearing metal with a proportionately higher endurance limit. However, even in this case the presence of an oil film is useful as a shock absorber.

Design basis. From the discussion of causes of bearing failure it is evident that reducing friction by proper lubrication is one of the main problems of bearing design. Proper lubrication means making provision to interpose a liquid film between the rubbing surfaces, and thus to substitute fluid friction for frictional resistance between the metal surfaces of the journal and its bearing. Factors which must be considered in the design of a bearing are specific pressure, rubbing speed, viscosity of the lubricant, and heat dissipation.

23–2. Theory of journal lubrication. Most bearings supporting rotating machine parts are lubricated with oil. Some are lubricated with heavy grease; others are lubricated with water; and a few can run apparently dry if the surface is impregnated with oil or graphite.

According to the hydrodynamic theory of lubrication of rotating journals with oil, the following three phases of lubrication may be considered: (a) starting of the journal from rest; (b) operating with imperfectly lubricated surfaces; (c) running with perfectly lubricated surfaces.

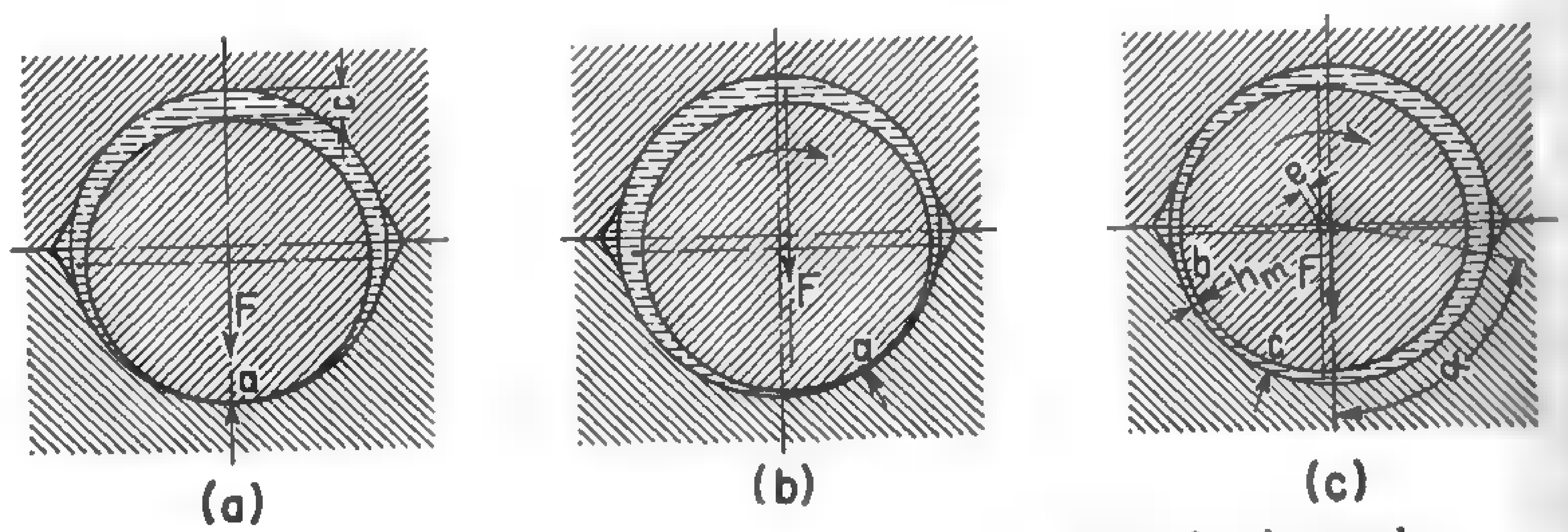


FIG. 23–1. Oil film at various phases of rotation of a journal.

Starting. If two lubricated surfaces are pressed together by a load, the pressure tends to expel the lubricant from between the surfaces. When machinery stands at rest, a large amount of the lubricant will be squeezed out and the metal surfaces will come more or less into contact, as in Fig. 23–1a at point *a*. When the shaft starts to rotate, the friction of the journal against the bearing is high and a certain amount of abrasion will always occur.

Imperfect lubrication. As the journal begins to rotate, Fig. 23–1b, it tends to roll up toward the right side of the bearing, the contact being at some point *a*. Because of molecular attraction, the wedge-shaped oil film is drawn in between the rubbing surfaces, at first in the shape of a thin film. In *thin-film lubrication*, or *imperfect lubrication*, there exists an unstable condition, and the metal surfaces may therefore touch each other from time to time.

Under certain conditions, such as low rubbing speeds or high unit loads, thin-film lubrication may continue indefinitely.

Perfect lubrication. The rotating journal acts as a pump, and the oil pressure increases with an increase of the speed. If the surface velocity of the journal increases above that of imperfect lubrication and there is a sufficient oil supply, enough oil pressure will be developed to raise the shaft completely off the bearing and to make it float upon the lubricant. Oil carried by the pumping action pushes the shaft to the left, and the point of nearest approach to the bearing surface moves to *b*, Fig. 23–1c. The point *c* of maximum pressure is somewhere between point *b* and the bottom of the bearing.

Where there is no metal-to-metal contact, lubrication is termed *thick-film lubrication*, or *perfect lubrication*. Since clearance must exist in order to obtain the eccentric position of the shaft necessary for the pumping action, part of the oil is discharged at the ends of the bearing, and the oil pressure decreases from the middle of the bearing toward each end. In Fig. 23–2 is shown the pressure distribution in the plane of maximum pressure for four different loads with the same oil and constant speed.[1] Theoretically each pressure curve should be a parabola. The deviations from parabolas are due to deflection of the shaft.

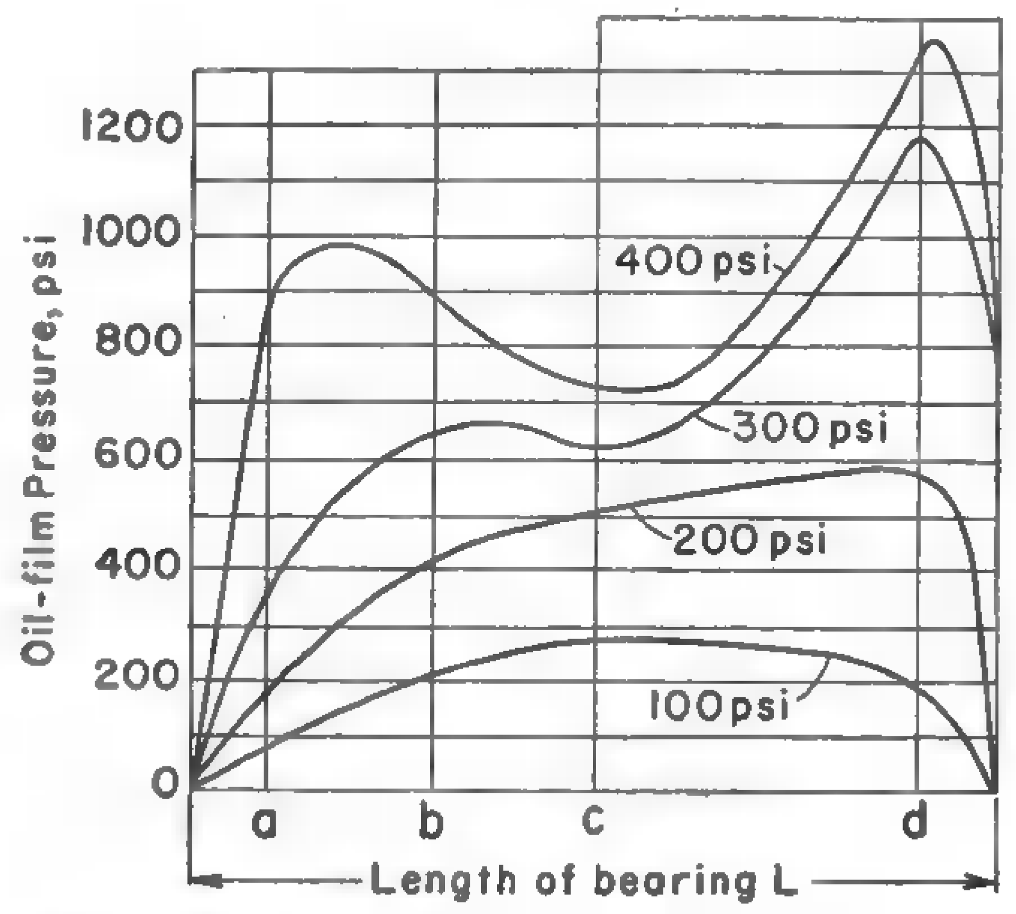


FIG. 23–2. Longitudinal distribution of oil pressure.

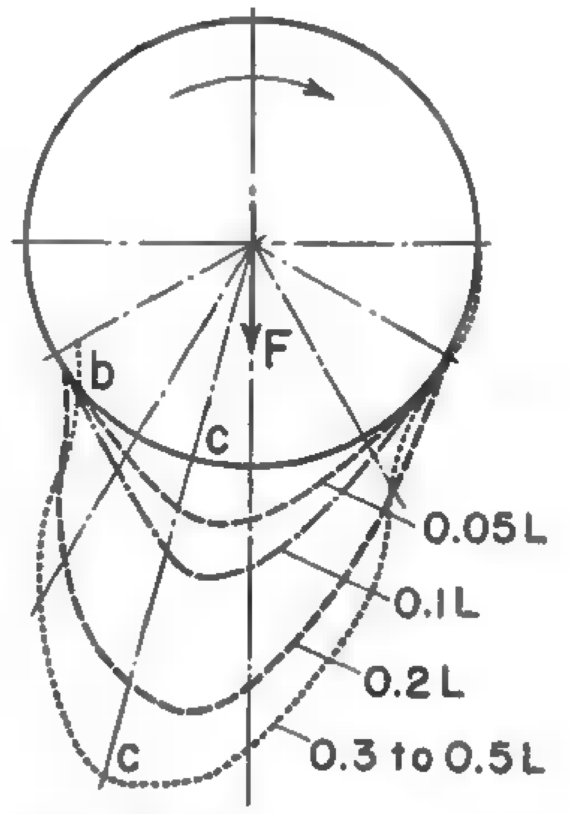


FIG. 23–3. Oil-pressure distribution around the circumference of a bearing.

Typical pressure distribution around the circumference at different planes of rotation is shown in Fig. 23–3. The curve marked $0.05L$ gives the pressure around the journal at a distance of $0.05L$ from the end of the bearing, where L is the length of the bearing; the curve marked $0.5L$ represents the pressure distribution at the center of the bearing. The points of maximum pressure along the bearing lengths are not in the same plane with

[1] L. J. Grunder and L. J. Bradford, *Oil-Film Pressure in a Complete Bearing*, Bulletin No. 50, Pennsylvania State College Engineering Experiment Station (Sept. 8, 1930).

the maximum pressure at point c near the middle of the bearing. The pressure drops to atmospheric or below at some point in the diverging channel after passing point b of minimum approach.

Surface conditions. In order to insure perfect lubrication, the bearing surfaces must be true and smooth. Bearings are finished with reamers or broaches, and in larger sizes are hand-scraped. Journals are polished or accurately ground. However, even the most carefully finished bearings and journals are improved by running in.[2] The improvement is particularly noticeable in the region of thin-film lubrication.

Oiliness. Oiliness is a joint property of a lubricant and metal surfaces in contact. It is an important factor for boundary and thin-film conditions, when actual metal-to-metal contact is prevented only by the absorbed oil film. There is no absolute measure of oiliness—values are only comparative. Lard oil has better oiliness than mineral oils. Babbitts favor the establishment of an absorbed film.

23–3. Viscosity. The most important property of the lubricating oil is its internal friction, or *viscosity*. Viscosity is expressed numerically by a coefficient which represents the resistance offered by a layer of the liquid of unit area to motion parallel to the area of another layer of the liquid, at unit distance, moving with unit velocity with respect to the first layer. In cgs units this coefficient is known as the *absolute viscosity*, and it is equal to the force in dynes per sq cm at a velocity of 1 cm per sec at a distance of 1 cm. The unit of absolute viscosity is termed a *poise*. For practical purposes the unit is too big, so one one-hundredth of it, the *centipoise*, is commonly used. It so happens that the viscosity of water at 68.4 F is exactly one centipoise. In English units viscosity is measured in pounds per sq in. per inch per second per inch of film thickness, or in pound-seconds per sq in., and the unit is called a *reyn*. If the viscosity in centipoises is designated by Z, and that in *reyns* by μ_o,

$$\mu_o = 1.45 \times 10^{-7} Z \tag{23–1}$$

In practice, viscosity is measured by an instrument called a *viscosimeter*, or *viscometer*. The viscometer used in this country is the Saybolt Universal. It gives the time in seconds required for a certain volume of the oil to flow under a certain head through a tube of a standard diameter and length. Since the viscosity of an oil varies with the temperature, the reading must be made at a certain temperature, usually at 100 F or 210 F or both. The viscosity thus determined is called *specific viscosity* and is designated by SSU, which stands for Saybolt Seconds Universal, or by S. In order to convert specific viscosity S to absolute viscosity, it is necessary to introduce another coefficient known as the *kinematic viscosity*. Kinematic viscosity Z_k is the ab-

[2] S. A. McKee, "The Effect of Running-in on Journal Bearing Performance," *Mechanical Engineering*, Vol. 49 (1927), p. 1335.

TABLE 23–1

SPECIFIC GRAVITY OF OILS AT 60 F

No.	Oil Characteristic	γ_{60}
A	Turbine oil, ring-oiled bearing	0.8877
B	Turbine oil, ring-oiled bearing, SAE 10	0.8894
C	All-year automobile oil, SAE 20	0.9036
D	Ring-oiled bearing oil, high-speed machinery	0.9346
E	Automobile oil, SAE 20	0.9254
F	Automobile oil, SAE 30	0.9263
G	Automobile oil, SAE 40, medium-speed machinery	0.9275
H	Airplane oil 100, SAE 60	0.8927
I	Transmission oil, SAE 110, spur and bevel gears	0.9328
J	Gear oil, slow-speed worm gears	0.9153
K	Transmission oil, SAE 160, slow-speed gears	0.9365

solute viscosity Z in centipoises, divided by the density, or mass per unit volume, ρ. However, if the absolute viscosity Z is expressed in centipoises, the density ρ of oil and its specific gravity γ relative to water have the same numerical value, and the latter is commonly used. Then

$$Z_k = \frac{Z}{\gamma} \tag{23–2}$$

The unit of kinematic viscosity is termed a *centistoke.*

If absolute viscosity is expressed in lb-sec per sq in.,

$$Z_k = \frac{\mu_o}{\rho} \tag{23–3}$$

where $\rho = w/g$.

The relation between the kinematic viscosity Z_k, in centistokes, and the specific viscosity S, in seconds, obtained from the Saybolt Universal viscometer, is given with sufficient accuracy by the equation

$$Z_k = 0.22S - \frac{180}{S} \tag{23–4}$$

Values of the specific gravity γ_{60} of several representative mineral oils at 60 F compared to that of water at 60 F are given in Table 23–1. In order to find the absolute viscosity of an oil at another temperature t, by equation 23–2, it is necessary to find the specific gravity γ_t at that temperature. This conversion may be made using the equation

$$\gamma_t = \gamma_{60} - 0.00036(t - 60) \tag{23–5}$$

A special semilogarithmic chart, Fig. 23–4, in which oil viscosities are represented by straight inclined lines, makes it possible to find the viscosity of an oil at any temperature if its viscosities at some two temperatures are known.

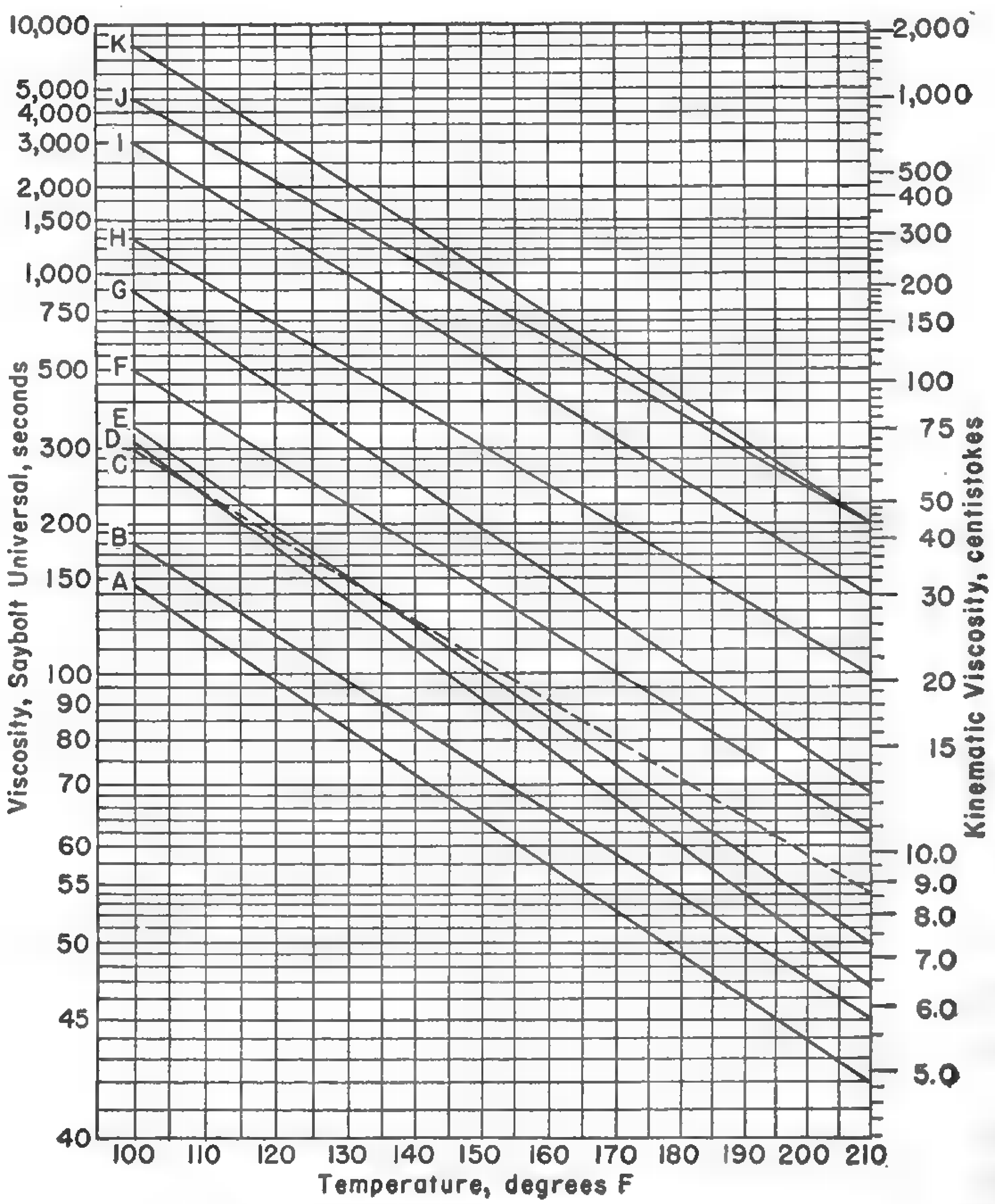


FIG. 23–4. Viscosity-temperature chart.

23–4. Bearing characteristic number. As shown in section 23–2, the oil pressure between a bearing and rotating journal varies both axially and circumferentially, the variation following a parabolic curve more or less. The design of bearings is materially simplified if the variation in pressure is taken into account by introducing the concept of an average *pressure* p. This average pressure is found by dividing the bearing load F by the projected area of the bearing, which area is the product of its length and its diameter. Thus,

$$p = \frac{F}{ld} \tag{23–6}$$

The average pressure p can also be computed by the equation

$$p = \frac{Zvdk}{c^2} \tag{23–7}$$

where Z is the absolute oil viscosity, at the temperature of the oil in the bearing, in centipoises;
v is the peripheral velocity of the journal, in feet per second;
k is a numerical factor which depends on the bearing construction and the ratio of its length to its diameter;
c is the diametral clearance between the journal and the bearing, in inches.

If the peripheral velocity v is replaced by its value in terms of the journal speed n in revolutions per minute—that is, if $\pi dn/12 \times 60$ is substituted for v—and the constant $720/\pi k$ is replaced by k', equation 23–7 becomes

$$p = \frac{Znd^2}{k'c^2} \tag{23–8}$$

Then

$$\frac{Zn}{p} = k'\left(\frac{c}{d}\right)^2 \tag{23–9}$$

The value of Zn/p is very helpful in designing bearings. It is called the *bearing characteristic number* and is a function of the relative clearance c/d, or the clearance per inch of diameter, and of the bearing construction in general, through the value of the factor k'.

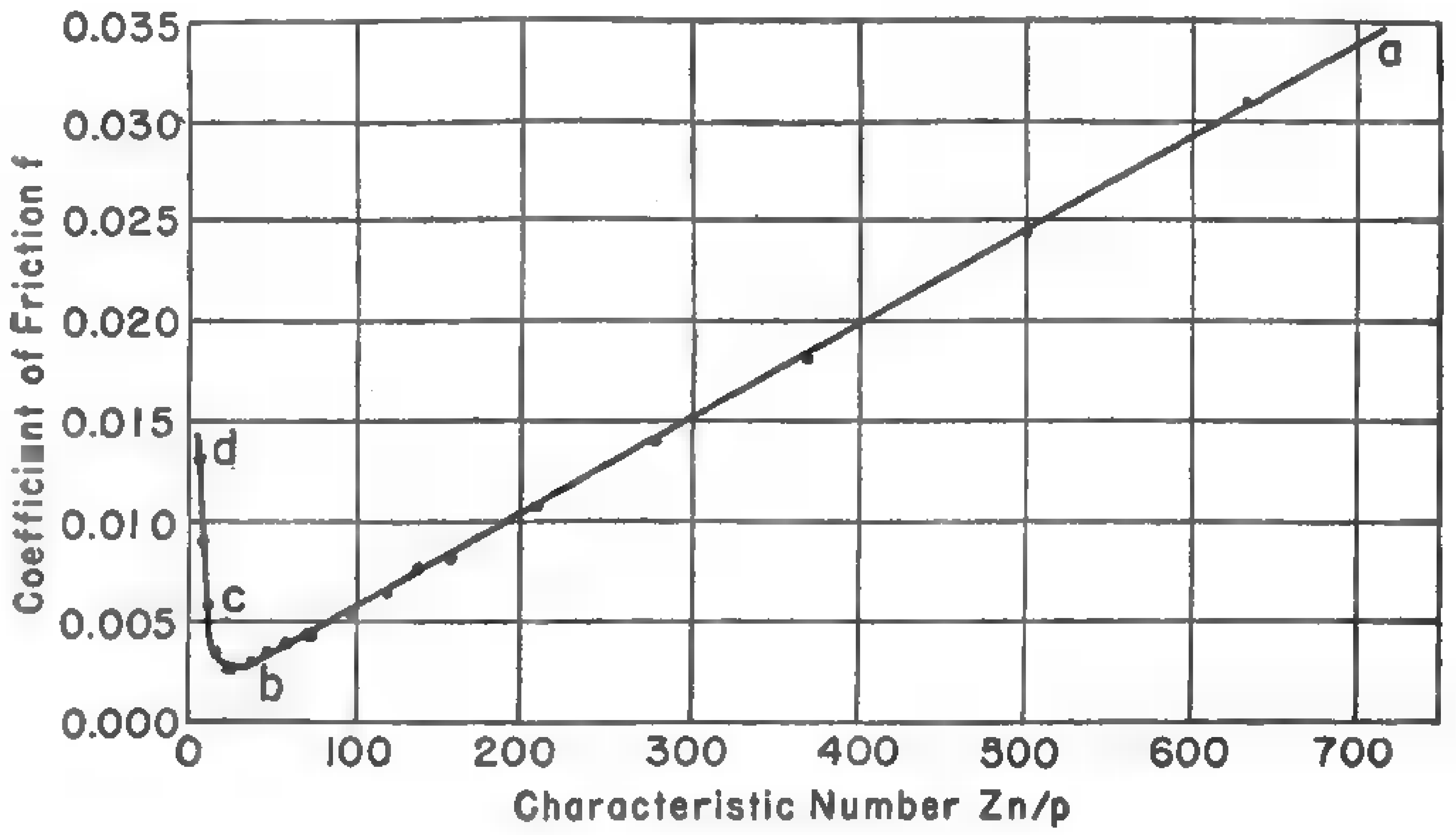


FIG. 23–5. Journal friction for a bearing with $l/d=1.0$ and $c/d=0.001$.

23–5. Friction. A very important factor in the design of bearings is the coefficient of friction f. It has been established that f is a function of the bearing characteristic number Zn/p and the relative clearance c/d. If a given bearing is operated with oils of different viscosities under various conditions of speed and load, and the values of the observed friction coefficients are plotted against the corresponding values of Zn/p, a curve such as *abcd* in Fig. 23–5 is obtained. The portion *ab* corresponds to thick-film lubrication. The portion *cd* corresponds to thin-film lubrication. The segment

bc represents a transition condition.[3] The actual value of f depends on many other factors, such as the materials of the bearing and journal, the condition of the surfaces, and the oil viscosity, which in turn depends on the temperature of the surfaces. The curve in Fig. 23–5 indicates that neither a thin-film condition nor the transition state is stable. Therefore it is advisable to have the value of Zn/p, for a bearing, far enough above the value corresponding to the minimum value of f. On the other hand, it is desirable to have the value of Zn/p as low as possible in the thick-film region in order to have a low friction coefficient and a small loss of power.

Experiments have shown that the coefficient of friction decreases with an increase of the relative clearance c/d. By using data obtained from these tests and plotting the friction coefficient f against the ratio of Zn/p to c/d, it was established that the relation is represented by an inclined straight line. The equation of this line may be taken as

$$f = K_a\left(\frac{Zn}{p}\right)\left(\frac{d}{c}\right)10^{-10} + \Delta f \tag{23–10}$$

The first term of the second member of this equation is the expression obtained by the hydrodynamic theory. The second term is a correction which takes into account the effect of the ratio l/d. Therefore, although the tests were conducted only with small-bore full-journal bearings having perfect lubrication, it seems justified for practical purposes to use this equation wherever a bearing operates with a thick film, the characteristic number $Zn/p \geqq 40$, and $f > 0.001$. The constant K_a may be set equal to 1.31β, where β is the circumferential length of the bearing, in degrees. For a full bearing, $\beta = 360°$ and $K_a = 473$.

The term Δf in equation 23–10 takes into account the effect of end leakage of lubricant. For bearings for which l/d ranges from 0.75 to 2.8, the term Δf may be considered to have the constant value 0.002.[4]

When a journal and its bearing are run in, the curve in Fig. 23–5 is altered so that the point for which f is a minimum is moved to the left. Oiliness has the effect of making the hook in the curve flatter and of moving the portion cd further to the left.

Work of friction. The work of friction W_f is equal to the product of the frictional force and the distance through which the force moves. The frictional force may be taken as the load on the bearing times the coefficient of friction, or $fpld$. The distance through which the force moves in a unit of time is the velocity of a point on the surface of the journal, or $\frac{1}{12}\pi dn$ fpm. Thus,

$$W_f = \tfrac{1}{12}\pi fpld^2n \tag{23–11}$$

[3] S. A. McKee and T. R. McKee, "Friction of Journal Bearings as Influenced by Clearance and Length," *Transactions of the American Society of Mechanical Engineers*, Vol. 51 (1929), APM-51-15, p. 164.

[4] *Ibid.*

TABLE 23–2

VALUES OF FACTOR C_1 IN EQUATION 23–12

Method of Lubrication	Workmanship	Attendance	Operating Conditions	C_1
Bath; flooded; oil ring	Very good	Good	Clean and dust-free	1
Oil, by constant drop feed	Good	Satisfactory	Ordinary conditions	2
Oil, by intermittent feed; grease cup	Fair	Poor	Exposed to dirt	4

23–6. Imperfect lubrication. When the characteristic number Zn/p is too low, or when the supply of lubricant to the bearing is insufficient to maintain fluid-film lubrication, thin-film lubrication and metal-to-metal contact will exist. This condition is represented by the part of the curve in Fig. 23–5 from b to c. The coefficient of friction in this case may be estimated by the following formula:[5]

$$f = 0.004C_1C_2\sqrt[4]{\frac{p}{v_m}} \tag{23–12}$$

where C_1 and C_2 are factors; p is the pressure on the projected area, in pounds per square inch; and v_m is the rubbing velocity, in feet per minute. Values of C_1 are given in Table 23–2, and values of C_2 are given in Table 23–3.

TABLE 23–3

VALUES OF FACTOR C_2 IN EQUATION 23–12

Types and Examples of Bearings	C_2
Rotating journals in rigid bearings and crankpins	1
Oscillating journals, such as rigid wrist pins and block pintles	1
Rotating journals, not very rigid, such as eccentrics	2
Rotating surfaces lubricated from center, such as annular step and pivot bearings	2
Reciprocating crosshead shoes wiping over ends of long guides	2
Reciprocating crosshead shoes wiping over ends of short guides	3
Wiping surfaces, such as marine thrust bearings and worm gears	3–4
Sliding parts, such as long nuts for power screws	4–6

If the load varies, the average pressure is used for p, but it must not be less than one-half the maximum pressure.

A journal in sintered porous bearings is an example of thin-film lubrication. At low and moderate speeds and low loading these bearings are quite satisfactory. The bearing clearance c in such bearings should be slightly greater than that in bearings with thick-film lubrication. Bearings with thin-film lubrication are often called *oilless bearings*, or *self-lubricating bearings*.

23–7. Factors in bearing design. Three factors or ratios that must be considered carefully in designing a bearing are the following: (a) the bear-

[5] Louis Illmer, "High-Pressure Bearing Research," *Trans. ASME*, Vol. 46 (1924), p. 833.

TABLE 23–4

DESIGN DATA FOR BEARINGS

No.	Machinery	Bearing	Maximum p (psi)	Suitable Z (centipoises)	Minimum $\frac{Zn}{p}$	Clearance Class	$\frac{c}{d}$	$\frac{l}{d}$
1	Automobile and aircraft engines	Main	800* –1,700†	7–8	15	2	......	0.8–1.8
2		Crankpin	1,500‡, *–3,500†		10	2	......	0.7–1.4
3		Wrist pin	2,300‡, *–5,000†		8	3	......	1.5–2.2
4	Gas and oil engines, four-stroke	Main	700‡, *–1,200†	20–65	20	2	0.001	0.6–2.0
5		Crankpin	1,400‡, *–1,800†		10	2	<0.001	0.6–1.5
6		Wrist pin	1,800‡, *–2,200†		5	3	<0.001	1.5–2.0
7	Gas and oil engines, two-stroke	Main	500‡, *– 800†	20–65	25	2	0.001	0.6–2.0
8		Crankpin	1,000‡, *–1,500†		12	2	<0.001	0.6–1.5
9		Wrist pin	1,200‡, *–1,800†		10	3	<0.001	1.5–2.0
10	Marine steam engines	Main	500	30	20	2	<0.001	0.7–1.5
11		Crankpin	600	40	15	2	<0.001	0.7–1.2
12		Wrist pin	1,500	30	10	3	<0.001	1.2–1.7
13	Stationary slow-speed steam engines	Main	400	60	20	2	<0.001	1.0–2.0
14		Crankpin	1,500	80	6	2	<0.001	0.9–1.3
15		Wrist pin	1,800	60	5	3	<0.001	1.2–1.5
16	Stationary high-speed steam engines	Main	250	15	25	2	<0.001	1.5–3.0
17		Crankpin	600	30	6	2	<0.001	0.9–1.5
18		Wrist pin	1,800	25	5	3	<0.001	1.3–1.7
19	Reciprocating pumps and compressors	Main	250‡	30–80	30	2	0.001	1.0–2.2
20		Crankpin	600‡		20	2	<0.001	0.9–1.7
21		Wrist pin	1,000‡, *		10	3	<0.001	1.5–2.0
22	Steam locomotives	Driving axle	550	100	30	2	0.001	1.6–1.8
23		Crankpin	2,000	40	5	2	<0.001	0.7–1.1
24		Wrist pin	4,000	30	5	3	<0.001	0.8–1.3
25	Railway cars	Axle	450	100	50	2	0.001	1.8–2.0
26	Steam turbines	Main	100‡–275†	2–16	100	2	0.001	1.0–2.0
27	Generators, motors, centrifugal pumps	Rotor	100‡–200‡	25	200	2	0.0013	1.0–2.0
28	Gyroscope	Rotor	850	30	55	2	0.0013	...
29	Transmission shafting	Light, fixed	25‡	25–60	100	2	0.001	2.0–3.0
30		Self-aligning	150‡		30	2	0.001	2.5–4.0
31		Heavy	150‡		30	2	0.001	2.0–3.0
32	Cotton mill	Spindle	1	2	10,000	1	0.005	...
33	Machine tools	Main	300	40	40	2	0.001	1.0–4.0
34	Punching and shearing machines	Main	4,000‡	100	..	1	0.001	1.0–2.0
35		Crankpin	8,000‡	100	..	1	0.001	1.0–2.0
36	Rolling mills	Main	3,000	50	10	1	0.0015	1.1–1.5

* Splash or scraper lubrication. † Force-feed lubrication. ‡ Ring or drop oiler.

ing pressure, or the ratio of the load to the projected bearing area; (b) the ratio of the bearing length to its diameter; and (c) the relative clearance, or the ratio of the clearance to the bearing diameter.

Bearing pressure. Experience shows that the pressure p is important in bearing design not only because it affects the characteristic member Zn/p but also because there are limits for p which should not be exceeded if thick-film lubrication is to be maintained. These limit values are given in Table 23–4 for various types of machinery and bearings. In some instances, as for automobile engines, locomotives, and punching and shearing machinery, there is not enough room for larger bearing surfaces, and the pressure limits used in practice are higher than those corresponding to thick-film lubrication. These high pressures result in thin-film lubrication. However, they are comparatively safe, as far as friction and wear are concerned.

In the design of motor and generator bearings, the General Electric Company has successfully used the equation[6]

$$p = 15.5\sqrt[3]{v_m} \qquad (23\text{–}13)$$

This equation corresponds to the use of a safety factor of about 2.

Finally, Table 23–5 gives safe bearing pressures for crossheads and trunk pistons of various machines as found in actual operation.

Bearing length. Experiments have shown that for a given characteristic number Zn/p, and for values of the ratio of the bearing length l to its diameter d greater than 0.75, changes in l/d do not materially affect the coefficient of friction. However, the bearing length may affect friction indirectly. If the bearing is long, the deflection of the shaft may result in points of excessively high bearing pressures, which will cause a breakdown of the oil film and therefore cause metal-to-metal contact. This undesirable condition may be remedied by the use of self-aligning bearings with spherical seats. Although an end bearing a, Fig. 23–6, may adjust itself to the deflection, a spherical seat in an inner bearing b may be of little advantage. Therefore, increasing the ratio l/d in order to lower the pressure with a given diameter d is advisable only with a rigid shaft.

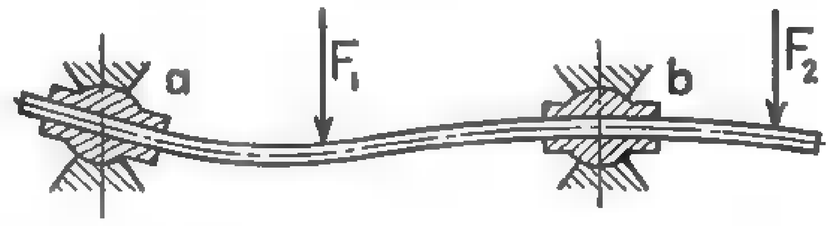


FIG. 23–6. Self-aligning bearings.

Commonly used values of the ratio l/d are given in the last column of Table 23–4. It should be noted that this ratio is governed chiefly by space limitations, rather than by some other factor.

Clearance. The number in the seventh column in Table 23–4 indicates the class of running clearance recommended for each bearing. The corresponding value of c may be computed by using data of Table 13–1.

[6] D. S. Kimball and J. H. Barr, *Elements of Machine Design*, 3d ed. (New York: John Wiley & Sons, Inc., 1935), p. 139.

TABLE 23–5

ALLOWABLE BEARING PRESSURE, RECIPROCATING MOTION

Type of Bearing	Type of Machinery	Pressure p (psi)
Crosshead	Steam engine, stationary	35– 60
	Steam engine, marine	55–100
	Steam engine, locomotive	70– 90
	Gas and oil engines, stationary	40– 70
	Compressors and pumps	50– 90
Trunk piston	Gas and oil engines, stationary	20– 25
	Automotive and aircraft engines	25– 40

There is no definite relation between the relative clearance c/d and the shaft diameter d. Some engineers believe that the relative clearance can be gradually decreased with an increase of the journal diameter, in order to decrease oil leakage. Curves *1* and *2* in Fig. 23–7 show clearances and corresponding tolerances as used for journals of different machines with babbitted bearings in one plant of the Westinghouse Electric Corporation.

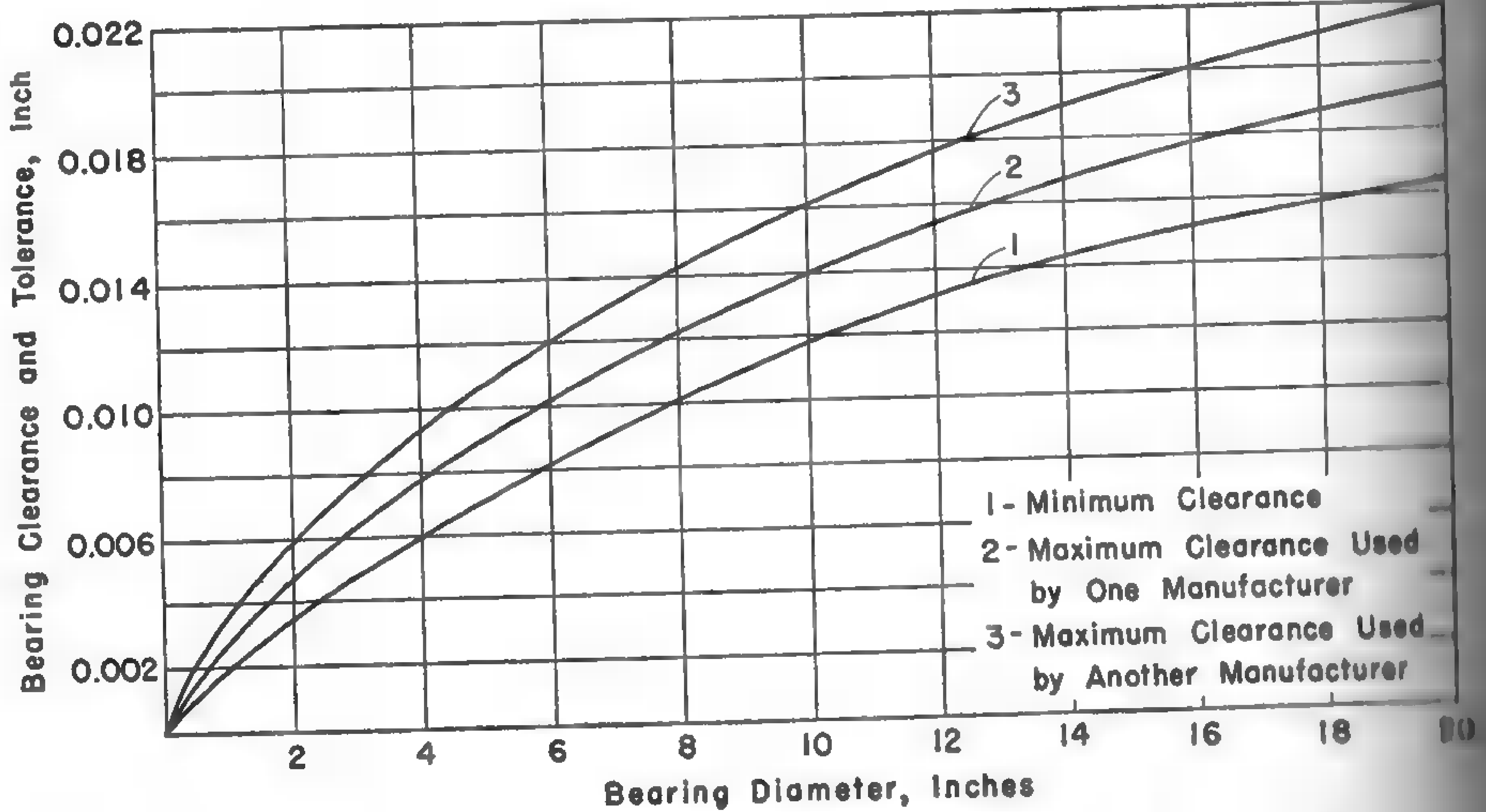


FIG. 23–7. Clearances used in babbitted bearings.

In another plant of the same concern the relation $c/d = 0.002$ is used for the maximum clearance for all shaft sizes. Another manufacturer also uses minimum clearances corresponding to curve *1* of Fig. 23–7, but follows closer tolerances that result in curve *3* for the maximum clearances. Still another manufacturer even recommends that the relative clearance be increased slightly with an increase in shaft size.

Table 23–6 gives running clearances, as used in various industrial applications by the majority of manufacturers, for five typical shaft sizes. This table shows the large tolerances used in practice. For other diameters the clearances may be obtained by interpolation.

TABLE 23–6

BEARING CLEARANCES IN INDUSTRIAL APPLICATIONS

Type of Service, Material, and Finish of Journal and Bearing	Running Clearance C, in Inches, for Shaft Diameter d				
	$d = \frac{1}{2}$ in.	$d = 1$ in.	$d = 2$ in.	$d = 3\frac{1}{2}$ in.	$d = 5\frac{1}{2}$ in.
Precision spindle, hardened and ground steel, lapped-in bronze bearing, rubbing speed under 500 fpm, pressure $p < 500$ psi	0.00025–0.00075	0.00075–0.0015	0.0015–0.0025	0.0025–0.0035	0.0035–0.005
Precision spindle, hardened and ground steel, lapped-in bronze bearing, rubbing speed over 500 fpm, pressure $p > 500$ psi	0.0005–0.001	0.001–0.002	0.002–0.003	0.003–0.0045	0.0045–0.0065
Electric motors and generators, ground journals in broached or reamed bronze bearings or reamed babbitt bearings	0.0005–0.001	0.001–0.002	0.002–0.0035	0.0035–0.005	0.005–0.007
General machinery, continuous rotating or oscillating motion, turned or cold-rolled steel journals in bored and reamed bronze bearings or poured and reamed babbitt bearings	0.001–0.002	0.002–0.003	0.003–0.005	0.005–0.007	0.006–0.008
Rough machinery, turned or cold-rolled steel journals in poured babbitt bearings	0.003–0.005	0.005–0.008	0.008–0.012	0.012–0.016	0.016–0.020

23–8. Heat dissipation. The work of friction in a bearing is transformed into heat which raises the temperature of the oil and of the journal and bearing. This heat must be dissipated in order to prevent an excessive temperature rise. The amount of heat that a bearing will dissipate depends on the difference in temperature of the bearing and the surrounding air, the area of the exposed surface of the bearing and its pedestal, and the rate of movement of air or other gases over the bearing, or of water if cooling by water circulation is used. When a journal begins to rotate, the heat generated is greater than the heat dissipated, and the temperature of the bearing rises until a steady state is reached. The temperature of a bearing should be kept below 150 F if possible, and it should never exceed 180 F. If the equilibrium temperature exceeds the safe limit, artificial cooling must be resorted to. It is possible to pass a stream of air across the bearing or to provide coils circulating cooling water in its body.

The heat-dissipating capacity Q of a bearing, in Btu per min, may be computed by the general expression

$$Q = hA(t_b - t_a) \qquad (23\text{–}14)$$

where h is the film coefficient, in Btu per sq in.-deg F-min;
A is the exposed bearing area, in square inches;
t_b is the temperature of the exposed surface, in degrees F;
t_a is the temperature of the surrounding air, in degrees F.

The film coefficient can be computed by the relation[7]

$$h = mv^{0.89} \tag{23-15}$$

where the constant m may be taken for the selected units of h as 2.6×10^{-6} and v is the air velocity in feet per minute. The air velocity v may be taken as 150 fpm for normal conditions without ventilation, and about 500 fpm for a well-ventilated bearing. The corresponding values of h are 0.00022 and 0.00065, respectively. These values of h are in agreement with data obtained from special bearing tests.[8]

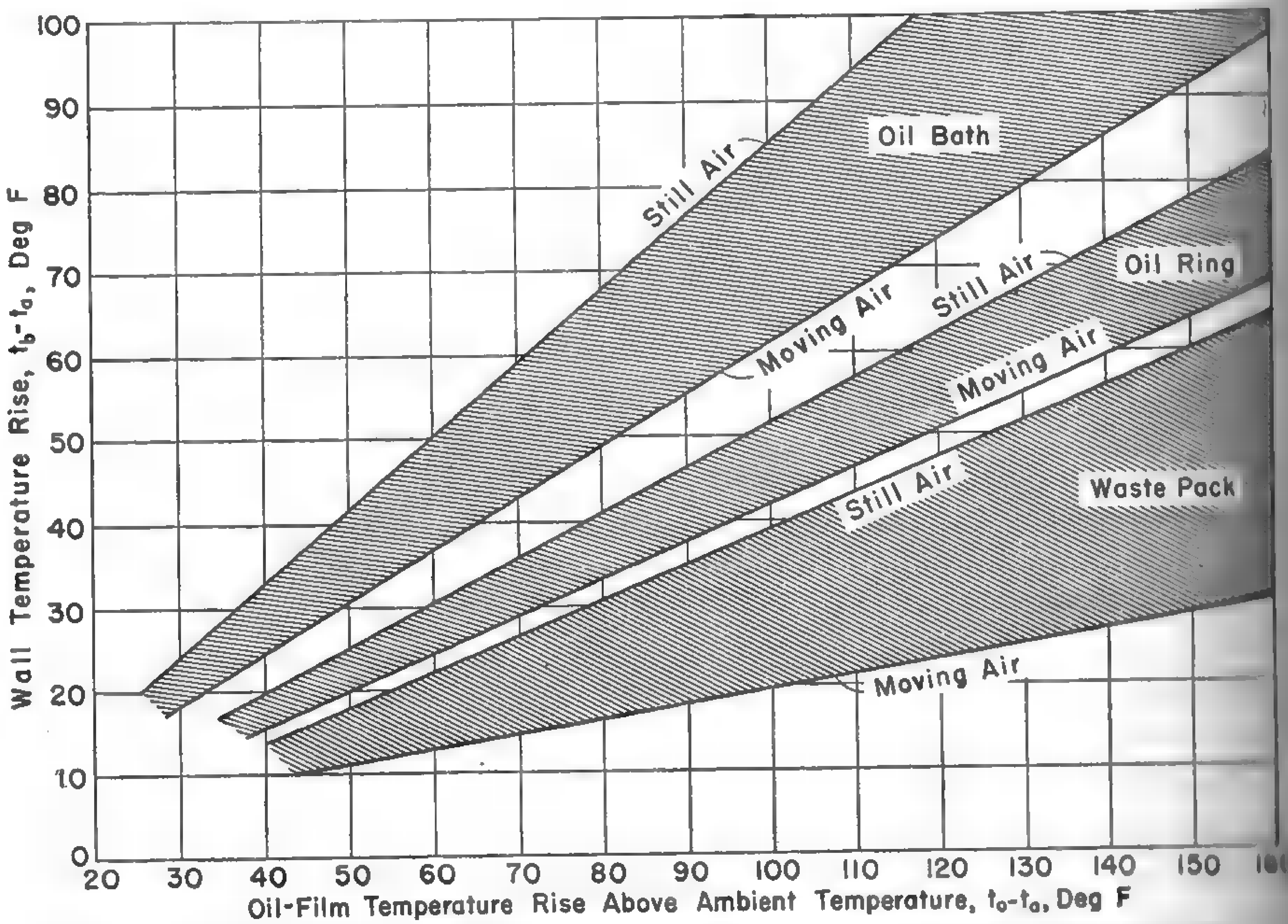


FIG. 23-8. Relation between oil-film temperature and bearing-wall temperature.

Bearing temperature. The temperature of the bearing is lower than the oil temperature, because of heat dissipation. The difference between the bearing-wall temperature t_b and the ambient temperature t_a may be taken from the chart of Fig. 23-8, which gives curves for the three main types of lubrication—by oil bath, by an oil ring, and by waste pack or drop feed.

[7] V. L. Maleev, *Internal Combustion Engines*, 2d ed. (New York: McGraw-Hill Book Company, Inc., 1945), p. 376.

[8] G. B. Karelitz, "Performance of Oil Ring Bearings," *Trans. ASME*, Vol. 52 (1930), APM-52-5, pp. 57-70.

The chart gives two limit conditions for the surrounding air. Data for intermediate conditions may be estimated by interpolation.

Exposed area. In computing the exposed area A of a bearing, it may be assumed that this area includes all parts of the bearing which lie within three diameters of the shaft axis. In a ring-oil bearing the exposed area is the part within 4 in. of the bottom of the oil reservoir.

Use of projected journal area. Another method of computing the heat-dissipating capacity of a bearing is to apply the relation

$$Q = cld \tag{23-16}$$

where ld is the projected journal area and c is a coefficient found by using the chart[9] in Fig. 23-9. To determine c, the expected temperature rise $(t_b - t_a)$ is selected from Fig. 23-8 and is taken as the ordinate in Fig. 23-9; and the corresponding abscissa for the proper curve is the desired value of c.

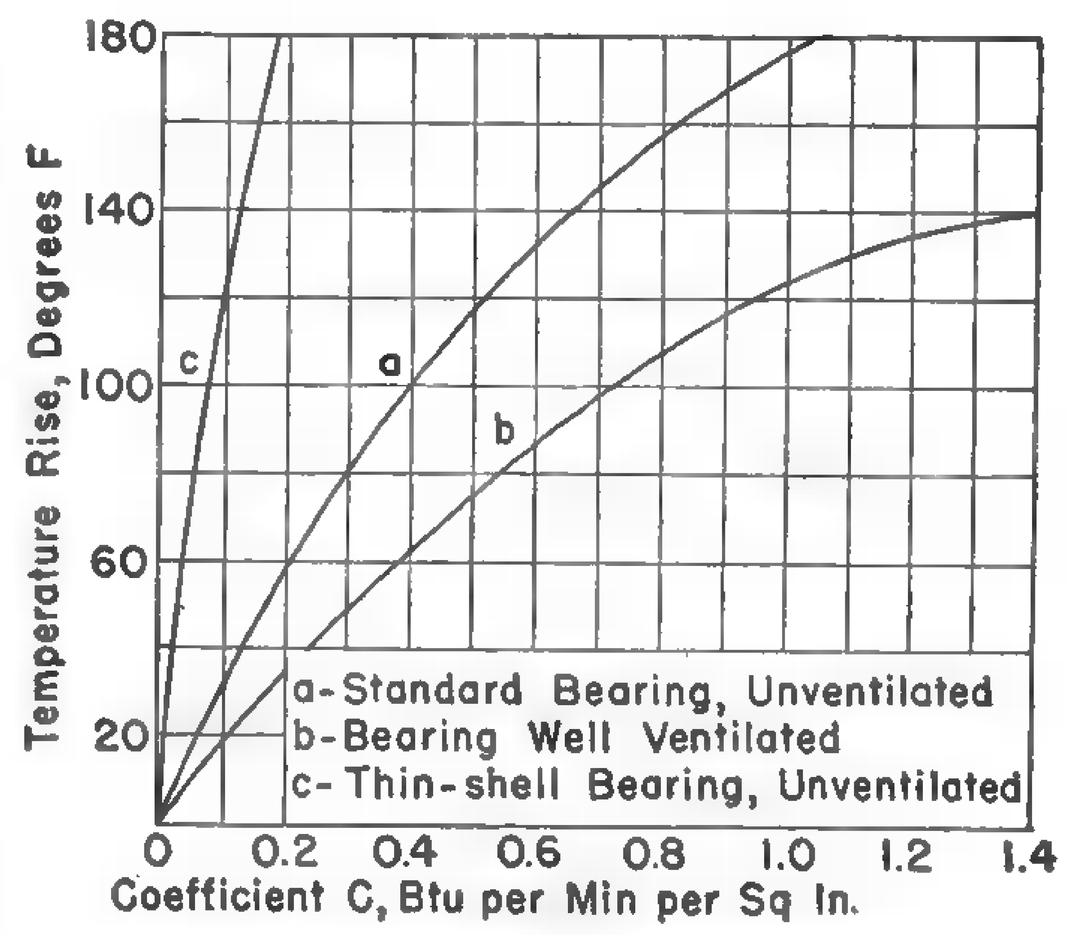


FIG. 23-9. Heat dissipation by bearings.

Instead of using Fig. 23-9 the coefficient c may be computed from the relation[10]

$$c = k(t_b - t_a + 33)^2 \tag{23-17}$$

where the coefficient k is 2.33×10^{-5} for bearings in still air, and 4.15×10^{-5} for well-ventilated bearings.

23-9. Oil-film thickness. The smallest oil-film thickness h_m, Fig. 23-1c, when the journal is turning, can be expressed in terms of the clearance c, Fig. 23-1a, and the *eccentricity* e, Fig. 23-1c, which is the distance between the center of the journal and the center of the bearing. From the diagram, $h_m = 0.5c - e$, or

$$h_m = 0.5c\left(1 - \frac{2e}{c}\right) \tag{23-18}$$

The magnitude of e is a function of the pressure p on the projected actual bearing area, the relative clearance c/d, the oil viscosity Z, the rotative speed n of the journal, and the angle α, Fig. 23-1c, between the edge of the bearing relief where the oil film starts and the direction of the load. The ratio e/c, which is termed the *eccentricity coefficient*, can be found by means of

[9] O. Lasche and W. Kieser, *Materials and Design in Turbo-Generation Plant* (London: Oliver and Body, 1927).

[10] Axel K. Pederson, "Charts for Journal Bearings," *American Machinist*, Vol. 37 (October 10, 1912), p. 599.

the chart in Fig. 23–10, in which the abscissas are values of a quantity designated as K. The latter, determined from theoretical considerations and corrected by experimental data, is computed by the relation[11]

$$K = 198\frac{\left(1{,}000\frac{c}{d}\right)^2}{\frac{Zn}{p}} \qquad (23\text{–}19)$$

Comparison of equations 23–19 and 23–9 shows that $K = 198 \times 10^6/k'$. When the quantity K has been computed by equation 23–19 for a given bearing with a certain angle α, Fig. 23–1c, and a certain oil viscosity Z, the ordinate of the corresponding curve in Fig. 23–10 gives the eccentricity coefficient e/c directly. Equation 23–18 can then be applied to find the expected smallest oil-film thickness h_m for this bearing.

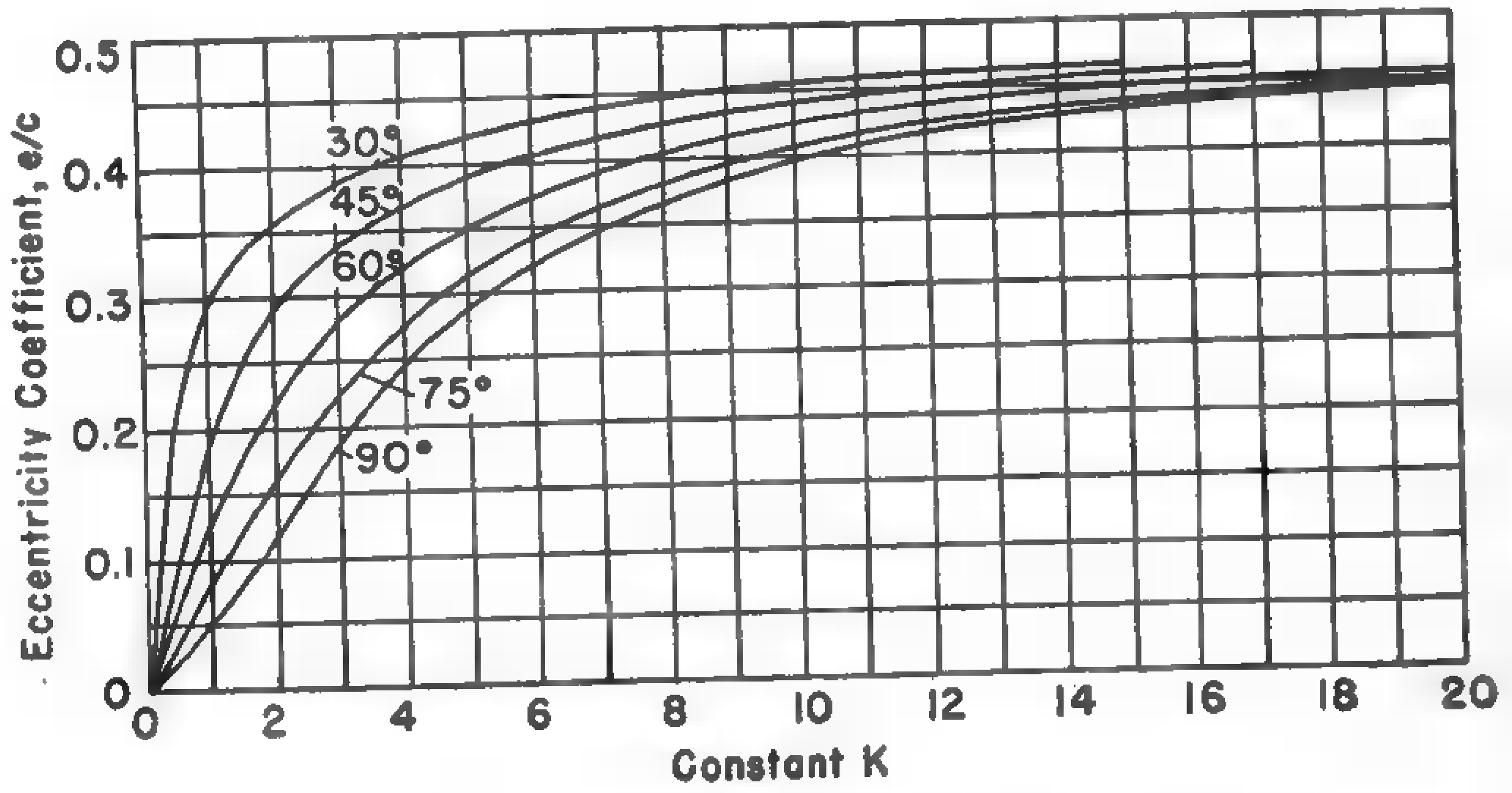


FIG. 23–10. Chart for eccentricity coefficient.

The minimum thickness h_m' that *must* be maintained for perfect lubrication depends on the surface finishes of the journal and the bearing. For a finely bored small bronze bushing and a polished journal, this thickness is of the order of 0.0001 in. In the case of a steel journal in an ordinary babbitted bearing, the thickness h_m' should be at least 0.00075 in. if n is high, and should not be less than one-half this value for lower speeds.[12] With large steel shafts of turbogenerators, fans, and the like, h_m' should be from 0.0003 to 0.0005 in. For journals of diesel engines running with peripheral speeds of 500 to 1,200 fpm, a thickness h_m' of 0.0004 to 0.0006 in. is recommended for 5-in. to 10-in. bearings.[13]

[11] Karelitz, *loc. cit.*, p. 59.

[12] *Ibid.*

[13] E. S. Dennison, "Film-Lubrication Theory and Engine-Bearing Design," *Trans. ASME*, Vol. 58 (1936), p. 25.

In general the safe thickness for a bearing in good condition and $v_m \geqq 200$ fpm is[14]

$$h_m' = 0.000026 v_m^{0.4} A^{0.2} \qquad (23\text{–}20)$$

where v_m is the peripheral, or rubbing, velocity of the journal, in feet per minute, and A is the projected area ld of the bearing, in square inches.

Oil supply. The approximate amount of oil G, in gallons per minute, that must be supplied to a bearing to make up for leakage and maintain thick-film lubrication may be determined by the formula[15]

$$G = 0.0034cldn \qquad (23\text{–}21)$$

where n is the journal speed, in revolutions per minute, and c, l, and d are bearing dimensions, in inches.

23–10. Methods of lubricating bearings. Bearings may be lubricated intermittently, continuously with a limited supply of lubricant, or continuously with an abundant amount of lubricant.

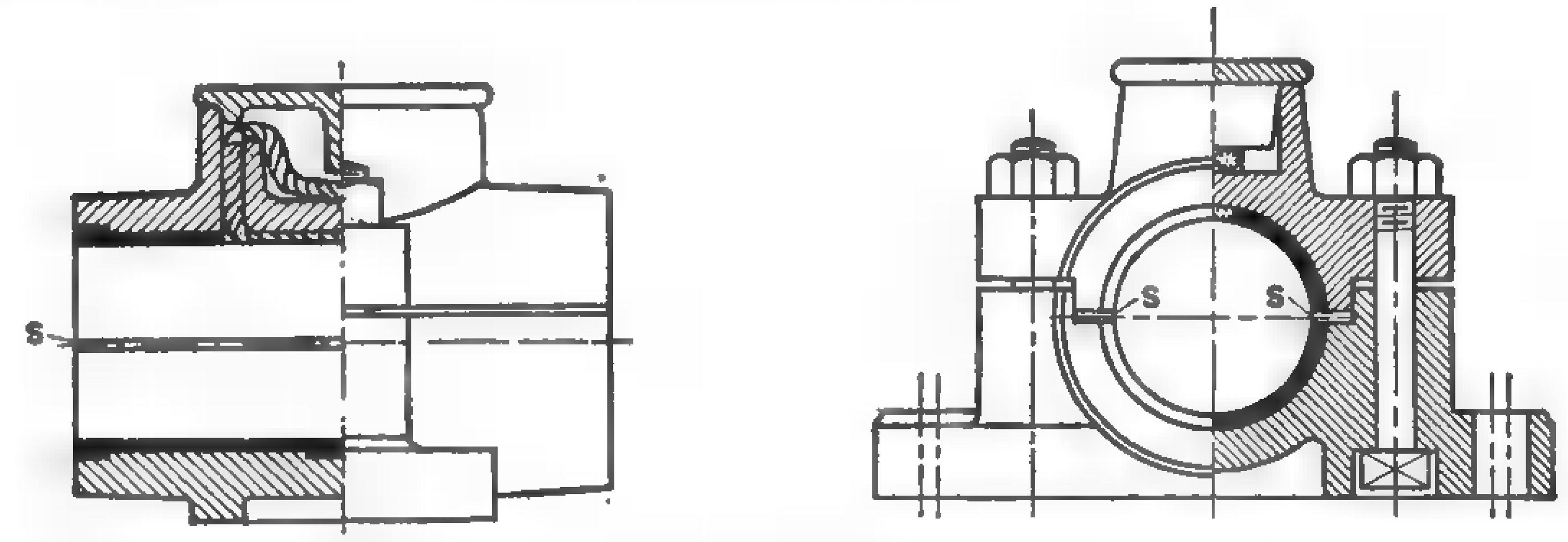


FIG. 23–11. Two-piece bearing with wick lubrication.

Intermittent lubrication. The lubricant for intermittent lubrication may be oil or grease. Oil may be applied by dropping it from an oil can into an oil hole in the cap of the bearing or by dropping it from an oil can into a felt plug or a wad of cotton waste covering the oil hole. Grease may be applied by forcing it from a compression grease cup screwed into the bearing cap or by forcing it into a hollow space in the bearing by means of a pressure gun.

Any of these methods, generally speaking, provides only thin-film lubrication. The coefficient of friction is variable and uncertain. To be on the safe side, it may be assumed to be 0.12 to 0.15.

In designing bearings that must work under conditions of thin-film lubrication the allowable mean pressure, $p = F/ld$, may be taken from Table 23–7.

Limited continuous lubrication. Continuous lubrication with a limited supply of lubricant may be obtained by use of a grease cup with spring action;

[14] Albert Kingsbury, "Optimum Conditions in Journal Bearings," *Trans. ASME*, Vol. 54 (1932), pp. 123 ff. These data were coordinated with other data available in the literature.

[15] S. J. Needs, "Effects of Side Leakage in 120° Centrally Supported Journal Bearings," *Trans. ASME*, Vol. 56 (1934), p. 721, and Vol. 57 (1935), p. 135.

TABLE 23-7

ALLOWABLE BEARING PRESSURES FOR SEMIFLUID LUBRICATION

Journal Material	Bearing Material	Allowable Pressure p (psi)
Hardened tool steel	Lumen of phosphor bronze	2,500
Hardened alloy steel	Hardened steel	2,100
SAE 1050 steel	Hard babbitt	1,500
Hardened alloy steel	Bronze	1,300
Cast iron	Cast iron	1,100
Alloy steel	Bronze	850
SAE 1040 Steel	Babbitt, soft	750
Mild steel, smooth finish	Bronze	550
Mild steel, ordinary finish	Bronze	400
Cast iron	Bronze	400
Mild steel	Cast iron	350
Mild steel	Lignum vitae, water-lubricated	350

by use of an oil reservoir with a wick which carries the oil by capillarity, as in Fig. 23-11; or by use of a sight-feed drop oil cup, as in Fig. 23-12.

All three methods are suited to light duty only. Only the third method may approach thick-film lubrication, and the computed value of the coefficient of friction should be used very cautiously.

Abundant lubrication. Only continuous lubrication with an abundant supply of lubricant insures thick-film lubrication. It may be obtained by means of ring, chain, or collar oiling; splash lubrication; bath lubrication; or flooded lubrication, under pressure or without it.

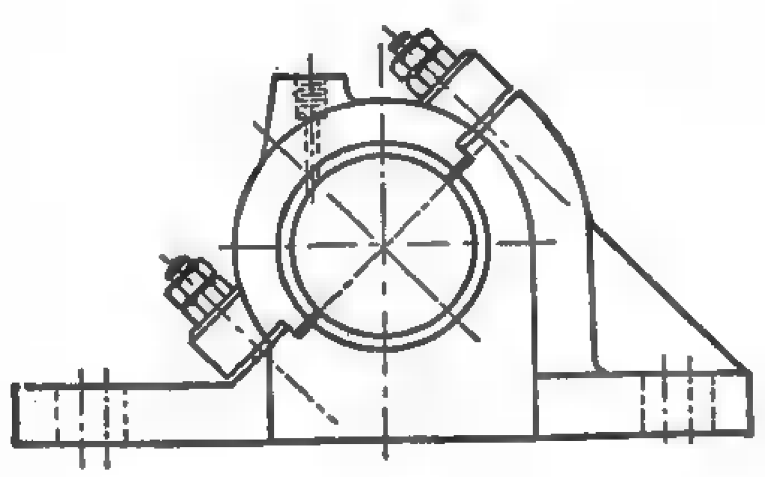


FIG. 23-12. Angle bearing.

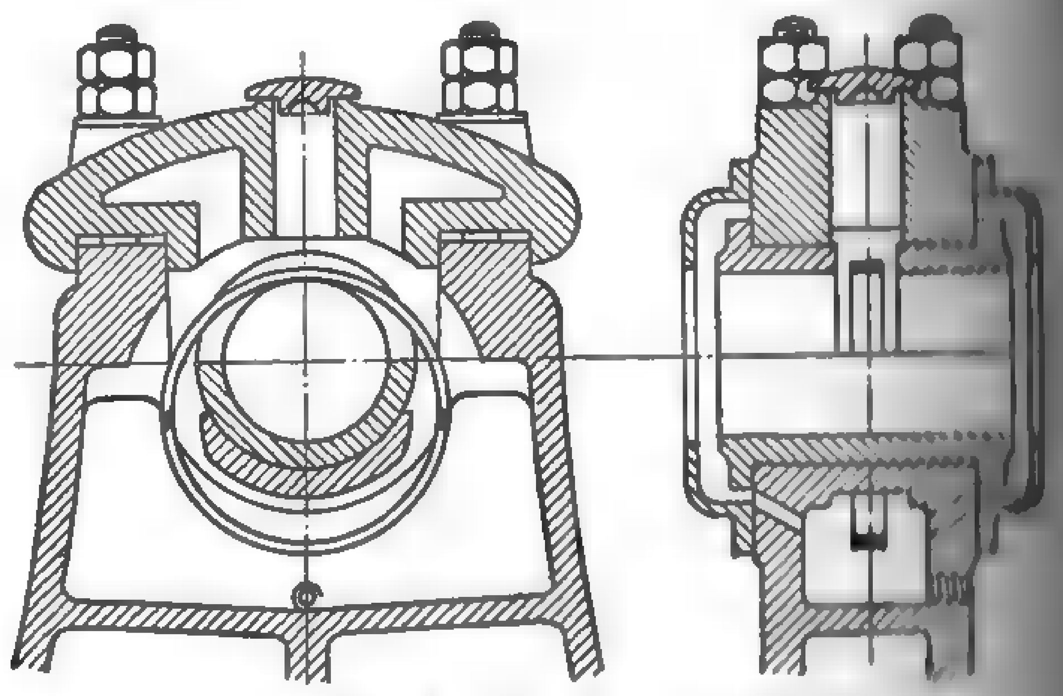


FIG. 23-13. Oil-ring bearing.

Ample quantities of oil at low and medium speeds up to a peripheral journal speed of 2,000 to 2,500 fpm are furnished by *ring oiling.* The amount of oil delivered to the journal is approximately proportional to the width of the ring, heavy rings delivering more oil than light ones. At high speeds the oil is thrown from the ring by centrifugal force on the upward journey, and special grooves must be used to collect the oil and return it to the journal. Typical bearings with ring oiling are shown in Figs. 23-13 and 23-14.

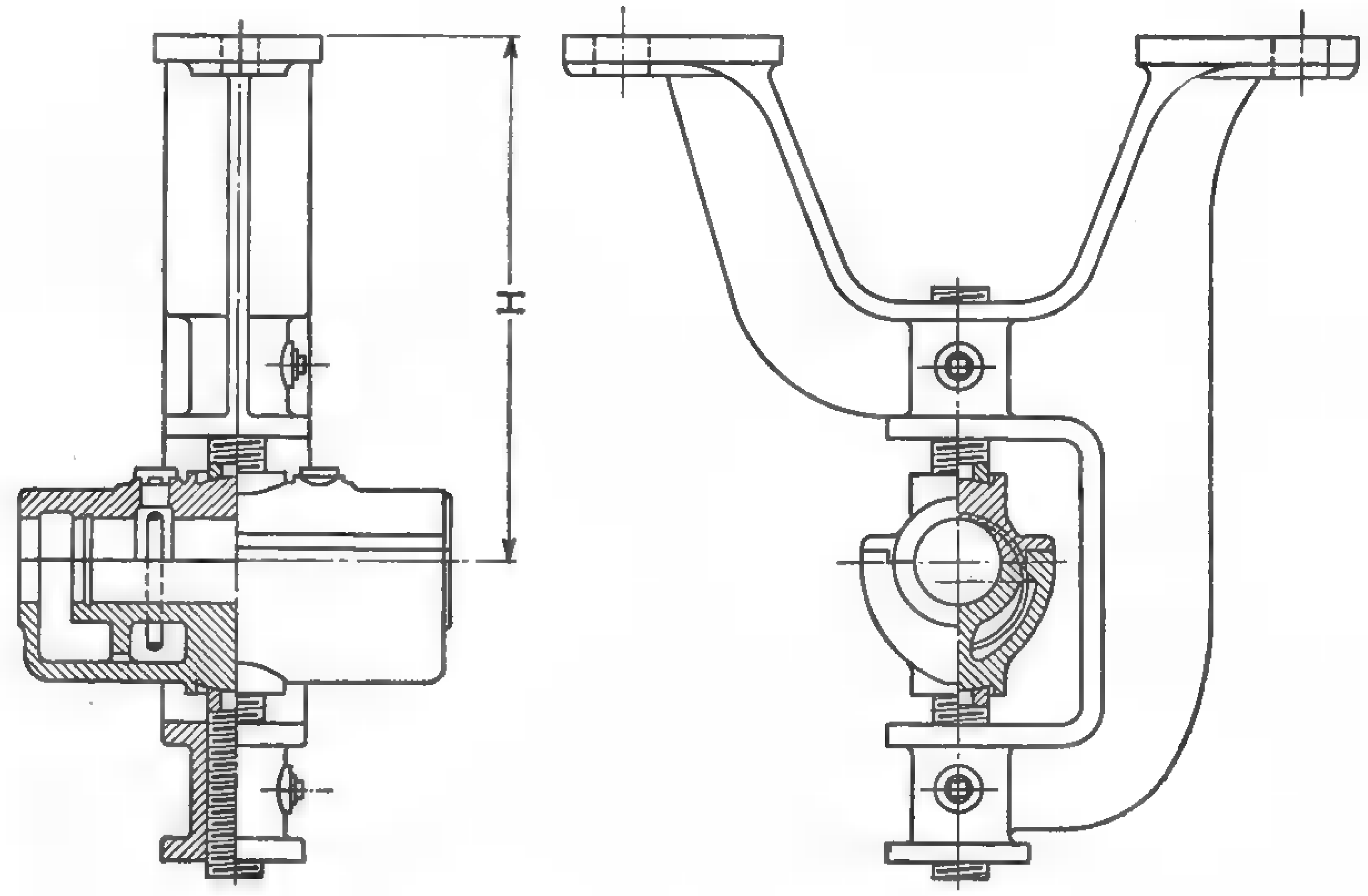


FIG. 23-14. Sellers oil-ring bearing in a hanger.

A variation of the idea of the ring oiler, with a chain substituted for the ring, is *chain oiling.*

With *collar oiling* (a bearing with such oiling is shown in Fig. 23-15), a split collar that is clamped to the shaft dips into the oil reservoir and carries oil to the top of the bearing, where the oil is wiped off and led to the journal. The disadvantage of the collar is that it divides the bearing into two parts, making the bearing equivalent to two bearings as far as end leakage and longitudinal pressure distribution are concerned.

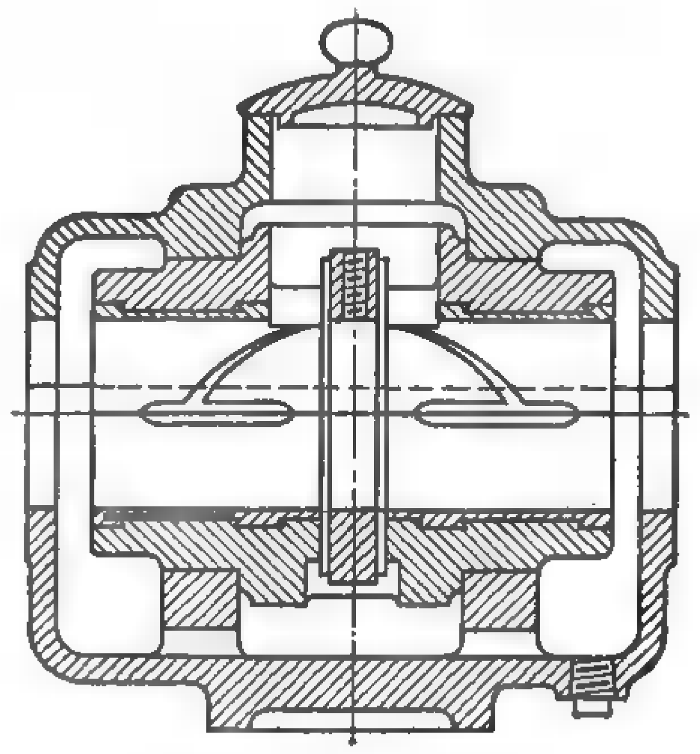


FIG. 23-15. Rigid collar-oiled bearing.

The method known as *splash lubrication* may be used in a fully enclosed mechanism. A crank or a similar part dips into an oil reservoir at each revolution.

In *bath lubrication* the journal is partly submerged in an oil reservoir. It is particularly suitable for a bearing that carries the load on the top half, because it admits oil at the point of minimum pressure.

In *flooded lubrication* the oil flows by gravity from an overhead tank or is pumped continuously to the bearing under a moderate head. In *pressure lubrication* the oil is pumped from an oil sump to the bearing, from which it flows back to the sump. Since the high pressures in the oil film are produced by the pumping action of the rotating shaft, the pressure under which the oil is supplied does not need to be high; a pressure of 5 to 30 psi is sufficient. Generally, the oil may be admitted at any point of the bearing. However,

if the shaft must be floated before it starts to rotate, the oil must be introduced at the bottom into the high-pressure region.

When oil is circulated under pressure, an oil cooler, with water or air as the cooling medium, usually is included in the system. The cooling medium takes away the heat that is generated in the bearing by friction and picked up by the oil. The oil temperature is thus lowered to its initial value, and the desired oil viscosity is maintained.

23–11. Design procedure. Because the design of a journal bearing involves many variables, the proper procedure is first to make certain assumptions and later to check the assumed values and, if necessary, to correct them. The following procedure is suggested, when the bearing load F, in pounds, the journal diameter d, in inches, and its speed n, in revolutions per minute, are known.

a) Select the mean bearing pressure p by using Table 23–4 and equation 23–13 as guides; determine the necessary length l by applying the equation $p = F/ld$; and check the obtained ratio l/d by Table 23–4.

b) Select a value for the relative clearance c/d in accordance with the recommendations in Table 23–4, and find the clearance c.

c) Select from Table 23–1 a suitable lubricating oil; assume its operating temperature t_o in the bearing as about 140 F to 150 F, or slightly higher; determine the viscosity Z of the oil, in centipoises, at this temperature by using Fig. 23–4 and equations 23–2 to 23–5; and compare this value of Z with the value given in Table 23–4.

d) Compute the characteristic number Zn/p and check it with the minimum data of Table 23–4.

e) Select the angle of relief γ which gives the angle of support $\alpha = 90 - \gamma$, Fig. 23–1c; compute the friction coefficient f by equation 23–10; and compute the work of friction W_f by equation 23–11.

f) Determine the heat-dissipating capacity Q of the bearing at the assumed oil temperature t_o by using equation 23–14 and 23–15, and check it by using Fig. 23–9 and equation 23–16.

g) The heat-dissipating capacity Q must be equal to the work of friction W_f converted to Btu by multiplying it by the reciprocal of the mechanical equivalent of heat $A = 1/778$. If it is not equal, the desired equilibrium can be obtained by assuming a new oil temperature t_o and finding the new values for Z, Zn/p, and f.

A method of finding the exact equilibrium temperature is shown in Fig. 23–16. For the first assumption of $t_o = 140$ F, $AW_f = 3.86$, and $Q = 3.68$; for the second assumption of $t_o = 145$ F, $AW_f = 3.62$ and $Q = 3.90$. The intersection of a line through points a and c and a line through b and d is point e. At this point the temperature t_o is 141.7 F, and $AW_f = Q = 3.75$ Btu per min.

h) Check the expected minimum oil-film thickness h_m, found by equation 23–18, against the safe thickness h_m', found by equation 23–20.

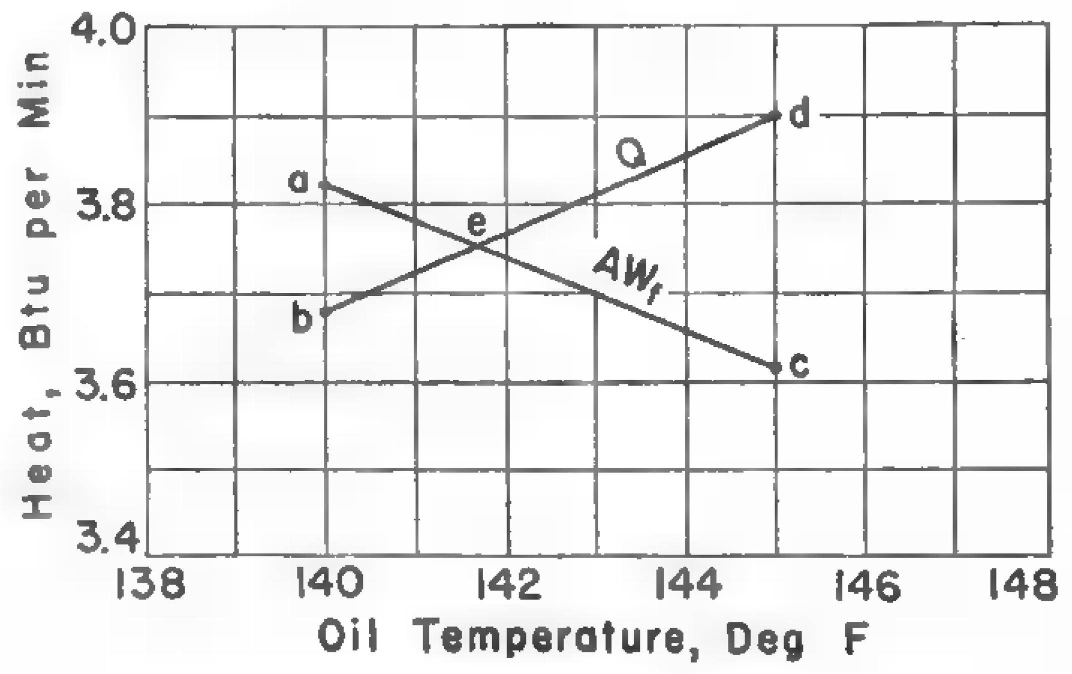


Fig. 23–16. Finding of equilibrium oil temperature.

If the equilibrium temperature t_o is found to be too high, or the film thickness h_m too small, the designer must either use another oil with a higher viscosity or increase the bearing dimensions l, c, α, and even d.

The outlined procedure is simple and gives satisfactory results. There exist more elaborate procedures,[16] but their advantage is doubtful, mainly in view of unavoidable inaccuracies in machining and of deflection in actual bearings and journals.

Example 23–1. Determine the length and clearance for an oil-ring bearing to support a $3\frac{11}{16}$-in. belt-driven jackshaft. The shaft transmits 120 hp at 230 rpm, and the belt pulley is close to the bearing and has a diameter of 52 in.

The net belt tension, found as shown in example 20-1, is

$$F = F_1 - F_2 = \frac{33{,}000 \times 120 \times 12}{\pi \times 52 \times 230} = 1{,}265 \text{ lb}$$

and the pull of the belt on the bearing is approximately

$$F_1 + F_2 = 3F = 3 \times 1{,}265 = 3{,}795 \text{ lb}$$

In accordance with the recommendations in Table 23–4, an average pressure of $p = 110$ psi is selected for heavy shafting. The required length, from equation 23–6, is

$$l = \frac{3{,}795}{110 \times 3.685} = 9.33, \text{ or } 9\tfrac{3}{8} \text{ in.}$$

The ratio $9.375/3.685 = 2.54$ is within the limits for l/d in Table 23–4. The final pressure, with $A = 3.685 \times 9.375 = 34.55$ sq in., becomes

$$p = \frac{3{,}795}{34.55} = 109.9 \text{ psi}$$

The diametral clearance, by Table 23–4, should be

$$c = 0.001 \times 3.685 = 0.004 \text{ in.}$$

Oil D, Table 23–1, should be used. For $t_o = 140$ F, equation 23–5 gives

$$\gamma_{140} = 0.9346 - 0.00036 \times (140 - 60) = 0.9058$$

From Fig. 23–4, the Saybolt viscosity at 140 F is SSU = 110 sec. By equations 23–2 and 23–4, the absolute viscosity is $Z = [0.22 \times 110 - (180/110)] \times 0.9058 = 20.4$ centerpoises.

Therefore the bearing-characteristic number is

$$\frac{Zn}{p} = \frac{20.4 \times 230}{109.9} = 42.8$$

[16] R. T. Kent, *Mechanical Engineers' Handbook*, 12th ed., Vol. 2, *Design and Production*, ed. by Colin Carmichael (New York: John Wiley & Sons, Inc., 1950), pp. 12–14 to 12–31.

This is higher than the minimum value 30 give in Table 23–4 and therefore seems to be satisfactory.

EXAMPLE 23–2. For the bearing of example 23–1, determine (a) the equilibrium temperature and (b) the oil-film thickness.

a) With a horizontal pull by the belt, an oil-ring bearing, Fig. 23–13, will not have any side relief, and angle $\alpha = 90°$. The coefficient of friction will be, by equation 23–10,

$$f = 473 \times 42.8 \times \frac{3.685}{0.004} \times 10^{-10} + 0.002 = 0.0039$$

The heat equivalent of the work of friction, found by equation 23–11, in which $pld = F_1 + F_2$, is

$$AW_f = \frac{0.0039 \times \pi \times 3{,}795 \times 3.685 \times 230}{12 \times 778} = 4.23 \text{ Btu per min}$$

The heat-dissipating capacity is found as follows: From a sketch of the bearing, the exposed area, extending 4 in. down from the oil reservoir, is about 760 sq in. Also, from Fig. 23–8, with $t_o - t_a = 140 - 70 = 70$ F, the rise in wall temperature for still air is $t_b - t_a =$ 35 F. Therefore, by equation 23–14, the heat-dissipating capacity, without ventilation, is

$$Q = 0.0002 \times 760 \times 35 = 4.41 \text{ Btu per min}$$

This is in good agreement with $AW_f = 4.23$ Btu per min. A check by using Fig. 23–9, and interpolating between curves a and b, gives

$$Q = 0.12 \times 34.55 = 4.15 \text{ Btu per min}$$

This is also a satisfactory agreement.

b) In order to find the minimum oil-film thickness, it is first necessary to compute the constant K by equation 23–19. Thus,

$$K = \frac{198 \times \left(1{,}000 \times \frac{0.004}{3.685}\right)^2}{42.8} = 5.45$$

From Fig. 23–10, for $\alpha = 90°$, $e/c = 0.30$. By equation 23–18,

$$h_m = 0.5 \times 0.004(1 - 2 \times 0.30) = 0.00080 \text{ in.}$$

The rubbing velocity is

$$v_m = \frac{\pi \times 3.685 \times 230}{12} = 221 \text{ fpm}$$

and the safe thickness, by equation 23–20, is

$$h_m' = 0.000026 \times 221^{0.4} \times 34.55^{0.2} = 0.00046 \text{ in.}$$

Hence the whole design is satisfactory.

23–12. Journal bearings. There exist so many different bearing constructions that only a few of the more typical ones can be discussed here. *Bushings* for small bearings are of either brass or bronze and are often made in one piece. The bushing is pressed into place and reamed to size. After excessive wear has taken place, the bushing is replaced.

A two-piece babbitted bearing with wick lubrication is shown in Fig. 23–11. The shims s serves to take care of the wear. They are made of strips of wood, or of brass or steel shim stock. The line of action of the load should be normal to the plane of separation. If this is not feasible, a satisfactory expedient is to place the bearing so that the line of action of the load

is not more than 30 deg from the bottom of the bearing. If the line of action of the load forms a large angle with the vertical, an angle bearing is used. In Fig. 23–12 is shown such a bearing, with a hole in the cap to take a grease cup or sight-feed oiler.

In Fig. 23–13 is shown a rigid bearing with ring oiling. It is used for heavy shafts and as an outboard bearing for an engine stub shaft.

A self-aligning bearing with ring oiling is shown in Fig. 23–14. It is used for transmission shaftings.

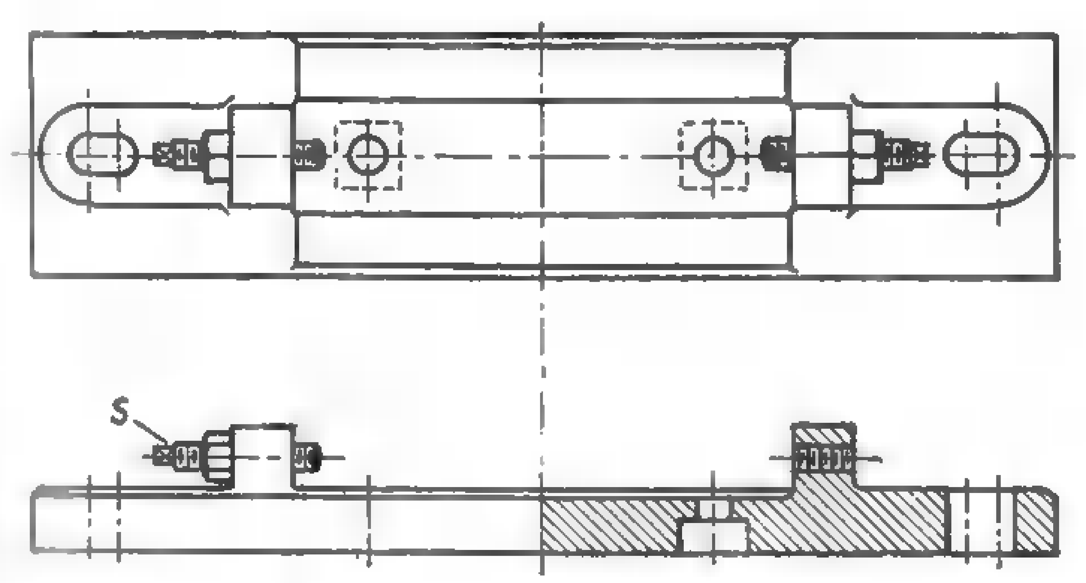


FIG. 23–17. Plain sole plate.

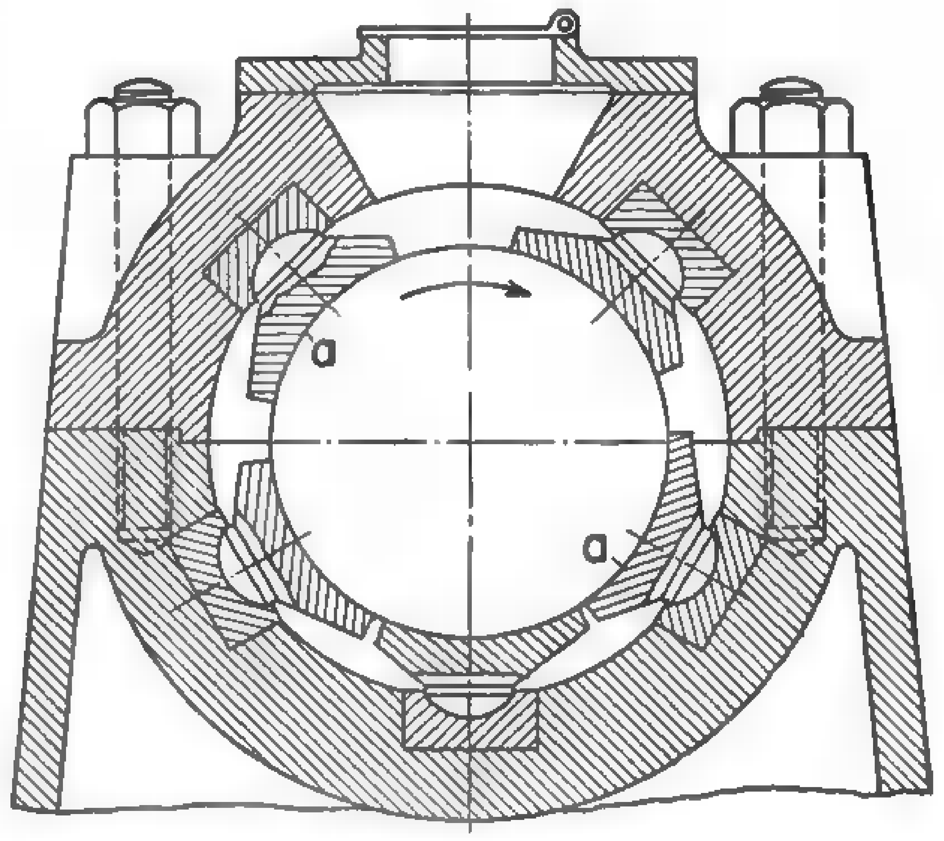


FIG. 23–18. Michell multipad bearing.

A rigid bearing with collar oiling is shown in Fig. 23–15. It has cast-iron babbitted shells.

Bearing supports. A sole plate which takes practically any type of bearing is shown in Fig. 23–17. The horizontal shaft alignment is made by means of the screws s. The vertical alignment is obtained by means of steel shims between the plate and the bearing.

A drop hanger for a two-point adjustable pivoted bearing is shown in Fig. 23–14. The hanger is made of cast iron and has different drops H for bearings of different sizes up to $4\frac{15}{16}$-in. bore.

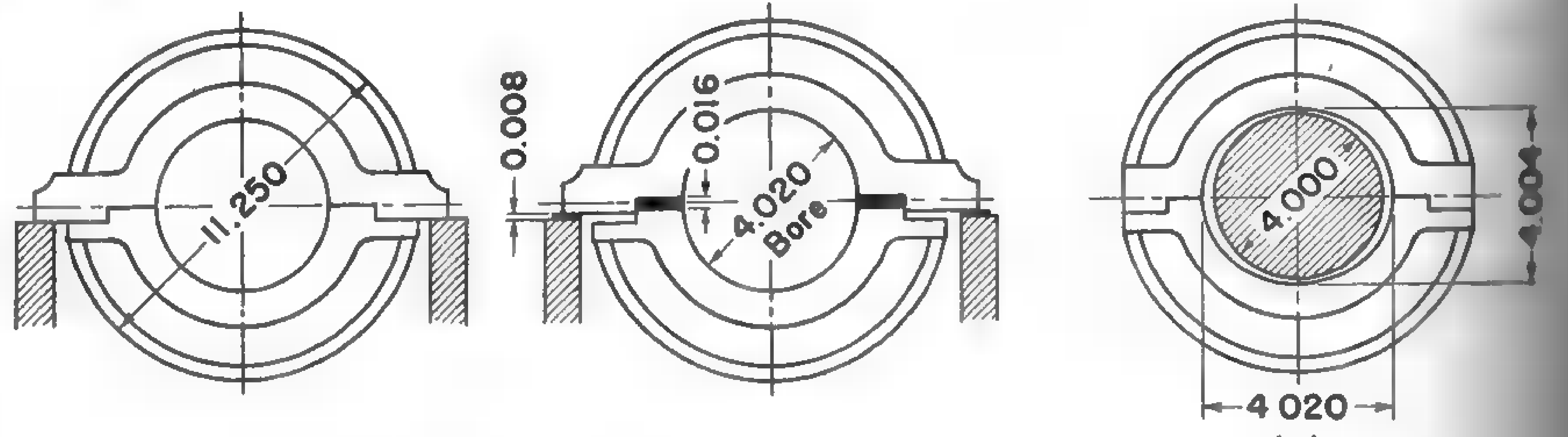
A *Michell multipad journal bearing* is made up of several pivoted segments a, Fig. 23–18, of the same curvature as the journal. Five are shown in the illustration. Because of the unsymmetrical shape of the segments, the oil pressure built up by the rotating shaft tips the segments slightly in their seats and thus forms the converging oil film characteristic of perfect lubrication. A bearing of the type shown in Fig. 23–18 was built for a 21-in. shaft of a certain turbogenerator, with the result that the over-all length of the machine was shortened from 32 ft to $23\frac{1}{2}$ ft.[17]

For smaller journals, similar results may be obtained much more cheaply by eccentric boring of the bearing.[18] Figure 23–19 illustrates the method for

[17] A. G. M. Michell, "Progress of Fluid-Film Lubrication," *Trans. ASME*, Vol. 51 (1929), MSP–51–21, p. 156. The same paper is condensed in *Mechanical Engineering*, Vol. 52 (1930), p. 115.

[18] E. A. Kraft, *The Modern Steam Turbine* (Berlin: VDI-Verlag, 1931), p. 63.

a 4.000-in. journal. The two halves of the bearing are bolted firmly together, as shown in Fig. 23–19a, and are machined to final dimensions, except for the inside surface. They are then separated, shims 0.016 in. thick are inserted, and the bearing shells are bored to a diameter of 4.020 in., as indicated in Fig. 23–19b. When the shims are taken out and the shells are put in place, Fig. 23–19c, the clearances become 0.004 in. vertically and 0.020 in. horizontally.



(a) (b) (c)

FIG. 23–19. Boring of a bearing shell for wedge lubrication.

23–13. Bearing materials. Table 4–4 gives information regarding the metals most generally used for bearings. Nonmetallic materials are now coming into use (see Table 4–8).

Babbitts. SAE 11 babbitt is used for bearings which are subjected to heavy pressures. SAE 10 babbitt is also suited for heavy pressures; it is very fluid and can be applied for thin linings of bronze-backed or steel-backed bearing shells like those used in automotive and aircraft engines.

Asarcoloy No. 7 is an alloy about 50 per cent stronger than tin-base babbitts, and it may be used for bearings subjected to shock action.

Lead-base babbitts may be used for larger bearings when the maximum pressures are below 500 psi. However, a lead-base alloy with the trade name Magnolia Metal, the analysis of which is about the same as that of SAE 14 babbitt, seems to give good service even in heavy-duty bearings if they are not subjected to pounding.

The advantage of babbitts is that if lubrication fails they simply melt out without scoring the journal.

Brass and bronze. Brass is used where the pressure is too high for babbitt but where the service is not severe enough to call for a more expensive bearing metal. Bronzes are used where the pressures are so high that thin-film lubrication may occur.

Hard bronzes, known under the trade names Nida-bronze[19] and Carobronze,[20] are melted in electrical induction furnaces. Their allowable work-

[19] Vereingte Deutsche Metall-Werke A.-G., *Nida Bronze* (Frankfurt am Main-Heddernheim: 1938).

[20] Carobronze G.m.b.H., *Carobronze* (Berlin: 1937).

ing pressures are so high—up to 8,000 psi—that bearings made of them are interchangeable with ball and roller bearings. With special precautions in regard to lubrication these bearings seem to work with a perfect oil film and do not show any wear.

Copper-lead alloys. Copper alloys having a high lead content—20 to 50 per cent—are of special interest. These alloys, put on the market under different trade names, have a low coefficient of thin-film-lubrication friction, about 0.005; and like babbitts, they do not score a journal when lubrication fails. The allowable pressure is 1,000 to 1,500 psi. In order to obtain a good copper-lead bearing alloy, it must be cast centrifugally. Such bearing shells and bushings have a uniform structure with a fine lead distribution and form a good bond with preheated steel shells.[21] With such bearings, particularly good results have been obtained in Germany. The allowable bearing pressure is 5,000 psi at a rubbing speed as high as 2,000 fpm.[22] The bearings are especially suitable where heavy pounding occurs, as in connecting-rod bushings of aircraft engines.

Aluminum alloys. Aluminum alloys are remarkable for their great resistance to scuffing, their low friction, and their high wear resistance under conditions of boundary and thin-film lubrication. The high coefficient of heat conductivity k helps to carry away the heat of friction. However, aluminum begins to lose its strength at about 225 F, and therefore the temperature of aluminum shells should not exceed 225 F, or 250 F if they have been cold-worked. Otherwise there is a danger of losing the necessary pressure between the contact surfaces of the two shells. Aluminum bearings can operate with pressures up to 3,000 psi, or even 4,000 psi, and with peripheral velocities up to 2,000 fpm.

Cast iron. For hardened steel journals, cast iron is a very good bearing material in regard to friction and wear, even if lubrication is in the thin-film region. However, this combination is suited only for light service where the pressures do not exceed 40 psi.

Oilless bearings. An oilless bearing depends on a lubricant incorporated in the bearing during its manufacture. This is done in several ways.

a) In some types, flaked graphite is inserted into the metallic (usually bronze) surface in the shape of spirals or studs.

b) Another development consists in suspending graphite in a babbitt alloy under a high pressure at the fusing temperature of the babbitt.

c) In other types, oil is impregnated into wood or some other porous or fibrous carrier.

[21] F. R. Hensel and L. M. Tichvinsky, "Copper-Lead Bronze for Bearings," *Trans. ASME*, Vol. 54 (1932), IS-54-3, p. 11.

[22] Manufactured according to patents of Glyco-Metall-Werke, Wiesbaden-Schierstein, Germany.

d) In powder-metal bearings, pulverized graphite and bearing bronze or iron are mixed with a binder and are pressed into molds with the application of heat. This process is called *sintering*. A sintered bearing is porous. It can absorb an amount of lubricating oil up to 30 per cent of its own weight, and can give the oil up very slowly in operation. Sintered bearings maintain a thin oil film for a long time and are correctly called *self-lubricating bearings*, not oilless bearings.

Oilless and self-lubricating bearings may be used in places where little or no attention can be given to the bearings and when the load and speed are low. Even under such conditions, however, the wear is relatively rapid. The clearance on bronze bearings with graphite-filled grooves and on sintered bearings should be 0.002 in. greater than for ordinary journal bearings.

Plastics. Celoron, formica, and micarta bearings are made from a special woven duck impregnated with phenolic resin. The materials are fused together under a very high pressure and at a high temperature. These materials have good mechanical properties (see Table 4–8) and great resilience. Such bearings can be lubricated with oil, grease, or water. They are used on heavy rolling mills as a replacement for bearings of bronze or lignum vitae without any changes in the roll stand itself. With water lubrication they can stand pressures up to 5,000 psi, and peripheral velocities of 2,000 fpm, with a friction coefficient f less than 0.007.[23] In general, the coefficient of friction is of the same order as that of tin-base babbitt.[24] Their drawback is a low heat conductivity, which necessitates the use of water or oil to cool them.

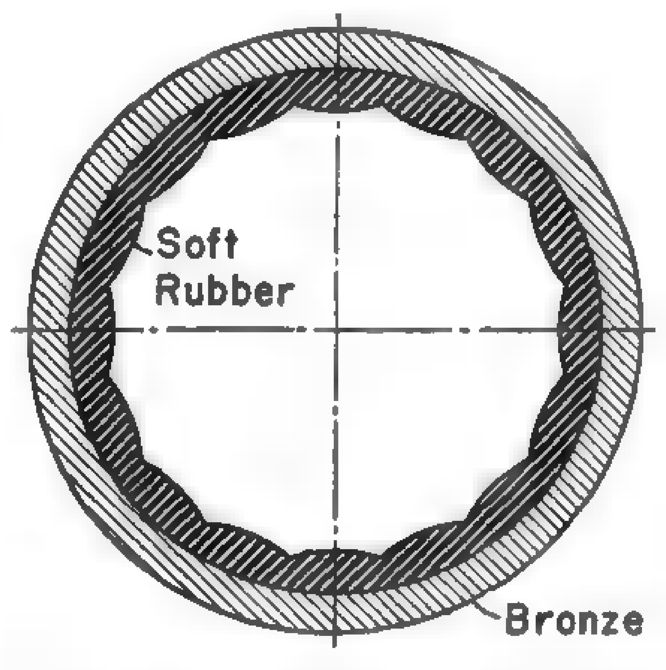


FIG. 23–20. Fluted rubber bearing.

Rubber. In hydraulic turbines, in stern bearings of ships, and in other machines where water is available, rubber bearings lubricated with water are used with increasing success instead of bearings with lignum vitae lining. The bearing is made of soft rubber and the inner surface either is fluted, as in Fig. 23–30, or has helical grooves. The softness of the rubber enables these bearings to stand up in the presence of sand and grit. They have a low coefficient of running friction, 0.02 to 0.04,[25] can carry loads up to 850 psi, and run at speeds up to 4,300 fpm with very little wear, if any. Rubber bearings are particularly suitable for use on shafts running at high speeds,

[23] O. K. Graef, "Phenolic Plastics—Design's Latest Bearing Materials," *Machine Design*, Vol. 8 (September, 1936), p. 34.

[24] L. M. Tichvinsky, "Properties and Performance of Plastic Bearing Materials," *Trans. ASME*, Vol. 62 (1940), p. 461.

[25] W. F. Busse and W. H. Denton, "Water Lubricated Soft Rubber Bearings," *Trans. ASME*, Vol. 54 (1932), IS–54–2, p. 3.

because the soft rubber allows the shaft to turn about the axis going through its center of gravity, even though this axis differs slightly from the geometrical axis. As a result the dynamic load on the bearing is reduced and there is less vibration in the machine. The main requirements for insuring small friction and low wear are the circulation of enough water to keep its temperature below 210 F, and a very smooth surface of the shaft. Rubber bearings are obtainable in stock sizes for shafts from 1 to 22 in. in diameter.

23–14. Mechanical design. In addition to determining the diameter, length, and clearance of a bearing, proper design involves:

a. Selection of a suitable bearing material
b. Provision for sufficient lubrication
c. Prevention of excessive oil leakage
d. Dissipation of heat generated by friction
e. Sufficient strength and rigidity of shells, caps, and bolts
f. Small wear and provision for taking it up in large bearings
g. Preservation of proper shaft alignment

Bearing materials. The right-hand column in Table 4–4 and information given in section 23–13 may serve as a basis for selection of the bearing material for any specific case.

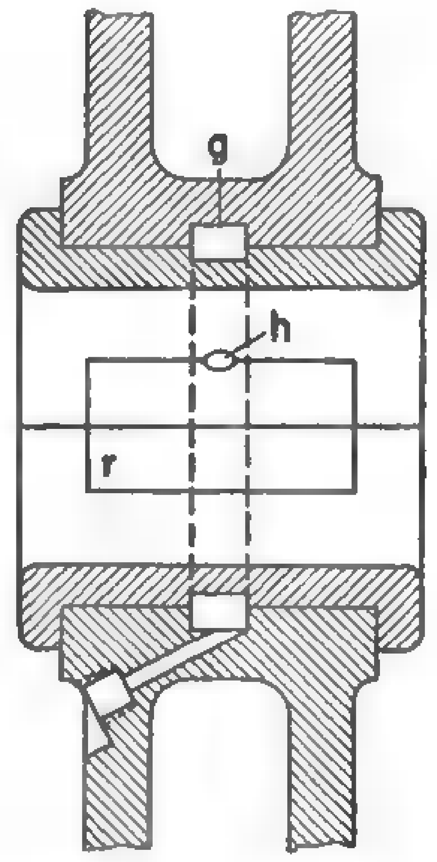


FIG. 23–21. Oil admission to main bearing of a crankshaft.

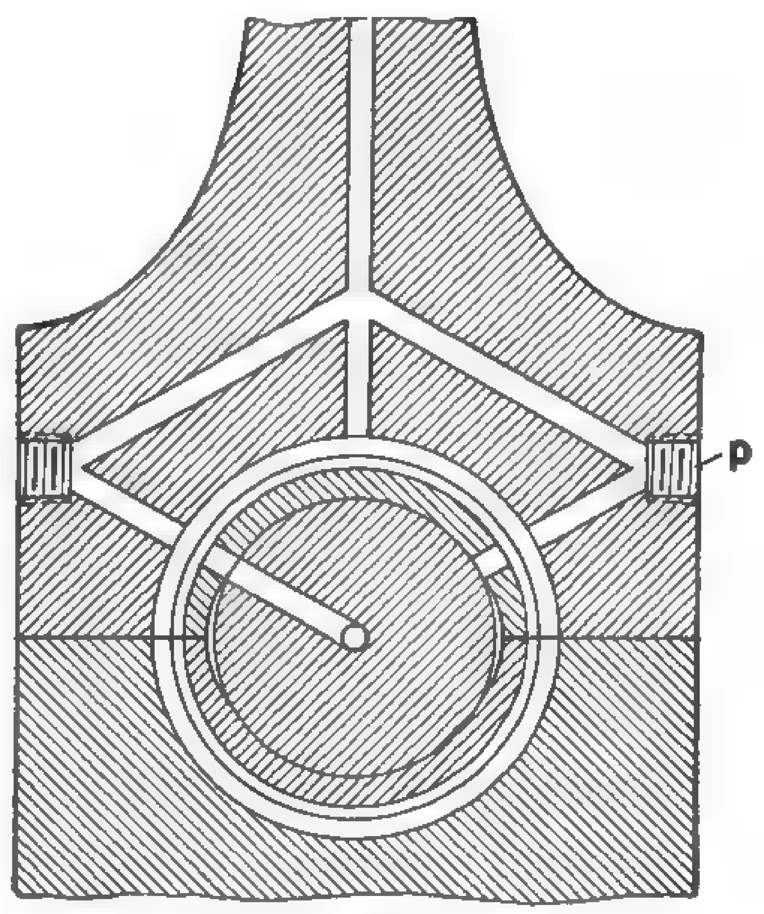


FIG. 23–22. Oil delivery to a piston pin.

Lubrication. Oil admission must be in the region of low pressure. As shown in Fig. 23–21, a circumferential groove g on the outside of the bearing shell with holes h drilled into the reliefs r will deliver oil to the crankshaft-journal surface where there is no pressure in the oil film. The detail in Fig. 23–22 illustrates how oil delivery from the crankpin to the piston pin through a rifle bore is increased by an outside groove with two cross-drilled oil holes in the big end of the connecting rod.

Oil grooves are useful in bearings that operate all or part of the time with thin-film lubrication, as in bearings with oscillatory motion in which the relative speed between the journal and bearing is low. In such cases properly cut oil grooves in the load-carrying surface help to distribute oil to the entire bearing surface. With thick-film lubrication, oil grooves placed in the high-pressure region are only harmful, since they interrupt the continuity of the oil film and may thus cause thin-film lubrication. The

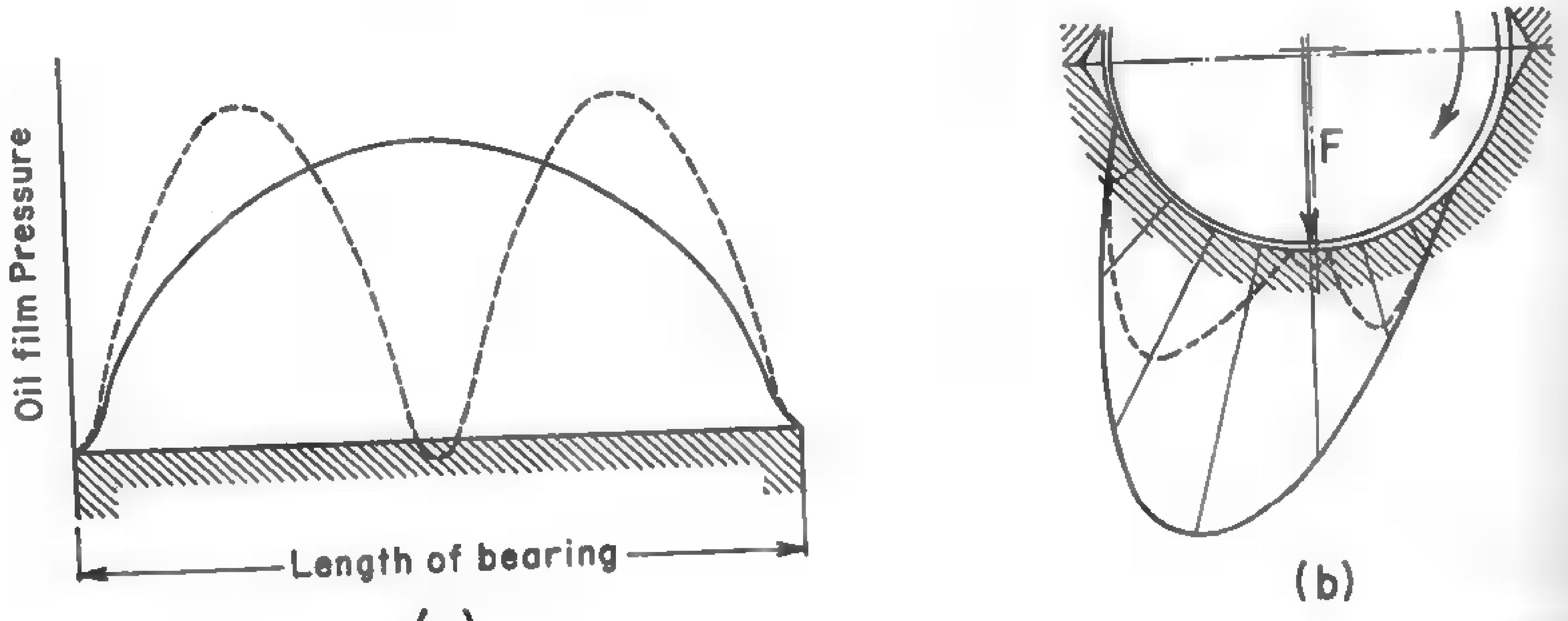


FIG. 23–23. Oil-pressure distribution without and with oil grooves.

full-line curve in Fig. 23–23a shows the lengthwise pressure distribution in a normal journal bearing; the dotted-line curves show the pressures when an oil groove is cut around the middle of the bearing. In order to support a given load, the pressure goes up. Theoretically this should decrease the oil-film thickness. However, tests indicate that actually the load capacity increases somewhat, probably because of an increase in oil flow which lowers the oil temperature and raises the oil viscosity.

A lengthwise oil groove at the point of maximum pressure is more harmful, as indicated in Fig. 23–23b, and may cause metal-to-metal contact. However, a longitudinal groove cut in the low-pressure region helps to dis-

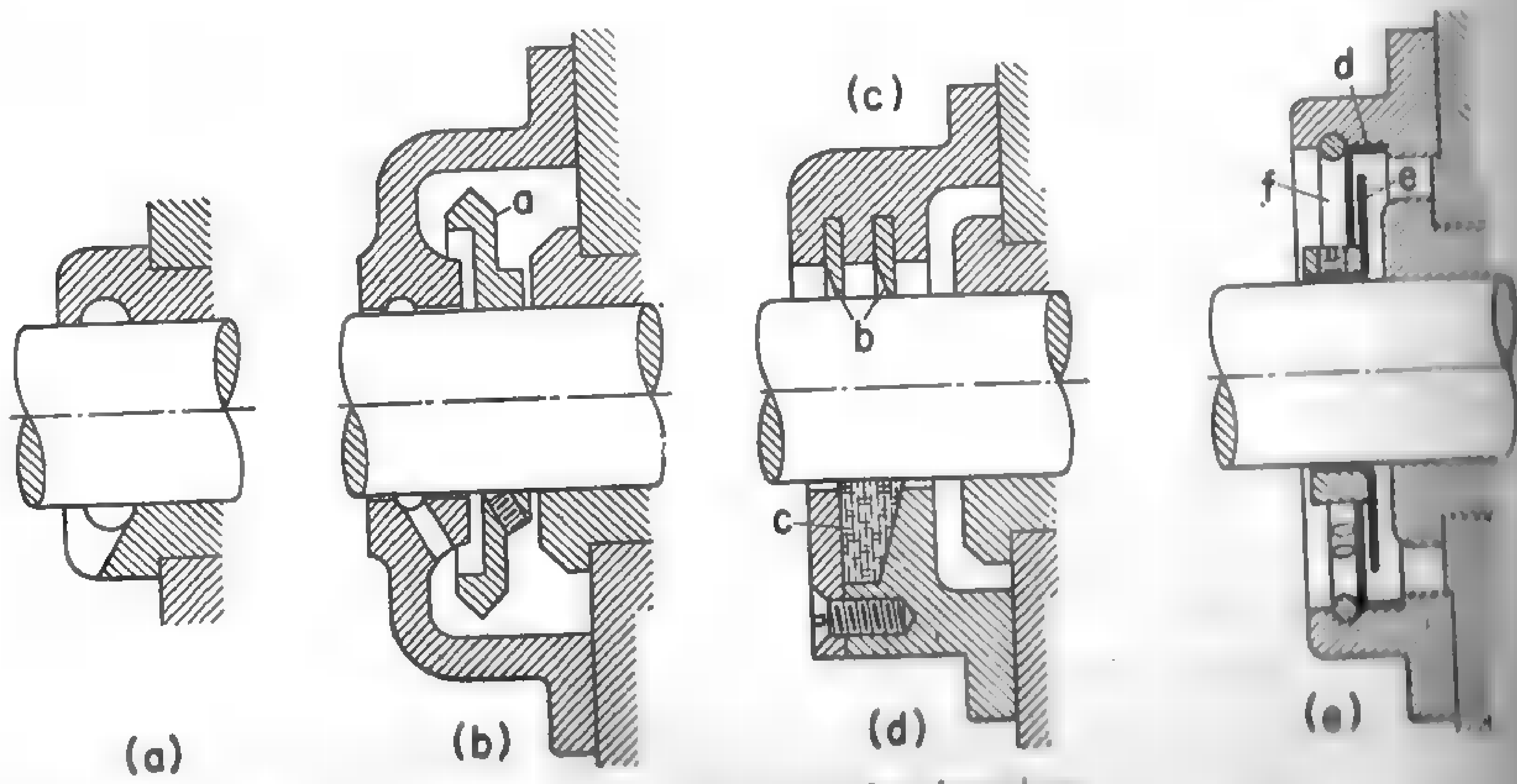


FIG. 23–24. Oil seals for bearings.

tribute the oil over the whole bearing. Side reliefs *a*, Fig. 23–25, are useful, as they help to wedge in the oil film and act as oil reservoirs.

All oil grooves should have the edges well rounded off, in order to facilitate the admission of oil into the pressure region.

Oil seals. Provisions for preventing oil leakage to the outside, and penetration of foreign matter, such as dust or fumes, to the inside of a bearing, are shown in Fig. 23–24. In Fig. 23–24a oil leakage about the shaft is stopped by a circular groove with sharp edges acting as a scraper; in Fig. 23–24b the main part of the oil is thrown off by centrifugal force by means of the throw ring *a*; in Fig. 23–24c oil is stopped by the sharp-edged brass rings *b* of the end cap; in Fig. 23–24d there is a split cap with a felt or leather ring *c*, which seals the bearing in both directions; in Fig. 23–24e the bearing is sealed by a stationary flanged tin ring *d* held in place by a retainer spring *f* and slightly pressed against the tin ring *e* acting as a throw ring.

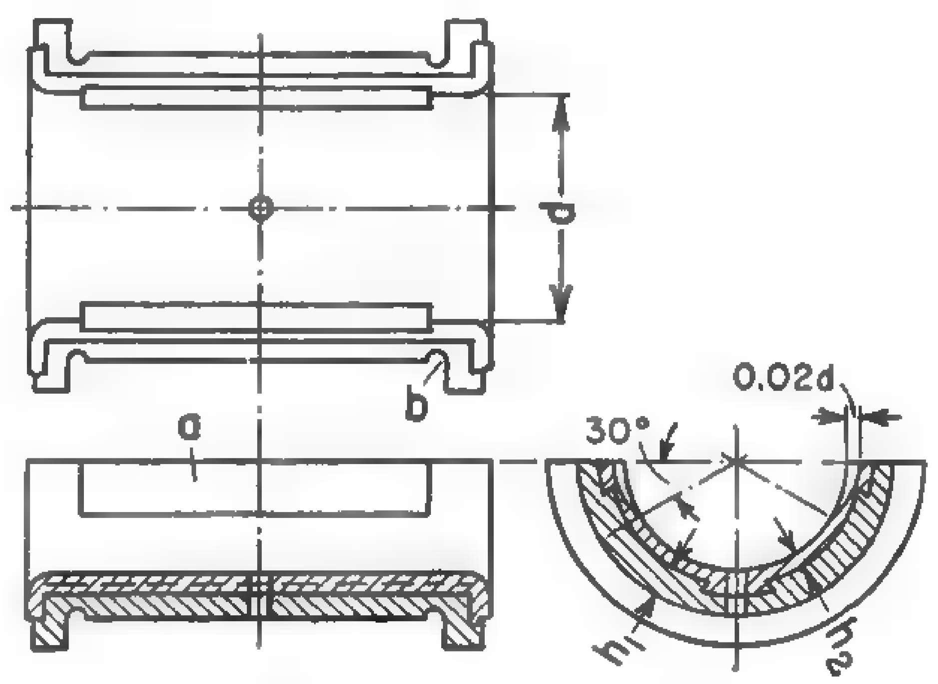


FIG. 23–25. Main-bearing shell.

Heat dissipation. The rate of dissipation of heat is materially increased by grinding and scraping the shells to a snug fit in the bearing body. Caps with ribs have a greater area and better heat dissipation than box-shaped caps. Water-cooling of the bearings is used only in special cases, as in roll-neck bearings of rolling mills or in main bearings of two-stroke-cycle oil engines. Bearings with pressure-feed lubrication often have an oil cooler inserted in the oil-circulation system.

Strength and rigidity of bearing shells. The thickness h of a small brass or bronze shell can be taken as $0.08d+0.1$ in. Shells of medium-size bearings, about 3 in. and up, are made as shown in Fig. 23–25. They have a cast-iron layer with a thickness h_1 equal to $0.20d$, and a babbitt layer having a thickness h_2 equal to $0.02d+0.125$ in. Engine main-bearing shells for 9-in. journals and up are made of a cast-steel layer for which $h_1=0.15d$, and a babbitt layer for which $h_2=0.01d+0.125$ in.

Lengthwise, swallow-tailed, babbitt anchor grooves are shown in Fig. 23–25, while Fig. 23–15 shows button-type anchors on the cylindrical part and round swallow-tail anchors at the ends.

An aluminum shell, when made solid and subjected to dynamic loads, should have a thickness h equal to $0.044d+0.020$ in. When the shell must be prevented from turning or from moving longitudinally by means of a dowel pin, the minimum thickness h is $\frac{5}{32}$ in. A bearing with a concentric bore should have a relief a few thousandths of an inch deep and extending about $\frac{1}{4}$ in. from the parting line and blending into the bearing surface to

compensate for a slight distortion which may be caused by the crush. The recommended relative clearance is $c/d = 0.002$ for small bores, up to 1.5 in.; $c/d = 0.00175$ for $d = 2$ in.; $c/d = 0.0015$ for $d = 5$ in.; and $c/d = 0.00125$ for $d = 8$ in.

In order to maintain the desired clearance c and uniform pressure distribution, it is essential that the shell be rigid and be supported rigidly.

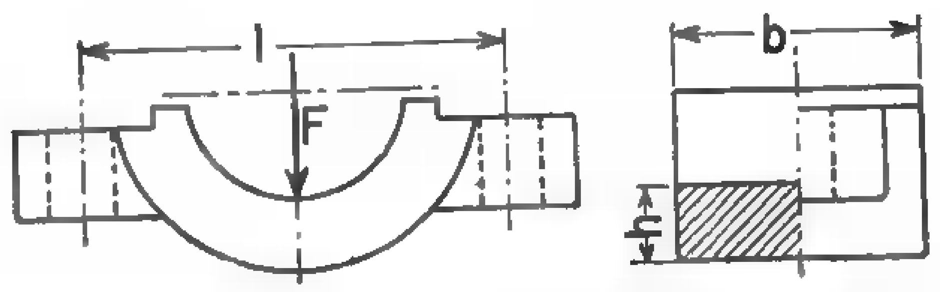


FIG. 23-26. Bearing cap.

Strength and stiffness of cap. The cap of a bearing is not usually subjected to a heavy load. However, sometimes the load acts upon the cap, as in the case of automobile main bearings. In such a case the cap may be regarded as a simple beam loaded at the center and supported by the holding-down bolts, Fig. 23-26.

The deflection of the cap may be computed by applying the expression for case c in Table 2-4 and substituting $\frac{1}{12}bh^3$ for I. Usual practice limits the permissible deflection y to 0.001 in.; in small sizes it should be only 0.0005 in.

Holding-down bolts. The bolts, studs, or screws which hold down the cap may be assumed to be subjected to simple tension. Each bolt must be designed for a load $1.33F/i$, where i denotes the number of bolts. The coefficient 1.33 takes into account the uneven load distribution on the bolts due to friction of the journal.

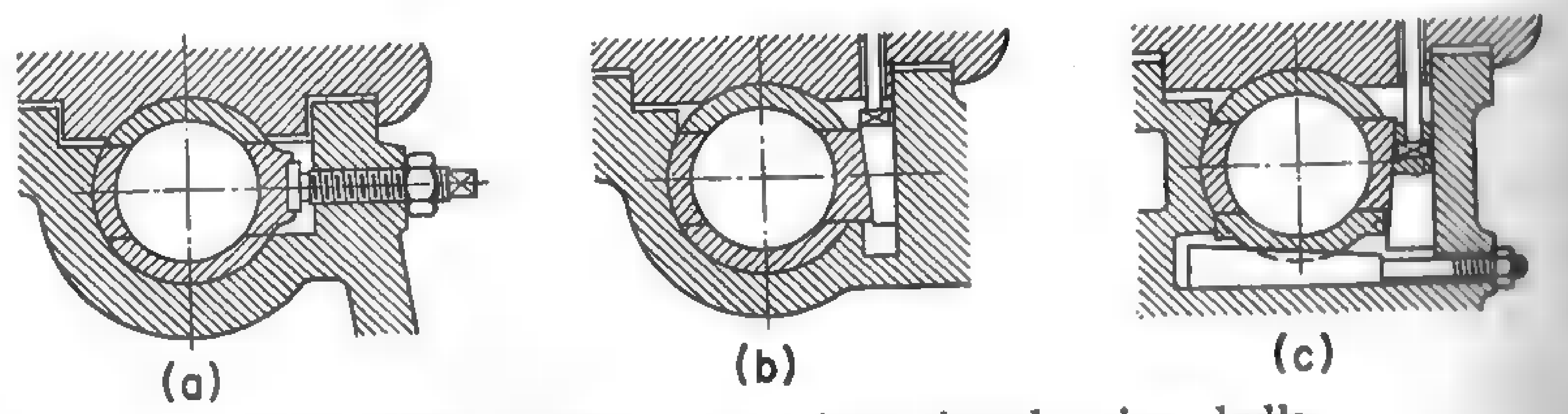


FIG. 23-27. Adjustment of four-piece bearing shells.

Take-up of wear. With thick-film, or perfect, lubrication, theoretically there should not be any wear at all. Practically, most bearings eventually run with imperfect lubrication and metal-to-metal contact, which results in wear of the rubbing surfaces. In order to reduce wear to a minimum, it is desirable to keep the bearing pressure low. It should never be higher, and if possible should be lower, than the value given in Table 23-4.

In a two-piece bearing the simplest means of taking up the unavoidable wear is by removing some of the shims inserted between the bearing halves. Shims are used also in an angle bearing, Fig. 23-12. If the force acts horizontally, a four-piece shell is used in which the wear may be taken up

either by means of a jackscrew, as in Fig. 23-27a, or by means of wedges, as in Figs. 23-27b and 23-27c.

Shaft alignment. Transmission bearings and pillow blocks are usually fastened to sole plates, the adjustment being by means of jackscrews in the horizontal direction, as in Fig. 23-17, and by shims between the plate and the bearing in the vertical direction. In a hanger bearing, both adjustments may be obtained by means of screws. In a large engine main bearing the horizontal adjustment may be obtained by means of two wedges. A vertical adjustment is shown in Fig. 23-27c.

23-15. Step, vertical, and collar bearings. A horizontal shaft is often subjected to an axial force, and a vertical shaft always. This force is taken by thrust bearings of various types.

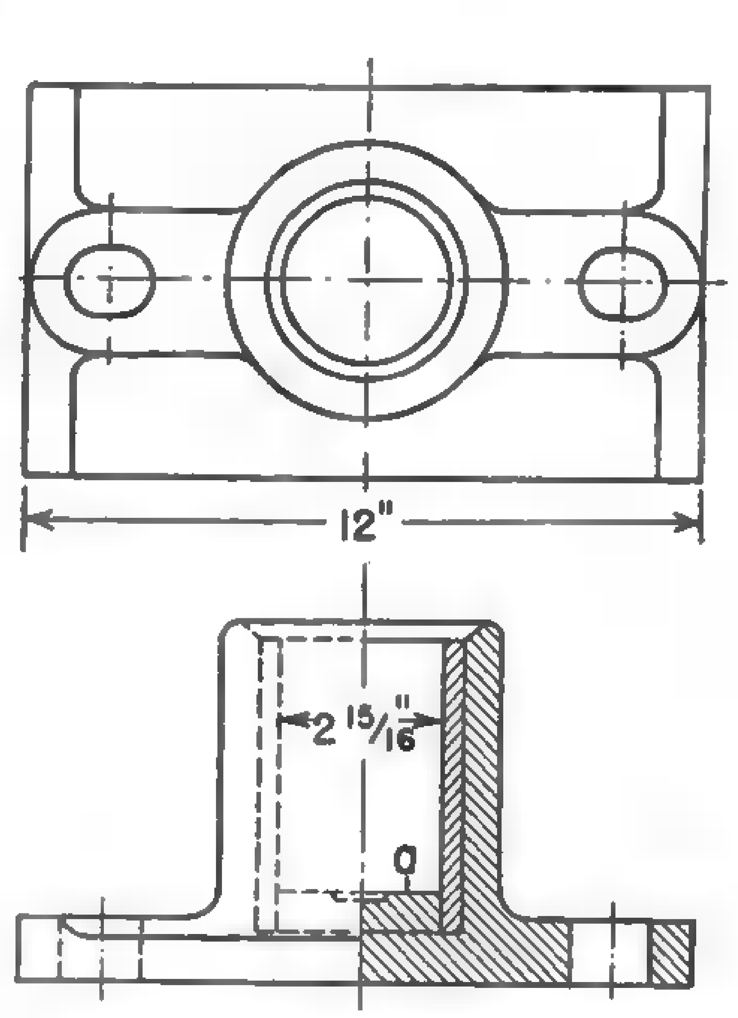


FIG. 23-28. Plain step bearing.

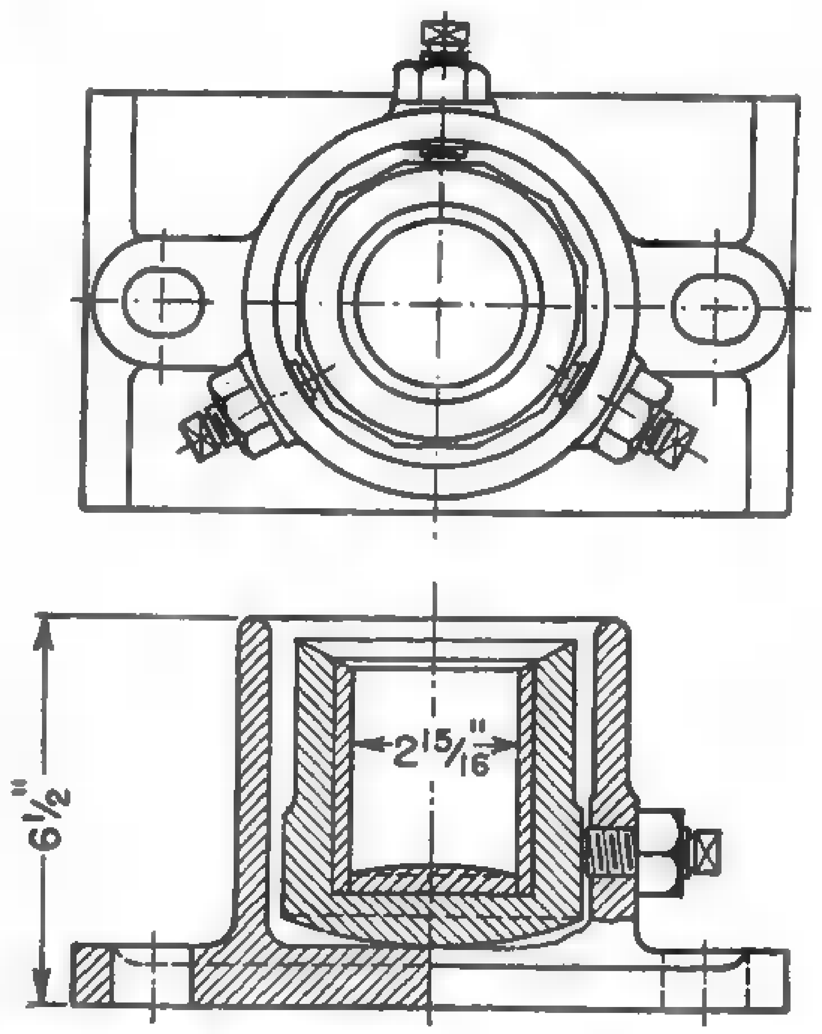


FIG. 23-29. Adjustable step bearing.

Step bearings. The simplest thrust bearing is the plain step bearing, Fig. 23-28, which is used for vertical shafts. The greatest wear in such a bearing occurs at the outer radius because the maximum velocity occurs there. The center portion is thus left higher. This wear eventually produces an excessive pressure at the center and causes overheating and failure of lubrication. In order to eliminate this trouble, the thrust disk a can be made with a hole in the center, or the shaft can be counterbored with a shallow hole at the end. Another method of reducing the danger of lubrication failure is to use a number of bronze and steel disks arranged alternately, in which case each disk rotates at a fraction of the shaft speed and reduces and distributes the wear. Should a pair of adjacent disks stick because of faulty lubrication, the bearing will not be damaged since the remaining disks are free to turn.

An adjustable step bearing which is used when side adjustment is required is shown in Fig. 23–29. A bronze bushing receives the lateral wear from the shaft, while the thrust is carried on a removable hardened-steel disk. The bearing can also adjust itself if the axis of the shaft is not quite normal to the plane of the bearing base.

Vertical bearings. For supporting the upper end of a vertical shaft any rigid bearing similar to a standard bearing may be used, but a lubrication system must supply the oil to the top part. This may be done by means of a wick-feed or sight-feed oiler. The loss of power is due only to the oil friction and may be computed in horsepower by the expression[26]

$$P=\frac{3.8\times10^{-8}Zd^3ln^2}{c} \tag{23–22}$$

where the notations are the same as previously used.

If the journal and the bearing are eccentric and the distance between their axes is e, the power loss will be

$$P=\frac{3.8\times10^{-8}Zd^3ln^2}{c\sqrt{1-\left(2\frac{e}{c}\right)^2}} \tag{23–23}$$

Collar bearings. Collar bearings are used chiefly on a horizontal shaft which must carry a large axial load, such as is created by a ship's propeller. In Fig. 23–30 is shown a thrust bearing with wick lubrication for a small propeller shaft. The lower half of the bearing is water-cooled.

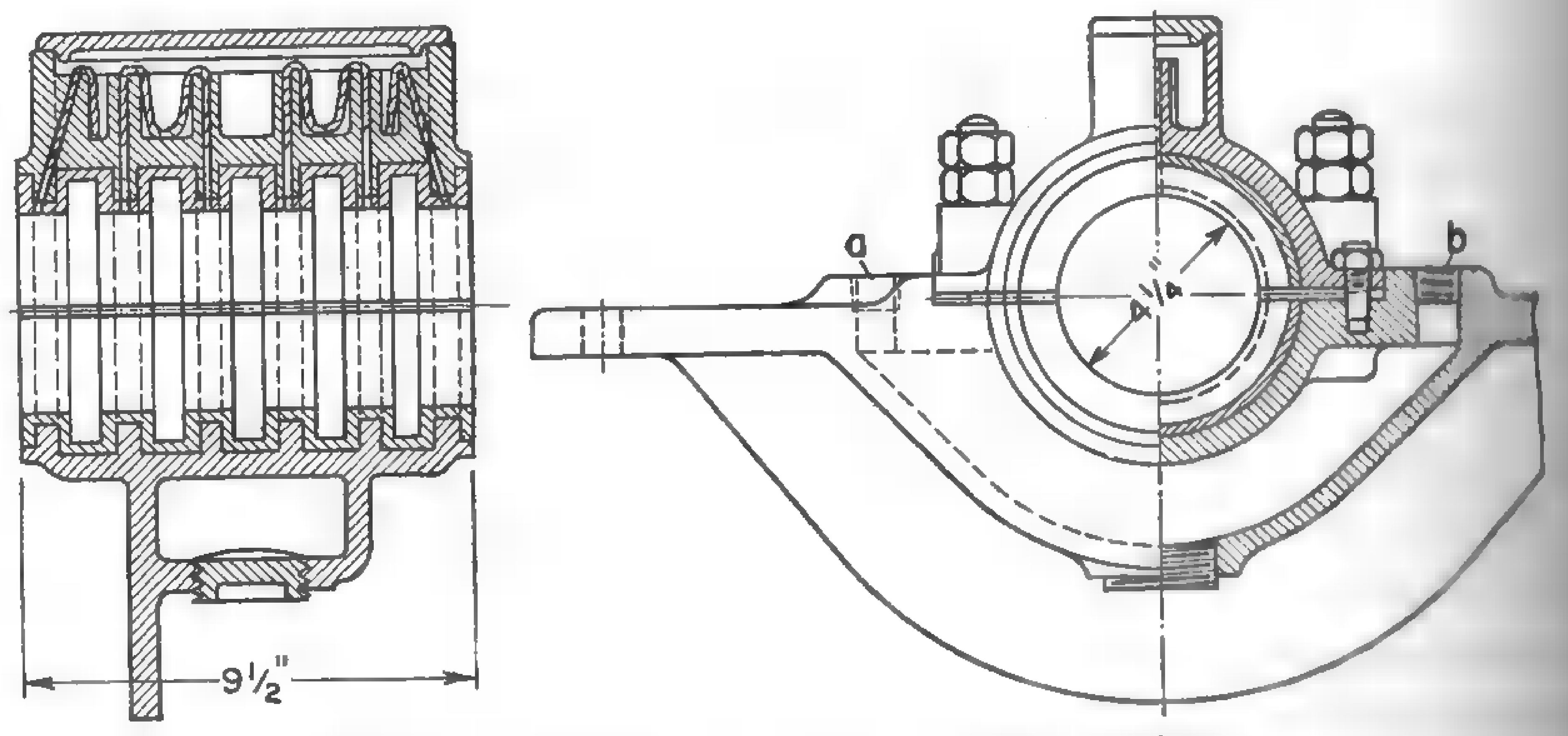


FIG. 23–30. Marine multicollar thrust bearing.

If feasible, the collars should be placed near the point of application of the axial load in order to avoid a column action upon the shaft. The allowable bearing pressure for the collars should be slightly smaller than for a step bearing, since the load is not likely to be evenly distributed among all

[26] A. I. Ponomareff and E. D. Howe, "Some Problems in Lubrication of Vertical Bearings," *Trans. ASME*, Vol. 55 (1933), PME–55–4, pp. 27 ff.

collars. The diameter of the collars is made equal to $1.3d$ to $1.7d$, where d is the shaft diameter.

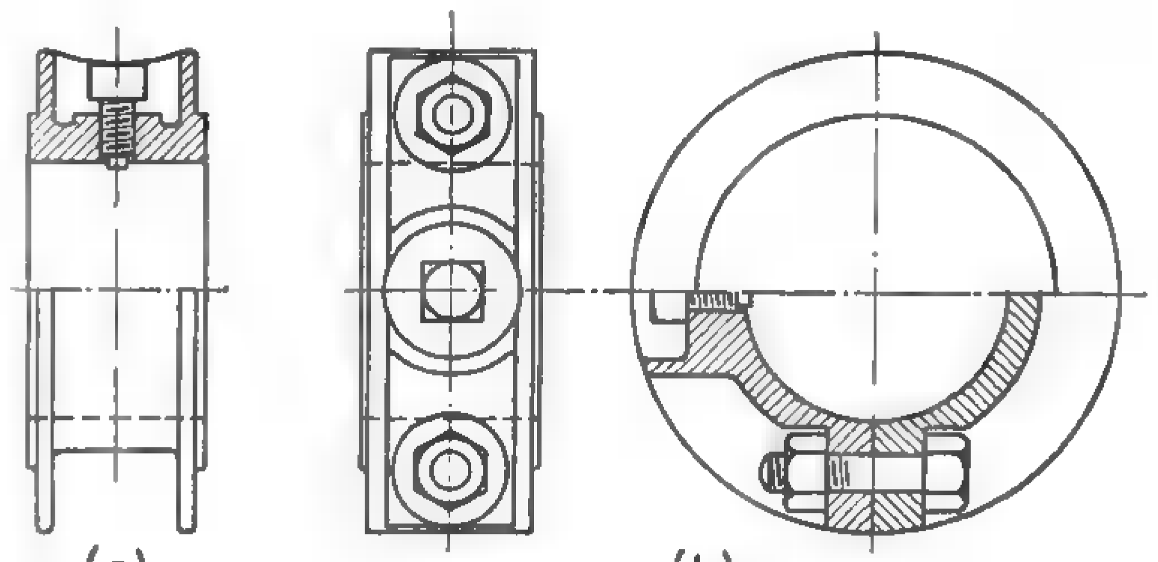


FIG. 23–31. Safety set collars.

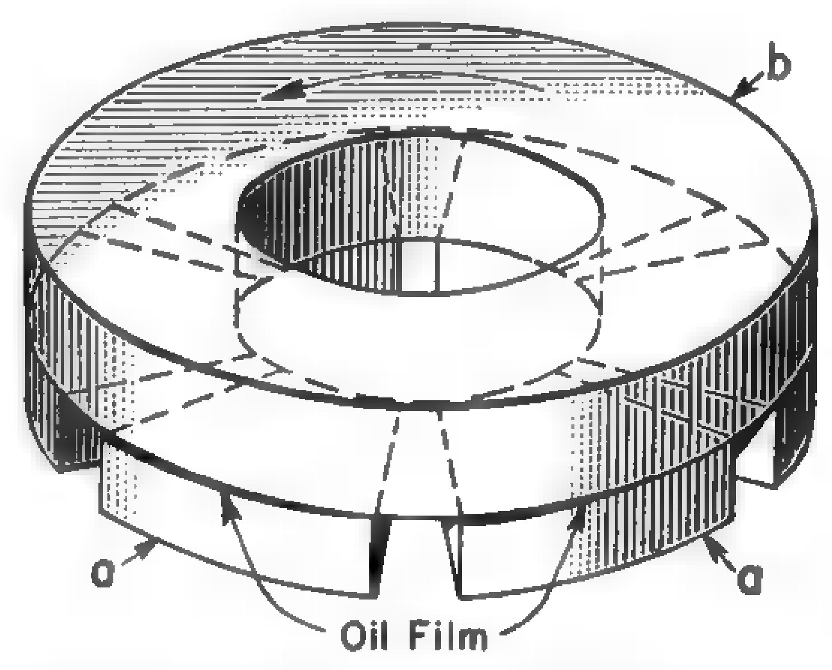


FIG. 23–32. Kingsbury thrust bearing.

Set collars. A small axial thrust can be taken by a set collar pressed against the end of a journal bearing. Set collars are used chiefly on transmission shafting. The head of the setscrew which fastens the collar to the shaft should be below the outside diameter of the flange, in which case the collar is termed a *safety collar*. Set collars are made of cast iron, malleable iron, or pressed steel. A solid collar is shown in Fig. 23–31a, and a split collar is shown in Fig. 23–31b.

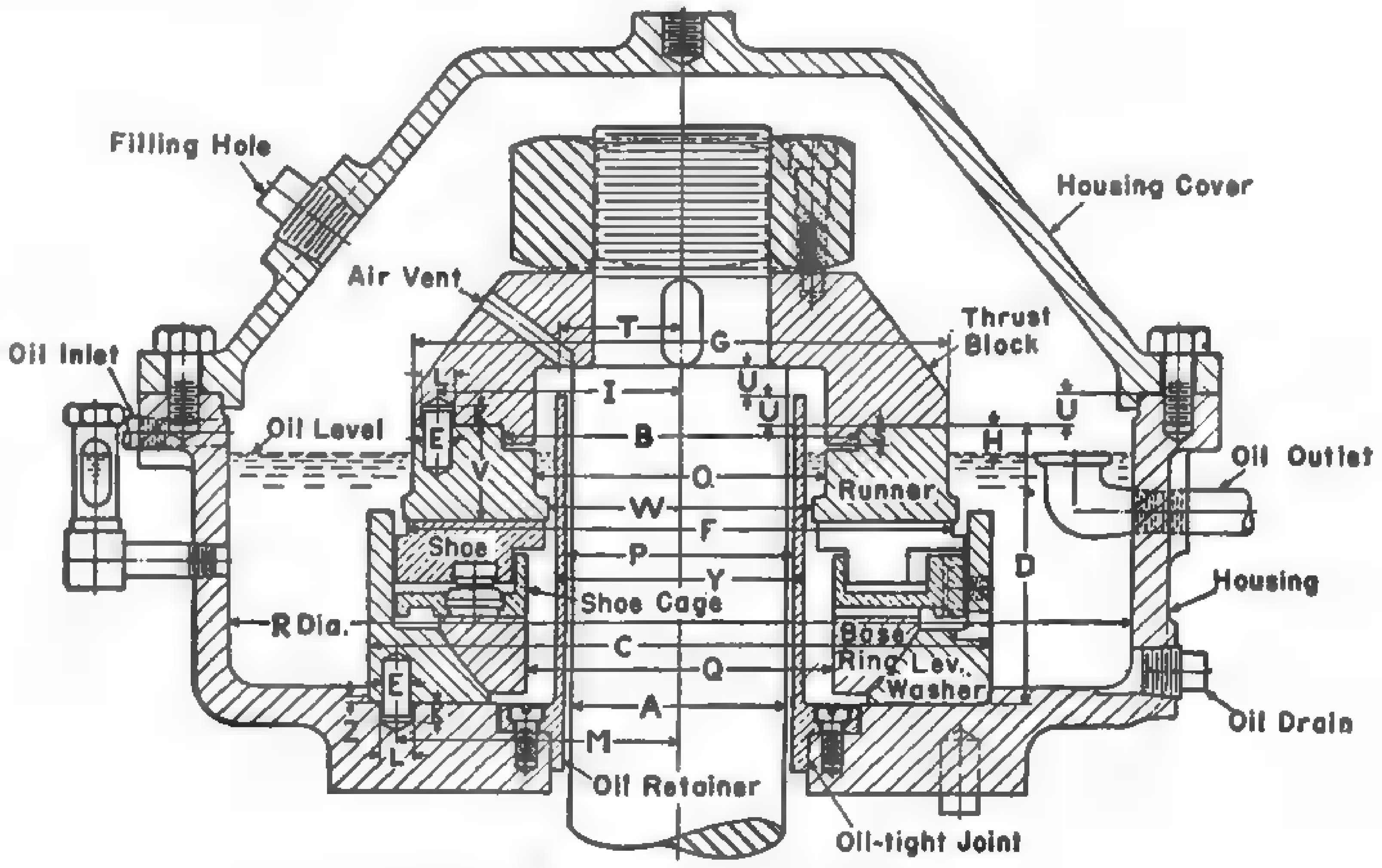


FIG. 23–33. Kingsbury thrust bearing.

Kingsbury thrust bearing. The principle of the Kingsbury thrust bearing is similar to that of the Michell bearing, Fig. 23–18. The Kingsbury bearing, Fig. 23–32, consists of several stationary pivoted segments, or shoes, a, against which is pressed the thrust collar b fastened to the rotating shaft. The shoes rest on spherical supports, or pivots, and are thus free to tilt in any direction. The pivots support the segments not at the center of gravity but slightly forward in the direction of rotation, as explained for the segments in

Fig. 23–18. Therefore the pressure on the back edge of each segment is lower than that on the forward edge. The collar runs in an oil bath, and its rotation draws an oil film into the spaces between it and the shoes, which tilt automatically so that a fluid wedge is formed at the back edge.

A Kingsbury thrust bearing for a vertical suspended shaft with a self-aligning spherical seat is shown in Fig. 23–33. The bearing is self-lubricating but is arranged for external oil circulation and cooling. A uniform distribution of load among the shoes is very important for proper operation of the bearing. It may be obtained by making the shoe supports adjustable, or by resting them upon a ring formed by equalizing levers. These bearings are made in types suitable for vertical or horizontal shafts, and for carrying thrusts in both axial directions, or in only one direction.

23–16. Design of thrust bearings. The friction coefficient of a well-lubricated step bearing may be taken as 0.015 to 0.02. The average friction coefficient of a collar bearing depends on the oil viscosity, speed, and load as expressed by the characteristic number Zn/p. A relation based on actual tests[27] is

$$f = 0.016v^{0.5}p^{-0.67} \qquad (23\text{–}24)$$

where v is the average peripheral speed, in feet per minute, and p is the specific pressure, in pounds per square inch.

The friction coefficient of a Kingsbury bearing depends on the characteristic number Zn/p. However, because of the presence of a perfect oil film and the allowable high specific pressure, it is about $\frac{1}{10}$ the friction coefficient in a collar thrust bearing, as found from tests. Under favorable conditions, Kingsbury thrust bearings have values of f as low as 0.0015, and 0.003 is a conservative figure.[28]

Friction loss in a pivot bearing. If a uniform pressure distribution is assumed in a pivot bearing, the friction torque, referred to the mean diameter d_m, is

$$T_f = \frac{fF_a d_m}{2} \qquad (23\text{–}25)$$

where F_a is the axial force. Since $d_m = \frac{2}{3}d$, the power absorbed by friction, by equation 2–17, is

$$P_f = \frac{fF_a dn}{189{,}090} \qquad (23\text{–}26)$$

Friction loss in a collar bearing. Equation 23–25 may be used for the friction loss in a collar bearing. The collars are either integral parts of the shaft or rigidly fastened to it, and the axial force F_a may be assumed to be evenly distributed over all collars. If $d_2 \leqq 1.5d_1$, where d_2 is the outside diameter of

[27] J. E. Hamilton, "Ball versus Tapered Roller Bearings," *Journal of the American Society of Naval Engineers*, Vol. 44 (1932), pp. 407–29.

[28] Kraft, *op. cit.*, p. 59.

the collars and d_1 is the inside diameter, then it is sufficiently accurate to assume that $d_m = \frac{1}{2}(d_2 + d_1)$. For this condition

$$T_f = \frac{fF_a(d_2 + d_1)}{4} \qquad (23\text{–}27)$$

The friction horsepower may be found by the equation

$$P_f = \frac{fF_a(d_2 + d_1)n}{252{,}120} \qquad (23\text{–}28)$$

The average pressure, with i collars, is

$$p = \frac{F_a}{0.7854(d_2^2 - d_1^2)i} \qquad (23\text{–}29)$$

With proper lubrication and moderate pressures the wear of the bearing collars is small, and the assumption of uniform wear, used for brakes and friction clutches, is not justified.

Allowable pressures. For step bearings, single-collar bearings, and water-cooled multicollar thrust bearings, and for rubbing speeds v ranging from 50 to 200 fpm, the allowable pressure p may be taken so that

$$pv \leqq 20{,}000 \qquad (23\text{–}30)$$

For very low speeds, the pressure may be as high as 2,000 psi; for intermittent service, pressures up to 1,500 psi may be used; and for speeds over 200 fpm, the pressure should not exceed 100 psi.

For multicollar thrust bearings that are not water-cooled, the above values for p should be divided by 2.

Kingsbury pivoted thrust bearings operate satisfactorily with pressures from 300 to 1,000 psi.

Oil circulation. The usual practice is to allow 12 deg F for the temperature rise of oil circulating through low-speed and medium-speed thrust bearings. The amount of oil G, in gallons per minute, can be calculated by assuming that 1 gal of oil weighs 7.5 lb, the specific heat of oil is 0.5 Btu per lb, and 1 hp = 33,000/778 = 42.42 Btu per min. Thus

$$G = \frac{42.42}{7.5 \times 0.5 \times 12} = 0.94 \text{ gpm}$$

The approximate amount of oil is 1 gpm per horsepower of friction loss.

In bearings with high rubbing speeds, where turbulence is an important factor, the friction losses may be reduced by supplying less oil and allowing an oil temperature rise up to 25 deg F.

The passages in the bearings and in the oil pipes should be amply large, the velocity of oil going to the bearings should be not more than 240 fpm, and the velocity of the oil returning to the oil cooler should be not more than 150 fpm.

23–17. Design procedure for a multicollar bearing. For a given shaft diameter d_1, select d_2 not greater than $1.5d_1$. Take the mean diameter d_m as $\frac{1}{2}(d_1+d_2)$, and find the mean rubbing velocity v_m, in feet per minute.

Select the safe average pressure p and check it by equation 23–30. If several collars are used, it is advisable to use a value for p not more than one-half the limit given by equation 23–30.

Find the force F_a that each collar can take, and determine the total number of collars i required. Compute the friction coefficient f by equation 23–24. The loss of power is computed by equation 23–28.

If the bearing is not water-cooled, the heat-dissipating capacity should be estimated by equation 23–14 in conjunction with equation 23–15.

If water cooling is used, determine the amount of water that must be circulated to carry away the heat of friction. Allow a rise in water temperature of 10 to 20 deg F.

EXAMPLE 23–2. Determine the main dimensions of a multicollar thrust bearing for a propeller shaft of a 600-bhp marine oil engine. The engine makes 220 rpm; the shaft diameter is 6 in.; the propeller has a pitch of 100 in.

If a slip of 25 per cent is assumed, the boat will be driven 75 in., or 6.25 ft, for each revolution of the engine, or 1,375 fpm. The total axial force multiplied by the speed represents the work done. Therefore

$$F_a=\frac{600\times 33{,}000}{1{,}375}=14{,}400 \text{ lb}$$

For $d_2=1.5d_1=1.5\times 6=9$ in., the mean diameter is $d_m=\frac{1}{2}\times(6+9)=7.5$ in. The mean rubbing speed is

$$v=\frac{\pi\times 7.5\times 220}{12}=432 \text{ fpm}$$

If the thrust bearing is water-cooled, a pressure p of 80 psi may be assumed. The force that each collar can carry is

$$F_a=0.7854\times(9^2-6^2)\times 80=2{,}825 \text{ lb}$$

and the number of collars should be

$$i=\frac{14{,}400}{2{,}825}=5.1, \text{ or } 6$$

The actual pressure would be

$$p=\frac{14{,}400}{0.7854\times(9^2-6^2)\times 6}=68.0 \text{ psi}$$

The coefficient of friction computed by equation 23–24 is

$$f=0.016\times 432^{0.5}\times 68^{-0.67}=0.0196$$

By equation 23–12, with $C_1=2$ and $C_2=4$ from Tables 23–2 and 23–3,

$$f=0.004\times 2\times 4\sqrt[4]{\frac{68}{432}}=0.0202$$

This is a good agreement. To be safe, use $f=0.021$.

The loss of power, by equation 23–28, in which $F_a=14{,}400$ lb, is

$$P_f=\frac{0.021\times 14{,}400\times 15\times 220}{252{,}120}=3.95 \text{ hp}$$

CHAPTER 24

Bearings with Rolling Contact

24–1. General considerations. In a bearing with rolling contact the shaft is supported on rollers or balls. A bearing of this type has the theoretical advantage of reduced friction, but to be practical it must fulfill the following conditions:

a) Unavoidable sliding should be reduced to a minimum.
b) The rolling elements must be properly guided in their motion.
c) All rolling elements should be of exactly the same size.
d) The rolling elements and their guides, or raceways, must be extremely hard and very smoothly polished.
e) The pressure should be approximately normal to the surface of contact.
f) The rolling elements must not be overloaded.

Advantages. Well-manufactured bearings with rolling contact in properly designed applications have the following advantages over bearings with sliding contact:

a) They maintain accurate shaft alignment over long periods of time.
b) They can carry heavy momentary overloads without failure.
c) Power loss caused by friction is small except at high speeds.
d) They are particularly suitable for very low speeds.
e) Starting friction is very low.
f) Lubrication is simple and requires but little attention.
g) Replacement in case of failure is easy.

Disadvantages. Against the advantages just mentioned, rolling-contact bearings have the following disadvantages:

a) The design of the shaft and housing is more complicated.
b) The first cost is higher.
c) The housing diameter is larger, except with some needle bearings.
d) The resistance to shock loads is lower.
e) There is more noise, especially at higher speeds.
f) They are sensitive to dirt and grit.

Classification. Bearings with rolling contact may be divided into two main classes: *ball bearings*, and *roller bearings* with cylindrical, conical, spherical, or concave rollers.

Each of these classes may be subdivided into the following types: (a) radial bearings; (b) thrust bearings, and (c) radial-thrust bearings, or angular bearings, which can take both radial and axial forces.

24–2. Ball bearings. Each radial ball bearing consists of four elements: an inner ring, or *inner race*, grooved on its outer surface; an *outer race*, grooved on its inner surface; steel *balls;* and a *ball retainer*, or cage, for spacing the balls so that they do not touch each other, in order to reduce wear and noise.

Types. The various types of ball bearings are shown in Fig. 24–1. The single-row radial bearing, Fig. 24–1a, is usually made with a deep groove. The angular-type bearing, Fig. 24–1b, can take an axial load in addition to the radial load. In Fig. 24–1c is shown a double-row bearing for increased load capacity. Figure 24–1d shows a self-aligning double-row bearing, the inner surface of the outer race of which is part of a sphere.

Standardization. Through the efforts of the Society of Automotive Engineers, the manufacturers of radial ball bearings have adopted the international standard dimensions, according to which the bearings are divided into three series called the *light*, *medium*, and *heavy* series. The light series is designated by the number 200; the medium, by 300; and the heavy, by 400. The adapter type is designated by 500 or 600.

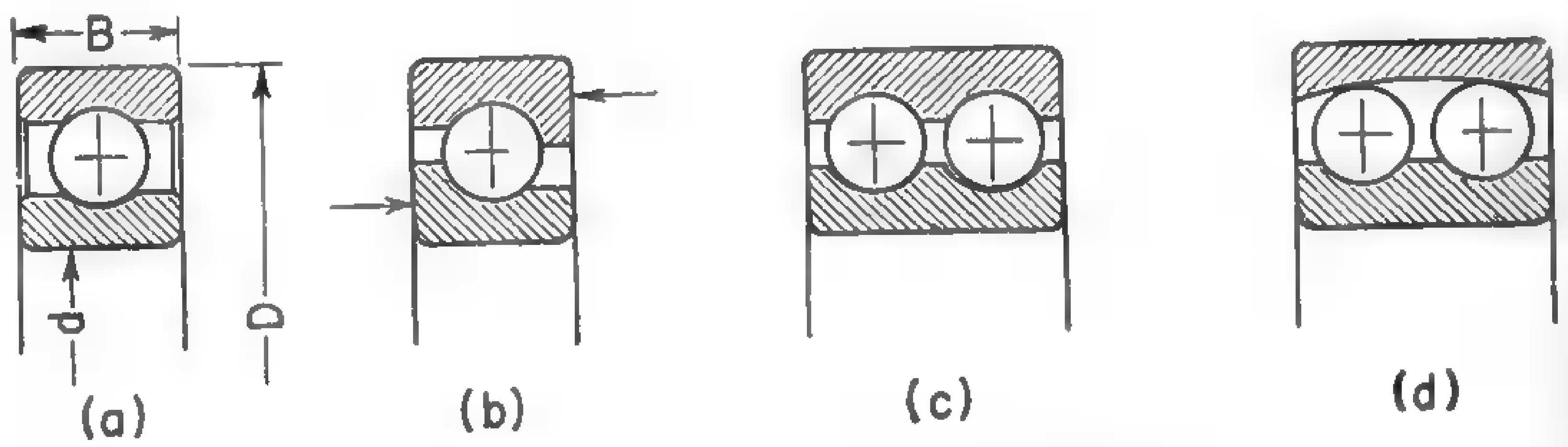


FIG. 24–1. Types of ball bearings.

The inside diameter d, Fig. 24–1a, the outside diameter D, and the width B are in millimeters. Some manufacturers make special, nonstandard bearings with the dimensions d, D, and B in inches. The last two digits of a standard bearing number designate the bore d, which comes in multiples of 5 mm, beginning with 10 mm, except for two intermediate sizes, 12 and 17 mm. Beginning with $d = 20$ mm, the last two digits in the bearing number multiplied by 5 give the bore d. The number before the serial number is given by manufacturers in accordance with their systems. Thus, an SKF bearing No. 6309-A has a single row of balls, its bore is $9 \times 5 = 45$ mm, and it is of the medium series with $B = 25$ mm. This bearing is interchangeable with the bearing No. 1309, which is a self-aligning, double-row bearing; that is, both bearings have the same values of d, D, and B. The bearing mentioned is also interchangeable with any bearing of another make which has the last three digits 309. The international standards do not specify either the size or the number of the steel balls.

Load capacity. Theoretically, the contact between a ball and a race is a point. Actually, however, because of the elastic deformations of the mate-

rial, a small area in the form of an ellipse supports the load. Races of radial ball bearings are always grooved, and the groove increases the area of contact of a ball under load. The groove has a radius about 4 per cent larger than the ball radius. The increased contact area permits the bearing to sustain larger loads with the same stress. Under normal conditions the failure of a ball bearing is caused by a flaking of the surfaces under repeated high stresses. Therefore, the larger the load, the shorter will be the bearing life, because it takes fewer repetitions of larger stresses than of lower stresses to produce an endurance failure.

There does not exist any standard method of rating the load capacity of ball bearings. According to research conducted by Stribeck,[1] a rational expression for static load capacity F is

$$F = KCid_o^2 \tag{24–1}$$

where K is a factor which depends on the material and is determined from compression tests; C is the conformity factor and depends on the curvature of the ball relative to both the transverse curvature of the race and its curvature in the plane normal to the axis of the bearing; i is the number of balls; and d_o is the ball diameter. The determination of the factors K and C is rather involved; only a specialist is in a position to use equation 24–1 correctly. The manufacture of ball bearings is a highly specialized field requiring both technical knowledge and special precision production methods. The problem confronting the general machine designer is to select the most suitable bearing from the catalogue of a reliable manufacturer in accordance with the instructions given in the catalogue.

As already mentioned, neither the diameter d_o of the balls nor their number i is standardized. Manufacturers use different values for d_o and i in their bearings. Moreover, there are some differences in the materials used and in the factor of safety assumed. These differences, when considered in conjunction with equation 24–1, explain the variations in the ratings of ball bearings of different manufacturers. Thus, the load ratings of some manufacturers, for interchangeable bearings with the same diameters d and D, are sometimes 100 per cent higher than those of other manufacturers.

Life of bearings. When a group of identical ball bearings is run under identical conditions of load and speed until they all fail by fatigue of the surfaces, it will be found that a few bearings will fail early, others will last longer, and some will last about four times as long as the average life L_m of the whole tested group. This dispersion in their lives is caused by unavoidable variations in the steel properties, heat treatment, surface characteristics, and dimensions of the balls.

[1] S. Stribeck, "Kugellager für beliebige Belastung," *Zeitschrift Verein Deutscher Ingenieure*, Vol. 45 (1901), p. 121.

Ball bearings fail due to repetition of compressive stresses created in the surfaces of the balls and races. Therefore the term *life* may be defined as the number of million revolutions or hours of operation at a given speed.

If the number of bearings tested is 30 or more, it is found that 90 per cent of the bearings have a life longer than one-fifth of the average life L_m. Thus the average life is a good criterion of the quality of the bearings and may be used for estimating the cost of replacement of bearings in a plant.

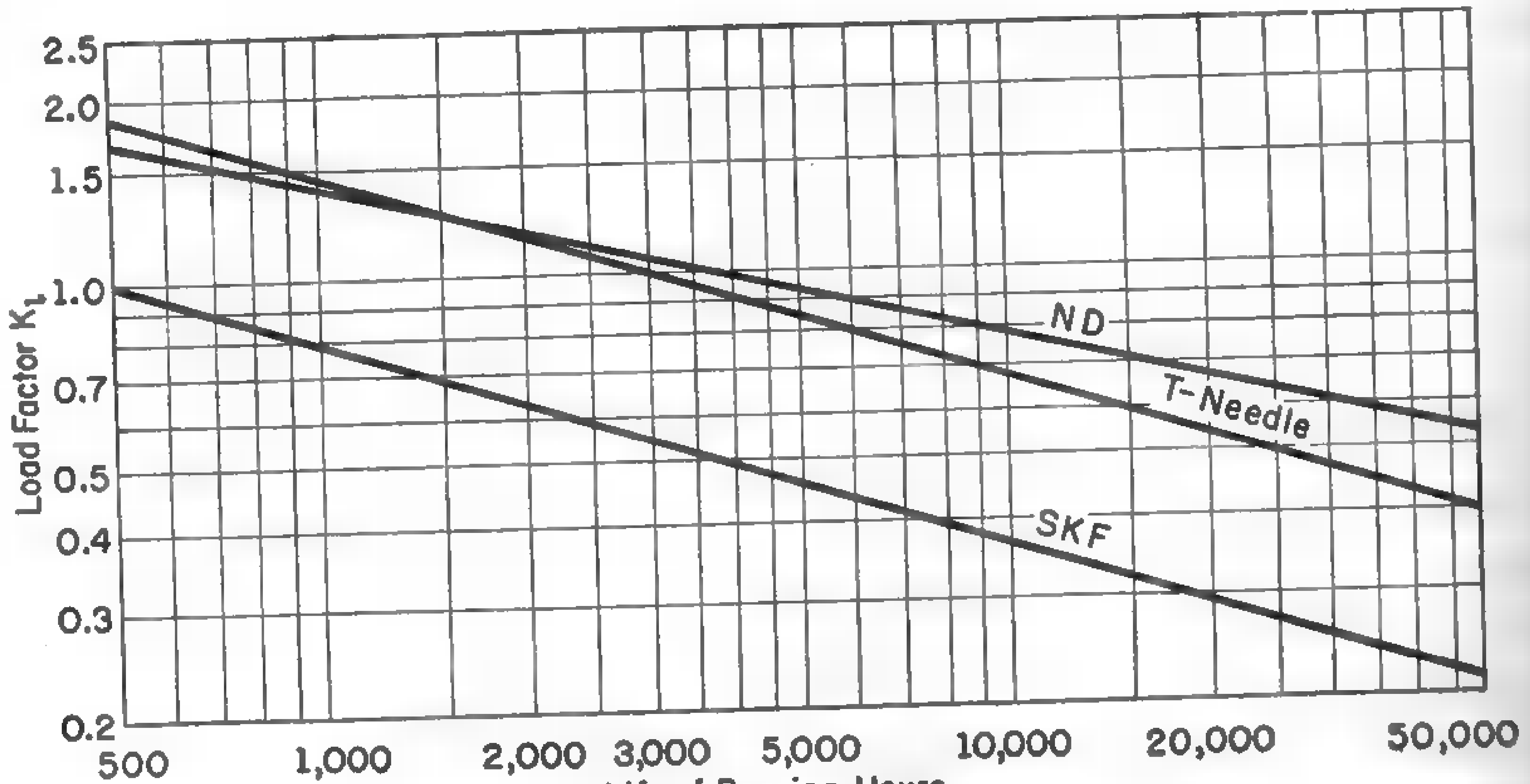


FIG. 24–2. Life curves of ball bearings.

Influence of load. Tests have shown that the life L of bearings is a function of the load. Tests also show that with sufficient accuracy, L may be considered to be inversely proportional to the third power of the load. Thus, if one group of test bearings of a certain type and size are run under a constant load F_1 and have a life L_1, and another group are run under a load F_2, but otherwise under identical operating conditions, and have a life L_2, then

$$\frac{L_1}{L_2}=\left(\frac{F_2}{F_1}\right)^3 \tag{24–2}$$

If L_2 is equal to one million revolutions under a constant load C, called the *dynamic specific capacity* of a bearing, then the life L_n of the bearing, also in millions of revolutions, under a constant load F can be presented by the more general equation

$$L_n=\left(\frac{C}{F}\right)^3 \tag{24–3}$$

The numerical values of C, in pounds, are given for all bearings in the SKF catalogue and are reproduced for certain more commonly used bearing types in Table 24–2.[2]

[2] SKF Industries, Inc., *General Catalog and Engineering Data* (Philadelphia: 1947), p. [illegible]

With a speed of 2000 rph, 1,000,000 revolutions are obtained in 500 hr, and equation 24–3 may be presented graphically, as shown in Fig. 24–2. Curve *SKF* shows that if the load is decreased to one-half, the life is increased from 500 to 4000 hr. The manufacturers of New Departure ball bearings use in equation 24–2 an exponent of 4. Also, they take as average life, 3,800 hr. As a result the curve *ND* has a different slope and runs higher.

The ratio of the actual load F to the basic load F_c given in a catalogue may be called the *load factor* K_l. Thus

$$K_l=\frac{F}{F_c} \tag{24–4}$$

Influence of speed. The number of load repetitions increases with an increase of the rotative speed, thus resulting in a shortened bearing life expressed in revolutions or in time units at a certain speed. The relation between the allowable load and the speed for a certain bearing life may be represented by the same curve, Fig. 24–2, but with a change in the scale of the abscissas. However, most bearing catalogues give the allowable load ratings directly for different speeds, from a minimum to the highest speed advisable for a given bearing.

The SKF engineers use a different procedure, introducing a speed factor f_n, which is given in Fig. 24–3a, and a life factor f_h, given in Fig. 24–3b. With these factors the permissible load F is determined by the relation

$$F=\frac{f_nC}{f_h} \tag{24–5}$$

EXAMPLE 24–1. Determine the load capacity of a single-row deep-groove SKF bearing—SAE 10, medium series—that must run 2000 hr at 700 rpm.

From Table 24–2, $C=10{,}400$ lb, and from Fig. 24–4, $f_n=0.362$ and $f_h=1.585$. Hence, by equation 24–2,

$$F=\frac{0.362\times 10{,}400}{1.585}=2{,}370 \text{ lb}$$

Materials. Under normal load conditions the magnitude of the compressive stresses occurring at the contact areas of balls and races varies from

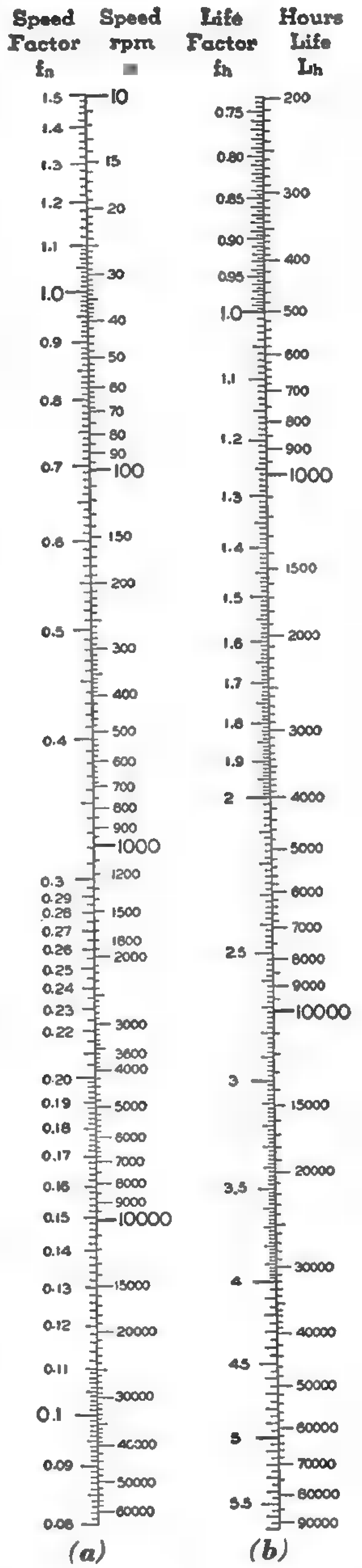


FIG. 24–3. Speed- and load-factor diagram.

TABLE 24–1

OVER-ALL DIMENSIONS OF RADIAL BALL BEARINGS

SAE Bearing Number	Bore of Inner Race d		Outside Diameter D (mm)			Width B (mm)		
	Mm.	In.	200 Series	300 Series	400 Series	200 Series	300 Series	400 Series
00......	10	0.3937	30	35	...	9	11	..
01......	12	0.4724	32	37	...	10	12	..
02......	15	0.5906	35	42	...	11	13	..
03......	17	0.6693	40	47	62	12	14	17
04......	20	0.7874	47	52	72	14	15	19
05......	25	0.9843	52	62	80	15	17	21
06......	30	1.1811	62	72	90	16	19	23
07......	35	1.3780	72	80	100	17	21	25
08......	40	1.5748	80	90	110	18	23	27
09......	45	1.7717	85	100	120	19	25	29
10......	50	1.9685	90	110	130	20	27	31
11......	55	2.1654	100	120	140	21	29	33
12......	60	2.3622	110	130	150	22	31	35
13......	65	2.5591	120	140	160	23	33	37
14......	70	2.7559	125	150	180	24	35	42
15......	75	2.9528	130	160	190	25	37	45
16......	80	3.1496	140	170	200	26	39	48
17......	85	3.3465	150	180	210	28	41	52
18......	90	3.5433	160	190	225	30	43	54
19......	95	3.7402	170	200	...	32	45	..
20......	100	3.9370	180	215	...	34	47	..
21......	105	4.1339	190	225	...	36	49	..
22......	110	4,3307	200	240	...	38	50	..

200,000 to 300,000 psi. It is therefore evident that only special alloy steels, with nickel and either chromium or molybdenum, that have been given extraordinary and uniform hardness and toughness by heat treatment can render satisfactory service.

Accuracy. The balls must be true spheres, and to secure an even load distribution the actual size should not vary from the nominal size by more than 0.00005 in. All other dimensions, such as the diameters d and D, and the radii of fillets, are machined to tolerances expressed in tenths of one thousandth of an inch. Balls and races are ground and highly polished.

Coefficient of friction. The coefficient of friction of ball bearings is practically independent of the load and the speed. It varies, according to operating conditions, from 0.0005 to 0.0030, and the general average is around 0.0015. The coefficient of friction of roller bearings is about five times as high.

24–3. Selection of radial ball bearings. In selecting a ball bearing from a trade catalogue for a specific installation, three main points must be considered:

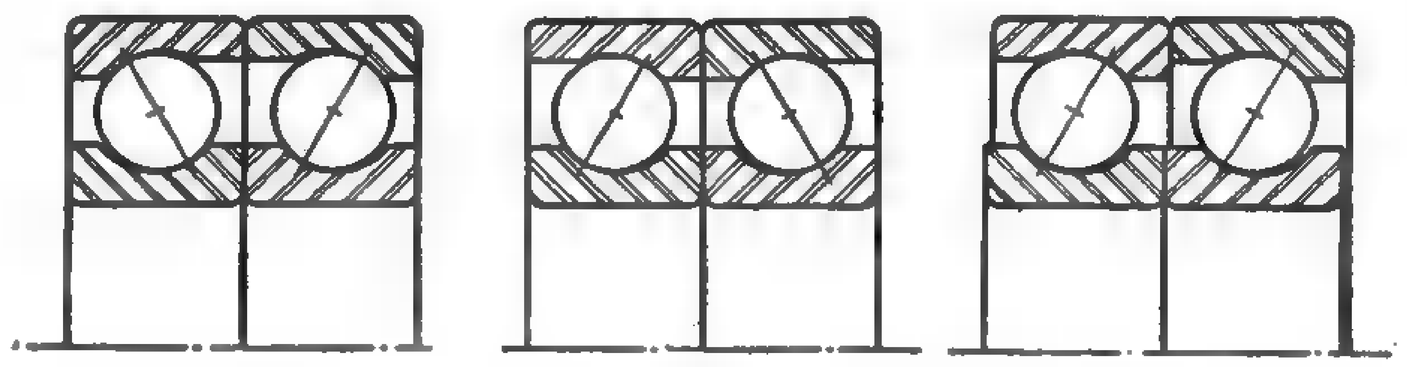
a) The bearing must be of the series best suited to the installation, in regard to both capacity and dimensions.

b) The type of bearing selected—radial, thrust, or combined—must be suitable for the type of imposed load.

c) The size of the bearing must be such as to give the required length of service with sufficient assurance.

Light series. Light-series bearings should be used where loads are moderate and shaft sizes are comparatively large, as for hollow shafts or for long shafts whose diameter is influenced not by strength but by requirements of rigidity. They should also be used where the housing space requires the narrowest bearing width and the smallest outside diameter available for a given bore size.

Medium series. Medium-series bearings provide a capacity increase of approximately 30 to 40 per cent. They should therefore be employed where loads are heavy and considerable bearing capacity in proportion to the shaft size is desirable.



(a) (b) (c)

FIG. 24–4. Methods of mounting two angular-contact bearings.

Heavy series. Heavy-series bearings have capacities approximately 20 to 30 per cent greater than those of the medium series. They are applied only to specially proportioned shafts and are recommended only for special installations.

Table 24–1 gives the over-all dimensions for all three bearing series.

Bearing types. Single-row bearings are used when the main load acts radially. However, they can also carry a certain axial load, usually not more than 50 per cent of the applied radial load.

Deep-groove single-row bearings have a slightly smaller radial capacity because of the use of fewer balls, but their axial capacity is about 75 per cent of the rated radial capacity.

Angular-contact bearings, Fig. 24–1b, have an axial capacity of 100 to 200 per cent of the rated radial capacity, but they resist axial thrust in only one direction. If an axial thrust occurs in both directions, then two angular-contact bearings must be used, as shown in Fig. 24–4a. If great radial rigidity only is desired, as in machine-tool spindles, the mounting shown in Fig. 24–4b is used. A tandem mounting, Fig. 24–4c, is used for an extremely large thrust load in one direction.

These bearings are not recommended for a light axial load.

TABLE 24–2

SPECIFIC DYNAMIC CAPACITY OF SKF BALL BEARINGS, IN POUNDS

SAE Number	Double-Row Self-Aligning			Single-Row Deep-Groove			Double-Row Deep-Groove			Single-Row Angular-Contact		
	Light	Medium	Heavy	Light	Medium	Heavy	Light	Medium	Heavy	Light	Medium	Heavy
00	865	.....	.....	750	1,460	.....	1,220	.....	.....	.....	.....	.....
01	915	1,500	.....	1,140	1,800	.....	1,860	.....	.....	.....	.....	.....
02	1,250	1,630	.....	1,250	1,960	.....	2,040	.....	.....	.....	.....	.....
03	1,400	2,120	.....	1,600	2,360	.....	2,600	3,800	.....	.....	.....	.....
04	1,830	2,240	.....	2,160	2,750	.....	3,550	4,500	.....	.....	2,700	.....
05	2,240	3,250	.....	2,320	3,650	6,300	3,800	6,000	.....	2,280	3,800	6,200
06	3,100	4,050	6,100	3,250	4,800	7,500	5,300	7,800	13,400	3,200	4,800	7,350
07	3,350	4,800	7,050	4,300	5,700	9,500	7,100	9,300	14,000	4,250	5,700	9,300
08	4,250	6,100	8,050	4,900	6,950	11,000	8,000	11,200	17,600	5,100	6,950	10,600
09	4,750	7,650	10,400	5,500	9,000	13,200	9,000	14,600	22,000	5,700	9,000	12,900
10	5,100	8,650	12,000	5,850	10,400	14,600	9,650	17,000	27,500	6,000	10,400	14,300
11	6,200	10,400	12,700	7,350	12,000	17,300	11,800	19,300	27,500	7,350	12,000	16,600
12	7,100	12,000	15,300	8,800	13,700	19,000	14,300	22,000	30,500	8,800	13,700	18,300
13	7,650	12,700	16,300	9,650	15,300	20,800	15,600	25,000	.....	10,000	15,300	20,000
14	8,500	15,300	20,000	10,400	17,000	26,000	17,000	27,500	.....	11,000	17,000	25,500
15	9,300	16,300	22,800	11,000	18,600	28,000	18,000	30,500	.....	11,400	19,000	27,000
16	9,800	18,000	25,500	12,200	20,800	.....	19,600	33,500	.....	12,700	20,800	.....
17	11,800	20,000	27,500	13,700	22,400	.....	20,400	33,500	.....	14,300	22,800	.....
18	12,900	22,800	31,000	15,600	24,500	.....	23,200	35,500	.....	16,600	24,500	.....
19	.....	25,500	.....	17,600	26,500	.....	28,500	.....	.....	17,600	26,500	.....
20	16,300	27,500	.....	19,600	30,500	.....	32,000	.....	.....	19,600	32,500	.....
21	.....	31,000	.....	21,600	32,500	.....	.....	.....	.....	.....	34,500	.....
22	20,400	33,500	.....	24,000	37,500	.....	.....	.....	.....	24,000	39,000	.....
24	.....	.....	.....	.....	.....	.....	.....	.....	.....	27,500	.....	.....
26	.....	.....	.....	.....	.....	.....	.....	.....	.....	31,000	48,000	.....
28	.....	.....	.....	.....	.....	.....	.....	.....	.....	33,500	.....	.....
30	.....	.....	.....	.....	.....	.....	.....	.....	.....	39,000	58,500	.....
32	.....	.....	.....	.....	.....	.....	.....	.....	.....	41,500	.....	.....
34	.....	.....	.....	.....	.....	.....	.....	.....	.....	43,000	.....	.....

TABLE 24–3

SAFETY FACTORS FOR MRB BALL BEARINGS

Load Conditions	Safety Factor, Life of 5 to 10 Years			Safety Factors, Life of 10 to 20 Years
	Intermittent	10 Hr per Day	Continuous	Continuous
Steady load	0.5–1	1.5	2	3
Light shock	1–2	2.5	3	4
Moderate shock	2–3	3.5	4	5
Severe shock	3–4	4.5	5	6

Double-row bearings are used either to obtain a larger radial and axial capacity, Fig. 24–1c, or to have the self-aligning feature, Fig. 24–1d, if the shaft is not very rigid.

To furnish a general idea of the loads which ball bearings can carry, SKF bearings are taken as an example. Table 24–2 gives the rated radial loads for the various types of SKF radial bearings at 500 rpm.

Size of bearing. In the majority of applications, ball bearings have to resist some combination of radial and thrust loads. The bearing rating is always referred to a radial load. Therefore the combined load must be reduced to an equivalent radial load. Various catalogues give different methods of reduction. In general, the equivalent load F_e may be computed by the equation

$$F_e = xF_r + yF_a \tag{24–6}$$

where F_r is the actual radial load, F_a is the actual axial load, and x and y are coefficients which depend on the ratio F_a/F_r. For most bearings, it may be assumed that $x = 1$ and $y = 1.5$. For angular-contact bearings, $x = 0.5$ and $y = 1$. For self-aligning ball bearings and roller bearings, $x = 0.5$ and $y = 2.5$.

Shock. The load capacities given in trade catalogues are based on a steady load. In the event of shock action the equivalent load F_e must be multiplied by a safety factor n_s. Values of n_s recommended by the Marvin-Rockwell Corporation are given in Table 24–3. It should be noted that these values also include life-load factors. If the latter are taken into account separately, the column for intermittent service should be used.

Speed influence. The wear of a ball bearing increases, and its life decreases, with an increase of the speed of one race relative to that of the other race. If n_1 is the speed of the inner race and n_2 is the speed of the outer race, the *effective speed* n_e which determines the life of the bearing may be found from the relation[3]

$$n_e = n_1 \pm n_2 \tag{24–7}$$

in which the plus sign must be used when the races rotate in opposite directions and the minus sign must be used when they rotate in the same direction.

Catalogue tables give bearing capacities for various effective speeds. However, the capacity corresponding to a certain speed may be calculated by multiplying the basic capacity by a speed factor such as given by Fig. 24–3.

Temperature. A ball bearing should not be exposed to a temperature over 200 to 220 F. A higher temperature may not affect the load-carrying capacity of the bearing, but at a higher temperature the lubricating oil will

[3] General Motors Corporation, New Departure Division, *New Departure Handbook*, 18th ed. (Bristol, Conn.: 1946), p. 9.

begin to evaporate and there will be danger that the bearing may run dry. With a special lubricant a ball bearing may operate, without any loss of load capacity, at temperatures up to 300 F or 325 F.

Bearing life. The life-load factor K_l, equation 24–4, may be taken from the corresponding curve, Fig. 24–2, New Departure bearings, or it may be taken from catalogue tables for bearings of other manufacturers.

Selection of size. After the equivalent load F_e, the safety factor n_s for shock, and the life-load factor K_l are established, the proper size of a bearing will be that whose capacity rating F_c, at the proper speed, as given in the manufacturer's catalogue, satisfied the condition

$$F_c \geqq \frac{n_s F_e}{K_l} \tag{24-8}$$

An SKF bearing is selected by using equation 24–5 in connection with Fig. 24–3, and either Table 24–2 or the SKF catalogue for other bearing types.

EXAMPLE 24–2. An SKF bearing must be selected for the end of a $3\frac{15}{16}$-in. shaft which can be reduced to $3\frac{1}{2}$ in. if necessary. The shaft speed is 230 rpm, and the shaft carries a radial load of 3,800 lb with light shocks, and also an axial load of 1,200 lb. A bearing life of 2 years with 10 hr of service a day, 5 days per week, is desired.

The equivalent radial load, by equation 24–6, is

$$F_e = 3{,}800 + 1.5 \times 1{,}200 = 5{,}600 \text{ lb}$$

From Fig. 24–3a, the speed factor f_n is 0.525. The desired minimum bearing life is $2 \times 52 \times 5 \times 10 = 5{,}200$ hr, and from Fig. 24–3b, the life factor $f_n = 2.19$. From Table 24–1, the safety factor for light shock may be taken as $n_s = 1.5$. Thus, by equation 24–5, the specific dynamic capacity must be

$$C \geqq \frac{1.5 \times 5{,}600 \times 2.19}{0.525} = 35{,}000 \text{ lb}$$

According to Table 24–2, a double-row deep-groove bearing No. 318, with a bore $d = 18 \times 5 \times 0.03937 = 3.5433$ in. and $C = 35{,}500$ lb, may be used, requiring the shaft to be slightly turned down.

24–4. Installation of ball bearings. In the installation of ball bearings the following points must be considered separately:

a) The shaft must be designed to take the inner race.

b) A suitable mounting must be designed for the outer race.

c) Lubrication must be provided for the bearing.

d) Methods of sealing the lubricant and preventing penetration of foreign matter must be provided.

Shaft design. It is very important that the inner race be a light press fit on the shaft, about class 6. If the fit is too loose, the inner race will slip; if it is too tight, the shaft may stretch the race, and the life of the bearing may be shortened.

The inner race must be clamped between a shoulder and a nut, as in Fig. 24–5a. The nut may be secured against unscrewing by the washer *a* with tongues, the outer one being bent into a slot of the nut; or a setscrew may be used, as in Fig. 24–5b at *b*. It is essential to provide a sufficiently

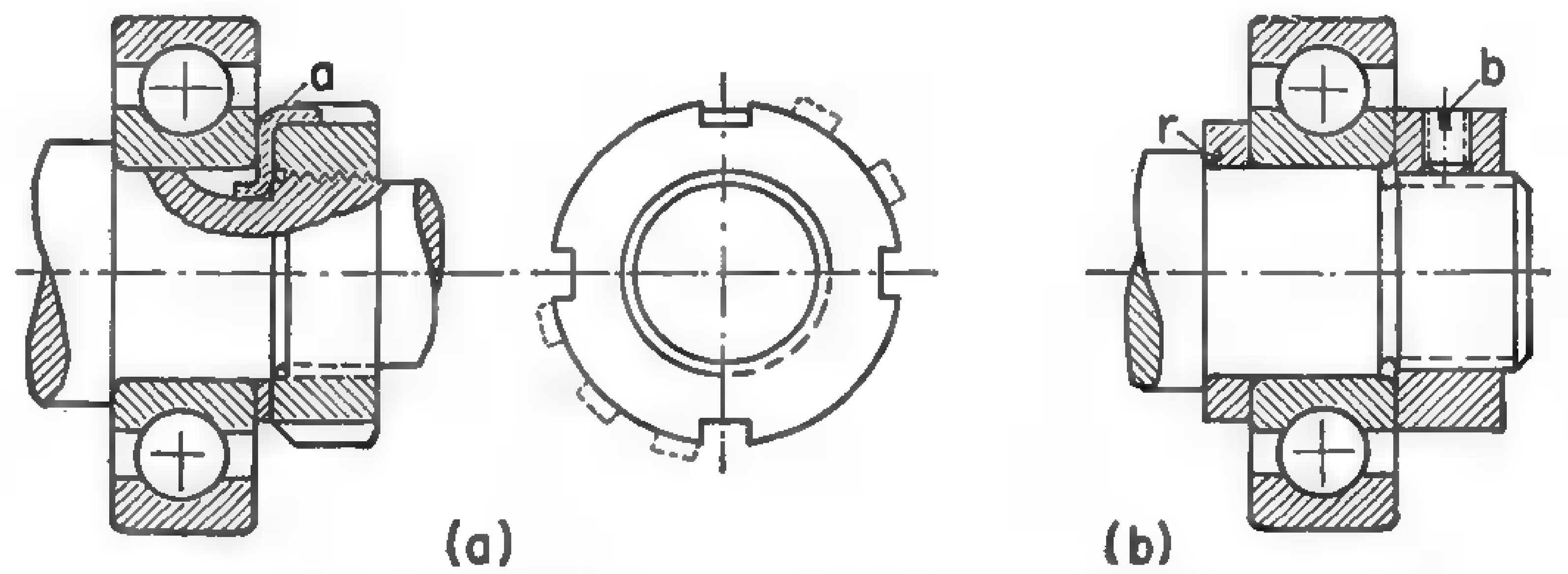


FIG. 24–5. Methods of fastening ball bearings to a shaft.

high shoulder on the shaft to locate the bearing positively. If the shoulder is so low that it would enter the corner radius of the bearing, a special shoulder ring must be made, as in Fig. 24–5b. This ring should be a tight fit, class 7, and should have a corner radius *r* exactly equal to the fillet radius on the shaft.

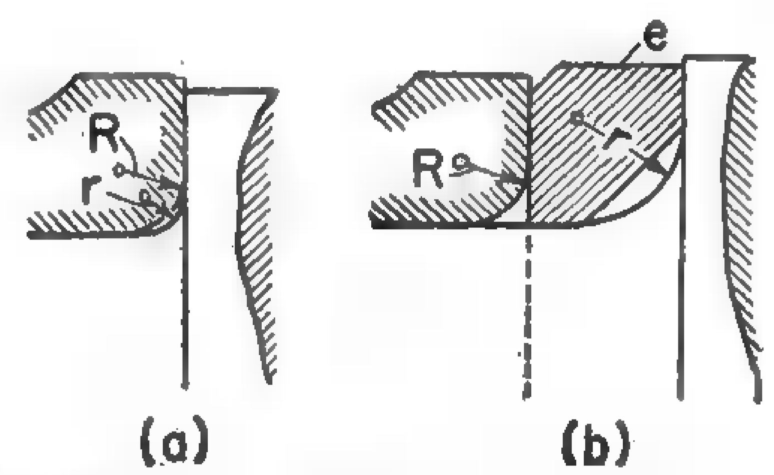


FIG. 24–6. Fitting of inner race to a shaft.

Corner fillet. For maximum strength the shaft requires a definite fillet at the junction of the bearing seat and the locating shoulder. However, it is very important that the shaft-fillet radius *r*, Fig. 24–6a, be smaller than the corner radius *R* of the bearing. If the bearing is located at a point of the shaft where the latter is under stress, it is imperative to have in the shaft a large fillet radius *r* to reduce stress concentration. In such a case a special shoulder ring *e*, Fig. 24–6b, should be used.

Shaft rigidity. Ball bearings give the best service where the shaft and its supports are rigid. When a shaft is comparatively long, its diameter between bearings must be large enough to properly resist bending. In general, the use of more than two bearings on any single shaft should be avoided, because of the difficulty of securing accurate alignment. With bearings mounted close to each other, inaccurate alignment may produce dangerously heavy bearing loads.

Bearing adapters. For shafts without shoulders, such as transmission shafts, bearing adapters are used. In Fig. 24–7a, a split conical sleeve *a* is tightened by a nut *b* and secured by a steel-wire snap ring *c*. In Fig. 24–7b, the inner race is extended and the end is turned eccentrically; the ring *a* has an eccentric counterbore fitted accurately over the end of the race; and a

slight turn of this ring creates enough pressure between the bearing race and the shaft to hold the race firmly by friction. In Fig. 24–7c is shown an adapter to be used if the bearing bore is larger than the shaft diameter.

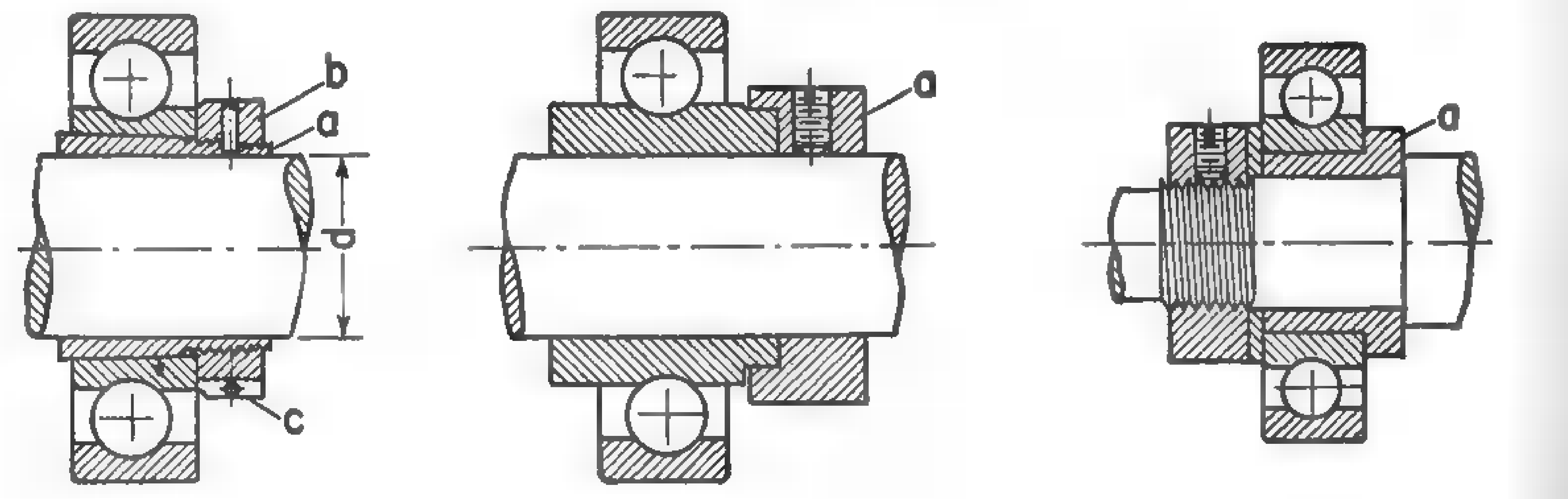


FIG. 24–7. Types of bearing adapters.

Housing design. Where two radial bearings are to be used, it is usually desirable to locate the shaft axially by clamping one of the bearings both on the shaft and in the housing. When this is done, the other bearing should have an unrestricted axial clearance, Fig. 24–8, of 0.010 to 0.015 in. In this manner, shaft expansion and variations in housing and shaft machining cannot so combine as to place the bearings under a useless, and possibly an injurious, thrust.

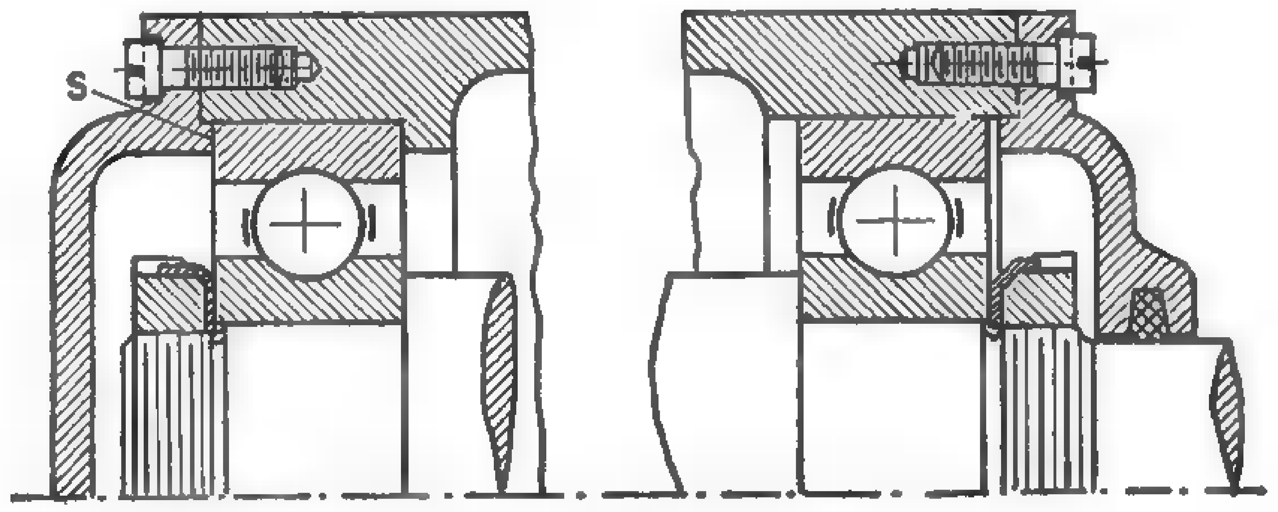


FIG. 24–8. Ball-bearing mounting.

In many cases where there is no thrust load and the axial location of the shaft need not be closely maintained, it is entirely practicable to bore both housings straight through without shoulders, and so machine the closure caps that the total axial movement of the bearings in the housing is from 0.015 to 0.020 in. In such a mounting the bearings may be simply pressed on the shaft.

Clamping to the housing. If there is an axial thrust or a fixed shaft location, the outer bearing race must be clamped to the housing. The simplest method of clamping is by means of a machined locating shoulder *s*, Fig. 24–8. The clamping cover must have a narrow register fitting into the housing bore. This register is very important when the clamping piece must also act as a closure about the shaft to insure a leakproof joint.

Lubrication. The main functions of ball bearing lubricants are: (a) to assist in the dissipation of heat caused by the deformation of load-carrying members; (b) to provide a lubricating film between the balls and the cage pockets; (c) to support any contact between the load-carrying members that is not pure rolling; (d) to protect the polished surfaces of the balls and races from rust and corrosion; (e) to assist the housing closure in excluding dust, dirt, and other foreign matter from the bearing.

In general, either mineral oil or grease may be used. Vegetable oils should never be used, since they turn rancid and tend to become acid. Where an economical means of supplying oil can be used and where a slight leakage past the bearing closure is not objectionable, oil is the most suitable lubricant. However, where a minimum of attention is essential, the use of grease is indicated, provided the speed is not too high.

Oil lubrication. For moderate speeds, the simplest method of lubricating a bearing of a horizontal shaft is to provide an *oil bath*, with the level reaching slightly above the center of the lowest ball. Another method is *wick-feed lubrication*. For higher speeds, to prevent churning of oil and the ensuing frictional heat generation, the oil should be supplied in small quantities, as by a *sight-feed drop oiler*, which can be adjusted to supply a drop every hour. The drop is broken up into mist by the rapidly rotating parts and thus reaches all points.

In enclosed assemblies where rotating parts require oil for their own lubrication, as in gearboxes, the ball bearings may be conveniently lubricated by the splash or mist of oil coming from these parts.

Grease lubrication. With grease lubrication, the use of closures and bearing housings of proper design reduces the loss of lubricant to a minimum. Renewal should not be required in less than six months, and the interval may be even greater.

Sealing devices. In general the devices employed for slide bearings, Fig. 23–24, may also be used for ball bearings. Because of absence of bearing wear, they all give much better results. Some manufacturers of ball bearings make bearings with closures as part of the bearing.

24–5. Thrust ball bearings. In Fig. 24–9 are shown the various types of thrust ball bearings. That in Fig. 24–9a is a one-direction thrust bearing with flat parallel seats; that in Fig. 24–9b is a self-aligning one-direction thrust bearing; and that in Fig. 24–9c is a two-direction thrust bearing with an inside shoulder.

The outside dimensions of one-direction thrust bearings have been standardized by the Society of Automotive Engineers for both flat-seat and self-aligning types, for light and medium series, with the dimensions given in inches. Two-direction thrust-bearing sizes are not standardized by the SAE. However, some manufacturers do not make any disk-race thrust

bearings at all, since angular-contact radial bearings have greater axial-thrust capacity than do thrust bearings of the types shown in Fig. 24–9.

All data necessary for the selection and mounting of thrust bearings may be obtained from catalogues of bearing manufacturers.

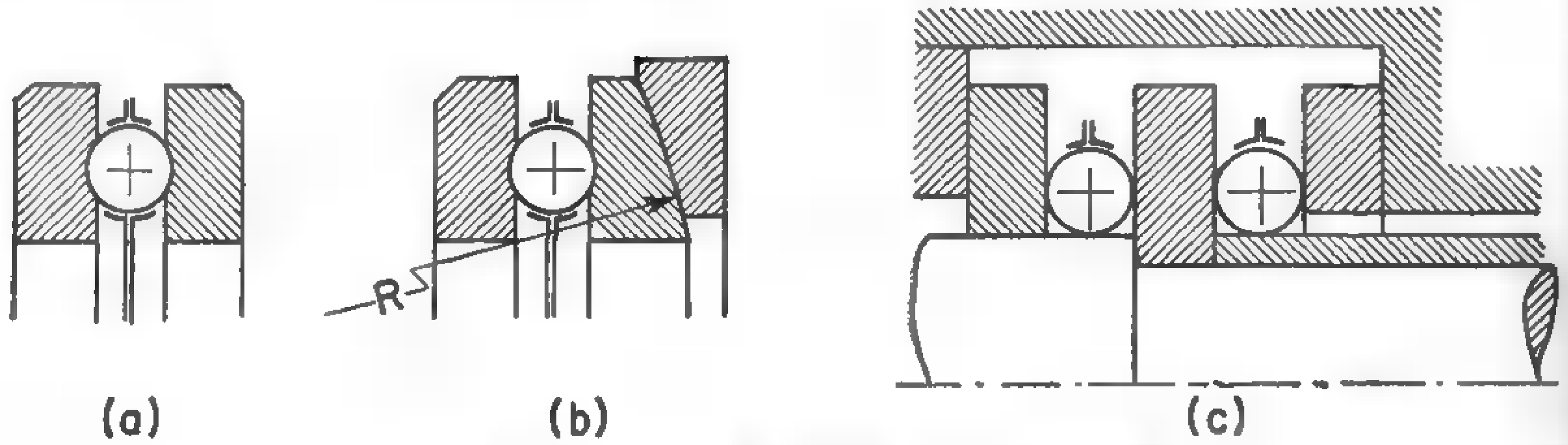


FIG. 24–9. Types of thrust ball bearings.

24–6. Roller bearings. According to the load application, roller bearings may be classified as radial, angular, and thrust. In this section, only radial roller bearings will be discussed.

The *Hyatt bearings*, Fig. 24–10a, have cylindrical hollow rollers that are wound helically from flat-strip chrome-nickel steel and are ground off on the ends to the proper length. The windings are alternately right- and left-hand. The rollers are flexible and adjust themselves to slight irregularities on the surfaces of the races. The hollow spaces within the rollers are used as containers for the lubricant, which the helical channels distribute uniformly over the bearing surfaces. In the *standard type* of Hyatt bearing the rollers are in direct contact with the shafts, the outer race being split. In the *heavy-duty type* the specific pressures are higher, and the shaft must be hardened by heat treatment at the place of contact with the rollers, or else a hardened-steel sleeve must form the bearing surface. Dimensions of these bearings and load capacities may be found in the manufacturer's catalogues or in handbooks.

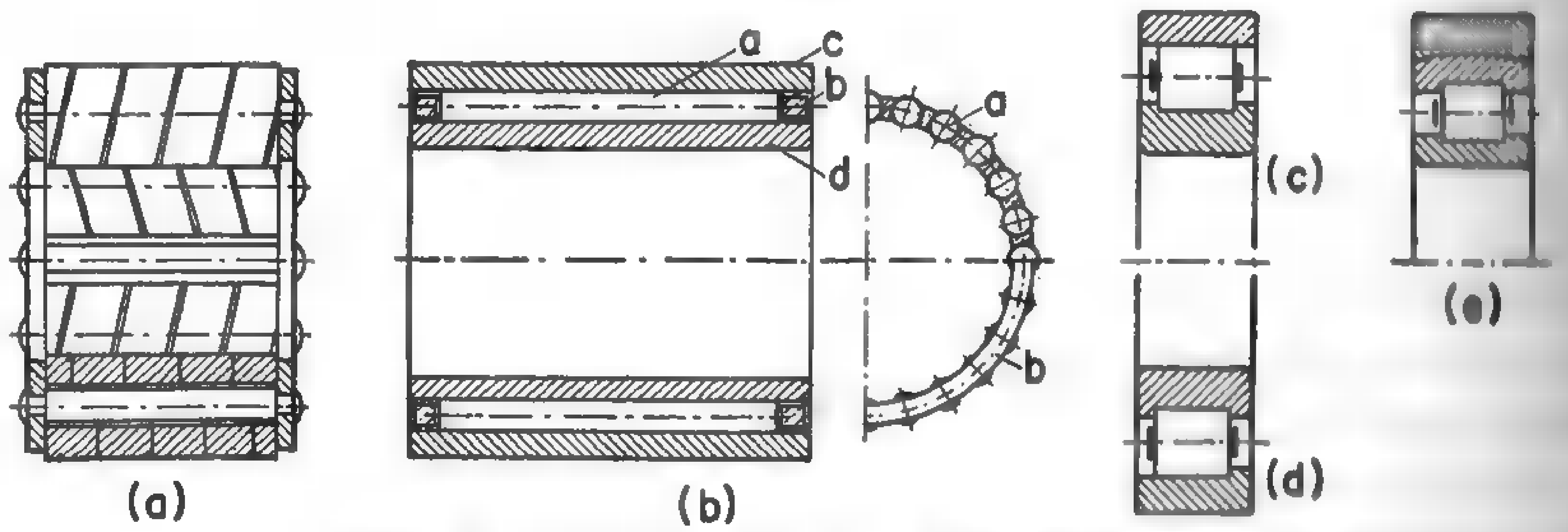


FIG. 24–10. Types of radial roller bearings.

Hyatt bearings with helical rollers are not affected by temperatures up to 350 F. At higher temperatures their load rating should be multiplied by a factor designated as k_t. It is 0.87 for 400 F, 0.73 for 450 F, and 0.60 for 500 F.

A roller bearing with long rollers is shown in Fig. 24–10b. The long rollers *a* are held in alignment by a bronze or steel cage *b*, and they roll between a solid inner and outer race *c* and *d*. The rollers and races are made of alloy steel that is hardened and ground. The inner race is sometimes omitted, and the rollers bear directly on the shaft. In this instance, however, the load capacity is lowered by the comparative softness of the shaft material.

The *Norma-Hoffman bearings* usually have short solid cylindrical rollers with lips on the inner race only, as shown in Fig. 24–10c. To carry a small one-direction thrust, there may also be lips on one side of the outer race, as in Fig. 24–10d; and for a small two-direction thrust, there may also be lips on both sides of the outer race. These bearings can be used up to 1,500 rpm for all sizes, and up to 5,000 rpm for the smaller sizes. The over-all dimensions are the same as those of the corresponding series of ball bearings, whereas the load capacities are about 100 per cent higher. In Fig. 24–10e is shown a self-aligning Hoffman bearing.

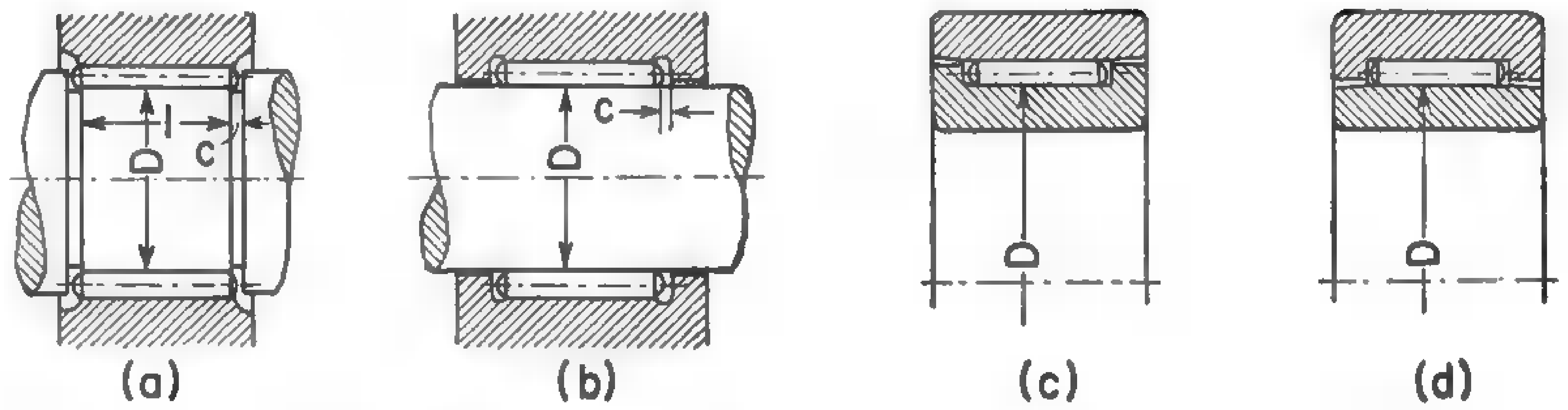


FIG. 24–11. Methods of applying needle rollers.

The *Rollway journal roller bearings* are also interchangeable with standard ball bearings. They have a load-carrying capacity from 50 to 100 per cent greater than the ball bearings.

The *SKF roller bearings* are built similar to those in Fig. 24–10e and d. They are interchangeable with corresponding ball bearings and have load capacities from 50 to 100 per cent greater than the latter.

Selection. Information necessary for the selection and application of roller bearings may be obtained from catalogues of manufacturers.

Comparison of roller and ball bearings. Compared with ball bearings, roller bearings that are interchangeable with them have the advantage of greater load capacity. Against this advantage are the following disadvantages: considerably smaller axial-thrust capacity, higher cost, and greater sensitivity to misalignment, dirt, and grit.

Needle rollers. Because of their comparatively small radial dimensions and their exceptionally high load capacity, particularly at low peripheral speeds, the use of needle bearings is rapidly increasing. Needle rollers are comparatively long rollers of hardened alloy steel of small diameter—2 to 4

TABLE 24–4

Torrington Needle-Roller Sizes, in Inches

Diameter*	Length†	Diameter*	Length†	Diameter*	Length†	Diameter*	Length†
0.0627	0.370	0.0939	0.687	0.1250	0.875	0.1875	0.745
0.0627	0.490	0.0939	0.975	0.1250	0.938	0.1875	1.000
0.0627	0.620	0.1250	0.385	0.1250	1.000	0.1875	1.187
0.0627	0.666	0.1250	0.520	0.1250	1.125	0.1875	1.375
0.0937	0.416	0.1250	0.615	0.1582	0.740	0.2182	0.752
0.0938	0.750	0.1250	0.750	0.1875	0.527	0.2500	1.250

* Tolerances +0.0000 to −0.0002 in. † Tolerances +0.000 to −0.020 in.

mm— which are used mostly without a cage. They may be used without a race, as in Fig. 24–11a and b; with inner and outer races, as in Fig. 24–11c and d; or with only an outer race.

Table 24–4 gives the stock sizes of needle rollers that can be obtained from Torrington and several other manufacturers.

The load capacity F, in pounds, is calculated on a projected-area basis. Thus,

$$F = K_h K_l p l D \tag{24–9}$$

where K_h is the hardness factor, given in Table 24–5;
K_l is the life-load factor, taken from the curve in Fig. 24–2 marked T needle;
p is the allowable pressure, in pounds per square inch;
l is the roller length, in inches;
D is the shaft or inner race diameter, in inches.

For wrist pins, rocker arms, and similar oscillating mechanisms, p may be as high as 5,000 psi. For rotary motion, p may be computed from the relation

$$p = \frac{5{,}500}{\sqrt[3]{D_1 n}} \tag{24–10}$$

where D_1 is the diameter of the revolving race, in inches, and n is its speed, in revolutions per minute.

TABLE 24–5

Hardness Factors for Needle-Roller Bearings

Rockwell C Hardness of Raceway	Approximate Brinell Hardness (Bhn)	Hardness Factor K_h	Rockwell C Hardness of Raceway	Approximate Brinell Hardness (Bhn)	Hardness Factor K_h
63	660	1.00	54	545	0.83
60	620	0.98	52	515	0.70
58	595	0.96	50	490	0.50
56	570	0.92			

TABLE 24–6

Design Data for Needle-Roller Bearings

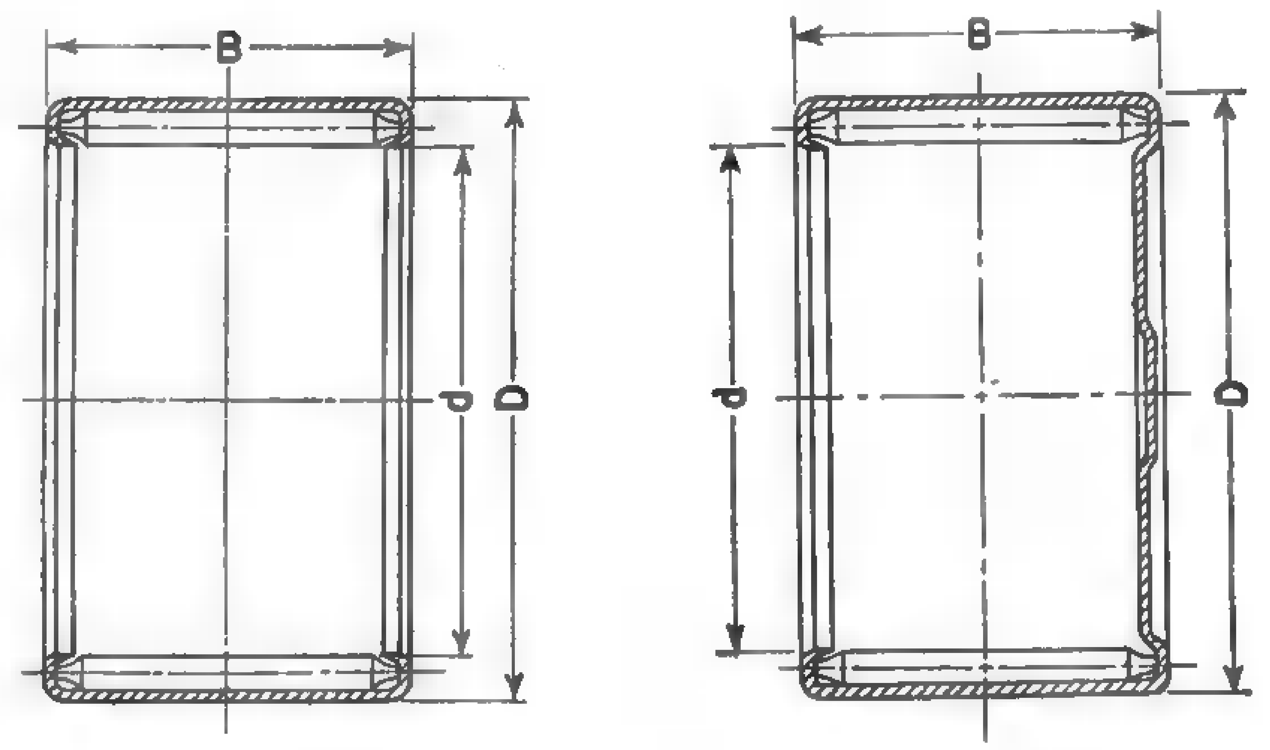
Journal or Race Diameter (in.)	Recommended: Total Radial Clearance (in.)	Recommended: Needle Diameter (in.)	Journal or Race Diameter (in.)	Recommended: Total Radial Clearance (in.)	Recommended: Needle Diameter (in.)
$\frac{3}{8}$–$\frac{3}{4}$	0.0005 to 0.0016	$\frac{1}{16}$	2–3	0.0010 to 0.0026	$\frac{1}{8}$
$\frac{3}{4}$–$1\frac{1}{4}$	0.0007 to 0.0020	$\frac{3}{32}$	3–5	0.0012 to 0.0030	$\frac{3}{16}$
$1\frac{1}{4}$–2	0.0008 to 0.0022	$\frac{1}{8}$	5–7	0.0014 to 0.0034	$\frac{3}{16}$

The total circumferential clearance can vary from 0.020 in. up to the diameter d_r of one roller, and it may be checked by the formula

$$c = \pi(D + d_r) - i d_r \tag{24–11}$$

where i is the number of needles.

Table 24–6 gives the recommended needle diameters and the total radial clearances for various diameters of the shaft or inner race.



(a) (b)

Fig. 24–12. Needle bearings.

Needle rollers are also used assembled with ground steel races as in other roller bearings. The main advantage of needle-roller bearings is their small outside diameter. In addition, needle-roller bearings are used rather extensively assembled in an outer pressed-steel race, either with both ends open, as in Fig. 24–12a, or with one end closed, Fig. 24–12b. Such bearings are available for shafts with diameters from $\frac{5}{32}$ in. to $2\frac{3}{4}$ in., and in lengths from about one-half to one shaft diameter. Dimensions and load capacities should be taken from manufacturers' catalogues.[4] The advantages of these bearings are small weight, compactness, and high radial-load capacity.

[4] The Torrington Company, Torrington, Conn.; Roller Bearing Company of America, Trenton, N. J.; Orange Roller Bearing Company, Orange N. J.; and others.

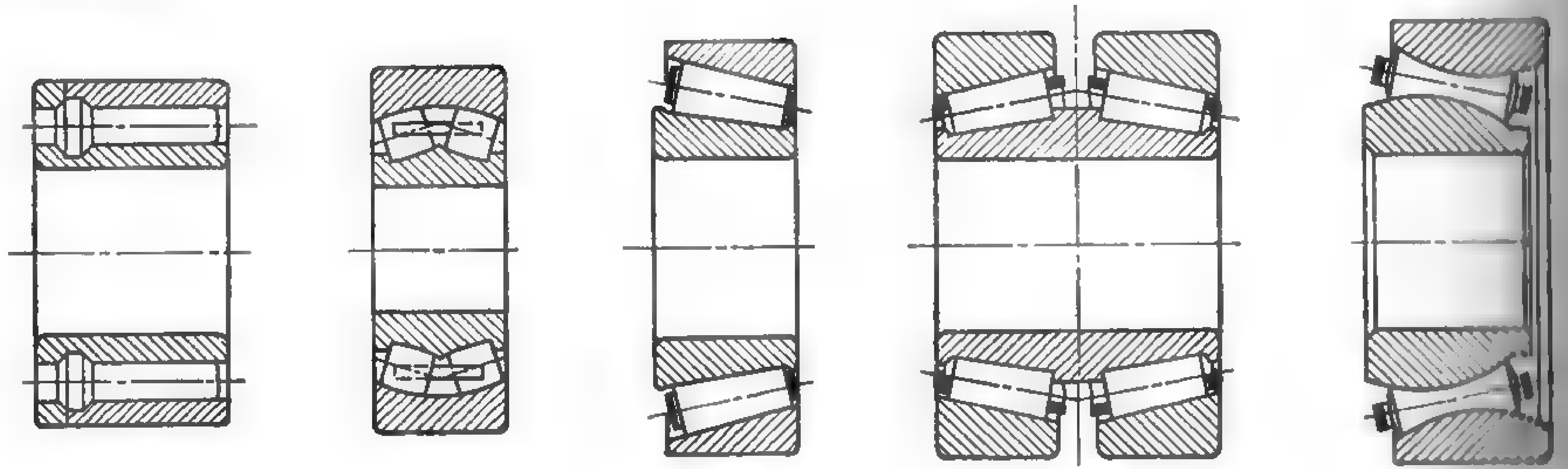
However, the coefficient of friction of needle bearings is considerably higher than that of short roller bearings. It may be six times as high.

Large rollers. For rollers of large diameters, such as those used on bridge turntables and in other places where the speeds are very low, the safe load is

$$F = Cild_r \tag{24-12}$$

where C is an empirical coefficient. The value of C is 360 for hard cast-iron rollers running on hard cast-iron tracks, and 850 for hard-steel rollers on hard-steel tracks.[5] For very large hardened steel rollers with $d_r > 10$ in., on hardened-steel tracks, $C = 8{,}000/d_r + 50$. If $l > 5d_r$, smaller values of C should be used.[6]

24-7. Thrust roller bearings. *Bower roller bearings,* in Fig. 24–13a, are designed to take both radial loads and thrust loads. The cylindrical rollers take the radial load, and the flanged roller heads take the thrust load, the total axial load being divided among all the rollers. The maximum thrust load is equal to one-third the radial rating capacity. Bower roller bearings are made interchangeable with single-row and double-row ball bearings.



(a) (b) (c) (d) (e)

Fig. 24–13. Types of thrust roller bearings.

SKF roller bearings, Fig. 24–13b, have a double row of barrel-shaped rollers. The rollers run in a spherical outer race, which makes the bearing self-aligning. These bearings are built for shaft diameters from 80 to 300 mm in light and medium series, and they are interchangeable with corresponding ball bearings. Their load capacities are from 50 to 100 per cent greater than those of ball bearings.

Timken roller bearings, Fig. 24–13c, have taper rollers whose races are frustrums of cones with their apexes coinciding on the axis of the bearing. Taper roller bearings should be used in pairs, each bearing supporting the end thrust in one direction. Axial adjustment is usually provided for in the

[5] Lionel S. Marks, ed., *Mechanical Engineers' Handbook*, 3d ed. (New York: McGraw-Hill Book Company, Inc., 1930), p. 1004.
[6] W. M. Wilson, *The Bearing Value of Rollers*, Bulletin No. 263, University of Illinois Engineering Experiment Station (1934), p. 4.

cone, or inner race. The axial-thrust capacity depends on the thickness of the cup, or outer race, and is approximately equal to the radial capacity.

Timken bearings are also made with two rows of rollers, as shown in Fig. 24–13d. The bearing may have a double cup and standard cones, or a double cone and standard cups. In either case the cones may be turned either toward or away from each other. This latter arrangement provides ease of assembling and is used in roller pillow blocks.

Timken bearings are made either with a straight bore, both in millimeter sizes and in inch sizes, or with a conical bore to be used with a split adapter, such as that shown in Fig. 24–7a, for use with ball bearings.

Shafer roller bearings, Fig. 24–13e, have concave rollers set at an angle, a convex cone which is a segment of a sphere, and a cup that is machined to fit the curved rollers. This construction allows the cone to oscillate in all directions, making it self-aligning. These bearings are suitable for heavy radial and thrust loads at moderate speeds.

24-8. Roller bearing application. An operating temperature up to 350 F does not affect the load capacity of roller bearings. The higher temperatures affect mainly the problem of lubrication. Up to 180 F either oil or special greases may be used. Above 180 F greases are not recommended; only mineral oils with a viscosity of at least 100 SSU at 210 F should be used.

Standardization. Roller bearings are made in standardized series. The SAE standards include light and medium series corresponding in outside dimensions to the same series in the wide-type radial bearings. Load ratings vary considerably for different makes, as do the sizes and numbers of rollers, and therefore data must be taken from catalogues of manufacturers.

Design of mountings. The general rules for mounting of ball bearings also apply to roller bearings. The revolving race must be a tight fit on the shaft and securely fastened in place; the stationary race should be a snug fit on its seat but need not have lateral play. The rollers must run in a bath of lubricant contained within the housing and protected from leakage and contamination.

CHAPTER 25

Crankshafts

25–1. General considerations. A shaft with a crank is used to transform a reciprocating motion into a rotary one, or vice versa. Regardless of its type and shape, a crankshaft may be considered as a beam with two or more supports. In addition to bending stresses, there are also shear stresses due to the torsional moment on the shaft. Both the bending moment and the torsional moment are caused by the main forces acting upon the crosshead connected to the crank, and by intertia and centrifugal forces of the moving parts. In addition, there may exist bending moments caused by the weight of the flywheel, the pull of the belt, or the weight of the rotor of an electrical generator.

Types. There are two main crankshaft types. The shaft may have a side crank, or overhung crank, as in Fig. 25–1a, or it may have a center crank, as in Fig. 25–1b. A shaft is often made with two side cranks, one on each end, or with two or more center cranks.

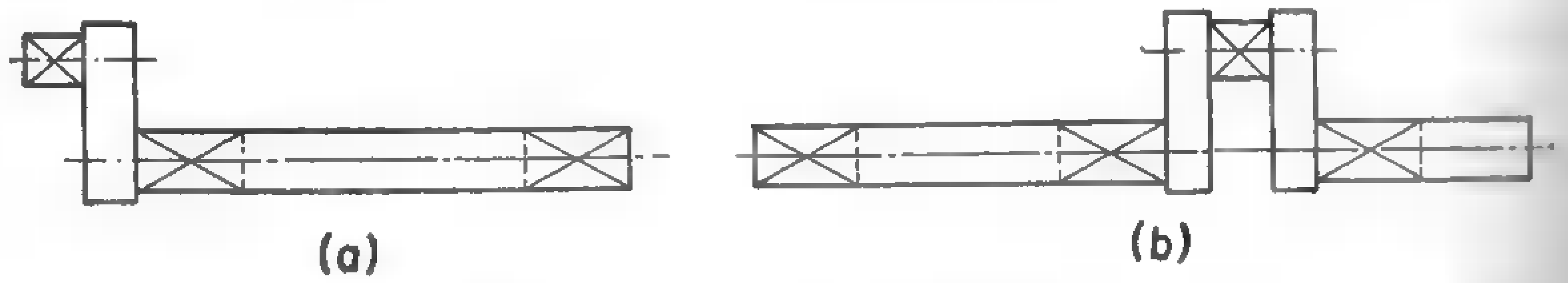


FIG. 25–1. Types of crankshafts.

Materials. One-piece crankshafts are usually forged of open-hearth steel, similar to SAE 1030, with an elastic limit in tension of 37,500 psi and an elongation of 25 per cent in 2 in. Crankshafts of marine engines are forged of steel with a lower carbon content, such as SAE 1020 to SAE 1025. Automobile and airplane-engine crankshafts are made of chrome-nickel steel, such as SAE 3140 or SAE 3240, heat-treated to obtain an elastic limit of 95,000 to 120,000 psi. Automobile crankshafts have of late been cast of a special iron alloy and iron-copper alloy, No. 6 in Table 4–1.

Built-up crankshafts have crankpins made of steel with a carbon content of 0.45 to 0.55 per cent, and the cranks are made either of cast steel or of a nickel cast iron.

Stresses. Since the failure of a crankshaft is likely to cause a serious engine wreck, and neither all acting forces nor all stresses can be determined accurately, a high safety factor, from 3 to 4, should be used. Stress concentration due to various discontinuities must be taken into account. Where

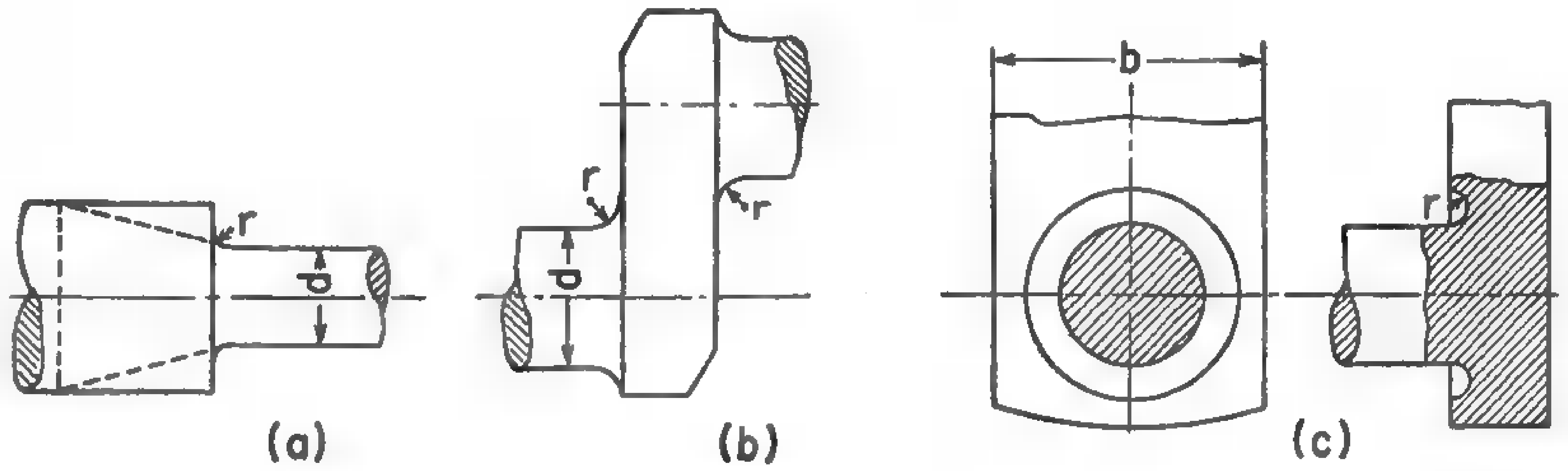


FIG. 25–2. Fillets at junctures of different sections.

possible, their effect should be reduced to a minimum. Radial holes and keyseats with sharp corners should be avoided; and if two different cross sections must be joined, they should be blended with a large fillet r, as in Fig. 25–2a or b. If possible, the radius r should be not less than $0.2d$. A gradual change of section, as shown by dotted lines in Fig. 25–2a, is still better. If there is no room for a regular fillet, it is advisable to undercut the cheek in order to obtain the fillet r, Fig. 25–2c. A weakening of the cheek is avoided by increasing its width b. Such undercut fillets in a built-up crankshaft of a large gas engine are shown in Fig. 25–3.[1]

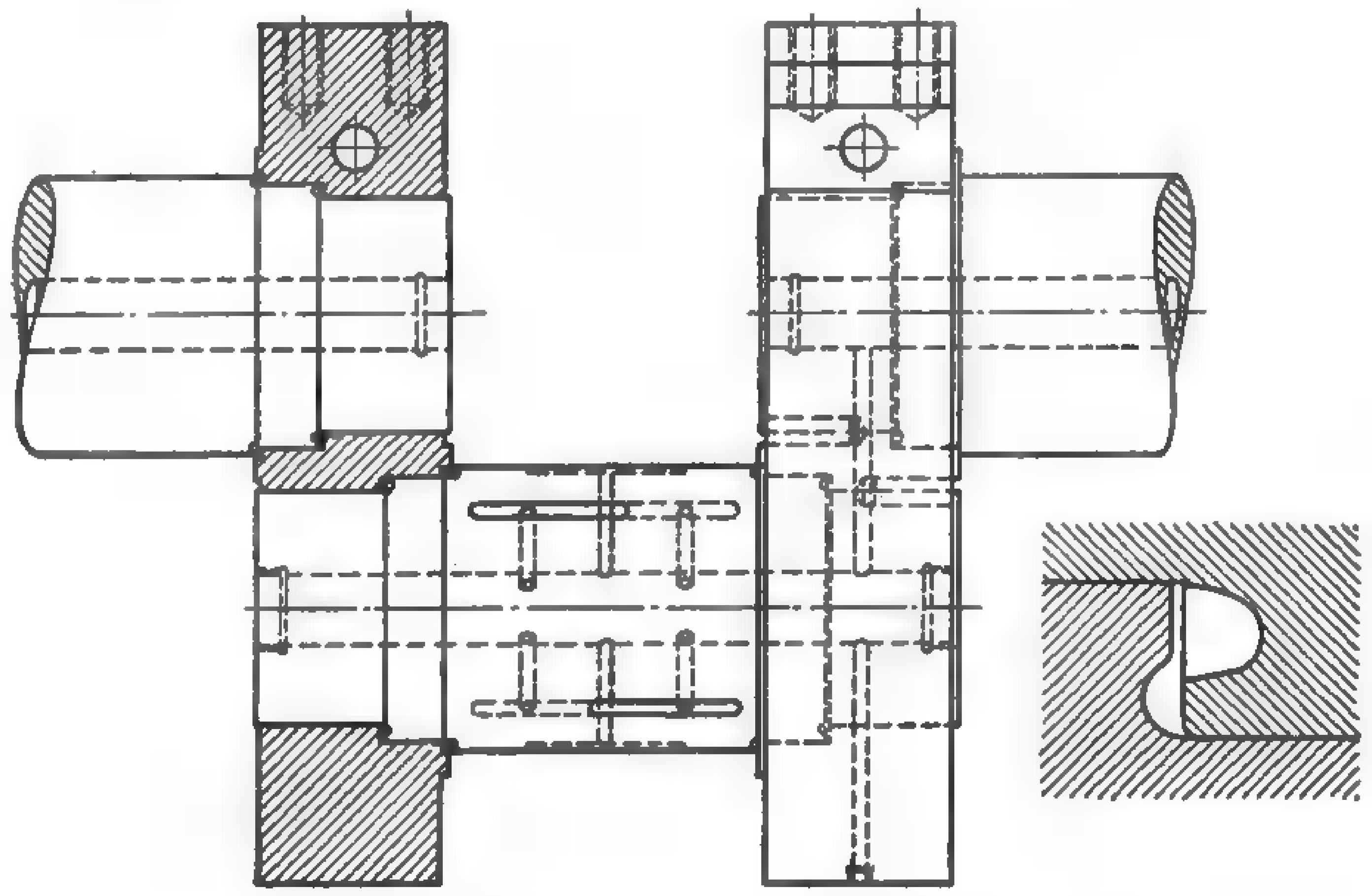


FIG. 25–3. Built-up crankshaft.

Repeated stresses. Most crankshaft failures are caused by a progressive fracture due to repeated bending or reversed torsional stresses. Reversed bending stresses exceeding the endurance limit may be produced in a crankshaft by one or more bearings being lower or higher than the rest. Dangerous torsional stresses may be produced in multithrow crankshafts because of torsional vibration at or near critical speeds. In single-throw or double-throw engine crankshafts these stresses may occur from counterweights set into the rim of the flywheel instead of being fastened to the crank cheeks.

[1] H. Dubbel, *Oel und Gas Maschinen* (Berlin: Julius Springer, 1926), p. 371.

In a revolving shaft the bending stress in the most-stressed fibers changes from compression to tension, and vice versa. The maximum compressive stress in general differs from the maximum tensile stress. However, to be on the safe side, it is advisable to use the endurance limit S_{en}, which corresponds to a complete reversal of the stresses and a mean stress $S_m = 0$. The same rule applies to the torsional stress.

EXAMPLE 25-1. Select the material and determine the allowable stresses in bending and torsion for a crankshaft of a medium-size gas compressor.

SAE 1025 steel is commonly used for similar applications, because it is relatively inexpensive and is easily forged and machined.

Since in a gas compressor the change of the load on the crankshaft is rather gradual, a safety factor n of 3 is sufficient. The endurance diagram in Fig. 4-3 gives $S_{en} = 27{,}500$ psi, and the allowable nominal stress is

$$S_d = \frac{S_{en}}{n} = \frac{27{,}500}{3} = 9.160 \text{ psi}$$

In torsion

$$S_{ds} = \frac{16{,}000}{3} = 5{,}330 \text{ psi}$$

The actual allowable stresses must be smaller because of the size influence, as given by equations 5-7 and 5-8.

25-2. Design procedure. The first step in the design of a crankshaft is to determine the magnitudes of the various loads on the crankshaft. The next step is to determine the distances between the supports and their positions with respect to the loads. For the sake of simplicity and also for safety, the shaft is considered supported at the centers of the bearings, and all forces and reactions are assumed to be acting at these points. The distances between the supports depend on the lengths of the bearings, which in turn depend on the diameter of the shaft because of the allowable bearing pressures. Therefore certain maximum bearing pressures p and the length-to-diameter ratios l/d are selected from Table 23-4, and these dimensions l and d are determined by considering the acting loads. Values thus obtained are checked by computing the bearing characteristic number Zn/p, as explained in section 23-4. If necessary, dimensions are changed to insure better operation. After this the thickness h of the cheeks, or webs, is selected, being made about $0.4d$ to $0.6d$. With these values determined, the distances between the supports are found; and after allowable bending and shear stresses are assumed, the main dimensions of the crankshaft are fixed.

When computing the bending moments at various cross sections of a crankshaft, two methods may be used. One method is to find, at the supports, the net reactions due to all loads, and then to determine the moments at the various points. Another method is to compute the reactions and the bending moments due to each load separately, and then to combine all moments either graphically or analytically. After all shaft dimensions have been determined, the shaft should be checked for rigidity by adopting a procedure similar to that outlined in section 20-2.

25-3. Shaft with side crank. This shaft type is used for medium-size and large horizontal engines. Its main advantage is that it requires only two bearings, in either the single-crank or two-crank construction. Two bearings present less danger of misalignment, which causes most shaft failures, than do three or more supports.

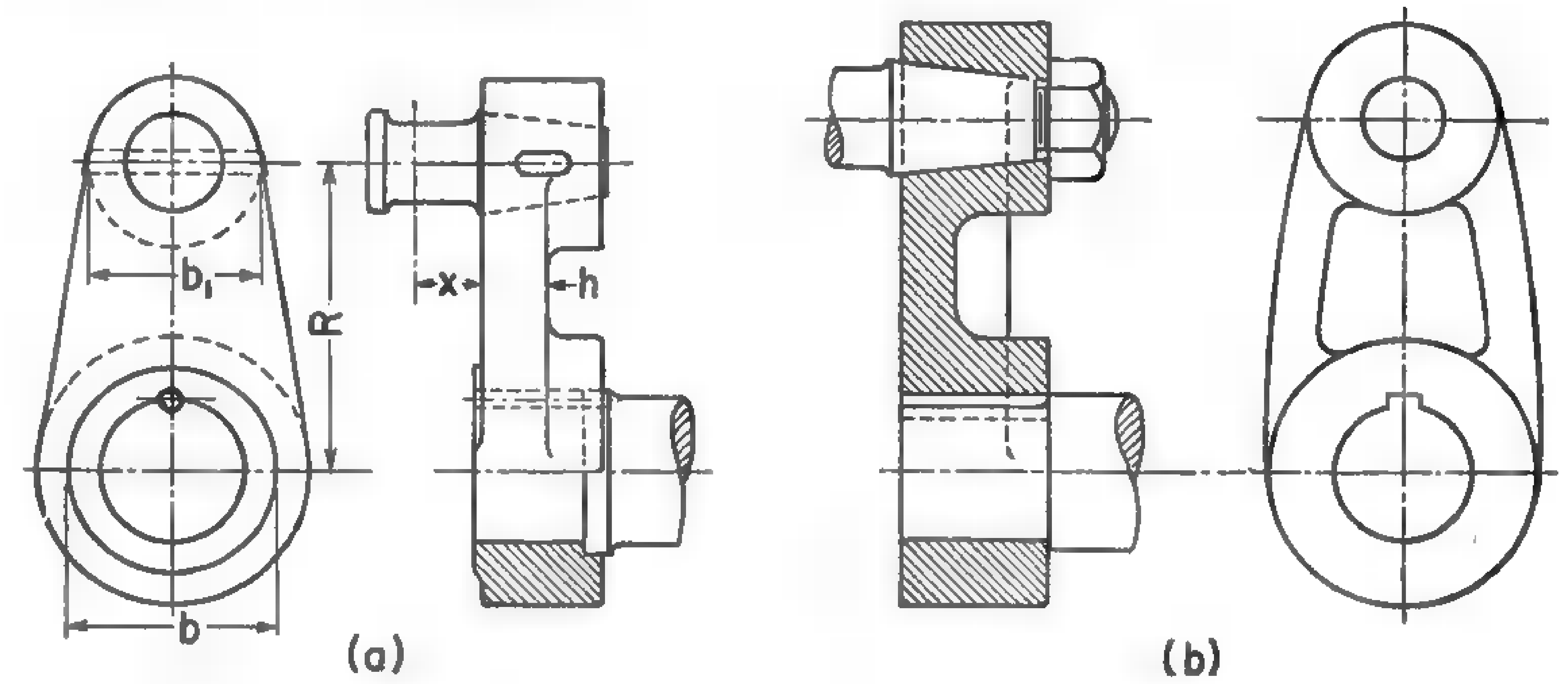


FIG. 25-4. Side cranks for built-up crankshafts.

Construction. Cranks for pumps, compressors, and other machines are either steel forgings, as in Fig. 25-4a, or steel castings, as in Fig. 25-4b or Fig. 25-5. Only in small, cheap machinery is the crank made of cast iron. The crankpin has a cone seat and is held by a cotter, as in Fig. 25-4a, or by

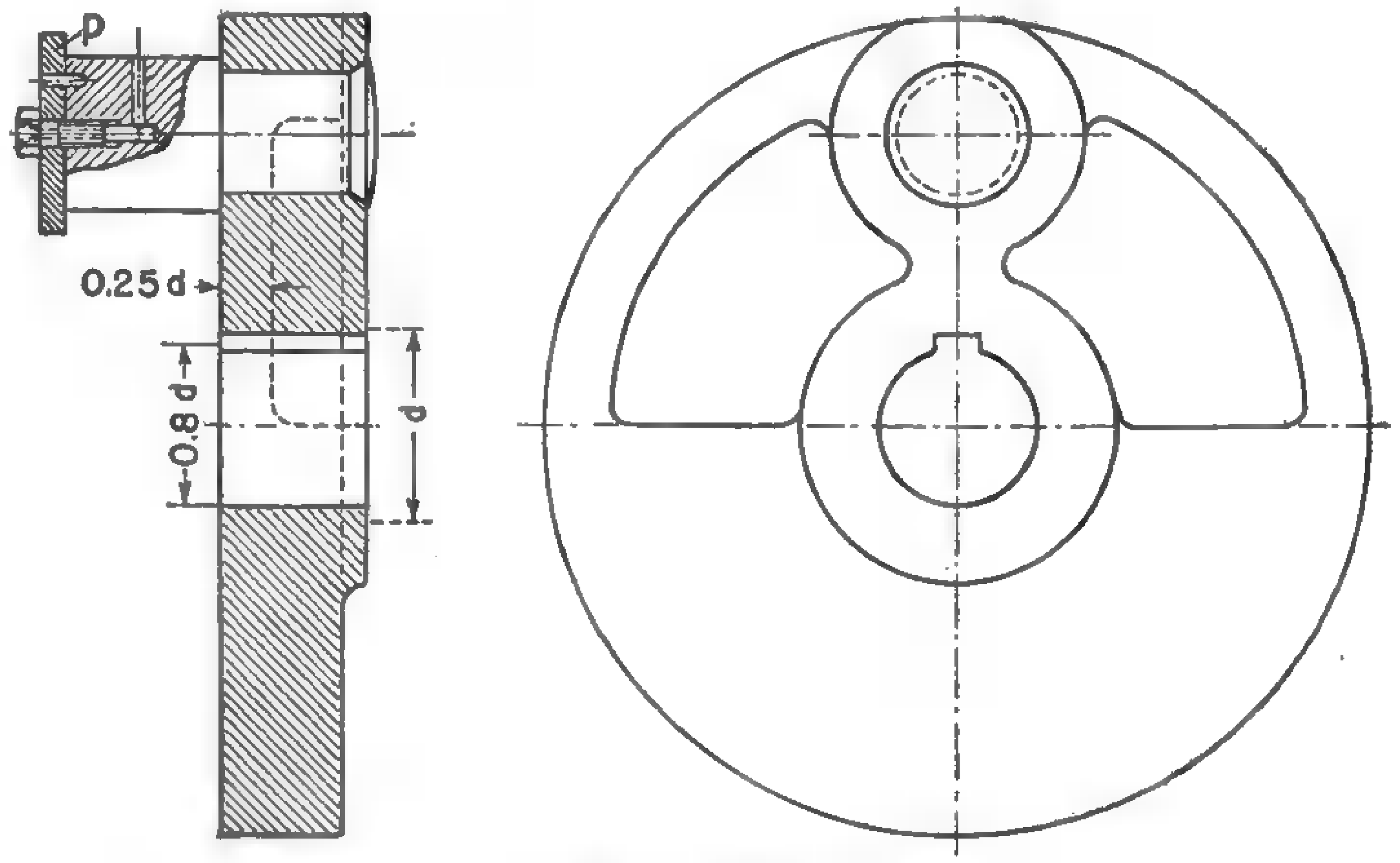


FIG. 25-5. Disk crank.

a nut, as in Fig. 25-4b. If the crankpin is not expected to wear much, it may be pressed in or shrunk in, and the end may be riveted over, as in Fig. 25-5. A pin with a collar, Fig. 25-4a, requires a connecting rod with an open end and a cap. A straight pin with a collar plate p, Fig. 25-5, takes a connecting rod with a closed end. Sometimes the crank and pin are a one-

piece steel casting. The drawback of this construction is that it requires an expensive steel for better wear of the crankpin.

The crank-to-shaft connection is a press or shrink fit with a rectangular or round key, Fig. 25–4a or b.

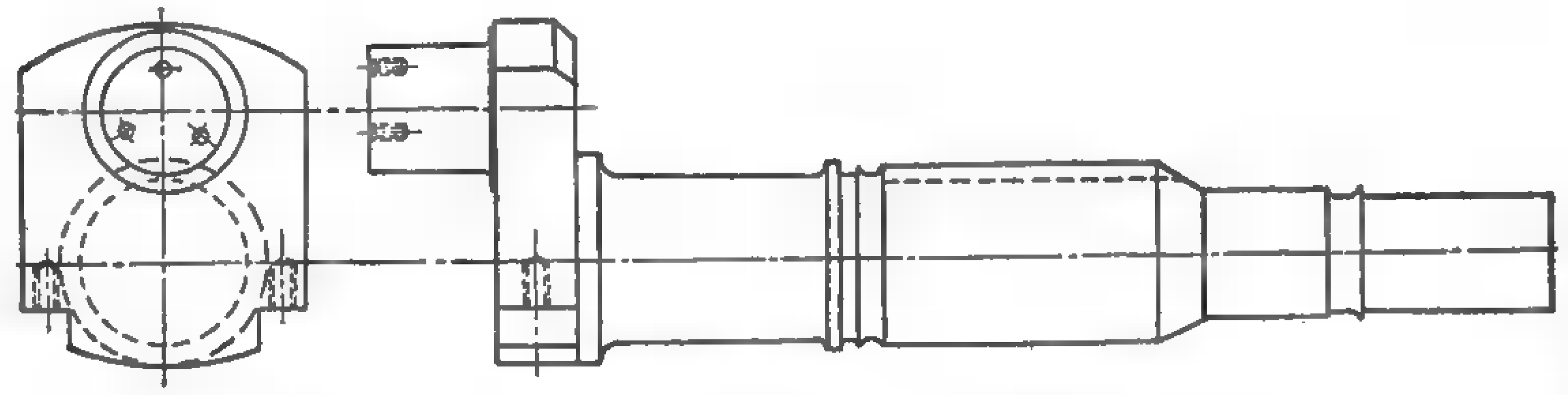


FIG. 25–6. One-piece crankshaft with side crank.

Smaller steam and internal-combustion engines usually have a one-piece forged crankshaft with a cast-iron counterweight bolted to the crank, as shown in Fig. 25–6. In larger engines the shaft, crank, and pin are made separate, as in Fig. 25–7, and are forced together with a hydraulic press. The crank and counterweight are usually a single steel casting. The key K is merely a safety device, as the press fit must be sufficient to transmit the maximum torque. The usual main proportions of a separate crank are indicated in Fig. 25–7.

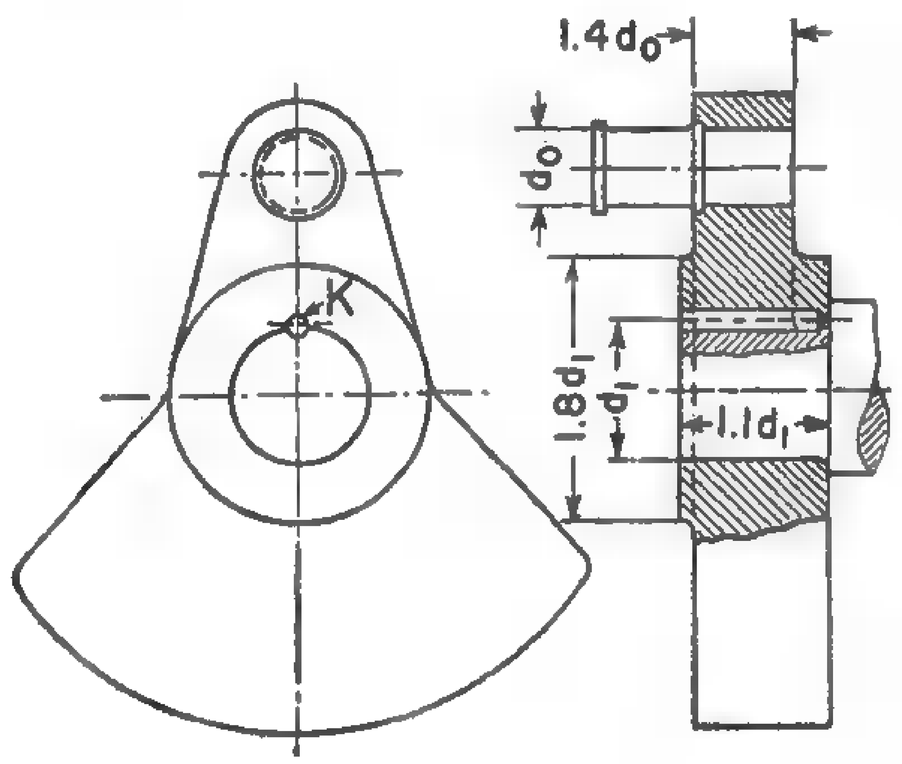


FIG. 25–7. Three-piece crankshaft with side crank.

Design calculations. Every crankshaft must be designed or checked for at least two crank positions—one when the bending moment is a maximum, and the other when the torsional moment is a maximum. In computing the bending moments, the crankshaft can be considered to be a straight shaft, even for close, long cranks and short pins.[2] In both positions the additional moments due to the flywheel weight, belt tension, and other forces must be considered. If a new crankshaft is designed, the first step is to determine the approximate values of the diameters and lengths of the crankpin and the journals.

25–4. Shaft with center cranks. The general procedure for the design of a shaft with center cranks is similar to that for a shaft with side cranks. To make the computations simpler without seriously impairing accuracy, it is customary to assume that the influence of any bending force does not extend beyond the two bearings between which that force is applied.

[2] F. Rötscher, "Die Ermittlung der Spannungsverteilung in Konstruktionsteilen durch Dehnungsmessungen," *Zeitschrift Verein Deutscher Ingenieure*, Vol. 77 (1933), p. 378.

TABLE 25–1

COEFFICIENT a IN THE AMERICAN BUREAU OF SHIPPING FORMULA

Type	Number of Cylinders		Ratio of Stroke to Distance over Crank Webs $= L/c$							
	Four-Stroke	Two-Stroke	0.7	0.8	0.9	1.0	1.1	1.2	1.3	1.4
Explosion engines	1, 2, 4	1, 2	1.17	1.17	1.17	1.17	1.17	1.17	1.17	1.17
	3, 5, 6	3	1.17	1.17	1.17	1.17	1.19	1.20	1.22	1.24
	8	4	1.17	1.19	1.21	1.23	1.25	1.28	1.30	1.32
	10, 11, 12	5, 6	1.18	1.20	1.23	1.25	1.28	1.31	1.33	1.35
Air-injection diesel engines	1, 2, 4	1, 2	1.17	1.19	1.22	1.25	1.28	1.31	1.34	1.36
	3, 5, 6	3	1.19	1.22	1.25	1.28	1.32	1.35	1.38	1.41
	8	4	1.20	1.24	1.27	1.30	1.33	1.37	1.40	1.43
	12	5, 6	1.22	1.25	1.29	1.32	1.36	1.39	1.42	1.45
	16	8	1.25	1.29	1.33	1.36	1.40	1.44	1.47	1.50

The shaft of a stationary engine with a center crank is usually a one-piece forging. With progress in the metallurgy of alloy cast irons and steels, cast crankshafts have begun to be used, even in larger machines. Built-up crankshafts with cast cranks similar to those in Fig. 25–4b and Fig. 25–5, with shrunk-in steel shaft ends and steel crankpins, are used for small pumps and compressors.

25–5. Multithrow crankshafts. To determine the stresses in a multithrow crankshaft, it is necessary to make certain assumptions in regard to the shaft axis, and the analysis is rather laborious if it is theoretically accurate. In these computations it is assumed that the crankshaft is correctly aligned and its axis is a straight line. When the engine begins to operate, one or several of the bearings will wear faster than the others; the crankshaft center line will sag at these bearings; and the bending stresses will be increased greatly, but indefinably, because the deflections are not known. Similar conditions arise if the bedplate or crankcase is not rigid. Thus the theoretically correct procedure does not insure the safety of the shaft against progressive fracture.

For this reason actual shaft dimensions are often based on experience, and the only calculations involve simple proportion. Another common method is to determine the dimensions of the crankpin, the crank webs, and the journals, as for a single-throw shaft, and to disregard the influence of the other throws except for the torsional moments. The use of a comparatively high factor of safety takes care of the simplifying assumption.

Empirical formulas. Large insurance companies have set up certain empirical formulas pertaining to the main dimensions of crankshafts of marine oil engines. Shafts complying with these rules give very satisfactory service, possibly because these rules give a stronger shaft than do usual com-

putations. These formulas differ one from another, but all give practically the same dimensions. Therefore only one set, that of the American Bureau of Shipping, will be given here.

The diameter d of the crankpins and journals is found from the relation

$$d = a\sqrt[3]{\frac{D^2pc}{S_d}} \tag{25–1}$$

where a is a coefficient the values of which are given in Table 25–1;
D is the cylinder bore, in inches;
p is the maximum gas pressure, in pounds per square inch;
c is the distance over the crank web plus 1 in., shown in Fig. 25–8;
S_d is the allowable fiber stress, in pounds per square inch.

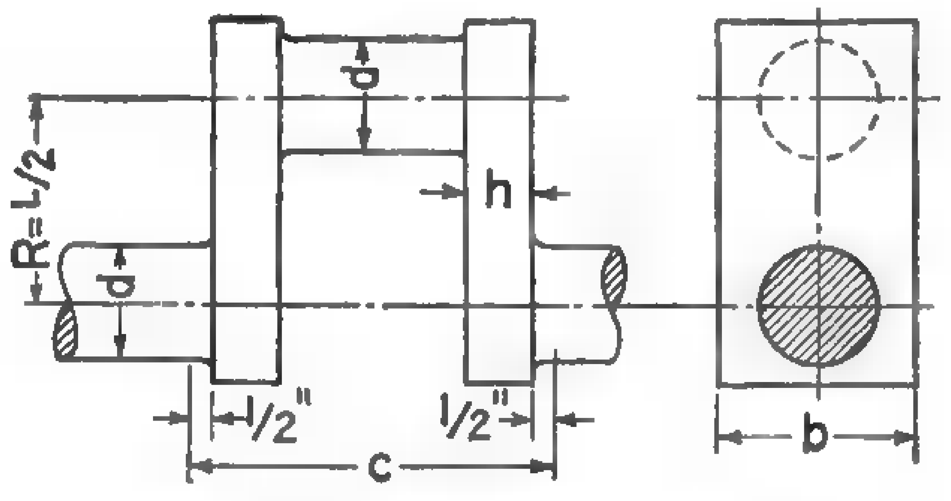


Fig. 25–8. American Bureau of Shipping method.

The value of S_d may be taken as 7,000 psi for cast steel, 7,500 psi for open-hearth steel, and 8,000 psi for the best grade of forged steel. In Table 25–1, L is the engine stroke, in inches.

The thickness h and the width b of the crank cheeks, also called *webs*, can be selected in the same way as indicated for a single-throw shaft, but they must satisfy the following conditions:

$$bh^2 \geqq 0.4d^3 \tag{25–2}$$

and

$$b^2h \geqq d^3 \tag{25–3}$$

The relations given here were set up for marine internal-combustion engines. However, they can be used satisfactorily for preliminary calculations of crankshaft dimensions for other reciprocating machinery, such as compressors or pumps.

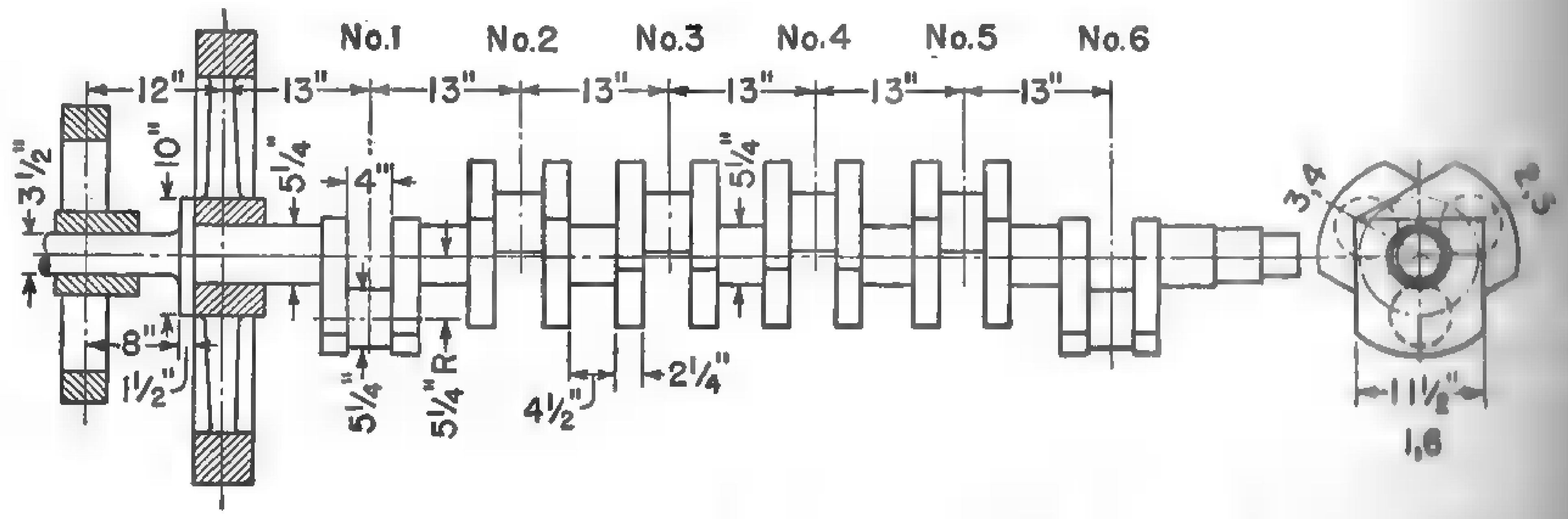


Fig. 25–9. Crankshaft for six-cylinder oil engine.

Constructions. In Fig. 25–9 is shown a drawing of a six-throw crankshaft for an $8\frac{3}{4} \times 10\frac{1}{2}$-in. oil engine. The shaft is drilled for delivering pressure lubrication to the crankpins and wrist pins. All junctions of different sec-

tion have fillets of $\frac{5}{8}$-in. radius, which according to Figs. 3–17 and 3–31 still leave high stress concentration. One of the journals, that between cylinders No. 3 and 4, locates the shaft lengthwise by means of an accurately machined distance between the cheeks. The other journals are made slightly longer in order that, with all bearing shells of the same length, free expansion of the shaft may be permitted.

Multithrow crankshafts of large engines are built up of several pieces, either shrunk together, as in Fig. 25–3, or bolted together by means of flanges.

25–6. Torsional vibration. In general a crankshaft is made up of parts having different cross sections. The computations are simplified if the shaft is replaced by a shaft with a constant cross section but the same frequency of vibration. Such a shaft is called an *equivalent shaft.*

Equivalent shaft. From equations 2–14 and 2–16, the angle of torsion of a solid round shaft is directly proportional to the length l and inversely proportional to the fourth power of the diameter d, or d^4. Thus a portion of a shaft of length l and diameter d can be replaced by a portion of length l_o and diameter d_o, where

$$l_o = l\left(\frac{d_o}{d}\right)^4 \tag{25–4}$$

The length h_o equivalent to each crank web may be found by the expression

$$h_o = \frac{RC}{B} \tag{25–5}$$

where R is the crank radius, as shown in Fig. 25–8;
$C = \frac{1}{32}\pi d_o^4 G$ is the torsional rigidity of the crankpin;
$B = \frac{1}{12}hb^3E$ is the flexural rigidity of the web.

Equations 25–4 and 25–5 give a correct equivalent length if the clearances in the bearings are such that free angular and axial displacements of the journals during twist are possible. Small bearing clearances and axial constraints increase the rigidity of the shaft and give a shorter equivalent shaft. However, the computations for a constrained crankshaft are rather involved.[3] Since the bearing clearances are variable because of wear, and their numerical influence upon the total length of the equivalent shaft is small, this influence may be neglected.

After the equivalent length of the whole shaft is found, equations 5–45 to 5–49 can be applied. However, it should be remembered that they give only approximate values.

[3] S. Timoshenko, *Vibration Problems in Engineering* (New York: D. Van Nostrand Company, Inc., 1928), p. 157; also Lionel S. Marks, ed., *Mechanical Engineers' Handbook*, 5th ed. (New York: McGraw-Hill Book Company, Inc., 1951), p. 494.

EXAMPLE 25–2. Analyze the crankshaft of a six-cylinder mechanical-injection oil engine, Fig. 25–9, in regard to danger from torsional vibration. Additional data are:

Engine speed	1,200 rpm
Weight of revolving parts for each crank	150 lb
Weight of reciprocating parts for each crank	80 lb
Weight of flywheel	420 lb
Weight of generator rotor	180 lb
Radius of gyration of cranks	$5\frac{1}{4}$ in.
Radius of gyration of flywheel	18 in.
Radius of gyration of rotor	12 in.

The journals and the crankpins have the same diameter. The equivalent length of a crank web reduced to a straight shaft $5\frac{1}{4}$ in. in diameter, as found by equation 25–5, is

$$h_o = \frac{5.25\pi \times 5.25^4 \times 11{,}500{,}000 \times 12}{2.25 \times 11.5^3 \times 30{,}000{,}000 \times 32} = 0.527 \text{ in.}$$

The equivalent length of the part of the extension shaft 8 in. long and $3\frac{1}{2}$ in. in diameter is, by equation 25–4,

$$l_o = 8 \times \left(\frac{5.25}{3.5}\right)^4 = 40.5 \text{ in.}$$

By equation 25–4, the equivalent length of the flywheel flange, for which $D = 10$ in. and $l = 1\frac{1}{2}$ in., is

$$l_o' = 1.5 \times \left(\frac{5.25}{10}\right)^4 = 0.114 \text{ in.}$$

If it is considered that one-half the reciprocating mass of a crank is added to the revolving mass, the polar moments of inertia, or flywheel effects, Wk_o^2 are as follows:

Cranks: $(150+0.5\times 80)\times 5.25^2 = 5{,}230$ lb-in.^{2}
Flywheel: $420 \times 18^2 = 136{,}000$ lb-in.^{2}
Rotor: $180 \times 12^2 = 25{,}900$ lb-in.^{2}

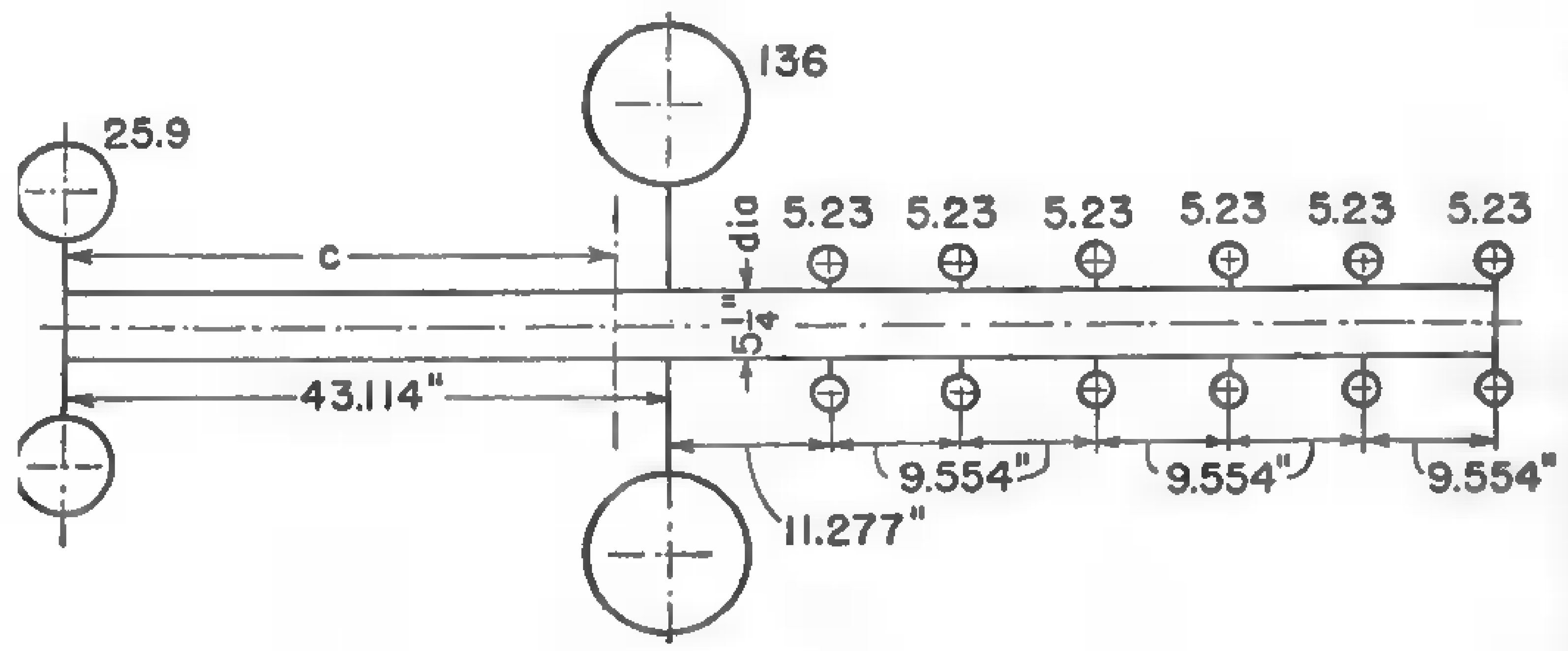


FIG. 25–10. Diagram of equivalent shaft.

The equivalent lengths of portions of a straight $5\frac{1}{4}$-in. shaft with flywheel effects, in thousands of pound-inches square, are shown in Fig. 25–10. If the distance from the end to the node is designated by c, its value can be found by proceeding as explained for equation 5–1. Taking the sum of the products of the flywheel effects and the corresponding distances from the left end over the sum of the flywheel effects, the result is

$$c = \frac{25.9\times 0 + 136 \times 43.114 + 5.23[(43.114+11.277)\times 6 + 9.554 \times 15]}{25.9 + 136 + 6 \times 5.23} = 43.04 \text{ in.}$$

The node is between the rotor and the flywheel.

The natural frequency can be found from equation 5–49. For the sake of simplicity, the part of the shaft to the left of the node will be considered. For this part,

$$J = \tfrac{1}{32}\pi d^4 = 0.0981 \times 5.25^4 = 74.6 \text{ in.}^4$$

and

$$\Sigma(Wk_o^2 l) = 25{,}900 \times 43.04 = 1{,}119{,}000 \text{ lb-in.}^3$$

Hence,

$$f = 3.125\sqrt{\frac{11{,}500{,}000 \times 74.6}{1{,}119{,}000}} = 86.8 \text{ vibr per sec}$$

A six-cylinder four-stroke engine has three firing events per revolution. Therefore the main critical speed will be $\frac{1}{3} \times 86.8 \times 60 = 1{,}736$ rpm, which is 44.6 per cent above the normal speed of 1,200 rpm. This is usually considered to be a safe margin. However, a reliable evaluation of the safety of such a system can be made only if the harmonic components of the engine torque are known.[4]

Elimination of resonance. To obtain a greater difference between the critical speed of a shaft and the engine speed, the natural frequency of the revolving shaft must be increased. As indicated by equation 5–49, the desired increase can be attained by one or more of the following methods:

a. Increasing the polar moment of inertia of the shaft
b. Decreasing the flywheel effects, Wk_o^2, of the rotating masses
c. Decreasing the length of the shaft

The first method means an increase of the shaft diameter. This method can be combined with that of making the shaft hollow. For a hollow shaft with an outside diameter d_1 and an inside diameter d_2, the polar moment of inertia is

$$J = 0.0981(d_1^4 - d_2^4) \qquad (25\text{–}6)$$

It is evident that removing even a considerable part at the center has little effect on J. Thus a hollow shaft with $d_2 = 0.31d_1$ has a moment of inertia only 1 per cent smaller, but a weight 10 per cent smaller, than a solid shaft of the same outside diameter.

The second method means that with a long shaft the flywheel should be made only heavy enough to obtain the required uniformity of rotation, and all other rotating masses should be made as light as possible.

The third method can be applied to a crankshaft connected to a large extension shaft. By inserting a flexible coupling between the crankshaft and the driven shaft, the length of each is decreased and the natural frequency is raised.

[4] Applying the accurate Holzer method, it will be found that $c = 40.1$ in., and the corresponding value of f is 89.8 vibr per sec. Thus the approximate method is in error by only 3.3 per cent.

CHAPTER 26

Flywheels

26–1. Flywheel action. The purpose of a flywheel is to keep the speed of a machine between given limits while the machine is doing work or receiving energy at a variable rate. A flywheel stores up energy when energy is supplied more rapidly than it is used, and it gives out energy when the reverse is the case. While a flywheel is storing energy, its speed is increasing; while it is giving out energy, its speed is decreasing. The allowable amount of variation of speed in any particular problem depends on the conditions of the problem.

In some machines, such as shears, presses, or punches, the work is done during a small part of the cycle. If such a machine is driven directly by an electric motor or a belt, the latter must be powerful enough to supply all the energy consumed by the machine during the working portion of the cycle, and will run idle during the remaining part of the cycle. By inserting a flywheel between the driving device and the driven machine, a much smaller motor or belt may be used to supply energy at a practically constant rate throughout the cycle.

Fywheel effect. The kinetic energy in the rim of a rotating flywheel is

$$K = \frac{Wv^2}{2g} \tag{26–1}$$

where W is the rim weight, in pounds, v is the mean velocity of the rim, in feet per second, and $g = 32.2$ fpsps. The amount of energy in the arms and the hub is so small that it may be neglected. If the velocity changes from v_1 to v_2, the energy released or absorbed during this change, which is called *excess energy*, is

$$E = K_1 - K_2 = W\frac{(v_1^2 - v_2^2)}{2g} \tag{26–2}$$

From equation 26–2, the rim weight is

$$W = \frac{2gE}{v_1^2 - v_2^2} \tag{26–3}$$

The velocity may be expressed by the relation

$$v = \frac{2\pi kn}{60} \tag{26–4}$$

TABLE 26–1

REQUIRED COEFFICIENT OF STEADINESS m

Driven Machinery	Type of Drive	m
Hammers, crushers, punch presses	Belt	5
Compressors, concrete mixers, excavators	Belt	7–10
Pumps, shears	Belt or flexible coupling	20–25
Metalworking and woodworking machinery	Belt	30
Flour, paper, and textile mills	Belt	40–50
Compressors, pumps, and similar machines	Gears	50
Spinning machinery, coarse to fine	Belt	50–65
D-C generators, single or parallel	Belt	35
D-C generators, single or parallel	Direct-coupled	70
A-C generators, single or parallel	Belt	60
A-C generators, single or parallel	Direct-coupled	100

where k is the polar radius of gyration of the rim, in feet, and n is the speed, in revolutions per minute. Then

$$Wk^2 = \frac{182.4gE}{n_1^2 - n_2^2} \tag{26–5}$$

The product Wk^2 is the polar moment of inertia. It is also known as the *flywheel effect.*

Speed fluctuation. The relative speed variation is determined from the relation

$$m = \frac{n}{n_1 - n_2} \tag{26–6}$$

where n is the mean speed of the flywheel, and n_1 and n_2 are the maximum and minimum speeds, respectively. The number m is called the *coefficient of steadiness*, and its reciprocal $u = 1/m$ is called the *coefficient of fluctuation of rotation.* The value of m, or u, depends on the nature of the service for which the machine is built; that is, on the permissible variation between the highest and lowest speeds during each operating cycle of the driven machine and on the method of connecting it to the driving motor. With a flexible connection, such as a belt drive, the coefficient m may be smaller than with a less flexible connection. An electric generator requires a more uniform drive, and a greater m, than a pump. Two or more generators operating in parallel require a still greater value of m. Table 26–1 gives minimum values of m for a number of typical drives. The maximum limit, as used in practice, is about 25 per cent higher.

The mean speed may be determined by the relation

$$n = \frac{n_1 + n_2}{2} \tag{26–7}$$

Equation 26-5, in which $n_1^2 - n_2^2$ is replaced by $(n_1 - n_2)(n_1 + n_2)$, and in which the values from equations 26-6 and 26-7 are substituted, becomes

$$Wk^2 = \frac{91.2gEm}{n^2} \tag{26-8}$$

Excess energy. The excess energy E, defined by equation 26-2, may be found either analytically or graphically. Often the graphical method is simpler, particularly when the work is done at a variable rate and sufficient information is available to draw a work diagram.

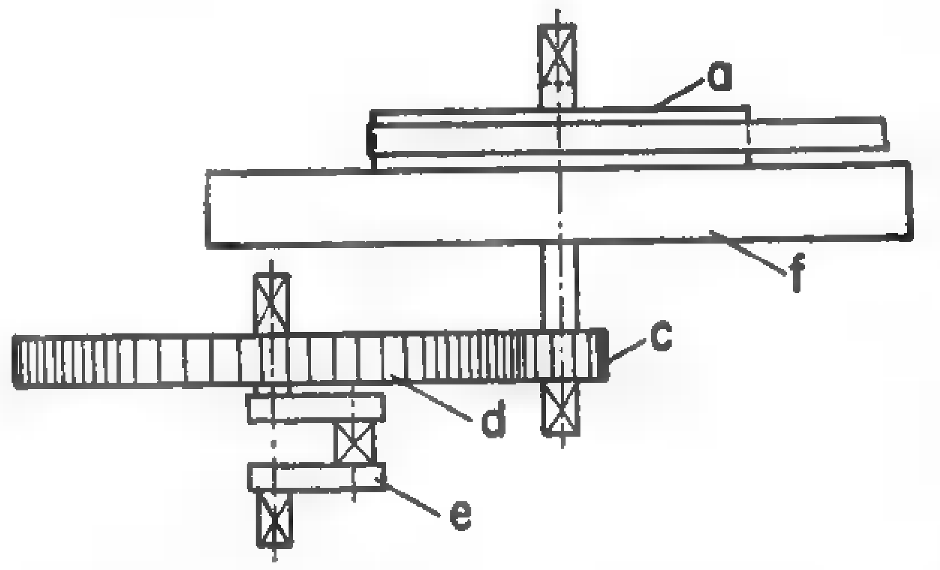


FIG. 26-1. Drive for a horizontal press.

EXAMPLE 26-1. Determine the necessary flywheel effect for a horizontal-press drive for which the plan view is shown in Fig. 26-1. The belt pulley a drives the press and makes 250 rpm; its diameter is 32 in.; the speed reduction between the spur gears c and d is 6.5 to 1; the actual work done during each revolution of the crankshaft e is equal to 6,800 ft-lb, and it is done during four-tenths of a revolution. Assume that the mechanical efficiency of the machine is 82 per cent.

The crankshaft speed is

$$n = \frac{250}{6.5} = 38.45 \text{ rpm}$$

The horsepower needed to drive the press is

$$P = \frac{6{,}800 \times 38.45}{0.82 \times 33{,}000} = 9.67 \text{ hp}$$

The belt speed is

$$v = \frac{\pi \times 32 \times 250}{12} = 2{,}095 \text{ fpm}$$

The net belt pull is

$$F_t = \frac{P \times 33{,}000}{v} = \frac{9.67 \times 33{,}000}{2{,}095} = 152 \text{ lb}$$

The total work to be done during one cycle is

$$W_c = \frac{6{,}800}{0.82} = 8{,}290 \text{ ft-lb}$$

The distance which the belt travels during the working portion of the cycle is

$$\frac{2{,}095 \times 0.4}{38.45} = 21.8 \text{ ft}$$

and the work done by the belt during this period is

$$W_b = 152 \times 21.8 = 3{,}315 \text{ ft-lb}$$

The energy to be stored by the flywheel is

$$E = W_c - W_b = 8{,}290 - 3{,}315 = 4{,}975 \text{ ft-lb}$$

From Table 26-1 the required coefficient of steadiness is $m = 5$. By equation 26-8, the required flywheel effect is

$$Wk^2 = \frac{91.2 \times 32.2 \times 4.975 \times 5}{250^2} = 1{,}168 \text{ lb-ft}^2$$

26-2. Stresses in flywheels. The stresses which are created by the centrifugal force in the rim and arms of a flywheel are rather complicated, and no entirely satisfactory method of computing them can be offered. In a wheel cast in one piece, unknown shrinkage stresses of great magnitude may exist, making futile any refined calculations. In general, the stresses in both the rim and the arms are tensile. However, since the rim and the arms are rigidly fastened together and stretch to a different degree, bending stresses are produced in the rim, as indicated in Fig. 26-2. Speed variations and consequent inertia forces act tangentially and produce bending stresses in the arms.

Stresses in the rim. The tensile stress created in each cross section of the rim by the centrifugal force is found by the equation 3-5, which is

$$s_1 = \frac{0.000914wr^2n^2}{g}$$

where r is the mean radius of the wheel, in inches.

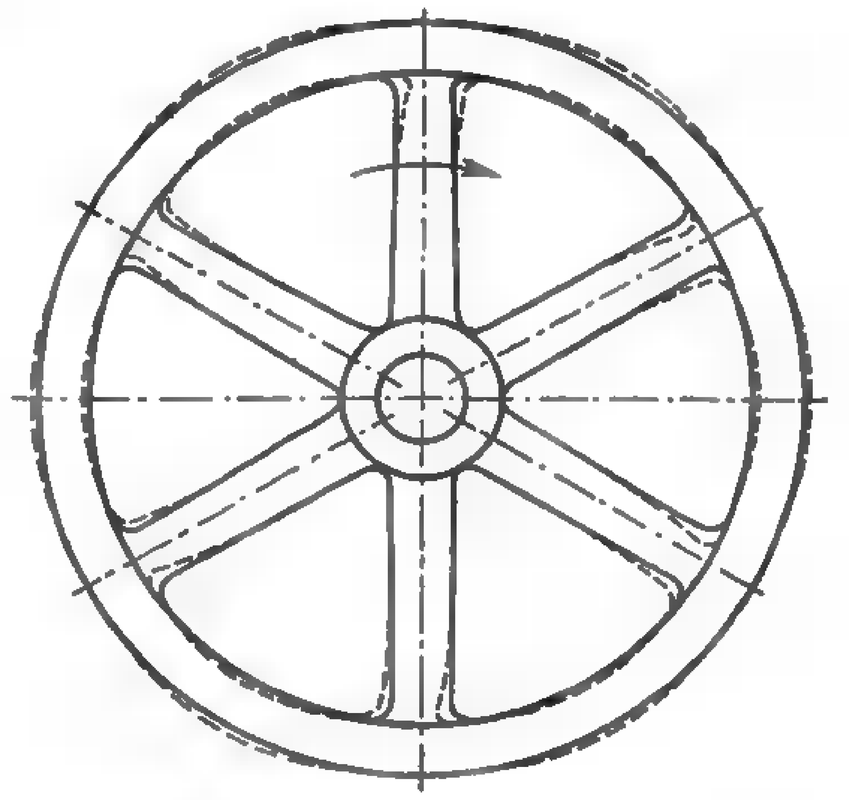


FIG. 26-2. Strains in a flywheel.

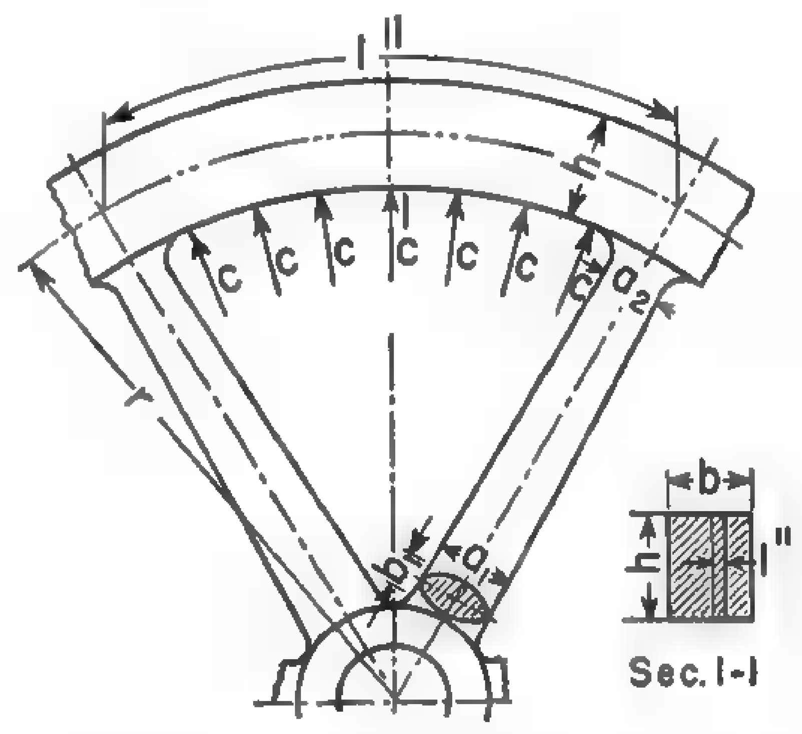


FIG. 26-3. Bending of flywheel rim.

The stress due to bending may be found by considering a strip of the rim with a face 1 in. wide (see Fig. 26-3). Since the area of the cross section for a strip of rim 1 in. wide is h, the centrifugal force per inch of rim width is, applying equation 3-2 with equation 3-3,

$$C' = \frac{0.000914wr^2n^2h}{g} \tag{26-9}$$

The rim is loaded by the force C' uniformly over the whole circumference, as indicated in Fig. 26-3.

The maximum bending moment in the rim occurs at the arms. If the rim is considered as a straight beam, this bending moment may be found by the expression in Table 2-4 for case i. It is

$$M = \frac{C'l^2}{12} \tag{26-10}$$

where l is the length of the rim between arms, in inches. The stress due to the moment M is

$$s_2 = \frac{M}{Z} \tag{26–11}$$

Substituting for M its value found by applying equations 26–10 and 26–9, replacing Z by $h^2/6$, and taking l as $2\pi r/i$, where i is the number of arms, results in

$$s_2 = \frac{0.0181wr^3n^2}{i^2hg} \tag{26–12}$$

The stretch of the arms may be taken as three-fourths that necessary for free expansion of the rim.[1] The combined tensile stress is then

$$s = 0.75s_1 + 0.25s_2 \tag{26–13}$$

Experience has shown that for cast-iron wheels the total stress s may be as high as 5,000 psi, or even 6,000 psi, but the tensile stress s_1 should not exceed 1,000 psi. For cast-steel wheels, s_1 should not exceed 4,000 psi.

Stresses in arms. When a flywheel is accelerated from rest, or when the energy supply is suddenly cut off, the arms may have to carry the full torque load. Each arm of a wheel with a rigid rim may be considered to act as a cantilever beam that is fixed at the hub end and carries a concentrated load at the free end at the rim. The bending moment in this case is a function of the transmitted torque T. Thus, $M = T(D-d)/D$, where d is the hub diameter. The stress is

$$s_1 = \frac{T(D-d)}{iZD} \tag{26–14}$$

where Z is the section modulus of the arm cross section at the hub. For cast iron this stress should not exceed 2,000 psi, because of the uncertainties of the material and possible shrinkage stresses. In the case of a sudden load a still lower stress should be used, reduced to 1,000 psi for very severe load conditions. However, the constraint of the rim reduces the stress s_1. A very heavy rim may reduce it almost to $0.5s_1$, where s_1 is found by equation 26–14.

If the flywheel is used as a belt pulley, the arms are bent not only by the variation in speed but also by the net belt tension $(F_1 - F_2)$. The moment due to this belt action is

$$M = \frac{(F_1 - F_2)(D-d)}{2i} \tag{26–15}$$

where D is the flywheel diameter and d is that of the hub. The stress at the hub is

$$s_2 = \frac{(F_1 - F_2)(D-d)}{2iZ} \tag{26–16}$$

[1] D. S. Kimball and J. H. Barr, *Elements of Machine Design*, 3d ed. (New York: John Wiley & Sons, Inc., 1935), p. 440.

In a thin-rim wheel, because of the absence of rigidity, the load is not distributed equally among the arms.[2] In such a case it is safer to assume that the maximum stress is twice as great as its average magnitude. Thus

$$s_2' = \frac{(F_1 - F_2)(D-d)}{iZ} \tag{26–17}$$

Finally, the arms are subjected to a tensile stress s_3 due to the centrifugal force acting upon the rim when the wheel is running at its maximum speed. This stress is evidently equal to s, determined by equation 3–5. The maximum tensile stress in an arm is at the hub end, and is

$$s_{max} = s_1 + s_2 + s_3 \tag{26–18}$$

For cast iron this stress should not exceed 3,000 psi.

Stress due to acceleration. When a large load slows down a machine, the flywheel tends to maintain its speed, and this action throws a considerable bending stress into the arms. A quick stopping of the machine may be considered as a limit case. The stress depends on the number of seconds t in which the wheel is stopped. The force F necessary to stop the wheel—whose rim weighs W lb— in that time is

$$F = \frac{Wa}{g} \tag{26–19}$$

where the negative acceleration a, or deceleration, is

$$a = \frac{v}{t} \tag{26–20}$$

The force F is applied at the center of gravity of the rim, and at the hub the bending moment due to this force is $M = F(r - d/2)$. The stress produced is found by proceeding as indicated for the stress s_1 in equation 26–14.

Example 26–2. Find the stress produced in the arms of a flywheel when it is stopped in two revolutions from a speed of 250 rpm. The rim weighs 382 lb, its radius of gyration is $k = 1.75$ ft, and the arms have an elliptical section $2\frac{1}{2} \times 1\frac{1}{8}$ in.

The normal velocity of the center of gravity is

$$v = \frac{2\pi kn}{60} = \frac{2\pi \times 1.75 \times 250}{60} = 45.8 \text{ fps}$$

The time of deceleration, which is the distance traveled divided by the mean velocity, is

$$t = \frac{2\pi \times 1.75 \times 2}{\frac{1}{2} \times (45.8 + 0)} = 0.961 \text{ sec}$$

By equation 26–20, the deceleration is

$$a = \frac{45.8}{0.961} = 47.65 \text{ fps}$$

Since the weight of the rim is $W = 382$ lb, the force F is, by equation 26–19,

$$F = \frac{382 \times 47.65}{32.2} = 565 \text{ lb}$$

[2] Lionel S. Marks, ed., *Mechanical Engineers' Handbook*, 5th ed. (New York: McGraw-Hill Book Company, Inc., 1951), p. 913.

If the hub diameter is $d = 6$ in., the moment is

$$M = 565 \times (21 - \tfrac{1}{2} \times 6) = 10{,}170 \text{ lb-in.}$$

The section modulus of the arm section is, from case i in Table 2–5,

$$Z = \frac{\pi \times 1.125 \times 2.25^2}{32} = 0.56 \text{ in.}^3$$

With six arms, the stress is

$$s = \frac{M}{Zi} = \frac{10{,}170}{0.56 \times 6} = 3{,}025 \text{ psi}$$

For cast iron this is a fairly high stress, but it does not reach the danger point.

26–3. Flywheel design. In finding the weight of a flywheel, the mass of the rim only is considered; the additional small effect of the arms and the hub is neglected. First the radius of gyration k in the expression Wk^2 is determined by assuming a proper rim speed $v = 2\pi kn$.

Rim speed. The rim speed should not exceed 5,000 fpm for cast-iron wheels of machines under 100 hp, and 6,000 fpm for larger machines. With a special rim design which prevents blowholes in the rim, v may be increased to 7,000 fpm; and with special arm designs rim speeds as high as 10,000 fpm are used.[3] Cast-steel wheels can operate with speeds up to 10,000 fpm. Large flywheels for steel mills, having a rim speed of 15,000 fpm at 375 rpm, are assembled by using a cast-steel spider with arms and a laminated rim made of rolled steel plates. In automotive engines, rim speeds of cast-iron flywheels reach 10,000 fpm; those of semisteel wheels, 15,000 fpm; and those of cast-steel wheels, 20,000 fpm.

Rim dimensions. The relation between k, in feet, and the outside diameter D of the rim, in feet, is

$$k^2 = 0.125\left[D^2 + \left(D - \frac{2h}{12}\right)^2\right] \tag{26–21}$$

If the rim thickness h, in inches, is small compared to D, it may be neglected. Then $k = 0.5D$.

After the weight W is found from equation 26–8, the cross-sectional area A of the rim, in square inches, can be computed from the equation

$$W = 2\pi k \times 12 \times Aw \tag{26–22}$$

where, for cast-iron, $w = 0.26$ lb per cu in., and for steel, 0.28 lb per cu in. The most common cross section is a rectangle with a width b and a height h, for which $A = bh$. The ratio b/h is selected between 0.65 and 2. If the flywheel is to be used also as a belt pulley, the rim face b must be made at least 1 or 2 in. wider than the belt. The outside diameter, in feet, can be found from equation 26–21; or its approximate value, in inches, may be taken as

$$D_0 = 2 \times 12k + h \tag{26–23}$$

[3] F. A. Halsey, *Handbook for Machine Designers*, 2d ed. (New York: McGraw-Hill Book Company, Inc., 1916), pp. 69–71.

The *hub diameter* may be made equal to two shaft diameters, and the hub length may be 2 to 2.5 shaft diameters.

Arms. The standard number of arms on a flywheel is six. Very wide flywheels serving as belt pulleys are made with two rows, or 12 arms; and flywheels with very large diameters are sometimes made with 8, 10, or even 12 arms. Some oil engines have flywheels with webs, as indicated in Fig. 26–6, instead of arms. The holes are made to facilitate handling.

The arms usually have an elliptic section, as shown in Fig. 26–3, with the major axis twice the minor, or $a = 2b$. From the hub down to the rim the arms taper from 10 to 25 per cent. Arms with H and I sections are used very rarely, but in very large flywheels hollow elliptic sections are sometimes used to prevent porous castings. For an elliptic section, $Z = \pi ba^2/32$. After Z has been found by equation 26–14, $a/2$ can be substituted for b, and the major axis can be computed from the relation

$$a = \sqrt[3]{\frac{64Z}{\pi}} \tag{26–24}$$

EXAMPLE 26–3. Find the weight and the main dimensions of the flywheel of example 26–1. The distance from the center of the shaft to the wall is 2 ft.

The maximum outside diameter of the wheel is $D < 4$ ft, or < 48 in. The radius of gyration must be assumed slightly smaller than $D/2$, and k will be taken as 21 in., or 1.75 ft. Since $Wk^2 = 1.168$ lb-ft²,

$$W = \frac{1{,}168}{k^2} = \frac{1{,}168}{1.75^2} = 382 \text{ lb}$$

By equation 26–22, the area of a cast-iron rim should be

$$A = \frac{W}{2\pi k \times 12w} = \frac{382}{2\pi \times 1.75 \times 12 \times 0.26} = 11.13 \text{ sq in.}$$

A suitable rim section would be a rectangle $5 \times 2\tfrac{1}{4}$ in. The outside diameter may be found by applying equation 26–21, in which $k = 21$ in. and $h = 2.25$ in., and solving for D. This results in $D = 44.18$ in. The approximate value, by equation 26–23, is

$$D = 2 \times 12 \times 1.75 + 2.25 = 44.25 \text{ in.}$$

The difference is negligible and the error is on the safe side if the approximate formula is used.

The arm section may be found by using equation 26–14. The moment is equal to $F_t(D - d)/2 = 152 \times (44.25 - 6)/2 = 2{,}910$ lb-in. If s_1 is taken as 1,000 psi, since shock action is present, and if six arms are used,

$$Z = \frac{M}{is_1} = \frac{2{,}910}{6 \times 1{,}000} = 0.485 \text{ in.}^3$$

with $b = a/2$, the major axis of the ellipse, by equation 26–24, should be

$$a = \sqrt[3]{\frac{64 \times 0.485}{\pi}} = 2.15 \text{ in., or } 2\tfrac{1}{4} \text{ in.}$$

Then $b = a/2 = 1\tfrac{1}{8}$ in. At the rim, if $a' = 0.9a$, the dimensions are $a' = 2$ in., and $b = 1$ in.

Construction. Flywheels up to 8 ft in diameter are cast in one piece; above this size they are usually made in halves. A split hub, Fig. 26–4,

bored 0.001 or 0.002 in. smaller than the shaft, clamps the shaft and prevents the key from working loose. A steel wedge driven into the hub slot is used to put the wheel on the shaft. The diameter of the clamping bolts is made about one-sixth the shaft diameter. Sometimes the hub is split clear through, as shown by the dotted lines below the horizontal center line in Fig. 26–4. This procedure has the additional advantage of relieving the shrinkage stresses in the arms to a certain extent.

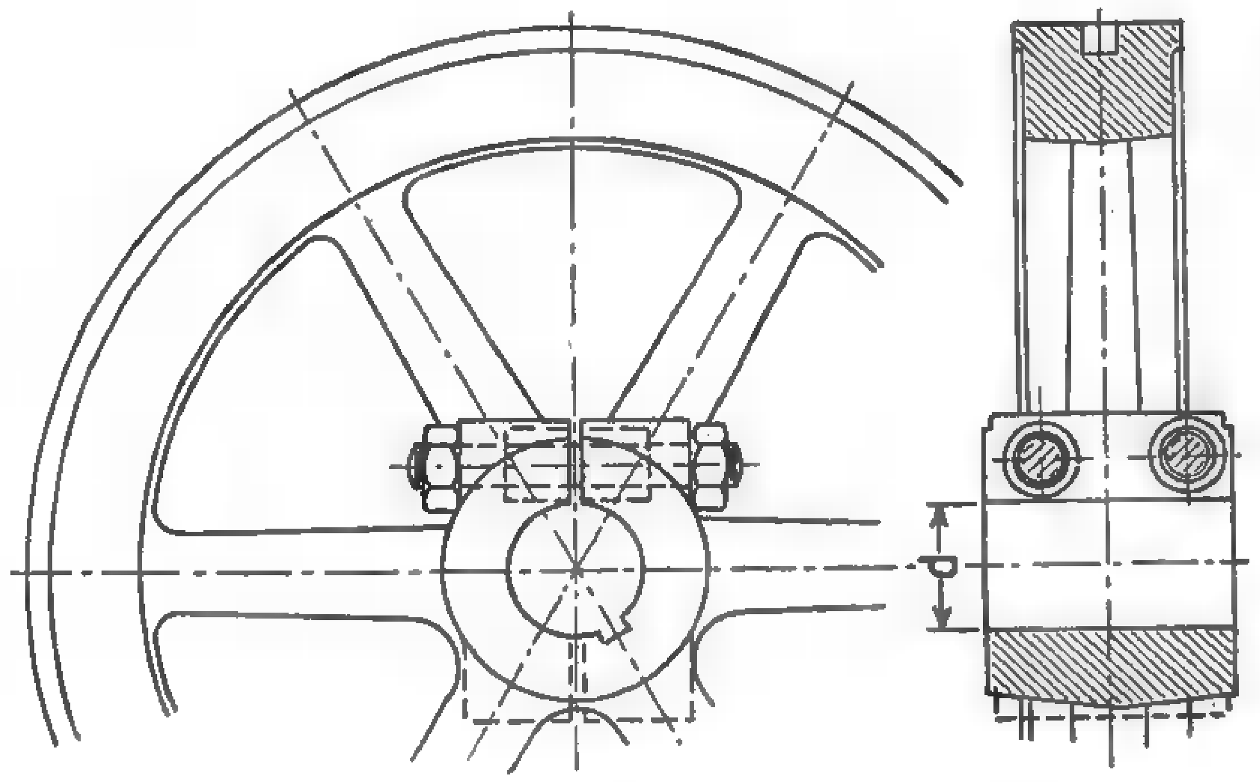


Fig. 26–4. Flywheel with a split hub.

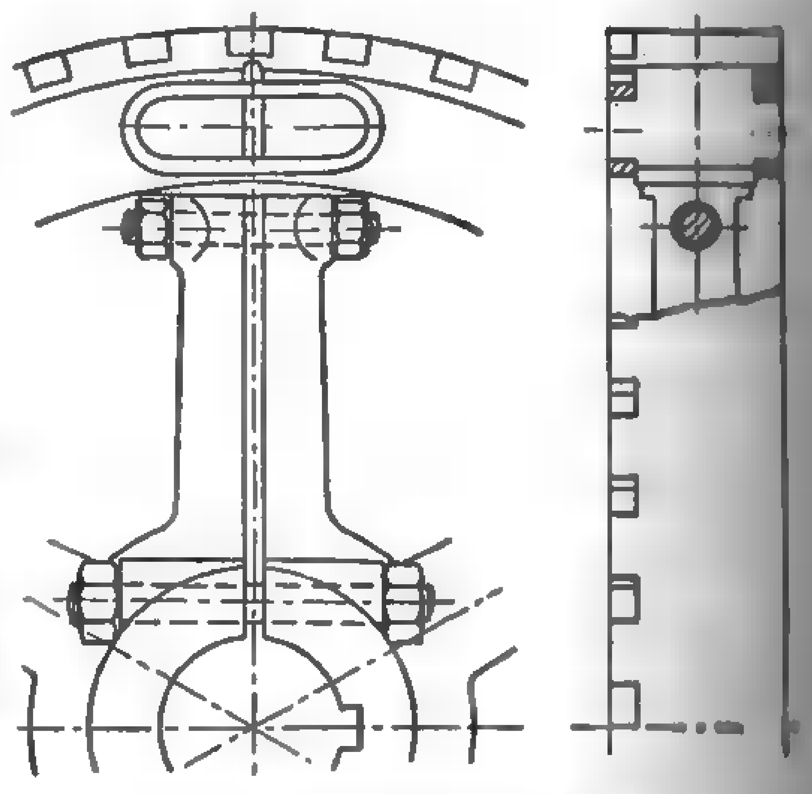


Fig. 26–5. Split flywheel.

A flywheel made in two halves should be parted on an arm rather than between arms, the latter method giving a joint only half as strong as the former, as shown in Table 26–2. The halves are connected by bolts through the hub and near the rim, and also by shrink links, as shown in Fig. 26–5, or by shrink anchors, as in Fig. 26–6. The anchor connection has the advantage of easier and more accurate machining, which assures that the desired force will be created when the anchor is shrunk into place. If the rim section is made I-shaped, as in Fig. 26–7, the anchors can be so proportioned that the joint will be as strong as the rim proper.[4]

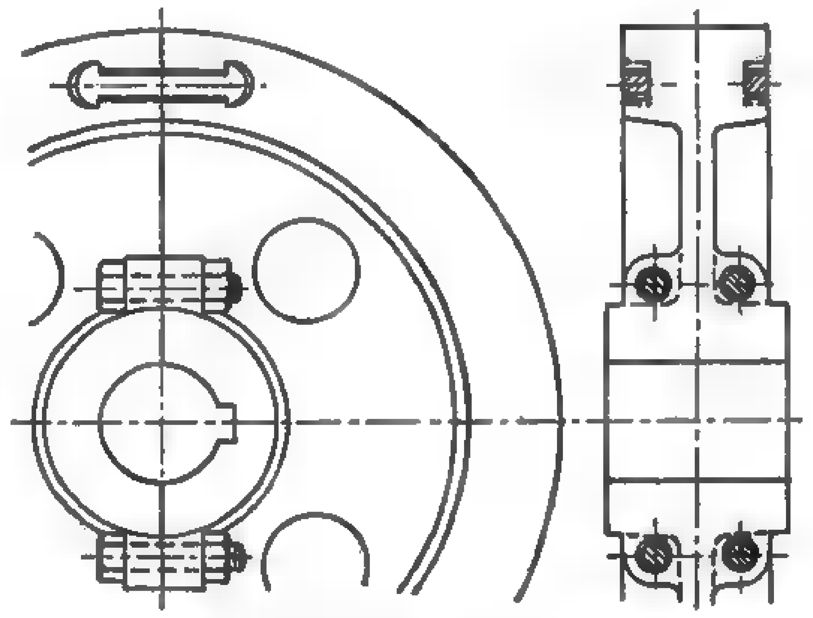


Fig. 26–6. Split disk flywheel.

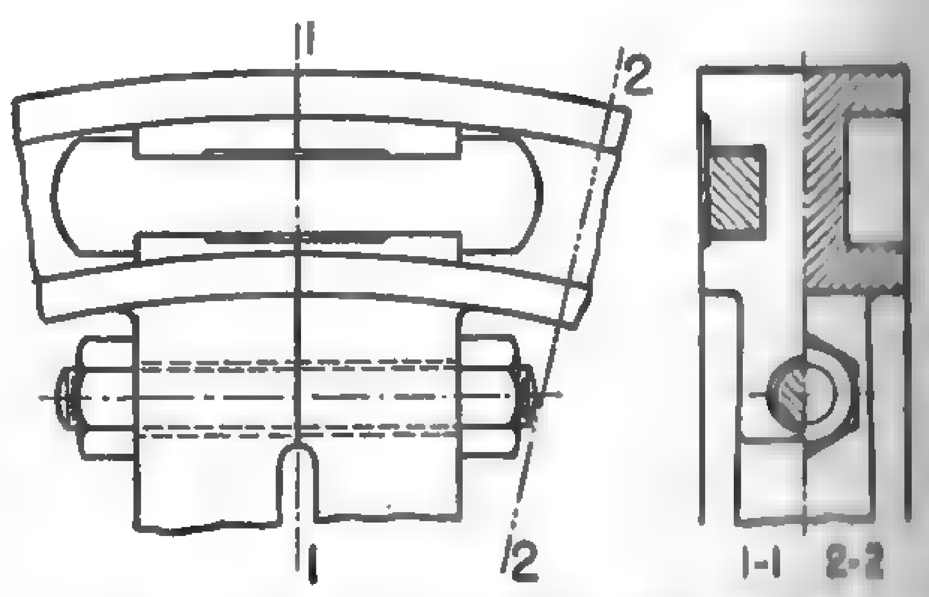


Fig. 26–7. The Haight rim joint.

[4] H. V. Haight, "A High Efficiency Flywheel Joint," *American Machinist*, Vol. [illegible] (February, 1907), p. 267; also Halsey, *op. cit.*, p. 73.

An anchor connection with wedge-shaped cotters, as in Fig. 26–8, eliminates the troubles encountered with shrink fits, especially if the flywheel must be taken off eventually.

While the relative strengths of a rim joint and the solid rim depend on the exact proportions used, average values confirmed by tests are given in Table 26–2.[5]

The force F which acts upon a rim connection is

$$F = 2sA \tag{26–25}$$

where s is the stress due to the centrifugal force, determined by equation 3–5, and A is the cross-sectional area of the solid rim.

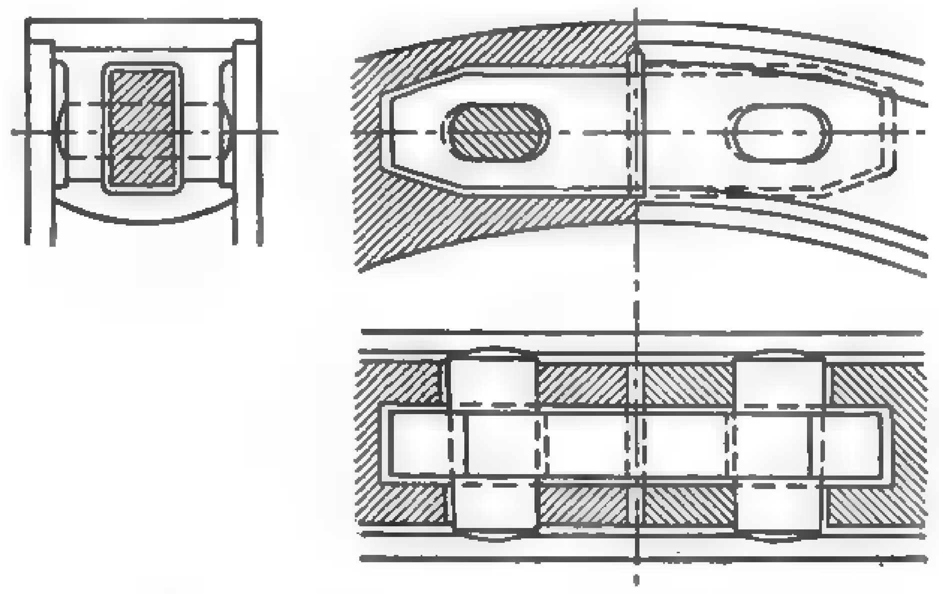


Fig. 26–8. Anchor with cotters for a split-flywheel rim.

Welded wheels. Large flywheels and those having high peripheral speeds are fabricated by welding. The hub is made of a steel forging; the rim is obtained by bending a rectangular bar with a suitable cross section into a ring and welding the ends together; and the arms are built up along the lines of Figs. 30–14 and 30–15.

TABLE 26–2

Relative Strengths of Flywheel Rims

Type of Construction	*Relative Strength*
Solid rims	1.00
Flanged joint, bolted, rim parted between arms	0.25
Flanged joint, bolted, rim parted on an arm	0.50
Shrink-link joint (Fig. 26–5)	0.60
Anchor joints (Figs. 26–6 and 26–8)	0.70
Haight joint (Fig. 26–7)	1.00

[5] C. H. Benjamin, "The Bursting of Small Cast-iron Flywheels," *Transactions of The American Society of Mechanical Engineers*, Vol. 20 (1899), p. 209, and Vol. 23 (1902), p. 168.

CHAPTER 27

Belt Drives

27–1. Belt materials. Belt drives are used to transmit power from one shaft to another when the shafts are some distance apart and it is not required that their velocity ratio be absolutely constant. The shafts may have any speed ratio, within reasonable limits; and while they are generally parallel, other arrangements are also used. With an open belt, Fig. 27–1a, the shafts turn in the same direction; with a crossed belt, Fig. 27-1b, they turn in opposite directions. Crossed belts are subjected to greater wear and tear and should be used only where the speed is low and small power is transmitted.

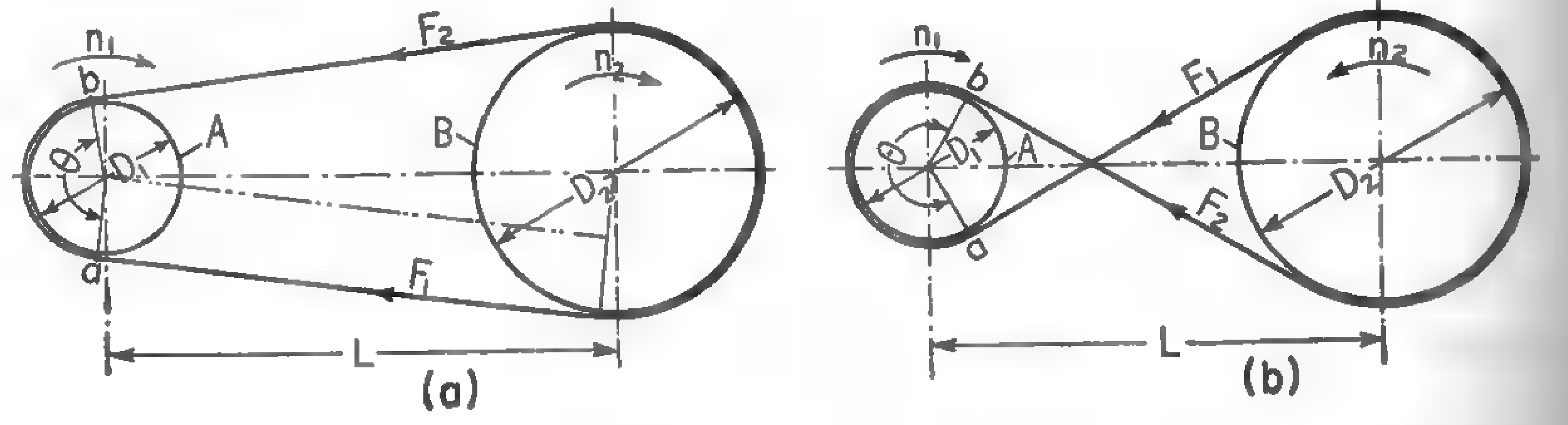


FIG. 27–1. Open and crossed belts.

Leather belts. Most belts, especially those for high speeds, are made of leather strips cemented together, the best quality being obtained from the parts of steerhides near the backbone. Leather for belting may be oak-tanned or chrome-tanned. Both classes are used for general service, but chrome leather is less affected by dampness, acid, fumes, or oil. The thickness at different places of a hide varies from about $\frac{1}{8}$ to $\frac{1}{4}$ in. The various belt thicknesses are obtained by using hides of different thicknesses and also by gluing several layers together, giving what are known as single, double, and triple belts. The thicknesses and widths in which belts can be obtained are shown in Table 27–1. The hair side of leather is smoother and harder than the flesh side, but the flesh side is considerably stronger. The hair side of a single belt should run in contact with the pulley face, since the flesh side is better adapted to stand the greater tensile stress when going around the pulley. The modulus of elasticity varies from 20,000 to 45,000 psi. The deformation of leather does not conform to Hooke's law, and the modulus of elasticity increases with an increase of the stress. The weight of leather for belting is 0.035 lb per cu in.

TABLE 27–1

LEATHER-BELT DATA

Grade of Belt	Approximate Thickness (in.)			Width (in.)	Ultimate Strength (psi)
	Single	Double	Triple		
Light......	$\frac{1}{8}$	$\frac{1}{4}$	..	$\frac{1}{2}$– 1, $\frac{1}{8}$-in. increment 1– 3, $\frac{1}{4}$-in. increment 3– 6, $\frac{1}{2}$-in. increment 6–10, 1-in. increment 10–56, 2-in. increment 56–84, 4-in. increment	Oak-tanned: 3,000 to 4,500 Chrome-tanned: 4,000 to 5,500
Medium....	$\frac{5}{32}$	$\frac{5}{16}$	$\frac{15}{32}$		
Heavy......	$\frac{3}{16}$	$\frac{3}{8}$	$\frac{17}{32}$		

Rubber belting. Rubber belting consists of cotton duck and is normally from three to ten plies in thickness, although heavier belting, up to 15 plies, can be obtained on special order. The duck is impregnated with rubber and is provided with a wearing surface of rubber on both faces. Table 27–2 gives data for thicknesses and widths of commercial rubber belting. Compared with leather belts of the same power-transmitting capacity, rubber belts are cheaper but have shorter life. Because they stand up better than leather belts under adverse atmospheric conditions, they are the most preferable for outdoor service, but they are quickly ruined by oil or grease. The modulus of elasticity of rubber belting varies from 130,000 to 210,000 psi, and the belting weighs 0.045 lb per cu in.

Balata belting. Balata is a gum similar to rubber and is used in the manufacture of fabric belts to render them acidproof and waterproof. However, balata belts cannot be used at temperatures above 100 F, at which temperature balata begins to soften.

Textile belts. Textile belts are made of cotton, being built up of three to ten layers of duck stitched together or solidily woven into a strip of the desired width and thickness. The fabric is treated with linseed oil to make it waterproof. Textile belts are used for temporary, rough service. They are more

TABLE 27–2

RUBBER AND BALATA BELT DATA

Thickness (in.)	Number of Plies	Width (in.)	Increment (by widths)	Ultimate Strength (psi)
[illegible] to [illegible] per ply	3	1– 8	1– 2, $\frac{1}{4}$-in. increment	900 to 1,500
	4	1–24	2– 5, $\frac{1}{2}$-in. increment	
	5	$1\frac{1}{2}$–36	5–16, 1-in. increment	
	6	2–60	16–72, 2-in. increment	
	7	4–60		
	8–15	6–72		

durable than either leather or rubber belts if they have to run in contact with dirt and grit or in a dusty atmosphere. The tensile strength of cotton belting varies from 5,000 to 7,500 psi, and the modulus of elasticity may vary from 70,000 to 200,000 psi. The material weighs 0.025 to 0.050 lb per cu in.

27–2. Belt fastenings. The *efficiency* of a joint is the ratio of its strength to the full strength of the belt.

Cemented joint. The best form of belt fastening is the scarfed splice glued under pressure, which makes a very strong lap joint with an average efficiency of 98 per cent. Its great disadvantage is that in most cases it has to be made with the belt in place. If it is not necessary to make the splice in place, a very accurate measurement of the required endless belt must be taken in order to obtain just the right tension when the belt is stretched over the pulleys.

Metal hinges. A strong joint, used rather extensively for narrow belts, is made by securing a row of wire loops in the belt ends and locking them together with a special steel-wire pin or a rawhide pin. If the loops are applied by a special machine, this joint has an efficiency of about 60 to 90 per cent. It is easily put together and taken apart. Also, since it is very flexible, it is suitable for small pulleys.

Rawhide lacing. A popular form of joint is made with rawhide lacing threaded through holes punched in the ends of the belt. It is easily made and is durable, but it is rather stiff and has an efficiency of only 40 to 60 per cent.

Metal clamps. Heavy rubber or canvas belts subjected to shock loads, such as are encountered in oil-field work, are often joined by clamps formed of two steel plates with holes and through-bolts. The efficiency of such a joint is from 50 to 70 per cent.

27–3. Stresses in a belt. All stresses produced in a belt are tensile stresses.

Tensions in a running belt. Flat belts are put on with a certain initial tension. When the driving pulley A, Fig. 27–1a, turns without transmitting any power, the tension on both the running-on and running-off sides of the pulley remains unchanged, if the small frictional resistance in the bearings is neglected. As soon as power is transmitted the tension F in the pulling side will increase to F_1, and that in the running-off side will decrease to F_2. The force causing the driven pulley to rotate is the difference between these tensions, or $F_1 - F_2 = F_t$. This value is sometimes termed the *net pull*, or *effective pull*.

Slip. Any belt transmitting a load will slip on the surfaces of the pulleys. This means that the belt will move somewhat slower than the face of the driving pulley and somewhat faster than the face of the driven pulley.

Creep. Creep action adds to the effect of slip, but it is entirely different. Because of the difference in tension in the various sections of the belt, a unit length of the belt in passing from the point a, Fig. 27-1a, to the point b, decreases in length owing to its elasticity. Therefore the driver A delivers a shorter unit length of belt at b than it receives at a, and the average velocity of the belt is slightly lower than that of the pulley surface. A similar action occurs on the driven pulley B, but here the average belt velocity is slightly higher than that of the pulley face. This action, known as *creep*, reduces the speed of the driven pulley and also causes some loss of power.

In experiments it is easier to measure the combined action of slip and creep, which is called simply slip. Most figures referring to percentage slip include creep as well. The normal range of slip is considered to be from 1.5 to 2 per cent. When a belt is overloaded, its slip increases. If excessive slip occurs, the heat generated by friction may damage the belt.

Belt section. If the allowable belt stress is designated by S_d, and the thickness and width of the belt, in inches, are represented by h and b, respectively, then for lower belt speeds up to 2,000 fpm the cross-sectional area of the belt may be found from the equation

$$hb = \frac{F_1}{S_d} \qquad (27\text{–}1)$$

Centrifugal force. For belt speeds above 2,000 fpm, the centrifugal force must be taken into account. This force increases the tension in the belt without increasing its driving power. The centrifugal force of a piece of belt 1 in. long may be determined from the general relation of equation 3–1. Thus,

$$c = \frac{12hbwv^2}{gr} \qquad (27\text{–}2)$$

where w is the specific weight of the belt material, in pounds per cubic inch;
v is the belt velocity, in feet per second;
g is the acceleration of the force of gravity, or 32.2 fpsps;
r is the radius of the smaller pulley, in inches.

The projected length of the belt on the pulley is $2r$, and the component C of the centrifugal force parallel to the line connecting the centers of both pulleys is

$$C = 2rc = \frac{24hbwv^2}{g} \qquad (27\text{–}3)$$

This force is taken up evenly by both sides of the belt, creating on each side an additional stress given by the relation

$$s = \frac{24hbwv^2}{g \times 2hb} = \frac{12wv^2}{g} \qquad (27\text{–}4)$$

This stress must be deducted from the nominal allowable stress S_d, and equation 27–1 becomes

$$hb = \frac{F_1}{S_d - \dfrac{12wv^2}{g}} \tag{27-5}$$

Allowable stresses. For leather belts the allowable stress S_d is taken equal to one-tenth to one-eighth of the ultimate strength as indicated in Table 27–1, or 300 to 500 psi. Rubber and balata belting have a more uniform structure, and the safe stress may be taken equal to one-eighth to one-sixth of the ultimate strength in Table 27–2, or 150 to 250 psi. These values must be multiplied by the efficiency of the joint used.

27–4. Belt capacity. A belt transmits power through its friction upon the faces of the pulleys. The transmitting capacity depends on the allowable maximum tension in the belt; the belt speed; the coefficient of friction between the belt and the pulleys; the angle of wrap, or contact, on the smaller pulley; and the service conditions as expressed by the load factor.

Tension. Belt tension is discussed in section 27–3.

Belt speeds. Based on equation 27–5, the limit speed for a leather belt with $S_d = 400$ psi and $w = 0.035$ lb per cu in. is about 10,500 fpm. At this speed the centrifugal force is so great that there is no traction left between the pulley face and the belt. However, experiments have shown that a large amount of power can be transmitted at speeds up to 12,000 fpm.[1] For maximum power and economy, speeds of 5,000 to 6,000 fpm may be used. It so happens that this is also about the limit of safety for ordinary cast-iron pulleys. For a longer belt life combined with efficiency, a speed between 3,000 and 4,000 fpm is advisable. Speeds of shafts, and pulley sizes, often limit the belt velocity to considerably lower values.

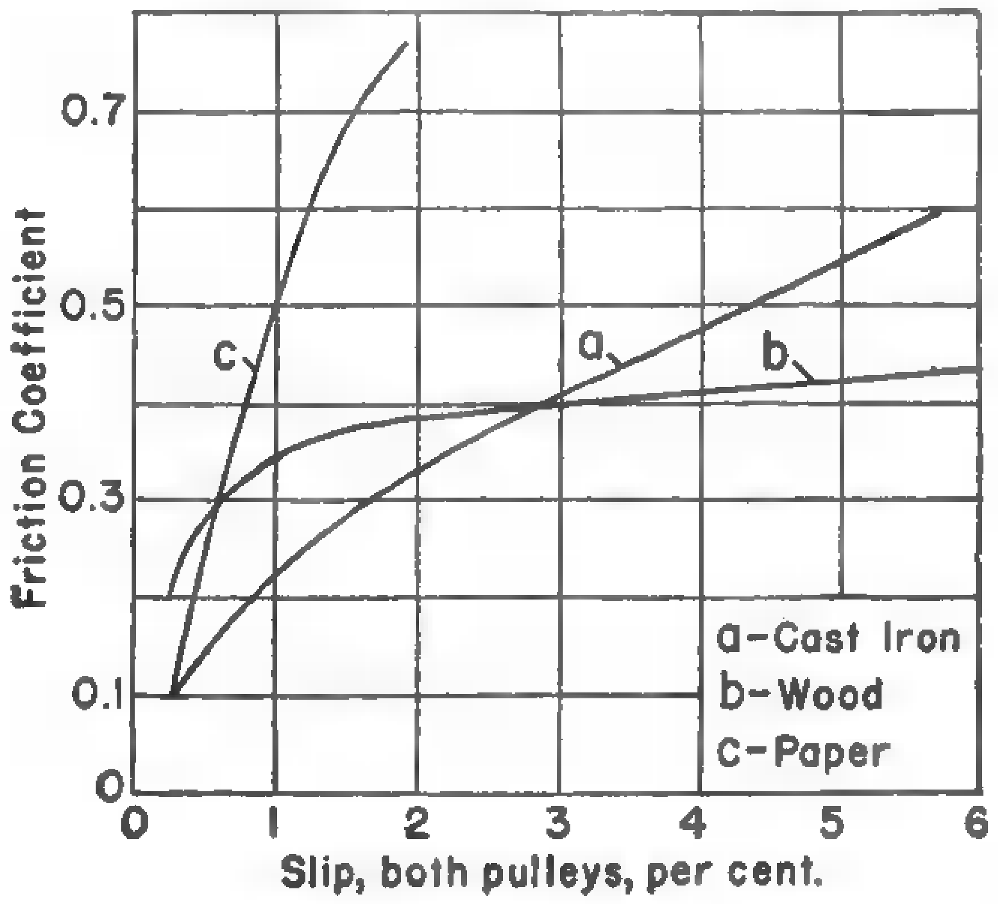


Fig. 27–2. Friction–slip curves.

Coefficient of friction. The coefficient of friction f is a function of the material of the pulley, the material of the belt, the rate of slip of the belt on the pulley, and the belt speed.

The influence of the material of the pulley on a leather belt may be seen from Fig. 27–2. The coefficient of friction of rubber belts is about 20 per

[1] C. H. Norman, *High-Speed Belt Drives*, Bulletin No. 83, Ohio State University Engineering Experiment Station (May, 1934).

TABLE 27–3

Average Coefficients of Friction for Leather and Rubber Belts

Belt Material	Pulley Material		
	Cast Iron	Wood	Paper
Leather	0.35	0.38	0.50
Rubber	0.30	0.33	0.42

cent lower than that of leather belts. The coefficient of friction increases with an increase of the rate of slip, but the latter should be limited to 1.5 or 2 per cent.

For leather belts on cast-iron pulleys the influence of speed is given by Barth's empirical formula, which is

$$f = 0.54 - \frac{140}{500 + v_m} \tag{27-6}$$

where v_m is the belt speed in feet per minute. This formula does not take into account the slip and should be considered only as giving the limit values which may be attained in operation under favorable conditions.

Values of the friction coefficient, often used in practice, are given in Table 27–3.

Angle of contact with tight and slack tensions. If the influence of centrifugal force is neglected, the ratio of the tight and slack tensions may be determined by equation 18–25, which is

$$\frac{F_1}{F_2} = e^{f\theta}$$

Also, the relation between the maximum pull F_1 and the net pull F_t may be found by equation 18–27. Thus

$$F_1 = \frac{F_t e^{f\theta}}{e^{f\theta} - 1}$$

Combining this equation and equation 27–5, and solving for F_t, gives

$$F_t = hb\left(S_d - \frac{12wv^2}{g}\right)\left(\frac{e^{f\theta} - 1}{e^{f\theta}}\right) \tag{27-7}$$

The angle of contact θ which must be used in equation 27–7, if the pulleys are of the same material, is the smaller of the two. In installations without an idler pulley, θ is the angle on the smaller pulley. If the pulleys are made of materials having different coefficients of friction, or if the angle of belt wrapping on the small pulley is increased by the use of an idler pulley, the design should be based on the pulley with the smaller value of $f\theta$.

For an *open belt*, Fig. 27–1a, the angles of contact, in radians, are approximately

$$\theta = \pi \pm \frac{D_2 - D_1}{L} \tag{27-8}$$

where the plus sign applies for the larger pulley and the minus sign applies for the smaller one.

For a *crossed belt*, Fig. 27–1b, the angles of contact are the same in both pulleys. Each is

$$\theta = \pi + \frac{D_2 + D_1}{L} \tag{27–9}$$

Equation 27–8 applies for a horizontal belt with the slack side on top. For belts with the slack side on the bottom, and for vertical or inclined belts, equation 27–8 gives values somewhat too large. For an inclined belt with an angle of 50° or more, the belt width b found by means of equation 27–7, in which θ is determined by equation 27–8, must be divided by a capacity coefficient given in Table 27–4.

TABLE 27–4

CAPACITY COEFFICIENT OF AN INCLINED BELT DRIVE

Angle with Horizontal (deg)	*Capacity Coefficient*	*Angle with Horizontal (deg)*	*Capacity Coefficient*
0	1.00	70	0.70
50	0.90	80	0.60
60	0.80	90	0.50

With idler pulleys, whether on a horizontal or vertical drive, the angle θ, Fig. 27–3, may be found graphically from a layout of the drive.

Sag and stiffness. Equation 27–7 is derived by taking into account only what happens when the belt passes over the pulley, and it must be considered a rough approximation, although on the safe side. Actually a centrifugal force acts upon the belt between the pulleys and sets up a tension equal to and oppositely directed from that set up on the pulley.[2] It is a known fact that fan belts can operate with an astonishing degree of slackness, the net pull F_t often being much greater than the standstill tension. Under such conditions the loose-side tension F_2, without regard to centrifugal force, might sink to zero, in which case F_1 should also become zero according to equation 18–27. This contradiction can be overcome by introducing in equation 27–7 a correction ΔS_1 to take into account the influence of belt sag, and a correction ΔS_k to allow for the still

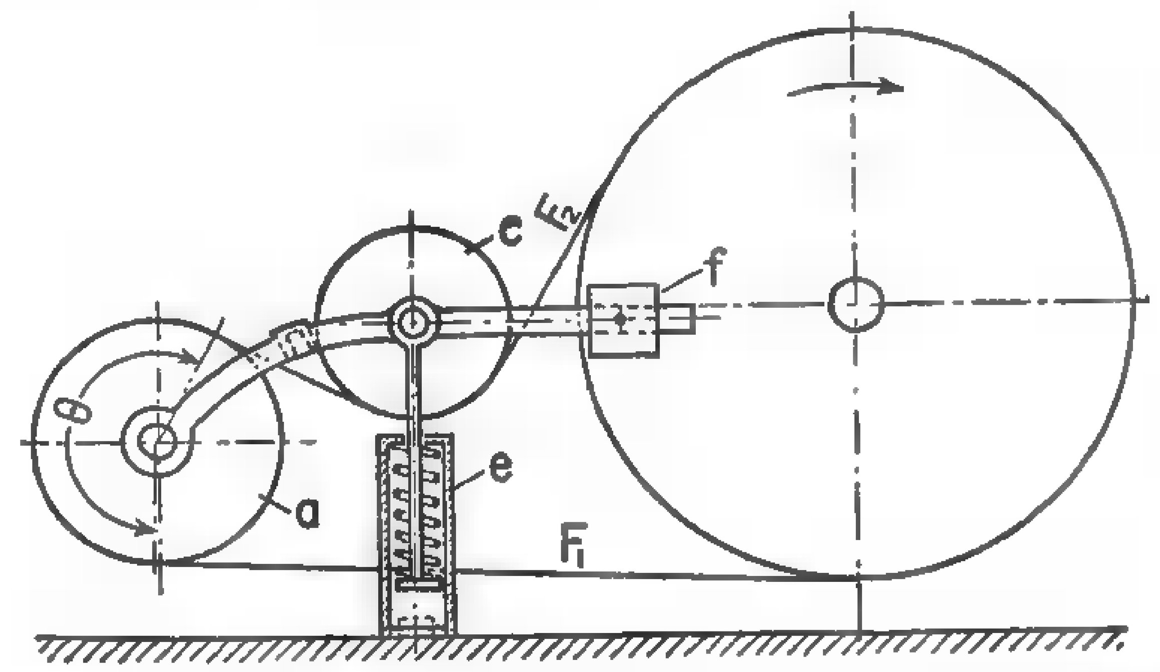


FIG. 27–3. Short-center drive with an idler pulley

[2] G. Schulze-Pillot, *Neue Riementheorie* (Berlin: Julius Springer, 1926).

ness of the belt.[3] However, for usual operating conditions these corrections are negligibly small.

Initial tension. It has been found that there is a definite relation between the tensions F_1 and F_2 and the initial tension F_0. This relation is expressed by the equation[4]

$$\sqrt{F_1} + \sqrt{F_2} = 2\sqrt{F_0} \tag{27–10}$$

Later experiments have brought out the value of this approximate equation, which seemingly can be applied for cases where F_2 becomes very small and nevertheless, contrary to equation 18–27, the pull F_1 is comparatively great. For leather belts a suitable initial tension is 200 to 225 psi,[5] and for rubber belts it is about 75 to 100 psi.

Load factor. A belt absorbs a large amount of shock, and the load factor K' for given conditions may be computed by the relation

$$K' = 1 + \frac{K_l - 1}{5} \tag{27–11}$$

where K_l is a factor the value of which may be taken from Table 20–3.

Horsepower. The horsepower that can be transmitted with a belt running at v_m fpm can be determined by taking into account the load factor K' of equation 27–11 and the force F_t of equation 27–7. Thus,

$$P = \frac{F_t v_m}{33{,}000K'} \tag{27–12}$$

27–5. Design of belt drives. The design of a belt drive is based to some extent on empirical rules.

Belt thickness. The selected belt thickness depends on the pulley size.

For *leather belts* a good practical rule is to determine the thickness h by the relation

$$h \leqq 0.02D \tag{27–13}$$

where D is the diameter of the small pulley. When the pulley diameter is too small for a given belt thickness, the bending of the belt will generate excessive heat and cause its rapid deterioration. If for some reason a smaller pulley diameter must be used, the limit is given by the equation

$$D = \frac{h}{0.03} \tag{27–14}$$

Also, in order to extend the belt life, a lower working stress, and hence a greater belt width, should be used in such a case.

[3] Norman, *op. cit.*

[4] C. Barth, "Transmission of Power by Leather Belting," *Transactions of the American Society of Mechanical Engineers*, Vol. 31 (1909), pp. 29–203.

[5] G. B. Haven and G. W. Swett, *Treatise on Leather Belting* (Cambridge, Mass.: Composition Company, 1931), p. 73.

A single belt should not be over 8 or 10 in. wide. Another rule is not to use a single belt where its width is more than four-thirds the diameter of the smallest pulley.[6]

For *rubber belts* a good practical rule is to have 3 in. of pulley diameter per ply of belt at lower speeds, and at higher speeds to have 4 in. of pulley diameter per ply. If the foregoing proportion cannot be maintained, a shorter belt life must be accepted, unless a special very flexible belt is used.

Belt width. The width of a belt can be determined from equation 27–7 after the belt thickness h has been selected as just explained and the other quantities have been computed.

Center distance. A good distance L between shaft centers is 20 to 25 ft. For single leather belts and 3-ply and 4-ply rubber belts the distance may be cut down to 12 ft. A further decrease of L with pulleys of different diameters decreases the angle θ and requires an increased initial tension F_0. The minimum value is sometimes given as $L \geqq 3.5D$, where D is the diameter of the larger pulley. When the belt has to stand shock action, an increase of belt length will give a longer belt life.

EXAMPLE 27–1. Determine the diameters of cast-iron pulleys and the thickness and width of a leather belt to transmit 175 hp from a shaft that is direct-connected to a steam engine turning at 300 rpm to a centrifugal pump running with a speed ratio of 1:3.5.

Assume a belt velocity of 5,000 fpm. Then the diameter of the driving pulley is

$$D_1 = \frac{12v_m}{\pi n} = \frac{12 \times 5{,}000}{\pi \times 300} = 63.7, \text{ or } 64 \text{ in.}$$

With an assumed total slip of 1.5 per cent, the diameter of the driven pulley must be

$$D_2 = \frac{64}{3.5 \times 1.015} = 18.1, \text{ or } 18 \text{ in.}$$

The general load factor K_l, from Table 20–3, is $1.5 \times 1.25 = 1.875$, and the load factor K' for the belt drive is, by equation 27–11,

$$K' = 1 + \frac{1.875 - 1}{5} = 1.175$$

The maximum net pull, by equation 27–12, is

$$F_t = \frac{175 \times 33{,}000 \times 1.175}{5{,}000} = 1{,}360 \text{ lb}$$

By equation 27–13, the thickness of the belt should be $h \leqq 0.02 \times 18 = 0.36$ in. From Table 27–1, a medium double belt with a thickness of $\frac{5}{16}$ in. may be selected. The allowable stress may be taken as $4{,}500/9 = 500$ psi. For a metal-hinge joint with an efficiency of 78 per cent, the design stress is $S_d = 500 \times 0.78 = 390$ psi.

The term $12wv^2/g$ is

$$12 \times 0.035 \times \left(\frac{5{,}000}{60}\right)^2 \times \frac{1}{32.2} = 90.6 \text{ psi}$$

The pulley-center distance may be taken as 24 ft. With an open belt, the angle of belt contact θ, by equation 27–8, is

[6] O. A. Leutwiler, *Elements of Machine Design* (New York: McGraw-Hill Book Company, Inc., 1917), p. 66.

$$\theta = \pi - \frac{64 - 18}{24 \times 12} = 2.98 \text{ radians}$$

The coefficient of friction, by Barth's formula (equation 27–6), is

$$f = 0.54 - \frac{140}{500 + 5{,}000} = 0.51$$

If this is considered as a high limit, and the value of 0.35 given in Table 27–3 for a cast-iron pulley as a rather low limit, a conservative design figure will be the mean value. Thus,

$$f = \frac{0.51 + 0.35}{2} = 0.43$$

The term $e^{f\theta} = e^{0.43 \times 2.98} = 3.63$, and the width b, from equation 27–7, is now

$$b = \frac{1{,}360 \times 3.63}{0.313 \times (390 - 90.6) \times (3.63 - 1)} = 20 \text{ in.}$$

Efficiency. The losses of power in a belt drive are due to slip; creep; bending over the pulleys; windage, or air resistance to the motion of the belt and the pulleys; and bearing friction. The loss due to slip and creep combined is, under normal conditions, about 2 per cent, and not over 3 per cent of the total power transmitted. The losses due to bending and windage are usually negligible. The bearing loss is about 1 per cent, and not over 2 per cent. Thus the losses are from 3 to 5 per cent, and the over-all efficiency is from 97 to 95 per cent.

27–6. Short-center drives. A shortening of the center distance of an open-belt drive with pulleys of different sizes decreases the angle of contact θ and increases the belt slip. By using an idler pulley c, Fig. 27–3, near the smaller pulley a, the angle of contact θ can be made greater than 180°. The idler pulley may maintain the necessary small tension F_2 either by its own weight or by additional springs e or by a weight f. In order to produce a smooth operation of the belt, idler pulleys must be machined all over and carefully balanced. The disadvantage of idler pulleys is the bending of the belt in two directions, which shortens its life. On the other hand, a smaller slack tension F_2 tends to lengthen the life.

The *Rockwood drive*, Fig. 27–4a, is a short-center drive which is superior to an idler drive because it eliminates the bending of the belt in opposite directions and automatically reduces the bearing pressure produced by the belt tension when the net belt pull decreases. It was originally designed as a mounting for electric motors, but it can be used for other drives as well. The platform f is so pivoted on the axis e that the weight of the motor produces the necessary belt tension. This tension is adjusted by moving the motor with respect to e and thus changing the moment arm a. The axis e, in turn, can be moved with respect to the axis of the driven shaft. The drive is made in sizes up to 100 hp. It can be used also for an inclined or vertical drive. To make the installation more compact the position of the platform can be made vertical, as in Fig. 27–4b.

In designing a Rockwood drive, it is first necessary to assume a suitable value for the ratio F_1/F_2. For a given power to be transmitted, the higher the value of this ratio, the lower will be the belt tensions and the bearing loads. For oak-tanned belts, recommended values of F_1/F_2 vary from 3 for $\theta = 120°$ up to 5 for $\theta = 180°$. For high-strength belts these values may be increased about 25 per cent.[7]

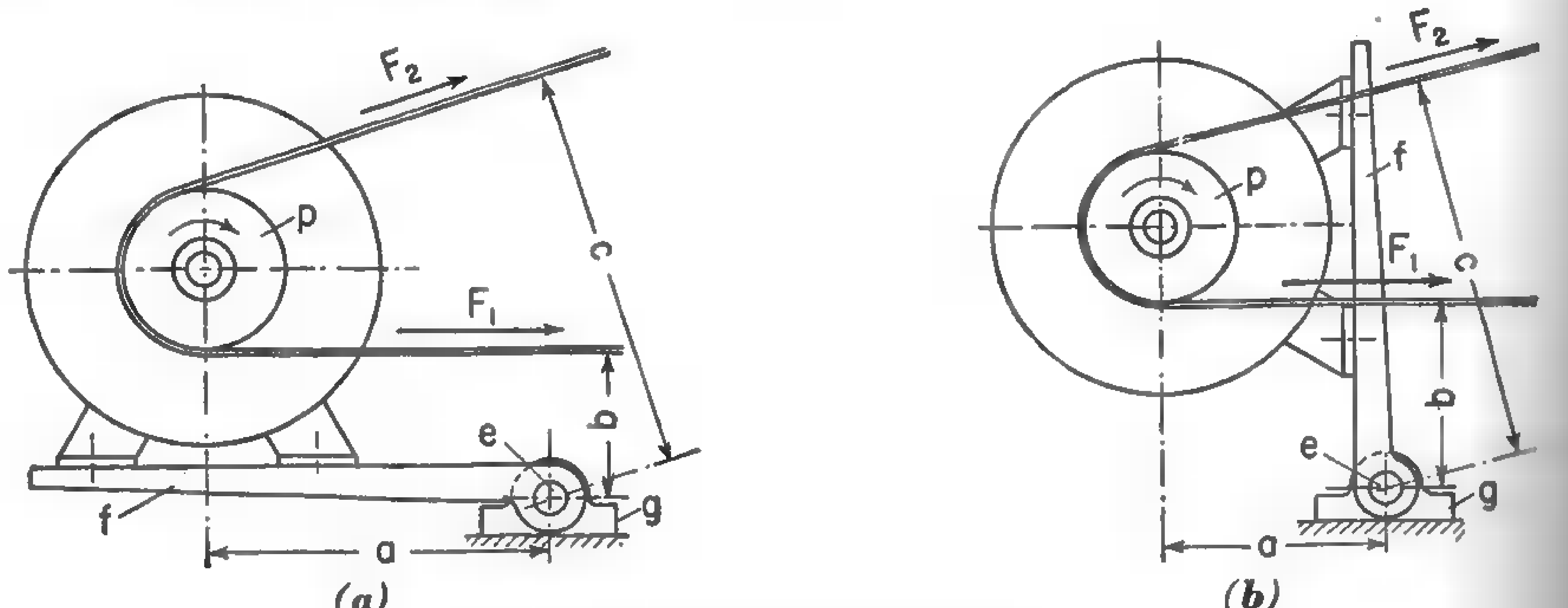


FIG. 27-4. Rockwood drives.

If F_t is the required net pull, and W is the weight of the motor, and a, b, and c are distances indicated in Fig. 27-4, and if moments are taken with respect to the pivot e, the following equations may be obtained for the tensions F_1 and F_2:

$$F_1 = \frac{aW + cF_t}{b + c} \tag{27-15}$$

and

$$F_2 = \frac{aW - bF_t}{b + c} \tag{27-16}$$

On the other hand, the required pivot-arm length a may be computed by dividing equation 27-15 by equation 27-16 and solving the resulting equation. Thus,

$$a = \frac{F_t(bF_1/F_2 + c)}{W(F_1/F_2 - 1)} \tag{27-17}$$

The distances b and c must be taken from a general layout; the approximate value for a is then computed from equation 27-17; and a final layout of the drive is made. This layout will give an accurate distance c. It is then advisable to check the arm length a by equation 27-17, and the tensions F_1 and F_2 by equations 27-15 and 27-16. In any case, the arm length a and the position of the pivot e must be made adjustable to take care of a possible variation in the weight W and a stretching of the belt.

EXAMPLE 27-2. Design a Rockwood drive for a 25-hp motor having a speed of 1,750 rpm and weighing 380 lb. The motor must drive a jackshaft at 600 rpm.

A belt speed v_m of 4,500 fpm may be selected. The motor-pulley diameter is then

[7] R. R. Tatnall, "The Pivoted Motor Drive," *Mechanical Engineering*, Vol. 57 (1935), p. 287.

$$D_1 = \frac{4{,}500 \times 12}{1{,}750\pi} = 9.8, \text{ or } 10 \text{ in.}$$

The net pull is found from the usual relation and is

$$F_t = \frac{33{,}000P}{v_m} = \frac{33{,}000 \times 25}{4{,}500} = 183.5 \text{ lb}$$

From a preliminary sketch, with $b = 12$ in. and $D_1 = 10$ in., it is found that $c = 25$ in. Next, select the ratio F_1/F_2 as 5. Then, by equation 27-17,

$$a = \frac{183.5 \times (12 \times 5 + 25)}{(5 - 1) \times 380} = 10.27 \text{ in.}$$

A final layout shows that a more accurate value for c is 24.75 in. This change reduces length a to 10.25 in.

The tensions found by equations 27-15 and 27-16 are

$$F_1 = \frac{10.25 \times 380 + 24.75 \times 183.5}{12 + 24.75} = 229.5 \text{ lb}$$

and

$$F_2 = \frac{10.25 \times 380 - 12 \times 183.5}{36.75} = 46.0 \text{ lb}$$

A check gives

$$F_t = F_1 - F_2 = 229.5 - 46.0 = 183.5 \text{ lb}$$

and

$$\frac{F_1}{F_2} = \frac{229.5}{46.0} = 4.99$$

These results are in satisfactory agreement with the design data.

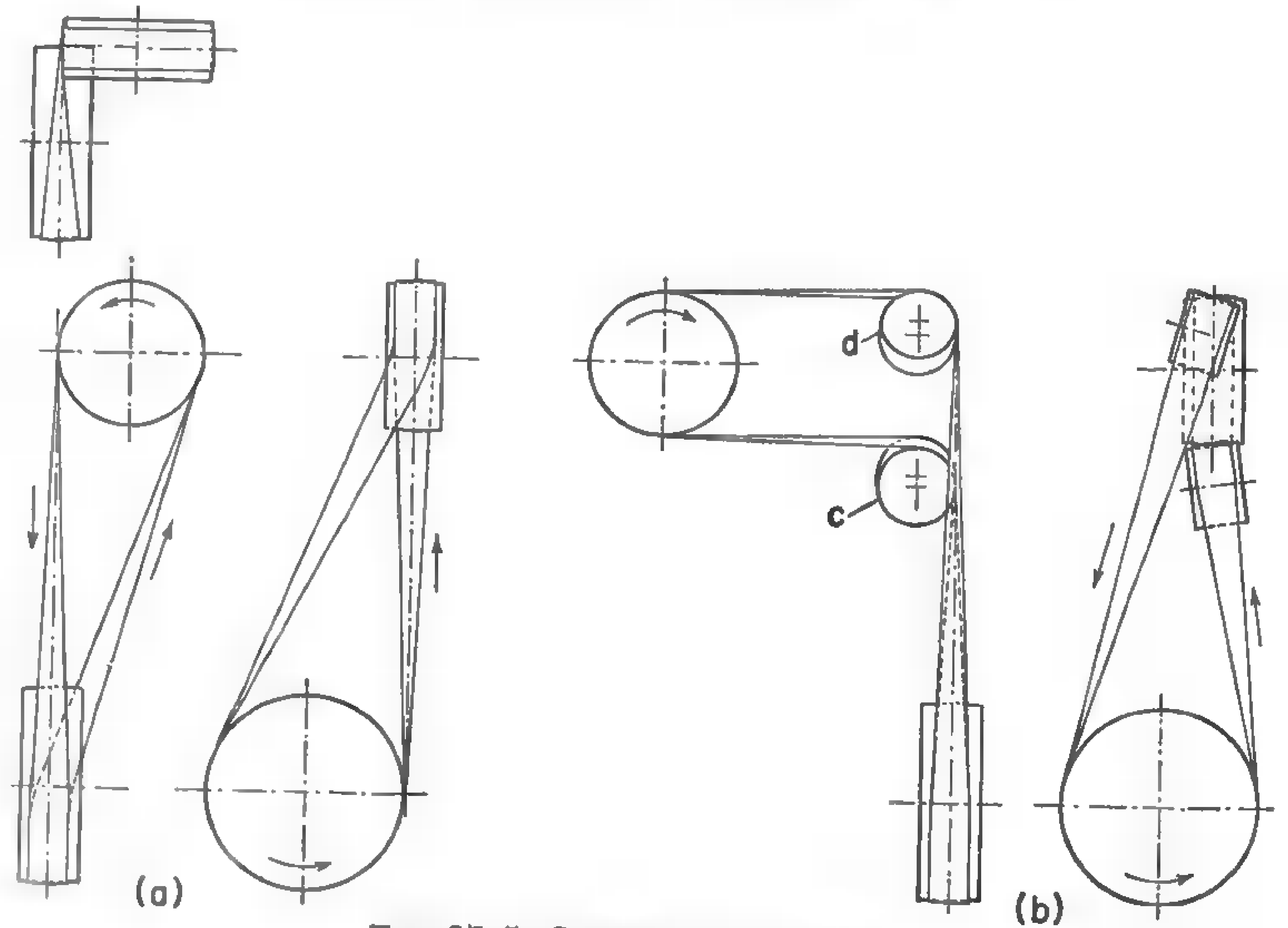


FIG. 27-5. Quarter-turn drives.

27-7. Quarter-turn drives. A quarter-turn belt is used to connect two shafts whose axes are in different planes. Usually one shaft is horizontal and the other is vertical.

Law of belting. In order to stay on the pulleys, a belt must approach each pulley in a plane normal to its axis of rotation. If the approaching

side is deflected, the belt will run off. In Fig. 27–5a this law is illustrated on a quarter-turn belt used for connecting two shafts with axes in planes normal to each other. A greater freedom in the location of pulleys is obtained by using guide pulleys, as c and d in Fig. 27–5b, and the drive may be made reversible.

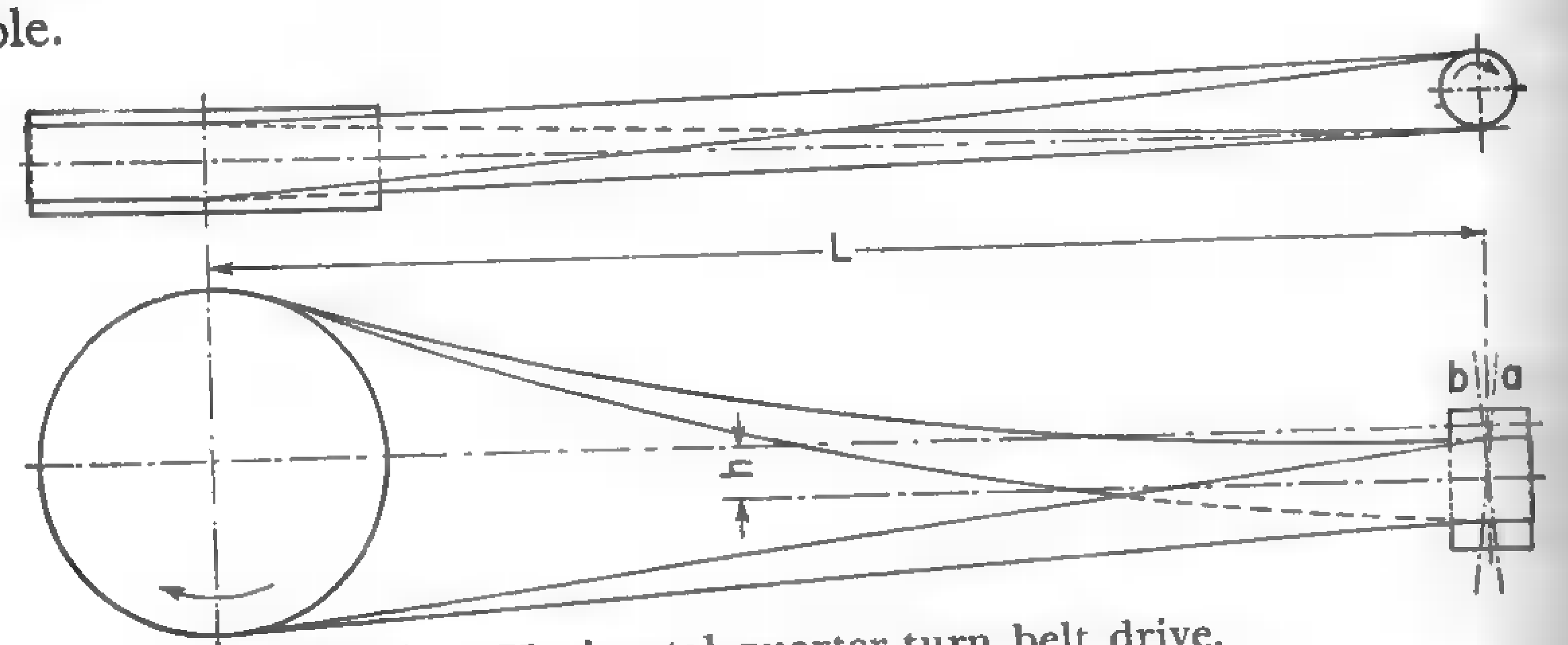


FIG. 27–6. Horizontal quarter-turn belt drive.

In a horizontal quarter-turn drive, such as is encountered in centrifugal deep-well pumps, Fig. 27–6, the height h at which the belt rides on the vertical pulley depends on the tension and stretching of the belt and also on the least deviation of the axis of the vertical pulley from a strictly vertical position. Thus if the axis assumes position a, the height h decreases and the belt goes up; if the axis moves to position b, the height h increases and the belt drops. A rule of thumb, good for normal conditions only, is to make h equal to $\frac{5}{8}$ in. per foot of center distance L. Thus,

$$h = 0.052L \qquad (27\text{–}18)$$

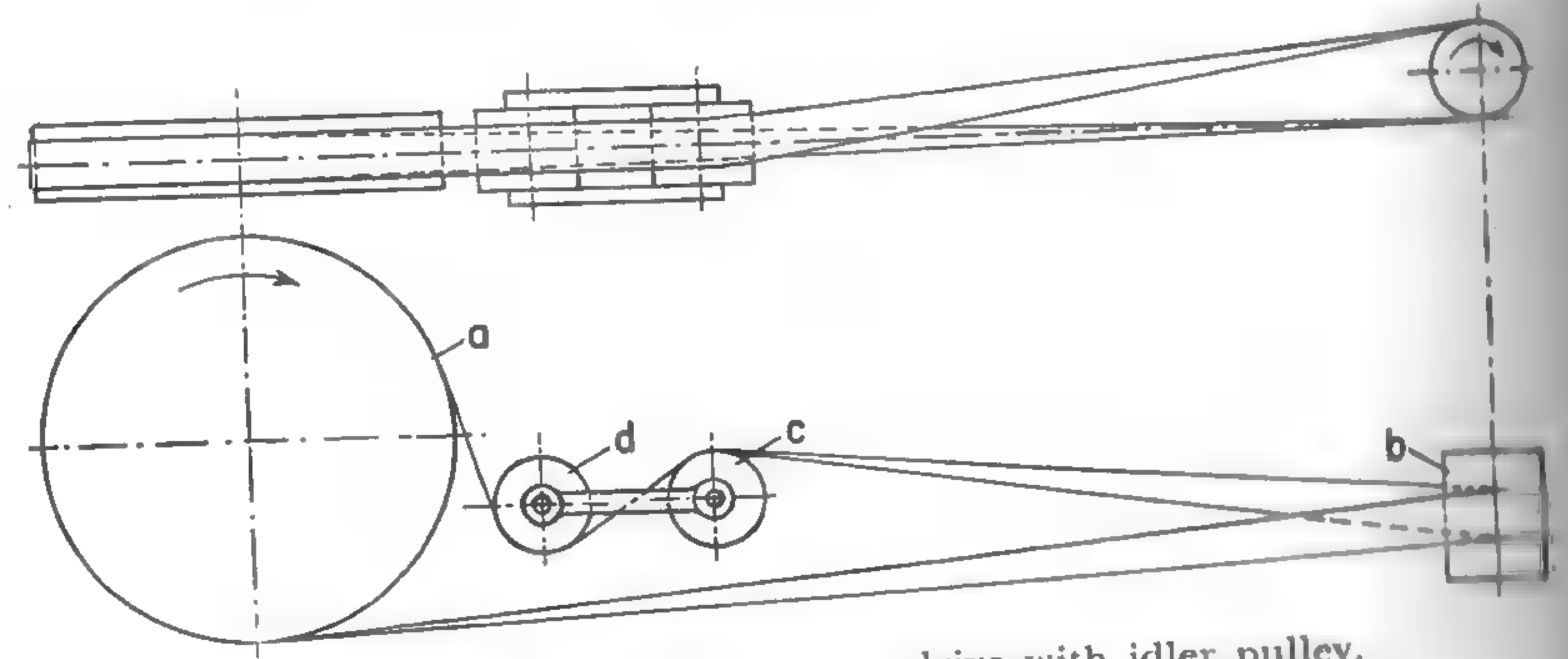


FIG. 27–7. Horizontal quarter-turn drive with idler pulley.

Because of this uncertainty in regard to h, the vertical pulley must have a very large face. The installation is much simplified by the use of a double pulley idler, Fig. 27–7. The pulley c, which is adjustable vertically, helps to bring the belt to the desired location on the driven pulley b; the idler pulley d is a regular idler pulley which, in this case, only maintains the necessary tension in the slack side of the belt.

27–8. Materials for pulleys. Cast iron is almost an ideal material for belt pulleys since it can be cast in any desired shape, is readily machined, and gives pulleys which do not change their form. Solid cast-iron pulleys can be used with a rim speed up to 5,000 fpm; when they are made of a better grade of cast iron the speed may be 5,500 fpm. Split pulleys with the split through the arms, as in Fig. 27–9c, can be used up to 4,500 fpm; and split types of ordinary construction, as in Fig. 27–9b, can be used up to 4,000 fpm.

Pressed-steel pulleys are lighter and less expensive than cast-iron pulleys but may not run quite as truly.

Wooden pulleys have a rim built up of maple segments. They are usually of the split type and can be obtained in sizes up to 48 in. in diameter, and up to a 12-in. face. They are apt to warp from change of atmospheric conditions and are therefore not suitable for high speeds.

Paper pulleys are used rather extensively where a higher transmitting capacity is essential. Such a pulley consists of a web built up of thin sheets of straw fiber cemented together under a high pressure and bolted to a cast-iron hub. They are obtainable in sizes up to 18 in. in diameter.

Cork-insert pulleys, made by pressing cork inserts into countersunk holes in cast-iron pulleys, are used to take advantage of the high coefficient of friction between a belt and cork. The inserts should not protrude over $\frac{1}{32}$ in. These pulleys are generally made in small sizes, up to 14 in. in diameter. Cork inserts do not help wood pulleys and paper pulleys.

27–9. Design of cast-iron pulleys. Only the features of cast iron pulley design will be considered here.

Face. After the diameter of a pulley has been determined by considering the selected belt velocity and taking the slip into account, the face B of the pulley, in inches, may be computed by Barth's formula. This formula is

$$B = 1.094b + 0.188 \qquad (27\text{–}19)$$

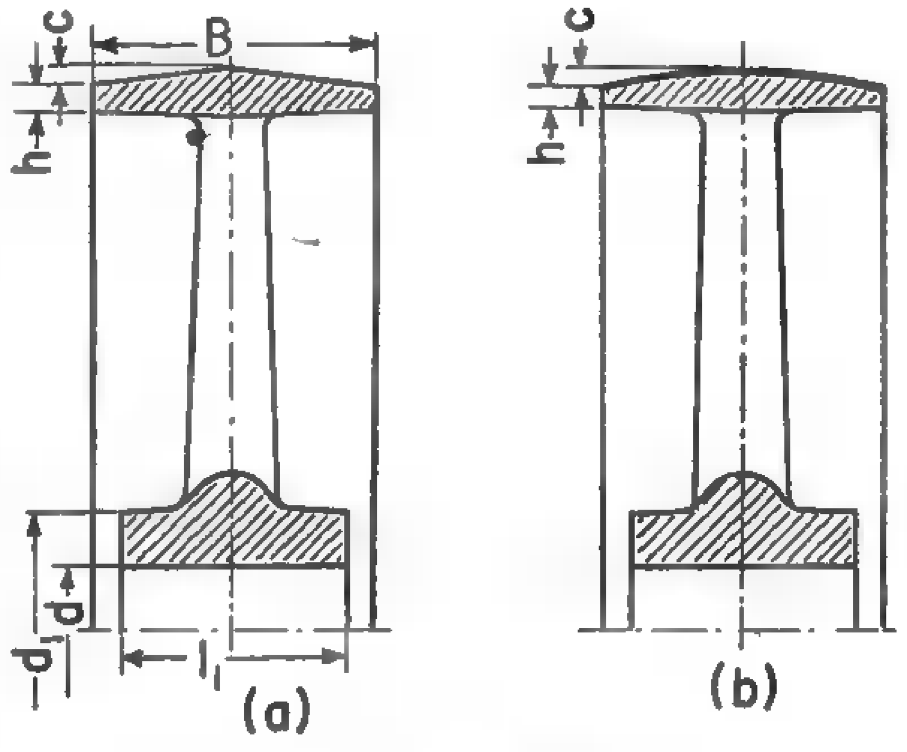


FIG. 27–8. Crowned pulleys.

where b is the belt width corrected to a commercial size. For simplicity, B may be taken as $1.1b$.

Crown. If a belt is led to a revolving conical pulley, it will tend, because of its lateral stiffness, to climb higher and higher upon the cone. This tendency is utilized in the principle of crowning, to keep belts in position. A crown may be formed either in a very flat inverted-V form, as in Fig. 27–8a, or with a convex curve, as in Fig. 27–8b. The V form is easier to machine, while the curved face has the advantage of stretching the belt

more uniformly. The usual crown height is $c = \frac{1}{8}$ in. per 12 in. of face width, but pulleys for very wide belts are given a relatively smaller crown height. Barth's empirical formula is

$$c = \frac{1}{32}B^{2/3} \tag{27–20}$$

This gives practically the same results as the first, simpler rules.

For rubber belts on well-aligned shafts,

$$c = \frac{B}{200} \tag{27–21}$$

For poorly aligned shafts,

$$c = \frac{B}{120} \tag{27–22}$$

An idler pulley should never be crowned, since its crown would bend the belt in the opposite direction. The pulley next to an idler should also have a straight face or only a small crown. When a pair of pulleys are connected by a belt, it is sufficient to crown only one pulley, usually the driven one. If both pulleys are crowned, the pulleys must be lined up very carefully so as to have both crowns in one plane and thus avoid sidewise distortion of the belt. Pulleys carrying shifting belts are not crowned.

Rim. For a light pulley, the thickness h of the rim at the edge, in inches, may be determined by the relation

$$h = 0.05\sqrt{D} + \frac{1}{16} \tag{27–23}$$

For a heavy-duty pulley for a triple belt, the thickness may be as great as

$$h = 0.075\sqrt{D} + \frac{1}{8} \tag{27–24}$$

The stress in the rim should be checked by equation 26–13 in conjunction with equations 3–5 and 26–12. If it is high, the thickness h, and perhaps also the number of arms, should be increased.

Hub. The diameter of the hub, in inches, is given by the relation

$$d_1 = 1.5d + 1 \tag{27–25}$$

where d is the shaft diameter. The hub length l_1 is made equal to $\frac{3}{4}B$, but not less than $1.5d$. The hub is made either solid, with a taper key, or split

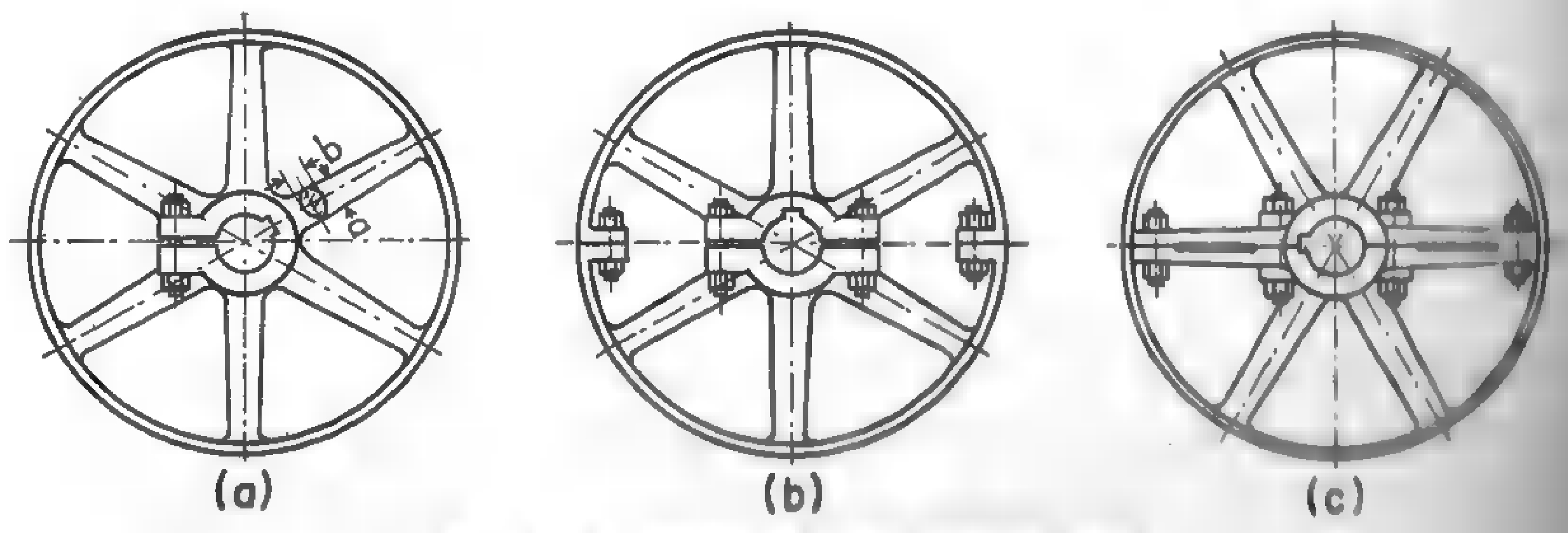


FIG. 27–9. Types of cast-iron pulleys

TABLE 27–5

RECOMMENDED TYPES OF KEYS FOR BELT PULLEYS

Diameter of Pulley (in.)	Split Pulleys, Hubs Bored for Clamping						Solid Pulleys			
	Width of Face (in.)						Width of Face (in.)			
	1–4	4–7¾	8–11¾	12–15¾	16–19¾	20–26	1–4	4–7¾	8–11¾	Over 12
Under 20	No Key, Only Setscrews	No Key, Only Setscrews	No Key, Only Setscrews	Flat Key	Flat Key	Square Key	Saddle Key	Saddle Key	Flat Key	Square Key
20–24	No Key, Only Setscrews	No Key, Only Setscrews	No Key, Only Setscrews	Flat Key	Square Key	Square Key	Saddle Key	Saddle Key	Flat Key	Square Key
25–32	No Key, Only Setscrews	No Key, Only Setscrews	Flat Key	Flat Key	Square Key	Square Key	Saddle Key	Flat Key	Flat Key	Square Key
32–40	No Key, Only Setscrews	Flat Key	Flat Key	Square Key	Square Key	Square Key	Flat Key	Flat Key	Square Key	Square Key
40–50	Flat Key	Flat Key	Flat Key	Square Key	Square Key	Square Key	Flat Key	Square Key	Square Key	Square Key
50–64	Flat Key	Flat Key	Square Key	Square Key	Square Key	Square Key	Square Key	Square Key	Square Key	Square Key
64–80	Flat Key	Square Key	Square Key	Square Key	Square Key	Square Key	Square Key	Square Key	Square Key	Square Key
Over 80	Square Key	Square Key	Square Key	Square Key	Square Key	Square Key	Square Key	Square Key	Square Key	Square Key

on one side, as in Fig. 27–9a. In this case it is bored slightly smaller than the shaft diameter (class 6 fit), and a wedge is used to open it when the hub is being put on the shaft. Sometimes a split-clamp hub, Fig. 27–9b, is used with a solid rim, the bore being made a class 6 or 5 fit.

Table 27–5 gives the type of key to be used with belt pulleys, according to German industrial standards (DIN).

Arms. Small pulleys are made with a web instead of arms. Pulleys from 6 to 18 in. in diameter have four arms; those from 19 to 60 in. in diameter have six arms; and those larger than 60 in. have eight arms. Pulleys with a face width up to 22 or 24 in. have one set of arms. Pulleys having a face of 24 in. or wider are made with two sets of arms.

The necessary arm section at the hub may be calculated by assuming that the arms act as cantilevers subjected to a load equal to the net belt pull F_t. If it is assumed that only one-half the total number i of the arms carry the load F_t, the bending moment on each arm is approximately

$$M = \frac{F_t \times 0.5D}{0.5i} = \frac{F_t D}{i} \tag{27–26}$$

By equation 2–22 $S_d = M/Z$, where Z is the section modulus of the arm at the hub. Therefore

$$Z = \frac{F_t D}{i S_d} \tag{27–27}$$

In the usual elliptic section of the arm, Fig. 27–9a, the major axis a is twice the minor axis b, or $a = 2b$, and the section modulus is

$$Z = \frac{1}{32}\pi a^2 b = \frac{1}{64}\pi a^3 \tag{27–28}$$

Substituting this value in equation 27–27, and solving for a, results in

$$a = 2.73 \sqrt[3]{\frac{F_t D}{i S_d}} \tag{27–29}$$

This determines the dimensions at the hub. The arms are tapered, with a decreasing at the rate of $\frac{1}{4}$ to $\frac{3}{8}$ in. per foot toward the rim. The design stress S_d may be taken from 2,000 to 3,000 psi.

Example 27–3. Determine the dimensions of a cast-iron pulley which is 48 in. in diameter and transmits 100 hp at 200 rpm. It is to be used with a heavy double belt.

The rim face, by equation 27–19, is

$$B = 1.094 \times 14 + 0.188 = 15\tfrac{1}{2} \text{ in.}$$

The thickness of the rim must be, by equation 27–24,

$$h = 0.075\sqrt{48} + \tfrac{1}{8} = 0.520 + 0.125 = 0.645, \text{ or } \tfrac{11}{16} \text{ in.}$$

If a $3\frac{7}{16}$-in. shaft is used, the hub diameter should be, by equation 27–25,

$$d_1 = 1.5 \times 3.437 + 1 = 6.16, \text{ or } 6\tfrac{1}{4} \text{ in.}$$

The hub length must be

$$l' = \tfrac{2}{3}B = \tfrac{2}{3} \times 15.5 = 10.34, \text{ or } 10\tfrac{1}{2} \text{ in.}$$

The pulley must have six arms. The belt velocity is

$$v_m = \tfrac{1}{12}\pi \times 48 \times 200 = 2{,}512 \text{ fpm}$$

and the net pull is

$$F_t = \frac{33{,}000P}{v_m} = \frac{33{,}000 \times 100}{2{,}512} = 1{,}313 \text{ lb}$$

If the safe stress is taken as $S_d = 2{,}500$ psi, the major axis of the elliptic section of each arm at the hub is, by equation 27–29,

$$a = 2.73\sqrt[3]{\frac{1{,}313 \times 48}{6 \times 2{,}500}} = 4.41, \text{ or } 4\tfrac{1}{2} \text{ in.}$$

Then
$$b = 0.5a = 2\tfrac{1}{4} \text{ in.}$$

At the rim, $a' = 4\frac{1}{2} - \frac{1}{4} \times 2 = 4$ in., and $b' = 0.5 \times 4 = 2$ in.

Example 27–4. Check the stress in the rim in example 27–3.

By equation 3–5, the tensile stress is

$$s_1 = \frac{0.000914 \times 0.25 \times (24 - 0.5 \times 0.688)^2 \times 200^2}{32.2} = 164 \text{ psi}$$

By equation 26–12, the bending stress is

$$s_2 = \frac{0.0181 \times 0.26 \times 23.656^3 \times 200^2}{6^2 \times 0.688 \times 32.2} = 3{,}120 \text{ psi}$$

In order to take into account the stiffening influence of the arms, the combined tensile stress is found by equation 26–13:

$$s = 0.75 \times 164 + 0.25 \times 3{,}120 = 123 + 780 = 903 \text{ psi}$$

The stiffening influence of the arms is greater on a pulley rim than on a heavy flywheel rim, and experience confirms that a pulley with the rim thickness just determined is entirely satisfactory.

Loose pulleys. By shifting a belt from a pulley a, Fig. 27–10a, fastened to the shaft, to a pulley b which is loose on the shaft, or back, the same result may be obtained as with a friction clutch. The hub of the loose pulley must have a bushing c. To decrease the wear the loose pulley is often mounted on ball bearings or roller bearings. In order to decrease the tension in the belt while it is standing still on the loose pulley, the latter's diameter may be made somewhat smaller than that of the tight pulley, as in Fig. 27–10b, and a taper flange may be provided for shifting the belt back to the tight pulley.

27–10. V belts. The gripping action between the belt and the groove enables ropes and wedge-shaped belts, running in V-grooved pulleys, to transmit large amounts of power with a relatively small initial tension. However, only after the development of the so-called Texrope Drive, which consists of special fabric-and-rubber belts of trapezoidal cross section running on sheaves with grooves, did this method come into wide use. At present a number of rubber-belt manufacturers make V belts. Although the belts of different makes differ in details, the principle is the same. A cord core a, Fig. 27–11, transmits the power, c is a cushion and compression member, and e is the outer rubber-and-fabric wrapping. The belts are made in five standard sections, as shown in Table 27–6. They are made endless at the factory in many standard lengths, the lower and upper limits also being given in Table 27–6, together with the maximum number of strands used.

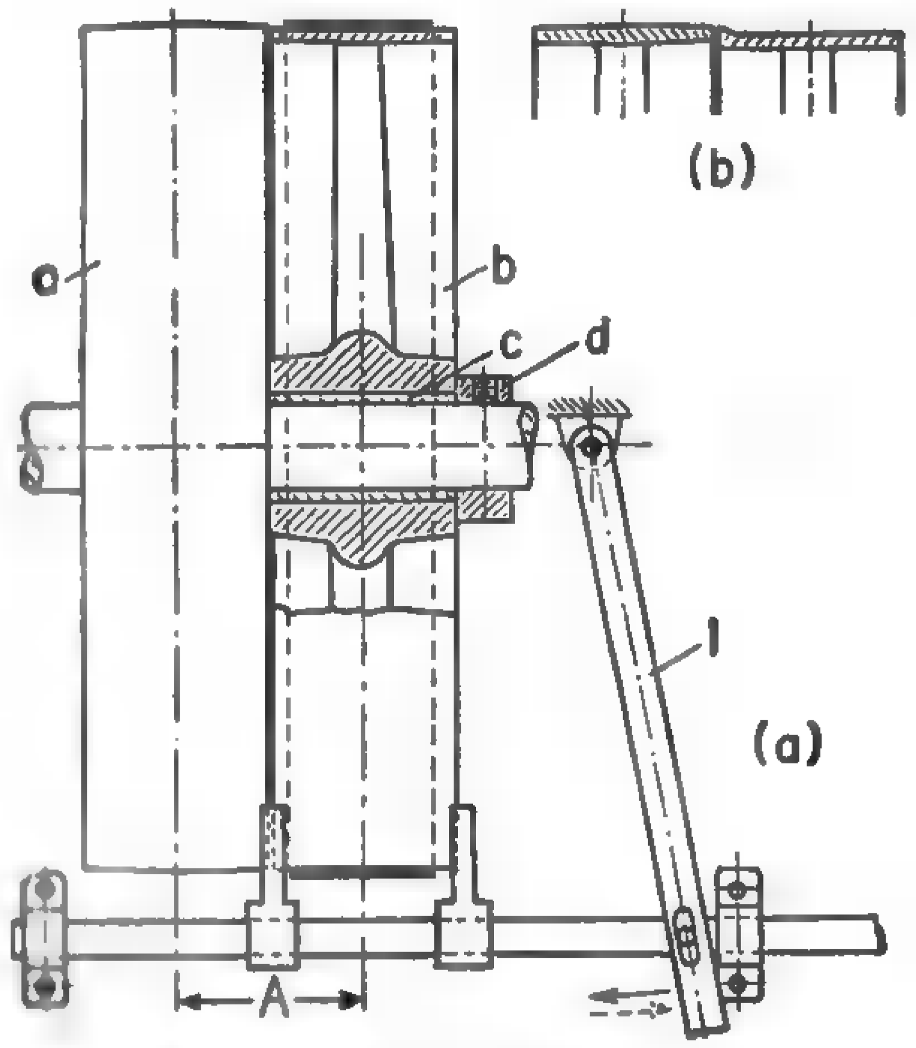


Fig. 27–10. Tight and loose pulleys.

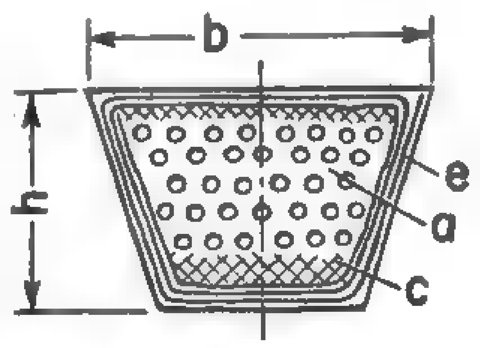


Fig. 27–11. V belt.

The wedging effect between a V belt and the sheave increases the friction considerably, so that the drive can work with small arcs of contact and a low initial tension. This makes the drive particularly suitable for a short-center arrangement of pulleys without an idler and with speed ratios as high as 10 to 1. The speed ratios are computed on the basis of the pitch diameters of the pulleys, each of which is equal to the outside diameter minus the belt

TABLE 27–6

STANDARD SIZES OF V BELTS

Section Number	Width b (in.)	Thickness h (in.)	Stock Pitch Length (in.) Minimum	Stock Pitch Length (in.) Maximum	Recommended Horsepower Ranges	Recommended Maximum Number of Strands
A	$\frac{1}{2}$	$\frac{11}{32}$	26.9	129	0.5–5	6
B	$\frac{21}{32}$	$\frac{7}{16}$	36.1	300	2–20	9
C	$\frac{7}{8}$	$\frac{17}{32}$	52.6	420	15–100	14
D	$1\frac{1}{4}$	$\frac{3}{4}$	122.4	600	50–200	14
E	$1\frac{1}{2}$	1	183.3	660	100–350	20

thickness. Furthermore, the position of the driving pulley with respect to the driven one makes no difference; the drive operates equally well with horizontal or vertical belts and with the slack side on either the top or the bottom. Other advantages are absence of vibration and noise; ability to absorb shocks; high efficiency (about 98 per cent), because of absence of slip; and great dependability, since the breakage of one of the strands merely shifts the load to the remaining ones until replacement can be made.

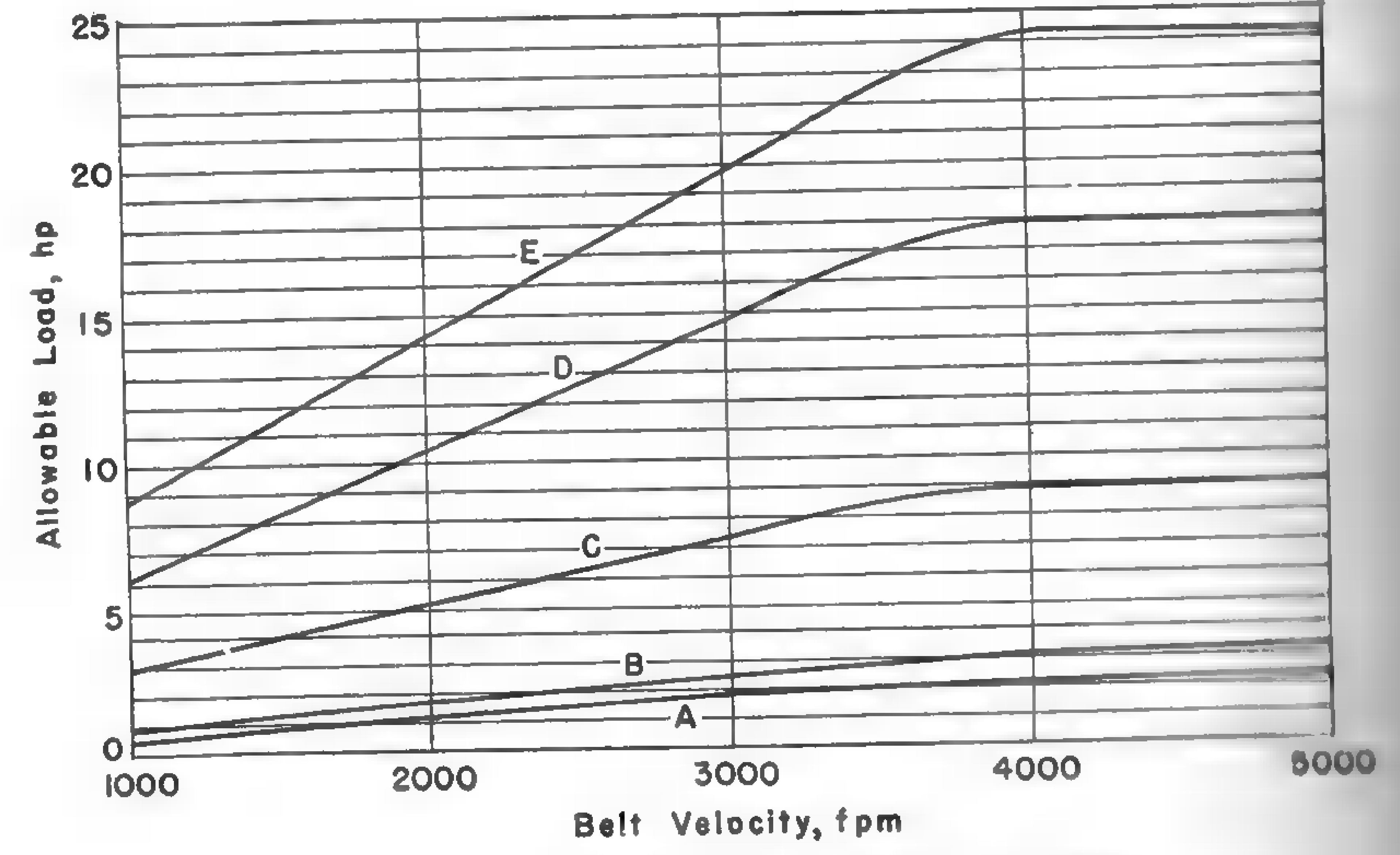


FIG. 27–12. Allowable loads per strand of V belts.

27–11. V-belt drives. In designing V-belt drives, the catalogues of manufacturers of belts and pulleys should be consulted.

Belt selection. Catalogues of the manufacturers of V belts give the necessary instructions for selecting the belt size, pulley sizes, center distance, and number of belts to be used. For a preliminary design the permissible loads per belt can be taken from Fig. 27–12. The curves given there take into

consideration the belt speeds, but they are plotted for an arc of contact of 180°. The capacities thus obtained must be multiplied by a correction factor found by the relation

$$C = \frac{(e^{f\theta} - 1)e^{f\pi}}{e^{f\theta}(e^{f\pi} - 1)} \tag{27–30}$$

where θ is the arc of contact of the smaller pulley and the friction coefficient f may be taken conservatively as 1.0. The presence of a great starting torque, of shocks, and of possible overloads is taken into account by dividing the capacity by a load factor K'. The values of K' may be computed by equation 27–11.

Pulleys. Pressed and welded steel pulleys can be obtained from several manufacturers in a great variety of diameters and number of grooves for the belt sections No. A and No. B. Cast-iron pulleys for all belt sections are also carried in stock and can be made by any machine shop in accordance with data in the catalogues of belt manufacturers. The angle of grooving a, Fig. 27–13, is made from 30° to 38°. In Fig. 27–14 the solid curves give the recommendations of one belt manufacturer, and the dotted lines give those of another one. In either case the angle depends on the pulley diameter. The width of the groove b, Fig. 27–13, is made equal to the width of the belt; the lands c are made $\frac{3}{32}$ in. for the No. A section and gradually increase to about $\frac{1}{4}$ in. for the No. E section; the depth h_1 is made $\frac{3}{16}$ to $\frac{1}{4}$ in. greater than the thickness h of the belt. The sides of the grooves must be finished and smoothly polished to prevent excessive wear of the belt.

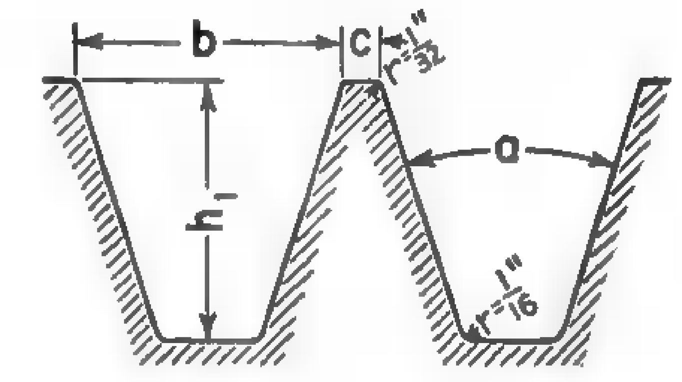


FIG. 27–13. Groove profile.

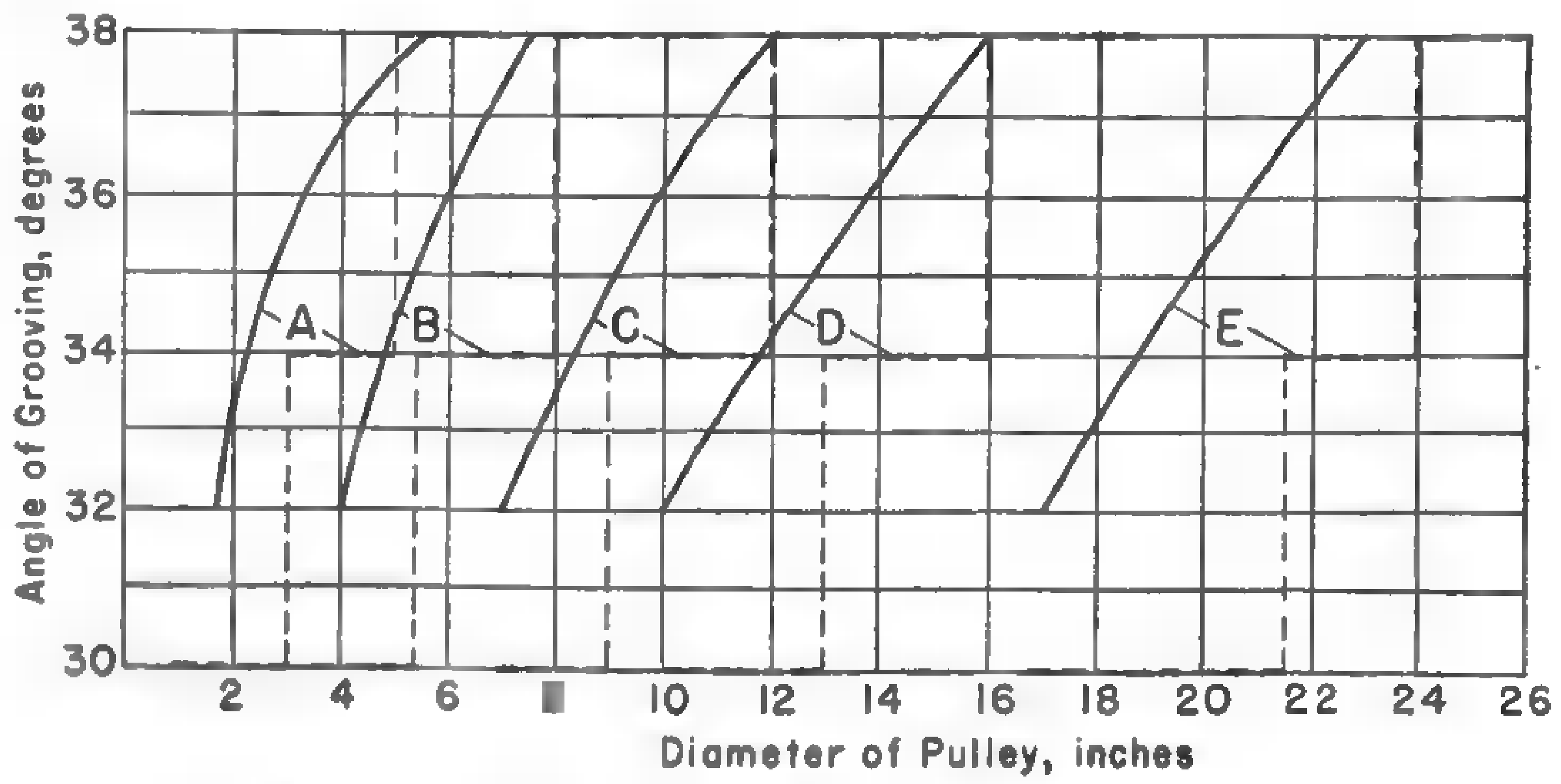


FIG. 27–14. Chart for determination of groove angle.

The recommended minimum pulley diameters, referred to the pitch line, are indicated in Fig. 27–14 for the five belt sizes. However, the use of larger pulleys will increase the belt life.

The *center distance* should be slightly larger than the diameter D_1 of the larger pulley and slightly smaller than the sum of the diameters of both pulleys. However, both shorter and longer center distances may be used if required. On high-speed drives short center distances give smoother running.

V flat drives. On V-belt drives with a speed ratio of 3 to 1 or more, and with short center distances, the grooving of the larger pulley can be omitted without sacrifice of power or efficiency. A belt has its maximum pulling power when the arc of contact on the large pulley is 240° to 250°. This angle is obtained when the following equation is satisfied:

$$\frac{D_2 - D_1}{L} \geqq 0.5 \qquad (27\text{–}31)$$

where D_2, D_1, and L have the meanings shown in Fig. 27–1a.

In calculating the speed ratios, it should be remembered that the pitch diameter of a flat pulley is equal to its outside diameter plus the belt thickness. The use of V flat belts is increasing very rapidly, particularly for low and medium torques.

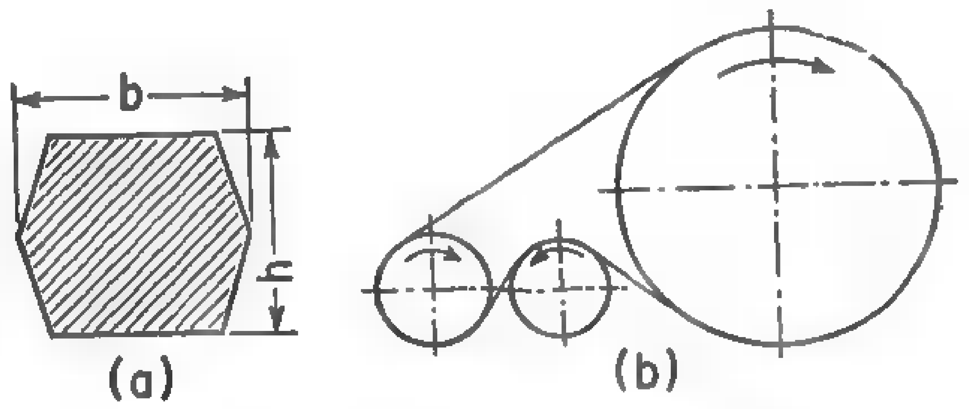


Fig. 27–15. Double-V belt drive.

Double-V belts. A double-V belt, Fig. 27–15a, is used when power must be transmitted to grooved pulleys from both the top and bottom of the belt, as in Fig. 27–15b. Such belts are made to fit standard V grooves.

Quarter-turn drives. Quarter-turn drives with V belts are being used both with and without idlers.[8] An idler may be omitted if the pulleys are relatively small and a take-up can be provided on one of the shafts. The minimum center distance must be equal to $6(D+B)$, where D is the diameter of the larger pulley and B is its width. With a larger speed ratio the shortest center distance becomes too long, and an idler should be used. The latter can also serve to obtain the necessary take-up for the belts. The center distance between the idler and the small pulley should be not less than $8B$. Flat-faced idler pulleys may be used when the belts move from the idler pulley toward the quarter turn. If the belts move from a quarter turn toward a flat-faced pulley, the belt strands will squeeze together and pile up. Therefore the idler must be grooved.

The horsepower rating of a V belt used on a quarter-turn drive should be taken as 75 per cent of that of a straight drive.

[8] The Gates Rubber Company, *Complete Guide for Selecting or Designing V-Belt Drives* (Denver: 1950), p. 44.

CHAPTER 28

Chain Drives

28–1. General considerations. The types of chains used for power transmission are the *block chain*, the *roller chain*, and the *inverted-tooth chain*, usually called the *silent chain.*

Block chains. Block chains, Fig. 28–1, are used only for transmitting power at a low rate and at moderate speeds, not exceeding 900 fpm. With a steady load they can be used up to 12 hp. With a fluctuating load, their limit is not over 3 or 4 hp.

Roller- and silent-chain drives. Advantages of roller-chain and silent-chain drives are as follows:

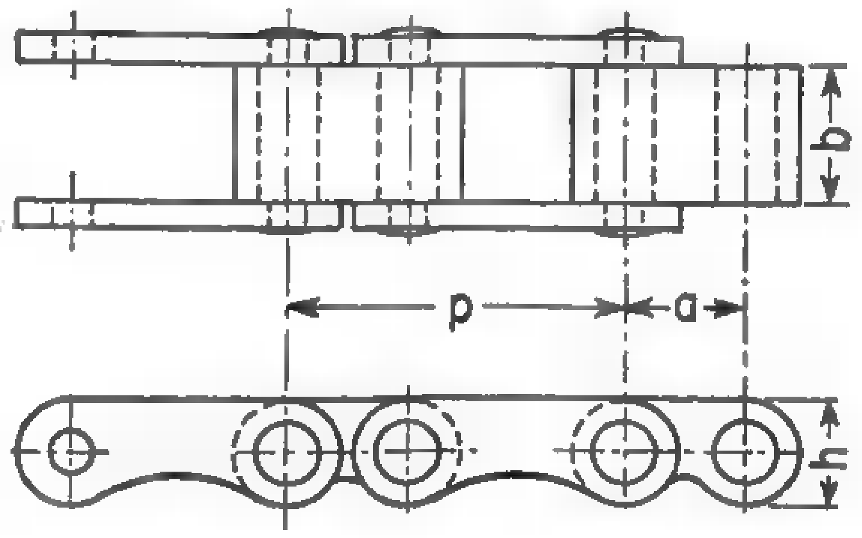


Fig. 28–1. Block chain.

a) They are compact because they make it possible to use very short center distances.

b) They permit greater flexibility in locating sprocket shafts, as the shafts need not be aligned perfectly.

c) They have a positive speed ratio, with no slippage.

d) They permit the use of a large speed ratio—8 to 1 or even 10 to 1—in one step.

e) They cause small pressures on the bearings, because the tension on the slack side is produced only by the weight of the chain.

f) Their mechanical efficiency is high, seldom less than 98 per cent.

g) They are immune to the effects of temperature changes, moisture, or fumes.

h) Their maintenance cost is low, adjustments are rarely necessary, and lubrication is simple.

Because of high speeds used in chain drives, all parts of these chains are accurately machined, and those parts which are subject to sliding action are hardened.

28–2. Roller chains. The construction of a roller chain is shown in Fig. 28–2. In order to obtain interchangeability of roller chains and sprockets, regardless of make, the American Society of Mechanical Engineers, the Society of Automotive Engineers, and the American Gear Manufacturers Association have jointly developed standard dimensions for roller chains. However, this standard list includes more sizes than are actually necessary.

TABLE 28-1

AMERICAN STANDARD STEEL ROLLER CHAINS

STANDARD NUMBER	PITCH p (IN.)	ROLLER SIZE: Diameter d (in.)	ROLLER SIZE: Width l (in.)	PIN DIAMETER d_2 (in.)	ULTIMATE STRENGTH FOR ALLOY STEEL (lb)	WEIGHT PER FOOT w_f (lb)	MAXIMUM SPEED n (RPM): 12 Teeth	18 Teeth	24 Teeth	NUMBER OF STRANDS MADE
35....	$\frac{3}{8}$	0.200	$\frac{3}{16}$	0.141	2,100	0.22	2,380	3,780	4,200	1–4(6)
41....	$\frac{1}{2}$	0.306	$\frac{1}{4}$	0.141	2,000	0.28	1,750	2,730	2,850	1
40....	$\frac{1}{2}$	0.312	$\frac{5}{16}$	0.156	3,700	0.41	1,800	2,830	3,000	1–4(7)
50....	$\frac{5}{8}$	0.400	$\frac{3}{8}$	0.200	6,100	0.66	1,300	2,030	2,200	1–4(6)
60....	$\frac{3}{4}$	0.469	$\frac{1}{2}$	0.234	8,500	1.03	1,030	1,620	1,700	1–4(6)
80....	1	0.625	$\frac{5}{8}$	0.312	14,500	1.69	750	1,160	1,310	1–4(6)
100....	$1\frac{1}{4}$	0.750	$\frac{3}{4}$	0.375	24,000	2.58	545	835	945	1–4
120....	$1\frac{1}{2}$	0.875	1	0.437	34,000	3.75	410	630	710	1–4
140....	$1\frac{3}{4}$	1.000	1	0.500	46,000	4.66	270	450	500	1–4
160....	2	1.125	$1\frac{1}{4}$	0.562	58,000	6.50	260	400	450	1–4
180....	$2\frac{1}{4}$	1.408	$1\frac{13}{32}$	0.687	74,000	9.06	215	330	370	1–4
200....	$2\frac{1}{2}$	1.562	$1\frac{1}{2}$	0.781	95,000	11.10	185	285	320	1–4
240....	3	1.875	$1\frac{7}{8}$	0.937	130,000	16.70	145	225	250	1–4

Table 28–1 contains a list of standard sizes which is recommended by one of the foremost chain-makers and which, in conjunction with multiple-strand construction, will meet practically all requirements for the roller chain as a power-transmission medium.

Multiple-strand chains. In order to increase the load capacity of a chain with a certain pitch, several chain strands are assembled side by side by using long through-pins. The capacity of a multiple-strand chain is equal to the capacity of a single strand multiplied by the number of strands. The usual limits for the number of strands are given in the last column of Table 28–1, although some chain-makers go as high as eight strands. If the capacity of a multiple-strand chain is not sufficient, two chains may be run side by side.

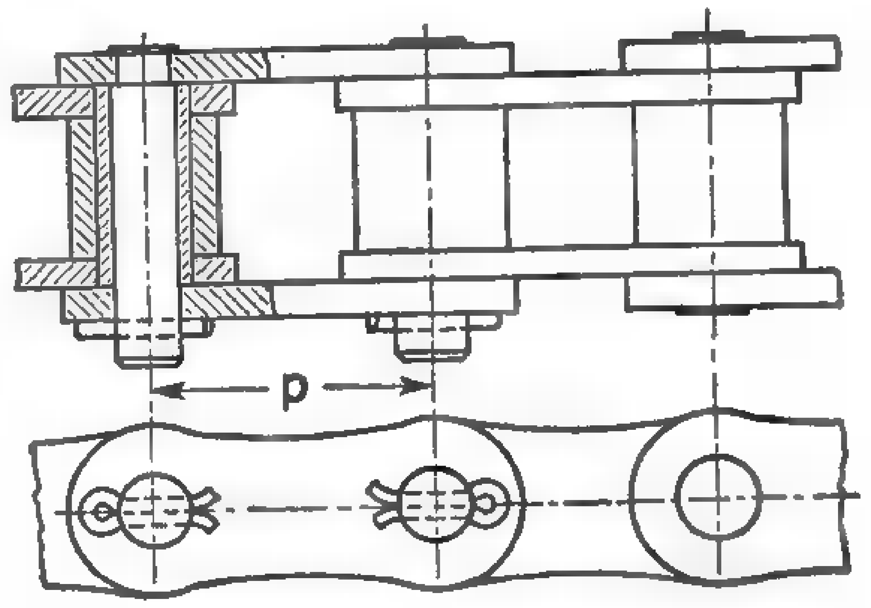


FIG. 28–2. Roller chain.

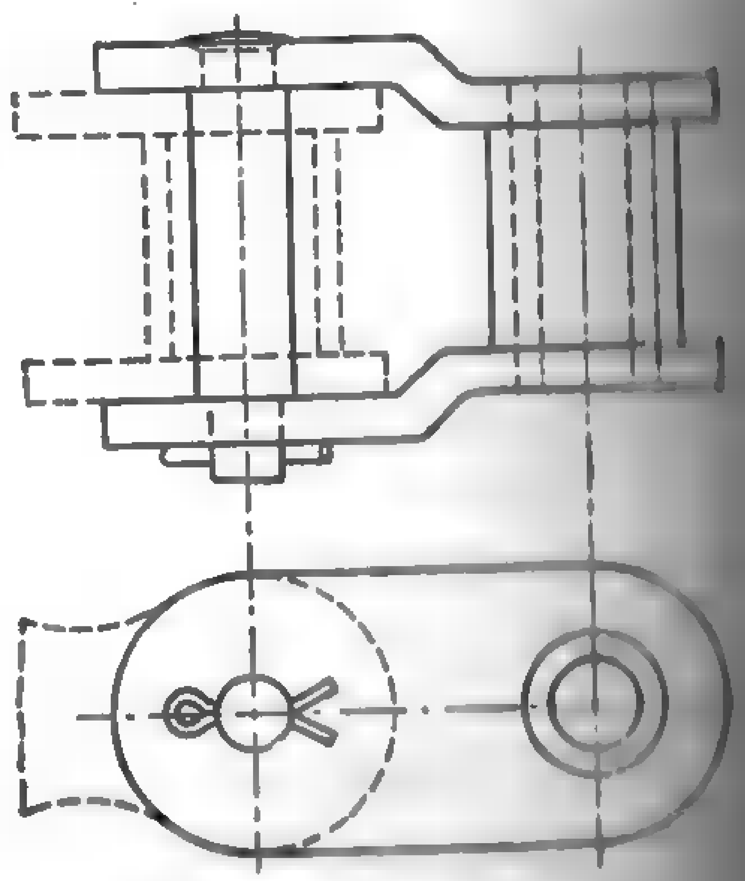


FIG. 28–3. Offset link.

Chain construction. Most chains are assembled by riveting the pins from both sides in all links except the last one. In the last connecting link one

end of each pin has a hole and a cotter pin. Sometimes the whole chain is assembled with cotter pins on one side, in order to make it easily detachable.

Offset link. In Fig. 28–3 is shown a link used in a roller chain when an odd number of links is necessary.

Chain selection. Noisy operation and rapid chain wear are caused chiefly by the impact between the sprocket and the rollers as the rollers seat themselves. The following empirical formula gives good results:

$$p \leqq \sqrt[3]{\left(\frac{900}{n}\right)^2} \tag{28–1}$$

where p is the pitch, in inches, and n is the speed of the small sprocket, in revolutions per minute. According to more recent investigations the relation between n and p can be based on the allowable amount of impact between a roller and a sprocket,[1] as shown by the equation

$$n \leqq \frac{1{,}920}{p}\sqrt{\frac{A}{w_f p}} \tag{28–2}$$

where A is the projected area of the roller, in square inches, which is equal to its diameter d times its width l; and w_f is the weight of the chain, in pounds per foot. All these values are given in Table 28–1. A comparison of equations 28–1 and 28–2 shows that the former, in spite of its simplicity, gives values within 5 or 7 per cent of those obtained by equation 28–2. If $12v_{max}/ip$ is substituted for n, where v_{max} is the maximum allowable chain velocity, in feet per minute, and i is the number of teeth of the small sprocket, equation 28–2 gives

$$v_{max} \leqq 160i\sqrt{\frac{A}{w_f p}} \tag{28–3}$$

If the maximum speed is based on the energy of impact per tooth per minute, the equation is

$$n \leqq \frac{2{,}000}{p}\sqrt[3]{\frac{A}{w_f}} \tag{28–4}$$

Substituting equation 28–4 for n in the equation $v_{max} = npi/12$ gives

$$v_{max} \leqq 166i\sqrt[3]{\frac{A}{w_f}} \tag{28–5}$$

Finally, if the maximum sprocket speed is based on the effect of centrifugal force, the equations are

$$n \leqq \frac{9{,}516}{p}\sqrt{\frac{A}{iw_f}} \tag{28–6}$$

[1] G. M. Bartlett, "New Basis for Rating Roller-Chain Drives," *Transactions of the American Society of Mechanical Engineers*, Vol. 57 (April, 1935), MSP-57-1, p. 98.

and

$$v_{max} \leqq 793\sqrt{\frac{Ai}{w_f}} \tag{28-7}$$

In an important design the allowable speeds n and v_{max} must be calculated by equations 28–2 to 28–7, and the minimum values should be used. It may be noted that values of n and v_{max} are the same whether the chain consists of one strand or more than one. As a general rule it is advisable to use the smallest pitch possible, in order to reduce surging; if necessary, a multiple-strand chain should be used in preference to a heavier single-strand chain.

Chain pull. The allowable pull F_d should be taken from the catalogue of the chain manufacturer, as it depends on many variables, such as material, workmanship, speed, bearing pressure, and number of teeth of the smaller sprocket. For preliminary computations F_d may be determined by the relation

$$F_d = \frac{F_u}{n_o} \tag{28-8}$$

where F_u is the ultimate strength given in Table 28–1, and n_o is a working factor. For large sprockets having over 40 teeth, and for low velocities of about 100 fpm, n_o may be taken as 5. It should be increased up to 18 for small sprockets having 10 or 11 teeth, and for high velocities of about 1,200 fpm. However, the best procedure is not to base the load capacity of a roller chain on its strength but to consider the allowable bearing pressure to which the roller pins can be subjected without undue wear. For low speeds this specific pressure should preferably be less than 6,400 psi, but it may be as high as 7,500 psi, referred to the projected area A of the pins. More conservative limits are 5,000 and 6,000 psi.[2]

The practice recommended by the American Gear Manufacturers Association is based on a bearing pressure of 4,333 psi for the pin, and a velocity factor $c = 600/(600+v)$. The allowable pull is then found by the equation

$$F_d = \frac{2{,}600{,}000 l d_2}{600+v} - \frac{v^2 w_f}{115{,}900} \tag{28-9}$$

where l is the length of the roller pin, d_2 is its diameter, and v is the actual chain velocity, in feet per minute, or

$$v = \frac{ipn}{12} \tag{28-10}$$

The second term in equation 28–9 is the influence of the centrifugal force.

28–3. Sprockets for roller chains. A sprocket for a roller chain is shown in Fig. 28–4, with the dimensions used in design.

[2] *Loc. cit.*, p. 102.

Number of teeth. The recommended number of teeth in the smaller sprocket is 15, 17, 19, 21, or 23 for moderately high speeds. Even numbers may be used, but they give less-uniform wear. A smaller number of teeth, down to 9, or even to 7, may be used, if necessary, for low-speed drives.

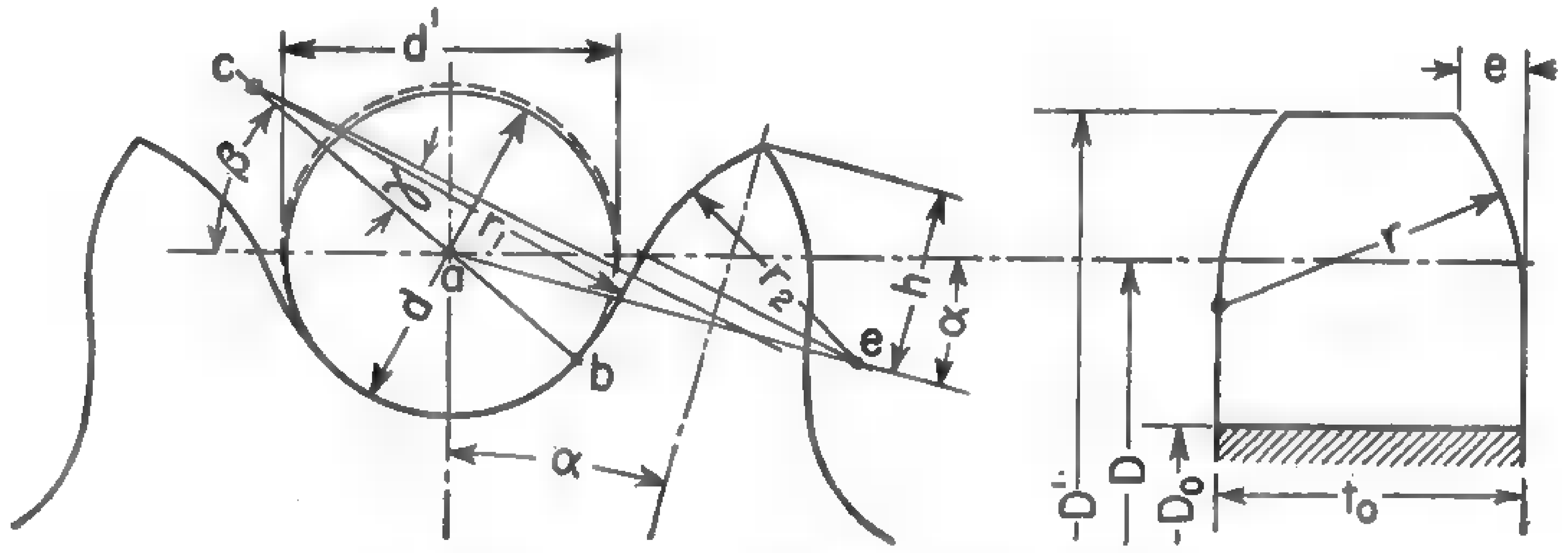


FIG. 28–4. Roller-chain sprocket teeth.

Main dimensions. For the notation in Fig. 28–4, $2\alpha \times i = 360°$ and

$$\alpha = \frac{180°}{i} \tag{28-11}$$

The pitch diameter is

$$D = \frac{p}{\sin\alpha} \tag{28-12}$$

and the root diameter is $D_o = D - d$.

The width t_o is

$$t_o = l - 0.045p \tag{28-13}$$

where l is the roller width given in Table 28–1.

The radius r is

$$r = 0.545p \tag{28-14}$$

and the corner relief e is

$$e = \tfrac{1}{8}p \tag{28-15}$$

According to the Diamond Chain Company, in order to obtain teeth giving maximum efficiency throughout the life of the drive, the angles and distances shown in Fig. 28–4 should have the following values:

The diameter d' of the seating curve, in inches, must be

$$d' = 1.005d + 0.003 \tag{28-16}$$

The required value of the angle β is

$$\beta = 35° + \frac{60°}{i} \tag{28-17}$$

The distance from a to c is made equal to $0.8d$, and the radius r_1 from c to b is $0.8d + 0.5d'$. Also, the angle γ is found from the relation

$$\gamma = 18° - \frac{56°}{i} \tag{28-18}$$

The height h is made equal to $0.3p$. With this value, the outside diameter D' is

$$D' = p\left[0.6 + \csc\left(\frac{180°}{i}\right)\right] \tag{28–19}$$

Sprockets cut according to Fig. 28–4 allow the chain, as it stretches, to ride higher on the teeth without catching on them.

In order to obtain a sufficient hub diameter, the pitch diameter D of the smaller sprocket should be determined by the relation

$$D \geqq 2d_o + p \tag{28–20}$$

where d_o is the shaft diameter. At the same time the diameter D should meet the requirement that

$$D \leqq \frac{5{,}727}{n} \tag{28–21}$$

28–4. Chain-drive design. The most economical chain and sprocket size may be selected most quickly if the following procedure is used: First, the maximum allowable pitch is determined from equation 28–1, and the next-smaller pitch is taken tentatively from Table 28–1. By using equation 28–10 the number of teeth i_1 and the chain speed v are selected. Simultaneously the number of teeth i_2 in the larger sprocket is determined. After this the selected chain pitch and number of teeth are checked by one or all of equations 28–2 to 28–7.

The next step is to find the allowable chain pull F_d by equation 28–9. Also, the required chain pull F_t is found from the equation

$$P = \frac{F_t v}{33{,}000 K_1 K_2} \tag{28–22}$$

where K_1 is a load factor and K_2 is a service factor. If the drive is used for a fluctuating load, the factor K_1 must be from 1.1 to 1.5, the exact value depending on the severity of the shock and being selected by using Table 20–3 as a comparative basis. The service factor K_2 depends on the length of daily operation. If the drive operates not more than 10 hr per day, $K_2 = 1$; for 24-hr operation, $K_2 = 1.2$.

If F_t from equation 28–22 is smaller than F_d found by equation 28–9, the drive may be improved by using a chain with the next smaller pitch, in accordance with the general rule given in section 28–2 that it is always desirable to use the smallest pitch possible. Naturally, all computations must be repeated.

If F_t is greater than F_d, a chain with several strands must be used. Then the number of strands j evidently is found by the relation

$$j = \frac{F_t}{F_d} \tag{28–23}$$

Horsepower. The horsepower which a chain can safely transmit under normal load conditions is given by the chain manufacturers in their catalogues. Unfortunately, the data of various manufacturers differ considerably, being based on different assumptions. However, a safe value will generally be obtained by using equation 28–22, in which F_d found from equation 28–9 is used instead of F_t.

Center distance. The proper center distance usually is considered to be equal to the sum of the sprocket diameters D_1 and D_2. The minimum value is $30p$, but if necessary it can be made smaller, down to $20p$, provided proper shaft and sprocket alignment is secured.

For speed ratios greater than 3.5 to 1, the center distance should be not less than $D_1 - D_2$, in order to have a sufficient arc of wrap of the chain on the small sprocket. This arc should not be less than 120°.

Short center distances should be avoided, as a long chain has greater elasticity, to absorb irregularities of motion, and has a longer life.

Chain length. The length of a chain may be computed from formulas given in catalogues and by using various constants given in special tables. However, satisfactory results are obtained from the simple approximate formula

$$L_p = 2C_p + 0.5(i_1 + i_2) + \frac{0.026(i_1 - i_2)^2}{C_p} \tag{28–24}$$

where L_p is the chain length, in pitches, and C_p is the center distance, also in pitches. Naturally, the next whole number above the calculated value of L_p must be used. The chain length in inches is $L = pL_p$.

Chain case and lubrication. Good lubrication should be provided for all chains. The oil drops should be led to the inside of the chain, because otherwise the centrifugal force will remove the oil before it has performed any service. If maximum life of the drive is essential, or if protection against dust or corrosive agents is important, the drive should be enclosed in a case. A case must have provision for lubrication and for easy inspection of the drive. A chain case is also desirable as a safety guard.

28–5. Silent chains. Silent chains have several advantages over roller chains:

a) They allow the use of higher linear velocities. Values up to 2,500 fpm are used even with large pitches, and values up to 5,000 fpm are used with some makes and under favorable conditions. To transmit power economically, however, the velocities should be between 1,500 and 2,000 fpm.

b) They can be built in great widths, up to 30 in., and one chain is thus able to transmit up to 1,700 hp.

c) The load is distributed equally among all sprocket teeth in contact with the chain, their wear thus being decreased.

Their main disadvantage, as compared with roller chains, is the absence of standardization and of interchangeability of different makes. They differ in the shape of the links and the profile of the sprocket teeth; in pitches; in the types of guides holding the chain on the sprockets; and finally in the types of pins which hold the links, and consequently in the capacity of chains of the same pitch and width.

Efficiency. Properly designed silent-chain drives have a very high efficiency, occasionally over 98 per cent.

General requirements. For satisfactory operation of a silent-chain drive, several conditions are essential. The prime requisite is a *true and rigid installation.* The shafts on which the sprockets are mounted must be parallel and properly supported so as not to tremble or pull together. The sprockets must be carefully aligned. An *odd number of teeth* in each sprocket and an *even number of links* in the chains is desirable. An odd number of teeth distributes the working contact over the whole face of each tooth, the wear thus being reduced to a minimum. If, because of the exact speed ratio required, one or both sprockets have an even number of teeth, a hunting link, as shown in Fig. 28–6, should be inserted to secure uniform wear.

The larger pitches should be used for large powers as well as low speeds. For moderate powers the smaller pitches are better suited. In most cases the largest pitch which can be used for the desired rotative speed will give a drive with the lowest first cost. However, at a given linear speed a smaller pitch will provide a quieter drive. Naturally, it will require a wider chain.

A silent-chain drive should not be used between shafts on which the torque is not uniform. Silent chains can absorb a certain amount of shock, but a more flexible drive, such as a V-belt drive, is preferable in such a case, as far as both quietness and life of the drive are concerned. However, if a silent chain is used for a fluctuating load, it should be figured with a load factor K_1 from 1.2 to 1.6, the exact value depending on the severity of the shock. Table 20–3 may serve as a guide for the amount of shock action. The values of K_1 just mentioned are for electric-motor drives. For oil-engine and gas-engine drives, these values should be increased by 25 per cent. As for roller chains, a service factor K_2 must be introduced if a drive must operate more than 10 hr per day. For 24-hr operation, $K_2 = 1.2$.

Silent chains must run with a small slack. Whenever possible it is desirable to have an adjustable center distance to take care of the lengthening of the chain by wear. However, slack in a chain drive is not objectionable in itself, provided there is no uneven running or rapid starting, which might cause a very loose chain to jump.

Center distance. The center distance for a silent-chain drive may be selected by following the rules given in section 28–4 for roller-chain drives. The chain length may be computed satisfactorily by equation 28–24.

Drive selection. For a given horsepower and given speeds of the driving and driven shafts, it is possible to design several drives with different combinations of pitch, width, and numbers of sprocket teeth. Usually, however, only one of these drives will give the maximum service for the smallest cost. This best drive can be selected after several combinations are tried.

Instead of furnishing engineering data similar to the values in Table 28–1, most silent-chain manufacturers give in their catalogues the horsepower that their chains of certain pitches can transmit, per inch of width, at various speeds of sprockets and with certain numbers of teeth. These ratings differ considerably, one from another, because of a great difference in the construction and material used, chiefly in the pins and bushings.

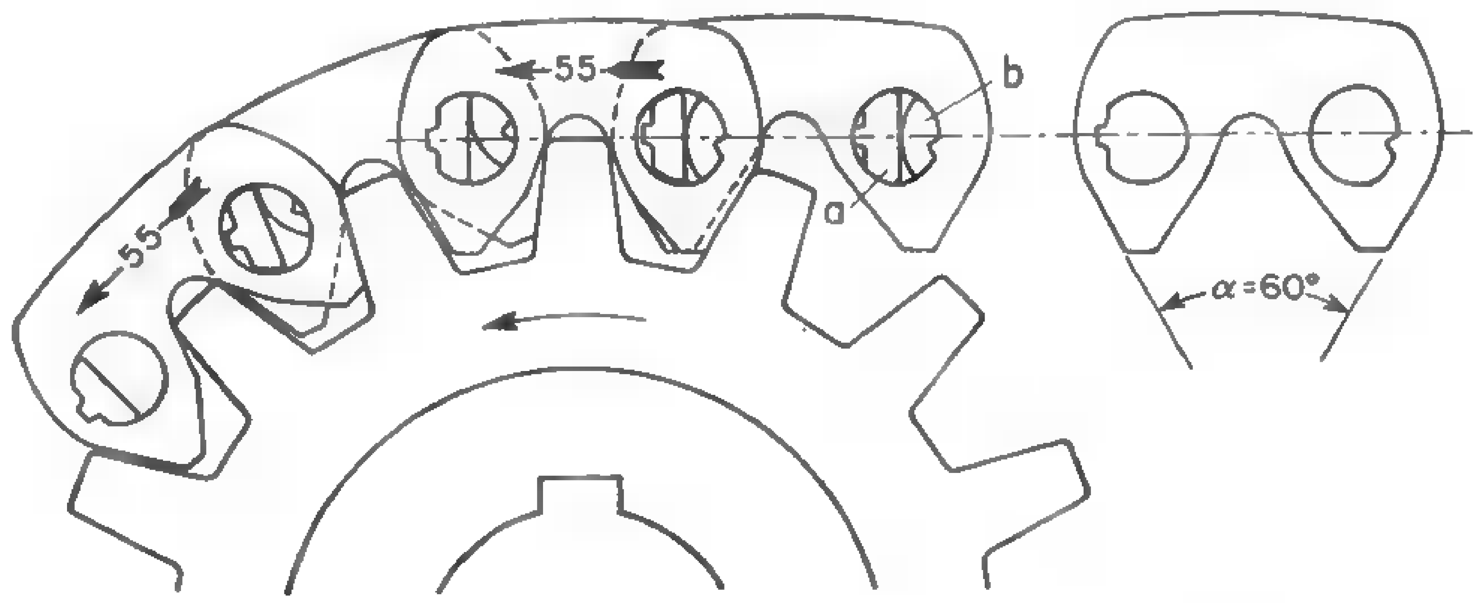


Fig. 28–5. Morse silent chain.

28–6. Morse chain. The Morse silent chain has unsymmetrical links, as shown in Fig. 28–5, because of the use of a split pin. The half *a* of the pin has a T section, and forms the seat for the rocker pin *b* held in the other link. This construction introduces rolling friction instead of the sliding friction of common pins, but requires that the chain be run in one direction only, as indicated by an arrow on each outside link.

TABLE 28–2

Design Data for Morse Silent-Chain Drives

Design Item	Pitch of Chain (in.)									
	$\frac{3}{8}$	$\frac{1}{2}$	$\frac{5}{8}$	$\frac{3}{4}$	0.9	1.0	1.2	1.25	1.5	2.0
Minimum teeth, driver	13	13	13	13	15	15	15	17	17	17
Desirable teeth, driven	17–25	17–25	19–25	19–25	19–25	19–25	21–25	21–25	21–27	21–27
Range of teeth, driven	21–120	21–130	23–150	23–150	23–150	23–150	25–150	25–150	27–150	27–150
Speed, normal (rpm)	3,000	2,400	1,800	1,200	1,100	1,000	800	800	600	450
Speed, maximum (rpm)	4,000	3,000	2,500	2,000	1,800	1,500	1,200	1,200	900	700
Normal tension (lb per in.)	75	100	125	150	185	205	265	265	335	600
Center distance (in.), minimum	6	8	14	18	21	24	30	30	36	48

TABLE 28–3

INFLUENCE OF ROTATIVE SPEED ON TENSION FACTOR

Per Cent of Normal Speed	Tension Factor K_t	Per Cent of Normal Speed	Tension Factor K_t	Per Cent of Normal Speed	Tension Factor K_t
10	3	60	1.40	110	0.84
20	3	70	1.25	120	0.68
30	2.35	80	1.15	130	0.54
40	2.0	90	1.05	140	0.40
50	1.65	100	1.00	150	0.25

Selection of pitch. Table 28–2 shows the normal and maximum rotative speeds recommended for each pitch. The use of a speed higher than the recommended normal speed shortens the life of the chain drive.

Number of teeth. The number of teeth on the driver should be as recommended in Table 28–2, so that a chain speed between 1,500 and 2,000 fpm is obtained. Under favorable conditions, such as uniform torque and good lubrication, a chain speed up to 3,000 fpm may be used.

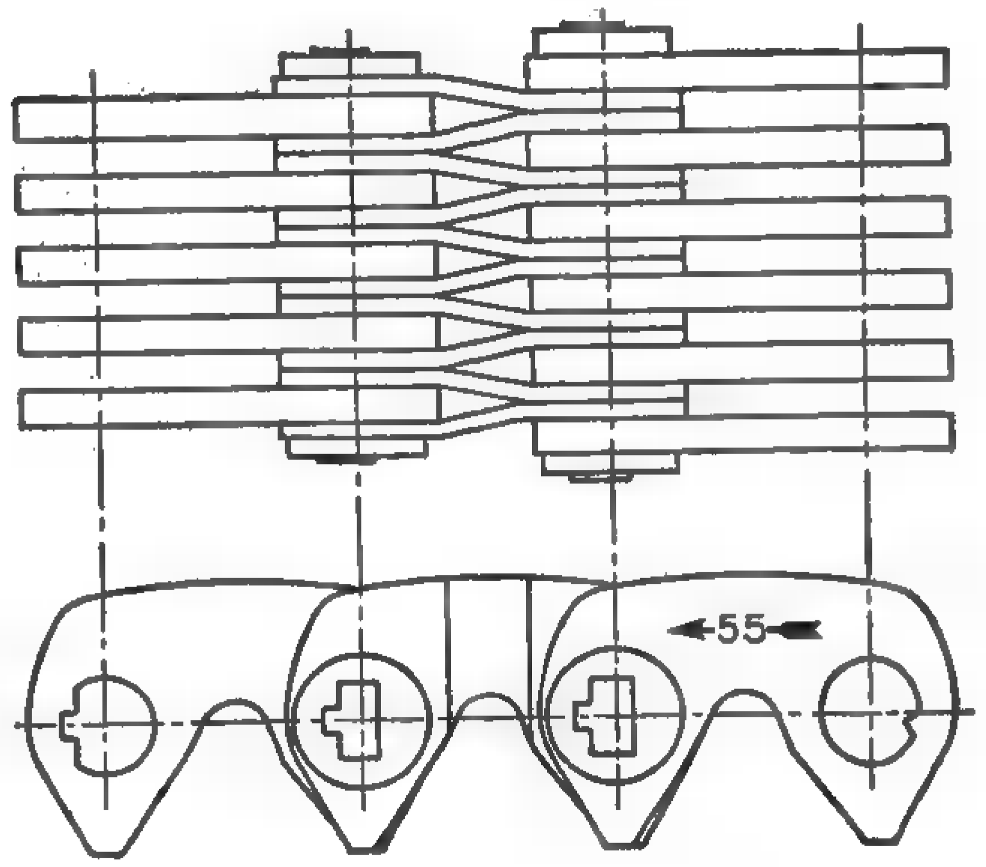


FIG. 28–6. Morse-chain hunting link.

Chain width. After selecting the pitch and number of the teeth in each sprocket, and the speeds of the sprockets, the total pull F necessary to transmit the horsepower is found. The width b is found by dividing F by the allowable tension F_a from Table 28–2, corrected for the actual sprocket speed in accordance with Table 28–3. The actual width will be the next-greater one, as given in the catalogue.

The range of width for each chain is approximately from p to $10p$.

Hunting link. If the length L_p comes out in an odd number of pitches, or if a chain has stretched and one pitch must be taken out, an offset hunting link, Fig. 28–6, is used.

EXAMPLE 28–1. Design a drive from an electric motor to a compressor to transmit 80 hp at 690 rpm of the motor and 135 rpm of the driven shaft. A short center distance is desired, and 40 in. is the limit for the diameter of the driven sprocket.

Table 28–2 shows that a $1\frac{1}{4}$-in. pitch is as large as should be used normally at 690 rpm. If a recommended 21-tooth sprocket is used, the number of teeth on the driven sprocket becomes

$$i_2 = \frac{21 \times 690}{135} = 107$$

The corresponding sprocket diameter is

$$D_2 = \frac{i_2 p}{\pi} = \frac{107 \times 1.25}{\pi} = 42.7 \text{ in.}$$

This is larger than can be used.

For the next-smaller odd number of driver teeth, or $i_1 = 19$, it is found that $i_2 = 97$, and thus

$$D_2 = \frac{97 \times 1.25}{\pi} = 38.60 \text{ in.}$$

The chain speed is

$$v_m = \tfrac{1}{12} i_1 p n_1 = \tfrac{1}{12} \times 19 \times 1.25 \times 690 = 1{,}370 \text{ fpm}$$

which is satisfactory. The total pull is

$$F = \frac{33{,}000P}{v_m} = \frac{33{,}000 \times 80}{1{,}370} = 1{,}930 \text{ lb}$$

The allowable tension is 265 lb per in. at a normal speed of 800 rpm. However, the actual speed is $(690/800) \times 100 = 86.2$ per cent of the normal speed. Therefore, according to Table 28–3, the tension may be increased 1.09 times. On the other hand, a fluctuating torque due to the compressor load calls for a load factor of about 1.4. Hence, the necessary width is

$$b = \frac{1.4 \times 1{,}930}{265 \times 1.09} = 9.35, \text{ or } 10 \text{ in.}$$

From Table 28–2, the minimum center distance is 30 in. Probably the shafts should not be brought so close together and a safer value would be

$$C = D_1 + D_2 = \frac{(i_1 + i_2)p}{\pi} = \frac{(19+97) \times 1.25}{\pi} = 46.2 \text{ in.}$$

By equation 28–24, in which $C_p = 46.2/1.25 = 36.95$ pitches, the chain length is

$$L_p = 2 \times 36.95 + 0.5(19+97) + \frac{0.026(97-19)^2}{36.95} = 136.2 \text{ pitches}$$

If a hunting link is inserted, 137 pitches may be used. Ordinarily, however, 138 pitches, or 69 double links, would be ordered.

28–7. Link-belt chain. Link-belt silent chains are made in nine pitch sizes and have hardened-steel round pins working in hardened-steel split bushings, as shown in Fig. 28–7. The use of thin-leaf links produces a more uniform chain.

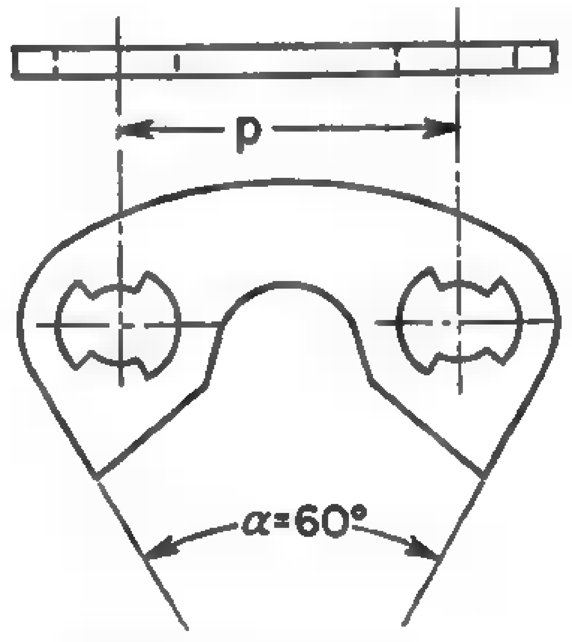


FIG. 28–7. Link-Belt silent chain.

The recommended chain speeds lie between 1,500 and 3,000 fpm, but encased drives with continuous lubrication can be designed to operate at chain speeds up to 5,000 fpm. Under favorable conditions speed ratios as high as 25 to 1 may be used.

The necessary chain width must be found from catalogue tables.[3]

Chain widths from $2p$ to $6p$ should be preferred from a mechanical standpoint, although the chains are made in widths up to $12p$. Load and service factors should be used to take into account torque fluctuation and long-hour operation.

[3] A useful condensation of Link-Belt's Catalogue 900, 1950 ed., can be found in R. T. Kent, *Mechanical Engineers' Handbook*, 12th ed., Vol. II, *Design and Production*, ed. by Colin Carmichael (New York: John Wiley & Sons, Inc., 1950), p. 15-78.

28–8. Silent-chain sprockets. The outside contours of the individual links of the various makes of silent chains are all similar. The angle α included between the working faces of a link is made 60° by all manufacturers, as shown in Figs. 28–5 and 28–7. Therefore the angle α, Fig. 28–8, included between the flanks of alternate teeth is also 60°, irrespective of the number of teeth in a sprocket. However, the angle β formed by the flanks of the same tooth varies with the number of teeth i. From the geometry of the triangle, it follows that

$$\beta = 60° - \frac{720°}{i} \tag{28–25}$$

Equation 28–25 shows that β decreases with a decrease of the number of teeth, and vice versa. In order to have sufficiently strong teeth, the smallest number of teeth in the small sprockets is usually limited to 15. Again, since the angle β increases with the number of teeth, it is necessary to limit the greatest number of teeth to about 150 because of the danger of the chain sliding over the teeth if β is greater than 55.2°.

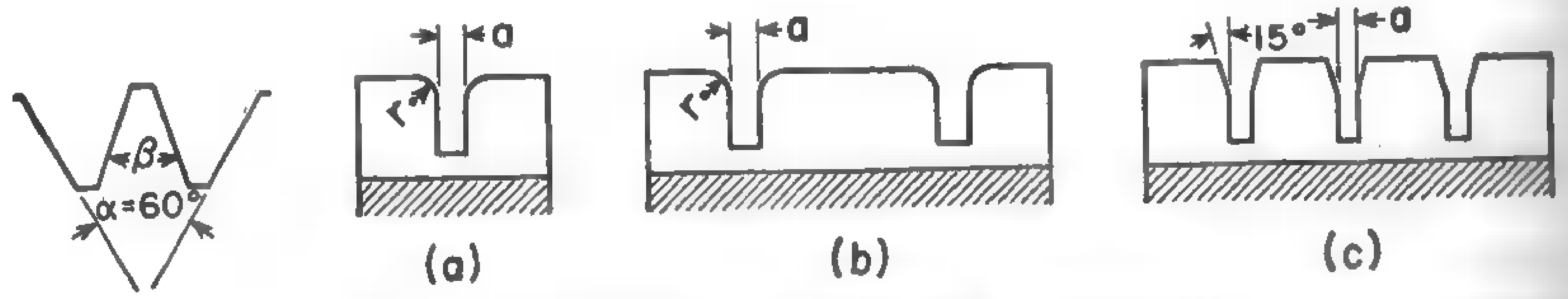


FIG. 28–8. Chain-guide grooves in sprockets.

Chain guides. The difference in the shape of sprockets which renders them uninterchangeable for chains of different makes comes chiefly from the method used for guiding the chains. Most chains have one, two, or three lines of straight links, without a cutout, and the links run in grooves cut across the sprocket teeth. In Fig. 28–8a is shown a single-groove guide used in Morse chain sprockets having up to 32 teeth; and in Fig. 28–8b is shown a double-groove guide for Morse chain sprockets having more than 32 teeth and a width of 3 in. and up.

Other manufacturers use one, two, or three guide grooves, the number depending only on the chain width, irrespective of the number of teeth. In Fig. 28–8c is shown the shape of grooves used for Ramsey silent chains. Still other chain-makers have guide plates on each side of the chain. Then the sprocket teeth have no grooves, but only rounded end corners, and the tooth length is slightly smaller than the nominal chain width. Finally, some chains are made straight and without any guides, but guides are put on the sides of the sprockets, whose teeth are made slightly longer than the nominal chain width. In Fig. 28–9a is shown a sprocket with screwed-on cast-iron ring flanges; and in Fig. 28–9b are shown guides formed by cramped wire driven between the teeth.

Material. Small sprockets are cut of steel, and sometimes they are made integral with the shaft. Occasionally the rims are steel rings bolted to cast-iron disks and hubs. In larger sizes the sprockets are of cast iron, which is strong enough because the pull is taken by all teeth in contact with the chain.

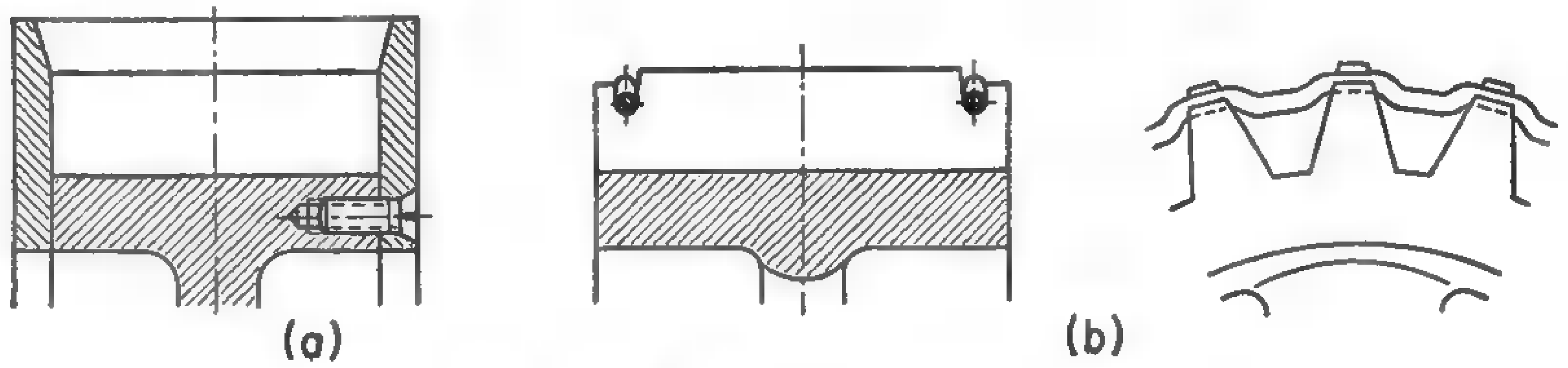


FIG. 28–9. Rim guides of Link-Belt sprockets.

Fastening to shaft. The common method of fastening a sprocket to a shaft is to use a key between the hub and the shaft. A large-diameter sprocket with a tapered key must be handled with caution in order that the sprocket will not be distorted and will not wobble when running. Sprockets that are split and then clamped on the shaft are convenient, but they should be secured by one or two setscrews.

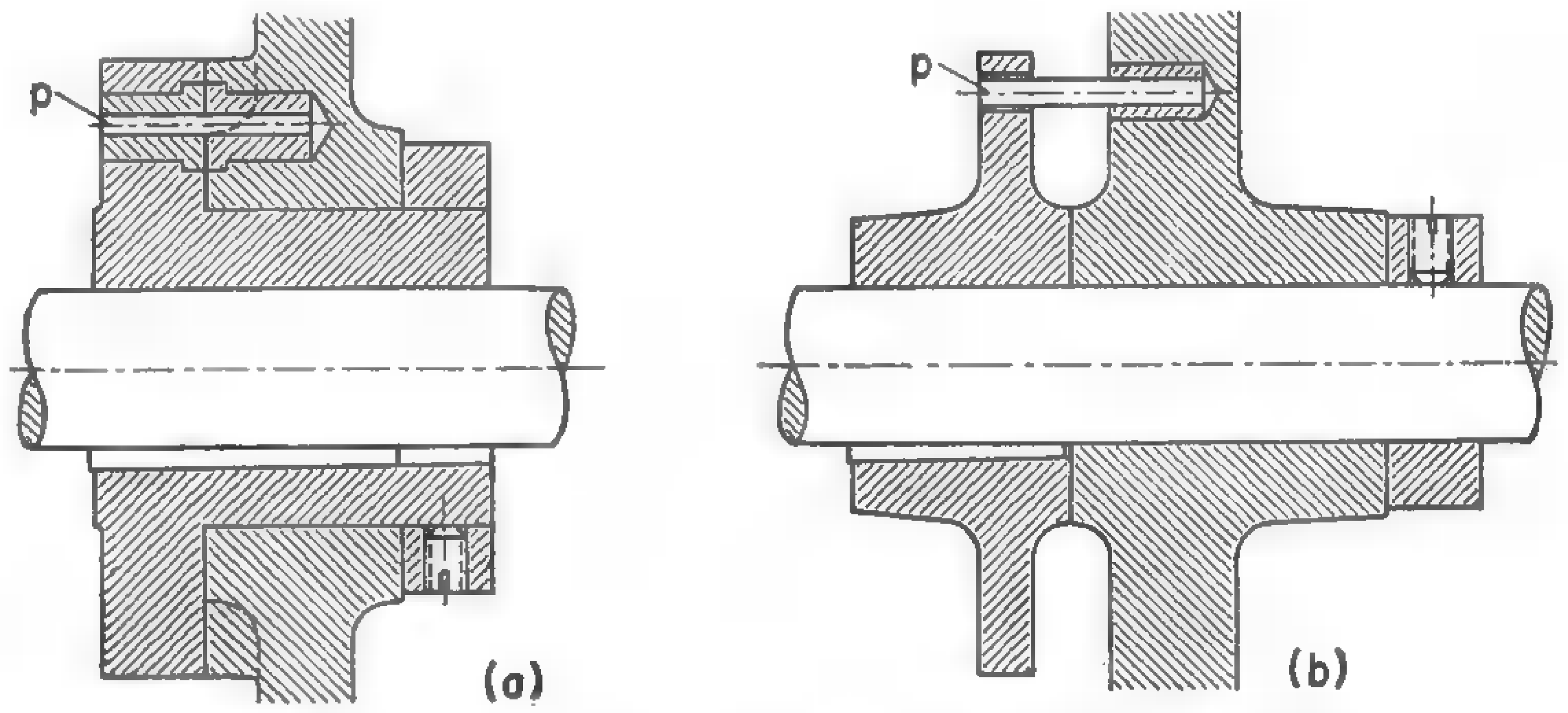


FIG. 28–10. Shear and breaking safety-pin connections.

When heavy shocks are expected, the sprocket may be connected with the hub by means of a shear pin, as in Fig. 28–10a, or a breaking pin, as in Fig. 28–10b. Naturally, either a shear pin or a breaking pin p may be used with either of the hub designs shown. In case there is an excessive overload, the pin shears off or breaks and thus protects the chain and other machinery. A spare pin is readily inserted.

Idlers. In general, idlers which bend the chain in the opposite direction are not desirable. However, if the installation requires an idler, as in the case of a vertical drive which cannot be run slack, one made of hard vulcanized fiber with cast-iron flanges, as in Fig. 28–11, may be used. The idler should bear against the back of the chain. Such idlers are made for chains

from 2 to 18 in. wide and with diameters of $2\frac{1}{2}$, 4, and 6 in. The diameter should be selected so as to avoid an excessive speed of the idler. It should not be over 1,000 rpm for the largest diameter, and not over 1,500 rpm for the smallest.

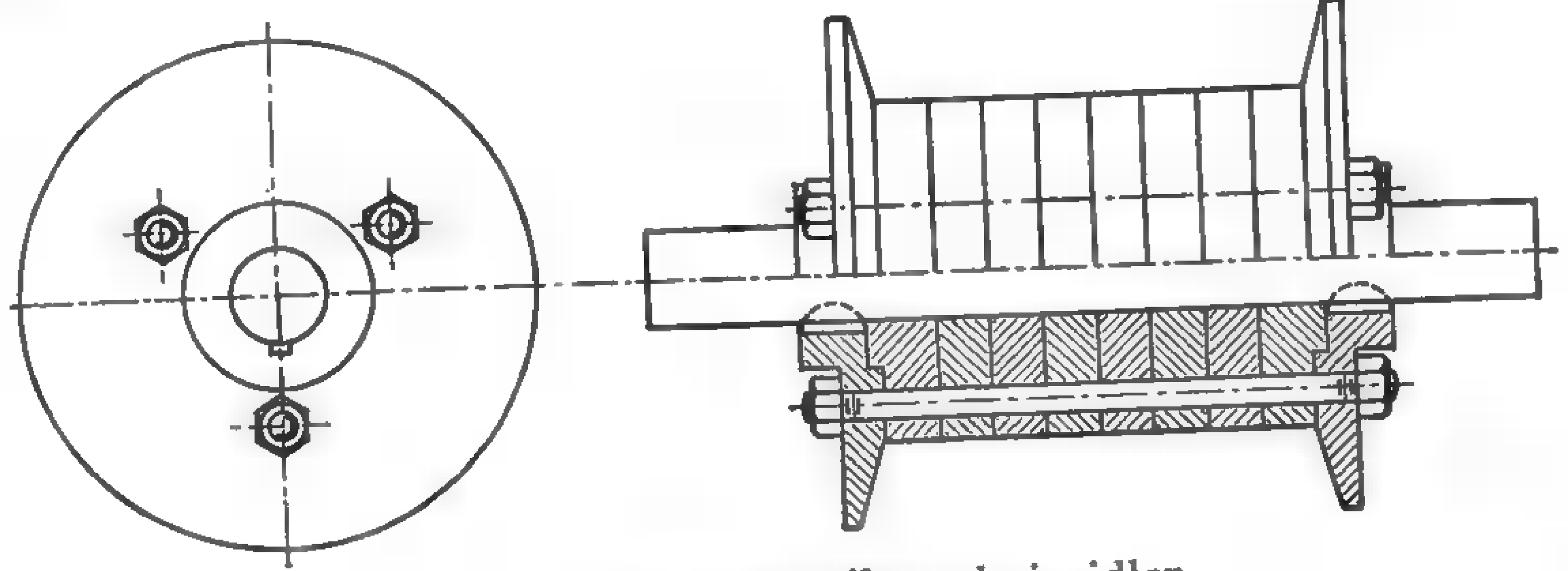


FIG. 28–11. Link-Belt silent-chain idler.

Casings. The best way to extend the life of a silent-chain drive is to have it properly lubricated. Running the chain in an oil bath is not feasible with normal chain speeds, because of excessive resistance. The use of a continuous small oil stream on the inside of the chain is about the best method; the next-best scheme is to use a sight-feed oiler. In any event, the whole drive must be enclosed. In designing a casing, particular attention should be given to oiltightness, ease of inspection of the drive, and ease of removal of the casing for chain adjustments or repairs.

A special method may be used for lubricating chains running at high speeds, above 3,750 fpm. The chain drive is enclosed in a perfectly airtight casing in which an oil level is maintained about $\frac{1}{4}$ in. below the lowest point of the chain. When the chain is running at such high speeds, it imparts motion to the air over the oil surface, and the air whips the oil into a spray. The spray is carried around and strikes a baffle in the top of the case. From the baffle, oil flows into a trough that distributes it over the lower strand of the chain.

CHAPTER 29

Friction Gearing

29–1. General considerations. Friction gears depend for their driving action upon the friction of the driving wheel, or *driver*, against its mate, or *follower*. The friction surface of a driver should be of a comparatively soft material, such as wood, fiber, leather, or rubber, while that of the follower is usually made of cast iron. This arrangement insures the maintenance of the correct shape of the friction surfaces. If the follower is of the softer material, its surface might be injured when the drive is started under load or when an excessive load brings it to a standstill.

Friction gears are used for light and medium powers in machinery which is frequently started and stopped, and also where provision must be made for a change of speed of the driven shaft, or for its reversed motion.

The disadvantages of friction gears are the thrust on the bearings, and slippage, resulting in a comparatively low efficiency. However, by using metal-to-metal-contact surfaces and ball or roller bearings, slippage and thrust drawbacks can be considerably reduced.

Experimental data. Tests conducted with strawboard driving wheels and a cast-iron follower gave the following results:[1]

a) Slippage increases gradually as the load increases, up to 3 per cent; after that it is likely to increase suddenly so much that the follower may stop.

b) The coefficient of friction seems to be constant for all pressures up to a safe limit of slightly over 150 lb per inch of face.

c) The coefficient of friction increases with slip, up to 2 per cent slip.

d) The coefficient of friction is not affected by the tangential velocity between the limits of 1,100 and 2,800 fpm.

Table 29–1 contains data for the design of friction gearing compiled from various sources.[2] In operation, leather fiber becomes glazed and hard, as a result of slippage, and its coefficient of friction drops. Tarred fiber and straw fiber are better if the drive is started and stopped frequently.

Rigidity. In order to obtain an even contact pressure across the whole face of each wheel of friction gears, the wheels must be rigid and must be rigidly supported by closely located bearings.

Classification. Friction gears are of the spur, bevel, and disk types.

[1] W. F. M. Goss, "Power Transmission by Friction Driving," *Transactions of the American Society of Mechanical Engineers*, Vol. 29 (1907), pp. 1093 ff.

[2] R. T. Kent, *Mechanical Engineers' Handbook*, 12th ed., Vol. II, *Design and Production*, ed. by Colin Carmichael (New York: John Wiley & Sons, Inc., 1950), p. 15-83.

TABLE 29–1

DESIGN DATA FOR FRICTION GEARING

Material of Driver	Allowable Pressure p (lb per in.)	Coefficient f with Cast Iron	Material of Driver	Allowable Pressure p (lb per in.)	Coefficient f with Cast Iron	Coefficient f with Aluminum
Cast iron.	3,000	0.15	Leather.	150	0.09	0.13
Cork composition. . .	50	0.21	Leather fiber.	240	0.18	0.18
Paper.	150	0.15	Straw fiber.	150	0.15	0.16
Rubber.	100	0.20	Sulfite fiber.	140	0.20	0.19
Wood.	150	0.15	Tarred fiber	250	0.28	0.28

29–2. Spur friction gears. The wheels of spur friction gears may be plain or grooved.

Plain spur wheels. The simplest type of friction gearing consists of two plain cylindrical wheels, Fig. 29–1, held in contact with each other by two forces F. In the following equations,

P is the horsepower transmitted;
v_m is the mean circumferential velocity, in feet per minute;
b is the width of face of the gears, in inches;
p is the permissible pressure, in pounds per inch of face;
f is the coefficient of friction;
F_t is the tangential force due to the pressure $F = bp$, namely,

$$F_t = fbp \tag{29–1}$$

and the horsepower transmitted is

$$P = \frac{F_t v_m}{33,000} \tag{29–2}$$

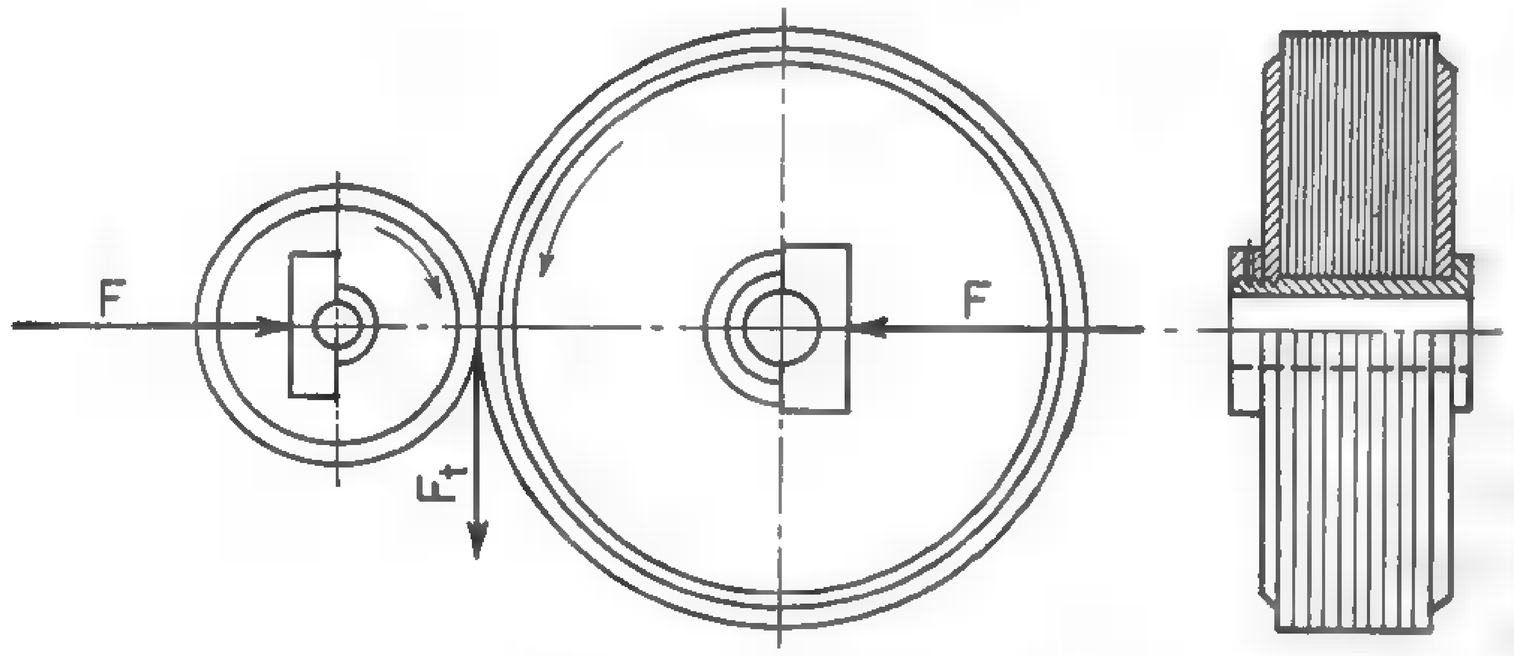


FIG. 29–1. Spur friction gears.

By substituting for F_t its value from equation 29–1, substituting for v_m its equivalent $\pi Dn/12$, where D is in inches, and solving for b, there results

$$b = \frac{126,000P}{fpDn} \tag{29–3}$$

Applications. Plain spur friction drives are used for driving light power hoists, coal screens, gravel washers, driers, etc., both at an increased speed and at a decreased speed. The commercial limits for the sizes of fiber-face pulleys are as follows: D ranges from 6 to 36 in., and b ranges from 4 to 18 in.

Grooved spur wheels. Grooved spur wheels are capable of transmitting more power with the same radial force applied than the cylindrical type. They are therefore used where a greater power is required, as in hoisting machines.

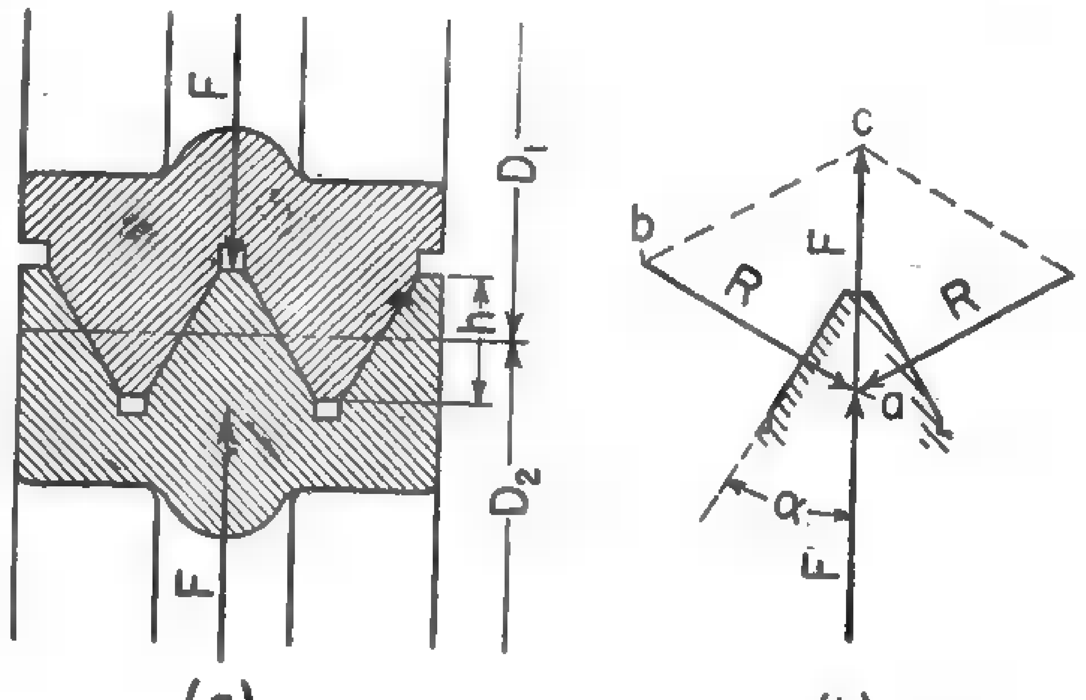


FIG. 29–2. Grooved spur friction gears.

Such wheels are formed as shown in Fig. 29–2a. In Fig. 29–2b the radial force F is held in equilibrium by the normal pressures R, and friction forces fR. Summing up along the vertical axis gives, for each groove,

$$F = 2R \sin \alpha + 2fR \cos \alpha \tag{29–4}$$

The tangential resistance F_t of each groove is equivalent to the normal pressure $2R$ multiplied by f, or

$$F_t = 2Rf \tag{29–5}$$

Also, from the geometry of the illustration, the normal pressure is

$$R = ph \sec \alpha \tag{29–6}$$

where p is the pressure per inch at the groove side, and h is the depth of the groove.

When the value of R from equation 29–6 is substituted in equations 29–4 and 29–5 and the results are simplified, the result is

$$F = 2ph(\tan \alpha + f) \tag{29–7}$$

and

$$F_t = 2phf \sec \alpha \tag{29–8}$$

By using the general equation 29–2, substituting iF_t for the total tangential force, where i is the number of grooves, and replacing F_t by its value from equation 29–8, we obtain the relation

$$ih = 63,030P\frac{\cos \alpha}{fpDn} \tag{29–9}$$

Since along the lines of contact the so-called *pitch point* is the only one at which the two gears have the same peripheral speed, at all other points there must be slippage. In order to avoid excessive friction and wear, the depth h

should be made comparatively small, usually not over $\frac{1}{4}$ in. A practical value, in inches, is

$$h=0.006D+0.15 \tag{29-10}$$

where D is the diameter of the smaller pulley, in inches. Generally, α ranges from 12° to 18°, and it should not be over 20°. Both gears are made of cast iron. The drive works satisfactorily only with a sufficiently high peripheral speed.[3] A recommended value is

$$v_m \geqq 1{,}200+4D \tag{29-11}$$

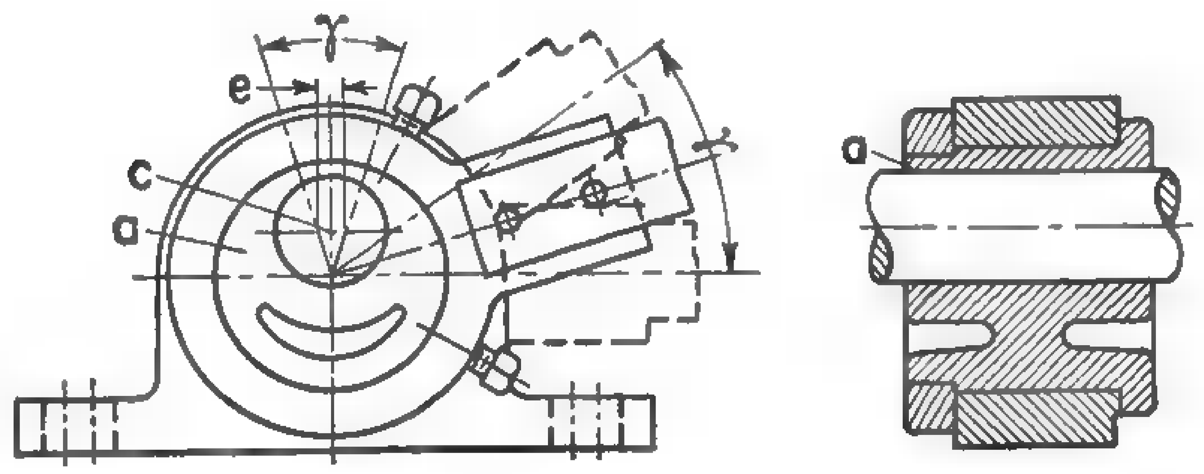


FIG. 29-3. Side-motion bearing.

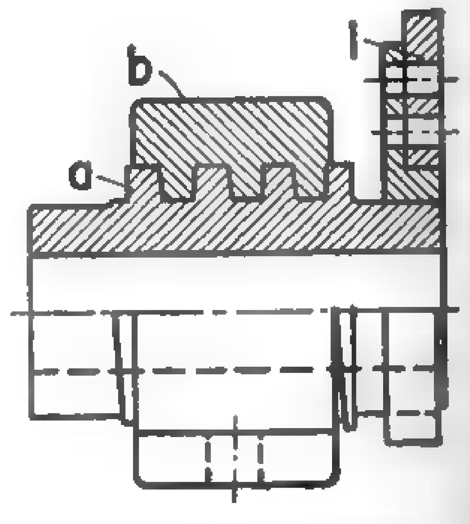


FIG. 29-4. Axial-motion bearing.

Bearings for friction gears. A change of the center distance necessary in engaging and disengaging a pair of cylindrical friction gears is obtained by means of suitable bearings, as shown in Fig. 29-3. The bearing sleeve a is bored eccentrically and may be rotated through a small angle γ by means of the attached lever. The center c of the shaft is thus shifted through a distance e. If axial displacement is needed, it can be obtained by means of a square thread on the outside of the bearing sleeve, as shown in Fig. 29-4.

29-3. Bevel friction gears. Bevel friction gears are suitable only for light power transmission, as it is difficult to maintain a uniform contact pressure. The larger gear is usually keyed rigidly to the shaft, while the smaller one is either splined to the shaft or can be moved axially together with the shaft. It should be noted that the starting conditions differ from the running conditions.

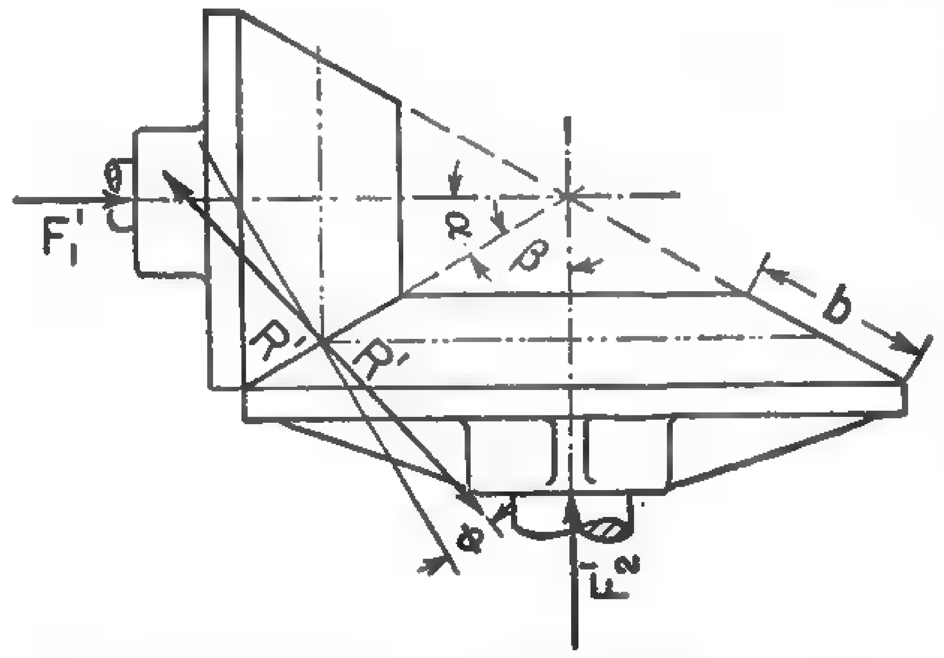


FIG. 29-5. Bevel friction gears.

Starting. It will be assumed that the drive is started under full load, as is often the case. As for grooved spur gears, the reaction R', Fig. 29-5, is inclined from the normal by the angle of friction ϕ for these conditions, and therefore

$$R'=\frac{F_1'}{\sin(\alpha+\phi)}=\frac{F_2'}{\cos(\alpha+\phi)} \tag{29-12}$$

[3] *Ibid.*, 10th ed. (1923), p. 1643.

The tangential force transmitted from one gear to the other is equal to the product of the normal force and the coefficient of friction. Therefore

$$F_t'=fR'\cos\phi=\frac{33{,}000P}{v_m} \tag{29-13}$$

Combining equations 29-12 and 29-13 gives, for the least axial thrust required,

$$F_1'=\frac{33{,}000P(\sin\alpha+f\cos\alpha)}{fv_m} \tag{29-14}$$

and

$$F_2'=\frac{33{,}000P(\cos\alpha-f\sin\alpha)}{fv_m} \tag{29-15}$$

Running. After the drive gets up to speed, the relative motion between the gears along the line of contact ceases, the reaction between the two surfaces in contact is normal, and the angle ϕ is 0. If the reaction in this case is designated by R, the following equations are obtained:

$$R=\frac{F_1}{\sin\alpha}=\frac{F_2}{\cos\alpha} \tag{29-16}$$

and

$$F_t=fR=\frac{33{,}000P}{v_m} \tag{29-17}$$

Combining these equations gives, for the least axial thrust,

$$F_1=\frac{33{,}000P\sin\alpha}{fv_m} \tag{29-18}$$

and

$$F_2=\frac{33{,}000P\cos\alpha}{fv_m} \tag{29-19}$$

Thus the thrust F_1 on the driver is less than F_1', and the thrust F_2 on the driven wheel is greater than F_2'. In equation 29-16, $R \leqq bp$, where p is the permissible unit pressure.

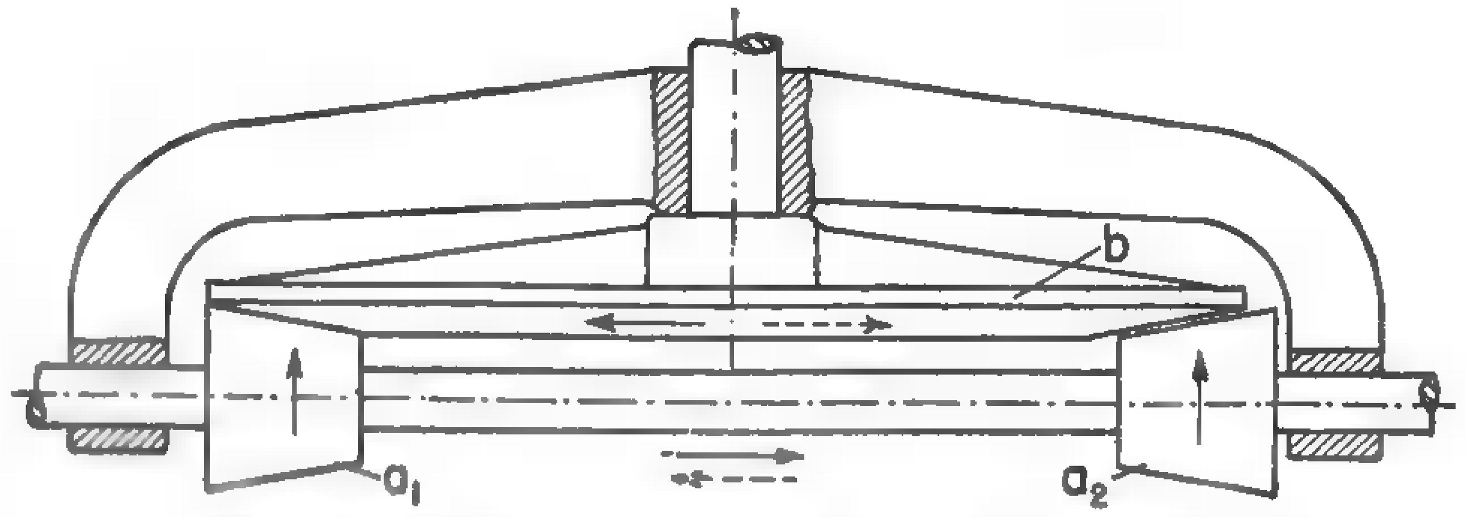


FIG. 29-6. Reversing-motion friction gearing.

Reversing-motion drive. A drive for reversing motion by means of bevel gears is illustrated in Fig. 29-6. When the driver a_1 is pressed against the follower b, the latter rotates in a certain direction. If the driving shaft is

shifted to the left, a_1 is disengaged and a_2 is pressed against b, and the follower begins to rotate in the opposite direction.

Bearings. To obtain the necessary pressure between the faces of two bevel gears, one of them must have a stationary thrust bearing, and the thrust bearing of the other gear must have provisions for an axial motion. In Fig. 29–4, for example, the bearing a has an Acme thread on the outside that meshes with a thread in the stationary support b, and the lever l serves to turn the bearing a and thus to move it axially.

29–4. Disk friction gears. The disk type of friction gear, Fig. 29–7, is a special kind of bevel-gear drive in which the angle α is 0. It is used mostly as a variable-speed drive. In a disk friction gear, unlike the spur friction gear, the usual practice is to make the driver a of cast iron or aluminum, and to face the driven wheel c with a composition material. The speed of the wheel c is changed by moving it across the face of the disk a, and the direction of its rotation may be reversed by moving it on the other side of the center of the disk a.

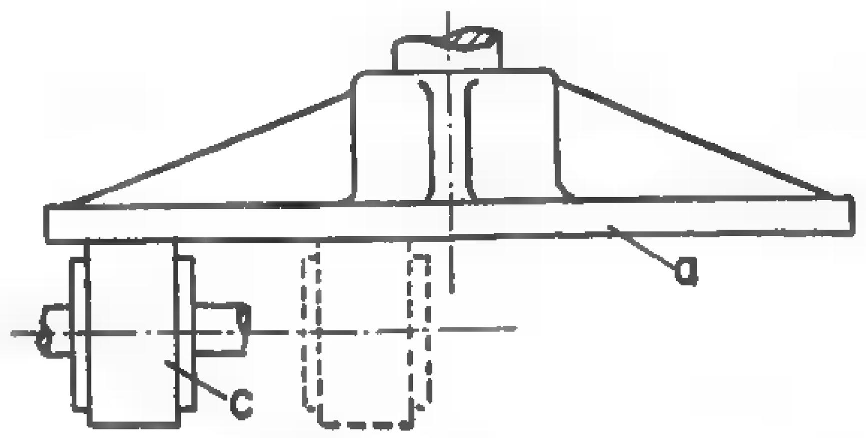


FIG. 29–7. Disk friction gearing.

Sometimes the wheel c is used as the driver, and the disk a is the driven member.

Force analysis. The torque on the driving shaft is $63{,}030P/n$. Hence the tangential forces acting upon the driven wheel for the two limiting speeds corresponding to a contact at the minimum and maximum diameters D_1 and D_2 are as follows:

At the minimum speed,

$$F_{t1} = \frac{126{,}060P}{nD_1} \tag{29–20}$$

At the maximum speed,

$$F_{t2} = \frac{126{,}060P}{nD_2} \tag{29–21}$$

The thrusts that may be applied to the disk for the two limiting speeds are found by dividing the values of F_{t1} and F_{t2} by the friction coefficient f. Therefore

$$F_1 = \frac{126{,}060P}{fnD_1} \tag{29–22}$$

and

$$F_2 = \frac{126{,}060P}{fnD_2} \tag{29–23}$$

Again $F_1 \leqq bp$, where b is the face of the driven cylindrical wheel.

Since there is relative motion between the faces of parts a and c, the actual tangential forces available at the driven wheel are found by multiplying the values just stated by the efficiency e. Approximately,

$$e = \frac{D}{D+b} \tag{29–24}$$

The efficiency varies from about 0.60 at low speeds, when $D = D_1$, to about 0.85 at high speeds, when $D = D_2$.

The axial force F_a required to shift the driven wheel c under load is found by multiplying the thrust F_1 or F_2 by the friction coefficient f and adding to the product the frictional resistance to axial motion of the shaft. Thus,

$$F_a = F_1(f + f_1) \tag{29–25}$$

where f_1 is the coefficient of friction between the shaft of wheel c and its bearings.

CHAPTER 30

Toothed Spur Gearing

30–1. General considerations. Toothed gears may be considered as friction gears in which the contact surfaces are provided with grooves and projections which prevent slip and insure positive means of transmitting rotation. Toothed gears are used when a constant speed ratio is desired and the distance between the shafts is relatively small. Toothed spur gears are used to connect parallel shafts and are suitable for transmission of any amount of power.

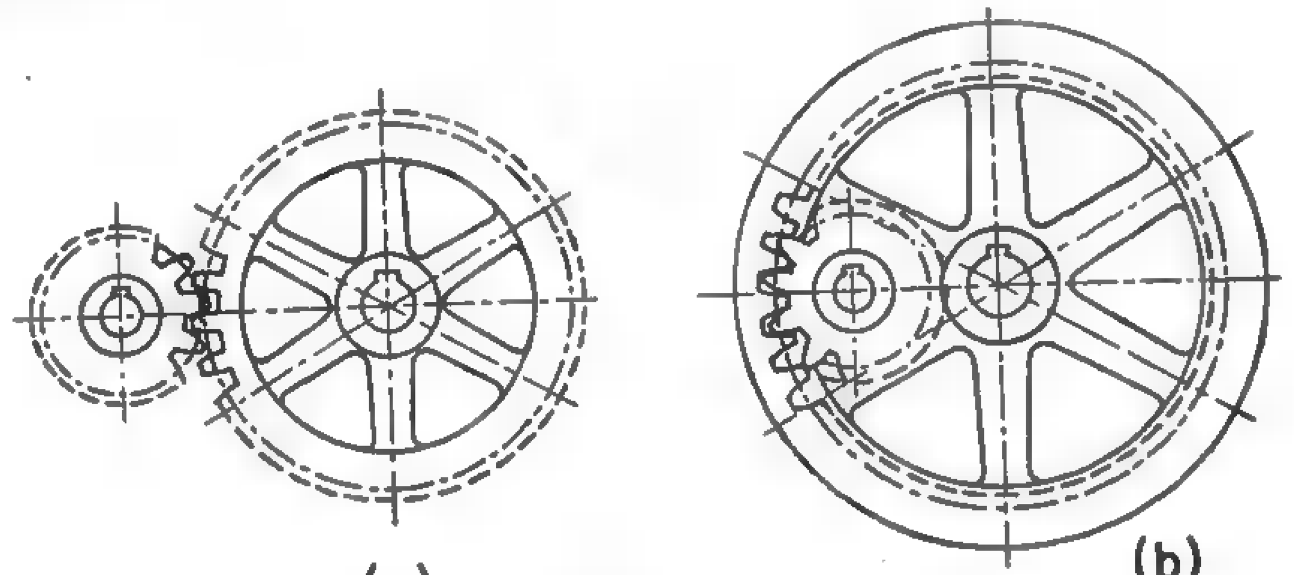


Fig. 30–1. External and internal spur gearing.

Fig. 30–2. Rack-and-pinion spur gearing.

Classification. Spur gearing may be classified as external, Fig. 30–1a; internal, Fig. 30–1b; and rack-and-pinion, Fig. 30–2. If the tooth elements are parallel to the shaft axis, the gears are termed *straight-tooth spur gears*; if the elements are helices, the gears are called *helical gears*. If the words spur gears are used without a modifying adjective, the more commonly used straight-tooth gears are signified.

Requirements. Toothed spur gears must meet the following requirements:

a) The teeth must have a profile which insures a constant velocity ratio.

b) The relative motion of one tooth upon the other should be more of a rolling nature than of a sliding nature.

c) The arc of engagement should be so long that at all times more than one pair of teeth is in mesh. In practice this is not always fulfilled.

d) The tooth profile should approach a cantilever beam of uniform strength.

Definitions. The information in Fig. 30–3 will help to explain the meaning of different terms used in connection with toothed gears.

When two gears are in mesh, the larger one is called the *gear* and the smaller one is called the *pinion*, regardless of which one is the driver.

The rubbing surfaces of the friction wheels have become the *pitch surfaces* of the toothed gear and pinion. The intersection of the pitch surface with a plane perpendicular to the axis of a gear or pinion is called the *pitch circle*. The contact point O, Fig. 30–3, of two pitch circles is called the *pitch point*. However, in involute gears the nominal pitch circles may not be tangent. The pitch circle is the basis of measurement of gears. The *size* of a gear is the diameter of its nominal pitch circle, in inches.

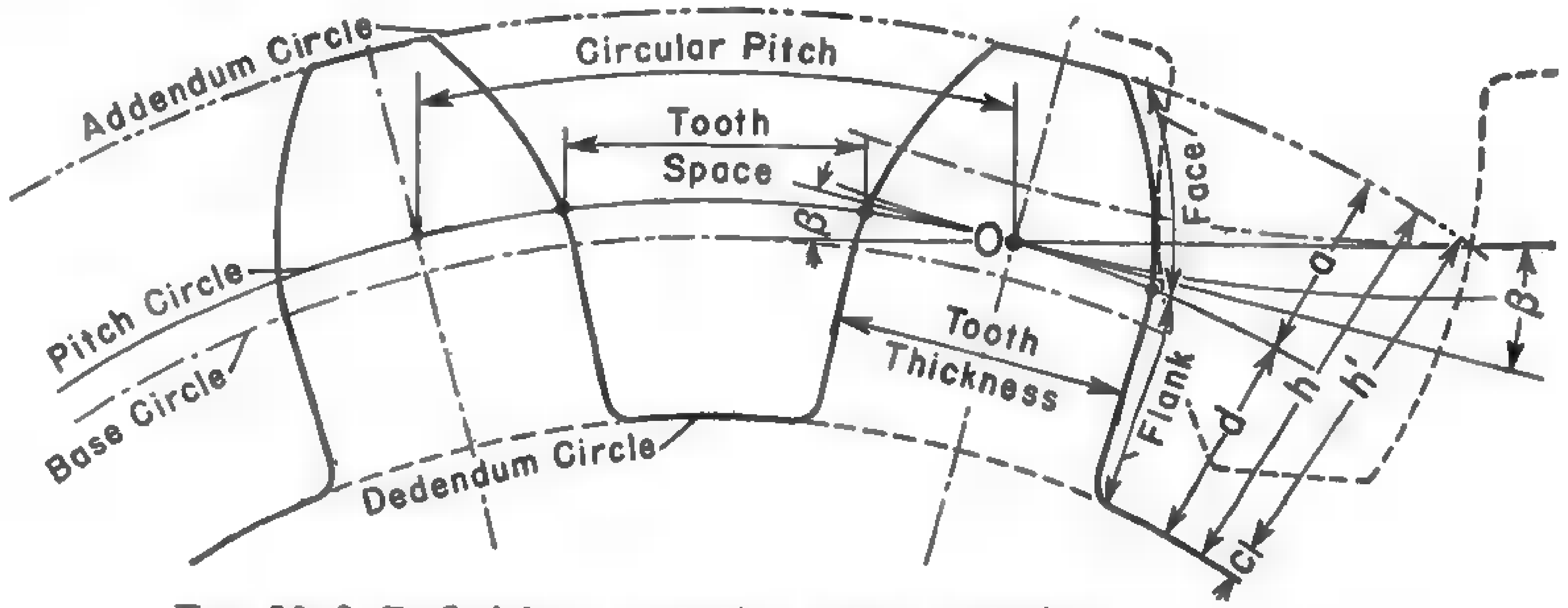


Fig. 30–3. Definitions and dimensions relating to spur gears.

The *addendum circle* is the circle which bounds the outer ends of the teeth, the addendum a being the radial distance between the pitch circle and the addendum circle.

The *dedendum circle*, or *root circle*, or *clearance circle*, bounds the bottom of the teeth, the dedendum d being the radial distance between the pitch circle and the dedendum circle.

Thus the *height* of the tooth is $h = a + d$.

The *clearance* c is the difference between the dedendum and the addendum; and for equal-addendum gears, $c = d - a$.

The *thickness* of the tooth is its thickness measured on the pitch circle, as shown in Fig. 30–3.

The *tooth space* is the width of the empty space on the pitch circle.

The *backlash* is the difference between the tooth space and the thickness of the tooth. Backlash is necessary to care for inaccuracies in the form and in the spacing of the teeth, and in the mounting of the gears. In gears with teeth cut very accurately, backlash may be practically zero.

The *face of the tooth* is the surface of the tooth between the pitch cylinder and the addendum cylinder; the *flank* is the surface between the pitch cylinder and the root cylinder.

The *face of the gear* is its width measured parallel to its axis.

The *line of centers* is the line connecting the centers of a pair of mating gears.

The *pressure angle*, or angle of obliquity, is the inclination of the line of action of the pressure between a pair of meshing teeth with respect to a line drawn tangent to the pitch circle at the pitch point, as angle β in Fig. 30–3.

The *base circle* is an auxiliary circle used in involute gearing to generate the tooth profile. It is tangent to the line representing the tooth thrust.

The *describing circle* is an auxiliary circle used in cycloidal gearing to generate the tooth profile.

The *arc of approach* is the arc measured on the pitch circle from the position of the tooth at the beginning of contact to the pitch point, as arc aO in Fig. 30-4. The *arc of recess* is the arc measured on the pitch circle from the pitch point to the position of the tooth where contact ends, as arc Or. The *arc of action* is the sum of the arc of approach and the arc of recess. Naturally it does not make any difference whether these arcs are measured on the pitch circle of the gear or that of the pinion. However, the corresponding angles of motion will be different.

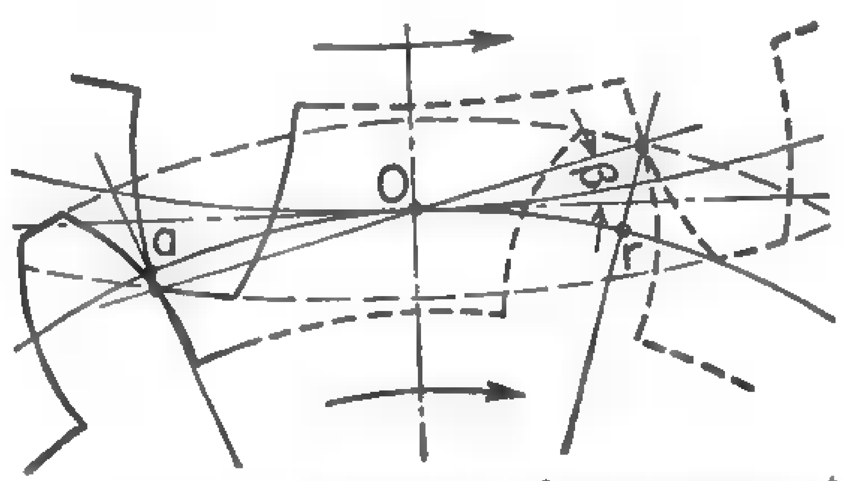


Fig. 30-4. Duration of engagement of a pair of gear teeth.

By *velocity ratio*, or *speed ratio*, r_v, is always meant the ratio of the number of revolutions of the driver to the number of revolutions of the driven gear. In a speed reducer, r_v is greater than 1; in a speed increaser, less than 1.

Pitch. The pitch is a measure of the size of a gear tooth. The pitches in common use are the circular pitch and the diametral pitch.

The *circular pitch*, designated p_c, is the distance, in inches, measured along the pitch circle from a point on one tooth to the corresponding point on an adjacent tooth, as shown in Fig. 30-3.

If D is the diameter of the pitch circle and i is the number of teeth, then evidently

$$p_c = \frac{\pi D}{i} \tag{30-1}$$

Circular pitch is used for cast-tooth gears because of its convenience when laying out the pattern. It is also used to a certain extent for cut-tooth gears and especially for large gears, when $p_c > 3$ in.

The *diametral pitch* p_d of a gear represents the number of teeth per inch of pitch diameter. Thus

$$p_d = \frac{i}{D} \tag{30-2}$$

Sizes of diametral pitches for which cutters may be obtained from stock are: from 1 to 4, by increments of $\frac{1}{4}$; from 4 to 6, by $\frac{1}{2}$; from 6 to 16, by 1; from 16 to 32, by 2.

Pitch relation. Multiplying equation 30-1 by equation 30-2 gives the relation for converting one pitch into the other one. This relation is

$$p_c p_d = \pi \tag{30-3}$$

30-2. Gear teeth. Two types of curves, the cycloidal and the involute, are now in general use for gear teeth. In regard to efficiency and strength, both forms are practically equal. An advantage of the cycloidal tooth over the involute one is that a convex surface is always in contact with a concave one; consequently, the wear is not as fast as with involute teeth, whose surfaces are either convex or straight. For this reason, gears transmitting very large amounts of power are sometimes cut with cycloidal teeth. On the other hand, the involute teeth have a decided advantage over the cycloidal ones in that the actual distance between the centers may deviate slightly from the theoretical distance without affecting the velocity ratio or general performance. Because of this distinct advantage, gears with involute cut teeth are used much more than those with cycloidal teeth.

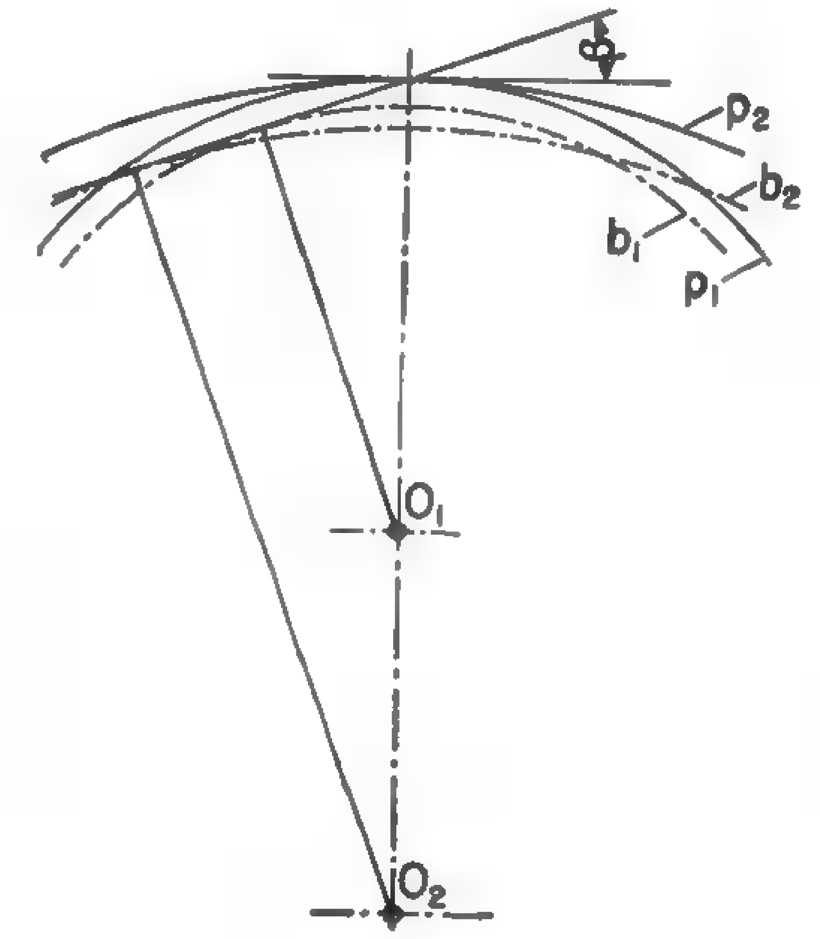


Fig. 30-5. Base and pitch circles.

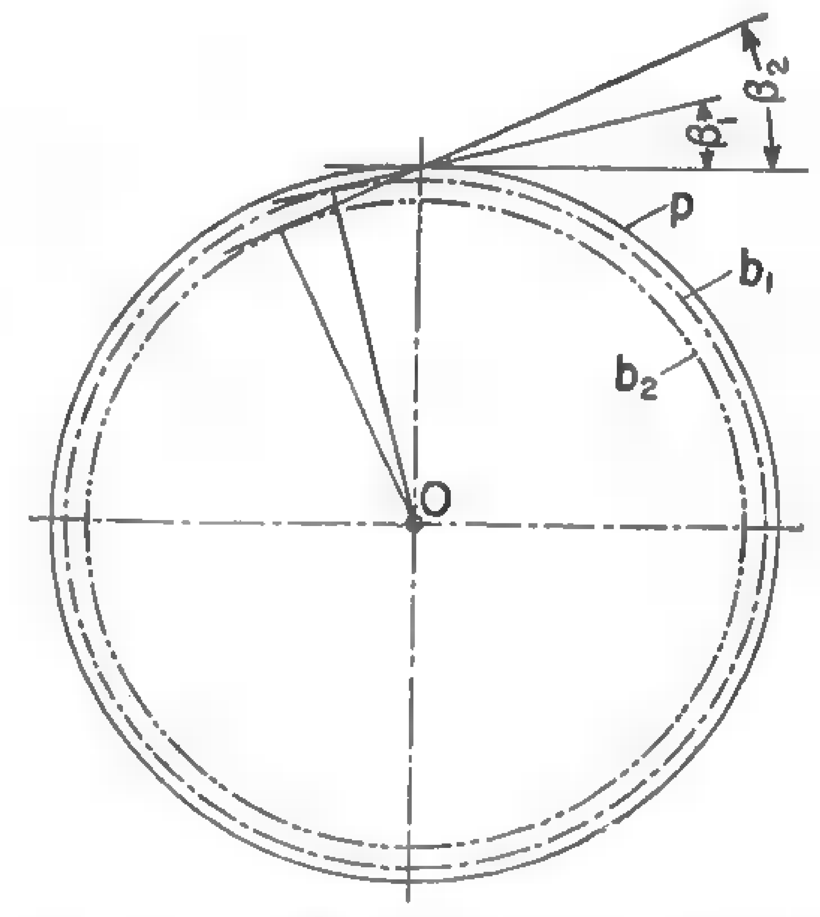


Fig. 30-6. Base circles and degrees of involute.

Involute-tooth action. It should be noted that an involute can exist only outside of the base circle. Therefore only that part of a tooth profile which lies outside the base circle is an involute and can mesh with another involute of the same pitch. The part of the flank of a tooth inside the base circle does not come in driving contact with the mating tooth. It may have any form, provided it does not interfere with the face of the mating tooth. Formerly this part was made radial. Modern methods of generating gear teeth automatically undercut this part of the flank.

For the same pressure angle β, the radial distance between the pitch circle and the base circle increases with an increase of the pitch diameter, as shown in Fig. 30-5. For a given pitch circle this radial distance increases as the pressure angle increases, as shown in Fig. 30-6. Pressure angles in common use are $14\frac{1}{2}°$, 15°, and 20°.

Since the pressure line coincides with the generating line of an involute and is normal to the involute, the point of contact of two mating teeth has

TABLE 30–1

PROPORTIONS OF INVOLUTE TEETH

Tooth Characteristics	Cast Teeth	AGMA Composite $14\frac{1}{2}°$ System	Full-Depth 20° System	AGMA Stub Teeth
Pressure angle (deg)	15	$14\frac{1}{2}$	20	20
Addendum (in.)	$0.3p_c$	$\frac{1}{p_d}$	$\frac{1}{p_d}$	$\frac{0.8}{p_d}$
Minimum dedendum (in.)	$0.4p_c$	$\frac{1.157}{p_d}$	$\frac{1.157}{p_d}$	$\frac{1}{p_d}$
Minimum total depth (in.)	$0.7p_c$	$\frac{2.157}{p_d}$	$\frac{2.157}{p_d}$	$\frac{1.8}{p_d}$
Minimum clearance (in.)	$0.1p_c$	$\frac{0.157}{p_d}$	$\frac{0.157}{p_d}$	$\frac{0.2}{p_d}$
Thickness of the tooth (in.)	$0.475p_c$	$\frac{1.571}{p_d}$	$\frac{1.571}{p_d}$	$\frac{1.571}{p_d}$
Backlash (in.)	$0.05p_c$	0	0	0

the pressure line as its common normal. Therefore two gears in mesh must have the same pressure angle, or must be of the *same degree of involute.*

Interference. If, with a certain angle of obliquity, the number of teeth of a pinion is decreased below a certain minimum, the tooth tips of the gear or rack will begin to interfere with the part of the tooth flank of the pinion below the base line. Thus, with a $14\frac{1}{2}°$ involute the smallest pinion of interchangeable sets that will mesh correctly with a rack of the same pitch contains 31 teeth. This difficulty is eliminated by slightly correcting the points of all the teeth in a set of cutters, so that a pinion of 12 teeth may still mesh with any of the gears of the same pitch.

For a 20° full-depth involute the smallest pinion that can correctly mesh contains 17 teeth; and for the 20° stub tooth, the pinion must have at least 14 teeth.

Methods of manufacture. Gear teeth are formed either by molding and casting or by machine cutting. Cast teeth are molded from complete gear patterns, or by means of gear-molding machines. The latter method gives better results. Nevertheless, cast teeth are always somewhat rough and warped out of shape, so the gears are noisy and are not suited for higher speeds.

Most gears of ordinary size are cut with a milling cutter, or by special gear planers, or else by means of hobs. This hobbing method of generating spur and helical gear teeth gives very accurate teeth. Other advantages of hobbing are that only one hob is required for all numbers of teeth of a given pitch, and the production cost is lower.

Interchangeability. In order to manufacture gears economically, it is necessary that any gear of a given pitch should work correctly with any other gear of the same pitch, thus making the gears interchangeable. Other conditions for interchangeability are that the gears must have the same pressure angle and that the addendum must equal the dedendum minus the clearance. These conditions are fulfilled by making the teeth according to certain standard proportions, which differ with the method of manufacture and the special requirements of strength. Table 30–1 gives proportions for the more commonly used systems.

30–3. Cast teeth. Cast-tooth gears are used for low-speed rough service, mostly in outdoor installations where noise is not objectionable. Because of the inaccuracy of forming and spacing the teeth, even with molding machines, it is safer to assume that the entire load is transmitted by one tooth.

It may be assumed that the load F_t acts at the upper corner of the tooth but is uniformly distributed along the length b of the tooth, as indicated in Fig. 30–7.

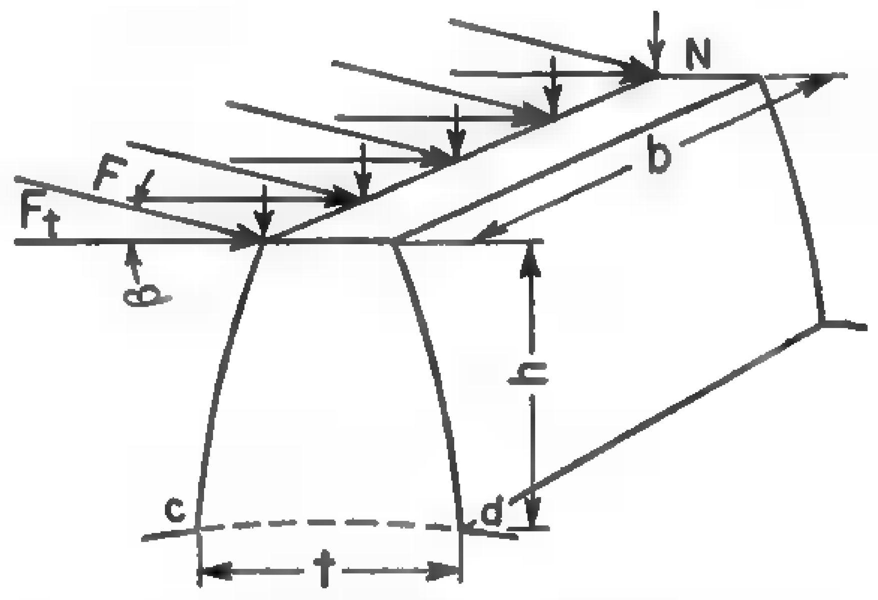


FIG. 30–7. Load action on a gear tooth.

If the bending moment at the root of the tooth is equated to the resisting moment and the stress is designated by the product S_oc, the result is

$$F_th = \tfrac{1}{6}S_ocbt^2 \tag{30–4}$$

in which S_o denotes the allowable stress in the material and c is the velocity factor.

According to Table 30–1, $h = 0.7p_c$ and $t = 0.475p_c$. Substituting these values in equation 30–4, and solving for F_t, gives

$$F_t = 0.054S_ocbp_c \tag{30–5}$$

The velocity factor c is introduced in order to take care of the impact when the teeth come in contact. It is computed by the following Barth's formula:

$$c = \frac{600}{600 + v_m} \tag{30–6}$$

where v_m is the pitch-line velocity. For cast teeth it should not exceed 450 fpm. The stress may be taken equal to the elastic limit in tension divided by a safety factor $n = 1.5$. The average values used are as follows: For ordinary cast iron, $S_o = 8{,}000$ psi; for high-grade or nickel cast iron, 12,000 psi; and for cast steel, 20,000 psi. The gear face b is made from $2p_c$ to $2.5p_c$. After p_c is found from equation 30–5, the next-larger standard circular pitch is used. The circular pitch of cast teeth varies in larger sizes in increments of ¼ in., and from 1 in. down in increments of [illegible] in.

With p_c found, the number of teeth i of a gear which has a peripheral speed of v_m fpm at n rpm is found from the relation

$$v_m = \frac{ip_cn}{12} \tag{30-7}$$

The pitch diameter D is found from equation 30–1. Then the outside diameter D', with the addendum $a = 0.3p_c$ from Table 30–1, is

$$D' = D + 2 \times 0.3p_c = D + 0.6p_c \tag{30-8}$$

30–4. Cut teeth. Three main profile systems are used for cut teeth. They are known as the standard $14\frac{1}{2}°$ involute system, the full-depth 20° involute system, and the stub 20° involute system.

Standard $14\frac{1}{2}°$ involute system. The standard angle of obliquity adopted by the manufacturers of gear cutters is 14° 28′ 40″, the sine of which is 0.25. The standard proportions of teeth for this system are given in Table 30–1.[1]

Full-depth 20° involute system. The tooth proportions for the full-depth 20° system are the same as those of the standard $14\frac{1}{2}°$ system, the only difference being in the pressure angle. The 20-deg angle reduces the interference, permitting the use of a 14-tooth pinion without any changes. Because of the increased pressure angle the teeth also become slightly broader at the root, as shown in Fig. 30–8, and stronger.

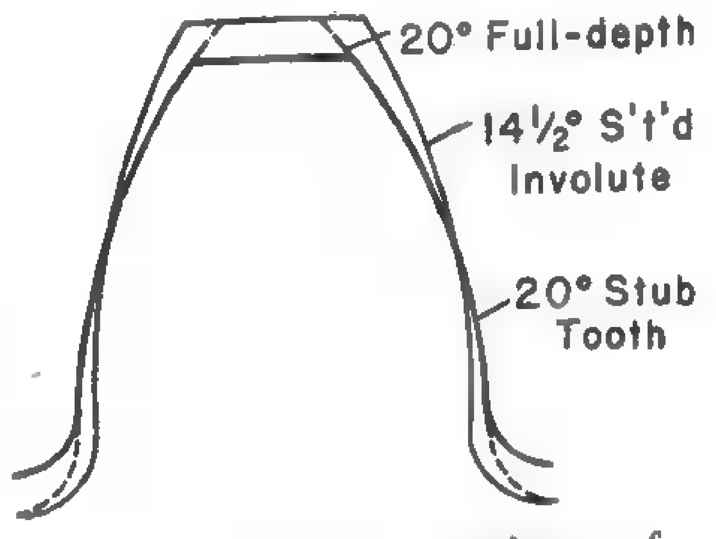


FIG. 30–8. Comparison of tooth shapes.

Stub 20° involute system. The 20° stub tooth is subject to a still-smaller interference because of a shorter addendum, as shown in Table 30–1. The interfering portion of the tooth is thus removed. Stub teeth are also considerably stronger than full-height teeth because of a smaller moment arm of the bending force, Fig. 30–7.

A third advantage of stub teeth is their lower production cost, as less metal must be cut away. The advantages of stub teeth apply chiefly to gears with a small number of teeth. With a large number of teeth, full-depth tooth gears having pressure angles of either $14\frac{1}{2}°$ or 20° perform better than stub-tooth gears.[2]

Several gear manufacturers have established their own standards for the proportions of stub teeth, particularly Nuttall (Westinghouse) and Fellows Gear Shaper Company. The American Standards Association has established proportions, given in Table 30–1, which make the gears interchangeable with other 20° stub-tooth gears.

[1] *Spur Gear Tooth Forms*, ASA B6.1–1932 (New York: American Standards Association, 1932).

[2] V. M. Faires, *Design of Machine Elements*, rev. ed. (New York: The Macmillan Company, 1942), p. 199.

The Fellows stub-tooth system uses a 20-deg pressure angle and a fractional designation for the pitch. The numerator of this designation is the actual diametral pitch of the gear, whereas the denominator indicates the pitch of the cutter used in cutting the teeth and thus determines their depth. The numerator is used in most equations, but the denominator must be used in the expression for the height of the tooth, as shown in Table 30–1, and in determining the outside diameter.

Design procedure. For many years the proportions of gear teeth were determined by the Lewis formula, which aims to obtain teeth sufficiently strong in regard to bending. With intermittent service, as gears were generally used in former days, the results were satisfactory. With the advent of machinery in which gear teeth transmit power continuously, such as in automobiles, the wear of the gear teeth became an important factor, and a new method of design was developed.

However, the older method is so much simpler that it seems advisable to continue its use for the preliminary design and later to check the dimensions thus obtained, by applying the newer formulas which take into consideration the service conditions and requirements.

30–5. Design for strength. In the derivation of the Lewis equation it is assumed that at the beginning of contact the load F is applied at the end of the tooth, with its line of action normal to the tooth profile, as shown in Fig. 30–7.

The normal force F is resolved at the end of the tooth into two components, N and F_t. The component N acts radially and produces pure compression, while F_t acts tangentially.

The dangerous section is at cd, where the stress may be determined by the equation

$$s = \frac{M}{Z} = \frac{6F_th}{bt^2} \tag{30-9}$$

The factor $t^2/6h$ is a geometrical property of the size and shape of the tooth. It may therefore be expressed as a function of the circular pitch p_c by the relation

$$\frac{t^2}{6h} = yp_c \tag{30-10}$$

where y is an abstract number known as the *Lewis form factor*.

From equation 30–10,

$$y = \frac{t^2}{6hp_c} \tag{30-11}$$

By introducing yp_c in equation 30–9, substituting the product S_oc for s, as was done in deriving equation 30–5, and solving for F_t, we obtain the original Lewis equation, which is

$$F_t = S_ocbyp_c \tag{30-12}$$

TABLE 30–2

VALUES OF Υ IN MODIFIED LEWIS FORMULA

Number of Teeth	Full-Depth 14½° System and Cycloidal	Full-Depth 20° System	Stub 20° System	Fellows Stub Teeth, 20° — Pitch 4/5	5/7	6/8	7/9	8/10	9/11	10/12	12/14
10	0.176	0.201	0.261	....	....	....	....	....	....	....	....
11	.192	.226	.289	....	....	....	....	....	....	....	....
12	.210	.245	.311	0.302	0.348	0.320	0.314	0.302	0.314	0.292	0.289
13	.223	.264	.324	.318	.361	.336	.332	.317	.327	.308	.302
14	.235	.276	.339	.330	.374	.352	.348	.332	.339	.320	.314
15	.245	.289	.349	.339	.386	.364	.361	.346	.348	.330	.324
16	.255	.295	.360	.348	.396	.374	.370	.355	.354	.340	.333
17	.264	.302	.368	.358	.405	.383	.380	.364	.366	.349	.342
18	.270	.308	.377	.368	.411	.390	.390	.374	.374	.358	.349
19	.277	.314	.386	.374	.414	.398	.398	.383	.380	.364	.355
20	.283	.320	.393	.380	.425	.405	.405	.390	.386	.371	.361
21	.289	.326	.399	.386	.431	.411	.411	.396	.392	.377	.366
22	.292	.330	.404	.391	.436	.417	.417	.402	.397	.382	.371
23	.296	.333	.408	.396	.441	.422	.422	.407	.402	.387	.377
24	.302	.337	.411	.401	.446	.427	.427	.411	.405	.392	.381
25	.305	.340	.416	.405	.449	.432	.432	.417	.409	.396	.386
26	.308	.344	.421	.409	.455	.436	.436	.421	.413	.401	.389
28	.314	.352	.430	.417	.461	.443	.444	.427	.421	.409	.396
30	.318	.358	.437	.425	.468	.449	.452	.433	.427	.415	.402
35	.327	.373	.449	.436	.480	.463	.465	.449	.438	.427	.415
40	.336	.389	.459	.446	.490	.475	.474	.458	.446	.440	.425
45	.340	.399	.468	.455	.500	.484	.484	.464	.455	.446	.433
50	.346	.408	.474	.461	.506	.490	.490	.471	.461	.452	.439
60	.355	.421	.484	.471	.515	.500	.500	.483	.471	.465	.449
70	.360	.429	.493	.480	.521	.506	.506	.490	.477	.471	.455
80	.363	.436	.499	.488	.528	.512	.512	.496	.483	.477	.461
90	.366	.442	.503	.492	.532	.517	.516	.499	.487	.481	.466
100	.368	.446	.506	.496	.536	.521	.521	.503	.490	.484	.471
150	.375	.458	.518	.509	.546	.534	.531	.515	.503	.496	.484
200	.378	.463	.524	.515	.553	.540	.536	.521	.509	.503	.490
Rack	0.390	0.484	0.550	0.543	0.578	0.562	0.553	0.540	0.534	0.528	0.521

Substituting π/p_d for p_c, from equation 30–3, and replacing πy by Υ, gives the modified Lewis equation, which is

$$F_t = \frac{S_o c b \Upsilon}{p_d} \tag{30–13}$$

The factor Υ is also called the *form factor*. The value of Υ depends on the shape of the tooth profile, including the influence of the number of teeth, and may be taken from Table 30–2 for any one of the various gear systems. Since the Lewis formula is used in designing gears that are cut with standard diametral pitches, equation 30–13 is more convenient than equation 30–12.

The values of Υ given for 20° stub teeth in Table 30–2 can be used also for Nuttall stub-tooth gears.

Gear face. There is no strict relation between the gear face b and the pitch. However, if the face is very long compared with the pitch, or the thickness of the tooth, it is likely that the pressure will not be uniformly distributed along the face. Since this is contrary to the assumption made in deriving the Lewis formula, the maximum stress probably will be greater than is assumed. Conversely, a very narrow face will require a coarse pitch and will give a less smooth action and poor wearing qualities. Practice indicates that the proper width of face is between $3p_c$ and $4p_c$, or in terms of the diametral pitch,

$$b > \frac{9.5}{p_d} \qquad b < \frac{12.5}{p_d} \tag{30–14}$$

However, under certain conditions, as where there are space limitations, the width b may be made as narrow as $6.3/p_d$ or as wide as $19/p_d$.

Design stress. The stress S_o in the Lewis formula should be taken, as for cast teeth, equal to the elastic limit of the material in bending divided by a factor of safety $n = 1.5$.

The velocity factor for ordinary cut gears running with a pitch-line velocity up to 1,500 fpm may be computed by Barth's formula (equation 30–6).

For carefully cut gears with a pitch velocity up to 2,500 fpm, Barth's formula may be modified to the relation

$$c = \frac{900}{900 + v_m} \tag{30–15}$$

For very accurately cut and ground metallic gears having a pitch velocity v_m from 1,200 to 4,000 fpm, Barth's formula may be modified by increasing the constant from 600 to 1,200. Thus

$$c = \frac{1{,}200}{1{,}200 + v_m} \tag{30–16}$$

For hardened-steel ground and lapped-in precision gears made for speeds over 4,000 fpm, the American Gear Manufacturers Association recommends the use of a velocity factor given by the relation

$$c = \frac{78}{78 + \sqrt{v_m}} \tag{30–17}$$

For convenience, values of safe static stress $S_o = S_e/n$ for materials commonly used for gears are given in Table 30–3. Nickel cast iron and chrome-nickel steels are used rather extensively.[3]

Noiseless, or silent, gears. In order to reduce the noise of the meshing teeth, especially at high pitch-line velocities, spur gears are made of non-

[3] J. W. Sands and F. J. Walls, "Nickel-Alloy Gear Materials and Their Heat-Treatments," *Product Engineering*, Vol. 6 (1935), p. 370; *Gear Materials and Blanks*, ASA B6.2–1933 (New York: American Standards Association, 1933).

TABLE 30–3

ALLOWABLE STATIC STRESSES FOR USE IN LEWIS FORMULA

Material	S_o (psi)	Material	S_o (psi)
Ordinary cast iron	8,000	Forged steel, about SAE 1030	25,000
Cast iron, about Class No. 35	12,000	Steel, SAE 1030, heat-treated	32,000
High-grade cast iron, about Class No. 50	15,000	Steel, SAE 1040, untreated	30,000
Cast steel, 0.20% C, untreated	20,000	Alloy steel, casehardened	50,000
Cast steel, 0.20% C, heat-treated	28,000	Cr-Ni steel, about SAE 3245, heat-treated	67,000
Bronze, SAE 62	10,000	Cr-Va steel, about SAE 6145, heat-treated	75,000
Phosphor gear bronze, SAE 65	12,000	Rawhide, Fabroil, etc.	6,000
Manganese bronze SAE 43	20,000	Laminated phenolic materials (Bakelite, Micarta, Celoron)	6,000
Aluminum bronze, SAE 68	22,000		

metallic materials such as rawhide, Fabroil, Bakelite, Textolite, or Celoron. Rawhide and Fabroil, which is cotton that is treated with oil under a high hydraulic pressure, are not self-supporting, and gears of these materials must be made with metal flanges, or shrouds, at both ends for teeth support. Bakelite, Celoron, Formica, Micarta, Textolite, and other materials which are made of laminated fibrous material impregnated with synthetic resin of the phenolic type, do not require a support for the teeth. The AGMA recommends for nometallic teeth the use of a velocity factor c found by the relation

$$c=\frac{150}{200+v_m}+0.25 \tag{30–18}$$

In cutting the teeth in phenolic materials it should be remembered that their coefficient of expansion from heat is slightly more than twice that of cast iron, and they therefore require a greater backlash. With all nonmetallic teeth, pitch-line speeds up to 3,000 fpm can be used; and with Celoron teeth, speeds up to 4,000 fpm are permissible.

The tooth load F_t is determined from the general equation

$$F_t=\frac{33{,}000Pc_s}{v_m} \tag{30–19}$$

where c_s is a service factor, the value of which may be taken from Table 30–4.

30–6. Dynamic load. Even with the best machine tools and most careful workmanship, gears will have inaccuracies in the teeth. These inaccuracies cause short periods of acceleration and deceleration, which are increased by the elastic deflections of the teeth. Also, even with exactly-spaced teeth there is always a sudden load application when two teeth come

TABLE 30–4

SERVICE FACTOR c_s FOR GEARS IN EQUATION 30–19

Type of Load	Type of Service		
	Intermittent, or 3 Hr per Day	8 to 10 Hr per Day	Continuous, 24 Hr per Day
Steady	0.80	1.00	1.25
Light shocks	1.00	1.25	1.50
Medium shocks	1.25	1.50	1.80
Heavy shocks	1.50	1.80	2.00

into contact. Thus, while the power transmitted may be constant, the load on the teeth varies.

A newer method of gear design considers that the maximum dynamic load F_d on the gear tooth consists of the transmitted, or useful, load F_t and an increment load F_i caused by inaccuracies of the teeth, errors in spacing, tooth deflection, unbalance, or power-flow fluctuation.[4] Thus

$$F_d=F_t+F_i \tag{30–20}$$

The increment load depends on the masses of the moving parts. For average conditions it may be found by the equation

$$F_i=\frac{0.05v_m(Ceb+F_t)}{0.05v_m+\sqrt{Ceb+F_t}} \tag{30–21}$$

where values of the coefficient C are given in Table 30–5 and the probable error e in the tooth profile may be taken from Fig. 30–9.

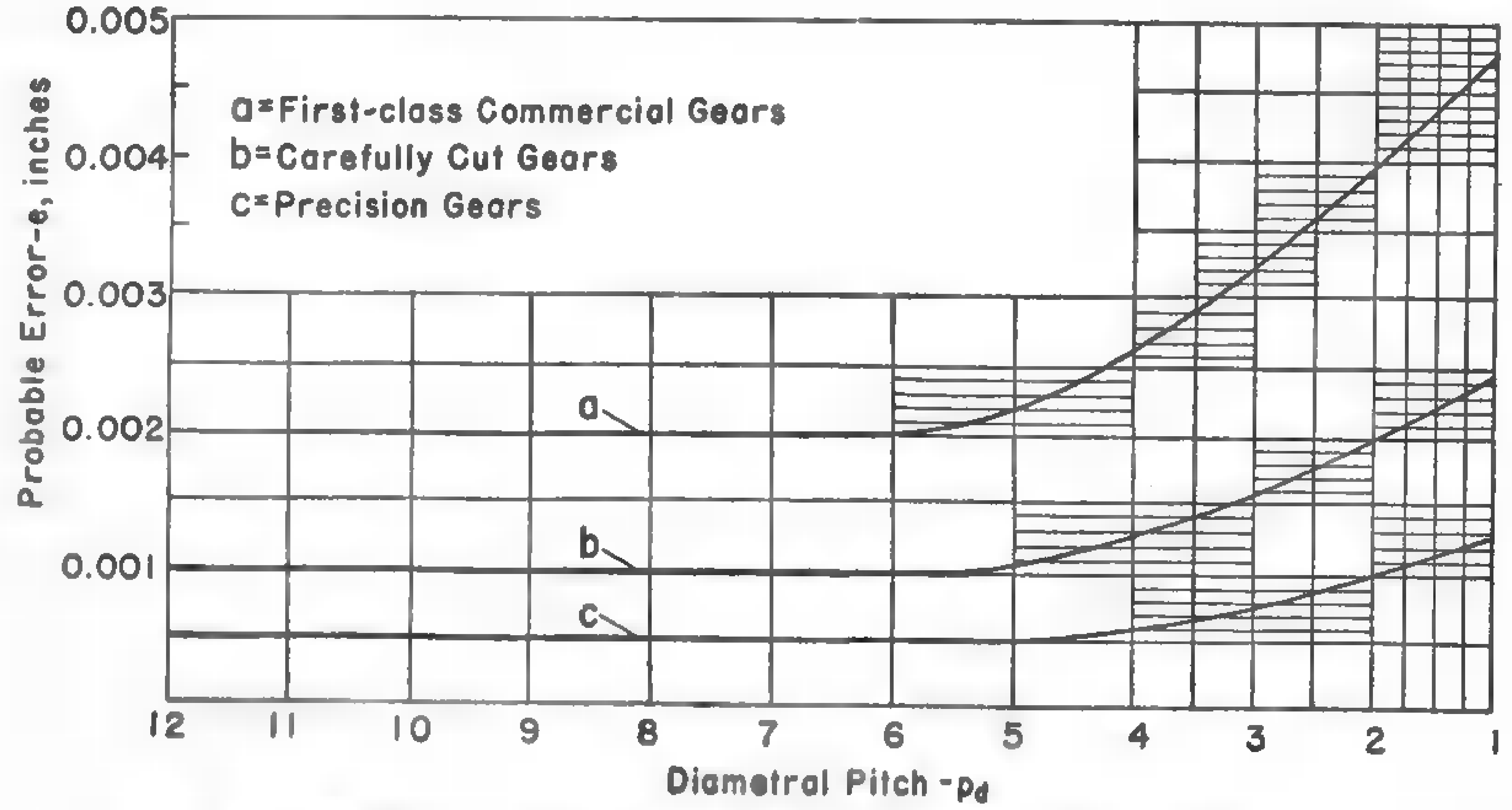


FIG. 30–9. Probable errors in tooth profiles and spacing.

[4] *Dynamic Loads on Gear Teeth*, Report of the ASME Special Research Committee on the Strength of Gear Teeth (New York: American Society of Mechanical Engineers, 1931); E. Buckingham, *Manual of Gear Design* (New York: The Industrial Press, 1953), Section 2, pp. 141 ff.

TABLE 30–5

VALUES OF COEFFICIENT C IN EQUATION 30–21

MATERIALS OF GEARS IN MESH	TOOTH FORM		
	$14\frac{1}{2}°$ Involute	20° Full-Depth	20° Stub Teeth
Gray iron and gray iron	800,000	830,000	860,000
Gray iron and steel	1,100,000	1,140,000	1,180,000
Steel and steel	1,600,000	1,660,000	1,720,000

For materials not shown in Table 30–5 the value C may be calculated from the relation

$$C=\frac{a}{\frac{1}{E_p}+\frac{1}{E_g}} \tag{30–22}$$

where E_p and E_g are the moduli of elasticity of the pinion material and the gear material, respectively, and a may be taken as 0.107 for the $14\frac{1}{2}°$ tooth form, 0.111 for the 20° full-depth form, and 0.115 for 20° stub teeth.

Nonaverage conditions, such as gears used in aeronautical work where the rotating masses are less than average, or gears connected with heavy flywheels where the masses are greater than average, require special, rather involved calculations.[5]

Silent gears, being made of materials whose resilience and flexibility are very high and whose specific weight is low, suffer from shock action much less than metal gears. Therefore their maximum load may be considered to be equal to the useful load divided by the velocity factor c, from equation 30–18, or

$$F_d=\frac{F_t}{c} \tag{30–23}$$

Endurance strength. In order to determine the degree of safety, the maximum load may be compared with the strength of the tooth found by applying the Lewis formula and using the endurance limit S_{en} instead of the permissible stress S_oc. This strength is

$$F_{en}=\frac{S_{en}bY}{p_d} \tag{30–24}$$

The permissible load computed by equation 30–24 is termed the *endurance strength* of the gear teeth. The use of the endurance limit S_{en} takes care of stress concentration at the base of the teeth. This method gives good results in practice. Values of S_{en} to be used in equation 30–24 are given in Table 30–6.

Safety Margin. The endurance load F_{en} found by equation 30–24 must be greater than the maximum load F_d determined by equation 30–20 or

[5] *Ibid.*

TABLE 30–6

ENDURANCE LIMITS FOR CHECKING GEAR TEETH

Material	Core (Bhn)	S_{en} (psi)	Material	Core (Bhn)	S_{en} (psi)
Gray iron	160	12,000	Steel	200	50,000
Semisteel	200	18,000	Steel, normalized	240	60,000
Manganese bronze, SAE 43	100	17,000	Steel, SAE 3140, heat-treated	280	70,000
Gear bronze, SAE 65	100	24,000	Steel, SAE 3240, heat-treated	320	80,000
Nonmetallic	...	6,000	Steel, oil-tempered	360	90,000
Steel	150	37,500	Steel, nitralloy	400	100,000

equation 30–23, as the case may be. The difference $(F_{en}-F_d)$ may be called the *safety margin.* This safety margin should be about 25 per cent of the actual load F_d for a steady transmitted load, about 35 per cent for a pulsating load, and 50 to 60 per cent for shock loads.[6]

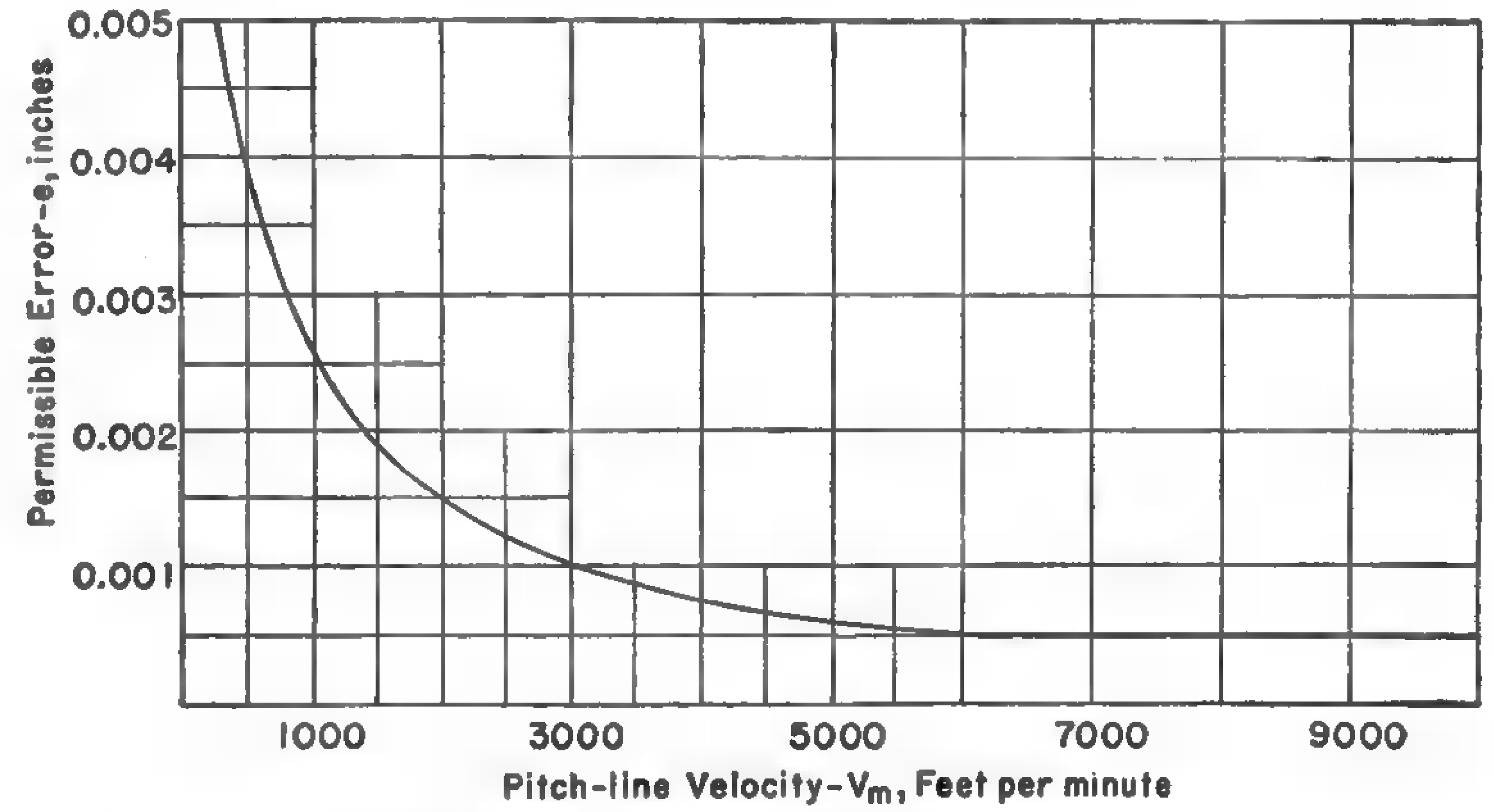


FIG. 30–10. Permissible errors in tooth profiles for quiet operation of gears.

Permissible errors. A very useful curve is given in Fig. 30–10. It shows the maximum error in the gear-tooth profile which a gear can carry for satisfactory operation at a given pitch-line speed. In connection with Fig. 30–9 the curve shows at a glance the class of workmanship required and the minimum value of p_d. Thus for a pitch-line speed of 3,000 fpm the permissible error is $e=0.001$ in., and it is necessary to use either carefully cut gears with $p_d=5.5$ or greater, or precision gears with $p_d=2$ or greater.

30–7. Check for wear. Gears in continuous service lose their usefulness because of excessive wear, not because of a sudden failure. Wear occurs in five ways: (a) by pitting of the tooth surface by repeated compressive stressing; (b) by abrasion caused by foreign matter; (c) by scoring caused

[6] *Ibid.*

by sharp and projecting edges and rough surfaces; (d) by scuffing, which results from the use of an improper lubricant; and (e) by seizing, due to a complete failure of the lubrication, accompanied by a locally generated heat sufficient to weld the surfaces one to another.

Tests have shown that in the order of increasing resistance to wear, the $14\frac{1}{2}°$ full-depth involute tooth comes first, the 20° involute stub tooth comes next, while the best is the 20° full-depth involute tooth.[7]

Pitting. The most serious cause of wear is pitting. It can be prevented only by proper design, while the other causes of wear can be overcome during operation of the gears. In order to prevent pitting, the compression endurance limit of the material must not be exceeded.

Pitting usually increases rather slowly. The *limiting load for wear*, F_w, is the load beyond which wear is likely to be rapid. It is found by the equation

$$F_w = KbD_pQ \tag{30–25}$$

where K is a load-stress factor, b is the gear face, D_p is the pitch diameter of the pinion, and Q is a function of the relative size of the gears.

The load-stress factor may be calculated by the equation

$$K = \left(S^2_{en,c} \times \frac{\sin \beta}{1.4}\right)\left(\frac{1}{E_p} + \frac{1}{E_g}\right) \tag{30–26}$$

where $S_{en,c}$ is the compressive endurance limit for the material of the pinion, β is the pressure angle, and E_p and E_g are the moduli of elasticity of the materials used. The value of K makes allowance for the cold working received by the more plastic material from the harder mating material. For a number of more commonly used materials the values of K computed by equation 30–26 are given in Table 30–7. This table shows that cast iron is a very good material as far as wear resistance is concerned.

In addition it must be stated that Table 30–7 confirms the general rule that less wear is incurred if the two surfaces in contact are of different materials. Exceptions to this rule are cast iron, which wears well on cast iron, and heat-treated steel, tempered to a very hard surface, against equally heat-treated hard steel.

The value of Q is found by the equation

$$Q = \frac{2i_g}{i_p + i_g} = \frac{2D_g}{D_p + D_g} \tag{30–27}$$

where i_p and i_g are the numbers of teeth and D_p and D_g are the pitch diameters of the pinion and gear, respectively.

If the gears are intended for continuous operation, the limit load for wear F_w must be greater than the maximum load F_d.

[7] G. H. Marx, L. E. Cutter, and B. M. Green, "Some Comparative Wear Experiments on Cast-Iron Gear Teeth," *Mechanical Engineering*, Vol. 48 (1926), p. 35.

TABLE 30–7

VALUES OF K FOR COMPUTING THE LIMITING WEAR LOAD

Gear		Pinion			Value of K	
Material	Hardness (Bhn)	Material	Hardness (Bhn)	Endurance Limit (psi)	$14\frac{1}{2}°$ Involute	20° Involute
Steel	150	Steel	150	50,000	30	41
Steel	150	Steel	200	60,000	43	58
Steel	150	Steel	250	70,000	58	79
Steel	200	Steel	200	70,000	58	79
Steel	200	Steel	250	80,000	76	103
Steel	200	Steel	300	90,000	96	131
Steel	250	Steel	250	90,000	96	131
Steel	250	Steel	300	100,000	119	162
Steel	250	Steel	350	110,000	144	196
Steel	300	Steel	300	110,000	144	196
Steel	300	Steel	350	120,000	171	233
Steel	300	Steel	400	125,000	186	254
Steel	350	Steel	350	130,000	201	275
Steel	350	Steel	400	140,000	233	318
Steel	350	Steel	450	145,000	250	342
Steel	450	Steel	450	170,000	344	470
Steel	450	Steel	500	175,000	364	497
Steel	450	Steel	600	180,000	385	526
Steel	500	Steel	500	190,000	430	588
Steel	600	Steel	600	230,000	630	861
Cast iron	180	Steel	150	50,000	44	60
Cast iron	180	Steel	200	70,000	87	119
Cast iron	180	Steel	250	90,000	144	196
Gear bronze	100	Steel	150	50,000	46	62
Gear bronze	100	Steel	200	70,000	91	124
Gear bronze	100	Steel	250	85,000	135	184
Cast iron	180	Cast iron	180	90,000	193	264
Metal	...	Nonmetallic	(34)	32,000	189	258

Scoring. Wear due to scoring progresses very rapidly if the operating conditions which cause it are not changed. Tests have shown that scoring starts when the tooth load reaches a certain value.[8] This limit value can be increased up to 10 to 15 per cent by using an oil with a higher viscosity.

30–8. Design procedure. In the preliminary design by the Lewis equation the following steps should be taken:

1) The peripheral velocity v_m should be selected. The value of v_m should be low—600 to 1,000 fpm—for a low rotary speed of the pinion and a low-to-moderate power transmitted. The velocity v_m should be increased for a higher pinion speed and greater power.

2) The materials of the pinion and gear and their static strengths S_o are selected from Table 30–3. Low-strength steel and cast iron are pre-

[8] V. N. Borsoff, J. B. Accinell, and A. G. Cattaneo, "The Effect of Oil Viscosity on the Power Transmitting Capacity of Spur Gears," *Transactions of the American Society of Mechanical Engineers*, Vol. 73 (1951), p. 687.

ferred for a low velocity v_m, and better-grade materials should be used for higher velocities and greater power.

3) The relative width of the face b is tentatively selected in terms of p_d in accordance with equation 30–14.

4) The tooth load and the velocity factor c are computed, the type of tooth profile is decided upon, and a preliminary value for the form factor Y of the pinion is assumed. It may be taken as 0.25 for a $14\frac{1}{2}°$ tooth form, 0.29 for a 20° full-depth form, and 0.35 for a 20° stub tooth.

5) Equation 30–13 is solved for p_d, and the nearest standard value is taken.

6) The approximate pinion diameter D_p is determined from the selected velocity v_m, and the numbers of teeth i_p and i_g are found.

7) The width b of the face is computed for both the pinion and the gear from equation 30–13, in which the corrected values for Y and c are used, and the greater value is taken.

8) The dimensions of the pinion and gear are checked for endurance strength and wear.

EXAMPLE 30–1. Determine the proper pitch, face, number of teeth, and outside diameters of a pair of $14\frac{1}{2}°$ spur gears to transmit 160 hp, from a pinion running at 750 rpm to a gear running at 140 rpm. The service is intermittent with light shocks.

Since no limitations are given for the largest diameter or the maximum center distance, the way to proceed is to select the materials of the pinion and gear, make some assumptions, calculate the drive, and if the results are not quite satisfactory, change the assumptions.

A suitable material for the pinion is forged steel SAE 1030. From Table 30–3, S_o = 25,000 psi. For the gear a suitable material seems to be cast iron of class No. 50, for which $S_o = 15{,}000$ psi.

It is next necessary to select a suitable pitch-line velocity. For gears operating at moderate speeds, a pitch velocity of 1,500 fpm seems to be suitable. The approximate pinion pitch diameter is then

$$D_p = \frac{12v_m}{\pi n_p} = \frac{12 \times 1{,}500}{\pi \times 750} = 7.13 \text{ in.}$$

If, in equation 30–19, c_s is taken as 1 from Table 30–4, the tangential tooth load is

$$F_t = \frac{33{,}000Pc_s}{v_m} = \frac{33{,}000 \times 160 \times 1}{1{,}500} = 3{,}320 \text{ lb}$$

The speed factor for the allowable stress, by equation 30–6, is

$$c = \frac{600}{600 + 1{,}500} = 0.286$$

Then the allowable stress is

$$S = 25{,}000 \times 0.286 = 7{,}150 \text{ psi}$$

The relative gear face may be assumed as $b = 9.5/p_d$, and the factor Y may be taken as 0.25. With these preliminary data the diametral pitch p_d, from equation 30–13, is

$$p_d = \sqrt{\frac{9.5SY}{F_t}} = \sqrt{\frac{9.5 \times 7{,}150 \times 0.25}{3{,}320}} = 2.26$$

The nearest standard pitch is 2.25. With a 7.13-in. pitch diameter, there should be 17 teeth, which is satisfactory. From Table 30–2, $Y = 0.264$.

The face b can be found from equation 30–13 by using the selected pitch and number of teeth and the corrected pitch-line velocity, the load F_t, the factor c, and the exact value for Y. This procedure gives

$$b = \frac{3{,}360 \times 2.25}{25{,}000 \times 0.287 \times 0.264} = 3.98 \text{ in.}$$

In accordance with equation 30–14, $b = 4.25$ in. should be used.

The number of teeth required in the gear will be

$$i_g = \frac{17 \times 750}{140} = 91$$

It is next necessary to find the required gear face. Here $S_o = 15{,}000$ psi; and for 91 teeth, $Y = 0.366$. Then, from equation 30–13,

$$b = \frac{3{,}360 \times 2.25}{15{,}000 \times 0.287 \times 0.366} = 4.8 \text{ in.}$$

A face b of 5 in. on both the gear and the pinion is permissible, according to equation 30–14.

The outside diameters can be determined by applying a relation similar to equation 30–8 and using data from Table 30–1. Thus

$$D_p' = \frac{i_p + 2}{p_d} = \frac{17 + 2}{2.25} = 8.444 \text{ in.}$$

and

$$D_g' = \frac{91 + 2}{2.25} = 41.333 \text{ in.}$$

EXAMPLE 30–2. Find the safety margin and suitability for continuous operation of the pair of gears computed in example 30–1.

In order to determine the dynamic tooth load by equation 30–20, the increment load F_i must be found first.

The value of C, from Table 30–5, is 1,100,000. Also, the probable error e, from curve b in Fig. 30–9, is 0.0019. In equation 30–21,

$$Ceb + F_t = 1{,}100{,}000 \times 0.0019 \times 5 + 3{,}360 = 13{,}800$$

Then, by equation 30–21, the increment load is

$$F_i = \frac{0.05 \times 1{,}483 \times 13{,}800}{0.05 \times 1{,}483 + \sqrt{13{,}800}} = 5{,}340 \text{ lb}$$

The total dynamic load, by equation 30–20, is

$$F_d = 3{,}360 + 5{,}340 = 8{,}700 \text{ lb}$$

For steel with a Bhn of 150, S_{en} from Table 30–6 is 37,500 psi. By equation 30–24, in which $Y = 0.264$, the endurance strength is

$$F_{en} = \frac{37{,}500 \times 5 \times 0.264}{2.25} = 22{,}000 \text{ lb}$$

The safety margin is 22,000 − 8,700 = 13,300 lb, or more than 150 per cent.

In order to check the limit load for wear by equation 30–25, the value of K from Table 30–7 is found to be 44. Then, by equation 30–27, the function Q is

$$Q = \frac{2 \times 91}{17 + 91} = 1.685$$

Therefore,

$$F_w = 44 \times 5 \times 7.55 \times 1.685 = 2{,}800 \text{ lb}$$

Since F_w is less than F_d, the pinion is not suitable for continuous work. If the pinion is made of heat-treated steel for which the Bhn is 250, Table 30–7 shows that $K = 144$.

Then F_w becomes 9,160 lb, which is greater than $F_d = 8{,}700$ lb. Therefore this design is satisfactory.

30–9. AGMA formulas. The American Gear Manufacturers Association has issued tentative standards for rating various toothed gearings. The rating for the beam strength of spur-gear teeth is a modified Lewis formula with several refinements. Factors for these refinements must be taken from numerical tables and graphs. The rating for wear is based on a formula similar to equation 30–25 but with numerous refinements.

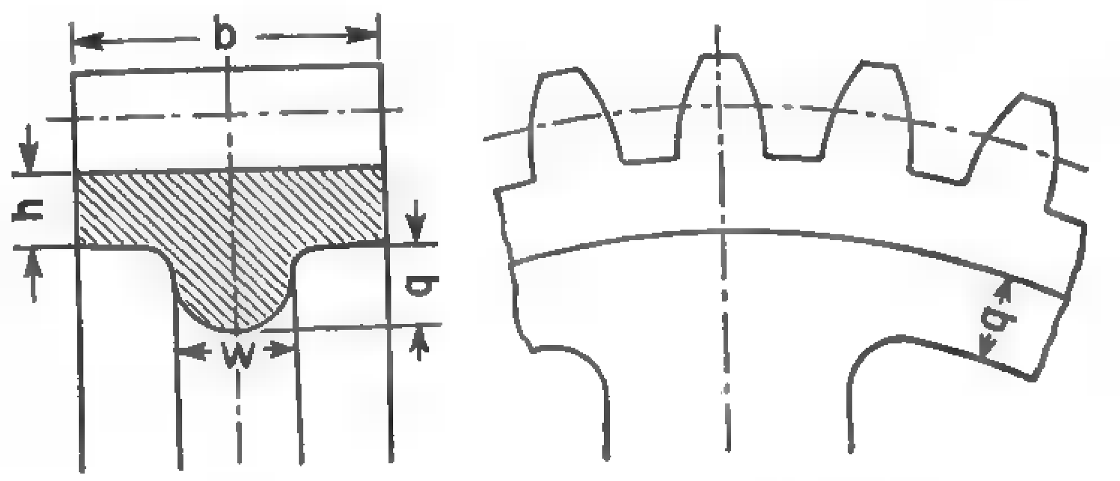


Fig. 30–11. Gear-rim dimensions.

For a more accurate, final design, the AGMA procedure should be used.[9] However, the much simpler procedure presented in sections 30–5 to 30–8 gives results sufficiently safe and has the advantage of bringing out more clearly the interrelation of the various factors.

30–10. Gear construction. The dimensions of the parts of a gear are determined largely by empirical rules.

Rim. The thickness h of the rim of a gear, Fig. 30–11, is made according to empirical rules from $0.5p_c$ to $0.6p_c$. The rim should taper at the rate of about 1 in. per ft toward the center, to secure draft in molding or forging.

The Westinghouse-Nuttall formula is

$$h = \frac{1}{p_d}\sqrt[3]{\frac{i}{2j}} \qquad (30\text{–}28)$$

where i is the number of teeth and j is the number of arms. This formula gives good results for larger gears; if it gives $h < 0.5p_c$, make $h = 0.5p_c$.

When arms are used, the stiffness of the rim should be increased by a rib having a depth $q = h$ and a width w equal to the thickness of the arms at this end.

Hub. Recommendations for the diameter and length of the hub of a gear are given in Table 30–8. However, the hub length should never be less than the gear face b. A small gear may be fastened to the shaft by a square key and a setscrew over it. A large gear requires a taper key which must be fitted very accurately to avoid excessive hammering that may distort the gear and result in noisy running and rapid wear.

[9] A comprehensive section on gear design, including AGMA recommended procedures and a good list of references, will be found in R. T. Kent, *Mechanical Engineers' Handbook*, 12th ed., Vol. II, *Design and Production*, ed. by Colin Carmichael (New York: John Wiley & Sons, Inc., 1950).

TABLE 30–8

Dimensions of Gear Hubs

Type of Service	Diameter		Length
	Cast Iron	Steel Casting	
Light load, no shock	$1.75d$	$1.6d$	$l \geqq 1.5d$
Medium load and shock	$1.85d$	$1.7d$	$l \geqq 1.75d$
Heavy load with shock	$2d$	$1.8d$	$l \geqq 2d$

The minimum thickness m of metal permissible above the keyway of a pinion may be determined by the empirical relation

$$m = \sqrt{\frac{0.2i_p}{p_d}} \qquad (30\text{–}29)$$

Solid web. In a pinion the pitch diameter is often so small that there is no space left between the rim and the hub. In such a case the pinion is made solid and of uniform thickness equal to b.

The diameter, in inches, for a solid pinion is given by the empirical equation

$$D \leqq 4.7p_c + 2.35 \qquad (30\text{–}30)$$

The limit for a pinion or gear with a web is given by the equation

$$D \leqq 7.5p_c + 3.35 \qquad (30\text{–}31)$$

The thickness, in inches, of a web may be found by the equation

$$h_w = 0.5p_c + 0.125 \qquad (30\text{–}32)$$

Arms. The number of arms is usually four for gears with diameters up to 15 in.; six for diameters from 15 to 120 in., in both the solid and split constructions; and eight for all larger diameters.

The usual cross section of the arms is an ellipse, as in Fig. 30–12a, with the major axis twice the minor one. Cross-shaped sections, as in Fig. 30–12b, and I-shaped sections, as in Fig. 30–12c, are also frequently used; and the H-shaped section, as in Fig. 30–12d, is used in very large gears.

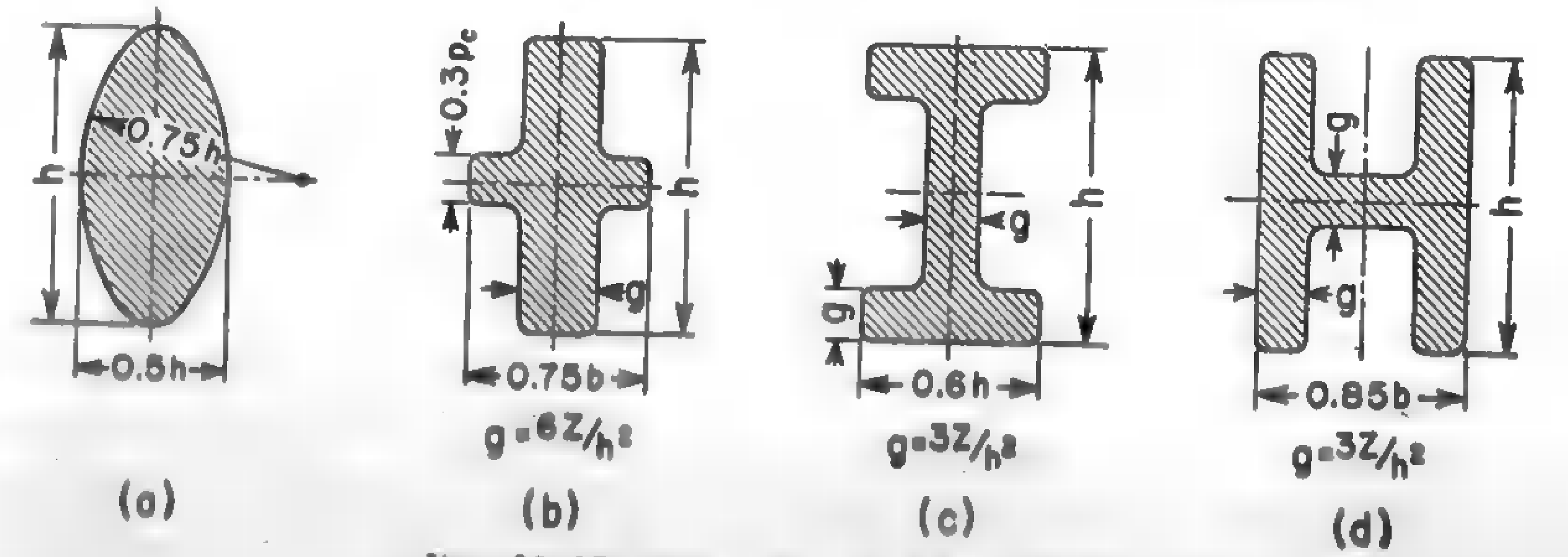


Fig. 30–12. Proportions of gear arms.

In deducing a formula for calculating the dimensions of a gear arm it is customary to assume that the rim has sufficient rigidity to distribute the load on the teeth equally among the arms, which are acting as cantilevers. In this case the required section modulus of the arm at the center of the hub may be determined by the relation

$$Z = \frac{F'D}{2jS_d} \tag{30-33}$$

where F' is the stalling load on the teeth at zero velocity, j is the number of arms, and S_d is the allowable stress. If the arm has the cross section shown in Fig. 30-12a, the arm width h at the hub may then be found from the formula for the section modulus of an elliptic section with the minor axis one-half the major axis. Thus

$$h = 2.73\sqrt[3]{Z} \tag{30-34}$$

The dimension h of any of the other cross sections of Fig. 30-12 is determined from the section modulus by using the proportions and the equation shown for the corresponding section. Care must be taken to obtain the necessary value of Z. The older practice of assuming that only one-half or one-third of all arms takes the load is unnecessarily conservative. The arms are tapered $\frac{3}{4}$ in. per ft, the smaller end being toward the rim.

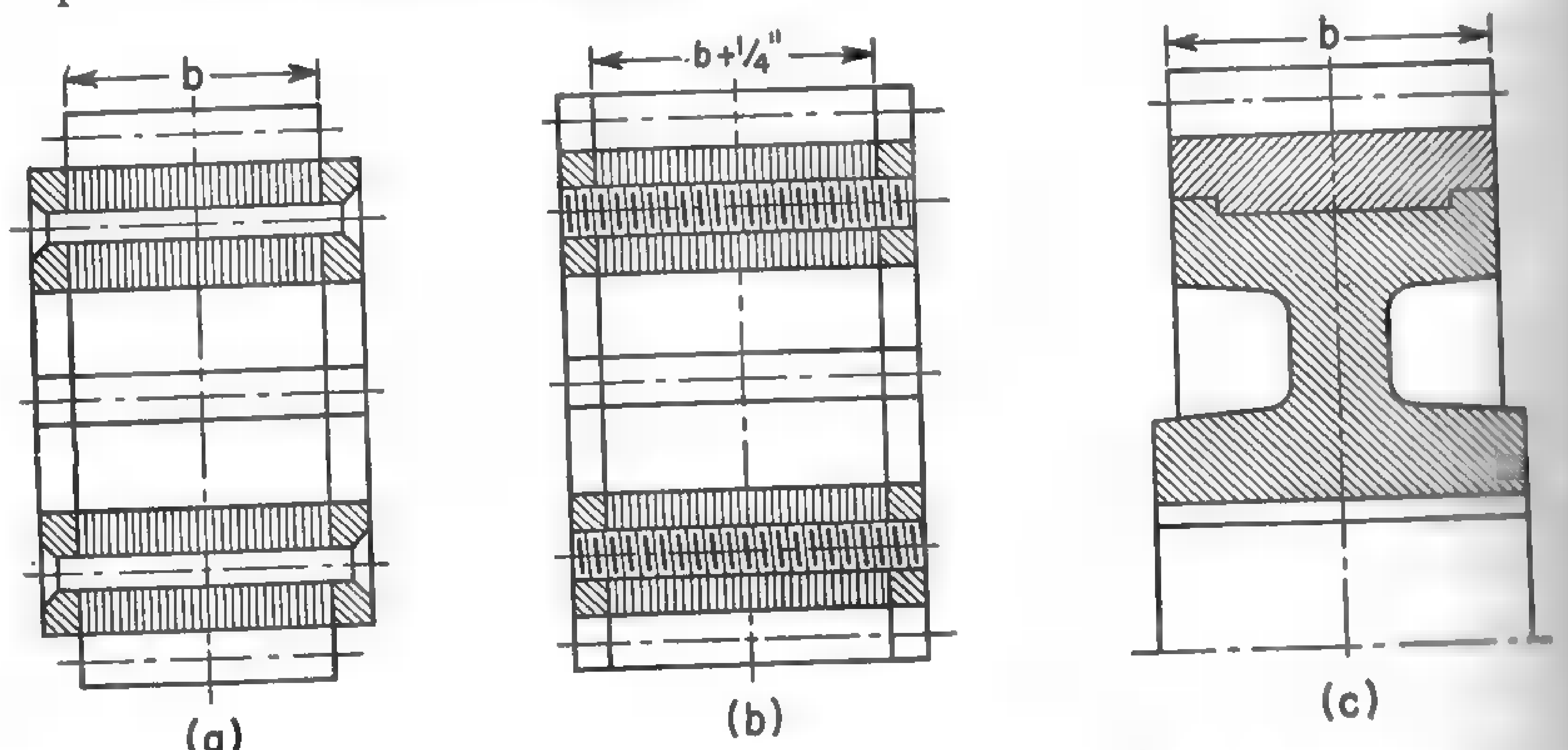


Fig. 30-13. Nonmetallic-tooth pinions and gears.

Nonmetallic-tooth pinions. Materials that are not self-supporting, such as rawhide or Fabroil, are fastened to steel flanges by rivets, as in Fig. 30-13a, or by threaded rods, as in Fig. 30-13b. The construction of Fig. 30-13b gives better support for the soft-material teeth but requires a wider face, to make sure that the teeth of the gear will not come in contact with the steel plates. Laminated phenolic materials are used without metal reinforcement, but a bushing is provided in the center to take the key. In larger diameters the phenolic material is molded over a cast-iron or steel center, as in Fig. 30-13c, to reduce the cost.

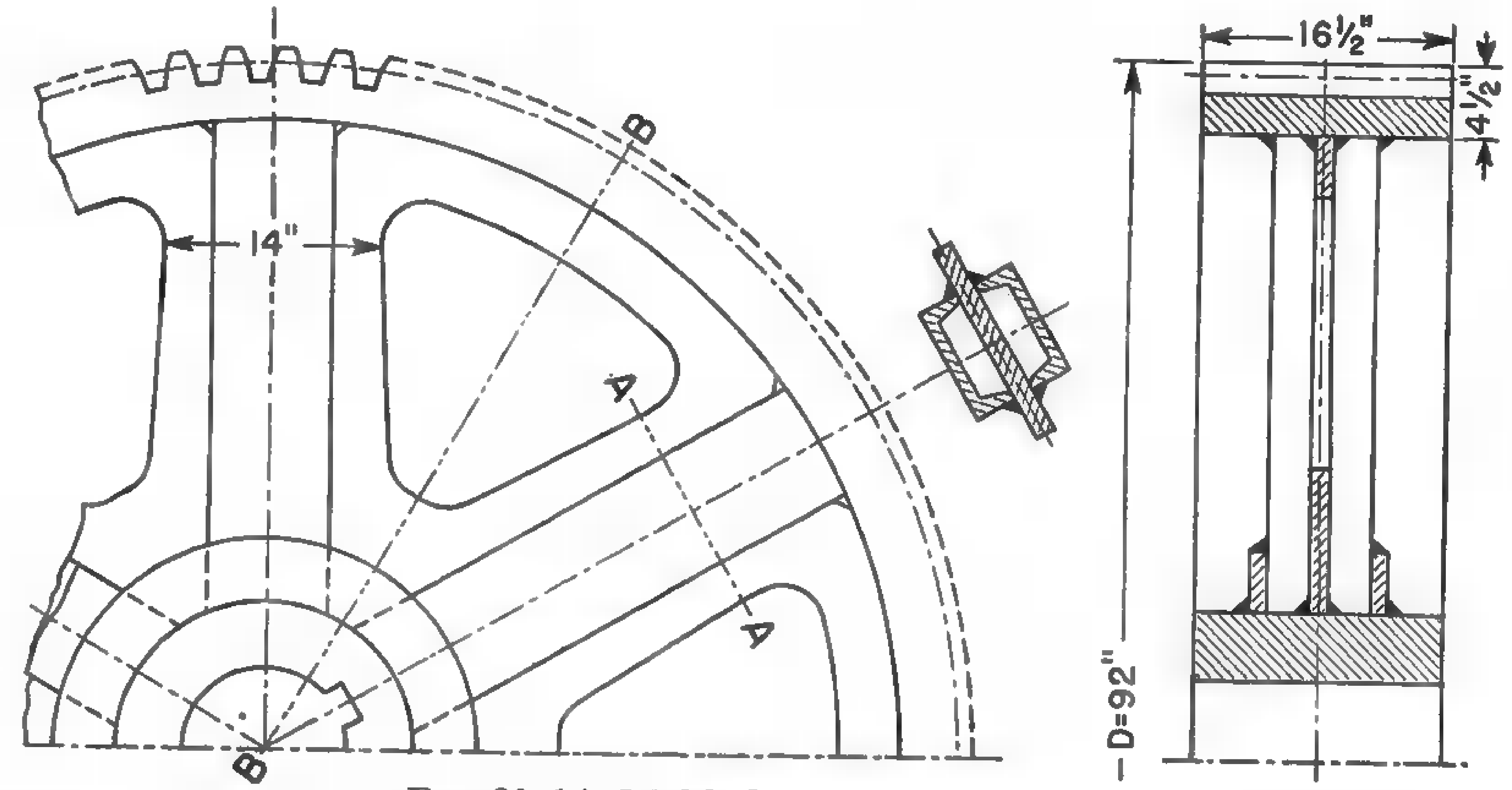


Fig. 30-14. Welded gear construction.

Welded gears. Large gear blanks are made by welding. In Fig. 30-14 is shown a spur gear whose rim is a steel plate bent into a ring and welded, the weld coming at a tooth space. The arms are U-shaped channels welded to a flat plate and reinforced at the hub by welded-on flat rings, and the hub is a forging. A wide-face herringbone gear with box-shaped arms is shown in Fig. 30-15. The construction may be varied to suit conditions. For a given strength, a welded construction is much lighter than a cast-iron or cast-steel blank.

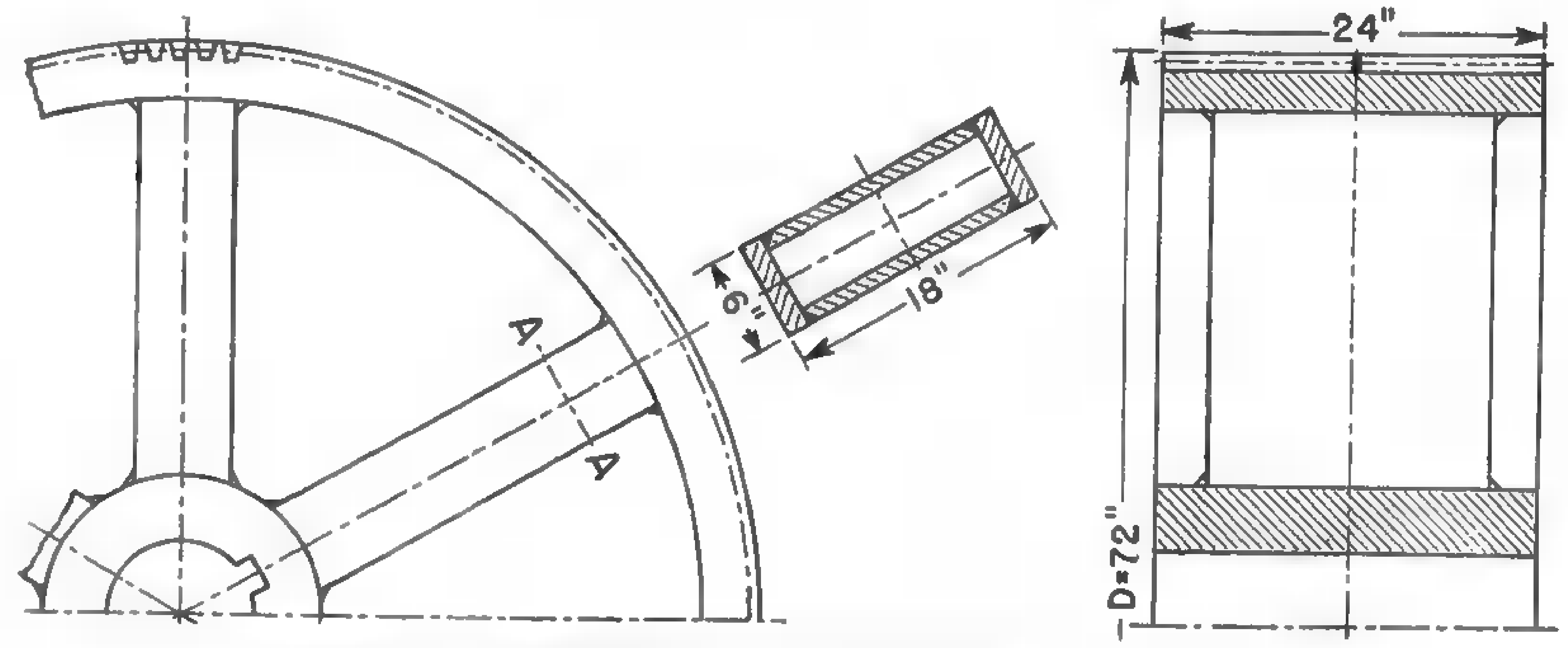


Fig. 30-15. Welded gear construction with wide face.

Split gears. Large gears were formerly cast in several pieces—two to six and more—which were bolted together or held by shrink anchors. With the introduction of welding, this type of construction, being much heavier, more expensive, and difficult to manufacture, is now hardly ever employed.

30-11. Other factors. There are a few other factors that should be considered in the design of gears.

Gear trains. When the velocity ratio is very high, it becomes desirable to reduce the speed in two or more steps. As an illustration, a triple reduction

is shown in Fig. 30–16. Ordinarily a speed ratio of about 6 for straight-tooth spur gears and a ratio of 12 for helical and herringbone gears are considered practical limits for a single reduction. The velocity ratio for a train of gears is equal to the product of the ratios of all pairs. Thus, for the train in Fig. 30–16 the ratio is

$$r = \left(\frac{n_1}{n_2}\right)\left(\frac{n_3}{n_4}\right)\left(\frac{n_5}{n_6}\right) = \left(\frac{D_2}{D_1}\right)\left(\frac{D_4}{D_3}\right)\left(\frac{D_6}{D_5}\right) \tag{30–35}$$

As the speed is reduced, the useful load on the tooth is increased. Therefore coarser teeth (i.e., a smaller diametral pitch) must be used. Because of the cumulative effect of lost motion, the shock action in the consecutive pairs of gears is increased, and a gradual decrease of the allowable stress, by about 10 or 15 per cent in each lower-speed step, is advisable.

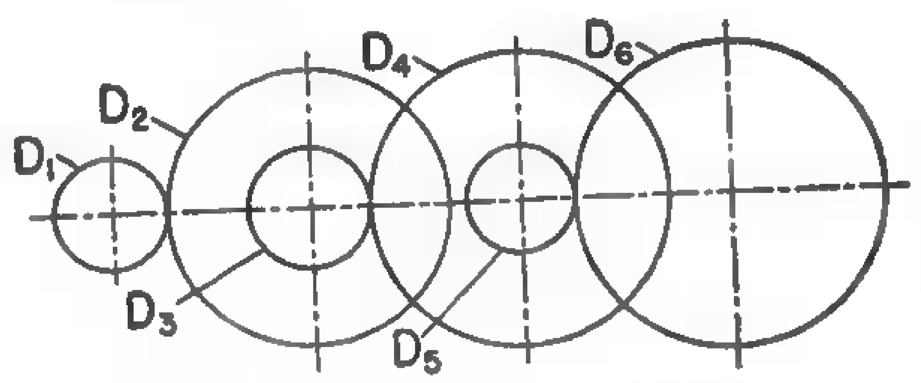


Fig. 30–16. Gear train.

Efficiency. There exist several theoretical formulas for expressing the efficiency of a toothed spur or bevel gear as a function of the coefficient of friction between the teeth, the pressure angle, and the number of teeth. However, they all are based on involved assumptions which are questioned by competent authorities. Experimental data form the most reliable guide. Considering tooth friction only, the efficiency of good unhardened spur gears is about 99 per cent. It is independent of the pitch-line velocity (at least within the range from 60 to 1,500 fpm), the pressure angle, the load transmitted, and the quantity of lubricating oil used, provided the quantity is sufficient to prevent cutting and heating.[10] The condition of the tooth surface is the most important factor. Tooth friction in accurately cut, hardened, and ground spur gears may be considered negligible. Table 30–9 shows the small losses that can be expected in various gear trains under the condition of best construction and care.[11] The starting losses are rendered higher by the greater viscosity of the cold oil.

Gear bearings. By decreasing the pitch-line velocity v_m for a given torque, the tangential tooth pressure F_t is increased in inverse proportion to v_m. Hence the bearing load, which is equal to $F_t \sec \beta$, is increased, and the bending moment on the gear shaft is likewise increased.

With electric-motor drives, and speeds higher than 1,300 fpm, overhung gears give trouble, and experience shows that an outboard bearing is a necessity and a good investment in diminishing repairs and shutdowns.

[10] C. W. Ham and I. W. Huckert, *An Investigation of the Efficiency and Durability of Spur Gears*, Bulletin No. 149, University of Illinois Engineering Experiment Station (1925), p. 21.

[11] W. H. Himes, "Modern Gear Efficiency Exceeds Limits Used in Most Designs," *Machine Design*, Vol. 4 (February, 1932), p. 29.

TABLE 30–9

Friction Loss in Spur and Helical Gears in Oiltight Cases

Number of Reductions for Straight Spur, Helical, or Herringbone Gears	Roller or Ball Bearings		Journal Bearings	
	Starting (per cent)	Running (per cent)	Starting (per cent)	Running (per cent)
Single	1–2	0.5–1	10–20	0.5–1.5
Double	2–4	1.0–2	15–25	1.0–3.0
Triple	3–6	1.5–3	20–35	1.5–4.0

In general, the more rigid the whole gear installation is, the more satisfactory will be its service.

30–12. Strengthening of gear teeth. Cut gear teeth may be strengthened by one of the following methods: (a) increasing the pressure angle; (b) using short teeth; (c) using stub teeth; (d) using teeth with an unequal addendum and dedendum; (e) crowning the teeth; (f) using helical teeth.

Increasing pressure angle. The gain in strength obtained by increasing the pressure angle has already been mentioned (see Fig. 30–8).

Short teeth. The Hunt and Logue standards use an addendum a of $0.25p_c$ and a dedendum d of $0.30p_c$, but Hunt has β equal to $14\frac{1}{2}°$, whereas Logue has it equal to 20°. The main objection to this method is that the gears are not interchangeable with other more widely used standards.

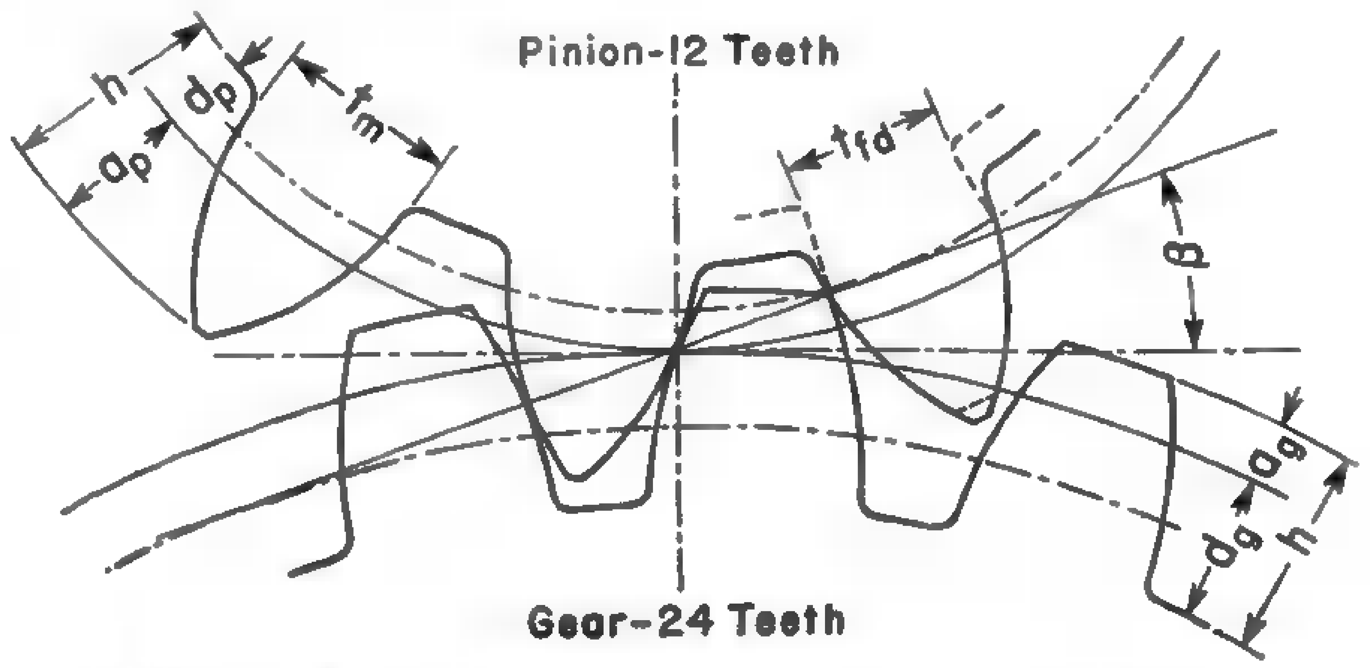


Fig. 30–17. Teeth with unequal addenda and dedenda.

Stub teeth. The stub-tooth method is actually a combination of the first two methods and is used rather extensively. In this country there are in use two main systems of stub teeth: the Nuttall, which practically coincides with the ASA standard; and the Fellows system, with the fractional designation for the pitch. Each system has a pressure angle of 20°. A still greater pressure angle, as $\beta = 22\frac{1}{2}°$, is not practical because of poorer wearing quality.

Unequal addendum and dedendum. A large part of the flank of the standard-tooth gears with small tooth numbers lies inside the base circle and is practi-

cally useless. It is therefore logical to use the same tooth height, but to change the shape of the pinion so as to reduce the dedendum and to increase the addendum. The mating gear must have its dedendum increased and its addendum decreased, as shown in Fig. 30–17.

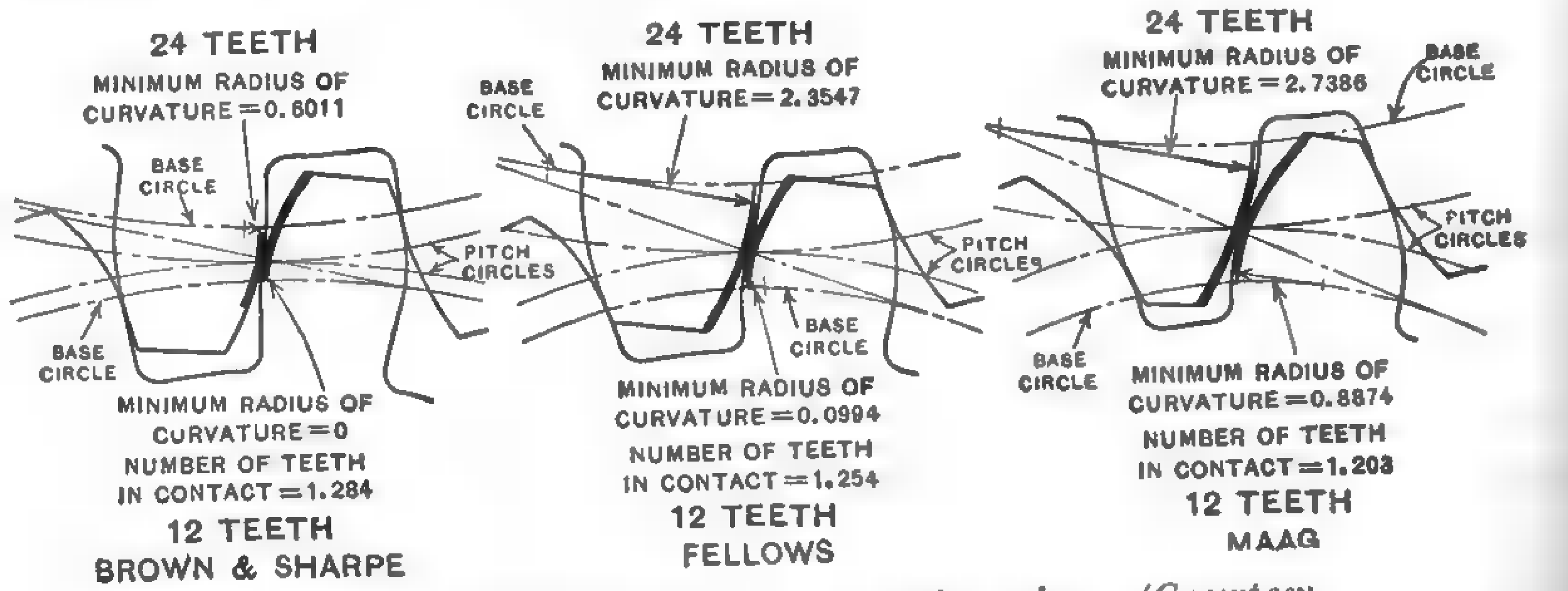


FIG. 30–18. Comparison of gear-teeth action. (*Courtesy* Niles Tool Works Company.)

The *Maag system* makes use of this method. However, it varies not only the addendum and dedendum but also the pressure angle, in order to obtain the best combination of all tooth elements, h, a, d, and β, in Fig. 30–17, in regard to running conditions. Therefore a 12-tooth pinion which runs with another 12-tooth pinion is not the same as a 12-tooth pinion mating with a 24-tooth gear. No rules for proportions of Maag gears exist. Therefore gears based on this system are not interchangeable. However, this disadvantage is not serious when quantity production is maintained and replacement stocks are available. A comparison of the teeth of the standard $14\frac{1}{2}°$ full-depth system, the Fellows stub-tooth system, and the Maag system for a 12–24 tooth drive is given in Fig. 30–18. The chief advantages of the Maag system are:

a) The teeth are much stronger than ordinary teeth.
b) The teeth of the pinion and gear can be made equally strong.
c) High reduction ratios, up to 20 to 1, are possible.
d) The profiles of the teeth eliminate any interference.
e) The gears run more quietly and last longer.

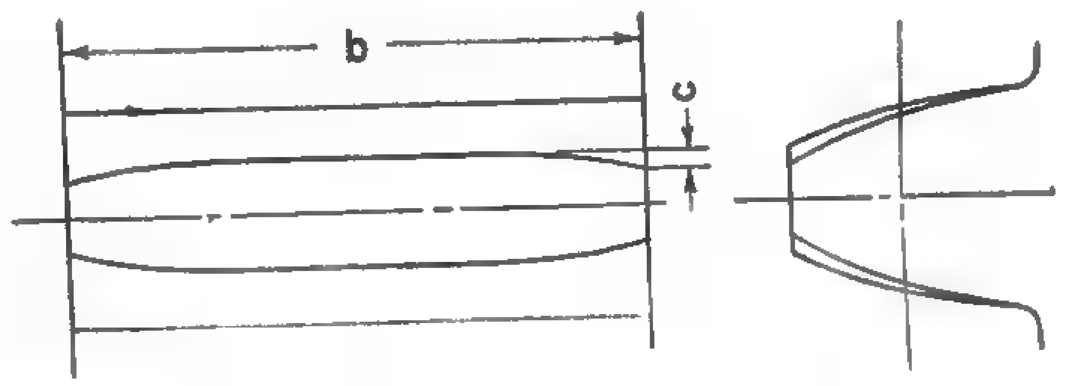


FIG. 30–19. Crowning of gear teeth.

Crowning. Inaccuracies in the teeth caused by machining and heat-treatment distortion, faulty assembly, and deflections at the shaft supports result in uneven pressures along the tooth face and throw toward the end of a tooth the maximum pressure on the tooth. The load can be moved toward the middle of each tooth by crowning or elliptoiding the teeth; that is, by making the ends slightly thinner, as

shown to an exaggerated degree in Fig. 30–19. The teeth become 20 to 100 per cent stronger, and their service life is increased up to 40 times.[12]

Crowning is done by shaving cutters. The advisable amount of crowning c, Fig. 30–19, depends on the deflection of the teeth in operation. However, it is very small, being only 0.0003 to 0.0005 in. per inch of face width.

30–13. Helical gears. A helical gear may be considered to be composed of an infinite number of infinitestimally narrow staggered spur gears. The result is that each tooth is slanting across the face, as shown in Fig. 30–20, so as to form a cylindrical helix.

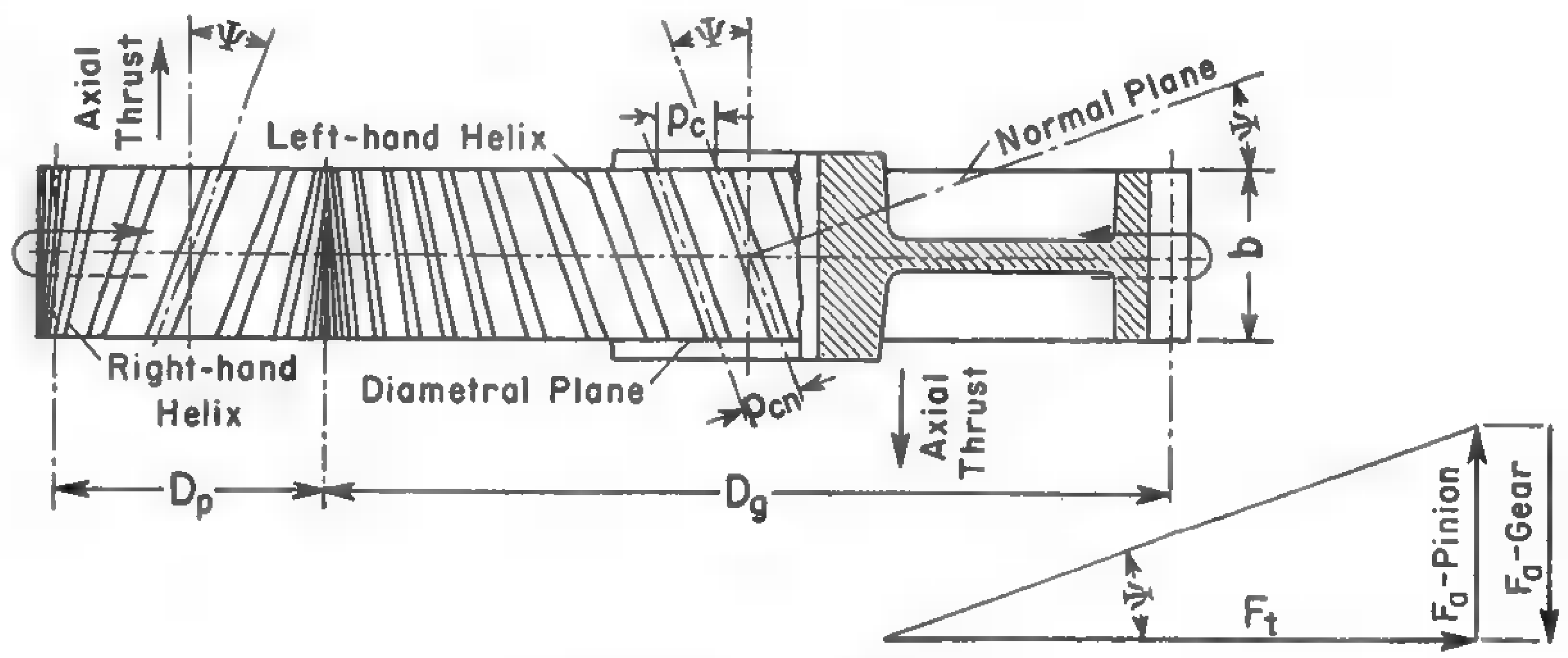


FIG. 30–20. Helical gears.

With one end of the tooth advanced over the other, engagement takes place progressively. The line of contact is a diagonal line extending from some point on the face of the advanced end to a point on the flank of the trailing end. As a result the engagement of the teeth on a helical gear is much smoother than that on a straight-tooth spur gear, and the gears run more quietly and operate satisfactorily at much higher pitch-line velocities, up to 6,000 fpm and more. The smooth engagement results in very slight wear and high efficiency, as may be seen from Table 30–9.

Strength. In straight-tooth spur gears there is a time in each period of contact when the load is concentrated at the upper edge of the tooth, thus acting with a leverage equal to the height of the tooth. With helical teeth the points of contact are at all times distributed over the entire working surface of the tooth. Therefore the mean lever arm of the bending action of the load is about one-half the height of the tooth. This makes helical-tooth gears much stronger than straight-tooth spur gears and also decreases the tooth deflection considerably.

Pitch. The *pitch of the gear* is the one in the diametral plane and is designated as p_d or p_c, as the case may be. The distance between the teeth

[12] National Broach & Machine Co., *Modern Methods of Gear Manufacture*, 3d ed. (Detroit: 1950), p. 50.

measured along a normal to the helix is called the *normal pitch* and is designated as p_{cn} or p_{dn}. Evidently

$$p_{cn} = p_c \cos \psi \qquad p_{dn} = \frac{p_d}{\cos \psi} \tag{30–36}$$

If helical gears are cut with standard hobs, the normal pitch p_{dn} must be specified, and the gear pitch p_d will contain a decimal fraction, as will also the pitch diameter.

Axial thrust. A drawback of helical gears is the axial thrust F_a equal to the axial component of the tooth pressure F_t. As may be seen in Fig. 30–20, the amount of this thrust may be expressed as

$$F_a = F_t \tan \psi \tag{30–37}$$

This thrust must be taken by the bearings and affects slightly the efficiency of the gear action.

Helix angle and face. According to the definition of the AGMA, the helix angle ψ, Fig. 30–20, is the angle between a tangent to a helix and an element of the cylinder. Unless otherwise specified, the *helix* is referred to the pitch circle. Since helical gears by their nature are not interchangeable, there are no standard helix and pressure angles. Current practice in helix-angle selection, for best over-all results, is to make this angle 20° to 35°.

Since the axial thrust F_a increases with an increase of the helix angle ψ, it is desirable that ψ be no greater than is necessary to obtain the advantages of the helical tooth; its value depends on the width b. For smooth operation one end of the tooth should be advanced over the other end a distance slightly greater than the circular pitch, $1.1p_c$ according to Fellows practice. Evidently, the narrower the face b, the greater the angle ψ must be to fulfill this requirement. The minimum value of b is given by the relation

$$b = \frac{1.15\pi}{p_d \tan \psi} \tag{30–38}$$

There is no definite limit for the maximum value of b. Sometimes it is suggested that b should not exceed $20/(p_d \tan \psi)$, but greater values are used.

The number of teeth for which the cutter must be selected is equal to the number of teeth with a normal pitch in a circumference corresponding to the helix. This number is called the *formative number of teeth* and is computed by the relation

$$i_f = \frac{i}{\cos^3 \psi} \tag{30–39}$$

EXAMPLE 30–3. A gear has 21 teeth, the normal pitch is $p_{dn} = 7$, and $\psi = 23°$. Find the pitch of the gear and the pitch diameter.

From equation 30–36,

$$p_d = p_{dn} \cos \psi = 7 \cos 23° = 7 \times 0.9205 = 6.644$$

and the pitch diameter is

$$D = \frac{i}{p_d} = \frac{21}{6.644} = 3.161 \text{ in.}$$

Some gear manufacturers make their cutters on the basis of a standard pitch in the diametral plane. Thus, helical gears can be obtained with either a standard normal pitch or a standard gear pitch.

30–14. Herringbone gears. As shown in Fig. 30–21, a herringbone gear is a double helical gear. The combination of right-hand and left-hand helices absorbs the axial thrust within the gear itself and eliminates the thrust on the bearings, which is a disadvantage of single helical gears. The disadvantage of herringbone gears is that axially they must be aligned very accurately if each half is to take its part of the load. In Fig. 30–21a is shown an ordinary herringbone gear with a relief groove for the cutting tool cut in the center; in Fig. 30–21b is shown a Wuest gear, which differs from the ordinary gear only in that the teeth are staggered; in Fig. 30–21c is shown a gear with continuous teeth cut in a cast blank with precast reliefs for the cutting tool; in Fig. 30–21d is shown a Sykes gear with continuous teeth cut by special machines. The accuracy obtained by these machines gives herringbone gears which have an efficiency of 99 to 99.5 per cent.

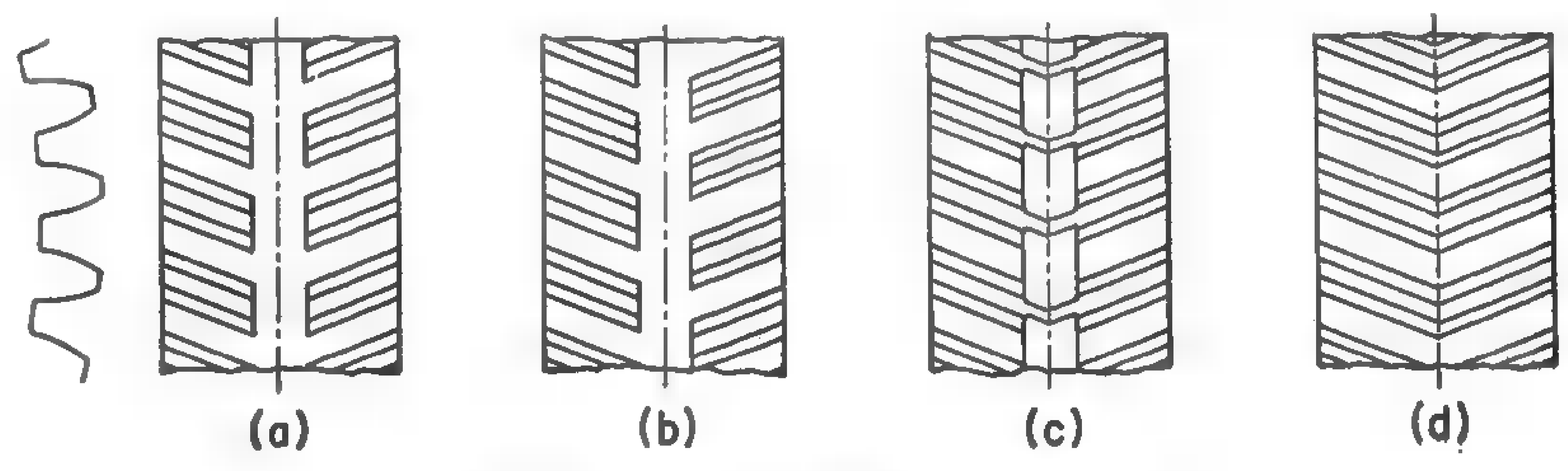


FIG. 30–21. Types of herringbone gears.

All discussion of single helical gears except that pertaining to the thrust applies also to herringbone gears. The helix angle is made up to 35°, since there is no axial thrust. Pitch-line velocities as high as 12,000 fpm have been attained with herringbone gears.

Strength of gears. A modified Lewis equation may be used to determine the pressure on a tooth of a helical or herringbone gear. This equation is

$$F_t = \frac{S_o c b Y}{p_d C_w} \tag{30–40}$$

in which S_o is the allowable static stress and c is the velocity factor. Values of S_o are given in Table 30–10. Values of c may be found as follows: For low-angle helical gears when v_m is under 1,000 fpm, by equation 30–15; for all helical and herringbone gears when v_m is 1,000 to 2,000 fpm, by equation

TABLE 30–10

DESIGN DATA FOR HELICAL AND HERRINGBONE GEARS

Material	Elastic Limit (psi)	Allowable Static Stress (psi)
Cast iron, ordinary	12,000	4,000
Cast iron, better grade	16,000	4,500
Laminated phenolic materials	(10,000)	4,000
Bronze, SAE 65	24,000	6,000
Cast steel, ASTM Class B, medium	35,000	7,500
0.4 to 0.5% carbon steel, not treated	40,000	10,000
0.4 to 0.5% carbon steel, heat-treated	50,000	12,500
High-carbon or alloy steels, heat-treated	60,000	15,000

30–16; for $v_m = 2{,}000$ to 4,000 fpm, using Barth's formula with a constant of 3,000 gives[13]

$$c = \frac{3{,}000}{3{,}000 + v_m} \tag{30–41}$$

For precision gears with v_m greater than 4,000 fpm, the velocity factor may be found by equation 30–17; and for nonmetallic gears, by equation 30–18.

Each value of S_o in Table 30–10 is practically equal to the elastic limit divided by a factor of safety n of 4. The active width of the gear face b does not include the groove around the center of a herringbone gear, Fig. 30–21a or Fig. 30–21b. The equation for the minimum value of b for herringbone gears given by AGMA is

$$b \geq \frac{2.3\pi}{p_d \tan \psi} \tag{30–42}$$

There is no definite limit for the maximum value of b. Some designers suggest for herringbone gears the relation $b < 30/(p_d \tan \psi)$. However, much greater values of b are used satisfactorily.

The Lewis factor Y in equation 30–40 is taken from Table 30–2 for the pressure angle in the plane of rotation and the number of teeth. There are no standard proportions for helical teeth, although they are usually made as stub teeth. For herringbone gears, the standard pressure angle[14] β in the plane of rotation is 20°. However, the AGMA gives for the pressure angle β the limits of 15° 23′ and 25°, and for the helix angle ψ the limits of 20° and 45°. The relation between the pressure angle β_n in the normal plane and β in the plane of rotation is

$$\tan \beta_n = \tan \beta \cos \psi \tag{30–43}$$

In equation 30–40, C_w is a wear and lubrication factor. For enclosed gears continuously lubricated with oil of the proper viscosity and character,

[13] C. D. Albert, *Machine Design Drawing Room Problems*, 4th ed. (New York: John Wiley & Sons, Inc., 1948), p. 381.

[14] Brown & Sharpe Mfg. Co., *Practical Treatise on Gearing*, 24th ed. (Providence: 1951), p. 133.

$C_w = 1.15$; for scant lubrication but regular, frequent inspection, $C_w = 1.25$; for indifferent lubrication, $C_w = 1.35$. To prevent oil from being thrown from the tooth surfaces, it should be introduced at the point where the teeth are beginning the engagement.

Equation 30–40 is based on tooth proportions which give 1.5 or more teeth in contact on the line of action in the plane of rotation. If the contact is less than 1.5 teeth, the tooth load should be reduced proportionally.

Some gear manufacturers use their own formulas for the tooth load and horsepower of herringbone gears of their type. In most cases these data are more conservative than the values obtained by using equation 30–40.

Dynamic load. Helical and herringbone gears are also subject to dynamic loads caused by inaccuracies in generating the teeth, although to a lesser degree than straight-tooth spur gears. For metal gears, the total dynamic load may be determined by equation 30–20, where the increment load may be computed by the relation[15]

$$F_i = \frac{0.05 v_m (Ceb \cos^2 \psi + F_t) \cos \psi}{0.05 v_m + \sqrt{Ceb \cos^2 \psi + F_t}} \tag{30–44}$$

As with spur gears, the total dynamic tooth load F_d must be smaller than the endurance strength F_{en} computed by equation 30–24, in which the value of S_{en} is taken from Table 30–6.

Wear resistance may be computed by the equation

$$F_w = \frac{K b D_p Q}{\cos^2 \psi} \tag{30–45}$$

This differs from equation 30–25 only because of the term $\cos^2 \psi$. The value of F_w must be greater than that of F_d found by equation 30–20. In most cases it will be found that a pair of gears properly designed by equation 30–40 will have both F_{en} and F_w greater than F_d.

The safety margin should be the same as for spur gears, or 25, 35, or 50 per cent, the value depending on the uniformity of the load.

Design procedure. Basically, the design procedure for helical or herringbone gears is the same as for straight-tooth spur gears. First it is necessary to select the pressure and helix angles, the materials, and tentatively the pitch velocity. The diametral pitch and the face b of the pinion are found by using equation 30–40, in which the safe static stress is taken from Table 30–10. The next step is to determine the actual pitch p_{dn} of the cutter to be used, the number of teeth in the pinion, and the number of teeth in the gear. After this the face of the gear is checked and, if necessary, both faces are increased to a safe value. Unlike for straight-tooth spur gears, there is no

[15] W. P. Schmitter, "Determining Capacity of Helical and Herringbone Gears," *Machine Design*, Vol. 6 (June and July, 1934), p. 40 and p. 33.

definite limit for the maximum face width b; but the minimum value of b is given by equation 30–38 or equation 30–42, as the case may be.

The final steps are to check the designed pinion and gear for dynamic load and resistance to wear. If the check gives unsatisfactory results, the design must be improved by using either a wider face or better materials, or by adopting both methods.

Example 30–4. Design a herringbone drive from a 3-hp steam turbine, running at 30,000 rpm, to a speed reducer that should run at 2,500 rpm.

Although the pinion speed is very high, the power is small and a pitch velocity 5,600 fpm may be selected. The pitch diameter of the pinion is then

$$D_p = \frac{5{,}600 \times 12}{\pi \times 30{,}000} = 0.714 \text{ in.}$$

The useful tooth pressure is found by equation 30–19, in which $c_s = 1$ from Table 30–4, for 10-hr service and steady load. Thus,

$$F_t = \frac{33{,}000 \times 3}{5{,}600} = 17.7 \text{ lb}$$

The velocity factor c, by equation 33–17, is

$$c = \frac{78}{78 + \sqrt{5{,}600}} = 0.510$$

If the number of teeth in the pinion is taken as $i_p = 15$, the diametral pitch is

$$p_d = \frac{15}{0.714} = 21.0$$

and the number of teeth in the gear is

$$i_g = \frac{15 \times 30{,}000}{2{,}500} = 180$$

The helix angle ψ may be selected so as to obtain a standard value for p_{dn}. From equation 30–36, with $p_{dn} = 24$,

$$\cos \psi = \frac{21}{24} = 0.875$$

Thus $\psi = 29°$, which is within the recommended limits. If a standard cutter with a pressure angle $\beta_n = 20°$ is used, then equation 30–43 gives

$$\tan \beta = \frac{0.364}{0.875} = 0.4160$$

Hence, $\beta = 22° \, 35'$. The formative number of teeth, by equation 30–39, is

$$i_{pf} = \frac{15}{0.875^3} = 22.4$$

For 15 teeth, the Lewis factor, from Table 30–2, is $Y = 0.349$.

The material for the pinion may be taken as high-carbon heat-treated steel, for which $S_o = 15{,}000$ psi from Table 30–10.

At $v_m = 5{,}600$ fpm the gears cannot run in an oil bath. Therefore the factor C_w should be taken conservatively as 1.35. With these figures the face, from equation 30–40, is

$$b = \frac{17.7 \times 21 \times 1.35}{15{,}000 \times 0.51 \times 0.349} = 0.188 \text{ in.}$$

By equation 30–42, the minimum width should be

$$b = \frac{2.3\pi}{21 \times 0.554} = 0.621 \text{ in.}$$

Even this seems to be too small, and tentatively b will be taken as 1.0 in.

The design will be checked for dynamic load capacity. By equation 30–44, in which the permissible and probable error is $e = 0.0005$ in. and $C = 1{,}720{,}000$, from Table 30–5,

$$F_i = \frac{0.05 \times 5{,}600 \times (1{,}720{,}000 \times 1.0 \times 0.875^2 + 17.7) \times 0.875}{0.05 \times 5{,}600 + \sqrt{658 + 17.7}} = 540 \text{ lb}$$

Then $F_d = 540 + 17.7 = 557.7$ lb

The endurance strength, with $S_{en} = 60{,}000$ psi from Table 30–10, is

$$F_{en} = \frac{60{,}000 \times 1.0 \times 0.349}{21} = 995 \text{ lb}$$

This corresponds to a safety margin of $(995/557.7 - 1) \times 100 = 78.8$ per cent.

The design must be checked for wear resistance by equation 30–45. From Table 30–7, for a pinion and gear with a Bhn of 300, $K = 196$. Also, the term Q, by equation 30–27, is

$$Q = \frac{2 \times 180}{180 + 15} = 1.85$$

Then the wear resistance becomes

$$F_w = \frac{196 \times 1.0 \times 0.714 \times 1.85}{0.875^2} = 338 \text{ lb}$$

Since F_w is less than F_d, the pinion and gear materials are not hard enough. The necessary hardness can be found from equation 30–45 by solving it for K. Thus,

$$K = \frac{F_d \cos^2 \psi}{b D_p Q} = \frac{557.7 \times 0.875^2}{1.0 \times 0.714 \times 1.85} = 323$$

Table 30–7 shows that a pinion with a Bhn of 450 and a gear with a Bhn of 350 will give $K = 342$.

To complete the design, the size of the gear blanks should be determined. The outside diameter of the pinion, with the addendum a taken from Table 30–1, is

$$D'_p = \frac{0.714 + 2 \times 0.8}{24} = 0.781 \text{ in.}$$

and that of the gear is

$$D'_g = \frac{180}{21} + \frac{2 \times 0.8}{24} = 8.639 \text{ in.}$$

CHAPTER 31

Toothed Bevel Gearing

31-1. General considerations. Bevel gears are used to connect two intersecting shafts with any given speed ratio. Two types of bevel gearing are in general use—straight-tooth gears and spiral-tooth gears. In the straight-tooth bevel gears, called *straight bevel gears,* several types of which are shown in Fig. 31-1, the elements of the teeth converge to a common point O, called the *apex,* which is the point of intersection of the gear axes. The form of tooth used for bevel gears is the involute. Spiral bevel gears are made with curved teeth, as shown in Fig. 31-8. Spiral bevel gears compare with straight bevel gears much as helical gears on parallel shafts compare with straight-tooth spur gears. Their advantages are smoother tooth engagement, quiet operation, greater strength, and higher permissible velocities.

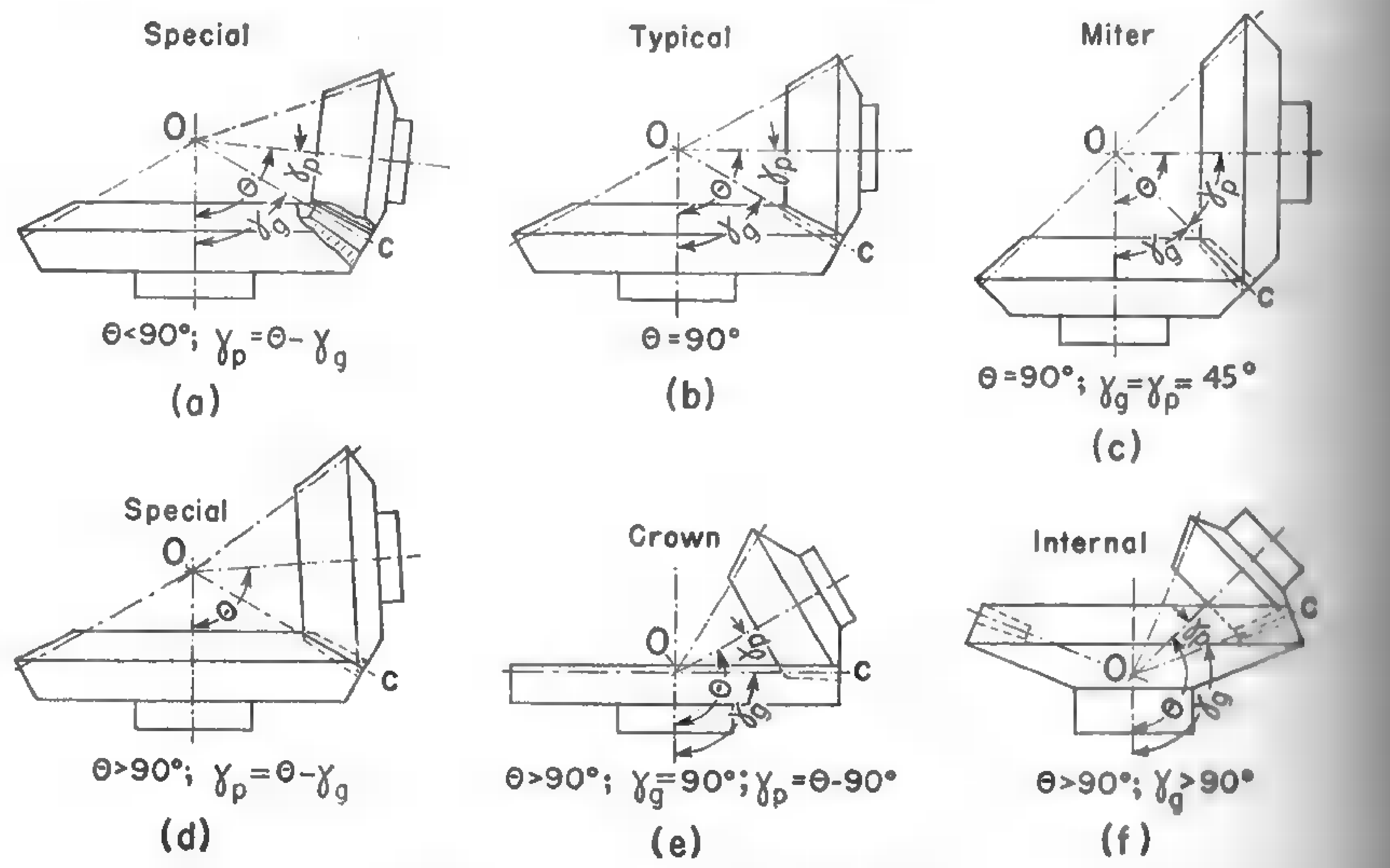


FIG. 31-1. Types of bevel gearing.

Bevel gears are not interchangeable and are designed in pairs. In the majority of cases the axes of the shafts form a right angle, but they may intersect at any desired angle, as shown in Fig. 31-1.

The names of bevel gears, based on the angles between the shafts and the pitch angles, are indicated in Fig. 31-1.

Definitions. As may be seen from Fig. 31-2, any group of tooth elements lie in the surface of an imaginary cone. Thus, lines containing the pitch elements of the teeth are elements of the *pitch cone.* The apex O of the pitch cone is called the *cone center.* The length l of a pitch-cone element is called the *cone distance,* or *pitch-cone radius.* The angle γ that the pitch line makes with the axis is called the *pitch angle* or *center angle.* The angle α is called the *addendum angle,* and the *face angle* evidently is equal to $\gamma+\alpha$. The angle δ is called the *dedendum* angle, and the *root angle,* also called the *cutting angle,* is equal to $\gamma-\delta$. In speaking of the pitch of a bevel gear, the pitch of the large end is meant. The *diameter* of the gear is the diameter D of the largest pitch circle. The addendum a and dedendum d are measured at the large end of the tooth. The outside diameter is designated D_o.

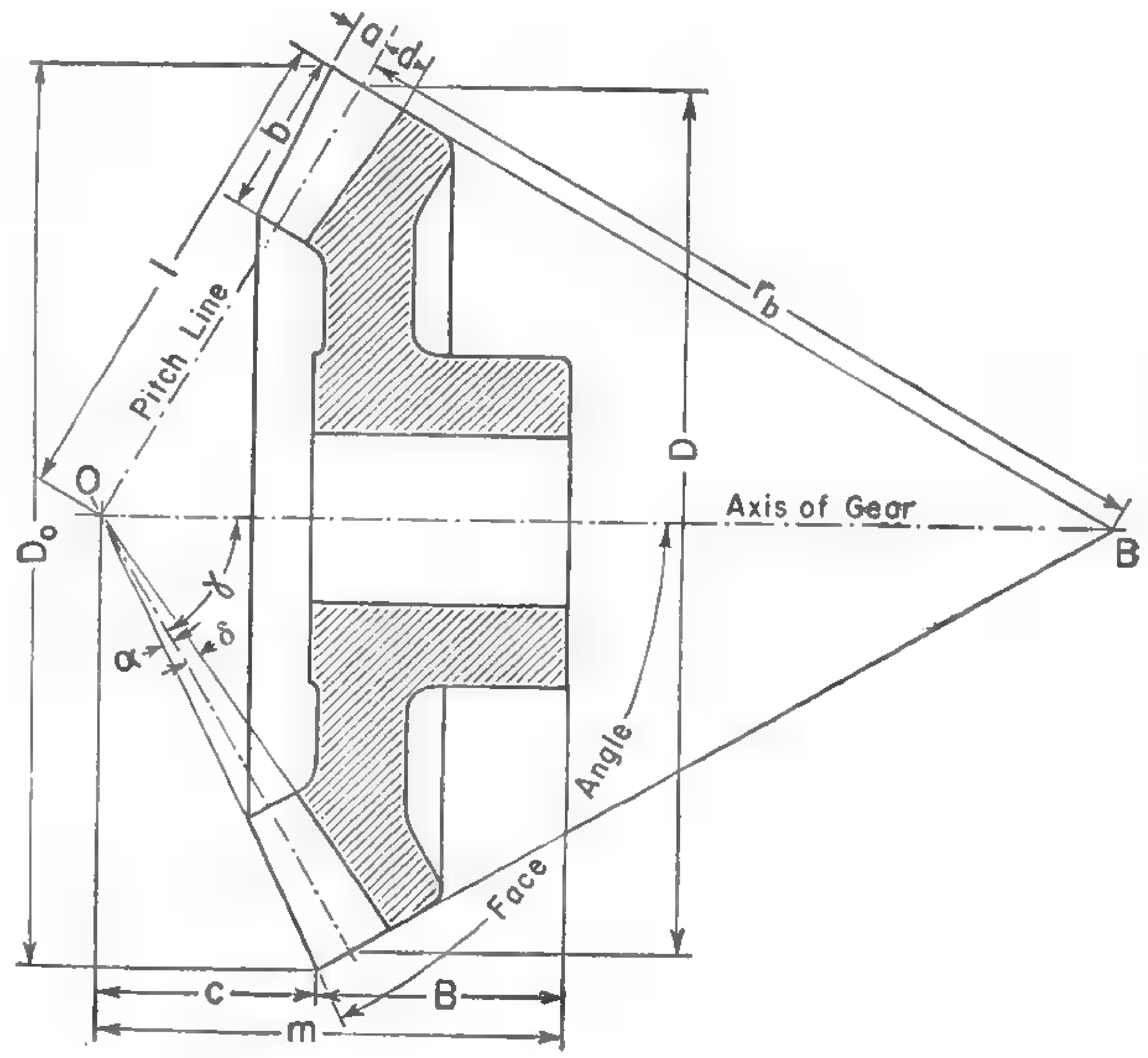


FIG. 31-2. Definitions and dimensions relating to bevel gears.

The *back cone* is an imaginary cone the elements of which are perpendicular to the elements of the pitch cone at the large end of the tooth. The length r_b of a back-cone element is called the *back-cone radius.*

The *formative number of teeth* i_f is the number of teeth of the given pitch which would be contained in a spur gear having a radius equal to the back-cone radius r_b.

The distance c is called the *crown height,* B is the *backing,* and m is the *mounting distance.*

The subscripts p and g will refer to the pinion and the gear, respectively.

Methods of manufacture. Bevel gears are either cast or cut. The casting process is similar to that used for making spur gears, but the cutting is made

much more difficult by the continuously changing size and shape of the tooth tapering from the large end toward the apex. There are several different methods of cutting the teeth, some of which produce only approximately correct forms and require hand filing for better results. The methods by which the teeth are formed with theoretical accuracy require special and rather complicated machines. The output of these machines for generating bevel-gear teeth is high and the cost of production is comparatively low. Therefore most bevel gears have generated teeth.

Efficiency. The efficiency of properly cut and well-lubricated bevel gears equipped with antifriction bearings is in general slightly higher than that of spur gears and runs up to 99 per cent.[1]

31–2. Angle relations. The shaft angle θ (Fig. 31–1) between the axes of the shafts may have any value up to 180° but is commonly 90°. It is always measured to include the pitch-cone element Oc, which is common to both the pinion and the gear.

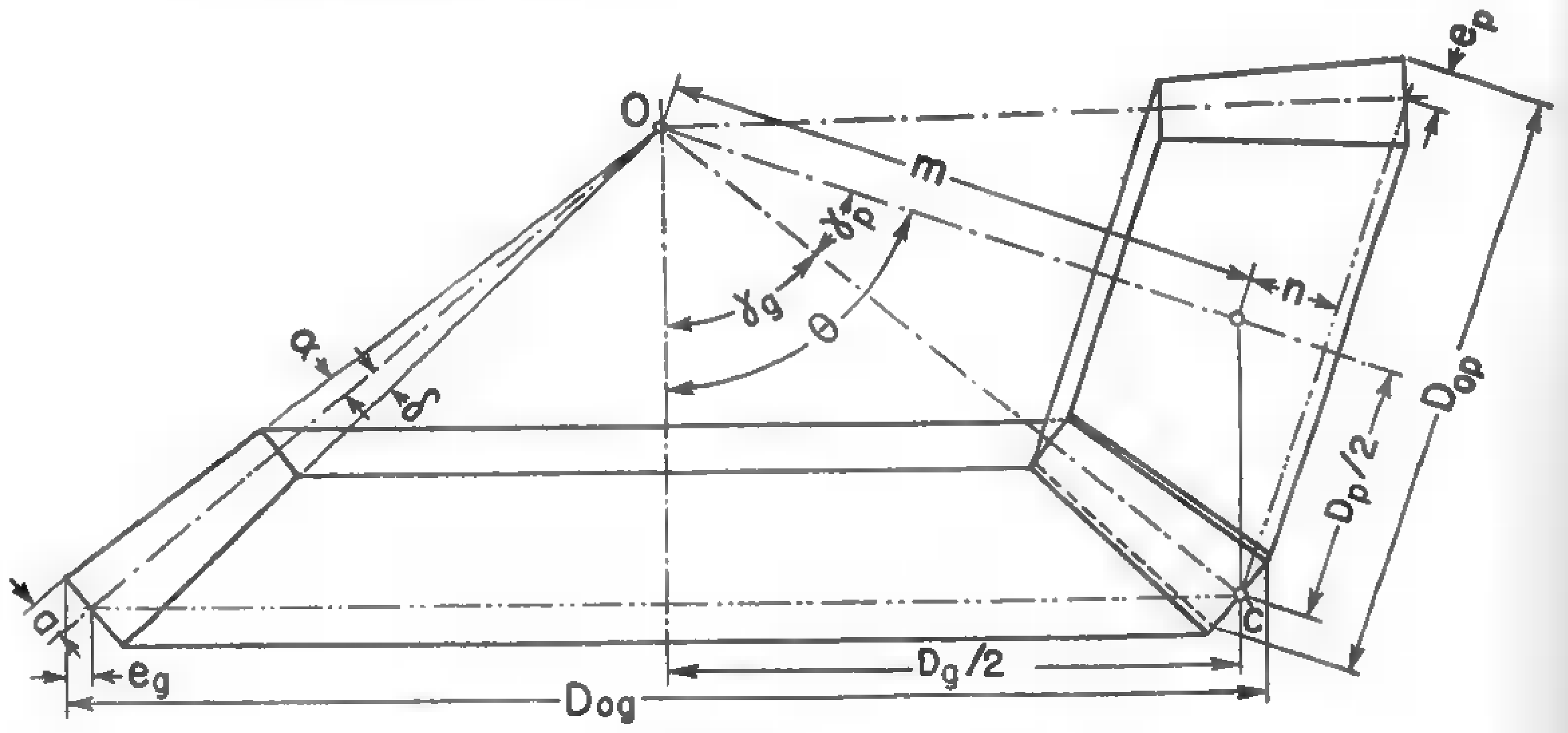


FIG. 31–3. Acute-angle bevel gears.

Acute-angle bevel gears. If the number of teeth are denoted by i_p and i_g, it follows from the geometry of Fig. 31–3 that

$$\tan \gamma_p = \frac{D_p}{2(m+n)}$$

where

$$m = \frac{D_p}{2 \sin \theta}$$

and

$$n = \frac{D_p}{2 \tan \theta}$$

[1] W. H. Kennerson, "Investigation of Efficiency of Worm Gearing for Automobile Transmissions," *Transactions of the American Society of Mechanical Engineers*, Vol. 34 (1912), p. 931; C. M. Allen and F. W. Roys, "Efficiency of Gear Drives," *Trans. ASME*, Vol. 40 (1918), pp. 106–9.

Therefore,

$$\tan \gamma_p = \frac{D_p \sin \theta}{D_g + D_p \cos \theta} = \frac{\sin \theta}{\dfrac{i_g}{i_p} + \cos \theta} \tag{31–1}$$

The center angle of the gear is $\gamma_g = \theta - \gamma_p$. By reasoning in the same manner as for equation 31–1, we can get

$$\tan \gamma_g = \frac{\sin \theta}{\dfrac{i_p}{i_g} + \cos \theta} \tag{31–2}$$

The addendum angle α, also called the *angle increment*, is found from the relation

$$\tan \alpha = \frac{2a \sin \gamma_p}{D_p} = \frac{2a \sin \gamma_g}{D_g} \tag{31–3}$$

The dedendum angle δ, also called the *angle decrement*, is found from the relation

$$\tan \delta = \frac{2d \sin \gamma_p}{D_p} = \frac{2d \sin \gamma_g}{D_g} \tag{31–4}$$

For turning the blanks it is necessary to know the outside diameters of the pinion and the gear. These diameters are equal to the pitch diameters plus twice the diameter increment. The diameter increment for the pinion is

$$e_p = a \cos \gamma_p \tag{31–5}$$

and the outside diameter of the pinion is

$$D_{op} = D_p + 2a \cos \gamma_p \tag{31–6}$$

For the gear, similarly, the diameter increment is

$$e_g = a \cos \gamma_g \tag{31–7}$$

and the outside diameter is

$$D_{og} = D_g + 2a \cos \gamma_g \tag{31–8}$$

Right-angle gears. When $\theta = 90°$, equations 31–1 and 31–2 reduce to

$$\tan \gamma_p = \frac{i_p}{i_g} \tag{31–9}$$

and

$$\tan \gamma_g = \frac{i_g}{i_p} \tag{31–10}$$

All other relations remain unchanged.

Obtuse-angle gears. In obtuse-angle gears, θ is greater than 90°. The three possible arrangements are illustrated in Figs. 31–1d, 31–1e, and 31–1f. A derivation similar to that just used gives for the pinion,

$$\tan \gamma_p = \frac{\sin (180° - \theta)}{\dfrac{i_g}{i_p} - \cos (180° - \theta)} \tag{31–11}$$

For the gear,

$$\tan \gamma_g = \frac{\sin (180° - \theta)}{\dfrac{i_p}{i_g} - \cos (180° - \theta)} \quad (31\text{–}12)$$

The remaining calculations are made by equation 31–3 to 31–8.

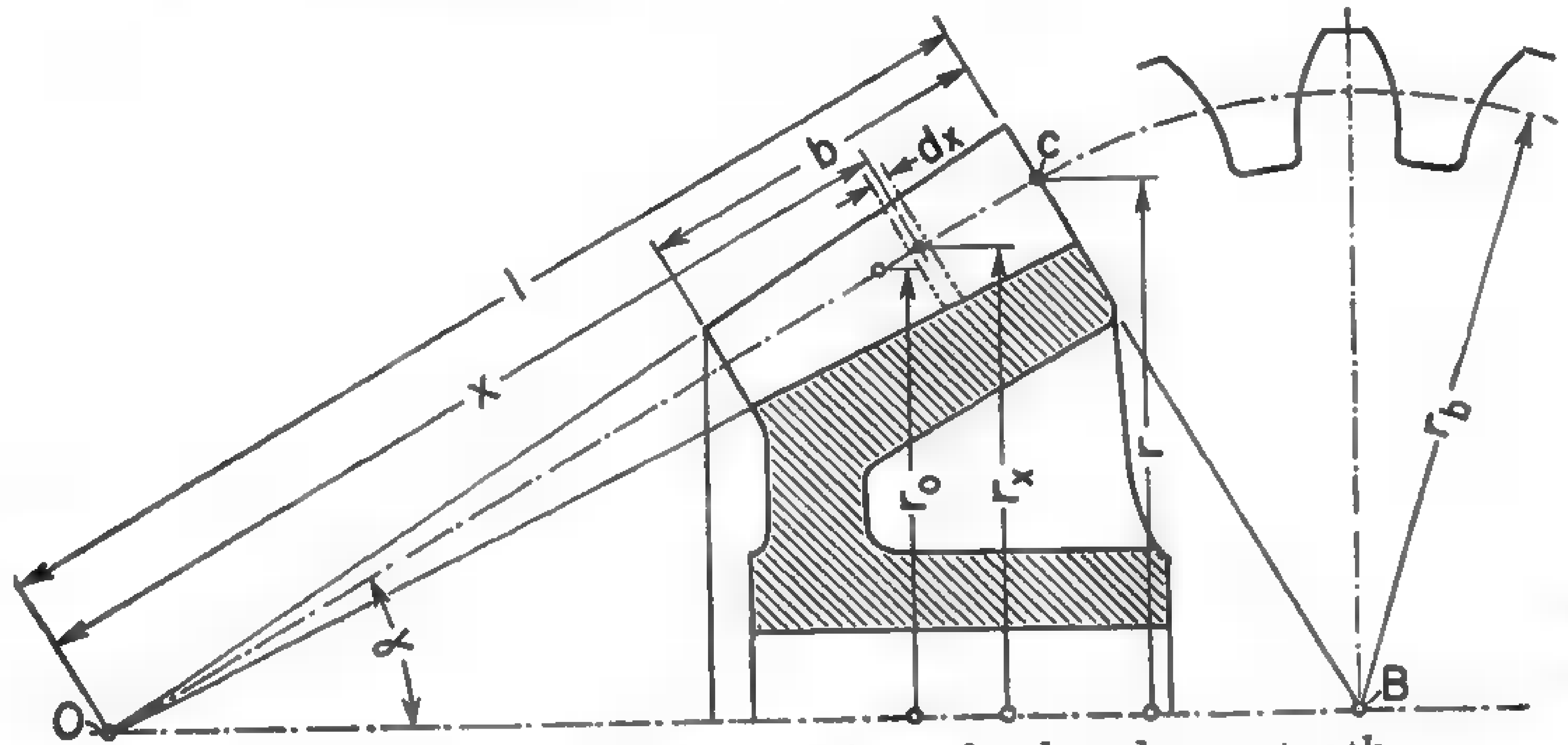


FIG. 31–4. Strength determination of a bevel-gear tooth.

31–3. Strength of cut teeth. The load on a bevel-gear tooth varies from the large end, along the face. The relation between the tooth strength and the tangential force corresponding to the horsepower transmitted at a given speed may be found by considering an infinitesimal tooth section at a distance x from the apex O, as indicated in Fig. 31–4. The infinitesimal face of this section is dx, and the force which acts upon it is dF_x. Applying equation 30–12 to this section gives

$$dF_x = S_o c y p_{cx}\, dx \quad (31\text{–}13)$$

where p_{cx} is the circular pitch at the distance x from the apex. Multiplying both sides of equation 31–13 by the gear radius r_x at this point, and integrating both sides, we get

$$\int r_x\, dF_x = \int S_o c y p_{cx} r_x\, dx \quad (31\text{–}14)$$

The left side represents a summation of the products of all elemental forces by their radii and is equal to the torque T transmitted by the gear. If all constant terms are put before the integral sign, the result is

$$T = S_o c y \int p_{cx} r_x\, dx \quad (31\text{–}15)$$

Since all tooth elements on a straight-tooth bevel gear converge to the cone center O, the circular pitch is proportional to the distance from O. Thus

$$p_{cx} = \frac{p_c x}{l} \quad (31\text{–}16)$$

Also, from the similar triangles in Fig. 31–4,

$$r_x = \frac{rx}{l} \quad (31\text{–}17)$$

Substituting values of p_{cx} and r_x from equations 31–16 and 31–17 in equation 31–15 gives

$$T = \left(\frac{S_o c y p_c r}{l^2}\right) \int x^2\, dx \quad (31\text{–}18)$$

In order to include the whole face b, the limits of integration for the integral in equation 31–18 must be $(l - b)$ and l. Integration then gives

$$T = \frac{S_o c y p_c r}{l^2}\left[\frac{x^3}{3}\right]_{l-b}^{l} = S_o c y b r\left(1 - \frac{b}{l} + \frac{b^2}{3l^2}\right) \quad (31\text{–}19)$$

Dividing both sides in equation 31–19 by the radius r, and noticing that $T/r = F_t$, and substituting for yp_c its equivalent Y/p_d, we get

$$F_t = \frac{S_o c Y b}{p_d}\left(1 - \frac{b}{l} + \frac{b^2}{3l^2}\right) \quad (31\text{–}20)$$

In actual gears the relative length of the face, $b/l \leqq \frac{1}{3}$, and the term $b^2/3l^2$ in the parentheses is usually neglected. The American Gear Manufacturers Association sanctions this omission, which is on the safe side. Equation 31–20 then becomes

$$F_t = \frac{S_o c Y b}{p_d}\left(1 - \frac{b}{l}\right) \quad (31\text{–}21)$$

It should be pointed out that F_t is not the actual load at the large end of the tooth but is simply an imaginary safe load to be compared with the transmitted load obtained from the horsepower equation when the speed at the large end is used.

The static stress S_o may be taken from Table 30–3. The velocity factor c should be computed by equation 30–6 if the teeth are cut by form cutters, and by equation 30–16 if they are generated with precision machines.

The form factor Y is obtained from Table 30–2, but it is based on the *formative number of teeth* i_f (see section 31–1) and not on the actual number of teeth in the gear. It follows from the definition of the diametral pitch that

$$i_f = 2 r_b p_d \quad (31\text{–}22)$$

In Fig. 31–5 the triangles OcB and OcE are similar. Hence, for the pinion, $r_b/l = 0.5D_p/0.5D_g$, or

$$r_b = \frac{l D_p}{D_g}$$

Substituting this value of r_b in equation 31–22 and noticing that $D_p p_d = i_p$, we get

$$i_{fp} = i_p\left(\frac{2l}{D_g}\right) \quad (31\text{–}23)$$

Similarly, the formative number of teeth for the gear is

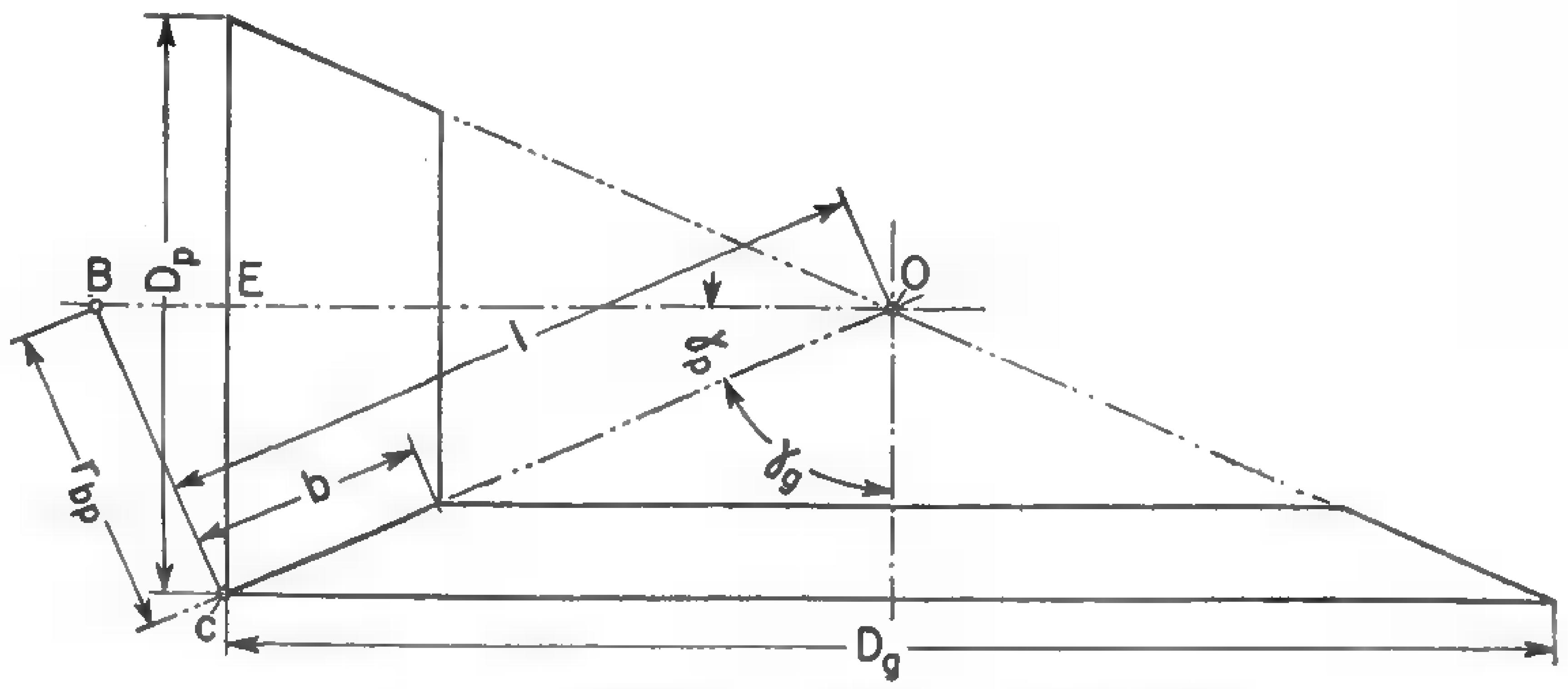


FIG. 31-5. Elements of a pair of bevel gears.

$$i_{fg} = i_g\left(\frac{2l}{D_p}\right) \qquad (31\text{-}24)$$

As can readily be seen,

$$\frac{D_g}{2l} = \cos\gamma_p \qquad (31\text{-}25)$$

and

$$\frac{D_p}{2l} = \cos\gamma_g \qquad (31\text{-}26)$$

Thus the relations in equations 31-23 and 31-24 may be given in the following general form:

$$i_f = \frac{i}{\cos\gamma} \qquad (31\text{-}27)$$

Equation 31-27 applies to any shaft angle θ and gives the formative number of teeth for acute-angle and obtuse-angle bevel gears as well.

Width of gear face. The maximum width b of the face of bevel gears should not be over one-third of the cone distance, or $b/l \leq \frac{1}{3}$. In addition, the practice of the Gleason Works is as follows:

If $l < 30/p_d$,

$$b = \frac{6}{p_d} \text{ to } \frac{7}{p_d} \qquad (31\text{-}28)$$

If $l > 30/p_d$,

$$b = \frac{7}{p_d} \text{ to } \frac{10}{p_d} \qquad (31\text{-}29)$$

The simplest way to find l is by the following relation, which is based on Fig. 31-5:

$$l = \sqrt{(0.5D_g)^2 + (0.5D_p)^2} = 0.5\sqrt{D_g^2 + D_p^2} \qquad (31\text{-}30)$$

Tooth proportions of ordinary bevel gears, at the large end, are made according to the AGMA composite $14\frac{1}{2}°$ standard as given in Table 30-1. Sometimes the gears are made with 20° stub teeth.

31-4. Design for service. The first step in the design of bevel gears should be to find a tooth size suitable for strength. Then, if the drive is for continuous service, its wearing qualities should be investigated. In regard to assumptions, if there are space limitations, the maximum gear diameter may be assumed and the pinion diameter may be found from the prescribed speed ratio. For straight bevel gears the minimum number of teeth may be from 12, for a velocity ratio of about 4, up to 18, for a velocity ratio of 1. For spiral gears, the minimum number of teeth may be slightly lower, ranging from 10 to 15 for the above velocity ratios. However, smoother and quieter operation is obtained by using more teeth.

When there are no space limitations, the starting assumption may be that of the pitch-line speed at the larger tooth end. This speed may be about 1,000 to 1,300 fpm for ordinary cut teeth and about 2,000 fpm for generated teeth.

Dynamic load. Until specific data for bevel gears are available, equations 30-20 and 30-21 may be applied for the dynamic load. In computing the load increase from equation 30-21, the velocity of a point on the largest pitch circle must be used for v_m, and the transmitted load F_d also must be based on this velocity. The rest of the procedure is the same as for spur gears.

Check for wear. The limit load for wear is given by the equation

$$F_w = \frac{KbD_pQ}{\cos\gamma_p} \qquad (31\text{-}31)$$

The factor K is the same as for spur gears and is given in Table 30-7. The value of Q must be determined for the formative number of teeth by the relation

$$Q = \frac{2i_{fg}}{i_{fp} + i_{fg}} \qquad (31\text{-}32)$$

Design procedure. The procedure in the design of bevel gears is the same as for spur gears. If an assumed pair of bevel gears with a certain combination of materials is found to be too small, as indicated by the fact that b is greater than $l/3$, or if the gears are found to be too big, because b is much less than $l/3$, the design is adjusted simply by changing the pitch diameters and, through them, changing the peripheral velocity v_m in the required direction. The number of teeth remains the same, and the pitch p_d changes automatically. Thus the preliminary design of a pair of bevel gears is simpler than that of a pair of spur gears.

31-5. Gleason systems of bevel gears. The Gleason Works have developed a system for generating bevel gears which combines the following qualities, in the order of their importance: quietness, strength, and durability. The Gleason system for bevel gears, combined with high-grade workmanship made possible by the use of special automatic tooth-generating machines, has

TABLE 31–1

SERVICE FACTORS FOR GLEASON GEARS

Character of Power Source	Character of Load on Driven Machine		
	Uniform	Moderate Shocks	Heavy Shocks
Uniform	1.00	1.25	1.75
Light shocks	1.10	1.35	1.80
Medium shocks	1.25	1.50	1.85

Data from an AGMA Report (which may serve as a guide)

Air compressor	1.35	Pneumatic tools	1.35
Airplane propeller	0.7–1.0	Pulverizers, coal or cement	1.0
Blowers, fans	1.0	Reciprocating pumps	1.5
Centrifugal extractors	1.0	Railway motor cars:	
Centrifugal pumps	1.0	a) Based on starting torque	0.5
Coal dryers, rotary	1.0	b) Based on normal running load	2.0
Coal and rock crushers	2.0	Road-building machinery	1.0–1.5
Conveyors	1.0–1.5	Rolling mills	2.0
Dredging machinery	1.5	Screens, coal or rock	1.0
Electric tools, portable	1.35	Speed reducers	1.0
Glass manufacturing machinery	1.0	Textile and woodworking machinery	1.0
Hoisting machinery	0.75–1.0	Washing machines	1.0
Machine tools:		Well-drilling machinery	1.35
a) Motor-driven	1.5	Wire-drawing machinery	1.0
b) Belt drive, direct	1.0		
c) Belt drive, transmission	0.8		
Mining machinery	1.35		

given such excellent results that it has been adopted as the recommended practice by the AGMA. This system takes full advantage of the fact that bevel gears are not interchangeable. The pressure angles and the addenda are varied in accordance with the ratios of the numbers of teeth in order to obtain the best results. The Gleason system is used for straight bevel gears and Zerol spiral bevel gears.

The difference between Gleason straight bevel gears and ordinary bevel gears is in the addenda and tooth depth. Zerol bevel gears, like spiral gears, have curved teeth, but the spiral angle α is 0°. Gleason spiral bevel gears have curved teeth with a spiral angle of 30° to 35°.

Although the same general principles are used in designing different Gleason gears, there are several practical differences which it is better to discuss separately.[2]

31–6. Gleason straight bevel gears. The usual data in a problem on gear design are the transmitted horsepower P, the character of the load, the rotative speed n_1 of the driving gear, and the gear ratio, n_1/n_2.

[2] F. E. McMullen and T. M. Durkam, "The Gleason Works System of Bevel Gears," *Machinery*, Vol. 28 (June, 1922), p. 788, and Vol. 29 (November, 1922), p. 227.

TABLE 31–2

MATERIAL FACTORS FOR BEVEL GEARS

Pinion			Gear			Durability C_m	Endurance C_{me}
Material	Hardness		Material	Hardness			
	Bhn	R-C		Bhn	R-C		
Cast iron or soft steel	160–200	.....	Cast iron	160–180	.....	0.30	0.10
Heat-treated steel	245–280	24–29	Heat-treated steel	245–280	24–29	0.35	0.50
Surface-hardened steel	480	50*	Cast iron	160–180	.....	0.40	0.10
Casehardened steel	550	55*	Cast iron	160–180	.....	0.40	0.10
Casehardened or oil-hardened steel	550	55*	Soft or cast steel	180–200	.....	0.45	0.10
Casehardened steel	.....	55*	Heat-treated steel	245–280	24–29	0.50	0.50
Oil-hardened steel	.....	.....	Oil-hardened steel	.....	.....	0.65	0.50
Surface-hardened steel	.....	50*	Surface-hardened steel	.....	50*	1.00	0.50
Casehardened steel	.....	55*	Surface-hardened steel	.....	50*	1.00	0.50
Casehardened steel	.....	55*	Casehardened steel	.....	55*	1.00	1.00

* Minimum values.

Design load. The effective load P_e, in horsepower, is taken as the greater of the two values found by the relations

$$P_e = C_s D \tag{31-33}$$

and

$$P_e = \frac{P_m}{2} \tag{31-34}$$

where C_s is the service factor found from Table 31-1, and P_m is the momentary peak horsepower.

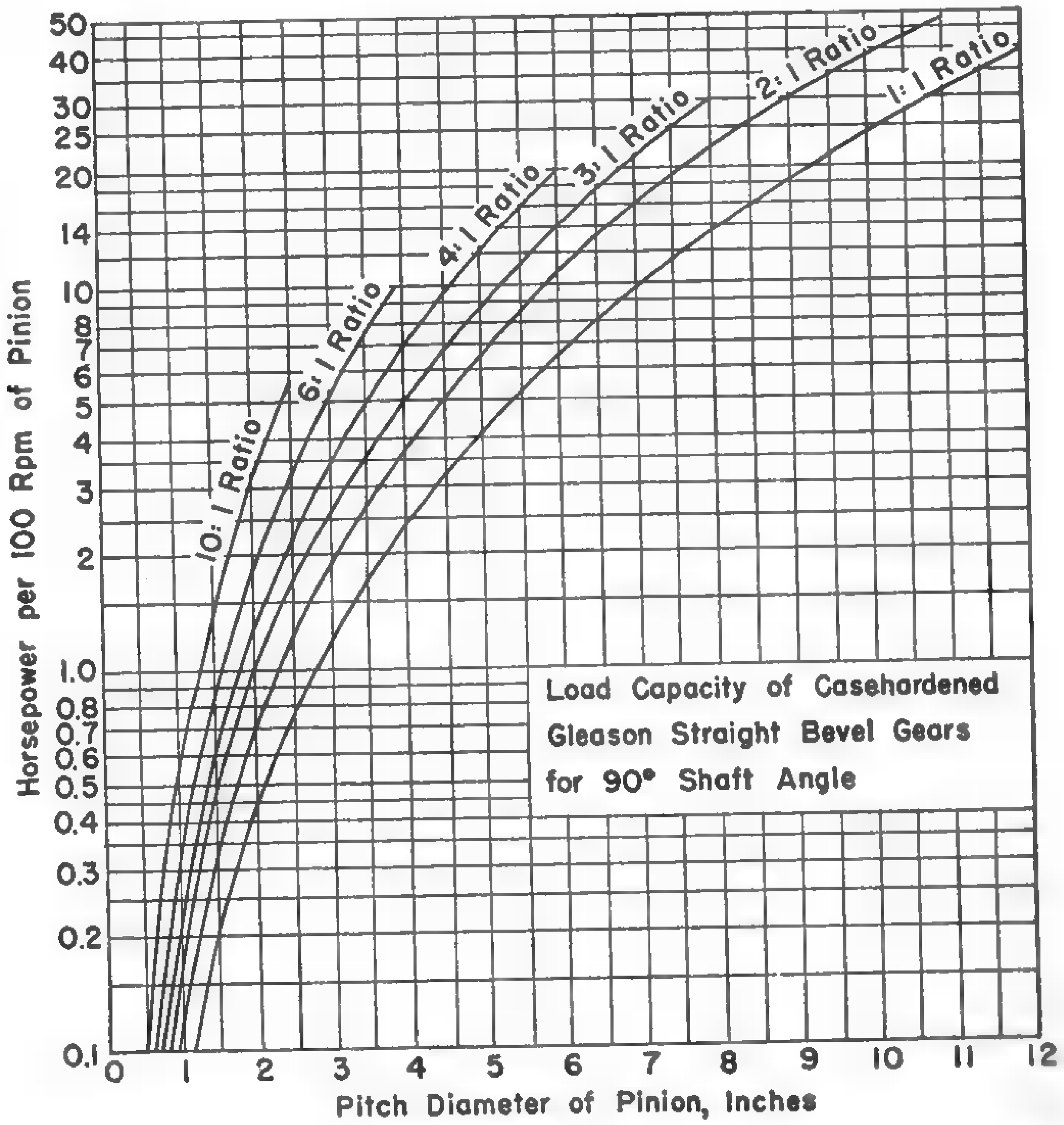


FIG. 31-6. Load capacity of casehardened Gleason straight bevel gears operating at 90° shaft angle.

The starting torque on the pinion of an electric motor must not exceed $3T_e$, where T_e is the effective design torque corresponding to the effective load P_e.

Pitch diameter of pinion. Except for cast-iron gears, or gears with great hardness, the tooth surface usually will fail before breakage occurs. Therefore the pinion size should be determined on the basis of surface durability and should then be checked for endurance strength.

The first step is to compute the rated power P_{100} per 100 rpm of the pinion by the relation

$$P_{100} = \frac{100P_e}{C_m n_p} \tag{31-35}$$

where C_m is a material factor, found from Table 31-2. Values of P_{100} found by equation 31-35 can be used with Fig. 31-6 for pitch velocities up to $v_m =$ 1,000 fpm, for which straight bevel gears are intended. For higher velocities the value of P_{100} from equation 31-35 must be multiplied by the ratio $(1{,}200 + v_m)/2{,}200$, and the result is used in Fig. 31-6. The gear ratio can be determined from the given speed ratio by the relation

$$\frac{i_g}{i_p} = \frac{n_p}{n_g} \tag{31-36}$$

The pitch diameter of the pinion is then found directly from Fig. 31-6.

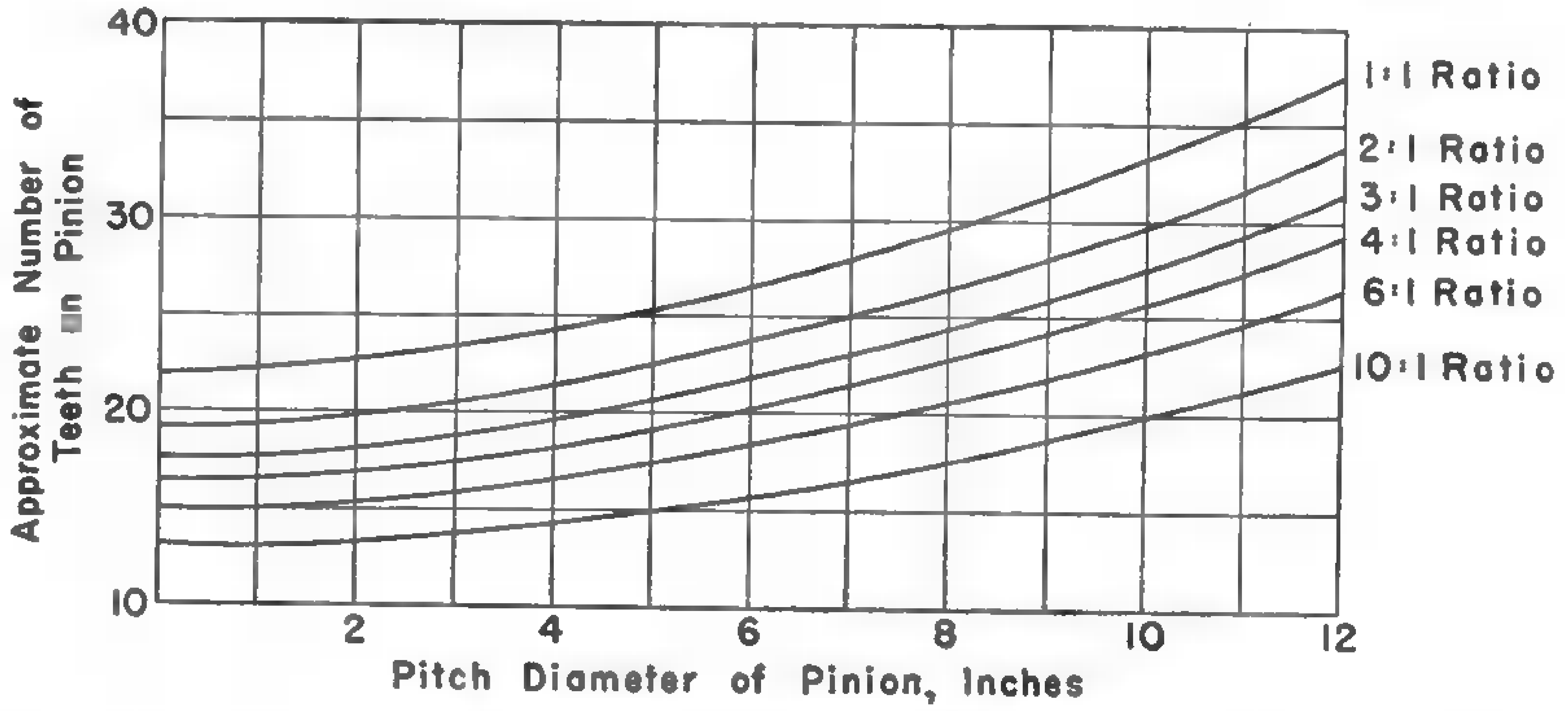


FIG. 31-7. Curves for selecting number of teeth in straight bevel gears.

Number of teeth. The number of teeth in the pinion is selected with the aid of the proper curve in Fig. 31-7. The number of teeth in the gear is determined from the ratio of equation 31-36. Instead of using Fig. 31-7, the number of teeth may be selected by using Table 31-3.

TABLE 31-3

MINIMUM NUMBER OF TEETH IN GLEASON STRAIGHT PINION

Pinion	*Gear*	*Pinion*	*Gear*
13	30 or more	15	17 or more
14	20 or more	16	16 or more

The diametral pitch at the large end is obtained by dividing the number of teeth by the pitch diameter.

Endurance strength. The endurance strength P_{en} is checked by the equation

$$P_{en} = \frac{bY_k v_m C_{me}}{6.3p_d} \tag{31-37}$$

TABLE 31–4

FACTORS Y_k FOR GLEASON 20° STRAIGHT BEVEL GEARS

NUMBER OF TEETH IN PINION	VALUES OF i_g/i_p FOR SHAFTS AT 90°											
	1.000–1.055	1.225–1.285	1.490–1.565	1.840–1.945	2.195–2.335	2.69–2.89	3.38–3.68	4.03–4.39	4.85–5.4	6.0–6.8	7.7–8.8	8.8–10.0
13...	....	....	....	....	0.701	0.730	0.760	0.779	0.799	0.819	0.840	0.850
14...	....	....	0.654	0.692	.721	.749	.779	.798	.818	.838	.859	.869
15...	....	0.636	.673	.711	.739	.767	.796	.815	.835	.855	.876	.886
16...	0.617	.655	.691	.728	.755	.783	.811	.831	.850	.871	.891	.901
17...	.635	.672	.708	.744	.770	.798	.826	.845	.864	.885	.906	.916
18...	.652	.689	.724	.759	.785	.812	.840	.859	.878	.899	.920	.930
19...	.668	.705	.739	.773	.799	.826	.853	.872	.891	.912	.933	.944
20...	.684	.720	.754	.787	.812	.839	.866	.885	.904	.925	.946	.957
21...	.697	.732	.766	.798	.823	.850	.876	.895	.914	.935	.956	.967
22...	.709	.744	.777	.809	.833	.860	.886	.904	.923	.945	.966	.977
23...	.721	.755	.788	.819	.843	.870	.895	.913	.932	.954	.976	.986
24...	.732	.766	.798	.829	.852	.879	.904	.922	.941	.963	.985	0.996
25...	.743	.777	.808	.839	.862	.888	.913	.931	.950	.971	0.994	1.005
26...	.754	.787	.818	.848	.871	.897	.921	.939	.958	.979	1.002	1.013
27...	.763	.796	.826	.856	.879	.904	.928	.946	.965	.986	1.009	1.020
28...	.771	.804	.834	.863	.886	.911	.935	.953	.972	.993	1.016	1.027
29...	.779	.812	.841	.870	.893	.918	.942	.960	.978	0.999	1.023	1.034
30...	.787	.819	.848	.877	0.900	0.924	0.948	0.966	0.984	1.005	1.029	1.040
31–32	.798	.830	.858	.887	....	....	....	....	....	....	....	....
33–34	.811	.842	.870	.898	....	....	....	....	....	....	....	....
35–36	.824	.855	.882	.909	....	....	....	....	....	....	....	....
37–38	.836	.866	.893	.920	....	....	....	....	....	....	....	....
39–41	0.851	0.881	0.907	0.933	....	....	....	....	....	....	....	....

where b is the face width, in inches (not to exceed $l/3$);

Y_k is the form factor for the pinion, found from Table 31–4;

C_{ms} is a material factor, found from the right-hand column of Table 31–2.

Table 31–4 is condensed from the original data, but the omitted values are easily found by interpolation. It should be noted that all Gleason formulas and tables use the actual number of teeth, not the formative number.

In determining values for Y_k the stress-concentration factor for the stress at the root fillet is based on an increased tool-edge radius of $0.240/p_d$ in. Values of Y_k from Table 31–4 should be decreased by 11 per cent if a standard tool-edge radius of $0.120/p_d$ in. is used.

Finally, it should be noted that equation 31–37 is based on a calculated stress of 11,000 psi, which corresponds to an endurance life of approximately $1,750 \times 10^6$ pinion cycles.

Tooth proportions. The present standard pressure angle for all Gleason gears is $\beta = 20°$, although smaller and larger angles are sometimes used.

The addendum of the gear, in inches, is

$$a_g = \frac{A}{p_d} \tag{31–38}$$

where A is the addendum of a gear with $p_d = 1$ and must be taken from Table 31–5.

The full tooth depth, in inches, is computed from the equation

$$h = \frac{2.188}{p_d + 0.002} \tag{31–39}$$

The working depth is

$$h_w = \frac{2}{p_d} \tag{31–40}$$

The addendum of the pinion is

$$a_p = h_w - a_g = \frac{2 - A}{p_d} \tag{31–41}$$

For any case, the dedendum is equal to the full depth h minus the corresponding addendum.

Backlash. The amounts of recommended backlash for different pitches are given in tables. However, for practical purposes the backlash may be made between $0.02/p_d$ and $0.03/p_d$. In case of choice, the smaller tolerance should preferably be used.

Localized contact. Gleason tooth-generating machines cut teeth with a very slight relief at the ends. The result is what the Gleason Works call *localized tooth contact.* The effect is identical with that of crowning or ellip-toiding of spur gears, discussed in section 30–12.

EXAMPLE 31–1. Find the pitch and the face of a pair of cast-iron Gleason straight bevel gears to transmit 10 hp at 600 rpm of the pinion in continuous operation with a steady load. The desired speed ratio is 3:1.

TABLE 31–5

FACTOR A FOR GLEASON STRAIGHT BEVEL GEARS

Gear Ratio	A	Gear Ratio	A	Gear Ratio	A	Gear Ratio	A
1.00–1.00	1.00	1.15–1.17	0.88	1.42–1.45	0.76	2.06–2.16	0.64
1.01–1.02	0.99	1.17–1.19	0.87	1.45–1.48	0.75	2.16–2.27	0.63
1.02–1.03	0.98	1.19–1.21	0.86	1.48–1.52	0.74	2.27–2.41	0.62
1.03–1.04	0.97	1.21–1.23	0.85	1.52–1.56	0.73	2.41–2.58	0.61
1.04–1.05	0.96	1.23–1.25	0·84	1.56–1.60	0.72	2.58–2.78	0.60
1.05–1.06	0.95	1.25–1.27	0.83	1.60–1.65	0.71	2.78–3.05	0.59
1.06–1.08	0.94	1.27–1.29	0.82	1.65–1.70	0.70	3.05–3.41	0.58
1.08–1.09	0.93	1.29–1.31	0.81	1.70–1.76	0.69	3.41–3.94	0.57
1.09–1.11	0.92	1.31–1.33	0.80	1.76–1.82	0.68	3.94–4.82	0.56
1.11–1.12	0.91	1.33–1.36	0.79	1.82–1.89	0.67	4.82–6.81	0.55
1.12–1.14	0.90	1.36–1.39	0.78	1.89–1.97	0.66	6.81–∞	0.54
1.14–1.15	0.89	1.39–1.42	0.77	1.97–2.06	0.65		

From Table 31-1, the service factor is $C_s = 1$; and by equation 31-33, $P_e = 10$ hp. The rated power P_{100} found by equation 31-35, in which the material factor is $C_m = 0.30$ (from Table 31-2), is

$$P_{100} = \frac{100 \times 10}{0.3 \times 600} = 5.55 \text{ hp}$$

From Fig. 31-6, $D_p = 3.65$ in. From Fig. 31-7, $i_p = 19.3$ teeth. If 20 teeth are used, $p_d = 20/3.65 = 5.48$.

For the gear,

$$i_g = 20 \times 3 = 60 \text{ teeth}$$

$$D_g = \frac{60}{5.48} = 10.95 \text{ in.}$$

The pitch-cone distance is

$$l = \tfrac{1}{2}\sqrt{3.65^2 + 10.95^2} = 5.76 \text{ in.}$$

For $b = 0.3l$, the face is

$$b = 5.76 \times 0.3 = 1.73, \text{ or } 1\tfrac{3}{4} \text{ in.}$$

To check the endurance strength by equation 31-37, the form factor Y_k must be found from Table 31-4 by interpolation. If the average ratios in the column headings are used,

$$Y_k = 0.839 + \frac{(0.866 - 0.839)(3.0 - 2.79)}{3.53 - 2.79} = 0.847$$

The pitch-line velocity is

$$v_m = \frac{\pi \times 3.65 \times 600}{12} = 573 \text{ fpm}$$

From Table 31-2, the material factor is $C_{me} = 0.10$. Therefore,

$$P_{en} = \frac{1.75 \times 0.847 \times 573 \times 0.10}{6.3 \times 5.48} = 2.46 \text{ hp}$$

In order to make the endurance strength $10/2.46 = 4.06$ times as great, the pinion diameter must be made $\sqrt[3]{4.06} = 1.59$ times as great. This change will increase b, v_m, and $1/p_d$ in the same proportion. The new pinion diameter should be $D_p = 3.65 \times 1.59 = 5.82$ (say 6.0) in. Then

$$p_d = \frac{20}{6} = 3.333 \text{ in.}$$

$$b = \frac{5.76 \times 0.3 \times 6}{3.65} = 2.85 \text{ in.}$$

$$v_m = \frac{\pi \times 6 \times 600}{12} = 942 \text{ fpm}$$

With these new values,

$$P_{en} = \frac{2.85 \times 0.847 \times 942 \times 0.10}{6.3 \times 3.333} = 10.82 \text{ hp}$$

which is satisfactory.

The addendum of the gear, by equation 31-38, in which $A = 0.59$ (from Table 31-5), is

$$a_g = \frac{0.59}{3.333} = 0.177 \text{ in.}$$

The working depth, by equation 31-40, is

$$h_w = \frac{2}{3.333} = 0.600 \text{ in.}$$

The addendum of the pinion teeth, by equation 31-41, is

$$a_p = 0.600 - 0.177 = 0.423 \text{ in.}$$

This example brings out clearly the great difference between Gleason gears and standard, or ordinary, bevel gears.

31-7. Gleason spiral gears. A spiral bevel gear and pinion are shown in Fig. 31-8. The teeth of the gear are curved on the arc of a circle with radius R. The teeth of the pinion are cut to mesh with those of the gear, but they are gradually relieved toward the ends to obtain a localized tooth contact, as mentioned in connection with straight bevel gears.

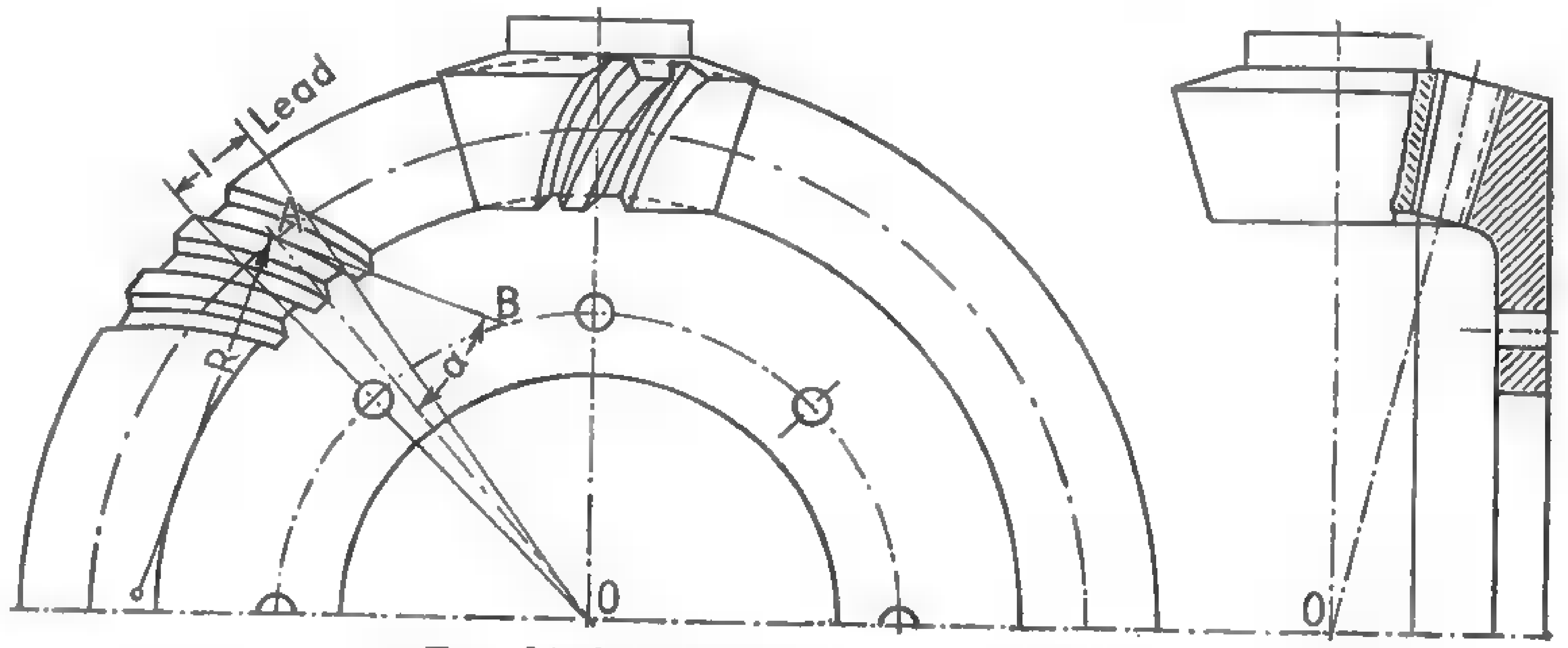


FIG. 31-8. Spiral-tooth bevel gears.

The angle α, which a tangent to the tooth at the middle point of the gear face makes with the element OA of the pitch cone, is called the *spiral angle* of the teeth. All latest data for Gleason spiral gears are based on $\alpha = 35°$.

The spirals may be left-hand and right-hand. A right-hand spiral pinion meshes with a left-hand spiral gear, as shown in Fig. 31-8, and a left-hand spiral pinion meshes with a right-hand spiral gear. The hand, or incline, of the spiral of a pinion is the same as the incline of a screw. The main advantages of spiral gears were discussed in section 31-1. This system permits the use of pinions with as few as five teeth. However, Gleason data are worked out only for applications where the minimum number of teeth in the pinion is 12, the pinion is the driver, and the shaft angle θ is 90°. Pressure angles of $14\tfrac{1}{2}°$, 16°, $17\tfrac{1}{2}°$, and 20° are used, the choice depending on the numbers of teeth in the pinion and in the gear.

Spiral gears are designed to operate with high pitch velocities, 1,000 fpm and higher. Gears operating with speeds in excess of 8,000 fpm should have ground teeth.

Design. Because the tooth proportions of spiral bevel gears depend on the method of generating the teeth, no formulas are given for the load capacity or size of spiral gears. Such information may be obtained from manufacturers of this type of gearing.

31-8. Hypoid gears. Hypoid gears, shown in Fig. 31-9, are also a development of the Gleason Works. They are similar in appearance to spiral bevel gears, but their axes do not intersect. The main feature of

hypoid gears is that the shafts of the pinion and the gear may continue past each other. These gears are so called because the correct pitch surface for such gearing is a hyperboloid of revolution. Their design is based largely on empirical data, and their manufacture requires the use of the same precision machines that are used for generating spiral bevel gears.

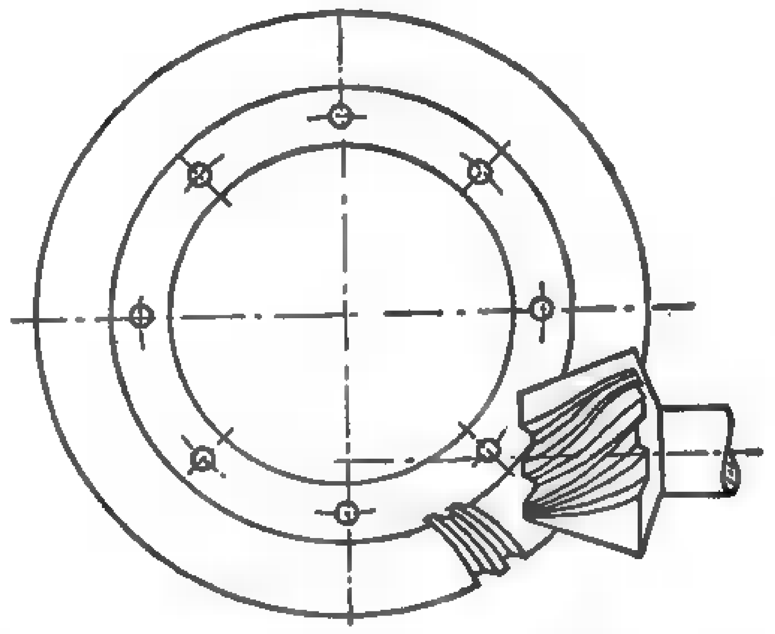


FIG. 31-9. Hypoid gearing.

31-9 Bearing loads and thrusts. As has been stated, the formulas used in designing the teeth of a bevel gear do not use the actual or tangential load on the tooth, but an equivalent load at the large end of the tooth. The effective normal tooth load and its point of application must be determined, in order to analyze the bearing loads and thrusts caused by the action of bevel gears.

Effective tooth load. By a method similar to that used in deriving equation 31-19 it can be shown that the radius r_o (Fig. 31-10) of the application of the effective tooth load on a bevel gear is

$$r_o = D\frac{1-\frac{b}{l}+\frac{1}{3}\left(\frac{b}{l}\right)^2}{2-\frac{b}{l}} \tag{31-42}$$

However, the distance to the midpoint of the face is

$$r_o' = D\frac{(l-0.5b)}{2l} \tag{31-43}$$

Since this distance differs from r_o by less than 1 per cent, it usually is taken as the radius of application of the effective tooth load.

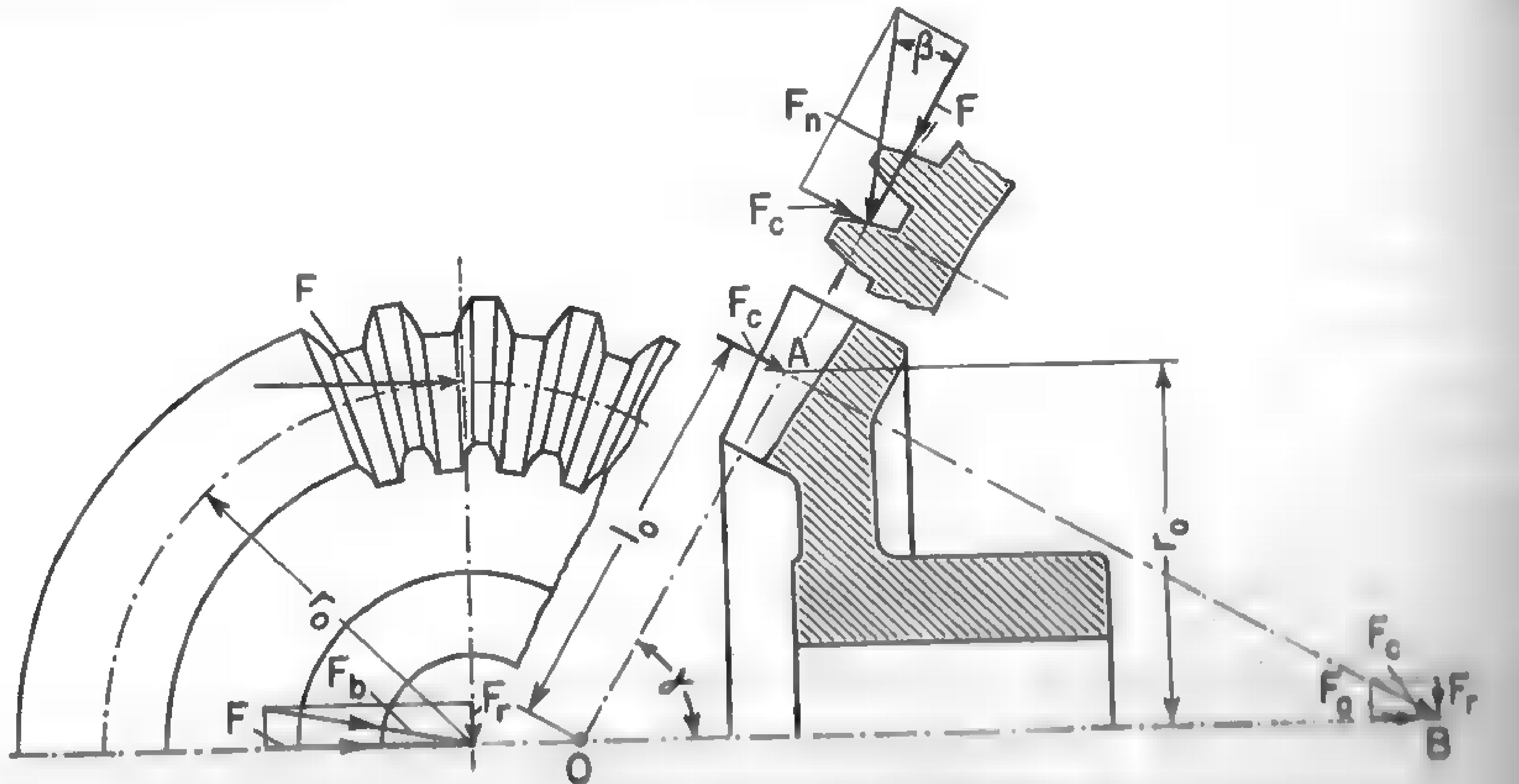


FIG. 31-10. Tooth load application.

The effective tooth load F may be obtained by simply dividing the torque T by the radius of application. Thus

$$F = \frac{T}{r_o} \tag{31-44}$$

If the tangential load F_t at the large end is known, however, the effective tooth load can be found from the relation

$$F = \frac{F_t l}{l-0.5b} \tag{31-45}$$

Bearing loads in straight bevel gears. If the effective normal tooth load F_n, Fig. 31-10, is resolved into components along and perpendicular to the tangent to the pitch circle, the tangential component is

$$F = F_n \cos\beta \tag{31-46}$$

The component of F_n at right angles to the element of the pitch cone, namely that along the cone line AB, is

$$F_c = F_n \sin\beta = F\tan\beta \tag{31-47}$$

The component F_c can be resolved into a lateral or radial load F_r and an axial thrust F_a, the magnitudes of which are

$$F_r = F_c\cos\gamma = F\tan\beta\cos\gamma \tag{31-48}$$

$$F_a = F_c\sin\gamma = F\tan\beta\sin\gamma \tag{31-49}$$

The force F produces only a lateral load upon the supporting bearings. As shown in Fig. 31-10, the total lateral load F_b on the bearings is the resultant of F and F_r.

Spiral bevel gears. The following analysis is based on the assumption that a tooth of a spiral bevel gear may be treated in a manner similar to a straight tooth having the same spiral angle α.[3]

For spiral gears the bearing loads and thrusts depend on the direction of rotation. If the driving member, which is nearly always the pinion, except when the speed ratio is 1.0, has a right-hand spiral and rotates the gear clockwise, as shown in Fig. 31-11, or if the pinion has a left-hand spiral and rotates the gear counterclockwise, the direction of rotation will be called *direct*. If the pinion has a right-hand spiral and the gear rotates counterclockwise, or if the pinion has a left-hand spiral and the gear rotates clockwise, the direction of rotation will be termed *reversed*. Also, the thrust will be considered *positive* if it acts away from the cone center, as in straight bevel gears, and the thrust will be considered *negative* if it acts toward the cone center.

[3] O. A. Leutwiler, *Elements of Machine Design* (New York: McGraw-Hill Book Company, Inc., 1917), p. 346.

Since it is easier to consider the various forces acting upon the tooth of a driven gear, the analysis will be conducted by using Fig. 31–11 for illustration. The pinion has a right-hand spiral, so the gear must be a left-hand spiral. For direct rotation the pinion rotates counterclockwise and the gear rotates clockwise, as shown in Fig. 31–11. The gear has a spiral angle α, a pressure angle β, and a pitch angle γ. It should be noted that in all following equations the angle γ always refers to the gear, even if the force is applied to the pinion.

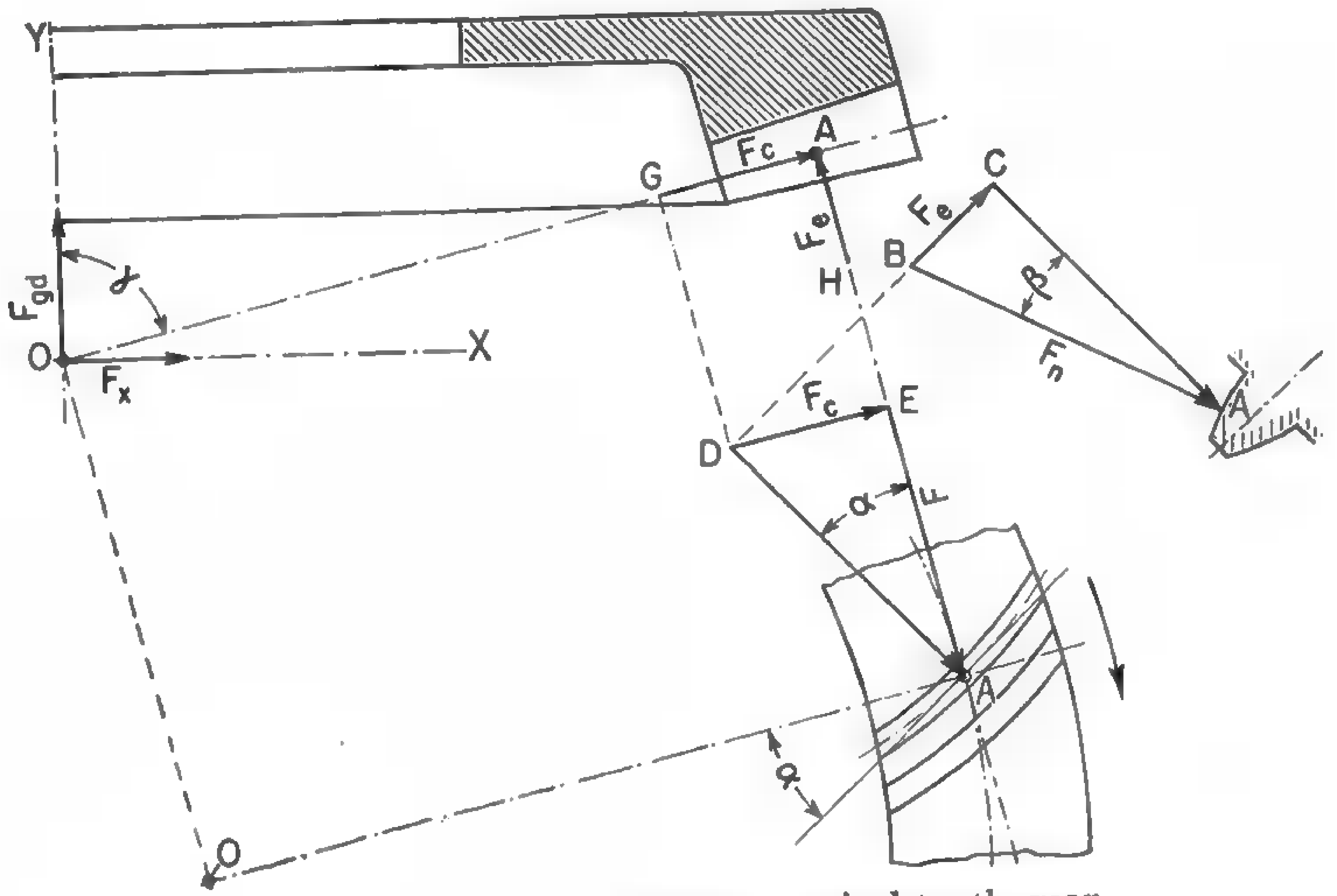


FIG. 31–11. Forces acting on a spiral-tooth gear.

Direct rotation. The effective normal tooth load F_n acting at the middle point A of a tooth may be resolved into the three components F, F_c, and F_e. The component F, which is the tangential force acting on the gear at A, is given by the equation

$$F = F_n \cos \alpha \cos \beta \tag{31–50}$$

The magnitude of the component F_c acting along the element of the pitch cone is

$$F_c = F \tan \alpha \tag{31–51}$$

The component F_e acting at right angles to the element of the pitch cone is

$$F_e = F_n \sin \beta = F \frac{\tan \beta}{\cos \alpha} \tag{31–52}$$

If the three forces F, F_c, and F_e are resolved into components whose lines of action are along the center line OY of the shaft and at right angles to it, or along OX, the thrust along the shaft of the gear is

$$F_{gd} = F_c \cos \gamma + F_e \sin \gamma = F \frac{(\sin \alpha \cos \gamma + \tan \beta \sin \gamma)}{\cos \alpha} \tag{31–53}$$

Also, the thrust along the line OX is

$$F_x = F_c \sin \gamma - F_e \cos \gamma = F \frac{(\sin \alpha \sin \gamma - \tan \beta \cos \gamma)}{\cos \alpha} \tag{31–54}$$

The thrust exerted by the pinion upon its shaft is numerically equal to F_x, but will be in the opposite direction. Therefore,

$$F_{pd} = F \frac{(\tan \beta \cos \gamma - \sin \alpha \sin \gamma)}{\cos \alpha} \tag{31–55}$$

Reversed rotation. If the direction of rotation is reversed, the component F_c reverses its direction and will act toward the apex O. Thus

$$F_c = -F \tan \alpha$$

The component F_e remains unchanged. Noticing that the force F again does not have components along OX and OY, resolving the forces F_c and F_e, and combining their components, we find that the thrust along the shaft of the gear is

$$F_{gr} = F \frac{(\tan \beta \sin \gamma - \sin \alpha \cos \gamma)}{\cos \alpha} \tag{31–56}$$

Also, the thrust along the shaft of the pinion, directly, is

$$F_{pr} = F \frac{(\tan \beta \cos \gamma + \sin \alpha \sin \gamma)}{\cos \alpha} \tag{31–57}$$

Experimental results. Tests conducted by the Gleason Works in regard to the thrusts of various types of bevel gears showed a good agreement with theoretical values calculated by the foregoing equations.[4]

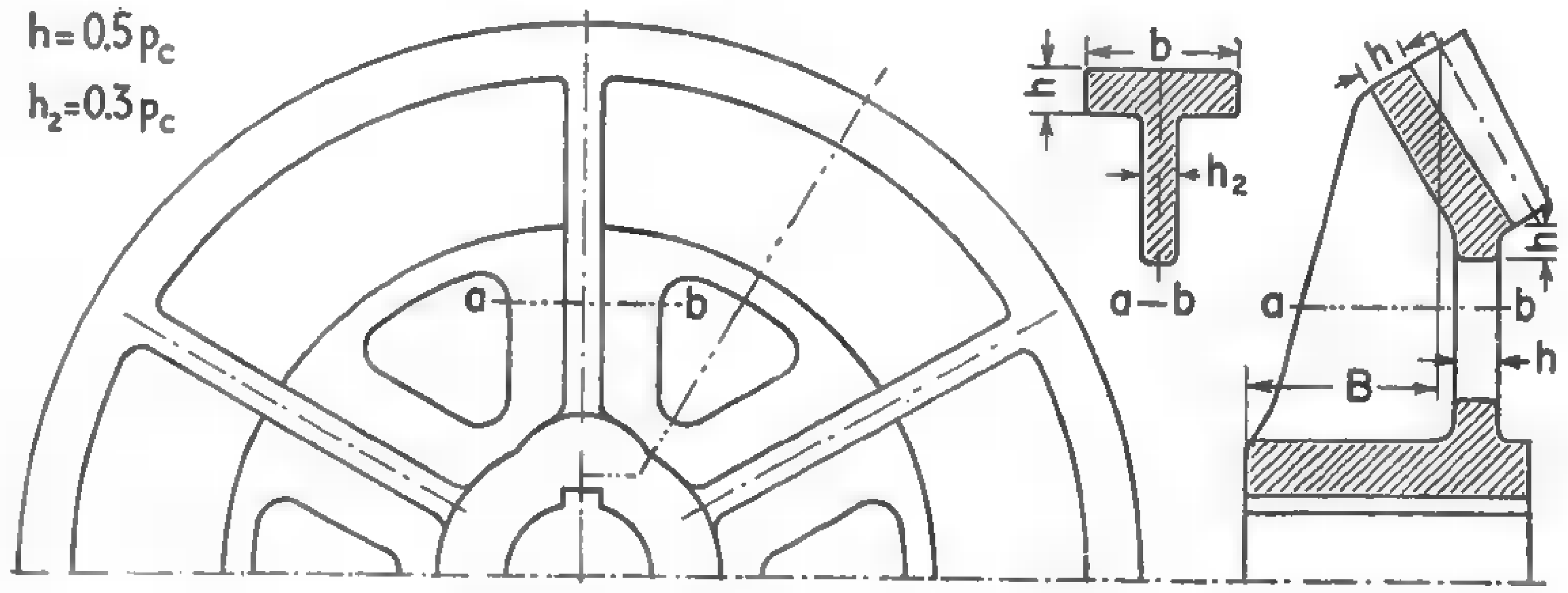


FIG. 31–12. Cast-iron bevel gear.

31–10. Construction details. Cast blanks for bevel gears have either solid webs or T-shaped arms, Fig. 31–12. The T shape is particularly well adapted for resisting the stress due to the thrust load and is used extensively in larger gears. The strengthening rib is not taken into account when computing the width b which resists bending in the plane of rotation. The

[4] *Ibid.*, p. 349.

general procedure used in the design of spur-gear arms may be followed in the case of bevel gears. However, because of an eccentric force application, it is advisable to assume that only half of the arms carry the tangential load. With the designations of Fig. 31–12, equating the external moment to the resisting moment of one-half of the number j of the arms gives

$$Fr_o = \frac{1}{12} Shb^2j \tag{31–58}$$

Solving equation 31–58 for the only unknown value gives

$$b = \sqrt{\frac{12Fr_o}{Shj}} \tag{31–59}$$

The rim, bead, and hub dimensions may be made to conform to Fig. 31–12 and to the rules given for spur gears. However, the hub must be long enough to give a positive backing B, Fig. 31–12. A practical rule is to determine the backing of the pinion by the equation

$$B_p = \frac{0.25D_gD_p}{D_g + D_p} \tag{31–60}$$

and to compute the backing of the gear by the relation

$$B_g = 0.25D_g - B_p \tag{31–61}$$

Cast iron and soft-steel bevel gears are used extensively. However, heat-treated alloy-steel gears are used when higher strength and wearing qualities are desired and the gears are not too large for heat treatment. Casehardened gears made of low-carbon steel, such as SAE 2315, combine a very hard wearing surface with a tough core and therefore are much used. Gears made of heat-treated alloy steels are usually designed in the shape of a separate ring bolted to a hub or center, as in Fig. 31–8 or Fig. 31–14b.

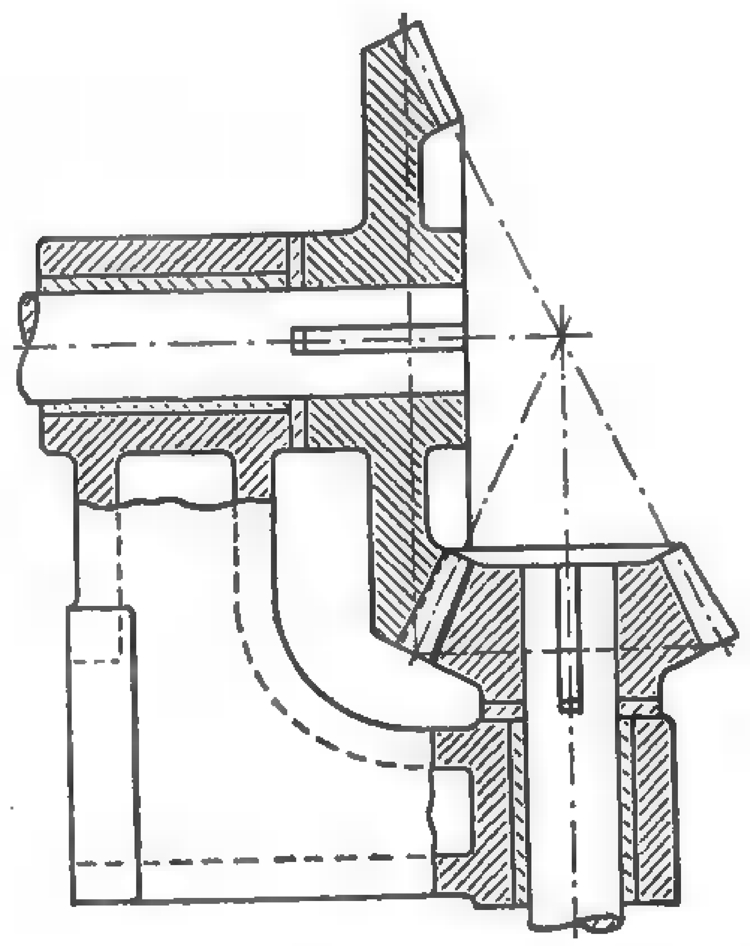
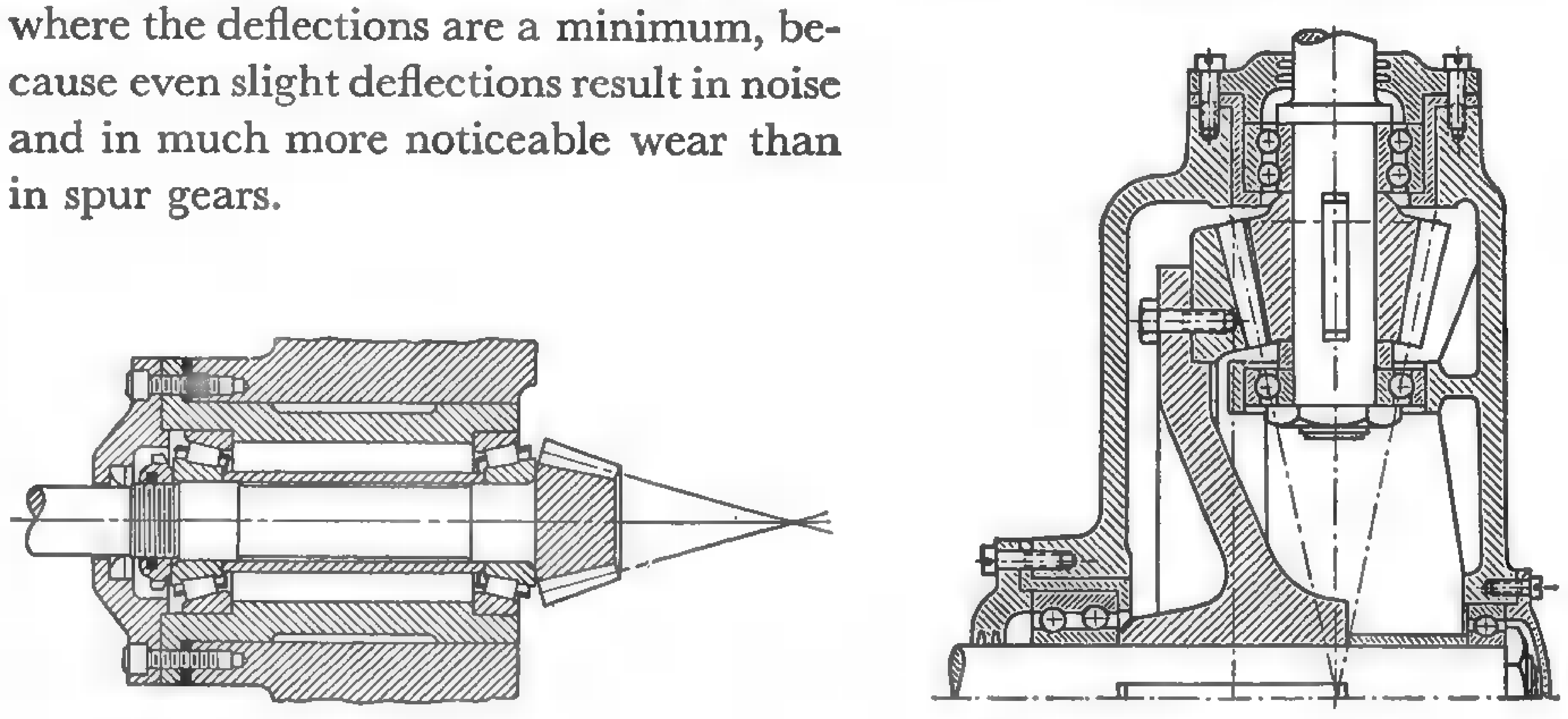


FIG. 31–13. Bevel-gear mounting with plain bearings.

Bevel-gear mounting should have some provision, such as spacing washers, for aligning the gears with respect to the apex. They should also provide for proper lubrication and, if the gears are not enclosed, for protective guards for the teeth. The bearing supports must be rigid, as in Fig. 31–13, which shows both the gear and the pinion mounted overhung. A better mounting, with the gear shaft supported on both sides of the gear, is shown in Fig. 31–14a. A still-better mounting, called *straddle mounting* because there are bearings on both sides of both the gear and the pinion, is shown in Fig. 31–14b.

Ball or roller bearings should always be preferred, since they are not subject to wear and they keep the proper gear alignment. Shafts carrying bevel gears should be sufficiently rigid. Also, the gears should be located where the deflections are a minimum, because even slight deflections result in noise and in much more noticeable wear than in spur gears.



(a) (b)

FIG. 31–14. Bevel-gear mountings with roller and ball bearings.

CHAPTER 32

Worm Gearing

32–1. General considerations. Worm gearing is a type of screw gearing used for transmitting power between nonintersecting shafts which are at right angles to each other. By this means higher speed reductions may be obtained in a minimum of space. There are two classes of worm gearing in common use, each of which has its advantages. One is a straight, or cylindrical, worm; the other is a worm with a hollow shape similar to that of an hourglass. Owing to its nature, worm gearing is used mostly as a speed reducer, the worm being the driving member.

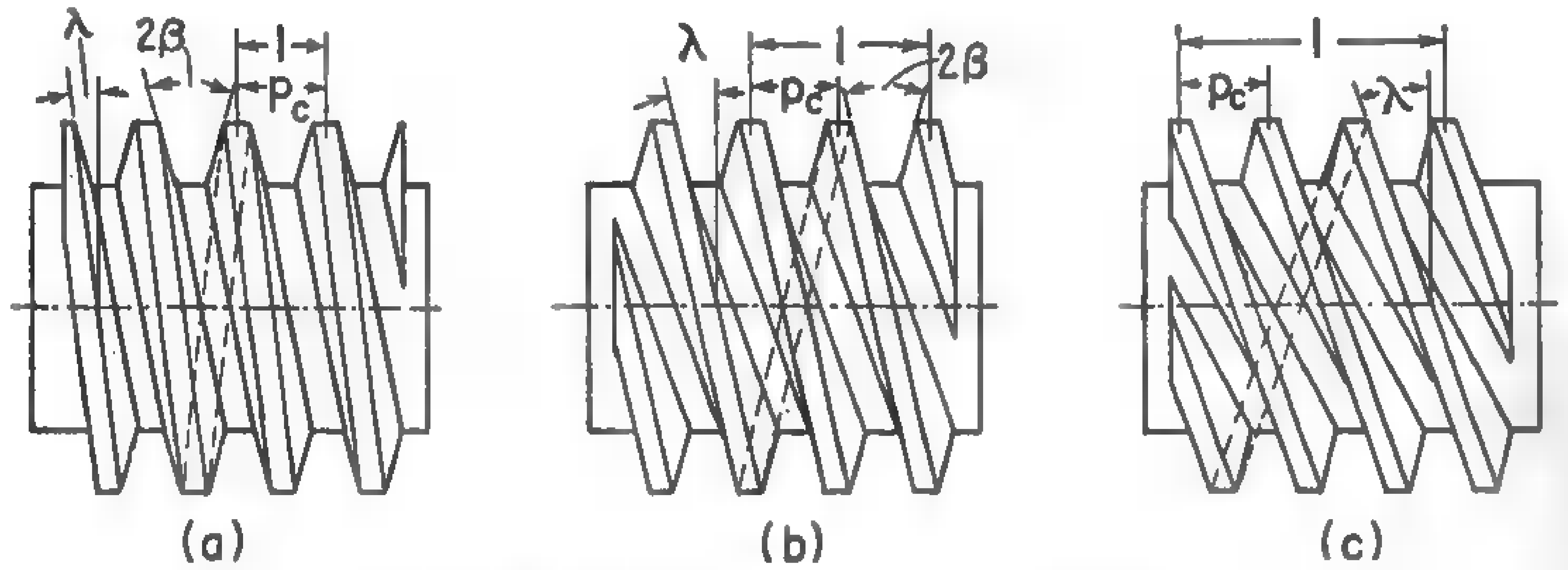


FIG. 32–1. Pitch and lead of worms.

Definitions. The *linear pitch* of a worm is the distance p_c, Fig. 32–1, measured axially from a point on one thread to the corresponding point on the adjacent thread. Evidently the linear pitch of the worm is equal to the circular pitch of the worm gear.

The *lead* is the distance l that a thread advances in one turn of the worm. Thus,

$$l = i_w p_c \tag{32-1}$$

where i_w is the number of threads of the worm. In Fig. 32–1a is shown a single-thread worm; in Fig. 32–1b, a double-thread worm; and in Fig. 32–1c, a triple-thread worm.

The *lead angle* λ is the angle between a tangent to the thread and a plane normal to the worm axis.

The *velocity ratio* r_v is equal to the pitch circumference of the worm wheel divided by the lead of the worm; or since the pitch can be canceled,

$$r_v = \frac{i_g}{i_w} \tag{32-2}$$

where i_g is the number of teeth in the gear.

32–2. Straight worm gears. The threads of a straight, or cylindrical, worm have an axial pitch that is constant for all points between the tops and the roots of the threads. The gear teeth are of the involute form. There exist two methods of cutting the worm-gear teeth. By the first method, which is used for ordinary worm gearing, the cutting hob has a constant diameter and is fed radially to the proper depth into the gear blank, both hob and blank being rotated in the required relation to each other. The teeth produced by this method are not theoretically correct but are sufficiently accurate for single-thread worm gears with a great number of teeth.

When a higher efficiency and better service are desired, the teeth are cut with a tapered hob fed into the gear blank longitudinally at right angles to the axis of the blank.

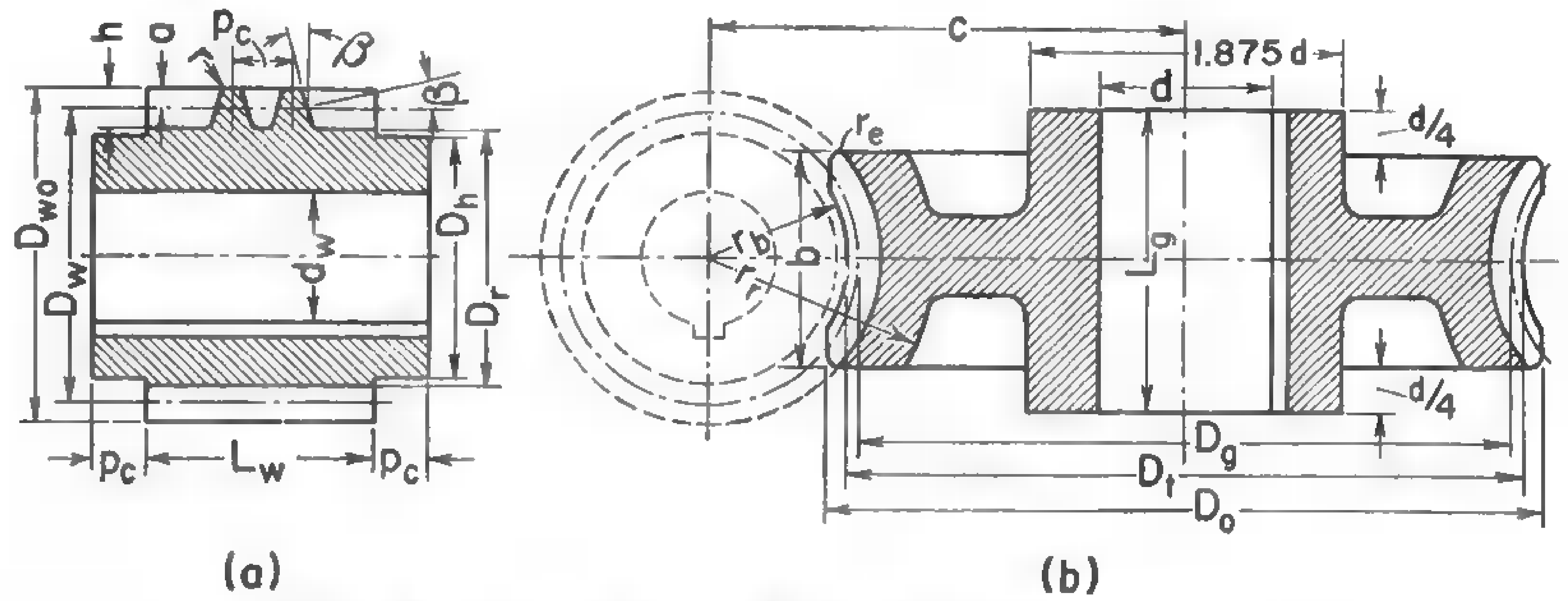


FIG. 32–2. Proportions of worms and worm gears.

Proportions of worm gears. Tables 32–1 and 32–2, with Fig. 32–2, give the proportions recommended by the American Gear Manufacturers Association for worms and worm gears for *industrial* use. The reason for using a larger pressure angle for triple and quadruple threads is that such threads have a large lead angle, and it is difficult to cut threads with a small pressure and large lead angles because of undercutting by the hob.

Pressure angles larger than 20° are often used. It is better to make the recommended pressure angle a function of the lead angle rather than of the number of threads. Thus, β may be 20° for values of λ up to 25°; 25° for values of λ up to 35°; and 30° for values of λ up to 45°. For *automotive* gears a large angle λ is desirable, and a value of β of 30° is recommended in order to obtain a high efficiency and to permit *overhauling* (which means to permit the worm to be turned by the wheel).

The teeth of single-thread and double-thread gears with a 14½° pressure angle are made with an addendum of the standard full-height tooth. Therefore, when the number of gear teeth is less than 32, they must be undercut. The teeth for the triple and quadruple threads are stubbed, but the proportions are smaller than those of the standard interchangeable stub tooth given in Table 30–1.

TABLE 32–1

PROPORTIONS OF WORMS RECOMMENDED BY AGMA

Dimension	Symbol (Fig. 32–2a)	Single and Double Threads	Triple and Quadruple Threads
Normal pressure angle (deg)	β	$14\frac{1}{2}$	20
Pitch diameter, bored for shaft (in.)	D_w	$2.4p_c+1.1$	$2.4p_c+1.1$
Pitch diameter, integral with shaft (in.)	D_w	$2.35p_c+0.4$	$2.35p_c+0.4$
Face length	L_w	$(4.5+0.02\ i_w)p_c$	$(4.5+0.02\ i_w)p_c$
Depth of tooth	h	$0.686p_c$	$0.623p_c$
Addendum	a	$0.318p_c$	$0.286p_c$
Top radius	r	$0.05p_c$	$0.05p_c$
Hub diameter (in.)	D_h	$1.66p_c+1$	$1.726p_c+1$
Maximum bore for shaft (in.)	d_w	$p_c+0.625$	$p_c+0.625$

The ends of the gear teeth are cut either parallel to the gear axis, as in Fig. 32–2b, or radially toward the worm axis, as in Fig. 32–3a, with a *face angle* 2δ of 60° to 75°. This shape is used for worms with a small lead angle. The recommended value for the face angle is[1]

$$\tan \delta \leqq \frac{\tan \beta}{\tan \lambda} \tag{32–3}$$

Straight teeth (Fig. 32–3b) cut with a form cutter are not efficient and are used only for intermittent service and small power transmitted.

Pitch. The following circular pitches are recommended by the AGMA as standard for industrial use: $\frac{1}{4}$, $\frac{5}{16}$, $\frac{3}{8}$, $\frac{1}{2}$, $\frac{5}{8}$, and $\frac{3}{4}$ in., and also from 1 in. to 2 in. in $\frac{1}{4}$-in. increments.

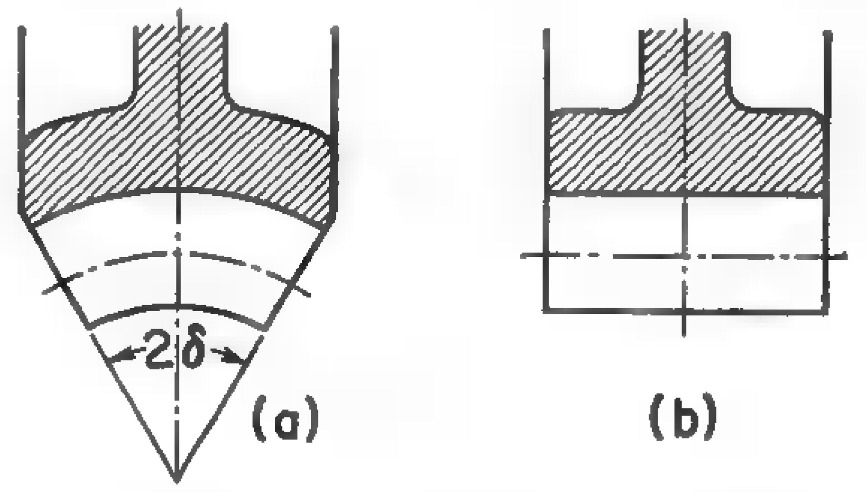


FIG. 32–3. Types of worm gears.

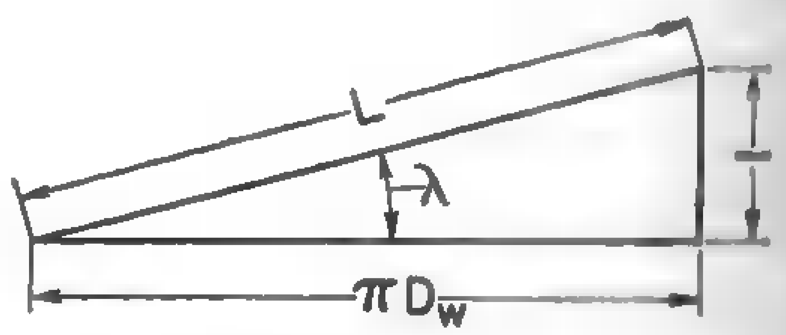


FIG. 32–4. Lead angle.

The lead angle may vary from 9° to 45°. Its magnitude may be found from Fig. 32–4, which shows the development of the worm. From Fig. 32–4 and equation 32–1, we can obtain the relation

$$\tan \lambda = \frac{i_w p_c}{\pi D_w} \tag{32–4}$$

[1] O. F. Shepard, "Worm Gear Proportions," *Transactions of the American Gear Manufacturers Association*, Vol. 11 (1927), p. 201.

Experience shows that an angle λ less than 9° results in rapid wear, and a safe value[2] is $\lambda \geqq 12\frac{1}{2}°$.

For a compact design the angle λ may be selected approximately from the relation[3]

$$\tan \lambda = \sqrt[3]{\frac{n_g}{n_w}} \tag{32–5}$$

Strength of worm-gear teeth. The teeth of a worm gear are weaker than the threads on the worm and should be checked by the Lewis formula (equation 30–13). Since both diametral and circular pitches are used for worm gearing, this equation may be given in the form

$$F = \frac{S_o c b Y}{p_d} = \frac{S_o c b Y p_c}{\pi} \tag{32–6}$$

Equation 32–6 assumes conservatively that the entire load is carried by one tooth. If it is desired to make allowance for the fact that the load is distributed, the allowable load F may be multiplied by the number of teeth in actual contact with the worm.

For the single-thread and double-thread worm gears having the proportions in Table 32–2 with $14\frac{1}{2}°$ and 20° pressure angles, the values of Y may be taken from the first two columns of Table 30–2. If the number of teeth in the gear plus the number of threads per inch in the worm is greater than 40, Y may be determined with safety for gears with any number of threads in the worm by the equation

$$Y = 0.314 + 0.0151(\beta - 14.5°) \tag{32–7}$$

The velocity factor c, which takes into account the dynamic load, may be computed by equation 30–16, in which v_m is the pitch-line speed of the gear.

TABLE 32–2

PROPORTIONS OF WORM GEARS GIVEN BY AGMA

Dimension	Symbol (Fig. 32–2b)	Single and Double Threads	Triple and Quadruple Threads
Normal pressure angle (deg)	β	$14\frac{1}{2}$	20
Outside diameter	D_o	$D_g+1.0135p_c$	$D_g+0.8903p_c$
Throat diameter	D_t	$D_g+0.636p_c$	$D_g+0.572p_c$
Face width (in.)	b	$2.38p_c+0.25$	$2.15p_c+0.2$
Radius of gear face (in.)	r_b	$0.882p_c+0.55$	$0.914p_c+0.55$
Radius of gear rim (in.)	r_r	$2.2p_c+0.55$	$2.1p_c+0.55$
Radius of edge	r_e	$0.25p_c$	$0.25p_c$

[2] F. A. Halsey, *Handbook for Machine Designers*, 2d ed. (New York: McGraw-Hill Book Company, Inc., 1916), p. 133.

[3] C. D. Albert, *Machine Design Drawing Room Problems*, 4th ed. (New York: John Wiley & Sons, Inc., 1949), p. 384.

TABLE 32-3

ALLOWABLE STATIC STRESSES FOR WORM GEARS

Material	S_o	Material	S_o
Ordinary cast iron	10,000	Leaded gun metal, SAE 63	8,000
High-grade cast iron or semisteel	15,000	Manganese bronze, SAE 43	20,000
Bakelite, textolite, rawhide, etc.	6,000	Phosphor bronze, SAE 65	15,000

For the allowable static stress S_o the values of Table 32-3 should be used.

The permissible tooth load F found by equation 32-6 must be greater than the actual tooth load F_t, which may be determined from the effective transmitted torque T_e and the selected suitable gear diameter D_g by the relation

$$F_t = \frac{2T_e}{D_g} \tag{32-8}$$

In this case the effective torque T_e is equal to the nominal torque T multiplied by a load factor K_l based on data in Table 20-3.

EXAMPLE 32-1. Design worm gearing to transmit 20 hp from an electric motor running at 1,165 rpm to a compressor which should run at 150 rpm.

In order to be sure that the gear teeth will not be undercut, assume that $i_g = 32$. By equation 32-2 the number of worm threads is

$$i_w = \frac{32 \times 150}{1{,}165} = 4.12, \text{ or } 4$$

The corrected number of teeth in the gear, by equation 32-2, is

$$i_g = \frac{4 \times 1{,}165}{150} = 31.2, \text{ or } 31$$

The corresponding compressor speed is 150.3 rpm.

If a gear diameter D_g of 15 in. is assumed tentatively, the circular pitch is

$$p_c = \frac{\pi \times 15}{31} = 1.52 \text{ in.}$$

The nearest AGMA standard pitch is $1\frac{1}{2}$ in., and the corrected gear diameter is $D_g = 31 \times 1.5/\pi = 14.82$ in. The corresponding pitch velocity is

$$v_m = \frac{\pi \times 14.82 \times 150.3}{12} = 582 \text{ fpm}$$

which seems to be satisfactory.

The effective torque on the gear shaft is, by equation 2-17, in which $K_l = 1.75$ (from Table 20-3),

$$T_e = \frac{63{,}030 \times 20 \times 1.75}{150.3} = 14{,}700 \text{ lb-in.}$$

The tooth load, by equation 32-8, is

$$F_t = \frac{2 \times 14{,}700}{14.82} = 1{,}982 \text{ lb}$$

The speed factor, by equation 30-16, is

$$c = \frac{1{,}200}{1{,}200 + 582} = 0.673$$

The face width, based on data from Table 32-2, is

$$b = 2.15 \times 1.5 + 0.2 = 3.425 \text{ in.}$$

The Lewis factor Y in equation 32-6, found from Table 30-2 by interpolation, is 0.361. If the gear is assumed to be of high-grade cast iron, $S_o = 15{,}000$ psi from Table 32-3. The maximum allowable load, by equation 32-6, is then

$$F = \frac{15{,}000 \times 0.673 \times 3.425 \times 0.361}{\pi} = 3{,}980 \text{ lb}$$

which is considerably greater than the tooth load of 1,982 lb. This indicates that the circular pitch $p_c = 1.5$ in. may be somewhat too great. However, the deciding factor in worm-gear drives is wear, not strength, and wear will be checked later.

With p_c, i_g, and i_w determined, all other dimensions can be found by means of Tables 32-1 and 32-2. Thus, the pitch diameter of the worm is

$$D_w = 2.35 \times 1.5 + 0.4 = 3.92 \text{ in.}$$

The face of the worm is

$$L_w = (4.5 + 0.02 \times 4) \times 1.5 = 6.87 \text{ in.}$$

The lead angle of the worm, found from equation 32-4, is

$$\tan \lambda = \frac{4 \times 1.5}{\pi \times 3.92} = 0.487$$

or $\lambda = 26°$, which seems satisfactory.

32-3. Cone gearing. Cone gearing, named after its inventor, Samuel I. Cone, is also called *double-enveloping worm gearing* because both the gear teeth and the worm teeth follow the shape of the other member. The gear has straight-sided teeth with a pressure angle of 20°. In general it resembles an ordinary worm gear, Figs. 32-2b and 32-3a. The main difference is in the shape of the worm, Fig. 32-5, which has teeth cut to conform to the shape of the gear.

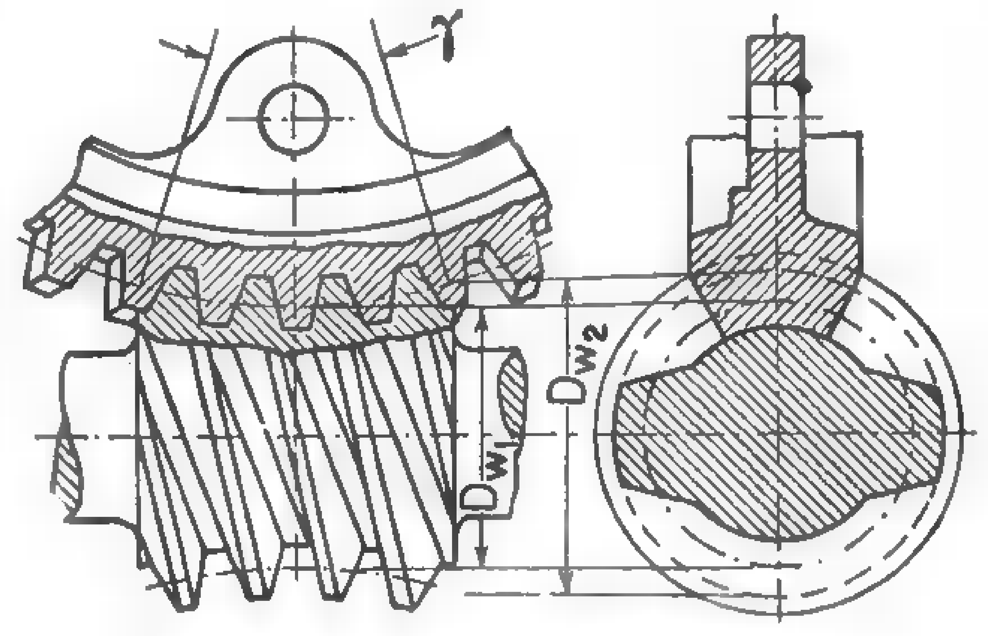


FIG. 32-5. Cone worm gearing.

The hourglass-shaped worms were first introduced by Hindley for the purpose of decreasing the wear of the worm. However, only the method of cutting the teeth in both the worm and the gear developed by Cone resulted in a practical type of gearing with a high load capacity and small wear. Since Cone gearing permits the use of shorter center distances, it needs only about two-thirds of the space and has about one-third of the weight of conventional worm gearing.[4]

Advantages. Cone drives that are properly designed and machined, and carefully assembled, show small wear and a high efficiency, combined with small size and weight. Their main disadvantage is the requirement of almost absolute accuracy in assembling and aligning. A small deviation

[4] F. E. Birth, "Double Enveloping Right Angle Gear Drives," *Product Engineering*, Vol 19 (August, 1948), p. 85.

from the correct center distance or the correct relative positions of the worm and gear results in the loss of the theoretical area of contact. However, wear does not affect a correctly assembled drive, as both the worm and the gear are regenerative and tend to correct themselves in case of a slight misalignment.

Design. The design procedure for a Cone drive is not complicated. The necessary formulas have been established by the manufacturer, but they require the use of special diagrams. Both the formulas and the diagrams are presented in a clear way in publications issued by the builder of the special machines for generating these gears.[5]

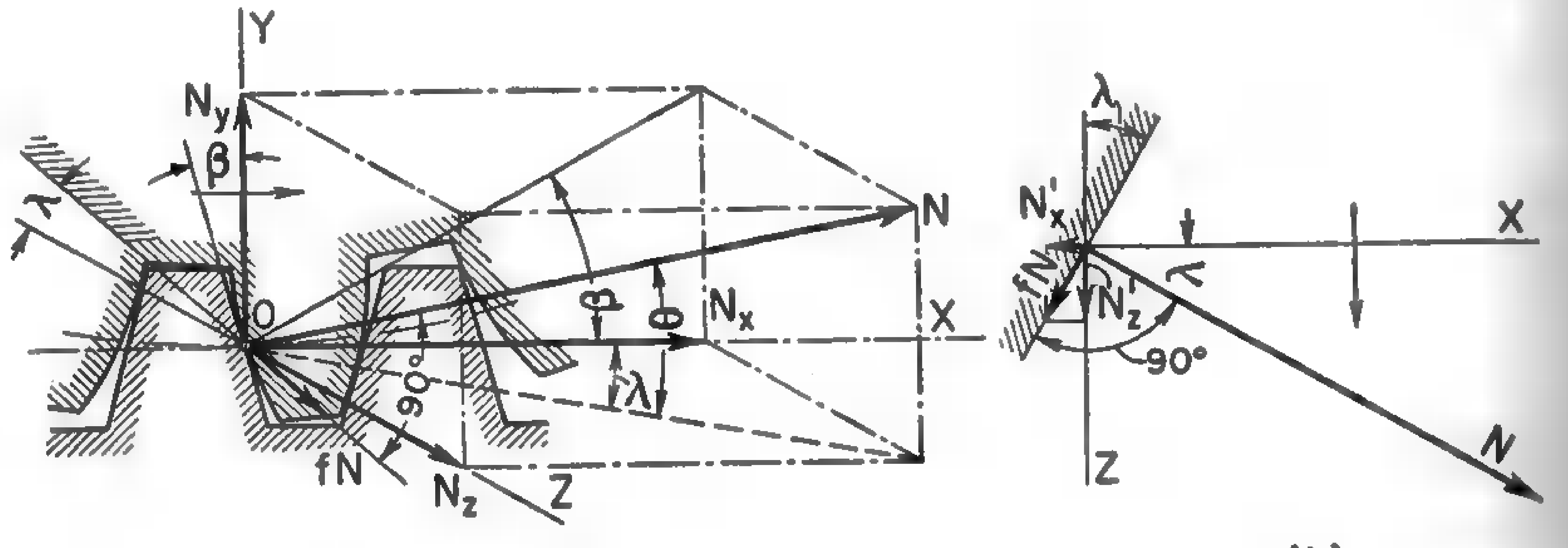


FIG. 32–6. Forces acting between the worm thread and the gear tooth.

32–4. Force analysis. The forces acting between a worm thread and a gear tooth are represented in Fig. 32–6.

Turning force and gear load. The relation between the turning force Q on a worm and the tangential load F_t on the worm gear may be found by using a perspective picture of the forces in a three-dimensional system of coordinates, as in Fig. 32–6a. The X axis is parallel to the axis of the worm, the Y axis is in the radial direction on the worm, and the Z axis is in the tangential direction for the worm normal to the plane X-Y. The vector N represents the normal reaction between the worm and a gear tooth at the point of contact O. If the frictional resistances are disregarded at first, the components of N along the axes OX, OY, and OZ are, respectively,

$$N_x = N \cos\theta \cos\lambda \qquad N_y = N \sin\theta \qquad N_z = N \cos\theta \sin\lambda$$

where the angle θ between N and the plane X-Z may be found from the relation

$$\tan\theta = \tan\beta \cos\lambda \tag{32–9}$$

If it is assumed that the worm rotates as indicated in Fig. 32–6b, the force $N' = fN$ due to friction between the worm and the gear teeth acts along

[5] Michigan Machine Tool Company, *Cone Drive Gears*, Catalog No. 50 (Detroit: 1951).

the tangent to the helix. The components of this force along the axes OX, OY, and OZ are

$$N_x' = fN \sin\lambda \qquad N_y' = 0 \qquad N_z' = fN \cos\lambda$$

When the components along the same lines of action are combined, the following results are obtained: The magnitude of the tangential driving force exerted by the worm upon the worm-gear teeth is

$$F_t = N_x - N_x' = N(\cos\theta\cos\lambda - f\sin\lambda) \tag{32–10}$$

The magnitude of the turning force Q required at the pitch radius of the worm is

$$Q = N_z + N_z' = N(\cos\theta\sin\lambda + f\cos\lambda) \tag{32–11}$$

The component along the axis OY represents the magnitude of either the downward pressure R upon the worm shaft or the upward pressure upon the worm-gear shaft. Thus,

$$R = N_y = N\sin\theta \tag{32–12}$$

Elimination of N by dividing equation 32–11 by equation 32–10 gives

$$Q = F_t \frac{\cos\theta\sin\lambda + f\cos\lambda}{\cos\theta\cos\lambda - f\sin\lambda} \tag{32–13}$$

Efficiency. If there were no friction, f would be 0 and the turning force would be reduced to

$$Q' = F_t \frac{\cos\theta\sin\lambda}{\cos\theta\cos\lambda} = F_t \tan\lambda \tag{32–14}$$

The efficiency is the ratio of the ideal effort without friction to the actual effort with friction, or $e = Q'/Q$. When the values from equations 32–14 and 32–13 are substituted for Q' and Q, and a few simplifications are made, the result is

$$e = \frac{\cos\theta - f\tan\lambda}{\cos\theta + f\cot\lambda} \tag{32–15}$$

Equation 32–15 differs slightly from equation 11–18, because the latter was deduced by using the approximate equation 11–15; that is, by assuming that $\cos\lambda = 1$. This is sufficiently accurate for single-thread screws. The efficiency of a worm gear is influenced greatly by the helix angle λ. The dotted curve 7, Fig. 32–7, shows values of e computed by equation 32–15, in which $\beta = 14\frac{1}{2}°$ and $f = 0.05$. The other curves, *1* to *6*, were found experimentally for different values of the rubbing velocity v from 5 to 200 fpm.[6] The influence of v is indirect, as there is a gradual change of the friction coefficient from about $f = 0.15$ at $v = 5$ fpm to $f = 0.02$ at 200 fpm. This variation is in agreement with the theory of lubrication. For single-thread worms of the irreversible or self-locking type, f may be assumed as

[6] W. Lewis, "Investigation of Worm Gear Drives," *Trans. ASME*, Vol. 7 (1885), p. 297.

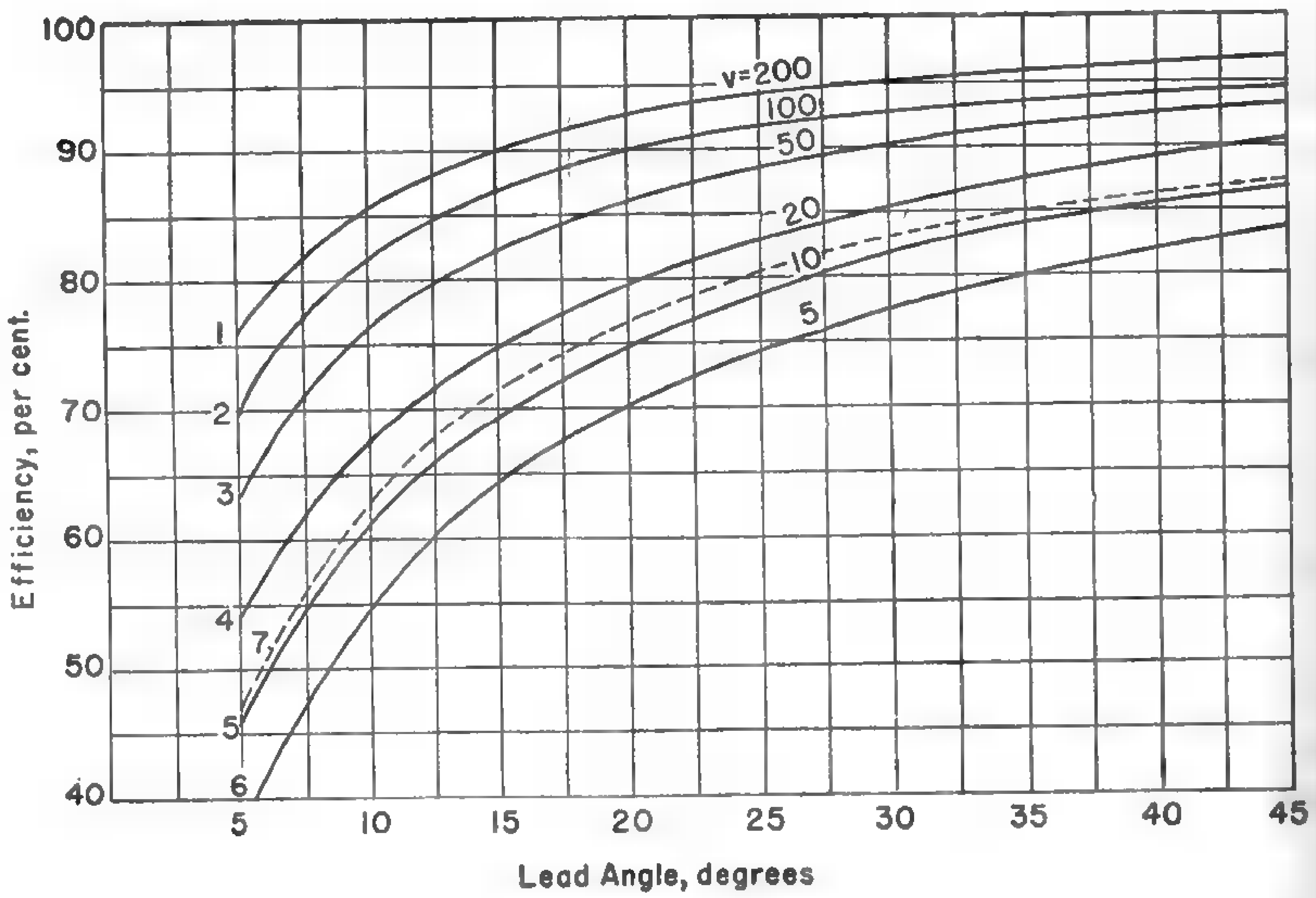


FIG. 32–7. Efficiencies of worm drives.

0.05. For multiple-thread worms operating at rubbing speeds from 40 to about 550 fpm, with good workmanship and proper running conditions, including adequate lubrication, a safe value of f may be estimated by the equation[7]

$$f=\frac{0.185}{v^{0.28}} \tag{32–16}$$

For rubbing speeds v greater than 550 fpm,

$$f=0.025+\frac{v}{60{,}000} \tag{32–17}$$

The friction coefficient f depends on the finish of the tooth surfaces and on the lubricant used. With the best materials for the worm and the gear, with precision machining, and with good lubrication, it is possible to obtain values of f as low as 0.025; with low speeds and indifferent lubrication, f may be as high as 0.10.

Rubbing velocity. It should be noted that the rubbing velocity v is different from the pitch velocity v_m. If the notation in Fig. 32–2 is used, the rubbing velocity may be computed by the relation

$$v=\frac{\pi D_w n_w}{12 \cos \lambda} \tag{32–18}$$

For ordinary industrial worms, the rubbing velocity should not exceed 1,200 fpm; for well-designed, precision-machined, hardened, and ground worms, it should not exceed 3,000 fpm.

[7] V. M. Faires, *Design of Machine Elements*, rev. ed. (New York: The Macmillan Company, 1942), p. 273. Modified on the basis of data from Lewis, *loc. cit.*

The curves in Fig. 32–7 show clearly how desirable it is to use a large angle λ. This angle should be 30° or higher, and such a large angle is obtained only with multiple-thread worms. Tests with steel worms and bronze worm gears and with values of λ ranging from 38° to 45° showed efficiencies as high as 98 per cent.[8] For values of the speed ratio greater than 8, the De Laval Company uses the following empirical formula:

$$e=1-0.005r_v \tag{32–19}$$

where r_v is the speed ratio of the worm and the gear. This formula presumes that the best helix angle, the lowest friction coefficient, the best thrust bearing, and the best lubrication are used.

EXAMPLE 32–2. Find the efficiency of the worm gear of example 32–1.
By equation 32–18, in which $\lambda=26°$ and $\cos\lambda=0.899$, the rubbing velocity is

$$v=\frac{\pi\times 3.92\times 1{,}165}{12\times 0.899}=1{,}330 \text{ fpm}$$

The coefficient of friction, by equation 32–17, is

$$f=0.025+\frac{1{,}330}{60{,}000}=0.047$$

Since $\beta=20°$ and $\tan\beta=0.364$, it is found from equation 32–9 that

$$\tan\theta=0.364\times 0.899=0.3275=\tan 18°\,8'$$

The efficiency found by equation 32–15, in which $\cos\theta=0.950$ and $\cot\lambda=2.05$, is

$$e=\frac{0.950-0.047\times 0.488}{0.950+0.047\times 2.05}=0.887$$

32–5. Design of worm gearing. Worm gearing should be designed to resist wear and provide for dissipation of the heat generated.

Wear. The determination of proper proportions of worm gearing for wear is a rather involved problem. The best proportions depend to a great extent on the capacity of the gear assembly for radiating the heat generated by friction. With correct alignment, excessive wear is the result of insufficient lubrication. Lubrication may be poor even if the worm runs in an oil bath, for it is caused either by excessive pressure or by a low viscosity of the lubricant resulting from insufficient heat dissipation. The limiting load F_w for wear may be computed by the relation[9]

$$F_w=D_g bK \tag{32–20}$$

where D_g is the pitch diameter of the gear; b is its face, in inches; and K is a constant which depends on the materials of the worm and gear. For a hardened-steel worm having a Brinell hardness number of 250 or more, as

[8] C. M. Allen and F. W. Roys, "Efficiency of Gear Drives," *Trans. ASME*, Vol. 40 (1918), pp. 110–11.

[9] R. T. Kent, *Mechanical Engineers' Handbook*, 12th ed., Vol. II, *Design and Production*, ed. by Colin Carmichael (New York: John Wiley & Sons, Inc., 1950), p. 14-43.

TABLE 32–4

ALLOWABLE SURFACE PRESSURES p_s IN EQUATION 32–21

MATERIAL		NUMBER OF TEETH IN GEAR							
Worm	Gear	10	20	30	40	50	60	70	80 and More
SAE 1020 steel*	Cast iron	75	225	425	750	900	1,080	1,250	1,350
SAE 1040 steel*	SAE 63 bronze†	112	340	625	1,075	1,350	1,625	1,900	2,000
SAE 1040 steel‡	SAE 63 bronze†	170	510	940	1,600	2,000	2,425	2,850	3,000
0.10 C alloy steel, carburized, hardened, and ground	SAE 65 bronze†	225	675	1,250	2,250	2,700	3,250	3,800	4,000
	SAE 65 bronze§	310	930	1,725	3,000	3,700	4,500	5,250	5,500
	Ni bronze†	375	1,125	2,150	3,600	4,500	5,450	6,350	6,700
	Ni bronze§	450	1,350	2,500	4,300	5,400	6,500	7,600	8,000

* Untreated. † Sand-cast. ‡ Heat-treated, ground. § Chill-cast.

required for continuous service, the values of K for various materials of the worm gear are:

For cast-iron or semisteel, $K = 50$
For manganese bronze, $K = 80$
For phosphor bronze, $K = 100$
For Bakelite or other similar materials, $K = 125$

Another formula for F_w, which takes into account the various gear data but assumes the use of a proper grade of lubricant, is[10]

$$F_w = \frac{A \cos \lambda \, p_s c}{c_s} \tag{32–21}$$

where A is the projected tooth area of contact, in square inches;
p_s is the allowable surface pressure, in pounds per square inch, as given in Table 32–4;
c is Barth's velocity factor, from equation 30–6, for the *worm*;
c_s is a service factor, which may be found by increasing the value in Table 30–4 by 25 per cent.

The projected tooth area may be calculated from the equation

$$A = \frac{hD_w\delta}{57.3} \tag{32–22}$$

where h is the tooth depth, in inches, as given in Table 32–1;
δ is one-half of the face angle, in degrees (see Fig. 32–3).

Values for allowable pressures in Table 32–4 are given for a $14\frac{1}{2}°$ pressure angle. For $\beta = 20°$ they can be increased by 5 per cent; and for $\beta = 30°$, by 10 per cent.

[10] Alex Vallance and V. L. Doughtie, *Design of Machine Members* (New York: McGraw-Hill Book Company, Inc., 1943), p. 417.

As may be seen from example 32–3, equation 32–21 gives considerably lower values than does equation 32–20. Which of these formulas gives more accurate information can be decided only by additional research work.

Heat dissipation. In order to prevent overheating of the lubricating oil, the work of friction, which may be calculated from the efficiency of the gear drive, must be dissipated chiefly by radiation. The heat-dissipating capacity depends on the size and surface of the housing and on the velocity of the air surrounding the housing. For average conditions the dissipating capacity Q may be taken equal to 0.46 Btu per hr per degree of temperature difference per square inch of projected area A of the worm and worm gear.[11] Thus

$$Q = 0.46A(t_2 - t_1) \tag{32–23}$$

where the difference between the gear temperature t_2 and the room temperature t_1 should not exceed 80 to 100 deg F.

Whether the worm runs in an oil bath or not, the gears should be entirely enclosed to prevent oil leakage and to protect them from dust. Ball bearings or roller bearings should be used in order to increase the efficiency and to maintain proper alignment.

Materials. It should be remembered that unlike metals are more satisfactory for sliding contact than are like metals. Thus, for light loads and low speeds the worm may be made of steel, such as SAE 1040, and the worm gear may be made of cast iron or leaded gun metal, SAE 63. For medium service conditions the worm may be made of SAE 2320 or SAE 3120 steel that is casehardened to a Brinell hardness number of at least 250, and the gear may be of phosphor bronze. For high speeds and heavy loads with shock action, the worm is made of molybdenum steel or chrome-vanadium steel and is hardened, and the gear is made of SAE 65 phosphor bronze, which may be chilled for hardness and refinement of grain structure. Often, in order to reduce the cost of a large gear, the rim alone is made of bronze, as in Fig. 32–5, and it is bolted to a cast-iron flange with arms and a hub.

EXAMPLE 32–3. Check the worm drive discussed in examples 32–1 and 32–2 for wear and overheating.

The pitch diameter of the gear is $D_g = 14.82$ in., and the face is $b = 3.425$ in. If it is assumed that two teeth are in mesh at one time, the limiting load for wear for a cast-iron gear is, by equation 32–20,

$$F_w = 14.82 \times 3.425 \times 50 \times 2 = 5{,}080 \text{ lb}$$

This is amply safe, since the transmitted tooth load is 1,982 lb.

The procedure in using equation 32–21 as a check is as follows: For a quadruple-thread worm, the tooth depth is

$$h = 0.623p_c = 0.935 \text{ in.}$$

[11] C. A. Norman, *Principles of Machine Design* (New York: The Macmillan Company, 1925), p. 386.

The angle δ, Fig. 32–3, can be found approximately from the relation

$$\sin\delta = \frac{0.5b}{0.5(D_w+2a)} = \frac{3.425}{3.52+2\times0.286\times1.5} = 0.717$$

Then $\delta = 45.6°$, and by equation 32–22 the projected tooth area, with two teeth in contact, is

$$A = \frac{0.935\times3.92\times45.6\times2}{57.3} = 5.84 \text{ sq in.}$$

Since $\lambda = 26°$, $\cos\lambda = 0.899$. For a cast-iron gear with 31 teeth, a worm of SAE 1020 steel, and $\beta = 20°$, the allowable surface pressure, found with the aid of Table 32–4, is

$$p_s = [425+\tfrac{1}{10}\times(750-425)]\times1.05 = 480 \text{ psi}$$

The pitch-line velocity of the worm is

$$v_m = \frac{\pi\times3.92\times1{,}165}{12} = 1{,}195 \text{ fpm}$$

and the velocity factor is

$$c = \frac{600}{600+1{,}195} = 0.335$$

The service factor for an air-compressor drive, taken from Table 30–4, is $c_s = 1.25 \times 1.25 = 1.56$.

Hence, by equation 32–21,

$$F_w = \frac{5.84\times0.899\times480\times0.335}{1.56} = 542 \text{ lb}$$

This is considerably less than the tooth load of 1,982 lb. For continuous operation the drive should therefore be redesigned. Changing the material of the worm to hardened 0.10 C alloy steel and changing the material of the gear to SAE 65 chill-cast phosphor bronze will increase F_w to 3,150 lb, which is satisfactory.

The heat that must be dissipated per hour is evidently

$$Q = \frac{33{,}000P(1-e)60}{778} = \frac{33{,}000\times20\times(1-0.887)\times60}{778} = 5{,}750 \text{ Btu per hr}$$

The projected area of the gear is

$$A_g = 0.7854\times14.82^2 = 173 \text{ sq in.}$$

and the projected area of the worm is

$$A_w = 3.92\times6.87 = 26.9 \text{ sq in.}$$

Solving equation 32–23 for $t_2 - t_1$ gives

$$t_2 - t_1 = \frac{5.750}{0.46(173+26.9)} = 62.5 \text{ deg F}$$

A temperature rise of 62.5 deg F is entirely satisfactory.

32–6. Mountings. The *worm shaft* is usually supported by two bearings. As indicated in Fig. 32–8a, the bearings a and b must take the pressures coming from the forces Q and R, and the thrust coming from the force F_t. If the shaft is mounted on ball bearings, a radial bearing can be installed to take care of the lateral and thrust loads simultaneously. Otherwise a special thrust bearing is necessary.

Bearing loads. If it is assumed that the turning effort Q and the upward reaction R are applied midway between the bearings a and b, which are c in. apart, each of these bearings must take a pressure equal to one-half of these forces. The forces Q and R being at right angles, their components at the

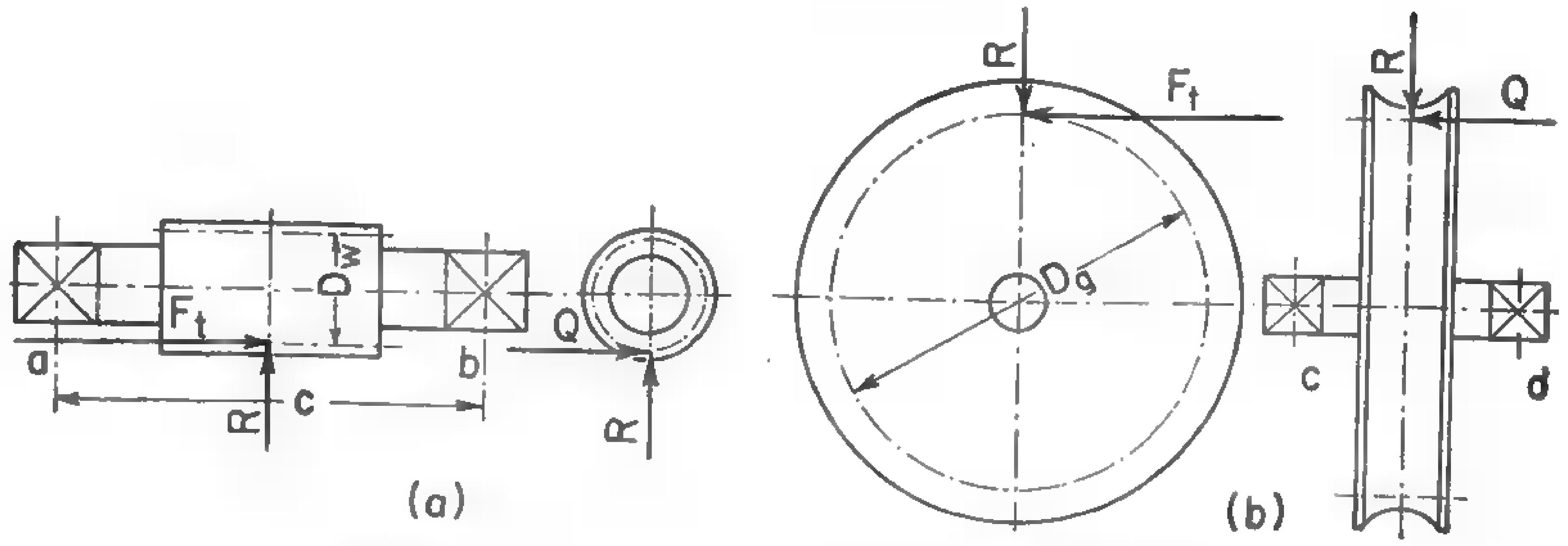


FIG. 32–8. Forces on worm and worm-gear shafts.

bearings are also at right angles to each other. The tangential force F_t causes an end thrust upon the bearing a and also exerts a pressure equal to $F_tD_w/2c$ upon each bearing, the pressure at a acting downward and that at b upward. Therefore the resultant radial, or lateral, load upon the bearing a is

$$A = \sqrt{\frac{Q^2}{4}+\left(\frac{R}{2}-\frac{F_tD_w}{2c}\right)^2} \tag{32–24}$$

and the resultant lateral load upon the bearing b is

$$B = \sqrt{\frac{Q^2}{4}+\left(\frac{R}{2}+\frac{F_tD_w}{2c}\right)^2} \tag{32–25}$$

Worm-gear shaft. If the bearings c and d, Fig. 32–8b, are located symmetrically with respect to the middle plane of the gear, the loads coming upon each bearing from the forces F_t and R will be equal to one-half of these forces. The force Q exerts a thrust on bearing c and also introduces lateral loads upon both bearings. The load upon bearing c is equal to $QD_g/2c$ and acts downward; the load upon bearing d is $QD_g/2c$ and acts upward. Since the components of F_t and those of R and Q act at right angles, the resultant radial load on the bearing c is

$$C = \sqrt{\frac{F_t^2}{4}+\left(\frac{R}{2}+\frac{QD_g}{2c}\right)^2} \tag{32–26}$$

and the resultant radial load on the bearing d is

$$D = \sqrt{\frac{F_t^2}{4}+\left(\frac{R}{2}-\frac{QD_g}{2c}\right)^2} \tag{32–27}$$

Mountings. A worm-gear drive should always be mounted in a dustproof casing which permits either the worm or the gear to run in an oil bath. The worm shaft is usually mounted on ball bearings, and there is often a double-row radial ball bearing on the side, where an axial thrust load exists. The gear shaft is supported either by ball bearings or by Timken adjustable roller bearings.

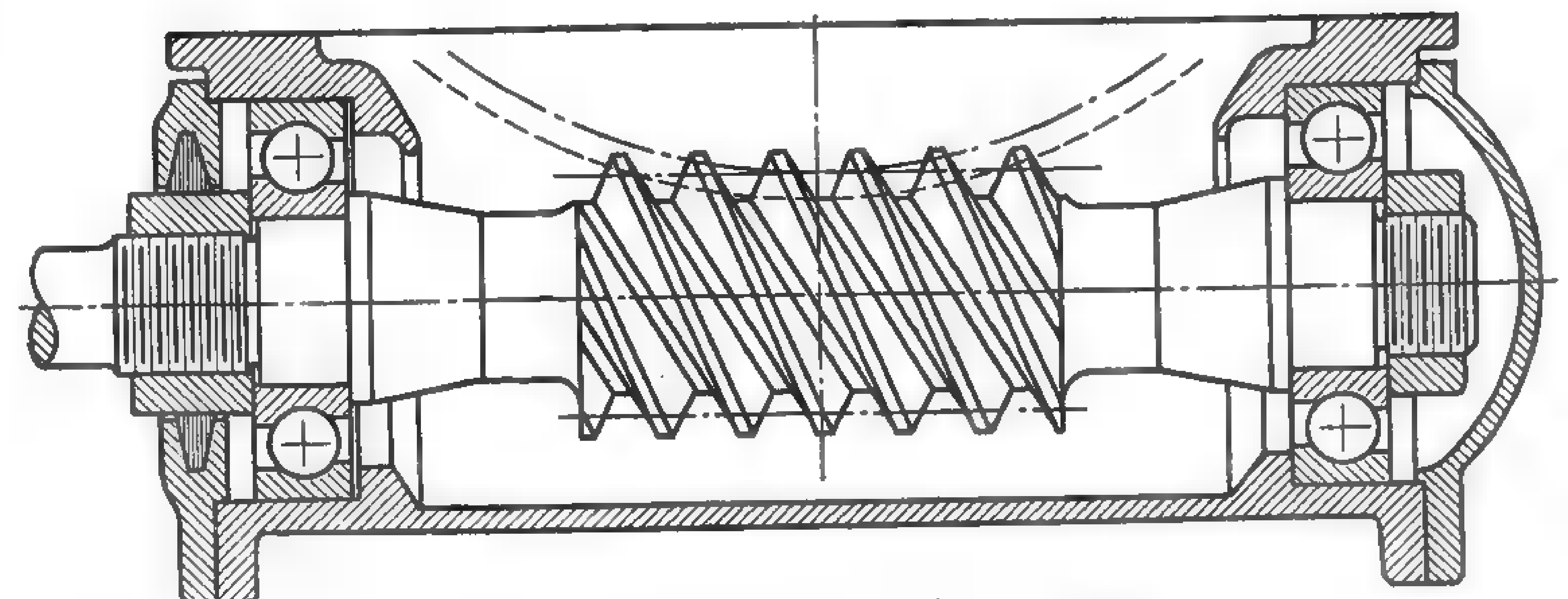


Fig. 32–9. Mounting of a worm.

If a worm is mounted as in Fig. 32–9, the axial thrust creates tension in the shaft, regardless of the direction in which the worm rotates.

CHAPTER 33

Screw Gearing

33–1. General considerations. Screw gears, or helical gears, often miscalled spiral gears, are used to connect shafts which are not intersecting. Their axes may be parallel, at right angles, or inclined at any angle to each other. Parallel gears are always called *helical gears* and are discussed in section 30–12. *Worm gears*, discussed in Chapter 32, are special types of gears with their axes at right angles. In order to preclude confusion with helical spur gears having parallel axes, a better name for those with nonparallel axes is *screw gears*. Screw gears resemble helical spur gears as far as the shape of the teeth are concerned, but their action is different. The engaging teeth slide over each other, instead of moving in the same direction at the pitch point. Such gears are suitable only for the transmission of light power at moderate speeds. They are used to convert rapid rotary motion into slow rotary motion or, when using a rack, into slow linear motion. When used as a speed increaser, they are subject to rapid wear.

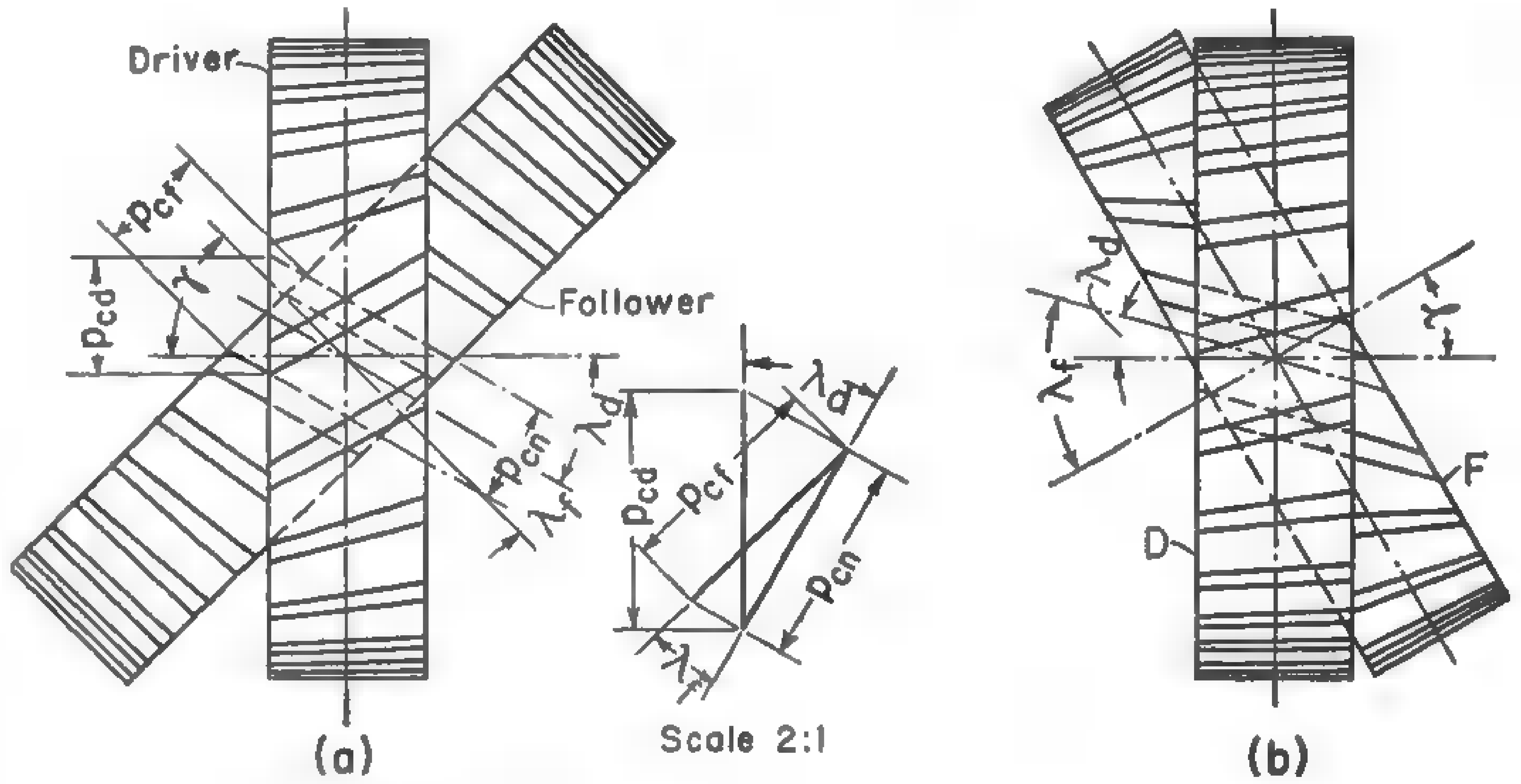


Fig. 33–1. Pitches and angles in helical gears.

Definitions. The two mating screw gears, which will be designated as the *driver* and the *follower* in order to differentiate them, may have different helix or pitch angles λ_d and λ_f and different circular pitches P_{cd} and P_{cf} measured on the pitch planes normal to the gear axes. However, the *normal circular pitch* p_{cn}, or the distance from the face of one tooth to the corresponding face of the next tooth, measured on the pitch cylinder normal to the tooth face, is the same for both gears, as shown in Fig. 33–1. The *normal diametral pitch* p_{dn} is related to p_{cn} by the equation $p_{dn}p_{cn} = \pi$. Any two screw

gears having the same tooth system and the same normal pitch will work together, regardless of tangential pitches. Thus, from Fig. 33–1,

$$p_{cn} = p_{cd} \cos \lambda_d = p_{cf} \cos \lambda_f \tag{33–1}$$

33–2. Geometric relations. If the pitch diameters of mating screw gears are designated by D_d and D_f, and the numbers of teeth are designated by i_d and i_f,

$$D_d = \frac{i_d p_{cd}}{\pi} = \frac{i_d p_{cn}}{\pi \cos \lambda_d} \tag{33–2}$$

and

$$D_f = \frac{i_f p_{cn}}{\pi \cos \lambda_f} \tag{33–3}$$

The velocity ratio r_v of any kind of tooth gearing is always equal to the inverse ratio of the number of teeth in the driver and follower, or

$$r_v = \frac{n_d}{n_f} = \frac{i_f}{i_d} = \frac{D_f \cos \lambda_f}{D_d \cos \lambda_d} \tag{33–4}$$

where n_d and n_f are the respective rotative speeds. Equation 33–4 shows that when it is desired to use screw gears for connecting two shafts so as to produce a desired velocity ratio, any one of a number of pairs of gears with various pitch angles and diameter ratios will give a satisfactory result.

By changing the pitch angles it is also possible to change the direction of rotation of the driven shaft. The pitch angle may be either right-hand or left-hand, the same as for screw threads, and the two helices of a pair of gears may be of the same hand or of opposite hands. Comparison of Figs. 33–1a and 33–1b shows that (1) when both helices are of the same hand, the shaft angle is $\gamma = \lambda_d + \lambda_f$; (2) when the helices are of opposite hands, $\gamma = \lambda_f - \lambda_d$.

Dimensions and angles for screw gears may be computed by using the formulas given in Table 33–1. The notations not shown in Fig. 33–1 follow: l_d and l_f are the leads of the tooth helix of the driver and the follower, respectively; a is the addendum of the normal pitch, which for the $14\frac{1}{2}°$ standard is equal to $1/p_{dn}$; and C is the center distance.

33–3. Design. Since screw gears have only a point contact, their wear is considerable and the efficiency cannot be very high. Therefore they are not used for large tooth loads. This makes possible the selection of the pitch and the face width from geometrical considerations, rather than from strength considerations. Generally p_{dn} is made from 5 to 10, and the face width b is made from $2p_c$ to $4p_c$. In order that the teeth may be cut with standard cutters, the normal pitch p_{dn} must be a simple number, and the center distance C of the gears should be allowed to vary somewhat.[1] If a comparatively high efficiency is desired, the two pitch angles λ_d and λ_f should be approximately equal.

[1] Brown & Sharpe Mfg. Co., *Practical Treatise on Gearing*, 17th ed. (Providence, R.I., 1935), p. 139.

TABLE 33–1

FORMULAS FOR SCREW-GEAR CALCULATIONS

Driver			Follower		
To Find	Formula		To Find	Formula	
λ_d	$\cos \lambda_d = \frac{p_{cn}}{p_{cd}} = \frac{i_d}{p_{dn} D_d}$	(33–5)	λ_f	$\cos \lambda_f = \frac{p_{cn}}{p_{cf}}$	(33–12)
	$\tan \lambda_d = \frac{\pi D_d}{l_d}$	(33–6)	λ_f	$\lambda_f = \gamma - \lambda_d$	(33–13)
p_{cn}	$p_{cn} = \frac{\pi D_d \cos \lambda_d}{i_d}$	(33–7)	p_{cn}	$p_{cn} = \frac{\pi D_f \cos \lambda_f}{i_f}$	(33–14)
p_{cd}	$p_{cd} = \frac{\pi D_d}{i_d}$	(33–8)	p_{cf}	$p_{cf} = \frac{\pi D_f}{i_f}$	(33–15)
l_d	$l_d = \pi D_d \tan \lambda_f = p_{cf} i_d$	(33–9)	l_f	$l_f = \pi D_f \tan \lambda_d = p_{cd} i_f$	(33–16)
D_d	$D_d = \dfrac{2C}{\dfrac{i_f \cos \lambda_d}{i_d \cos \lambda_f} + 1}$	(33–10)	D_f	$D_f = 2C - D_d$	(33–17)
	$D_d = 0.3183 i_d p_{cd}$	(33–11)		$D_f = 0.3183 i_f p_{cf}$	(33–18)

$$C = \frac{i_d}{2p_{dn} \cos \lambda_d} + \frac{i_f}{2p_{dn} \cos \lambda_f} \tag{33–19}$$

If the velocity ratio r_v, the shaft angle γ, and the approximate center distance C are given, the preliminary values for D_d and D_f are found by solving simultaneously equation 33–4, in which λ_f and λ_d are assumed to be equal, and equation 33–17 in Table 33–1. Then a suitable value for p_{dn} is assumed, the corresponding value of p_{cn} is found, and the numbers of teeth i_d and i_f are calculated by equations 33–7 and 33–14, respectively. The nearest whole numbers are taken for i_d and i_f, and either the preliminary values of D_d and D_f are corrected or new values are found for λ_d and λ_f. The tooth proportions are based on the relations in Table 30–1 for a pressure angle $\beta = 14\frac{1}{2}°$ and for the values of the *normal* circular and diametral pitches.

EXAMPLE 33–1. Design a pair of screw gears to have a velocity ratio of 1:3, with the shafts at an angle of 60° and approximately 5 in. apart. The teeth are to be $14\frac{1}{2}°$ involute with American Standard full-depth proportions.

Assume that $\lambda_d = \lambda_f$. Then, by equation 33–13, $\lambda = \gamma/2 = 30°$. By equation 33–17,

$$D_f = 2 \times 5 - D_d$$

Now, by equation 33–4, with $\lambda_d = \lambda_f$,

$$r_v = \frac{10 - D_d}{D_d} = \frac{1}{3}$$

From this relation, $D_d = 7.5$ in.; and $D_f = 10 - 7.5 = 2.5$ in.

Take p_{dn} as 8. Then $p_{cn} = \pi/8 = 0.3927$ and, by equation 33–14, the number of teeth on the follower is

$$i_f = \frac{\pi D_f \cos \lambda_f}{p_{cn}} = \frac{\pi \times 2.5 \times \cos 30^\circ \times 8}{\pi} = 2.5 \times 0.8660 \times 8 = 17.32, \text{ or } 17$$

By equation 33–4, the number of teeth on the driver is

$$i_d = \frac{i_f}{r_v} = 17 \times \frac{3}{1} = 51$$

The exact value of the pitch diameter D_d is, by equation 33–2,

$$D_d = \frac{51 \times 0.3927}{\pi \times 0.8660} = 7.360 \text{ in.}$$

Then

$$D_f = 7.360 \times \tfrac{1}{3} = 2.453 \text{ in.}$$

The center distance is, from equation 33–17,

$$C = 0.5 \times (7.360 + 2.453) = 4.907 \text{ in.}$$

By equation 33–8,

$$p_{cd} = \frac{\pi \times 7.360}{51} = 0.453 \text{ in.}$$

The face width b can be made between $2 \times 0.453 = 0.906$ in. and $4 \times 0.453 = 1.812$ in.; select 1.50 in.

The addendum, by Table 30–1, is

$$a = \frac{1}{p_{dn}} = \frac{1}{8} = 0.125 \text{ in.}$$

The outside diameters of the gears will be

$$D_d' = 7.360 + 2 \times 0.125 = 7.610 \text{ in.}$$

and

$$D_f' = 2.453 + 2 \times 0.125 = 2.703 \text{ in.}$$

Example 33–2. Design a pair of screw gears for the conditions of example 33–1 but with the shafts exactly 5.000 in. apart.

Equation 33–19, in conjunction with equation 33–4, gives

$$\frac{2Cp_{dn}}{i_d} = \sec \lambda_d + r_v \sec \lambda_f$$

For $C = 5$ in., $r_v = \frac{1}{3}$, $p_{dn} = 8$, $i_f = 17$, and $i_d = 51$, this equation becomes

$$\frac{2 \times 5 \times 8}{51} = 1.568 = \sec \lambda_d + \tfrac{1}{3} \sec \lambda_f$$

This equation can be solved by the trial method. For easier handling it can be rewritten in the form

$$3 \sec \lambda_d + \sec \lambda_f = 4.704$$

For the first try, it will be assumed that $\lambda_d = \lambda_f = 30^\circ$. Then

$$3 \times 1.1547 + 1.1547 = 4.6188$$

This shows that λ_d, which is multiplied by 3, must be increased and that λ_f must be decreased. When $\lambda_d = 33^\circ$ and $\lambda_f = 60^\circ - 33^\circ = 27^\circ$,

$$3 \times 1.1924 + 1.1233 = 4.7005$$

If $\lambda_d = 33^\circ\,9'$ and $\lambda_f = 26^\circ\,51'$,

$$3 \times 1.1944 + 1.1208 = 4.7040$$

A check by using equations 33–7 and 33–14 follows:

$$D_d = \frac{\pi \times 51}{8\pi \times 0.8372} = 7.62 \text{ in.} \qquad D_f = \frac{\pi \times 17}{8\pi \times 0.8922} = 2.38 \text{ in.}$$

$$C = 0.5 \times (D_f + D_d) = 0.5 \times (7.62 + 2.38) = 5.00 \text{ in.}$$

33–4. Shafts at right angles. In the most common case, the shafts of a pair of screw gears are at right angles to each other, as in Fig. 33–2. Such gears are used either as a speed reducer, as in a drive from a crankshaft to a camshaft in a four-stroke gas engine with a speed ratio of 2 to 1, or as a speed increaser, as in a drive from a crankshaft to the governor with a speed ratio of 1 to 2 or higher but always carrying a very small load.

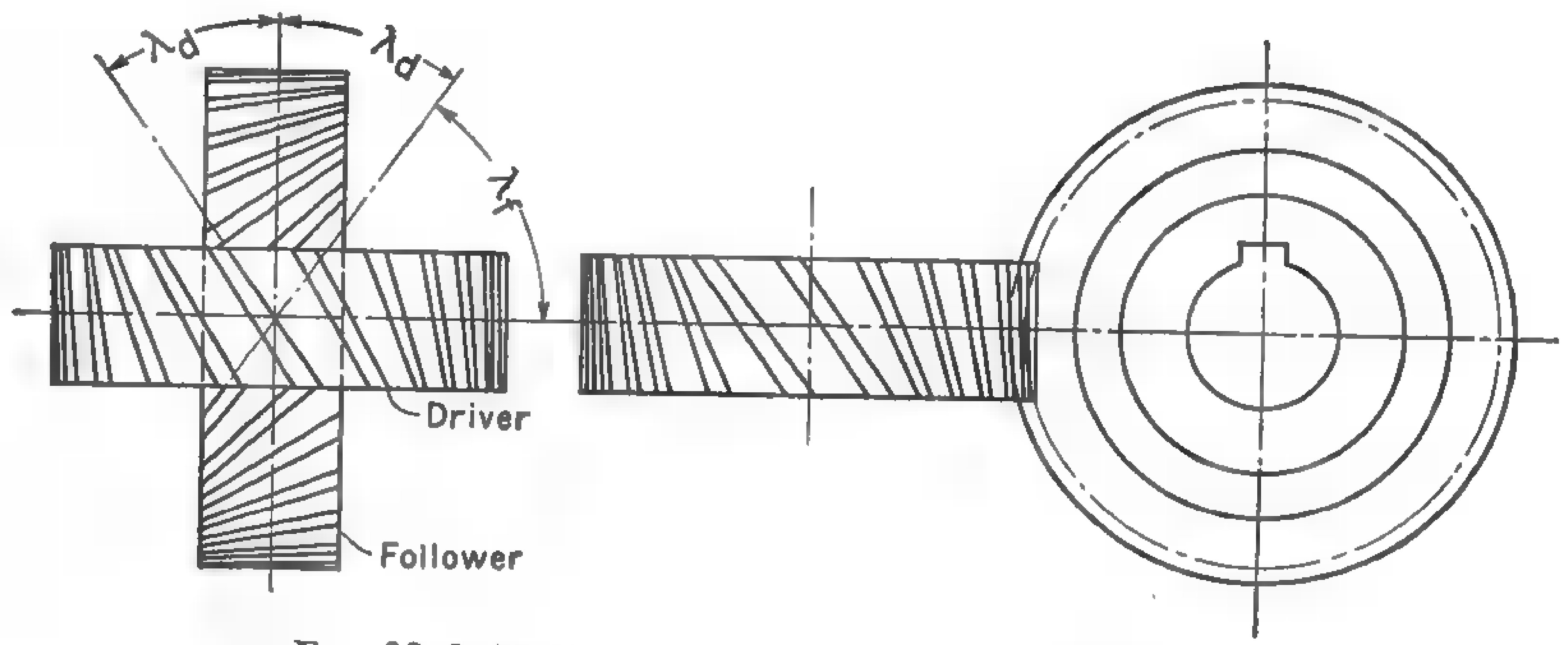


Fig. 33–2. Helical gears on shafts at right angles.

In principle this screw gearing is simply straight-tooth worm gearing having a short worm with a long lead so that only short segments of the thread appear on the surface.

Efficiency. Since screw gearing with the shafts at right angles is geometrically identical with worm gearing, equation 32–15 applies to it, with a slight difference in notation. In deriving equation 32–15 the worm was taken as the driver and the gear was the follower. A comparison of Figs. 32–1 and 33–2 shows that $\lambda = \lambda_f$. Also, in Fig. 33–2, $\lambda_f + \lambda_d = 90^\circ$. Equation 32–14 then becomes

$$e = \frac{\cos \theta - f \tan \lambda_f}{\cos \theta + f \tan \lambda_d} \tag{33–20}$$

The coefficient of friction f ranges from 0.05 to 0.10.

Design. The formulas of Table 33–1 become somewhat simpler when $\lambda_d + \lambda_f = \gamma = 90^\circ$. For the sake of efficiency and of smaller wear caused by sliding, the angles λ_d and λ_f should be about equal, and they must be kept between 20° and 70°.[2] The magnitude of the tangential effort F_t, Fig. 32–8, is found from the torque T_d that must be transmitted. Thus

$$F_t = \frac{2T_d}{D_d} \tag{33–21}$$

Strength of teeth. The strength of the gear teeth may be checked by the Lewis formula (equation 30–40), where the normal diametral pitch p_{dn}

[2] C. A. Norman, *Principles of Machine Design* (New York: The Macmillan Company, 1925), p. 389.

TABLE 33-2

VALUES OF K FOR SCREW GEARS

One Gear of Pair		Meshing Gear		K After Polishing Run	
Material	Bhn	Material	Bhn	Short	Careful
Steel	250	Steel	250	2	5
Steel	250	Bronze	100	4	12
Steel	500	Bronze	120	5	20
Steel	500	Cast iron	180	6	20
Steel	500	Steel	500	7	15
Cast iron	180	Cast iron	180	8	20
Nonmetallic	...	Steel or cast iron	...	10	25

should be used and the value of Y taken from Table 30-2 should be based on the formative number of teeth i'. This number is determined by the relation

$$i' = \frac{i}{\cos^3 \lambda} \tag{33-22}$$

The velocity factor c should be computed by equation 30-15. The value of the allowable stress S_o may be taken from Table 30-10. The effective width b of the teeth is rather uncertain in this type of gearing, but it may be taken safely as $2p_{cn}$. The values of the factor C_w are the same as those given in section 30-14.

The value of F_t found by equation 30-40 is the safe normal load on a tooth and should be greater than F_t found by equation 33-21.

Limit load. Because of the point contact of screw gears, even light tooth loads set up very high compressive stresses at the point of contact. Tooth loads smaller than those that may break the teeth may produce excessive wear caused by pitting and abrasion.

Wear can be considerably reduced, and the load capacity can be increased, if the gears are first broken in by operating them in their actual working positions under a light load until a polished line or narrow band appears on the teeth. A further increase of the load capacity—about 150 to 250 per cent—can be obtained if the gears, after being broken in as just described, are run carefully under a gradually increasing load until the narrow bands on the contact surfaces become appreciably wider. The load capacity of gears broken in by the described method may be determined by the relation

$$F_w = KQD_d^2 \tag{33-23}$$

Values for the factor K in this equation are given in Table 33-2, and the term Q is computed from the relation[3]

[3] R. T. Kent, *Mechanical Engineers' Handbook*, 12th ed., Vol. II, *Design and Production*, ed. by Colin Carmichael (New York: John Wiley & Sons, Inc., 1950), p. 14-16.

$$Q = \left(\frac{2D_f}{D_f + D_d}\right)^2 \tag{33-24}$$

To prevent excessively fast wear, the limit load F_w for wear should be greater than the maximum or dynamic load determined by the equation

$$F_d = \frac{F_t}{c} \tag{33-25}$$

where F_t is determined by equation 33-21 and c is found by equation 30-15.

In general, to avoid scuffing, soft steel and cast iron should not be used if the pitch-line velocity of either gear is considerably over 1,000 fpm. A bronze gear should not be used in combination with a nonmetallic gear, because of an excessive lapping in, or abrasion, of the bronze gear.

Bearing loads. The loads on bearings for screw gears may be found from equations 32-24 to 32-27 by substituting D_d for D_w and D_f for D_g.

The magnitude of the axial thrust Q on the follower is, by equation 32-13,

$$Q = F_t \frac{\cos\theta \tan\lambda_f + f}{\cos\theta - f\tan\lambda_f} \tag{33-26}$$

By eliminating N from equation 32-12 by means of equation 32-10, the downward pressure on the shaft of the follower is found to be

$$R = F_t \frac{\sin\theta}{\cos\theta \cos\lambda_f - f\sin\lambda_f} \tag{33-27}$$

Lubrication. Because the power transmitted by screw gears is usually small the problem of lubrication is often neglected, and as a result the screw gears wear out very rapidly. Screw gears, like worm gearing, should run in oil enclosed in a housing if proper efficiency and reasonably long life are desired. Since screw gears have comparatively low efficiency (not over 85 per cent) the heat generated by friction is comparatively great and provision should be made to dissipate it.

APPENDIX: PROBLEMS

APPENDIX

Problems

Many of the following problems require, in addition to a working knowledge of the data given in this textbook, the use of a mechanical engineering handbook and the catalogues of manufacturers of mechanical equipment, with which the young designer should become familiar.

CHAPTER 2: Static Stresses in Machine Parts

2–1. Calculate how much a standard 1-in. steel pipe (1.315 in. OD, 1.05 in. ID) 10 ft long will stretch when carrying a load in tension of 4 tons. Assume $E = 30{,}000{,}000$ psi.

2–2. After the nut on the $1\frac{1}{8}$-in. steel bolt with UNF thread in Fig. P2–1 is drawn snug, it is given one-quarter of a turn. The diameters of the round spacers on the bolt are $d_o = 2$ in. and $d_i = 1\frac{1}{4}$ in. The shorter one has a length l_1 of 4 in. and is of steel for which $E = 30{,}200{,}000$ psi. The longer one has a length l_2 of 12 in. and is of cast iron for which $E = 13{,}000{,}000$ psi. Data for the screw thread are given in Table 11–2. Find the force set up in the bolt.

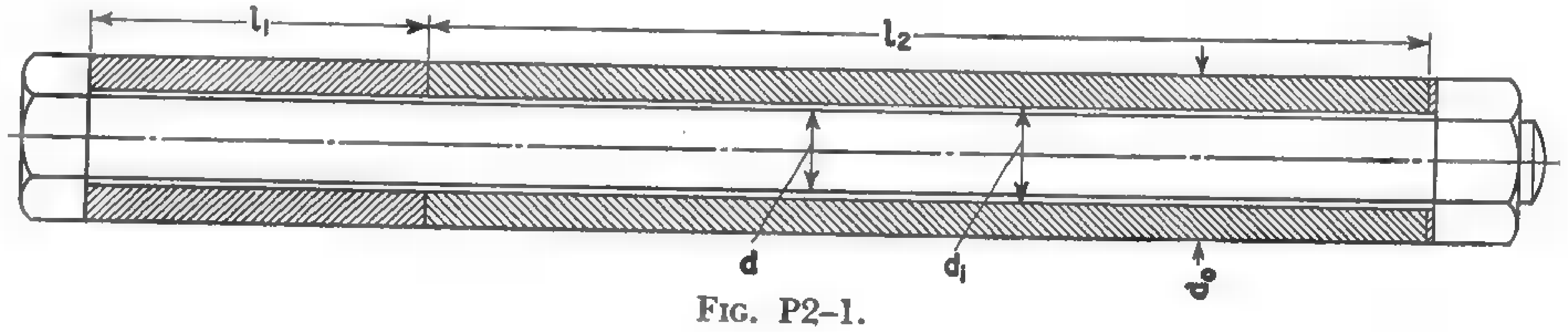


FIG. P2–1.

2–3. A $\frac{3}{4}$-in. steel bolt a (Fig. P2–2) with UNC thread holds two plates b in place, a 1-in. standard wrought-iron pipe being used as a spacer. After the nut is tightened just enough to take up the slack, it is given an additional one-fifth of a turn. Determine the stresses set up in the shank of the bolt and in the spacer, assuming that $E = 30{,}200{,}000$ psi for the steel and that $E = 27{,}000{,}000$ psi for the wrought iron. Also, $l = 12$ in. and $h = \frac{5}{8}$ in. Neglect the deformation of the plates b.

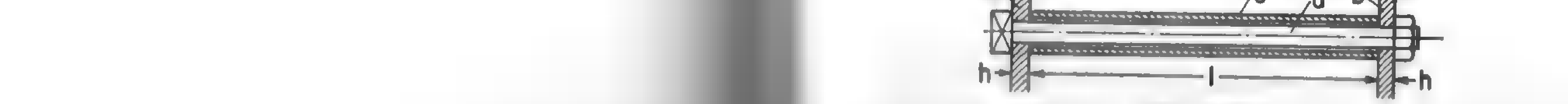


FIG. P2–2.

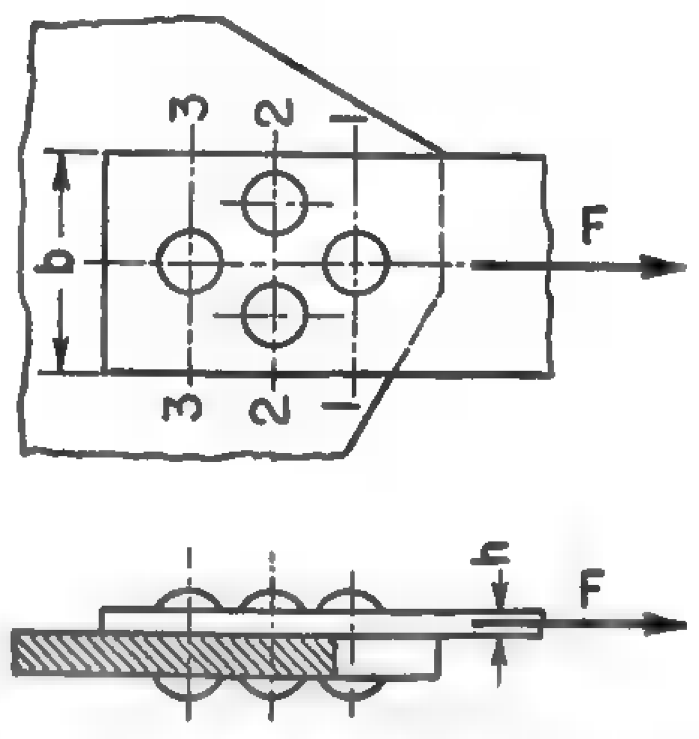


FIG. P2–3.

2–4. Determine the theoretical value of the transverse modulus of elasticity of a high-carbon steel bar having a diameter of 0.798 in. which, when subjected to a tensile load of 11,250 lb, stretched from 8.001 in. to 8.007 in. between the marks.

2–5. A tension member in a steel frame (Fig. P2–3) made of flat steel having the dimensions $b = 4$ in. and $h = \frac{1}{2}$ in. is fastened to the main plate by four $\frac{5}{8}$-in. rivets. Assuming that the load F of 12,000 lb is distributed equally between the rivets, determine the tensile stresses in the sections *1–1*, *2–2*, and *3–3*, and also the shear and bearing stresses in the rivets.

2–6. Find the maximum stress and the degrees of torsion of a hollow steel shaft, 5 in. in outside diameter, 2 in. in inside diameter, and 24 ft long, transmitting 600 hp at 240 rpm. The modulus of elasticity in shear of the material is 11,600,000 psi.

2–7. A solid round shaft $3\frac{7}{16}$ in. in diameter transmits 150 hp at 135 rpm. Determine (a) the torsional stress in the outer fibers, (b) the increase of stress if the shaft is made hollow with the same diameter and an inner diameter of $1\frac{1}{2}$ in., and (c) the stress in the inner fibers of the hollow shaft.

2–8. Find the maximum stresses in the long and short sides of a 1×2 in. rectangular bar 22 in. long, twisted by a weight of 200 lb acting with a lever arm of 15 in.

2–9. Find the load that can be put at the center of a simple cast-iron beam which rests on supports 3 ft apart. The section is uniform and has the shape of an inverted T. With the designations of Table 2–5, $a = \frac{3}{4}$ in., $d = 1$ in., $B = 2\frac{3}{4}$ in., and $H = 3$ in. The allowable stresses are 4,000 psi in tension and 30,000 psi for compression.

2–10. (a) Find the load which, put on the beam of problem 2–9, produces a maximum deflection of $\frac{1}{32}$ in. Assume for the modulus of elasticity a value of 13,000,000 psi. (b) Find the maximum stress created by this load.

2–11. A horizontal tension member of steel is $\frac{3}{4}$ in. thick, $2\frac{1}{2}$ in. deep, and 15 ft long. The ends are riveted rigidly to girders. Determine the maximum bending stress due to its own weight, assuming the weight of steel as 0.282 lb per cu in.

2–12. Find the radius of curvature of an American standard 7 in. $\times$ 15.3 lb beam 9 ft long used as a cantilever with a uniformly distributed load of 400 lb per ft (a) at the support and (b) at half-length of the beam. The modulus of elasticity of the steel is $E =$ 30,200,000 psi.

2–13. Find the radius of curvature of an American standard 6 in. $\times$ 12.5 lb **I** beam 12 ft long used as a simple beam with a uniformly distributed load of 500 lb per ft (a) at the middle and (b) at a point 25 per cent of the span from a support. Assume that $E =$ 30,200,000 psi.

2–14. Work problem 2–13, assuming that in addition to the uniform load there is a concentrated load of 1,000 lb applied at the center of the beam.

2–15. Find the deflection of the free end of a cantilever beam with the dimensions given in problem 2–12, but with a concentrated load F of 800 lb applied to the end, instead of the uniform load in problem 2–12. Neglect the weight of the beam itself.

2–16. Find the deflection of the free end of the beam in problem 2–12 by using equation 2–31, and check the result by data from Table 2–5.

2–17. Find the maximum deflection of a simple beam with the dimensions given in problem 2–13, but with a concentrated load of 3,000 lb applied at the center, instead of the uniform load. Neglect the weight of the beam itself.

2–18. The beam a, Fig. P2–4, rests on two struts b and carries the strut c in the center. Considering the upper and lower supports immovable, determine the force that the strut c will produce if it is made 0.018 in. longer than necessary to be assembled without a stress and is then forced into place. All parts are of steel for which $E = 30,000,000$ psi and $l_1 = 14$ in. and $l_2 = 15$ in.

2–19. A force F of 120 tons is transmitted from block B, Fig. P2–5, to block C through three steel struts, a, b, and c. All struts have the same cross-sectional area $A = 3$ sq in. The blocks can be considered as rigid. Because of inaccuracy in machining, one of the struts was made 0.005 in. shorter than the other two. Find the load carried by each strut (a) if the shorter strut is placed in the center and (b) if it is placed on the right-hand side.

2–20. Work problem 2–19, assuming that the center line of the middle bar is moved 3 in. to the left, thus making $l_1 = 12$ in. and $l_2 = 18$ in.

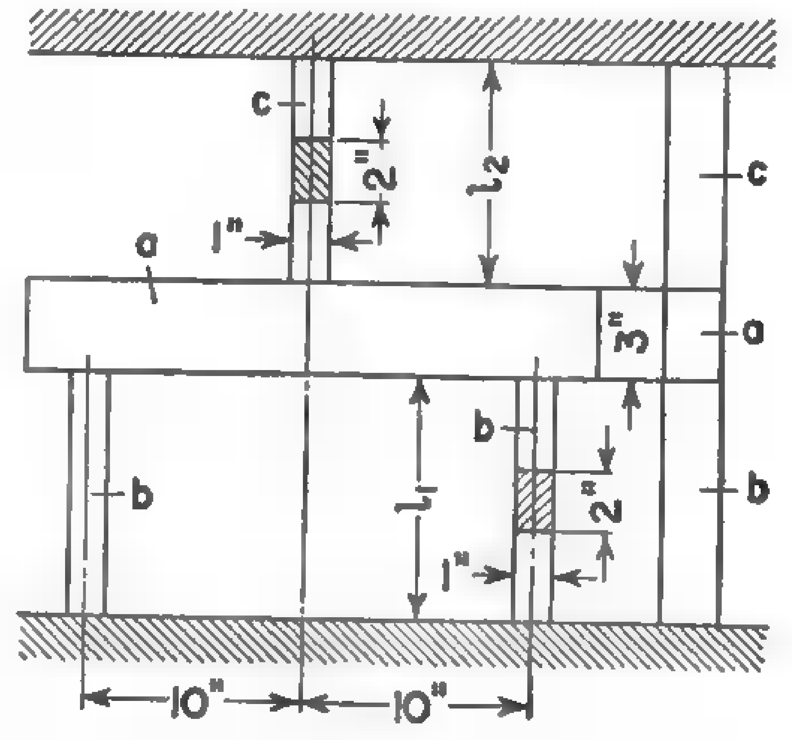


FIG. P2–4.

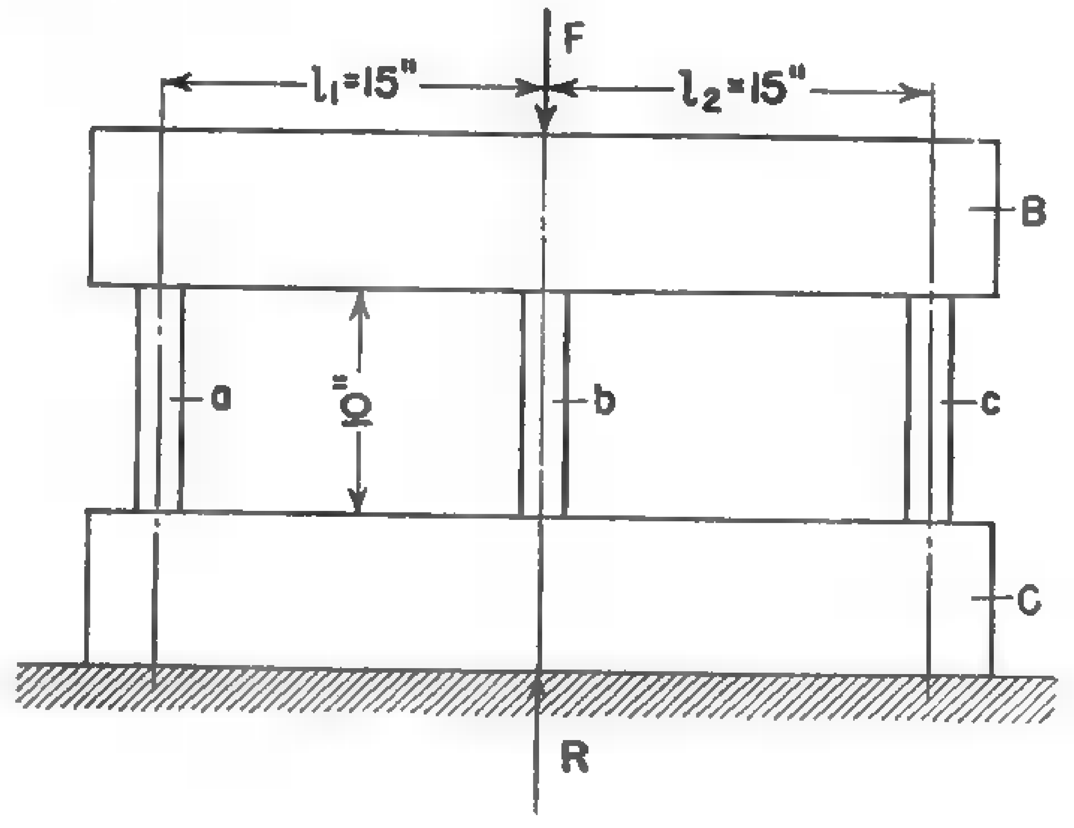


FIG. P2–5.

2–21. In the construction shown in Fig. P2–5 all struts are of steel for which $E = 30,200,000$ psi, they have exactly the same length, and they have the same cross-sectional area $A = 5$ sq in. The load F of 200 tons is applied 5 in. to the left of the center line. The reaction is on the same line with the load. Blocks B and C can be considered as rigid. Find the distribution of the load between the three struts.

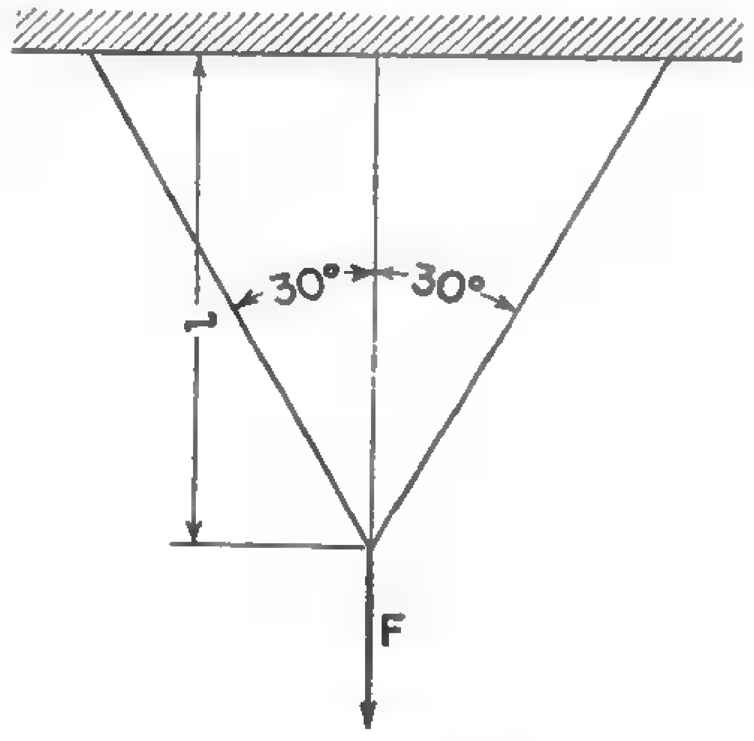


FIG. P2–6.

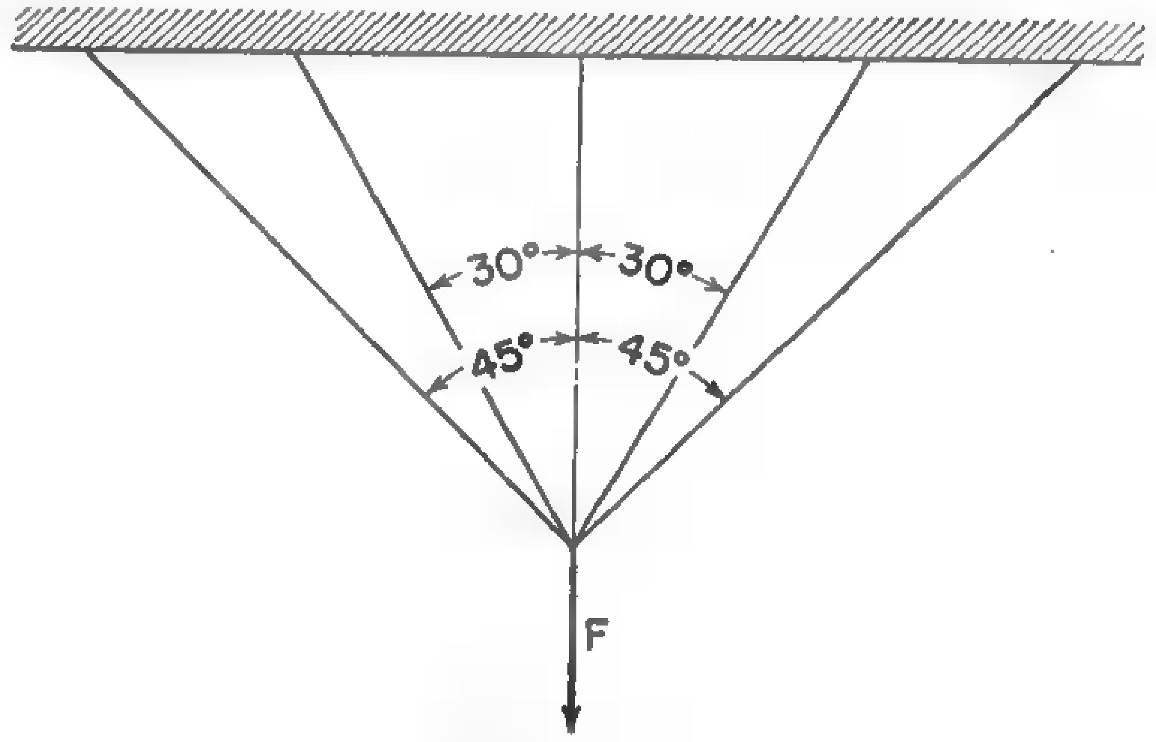


FIG. P2–7.

2–22. Each of the three steel rods of Fig. P2–6 has the same cross-sectional area. Their lengths are such that before the load F of 10 tons is applied, there is no stress in them. Find the force in each rod, taking E as 30,200,000 psi.

2–23. Work problem 2–22, assuming that the middle rod is made of ductile iron, Table 4–1, and that the side rods are of stainless steel SAE 30905, Table 4–2.

2–24. The rods of Fig. P2–7 are made of steel for which $E = 30,000,000$ psi, and they have equal cross-sectional areas. Their lengths are such that there is no stress in them before the load F of 12 tons is applied. Find the force acting on each rod.

2–25. Work problem 2–24, assuming that the cross-sectional area of each outer rod is one-half the area of each inner rod.

2–26. Find the distance l in Fig. P2–8 that will keep the rigid 2,000-lb weight level. The lower ends of the two suspension rods were at the same elevation before the two weights were applied. The rods are of the same material, and the cross-sectional area of rod *2* is 25 per cent smaller than that of rod *1*.

2–27. When assembled, the rigid beam b, Fig. P2–9, was level and there were no stresses in the steel suspension rods *1* and *2*. The weight of the beam is 3 tons. Find the stresses in the round rods under the action of the beam weight, taking E as 30,000,000 psi, (a) if both rods have a diameter d of $\frac{1}{2}$ in., and (b) if $d_1 =$ [illegible] in. and $d_2 = \frac{1}{2}$ in.

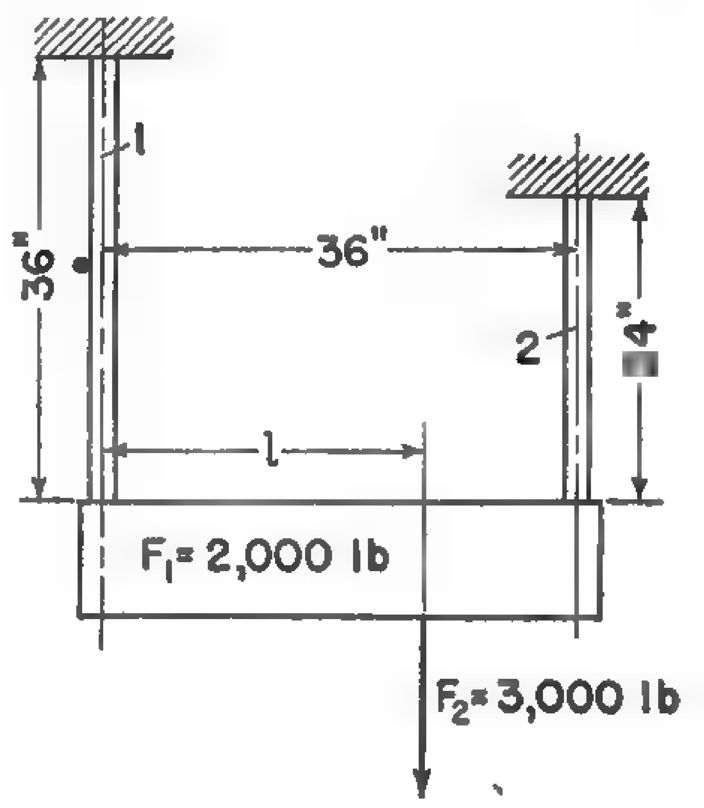


FIG. P2–8.

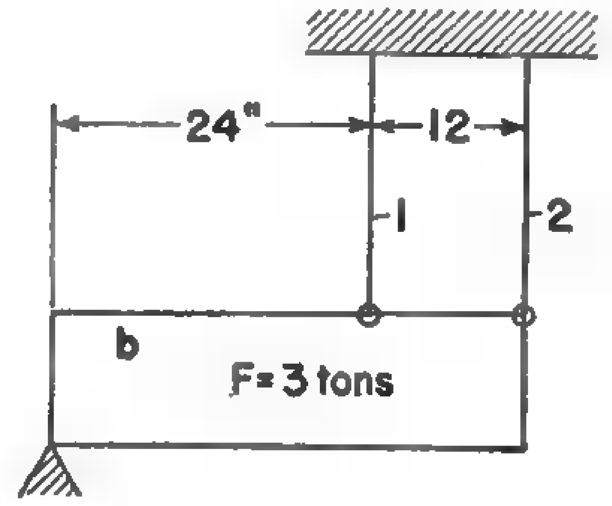


FIG. P2–9.

2–28. When assembled, the rigid beam *b*, Fig. P2–10, was level and there were no stresses in the steel suspension rods *1* and *2*. Find the stresses in the round rods under the action of the force F of 6 tons, assuming that $E = 30{,}000{,}000$ psi, (a) if both rods have the same diameter $d = 1$ in., and (b) if the diameters are $d_1 = \frac{3}{4}$ in. and $d_2 = 1$ in.

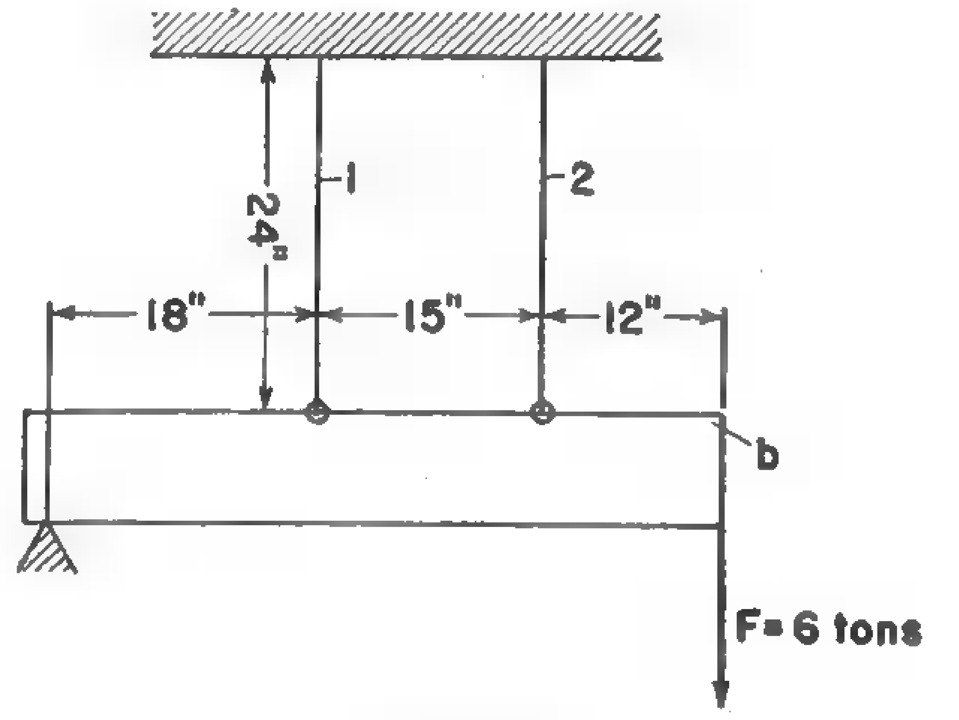


FIG. P2–10.

2–29. It is desired to have the same stress $s_1 = s_2 = 16{,}000$ psi in both rods of Fig. P2–10. Find the required rod diameters, and determine what other changes, if any, must be made in the rods before assembling.

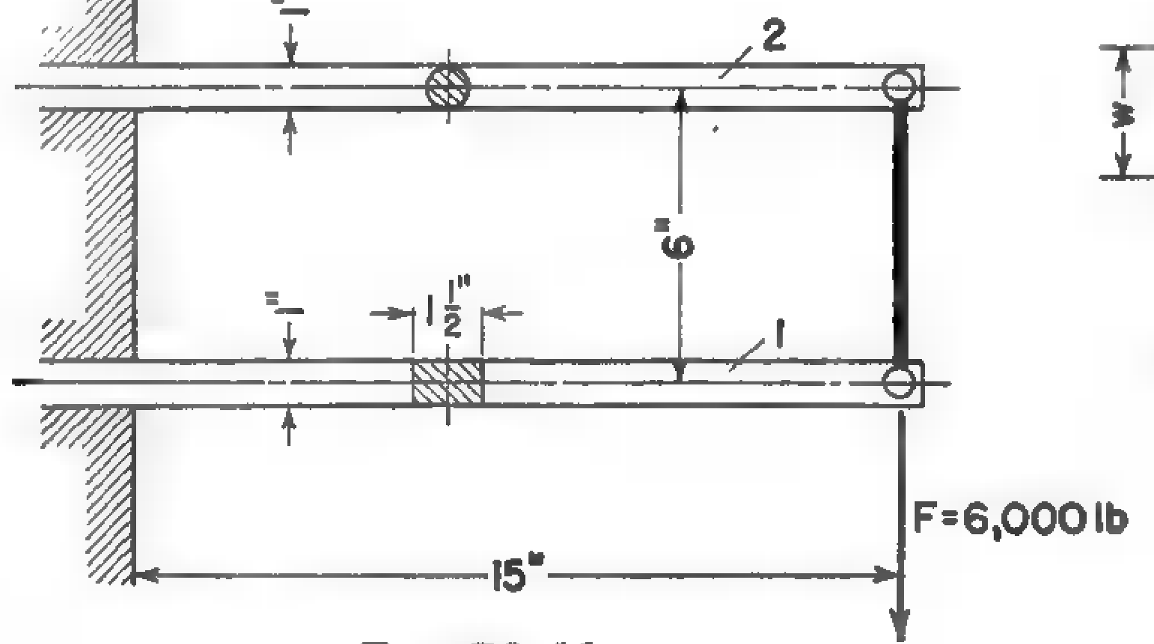


FIG. P2–11.

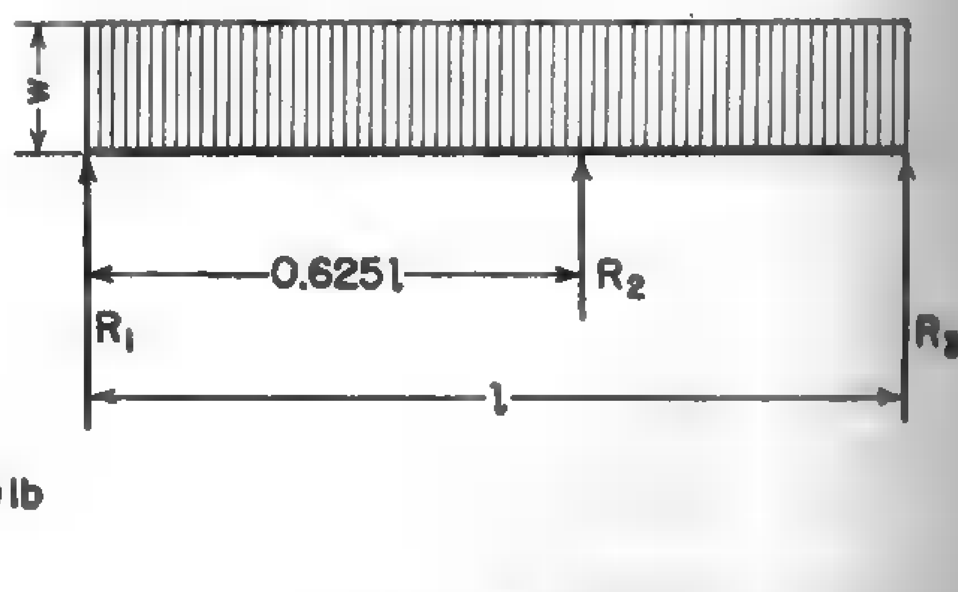


FIG. P2–12.

2–30. A cantilever *1*, Fig. P2–11, made of a $1 \times 1\frac{1}{2}$ in. flat steel bar, when acted upon by a force F of 6000 lb, was subjected to a somewhat excessive deflection. Find the amount by which the deflection will be reduced if the end of the cantilever *1* is tied to the end of a parallel cantilever *2* made of a 1-in. round rod of SAE 41 brass, Table 4–3.

2–31. Find the three reactions for the beam of Fig. P2–12. The uniformly distributed load is w lb per inch of length. All supports are level.

2–32. Because of lowering of the middle support under the beam in Fig. P2–12, the reaction R_2 is found to be $0.5wl$. Find (a) the amount by which the middle support is lower than the other two, and (b) the reactions R_1 and R_3.

2–33. Work problem 2–32 with the following data: An American standard 6 in. × 12.5 lb beam is used, the loading is $w = 1{,}200$ lb per foot of length, and $l = 8$ ft.

2–34. If the beam in problem 2–31 is an American standard 6 in. × 12.5 lb beam, if $w = 1{,}200$ lb per ft, and if $l = 8$ ft, find the maximum stresses (a) at the middle support and (b) and (c) between the supports.

2–35. Work problem 2–34 for the conditions of problem 2–32.

2–36. A force F of 12,000 lb acts simultaneously upon two cranks, as indicated in Fig. P2–13. The longer crank is fastened to one end of a steel bar 4 in. square and 36 in. long whose other end is fixed; the shorter crank is fastened to a round steel bar $2\frac{1}{2}$ in. in diameter, with the other end also fixed. Before the force is applied, the bars and the cranks lie in a horizontal plane. Friction in the joints is negligible. Neglect bending of the bars and the cranks, as the supporting bearings are close to the cranks. Taking G as 12,000,000 psi, determine (a) the stresses in the bars, and (b) the deflection of the ends of the cranks.

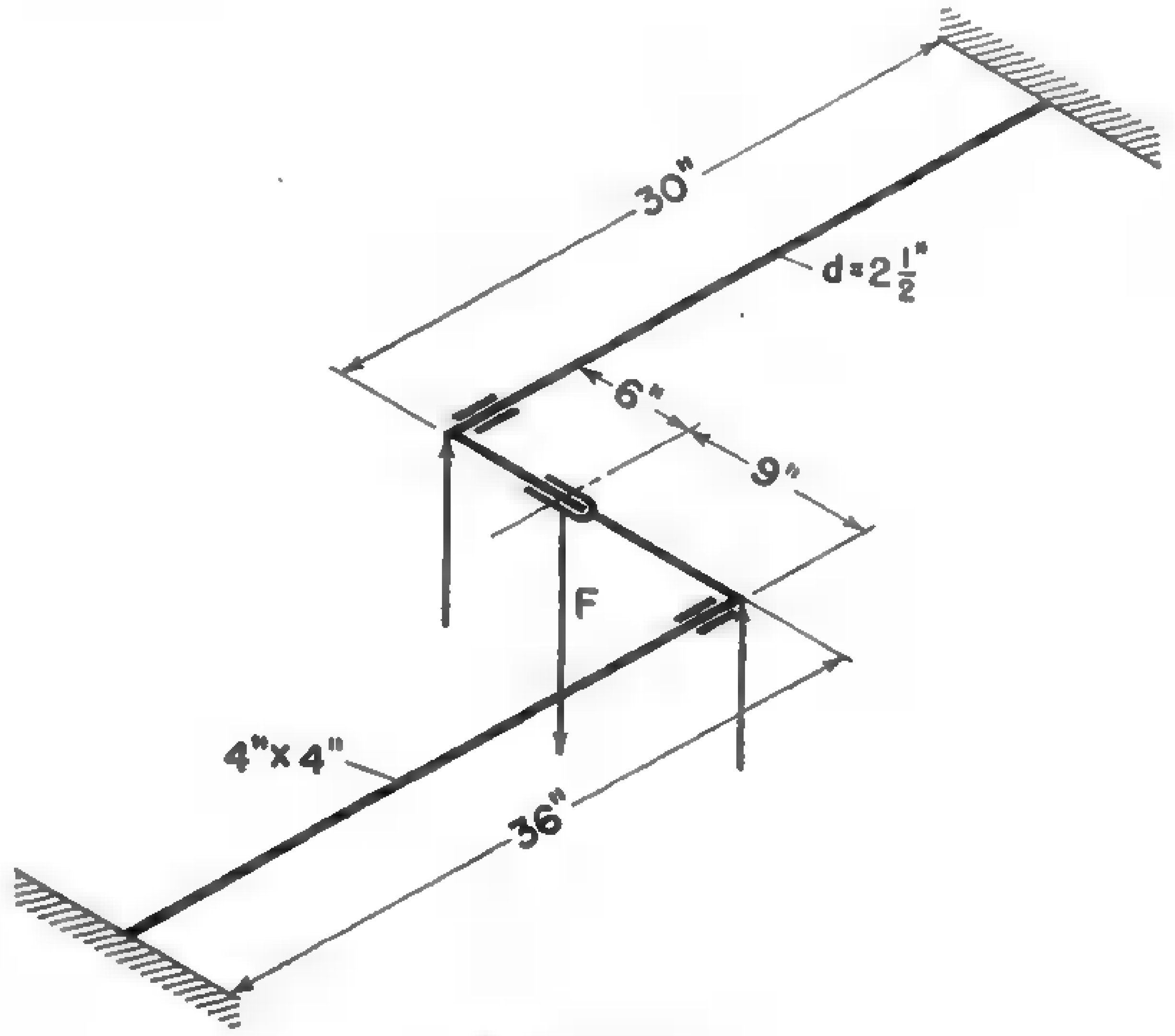


FIG. P2–13.

2–37. A force F of 2,500 lb is taken up in bending by a steel bar a (Fig. P2–14) which is 2 in. square. One end of this bar is fixed in the wall, while the other end is connected by a link b to a crank c fastened to one end of a round steel bar d, 2 in. in diameter. The other end of bar d is also fixed. Without loading, the bars and the crank are horizontal. Friction in the joints is negligible. Also, neglect bending in the bars c and d and stretching of the link b. Using $E = 30{,}000{,}000$ psi and $G = 12{,}000{,}000$ psi, determine (a) the stresses in the members a and d, and (b) the deflection of the end of bar a connected to link b.

2–38. Find the load which can be put on the beam of problems 2–9 and 2–10 if it is used as a column with both ends rounded or hinged. Assume that the elastic limit in compression is $S_e = 75{,}000$ psi. If Euler's formula should be used, consider the breaking load F_u to be 2.5 times as great as the safe load.

2–39. Find the load which a standard 3-in. wrought-iron gas pipe can support when used as a column with both ends screwed in firmly. The length of the pipe between the

supports is 2½ ft. Allow a compressive stress of 13,700 psi, and assume for wrought iron an elastic limit of 24,000 psi and a modulus of elasticity of 27,000,000 psi.

2–40. Find the maximum stress created in a low-carbon-steel bar 1 in. square when used as a column for a load of 5 tons. One end of the bar is fixed, and the other end is free but guided. The length of the bar is 18 in. The steel has an elastic limit of 30,000 psi and a modulus of elasticity of 30,300,000 psi.

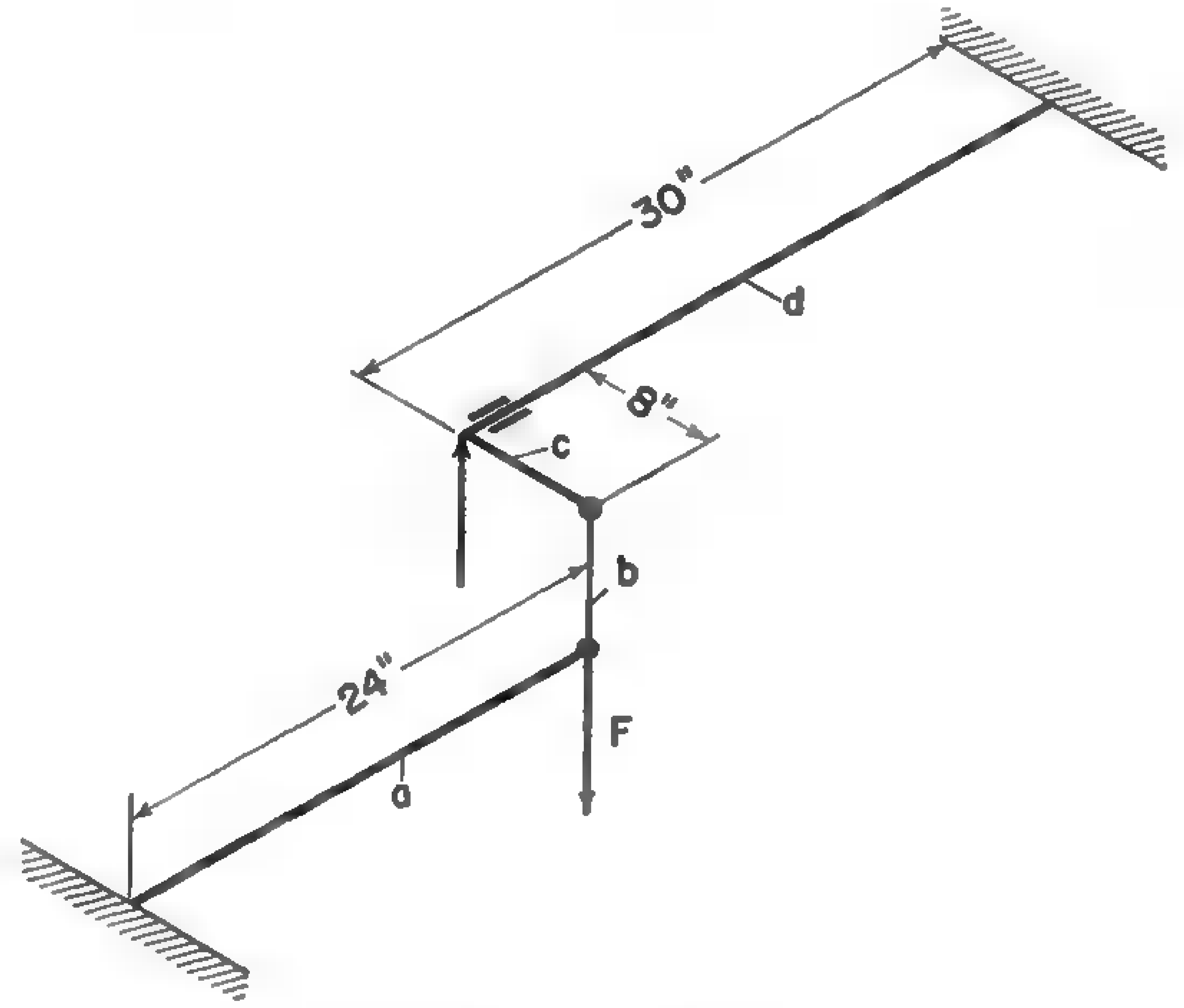


Fig. P2–14.

2–41. Determine the lateral stress in the bar of problem 2–40 in a section about 2 in. from one of the ends.

2–42. (a) Determine the maximum stress and the maximum deflection in a cast-iron simple beam, Fig. P2–15, with a hollow section. The loads are $F_1 = 2{,}000$ lb and $F_2 = 3{,}000$ lb. (b) Show by sketches the moment diagram and the shear diagram.

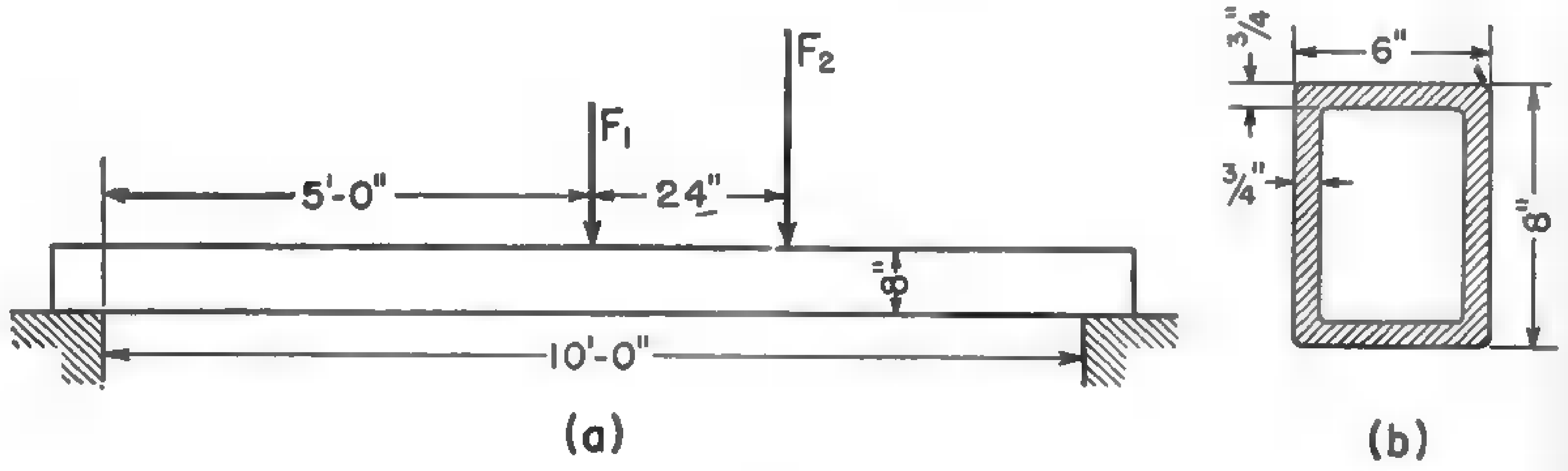


Fig. P2–15.

2–43. The crankshaft in Fig. 2–22 is of medium-carbon steel. If $D = 3\frac{1}{2}$ in., $F = 7{,}200$ lb, $r = 6$ in., and $l = 4\frac{1}{8}$ in., determine (a) the principal normal stresses created in the journal, and (b) the maximum stress, taking into account the induced strain.

2–44. Determine the resultant stress in a short worm shaft which is 2 in. in diameter and transmits 35 hp at 300 rpm. The axial thrust is 11,200 lb, and Poisson's ratio is $\mu = 0.3$.

2–45. Determine the principal stresses and the maximum shear stresses, and the directions of these four stresses, for a point on the vertical side of a steel beam at which there are a horizontal compressive stress of 6,000 psi, a vertical compressive stress of 1,200 psi, and a shear stress of 1,000 psi due to a positive vertical stress.

2–46. Work problem 2–45, assuming that the horizontal stress of 6,000 psi is tension.

2–47. Work problem 2–45, assuming that there is a horizontal tensile stress of 5,000 psi, a vertical tensile stress of 2,400 psi, and a horizontal shear stress of 700 psi.

2–48. Work problem 2–45 graphically by means of the Mohr circle.

2–49. Work problem 2–46 graphically by means of the Mohr circle.

2–50. Work problem 2–47 graphically by means of the Mohr circle.

2–51. At a certain point the maximum principal stress is $s_u = 9{,}800$ psi, and the minimum principal stress is $s_v = 4{,}000$ psi. Using a Mohr circle, find (a) the maximum shear stress, (b) the normal and shear stresses acting at an angle with the principal stresses of 11° counterclockwise, and (c) the same stresses acting at an angle of 22.5°.

2–52. At a certain point the principal stresses are $s_u = 7{,}500$ psi and $s_v = 2{,}100$ psi. Using a Mohr circle, find (a) the maximum shear stress, (b) the normal and shear stresses acting at an angle with the principal stresses of 7.5° clockwise, and (c) the same stresses acting at an angle of 30°.

2–53. At a certain point the principal stresses are $s_u = 6{,}200$ psi and $s_v = 1{,}200$ psi. Using a Mohr circle, find (a) the maximum shear stress, (b) the normal and shear stresses acting at an angle with the principal stresses of 15°, and (c) the same stresses acting at an angle of 40°.

2–54. (a) Determine the maximum stresses due to bending alone in a transmission shaft if its diameter is $2\frac{7}{16}$ in., the length of the shaft between the bearings is 8 ft 6 in., a belt pulley 36 in. in diameter is located 2 ft from one of the bearings, and the belt tension is 510 lb on the pulling side and 235 lb on the slack side. (b) Determine the maximum stresses, taking into account the shear stress from bending.

2–55. At a certain section of a $2\frac{15}{16}$-in. round solid steel shaft there exist a bending moment of 15,000 lb-in. and a torsional moment of 18,000 lb-in. Determine the maximum tensile, compressive, and shear stresses (a) in the outer fibers and (b) in the fibers $\frac{15}{32}$ in. from the surface of the shaft.

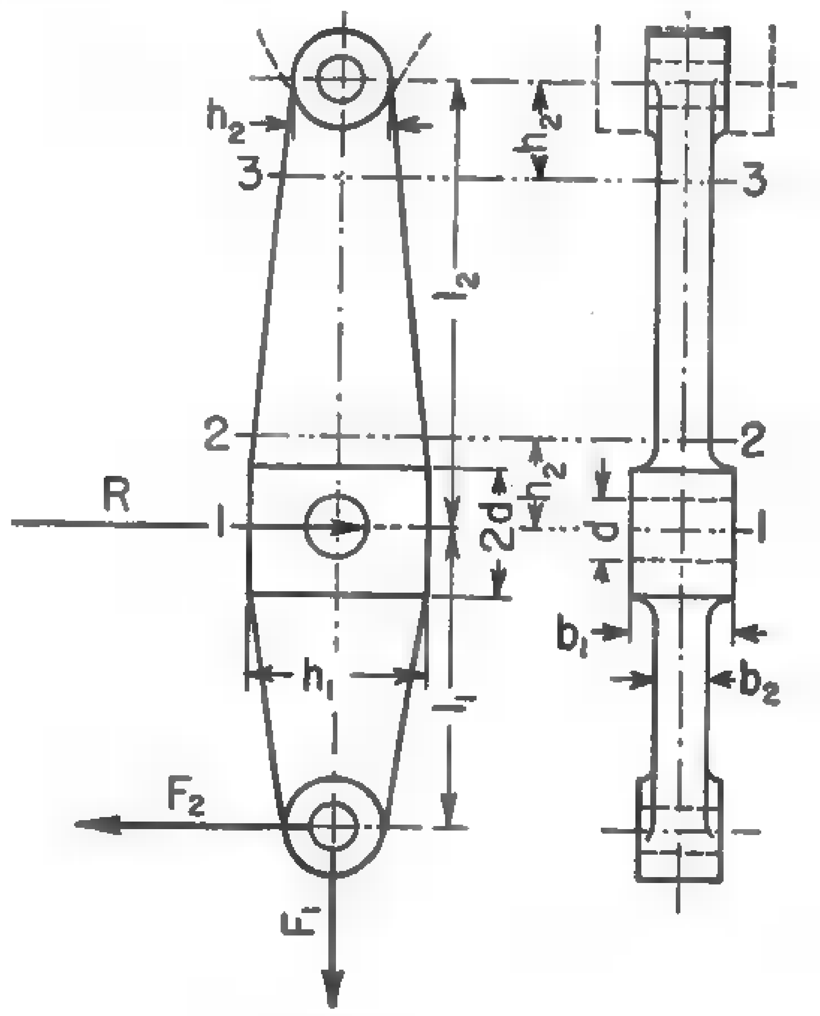


Fig. P2–16

2–56. A simple drop-forged steel lever, Fig. P2–16, is suspended from a fixed pivot and carries a load F_1 of 4,000 lb. The horizontal force necessary to overcome the reaction R is $F_2 = 6{,}500$ lb. Determine the stresses in the outer fibers of sections *1–1*, *2–2*, and *3–3* for the following lever dimensions: $l_1 = 6$ in. and $l_2 = 9$ in. Other dimensions are $d = 1\frac{1}{4}$ in., $b_1 = 2$ in., $b_2 = 1$ in., $h_1 = 3\frac{1}{2}$ in., and $h_2 = 2$ in.

2–57. A malleable cast-iron symmetrical link, Fig. P2–17a, transmits a steady pull F of 10,000 lb. The main dimensions are $b = \frac{3}{4}$ in., $h_1 = 1\frac{7}{8}$ in., and $d = 3$ in. Determine the load which the link could transmit with the same maximum tensile stress if the link

were changed to the unsymmetrical shape shown in dotted lines, the same width b being kept and the height being increased from h_1 to $h_2 = 2\frac{1}{8}$ in.

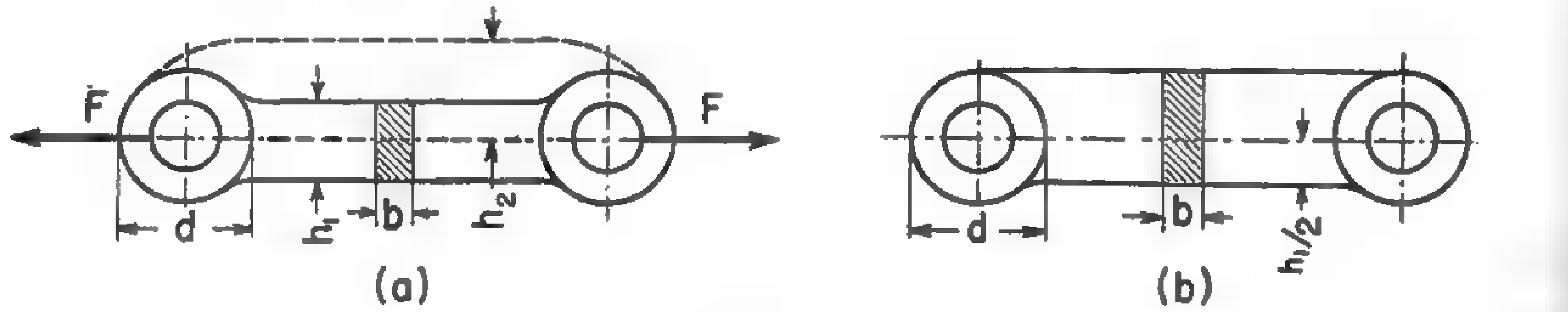


FIG. P2–17.

2–58. (a) Determine the stresses in the outer fibers of the link of problem 2–57 if its upper edge is raised tangent to the bosses, as in Fig. P2–17b. (b) Also find the load F which the link can transmit after the above change in its shape is made, if the maximum tensile stress will be the same as in the symmetrical link in Fig. P2–17a.

2–59. Determine the stresses in the outer fibers, top and bottom, of a cast-iron link, Fig. P2–18a, subjected to a steady pull F of 7,500 lb if the dimension a is 1 in.

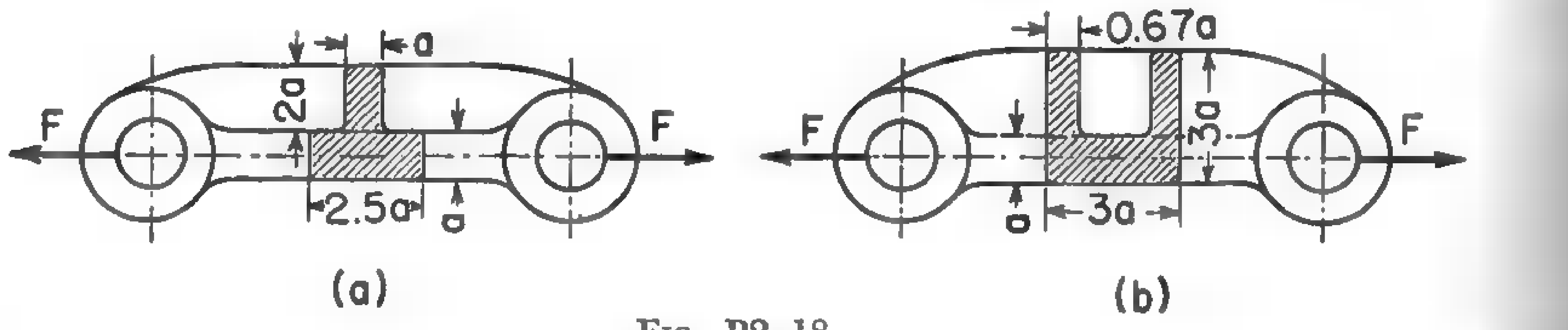


FIG. P2–18.

2–60. Determine the pull F which a cast-iron link, Fig. P2–18b, can transmit so that the tensile and compressive stresses will not exceed 4,000 and 20,000 psi, respectively. The dimension a is 1 in.

2–61. Determine the load which the bar of problem 2–40 can carry with the same maximum stress if the load acts ½ in. off the center line of the bar toward one side.

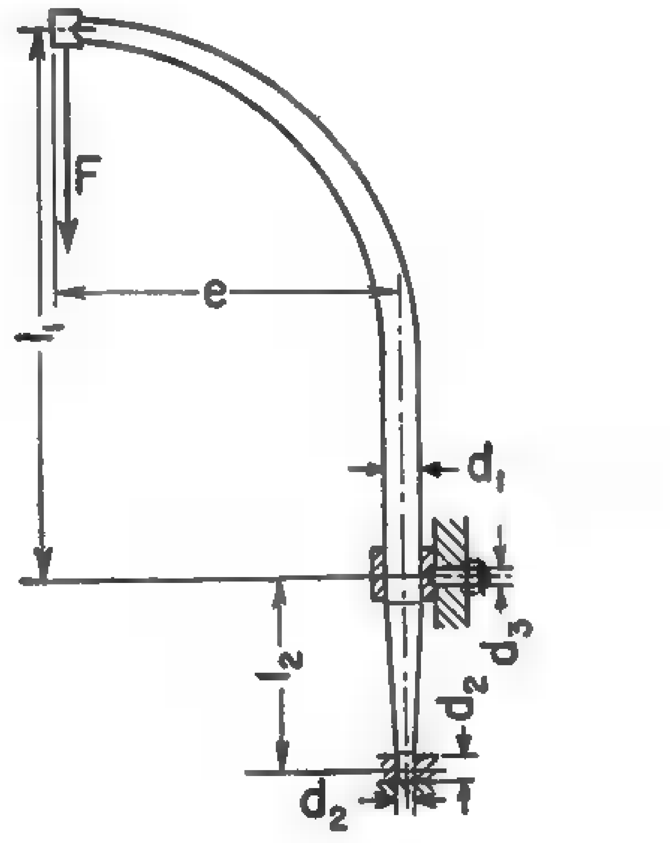


FIG. P2–19.

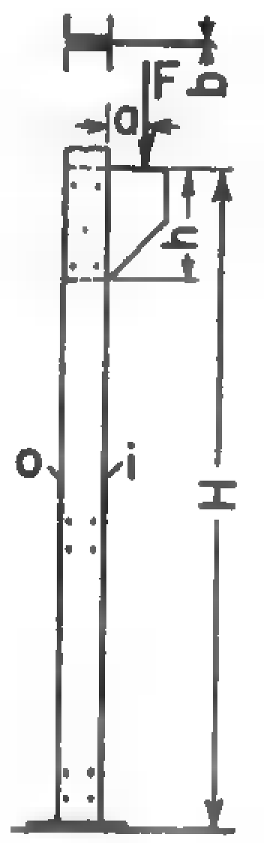


FIG. P2–20.

2–62. A boat davit, Fig. P2–19, carries a load of 6,000 lb. Its dimensions are: $e = 6$ ft, $l_1 = 10$ ft, and $l_2 = 3$ ft 6 in.; the diameters in the bearings are $d_1 = 7$ in. and $d_2 = 3$ in.; and the root diameter of the thread is $d_3 = 2.629$ in. Determine the maximum resultant stress (a) in the steel mast just above the upper bearing, (b) in the section just above the lower bearing, and (c) at the root of the thread of the holding eye bar. Use $S_e = 31{,}000$ psi and $E = 30{,}300{,}000$ psi.

2–63. Determine the maximum stresses in the inner fibers i and outer fibers o of two 5 in. × 6.7 lb American standard channels used as a crane column, as in Fig. P2–20. The load F is 5 tons, and $a = 5$ in., $b = 2$ in., and $H = 12$ ft. Consider the lower end of the column fixed and its upper end guided. Use $E = 30{,}200{,}000$ psi.

2–64. (a) Determine the maximum load F, Fig. P2–20, which can act on the column of a traveling crane formed by two 6 in. × 10.55 lb American standard channels, if $b = 3$ in., $a = 6$ in., and $H = 14$ ft. Allow a maximum tensile or compressive stress of 18,000 psi and a maximum shear stress of 12,000 psi. (b) Determine the maximum resultant stresses in the 1-in. plate supporting the load, considering it welded to the channels, if $h = 8$ in. Use $S_e = 31{,}000$ psi and $E = 30{,}300{,}000$ psi.

2–65. A cast-iron cylinder with an inside diameter of 6 in. and a wall thickness of 1 in. is subjected to an internal pressure of 1,000 psi. Find the hoop stresses along a radius from the bore to the outside, and also the normal radial stress s_r, (a) if there is an outside pressure of 500 psi and (b) if the outside pressure is negligible.

2–66. Work problem 2–65 for a steel cylinder with an inside diameter of ½ in. and an outside diameter of 1 in., if there is an internal pressure of 10,000 psi and an outside pressure of 1,000 psi.

CHAPTER 3: Dynamic Stresses and Stress Concentration

3–1. (a) Determine the stress caused by the centrifugal force in the rim of a cast-iron belt pulley. The pulley has an outside diameter of 24 in., a face of 8 in., and an average rim thickness of $\frac{5}{16}$ in., and it turns at 800 rpm. The specific weight of cast iron is 0.26 lb per cu in. (b) Find the stress if the speed is increased to 1,000 rpm.

3–2. (a) Determine the stress due to centrifugal action in the rim of a cast-iron belt pulley. The face of the pulley is 10 in., the thickness of the rim is ⅞ in., the pulley has an outside diameter of 60 in., and it turns at 240 rpm. Assume the weight of cast iron to be 0.26 lb per cu in. (b) Find the stress if the pulley speed is increased to 400 rpm.

3–3. A locomotive coupling rod connecting three wheels has a rectangular section 1½ in. thick and 3 in. deep, and it is 3½ ft long between each pair of centers. The cranks connected by the rod have a radius of 10½ in. and a maximum speed of 250 rpm. The maximum power developed is 1,200 hp and is applied to the rear axle. Assume that from there the power is distributed evenly between the six wheels. Determine (a) the tensile and compressive stresses created in the rod by the useful efforts, (b) the stresses created by the forces of inertia, and (c) the maximum resultant stress.

3–4. Determine the stress caused by bending by the force of inertia in a steel connecting rod 40 in. long having a round cross section, with a diameter d of 4 in. The piston stroke is 16 in., and the engine speed is $n = 325$ rpm.

3–5. (a) Determine the stress caused by bending by the force of inertia in a steel connecting rod 10½ in. long with an I-shaped cross section. The outside dimensions are 1½ in. and ¾ in., and the thickness of the flanges and of the web is $\frac{3}{16}$ in. The piston stroke is 5.5 in., and the normal speed is 3,200 rpm. (b) Find how much the stress will be increased if the engine speed is raised to 3,600 rpm.

3–6. A steel shaft $3\frac{3}{16}$ in. in diameter is supported on two bearings and has four 200-lb counterweights attached, as shown in Fig. P3–1. The shaft rotates at 300 rpm. Assuming that the supports are at the centers of the bearings, find the bending stress caused by the centrifugal forces.

3–7. Assume that a third bearing is placed at the middle point of the rotating shaft in Fig. P3–1. (a) Find the bearing reactions. (b) Draw a shear diagram and a bending-moment diagram for this shaft. (c) Determine the maximum bending stress.

3–8. A weight w of 300 lb falls through a height h of $\frac{1}{16}$ in. before the load strikes the square head a of a round steel bar ¾ in. in diameter and 18 in. long, Fig. P3–2. (a) Determine the stress induced, assuming a modulus of elasticity of 30,000,000 psi and neglecting the intertia of the bar. (b) Compare it with the stress from the same load acting statically. (c) Find how much the rod length can be changed before the impact stress exceeds 18,000 psi. (d) Determine how much the clearance can be increased before the impact stress

exceeds 18,000 psi. (e) Compare the influence of the inertia of the bar upon the impact stress, using $w = 0.282$ lb per cu in.

3–9. Assume that the square head *a* of the bar in problem 3–8 is replaced by a nut and that a $\frac{3}{4}$-in. UNC thread, Table 11–1, is used. Determine (a) the stress in the stress area of the thread, (b) the stress in the bolt if it is turned down to the minor diameter of the thread on a length of 8 in., and (c) the stress in the bolt if it is turned down on the full length of 18 in.

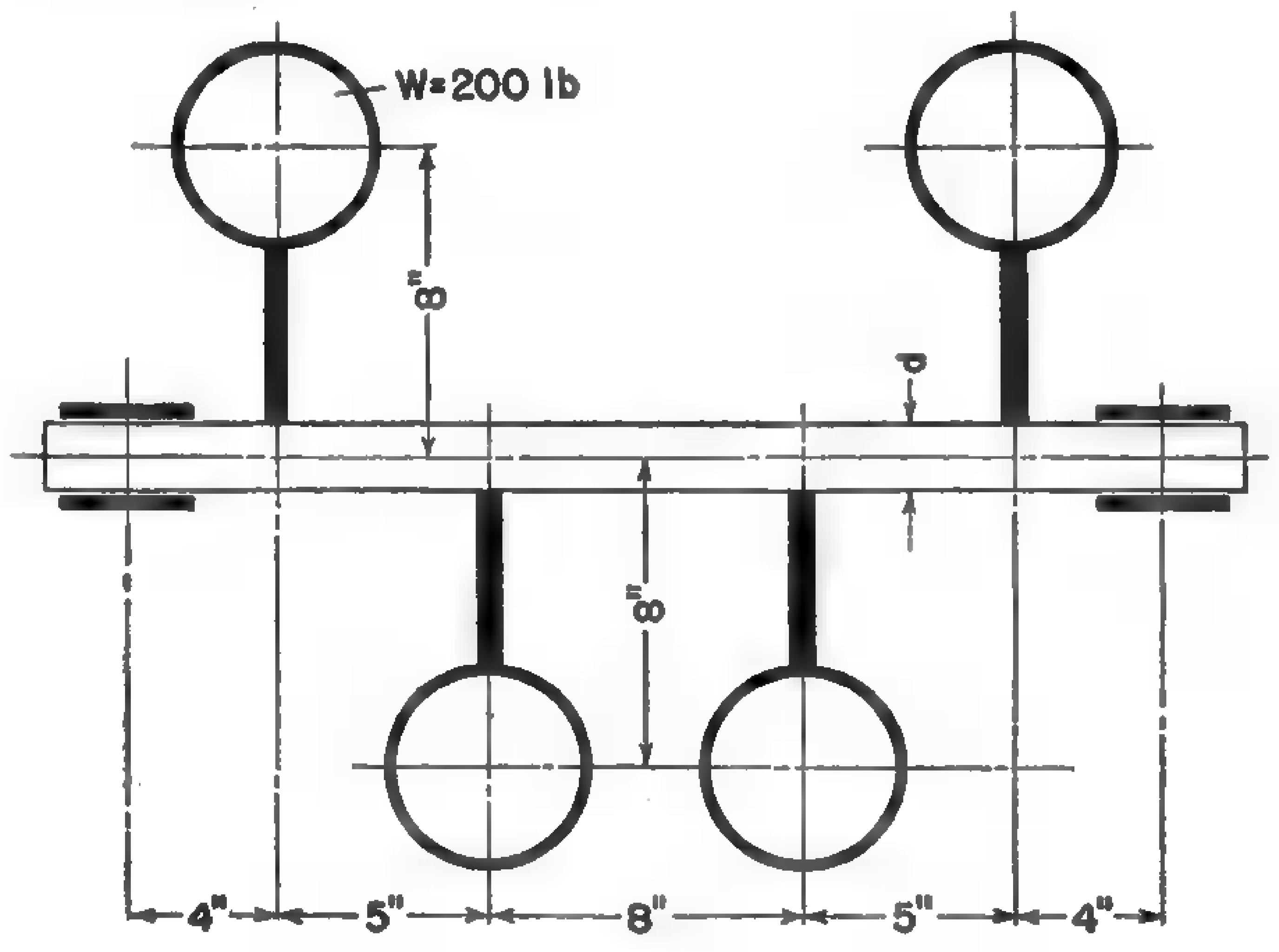


FIG. P3–1.

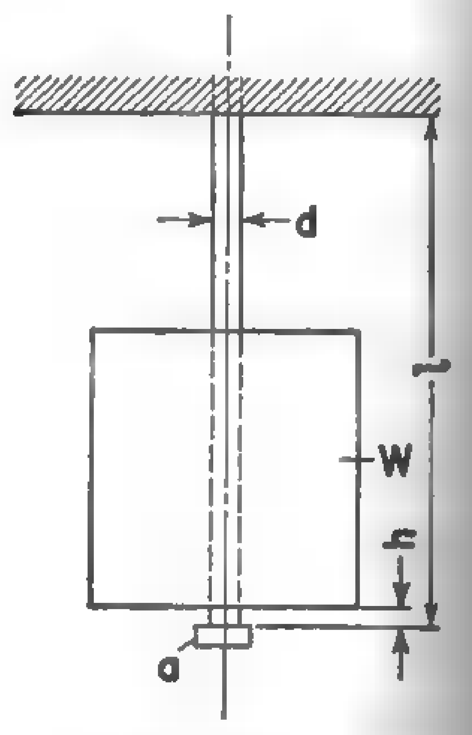


FIG. P3–2.

3–10. (a) Determine the maximum stress set up in, and the maximum deflection in, a simple beam made of a 4 in. × 7.7 lb American standard beam with a span of 8 ft, if a weight of 100 lb is dropped on the beam from a height of 2 in. Use $E = 30{,}300{,}000$ psi, and neglect the inertia of the beam. (b) Determine the stress and the deflection if the weight is placed on the beam suddenly but is not dropped from any height.

3–11. Determine how much the stresses and deflections found in both cases of problem 3–10 will be changed if the inertia of the beam is taken into account. The total beam length is 9 ft, and it extends 6 in. over the point of support on each end.

3–12. Work problem 3–11, assuming that the ends of the beam are fastened by rivets to rigid steel columns.

3–13. Find the compressive stress induced during the explosion in a connecting rod of a 10 × 18 in. natural-gas engine. The compression is 110 psi, gage, the explosion pressure is 300 psi, and the connecting rod is round, with a diameter of 2 in. (a) Consider the pressure increase due to combustion as a sudden load. (b) Compare with the stress induced by a static load of the same value. The length of the rod is 45 in., and the material is SAE 1030 steel.

3–14. Determine the resilience induced in the bar described in problem 2–4. Assume that the length of the stretched part of the bar is 9 in.

3–15. Assume that in example 3–2 in the text the speed of the rope is lowered to 3.4 mph, and the weight of the loaded car is increased to 0.9 ton. Determine the minimum length of the rope between the driving pulley and the point where the car is hooked in, if the maximum stress in the rope should not exceed 25,000 psi.

3–16. Determine the maximum impact load which an 8 in. × 18.41 lb American standard beam can take without receiving a permanent deformation. The span is 6 ft, the ends are clamped, and the load acts in the center. Use $S_e = 30{,}000$ psi and $E = 30{,}300{,}000$ psi.

3–17. Determine the resilience in torsion induced in the shaft of problem 2–6, and the impact energy in torsion which it can withstand if $S_s = 22{,}000$ psi.

3–18. Find the maximum theoretical stress in a cast-iron bar with a rectangular cross section $\frac{7}{8}$ in. thick and 4 in. wide and having a $\frac{1}{2}$-in. hole drilled through the center line of the larger side, when it is subjected to a load in tension of 6,000 lb.

3–19. A steel eye bar, Fig. 3–10, has the following dimensions: The outside diameter is $2\frac{1}{2}$ in., the inner eye diameter is $1\frac{1}{16}$ in., and the cross section of the body is 1 in. square. Determine the theoretical stresses at the points *d* and *l* under a pull of 12,000 lb.

3–20. Determine the theoretical stresses at the edges of rivet holes in the circumferential seam of a cylindrical pressure tank. The outside diameter of the tank is 36 in., the wall thickness is $\frac{1}{2}$ in., the rivet holes are 1 in. in diameter and are spaced $2\frac{3}{4}$ in. apart, and the inside pressure is 160 psi.

3–21. Find the maximum theoretical stress in a $3\frac{15}{16}$-in. shaft under an axial tensile load of 75,000 lb (a) if the shaft contains a radial hole $\frac{3}{16}$ in. in diameter, and (b) if there is a radial hole $\frac{3}{8}$ in. in diameter.

3–22. A $\frac{1}{4} \times 2$ in. flat steel bar with a protrusion $l = 1$ in., Fig. 3–18, is subjected to a tensile load F of 8,000 lb. Determine the maximum theoretical stress (a) if the fillet radius is $r = \frac{1}{16}$ in. and (b) if $r = \frac{1}{8}$ in.

3–23. (a) Find the maximum theoretical stress in a fillet of a cast-iron rib, Fig. 3–17, if $b = \frac{7}{8}$ in., $r = \frac{1}{4}$ in., and $B = 3\frac{1}{2}$ in. The tensile stress in the rib is 2,200 psi. (b) Find the maximum stress if r is increased to $\frac{3}{8}$ in.

3–24. A steel clamp, Fig. P3–3, has the following dimensions: $a = 2\frac{1}{2}$ in., $b = \frac{5}{8}$ in., $h = 2\frac{1}{4}$ in., and $r = \frac{1}{2}$ in. Assuming for both the maximum tensile and maximum compressive stresses a value of 16,000 psi and taking into account the theoretical value of stress concentration in the corners, determine the force F which can be applied by the screw.

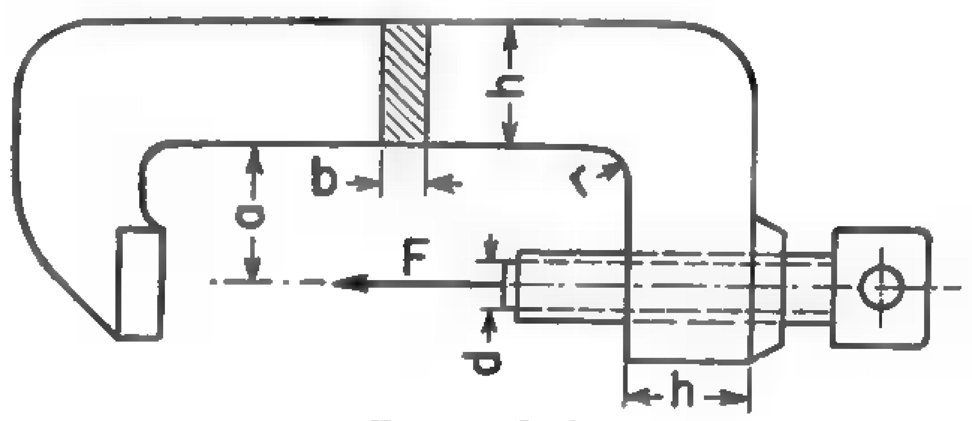


FIG. P3–3.

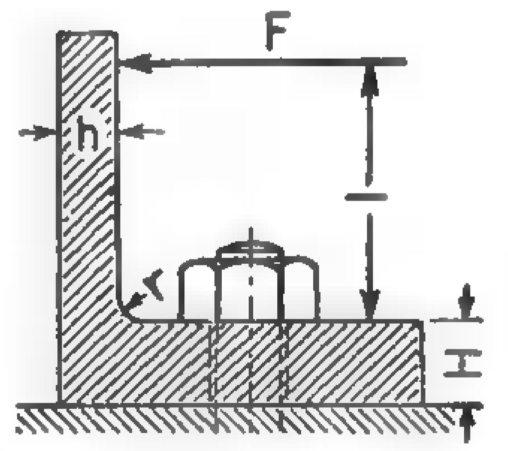


FIG. P3–4.

3–25. (a) Determine the maximum stress in a cast-steel angle bracket, Fig. P3–4, if $F = 1{,}000$ lb, $l = 3$ in., $h = \frac{3}{4}$ in., $H = 3$ in., $r = \frac{1}{4}$ in., and the width of the bracket is $b = 4$ in. (b) Find how much this stress will be changed with $r = \frac{3}{8}$ in.

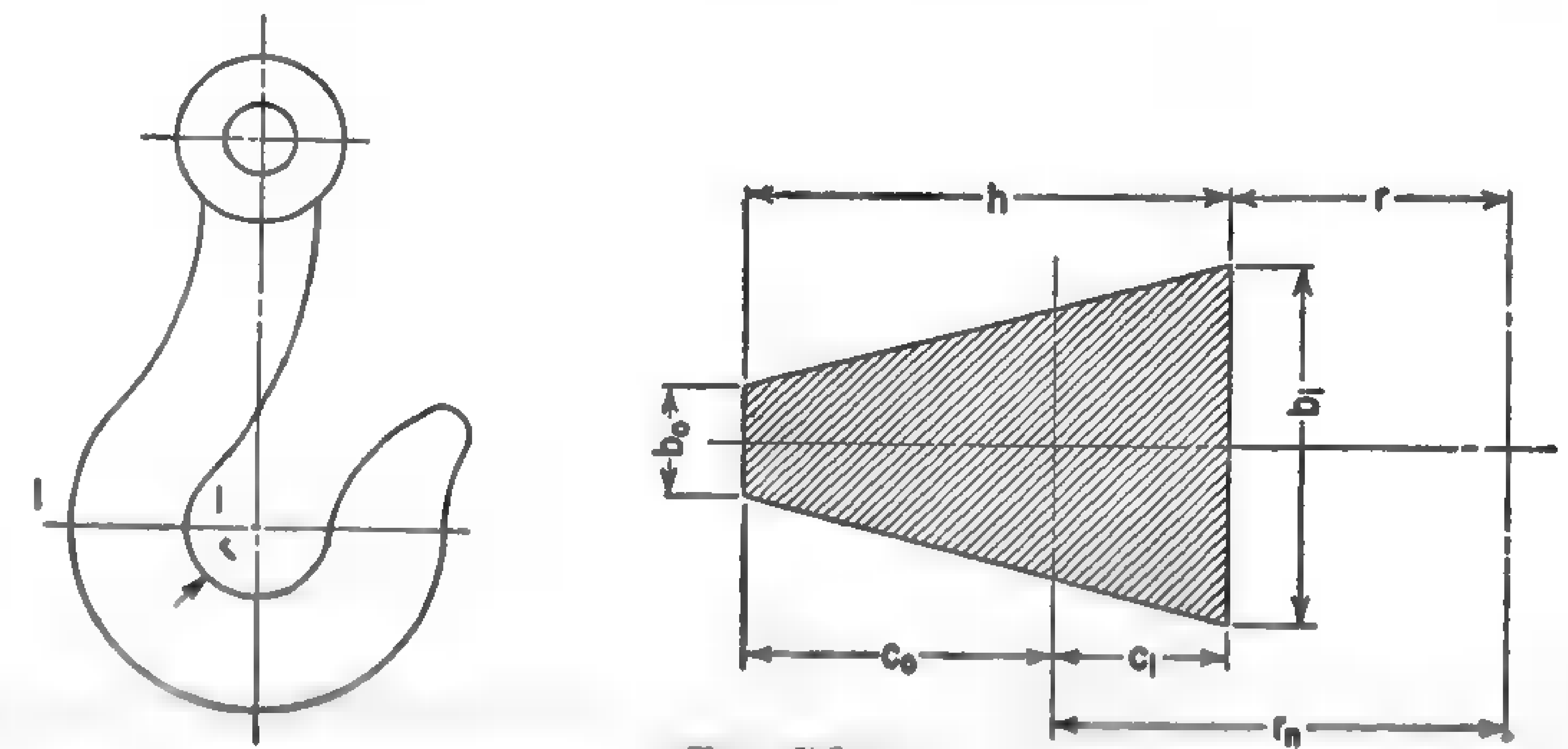


FIG. P3–5.

3–26. Determine the maximum shear stress in a shaft, Fig. 3–31, if $d = 2\frac{7}{16}$ in., $D = 3\frac{7}{16}$ in., and $r = \frac{1}{8}$ in., and if the shaft transmits 70 hp at 240 rpm.

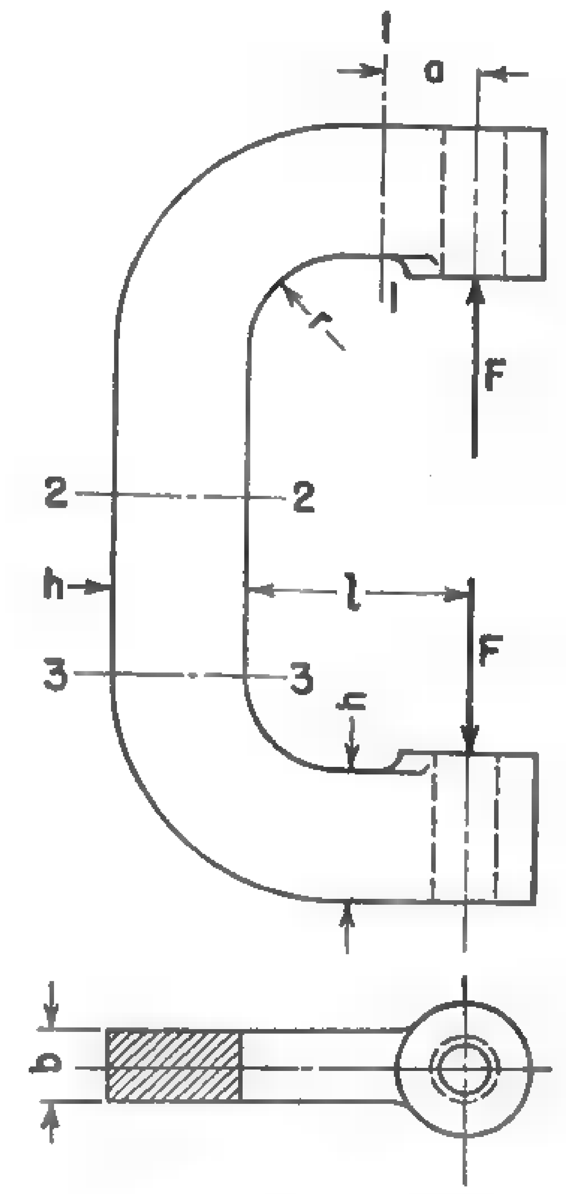


FIG. P3–6.

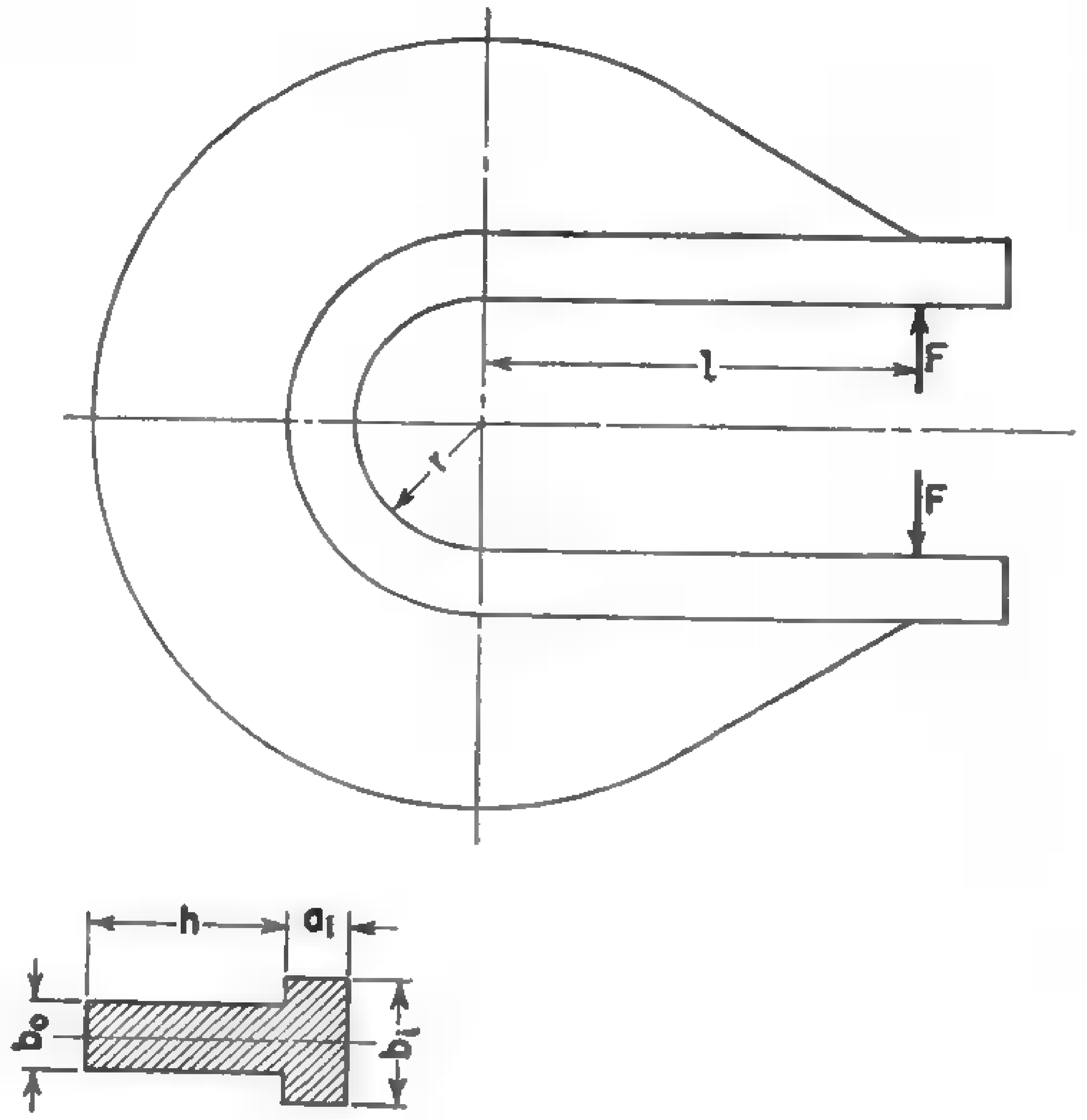


FIG. P3–7.

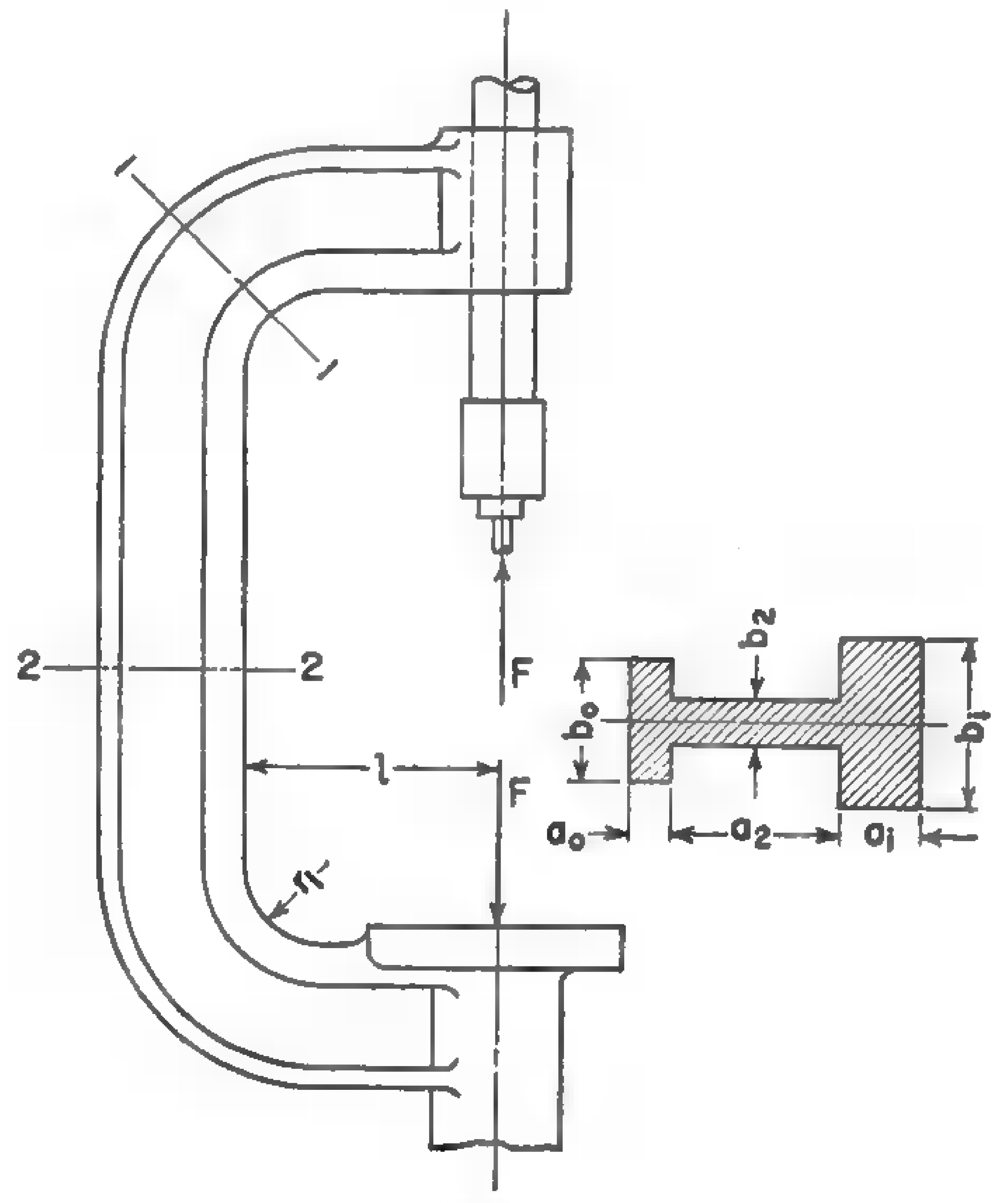


FIG. P3–8.

3–27. The main dimensions of a crane hook are designated in Fig. P3–5. Determine the stresses in the inner and outer fibers in the dangerous section *1–1* for a 10-ton hook for which $r = 2\frac{1}{2}$ in., $h = 4\frac{1}{2}$ in., $b_i = 3\frac{3}{8}$ in., and $b_o = 1$ in.

3–28. Work problem 3–27, but instead of a trapezoidal cross section use a rectangular one with $b = 2\frac{3}{16}$ in., all other data being unchanged.

3–29. In a crane hook similar to that in Fig. P3–5 but having a round cross section with $d = 3\frac{3}{4}$ in., the radius r is $2\frac{1}{4}$ in. (a) Determine the load which, when lifted by the hook, will produce a maximum stress of $s_i = 17{,}500$ psi in the inner fibers. (b) Determine the load that will produce the corresponding stress s_o in the outer fibers.

3–30. The load on the clamp in Fig. P3–6 is $F = 3{,}000$ lb. The dimensions are $a = 2$ in., $b = 1\frac{1}{2}$ in., $h = 3$ in., $r = 2$ in., and $l = 5$ in. Determine the stresses in the inner and outer fibers in the sections *1–1*, *2–2*, and *3–3*.

3–31. (a) Determine the force F which will produce a maximum stress of 10,000 psi in a class 50 cast-iron punch frame, Fig. P3–7, with the following dimensions: $r = 3$ in., $l = 10$ in., $b_i = 3$ in., $b_o = 1\frac{1}{2}$ in., $h = 4\frac{1}{2}$ in., and $a_i = 1\frac{1}{2}$ in. (b) Determine the corresponding compressive stress in the outer fibers.

3–32. A frame of a small drill press has the shape shown in Fig. P3–8. The dimensions of the cross section, with the designations of Table 3–4, type d, are: $r_i = 2$ in., $a_i = 1$ in., $a_2 = 2$ in., $a_o = \frac{1}{2}$ in., $b_i = 2$ in., $b_2 = \frac{1}{2}$ in., $b_o = 1\frac{1}{2}$ in. If $l = 6$ in., determine (a) the force F that will produce a maximum tensile stress of 8,000 psi in section *1–1*, (b) the corresponding compressive stress in section *1–1*, and (c) the stresses in the outer fibers of section *2–2*.

3–33. A ring is made of a 3-in. round bar, Fig. P3–9. The inside diameter $d_i = 4$ in. Calculate the stresses in the inside and outside fibers of section *1–1* under a load F of 5,000 lb.

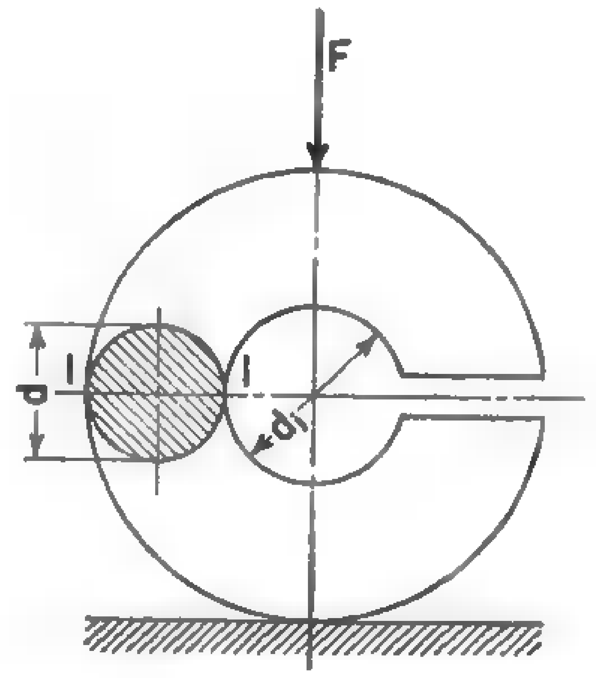


FIG. P3–9.

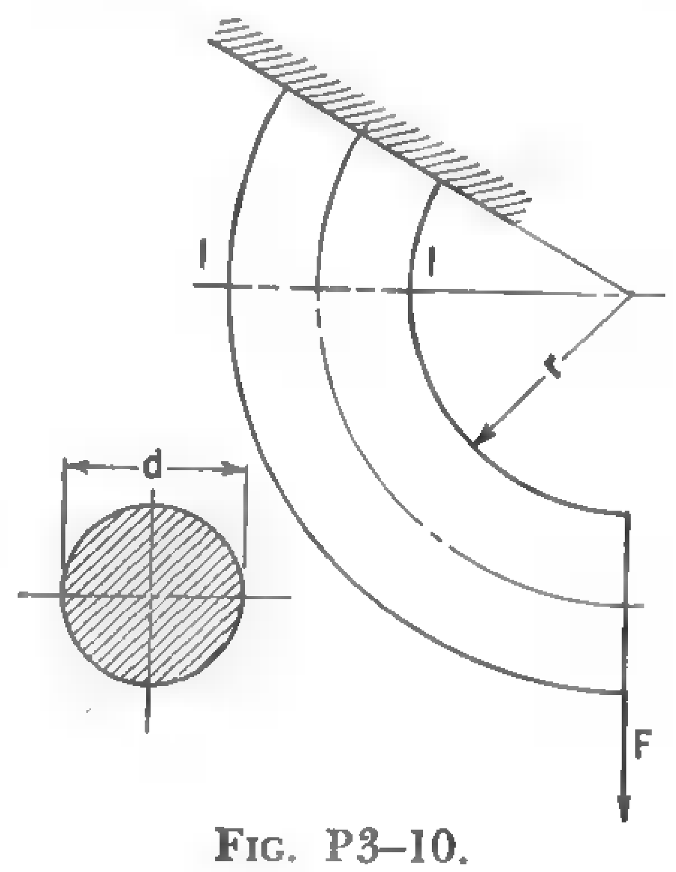


FIG. P3–10.

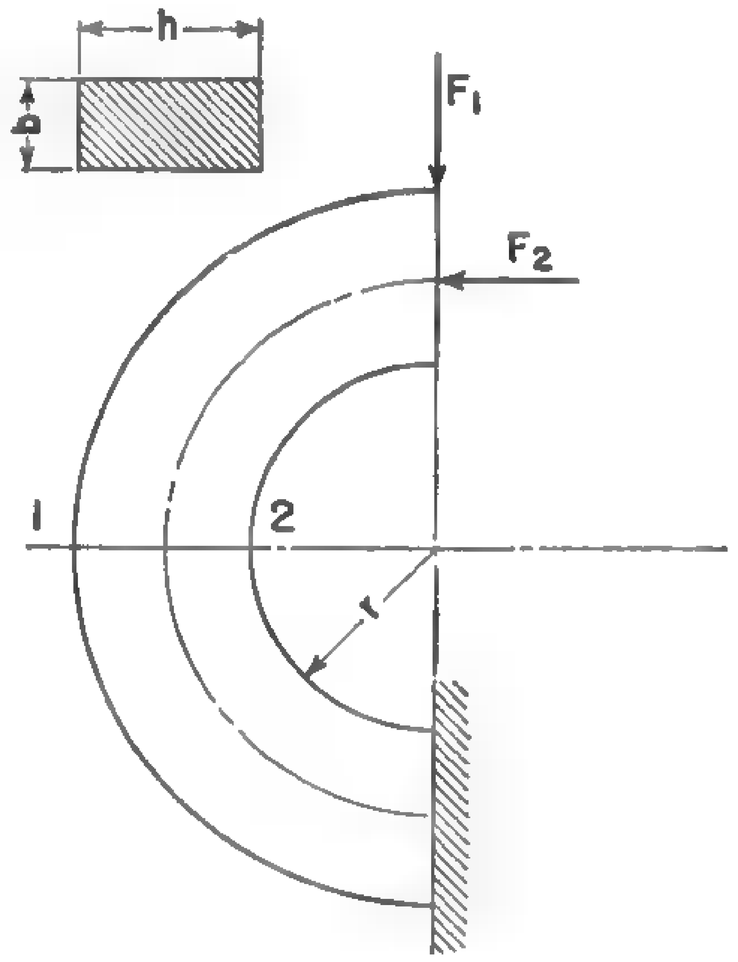


FIG. P3–11.

3–34. A curved member made of a 2-in. round steel bar, Fig. P3–10, carries a load F of 2,500 lb. If the radius of curvature is $r = 1.5$ in., what is the maximum stress in the section *1–1*?

3–35. A curved beam, Fig. P3–11, with the dimensions $h = 4$ in., $b = 2$ in., and $r = 4$ in. is loaded by a vertical force F_1 of 12,000 lb and a horizontal force F_2 of 5,000 lb. Determine the normal stresses in the horizontal section at points *1* and *2*.

3–36. The ultimate strength in tension of a certain steel is 65,000 psi. What are the approximate endurance limits of this material in bending, tension, compression, and shear, if the loads change from a maximum value in one direction to the same value in the opposite direction?

3–37. Determine the approximate endurance limits in tension, compression, shear, and bending of an SAE 68 aluminum bronze, whose ultimate strength in tension is 65,000 psi. Assume that the load changes from a maximum value in one direction to the same value in the opposite direction and that the relations of the various endurance limits are approximately the same as for steels.

CHAPTER 4: Engineering Materials

4–1. Find the Brinell hardness number for a steel sample in which a 10-mm steel ball under the action of 3,000 kg left an indentation 3.30 mm in diameter.

4–2. Find the Brinell hardness number for an aluminum-alloy sample in which a 10-mm steel ball under the action of 500 kg left an indentation 3 mm in diameter.

4–3. Express the hardness of the steel discussed in problem 4–1 in terms of the Rockwell–C number.

4–4. Define ductility, and enumerate some materials which are ductile.

4–5. Define brittleness, and enumerate five materials which are brittle.

4–6. State the reasons for the wide use of cast iron.

4–7. Compare the compositions of cast iron and carbon steel, and state the main differences between them.

4–8. State the advantages obtained through admixing nickel (a) to cast iron and (b) to steel.

4–9. State the main advantage of producing (a) iron castings in an electric furnace and (b) steel in an electric furnace.

4–10. Compare the main points of producing steel by various processes.

4–11. Enumerate with brief definitions the methods of heat-treating cast iron.

4–12. Explain how malleable cast-iron parts are made from white-iron castings.

4–13. Point out the advantages and disadvantages of cold-rolling of steel bars.

4–14. Discuss the influence of carbon content upon the properties of steel.

4–15. Enumerate with brief definitions the methods of heat-treating steels.

4–16. Explain the differences in objectives, procedures, and results between annealing and normalizing.

4–17. Discuss the effects of nickel, chromium, vanadium, tungsten, and molybdenum in steel.

4–18. Discuss the effect of a small quantity of aluminum in steel, and state in what special alloy it is used in a large amount.

4–19. Explain the principal differences between a brass and a bronze.

4–20. Enumerate the various light cast alloys and compare their physical properties with those of semisteel on the basis of the specific weights.

4–21. State what elements are alloyed with aluminum to increase its strength.

4–22. Indicate the approximate ultimate tensile strength, yield point, unit elongation, and hardness of the strongest cast aluminum alloy.

4–23. Answer the questions of problem 4–22 for a wrought aluminum alloy.

4–24. (a) Compute the necessary diameters of round bars made of 1020 steel, heat-treated aluminum alloy 24S, and SAE 41 rolled yellow brass, if each is stressed to its elastic limit by a load of 35 tons. (b) Determine their weights for lengths of 24 in. (c) Determine their relative weights.

4–25. A 4 in. × 7.7 lb American standard beam, 10 ft long and used as a simple beam loaded at the center, supports a load of F lb. Assume that the steel is SAE 1020 and the maximum stress is equal to one-half of the elastic limit. This beam must be replaced by a beam made of heat-treated aluminum alloy SAE 24 having the same depth, flange width, and web thickness. Determine (a) the flange thickness of the aluminum beam, assuming that its maximum stress is also equal to one-half of the yield strength, (b) the relative weight of the aluminum beam, and (c) the relative deflection of the aluminum beam.

4–26. Using the same data as in problem 4–25, and a force F of 1711.5 lb, determine (a) the flange thickness of the aluminum beam, assuming that its deflection is twice the deflection of the steel beam, (b) the relative weight of the aluminum beam, (c) the maximum stress in the aluminum beam and its percentage of the yield point.

4–27. (a) State the uses and advantages of phenolic resin materials. (b) Compare their physical properties with those of SAE 1020 steel on the basis of the specific weights.

4–28. State what materials are most suitable for (a) tension, (b) compression, and (c) repeated stress action, respectively. Give reasons for the selections.

CHAPTER 5: Machine Design Calculations

5–1. A tension member in a roof truss carries an axial static load of 20,000 lb. The member is to be made of flat steel, SAE 1010, $\frac{5}{8}$ in. thick, and is to be riveted to the main girder. Determine the number of $\frac{3}{4}$-in. rivets required and the width of the strip.

5–2. A load F of 10,000 lb is suspended from a plate a, Fig. P5–1, held by bolts *1*, *2*, *3*; the thickness of the plate is $h = \frac{1}{2}$ in.; and $l_1 = 10$ in. and $l_2 = l_3 = 4$ in. The bolts are made of SAE 1120 steel. Determine their size with a safety factor of 1.75. (b) Find the stress in each bolt. (c) Find the stresses in bolts *1* and *3* if bolt *2* is taken out.

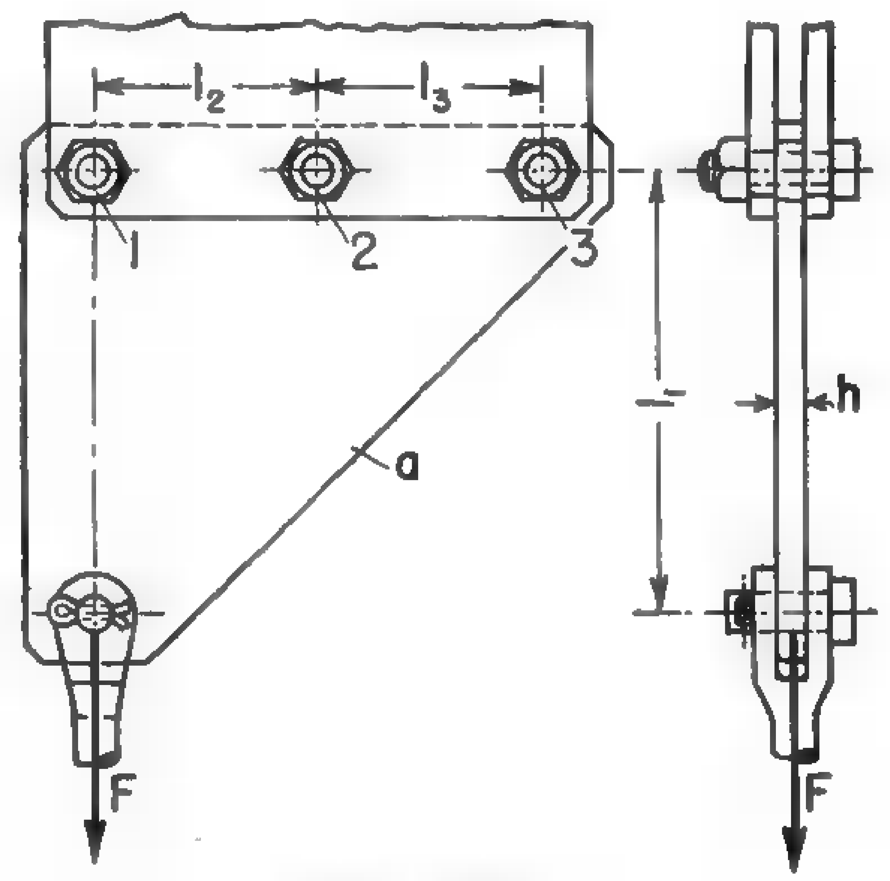


Fig. P5–1.

5–3. A comparatively short round shaft must transmit 150 hp at 120 rpm. Assuming a steady torque and using SAE 1030 steel with a suitable factor of safety, determine the diameter (a) of a solid shaft and (b) of a hollow shaft with the inner diameter one-half of the outer one. (c) Determine the saving of weight in per cent of the solid shaft.

5–4. Find the load which a standard 3-in. steel gas pipe can safely support acting as a column with both ends screwed in firmly. The length of the pipe between the supports is $2\frac{1}{2}$ ft. Use $S_e = 30,000$ psi, $n = 2$, and $E = 30,300,000$ psi, and see Table 11–3 for pipe dimensions.

5–5. Find the size of a square steel bar to act as a column for a load of 10,000 lb. One end of the bar is fixed, and the other is free but guided. The length of the bar is 18 in. The elastic limit is $S_e = 34,000$ psi and $E = 30,200,000$ psi. Use a safety factor n of 1.75.

5–6. (a) Find the size of the bar in problem 5–5 if the load acts $\frac{1}{2}$ in. off the center line of the bar toward one side. (b) Find the size of the bar if the load acts 1 in. off the center.

5–7. At a certain section of a round rotating shaft there are a steady bending moment of 12,000 lb-in. and a steady torsional moment of 18,000 lb-in. Determine the diameter of the shaft if it is made of Tobin bronze, SAE 73. Use a safety factor n of 2 and take Poisson's ratio μ as 0.333.

5–8. A simple drop-forged steel lever, Fig. P2–16, is suspended from a fixed pivot. It carries a weight F_1 of 4,000 lb and is also subjected to a horizontal force F_2 of 6,500 lb. Assuming that the lever and the pin are made of SAE 1040 steel, determine the dimensions h_1, b_1, and b_2. Use the following data: $b_1 = 1.6d$, $l_1 = 6$ in., $l_2 = 9$ in., and $h_1 = 2.5d$ to the nearest $\frac{1}{8}$ in. The diameter d is governed by the strength of the pin in shear.

5–9. A symmetrical link, Fig. P2–17, made of malleable iron transmits a steady pull of 12,000 lb. Using a safety factor of 2, determine (a) the dimensions h_1 and b, with $h_1 = 1.5b$, to the nearest $\frac{1}{8}$ in., and (b) the dimension h_2 after the link is changed to the unsymmetrical shape shown by dotted lines, keeping the width b unchanged and the same safety factor.

5–10. Work problem 5–9, using ductile iron of grade 80–60–05 instead of malleable iron, and determine all dimensions to the nearest $\frac{1}{16}$ in.

5–11. (a) Determine the dimensions of the main cross section of a link, Fig. P2–18a, made of cast iron of class 30 and subjected to a steady pull F of 8,000 lb. Use a safety factor n of 2.5. (b) Check the stress in the fibers furthest from the line of force application.

5–12. (a) Determine the dimensions of the main cross section of a link, Fig. P2–18b, made of ductile iron of grade 60–45–15 and subjected to a steady pull F of 12,000 lb. Use a safety factor n of 1.75. (b) Check the stress in the fibers furthest from the line of force application.

5–13. (a) Determine the width of the flat strip in problem 5–1, assuming that the load is decreased to 10,000 lb but instead of being steady is applied suddenly. (b) Find the number of $\frac{3}{4}$-in. rivets required.

5–14. A passenger electric locomotive has a side-rod drive. The distance between the wheel centers is 72 in. and the crank radius is 16 in. The maximum speed of the locomotive is 90 mph. The force in the coupling rods may be taken as 45,000 lb at starting and 15,000 lb when the locomotive is running at maximum speed. The rods must have a rectangular cross section with $h = 2b$, where h is the height in the plane of rotation. Determine the cross section of the rods, taking into account both the useful efforts and the forces of inertia. Use a steel for which $S_e = 55{,}000$ psi and $S_{en} = 45{,}000$ psi. The diameter of the drive wheels D is 5 ft.

5–15. Work problem 5–14, assuming that the rods will be made of wrought aluminum alloy with $S_e = 20{,}000$ psi and $S_{en} = 15{,}000$ psi.

5–16. Determine the size of an American standard beam used as a cantilever with a free length of 42 in. to take up the impact of a 200-lb weight dropping 6 in.

5–17. A spindle, Fig. P5–2, made of SAE 2320 steel drawn at 1,000 F, is acted upon repeatedly by a force F of 100 lb which each time travels through a clearance h of 0.031 in. The shaft diameter is $D = 2$ in., the spindle diameter is $d = 1\frac{3}{8}$ in., and $l = 4$ in. Using a safety factor n of 1.75, determine the minimum permissible fillet radius r. Assume that the support is rigid.

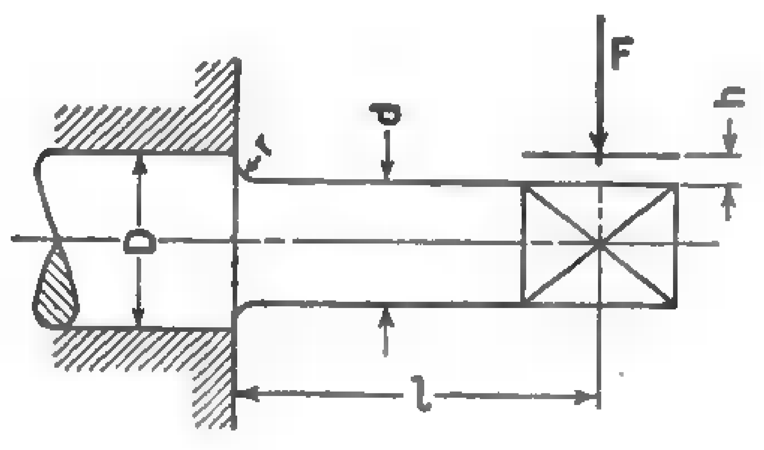


Fig. P5–2.

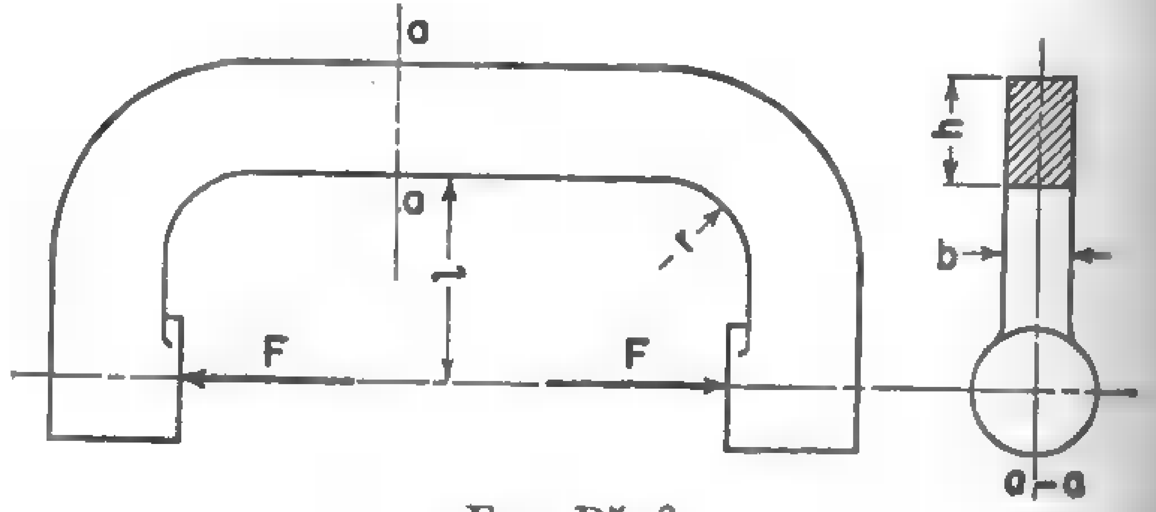


Fig. P5–3.

5–18. The load on the clamp in Fig. P5–3 is $F = 4{,}000$ lb. Assuming that $h = 2b$, $l = 6$ in., and $r = 3$ in., and using a safety factor n of 1.7, determine the necessary dimensions h and b if the clamp is made (a) of cast steel SAE 0022, and (b) of ductile iron of grade 60–45–15.

5–19. A link, Fig. P5–4a, made of ductile iron of grade 60–45–15 transmits a pull F of 14,000 lb. Assuming that $b = \frac{1}{2}$ in. and using a safety factor n of 2, determine (a) the necessary width h of the link, and (b) the thickness b if the same width h is used but the shape of the link is changed, as in Fig. P5–4b, to provide a clearance e of 1 in. when $r = 1$ in.

5–20. Determine the diameter of the shaft of problem 5–3 if the torque is not steady and may be applied suddenly.

5–21. Determine the diameter of the shaft of problem 5–3 if the torque is not steady but constantly fluctuates from the maximum value given in problem 5–3 to one-half of that value acting in the opposite direction.

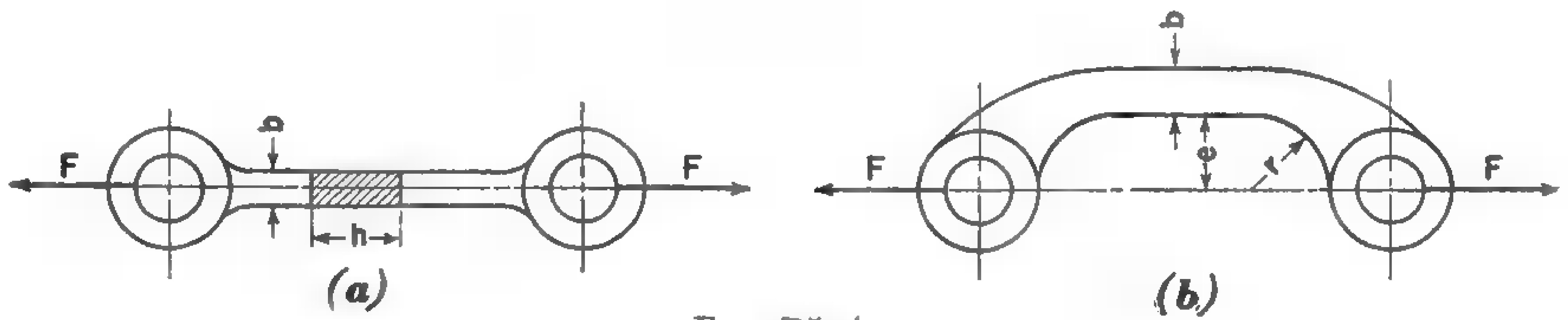


Fig. P5–4.

5–22. A stepped shaft, Fig. P5–2, made of SAE 3125 steel, rotates under a load F of 1,000 lb. If $D = 2$ in. and $l = 5$ in., determine the diameter d and the minimum fillet radius r, assuming a safety margin of 50 per cent.

5–23. A round shaft 11 ft long must transmit an average torque corresponding to 250 hp at 125 rpm. The torque fluctuates continually from a maximum value in one direction to one-half of that value acting in the opposite direction. Assuming that the shaft is of ductile iron of grade 60–45–15, determine (a) the diameter of a solid shaft and (b) the outside diameter of a hollow shaft with the inner diameter one-half of the outer one. Use an auxiliary endurance diagram constructed with data in Table 4–1, and assume a suitable safety margin.

5–24. Determine the main dimensions of the rod of example 5–4 if the impact load is also a repetitive one. Use the endurance diagram of Fig. 4–4.

5–25. Determine the diameter d_3 in Fig. 11–33, using the data of example 11–5.

5–26. Determine the natural frequency of vibration of a standard $2\frac{1}{2} \times 2\frac{1}{2} \times \frac{1}{2}$ in. steel angle laid horizontally on supports 4 ft 6 in. apart. One end is fixed, and the other end is simply supported. The angle is loaded (a) by its own weight only, and (b) by its own weight and a weight of 50 lb halfway between the supports.

5–27. Find the size of two standard steel channels to support a four-cylinder gasoline engine weighing 120 lb and running at 2,500 rpm. The webs of the channels are in vertical planes, the length of the channels between the supports is 36 in., one end of each channel is hinged and the other is supported, and the deflections of the channels must not exceed $\frac{1}{64}$ in. The channels must be safe with respect to resonance and first and second harmonics. The engine is not perfectly balanced and has a crankshaft with the cranks at 180°.

5–28. Determine the natural frequency of torsional vibration of a $2\frac{15}{16}$-in. SAE 1020 steel shaft 6 ft long, without any other parts attached to it.

5–29. (a) Prove that the natural frequency of torsional vibration of a bare solid shaft, without any parts attached to it, depends only on its material and length, the diameter not having any influence. (b) Prove the same phenomenon for a hollow shaft.

5–30. Find the length of a solid steel shaft 3 in. in diameter which will have a natural frequency of torsional vibration of 100 vibr per sec.

5–31. Find the stress set up in a mild-steel bar $\frac{3}{4}$ in. square and 5 in. long which is prevented from expanding when its temperature has increased from 68 F to 212 F.

5–32. (a) Find the stress set up in the cast-iron cylinder liner of a gas engine $\frac{7}{8}$ in. thick, with a mean outside wall temperature of 200 F and an average heat flow of 30,000 Btu per sq ft per hr. (b) Find the stress if the inside temperature goes up 20 deg F without any change in the temperature of the outside efficiently cooled surface. Use $E = 12{,}500{,}000$ psi. (c) Find the stress if the heat flow remains 30,000 Btu per sq ft per hr but the inside wall temperature goes up 20 deg F because of less-efficient cooling.

5–33. (a) Determine the diameter d, Fig. P5–5, of a cold-rolled Monel-metal tie rod a to clamp an aluminum shell b tightly between cast-iron covers c, if $D = 4$ in., $h_1 = \frac{1}{8}$ in., $l = 15$ in., and $h_2 = 1$ in. Assume that the pressure between the aluminum shell and the cast-iron covers must be kept as low as possible, but never less than 2,000 psi. The whole apparatus is kept at a temperature of 120 F, but occasionally its temperature will be lowered to 60 F. The minimum pressure between the shell and the covers must be present at all times without tightening the tie-rod nuts. Use UNF screw threads. (b) Determine

the maximum pressure between aluminum and cast iron which will be produced by the change of temperature. (c) Determine the stresses in the tie rod and the shell for both temperatures.

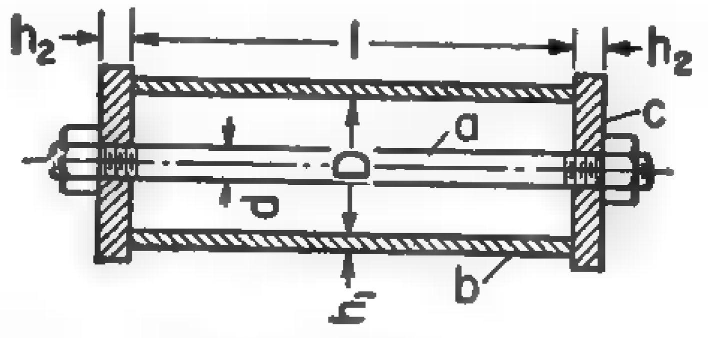


FIG. P5–5.

5–34. (a) Determine the stresses set up in the threads and in the unthreaded part of a 2-in. rod 48 in. long, made of screw stock, when during assembling the rod is heated to about 260 F on a length of 30 in. and then the nut is tightened so as to take up the slack. Room temperature is 70 F. (b) Determine the stress if the nut is turned an additional one-quarter turn after the slack is taken up.

CHAPTER 6: General Manufacturing Considerations

6–1. State the two main groups into which the methods of producing the shapes of various machine parts may be divided.

6–2. Enumerate the methods of preliminary, or rough, shaping of machine parts.

6–3. Enumerate (a) the various finishing operations used in machining surfaces of different machine parts, and (b) additional special machining processes necessary in the production of machine parts.

6–4. Show, by means of sketches, examples of how the cost of manufacturing can be reduced by proper design.

6–5. Show, by means of sketches, examples of how clearances may be provided for the runout of tools.

6–6. Explain what is called *grinding allowance*, and give the range of this allowance for various sizes of machine parts.

6–7. Explain what is called *tolerance*, and also explain why it is necessary to specify tolerances for certain machining operations.

6–8. Compute the basic tolerances, by the ISA formula, (a) for a $2\frac{3}{16}$-in. shaft, (b) for a 12-in. shaft, and (c) for a 36.250-in. cylinder bore.

6–9. Compute the basic tolerances, by the ISA formula, (a) for a $2\frac{15}{16}$-in. shaft, (b) for a 10-in. shaft, and (c) for a 42.000-in. cylinder bore.

6–10. Compute the shop tolerances for the machine parts of problem 6–8, considering that the fit must be (a) very fine, (b) free, and (c) loose.

6–11. Work problem 6–10 for the data in problem 6–9.

6–12. Explain what is termed a *unilateral method* of designating tolerances, and show how it would be applied to problem 6–10.

6–13. Explain what is termed a *bilateral method* of designating tolerances, and show how it would be applied to problem 6–10.

6–14. Explain how it is possible to obtain a greater magnification of the ordinates of a surface profile than of the abscissas.

6–15. Make a sketch, with all necessary dimensions and tolerances, of a $3\frac{3}{16}$-in. shaft 34 in. long. One end of it must be turned down and polished for a running, loose fit $2\frac{11}{16}$ in. in diameter and $4\frac{1}{2}$ in. long; the other end must be turned down and ground for a very fine fit with a nominal diameter of 2.000 in. on a length of 3 in. Indicate the proper surface finishes.

6–16. Work problem 6–15 for a $2\frac{15}{16}$-in. shaft 30 in. long. The end for the loose fit must have a diameter of $2\frac{3}{16}$ in. on a $3\frac{1}{2}$-in. length; the other, or ground. end must have a nominal diameter of 1.9685 in.

6–17. Give an example of a series of preferred numbers for flange couplings, gradually increasing in size from 6 in. to about 36 in. in diameter.

6–18. Work problem 6–17 for couplings ranging from 5 in. to about 50 in. in diameter.

CHAPTER 7: Design of Castings

7–1. Determine suitable cross sections x–x, y–y, and z–z for a class 30 cast-iron rocker arm, Fig. P7–1, assuming that $F = 1{,}200$ lb, $l_1 = 1\frac{3}{4}$ in., $l_2 = 6$ in., $l_3 = 9$ in., and $l = 7$ in.

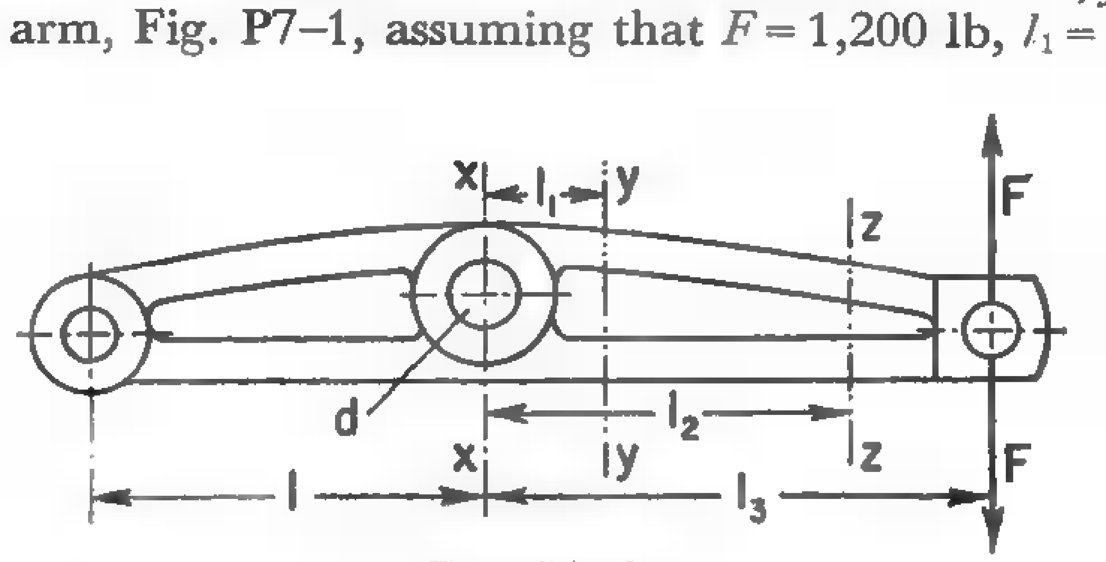


FIG. P7–1.

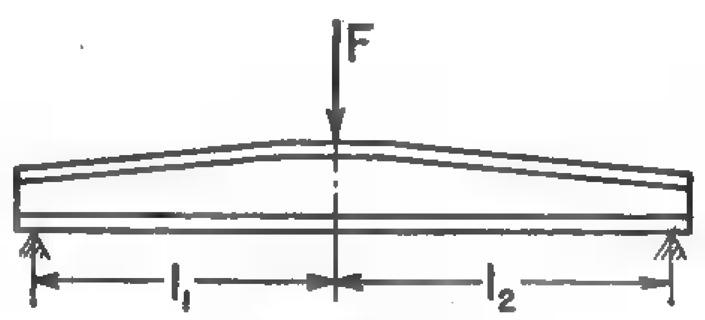


FIG. P7–2.

7–2. Using data of problem 7–1 but assuming that the rocker arm is cast of gun metal, determine suitable cross sections.

7–3. Using the data of problem 7–1 but assuming that the rocker arm is of 0.20 C cast steel, determine suitable cross sections.

7–4. Using the data of problem 7–1 but assuming that the rocker arm is of ordinary malleable iron, determine suitable cross sections.

7–5. Using the data of problem 7–1 but assuming that the rocker arm is made of ductile iron of grade 80–60–05, determine suitable cross sections.

7–6. Determine the proper cross sections and weight W of a cast-iron beam, Fig. P7–2, to carry a dead load F of 10,000 lb, if $l_1 = 3$ ft 8 in. and $l_2 = 4$ ft 4 in. Use class No. 30 cast iron, and design the beam so as to obtain a low weight.

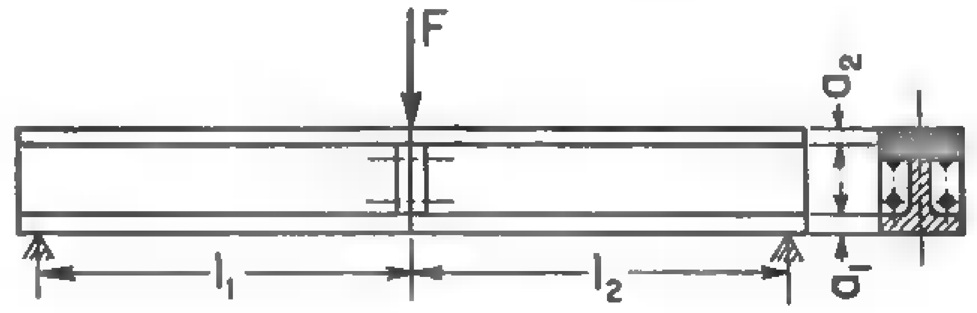


FIG. P7–3.

7–7. (a) Determine the proper cross section and the weight per lineal foot of a cast-iron beam of constant section, Fig. P7–3, to carry a load F of 5 tons. The spans are $l_1 = l_2 = 4$ ft. (b) Determine the number, size, and spacing of the bolts which connect the two halves of the beam. (c) Give the necessary sketches with all dimensions of the center joints, showing in detail the necessary machining surfaces.

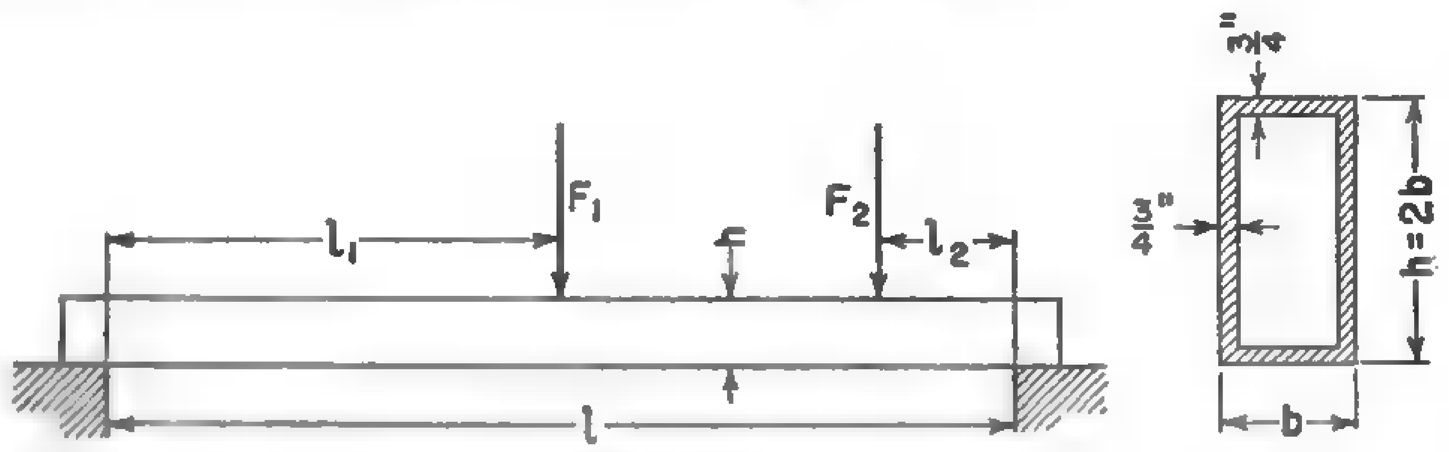


FIG. P7–4.

7–8 Determine the width b and the height h of a hollow beam of class 25 cast iron, Fig. P7–4, assuming that $h = 1.5b$. The span is $l = 10$ ft, $l_1 = 5$ ft, and $l_2 = 3$ ft, and the loads are $F_1 = 1$ ton and $F_2 = 1.5$ tons.

7–9. Work problem 7–8 for $l = 12$ ft, $l_1 = 6$ ft, $l_2 = 4$ ft, $F_1 = 2{,}500$ lb, and $F_2 = 3{,}200$ lb. Make the thickness of the beam walls such as to obtain as light a beam as possible.

7–10. Determine suitable cross sections x–x, y–y, z–z for an aluminum rocker arm, Fig. P7–1, assuming that $l_1 = 1\frac{1}{2}$ in., $l_2 = 3\frac{1}{2}$ in., $l_3 = 4\frac{1}{2}$ in., $l = 4$ in., and $d = \frac{7}{8}$ in., and that the load varies continuously and suddenly from zero to 300 lb. Select the alloy.

CHAPTER 8: Design of Weldments

8–1. (a) Determine the approximate static load which a welded class 1 joint, Fig. 8–5, can safely carry if it connects two $\frac{1}{4}$-in. plates and each fillet is 5 in. long and flush. The upper plate is of SAE 1020 steel and is 5 in. wide, and a shielded arc is used. (b) Find the efficiency of the joint. (c) Compare with results obtained by using the more accurate data of Table 8–3.

8–2. (a) Determine the safe static load for a class 2 welded joint, Fig. 8–5, if it connects two $\frac{1}{4}$-in. plates and each fillet is 3 in. long and concave. The plates are of SAE 1020 steel and 3 in. wide. (b) Find the efficiency of the joint.

8–3. (a) Determine the safe load for a class 3 welded joint, Fig. 8–5, if it connects two strips $\frac{1}{4}$ in. thick and 3 in. wide. The strips are of SAE 1010 steel and the weld is ground flush. (b) Determine the efficiency of the joint.

8–4. Determine the efficiency of the longitudinal seam in Fig. 8–13. The plates are of SAE 1020 steel, $t = \frac{7}{16}$ in., and $t' = 1.15t$.

8–5. Determine the necessary minimum distance c between the welds in Fig. P8–1 for a static load F of 8,000 lb, if $l = 4$ in. and $w = 6$ in. Use $\frac{5}{16}$-in. shielded-arc fillet welds, and assume that the direct shear stress is distributed uniformly over the throat area.

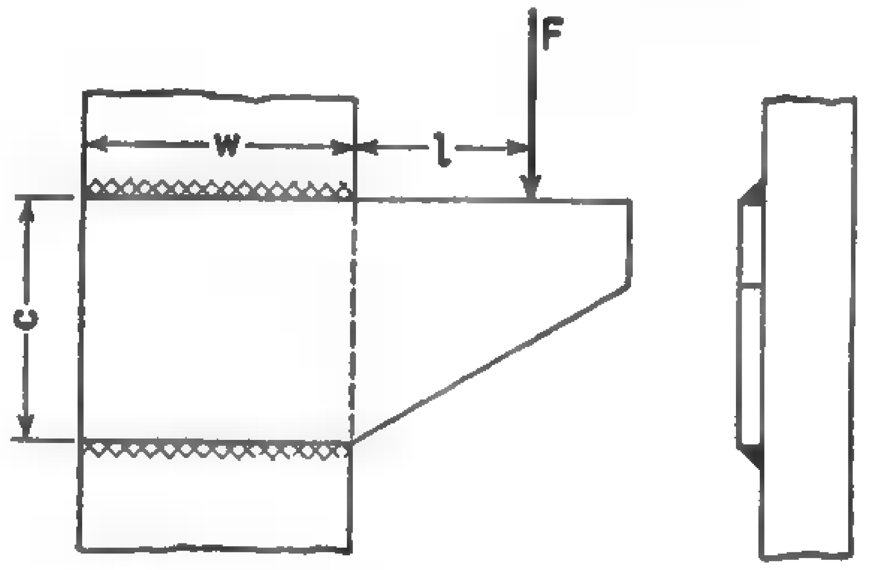


Fig. P8–1.

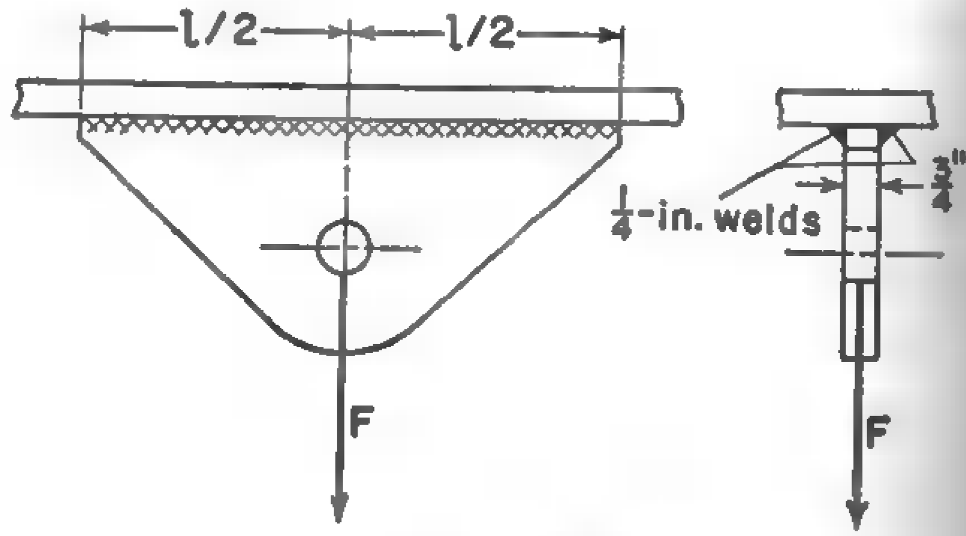


Fig. P8–2.

8–6. Work problem 8–5, assuming that the load F varies continually from zero to 8,000 lb. Increase the weld size to $\frac{3}{8}$ in.

8–7. Work problem 8–5 for the following conditions: $F = 9{,}000$ lb., $l = 9\frac{1}{2}$ in., $w = 7$ in., and the size of the welds is $\frac{3}{8}$ in.

8–8. Work problem 8–7, assuming that the load varies continuously from zero to 9,000 lb. Increase the weld size to $\frac{1}{2}$ in.

8–9. A clip angle is fastened to a column by two $\frac{3}{8}$-in. fillet welds loaded in transverse shear. (a) Determine the length of each weld to support a steady load F of 7,500 lb if a bare electrode is used. (b) Determine the length if a coated electrode is used.

8–10. Work problem 8–9 for the following conditions: $F = 5{,}000$ lb, and the size of the welds is $\frac{5}{16}$ in.

8–11. A load of 30,000 lb is suspended from a vertical $\frac{3}{8}$-in. plate fastened to a beam by two fillet welds so as to form a tee joint. Determine the necessary size and length of each weld, using (a) bare electrodes, (b) coated electrodes, and (c) oxyacetylene-gas welding.

8–12. Work problem 8–11 for a load of 24,000 lb.

8–13. Determine the size of the steel channel in Fig. 8–16b, and determine the dimensions of the welds, for a tensile load F that fluctuates from zero to 40,000 lb, using (a) flush fillet welds and (b) reinforced fillet welds. Assume first-class gas welding.

8–14. Determine the size of the steel angles in Fig. 8–17c in the text, and determine the dimensions of reinforced fillet welds, to transmit a load F that varies from 48,000 lb down to one-fourth of that value. Use (a) shielded-arc welds, (b) bare-arc welds, and (c) first-class gas welds.

8–15. Determine the size of the split H beam in Fig. 8–16c of the text, and determine the dimensions of flush welds, for a tensile load F of 68,000 lb. Use shielded-arc welds.

8–16. Determine the size of the angle in Fig. 8–17a in the text, and determine the dimensions of reinforced fillet welds to transmit a steady load F of 60,000 lb. Use (a) shielded-arc welds and (b) oxyacetylene welding. Assume good workmanship.

8–17. Find the permissible static load F for the connection of Fig. P8–2, if $l = 12$ in. and shielded-arc welds are used.

8–18. Work problem 8–17 for $l = 10$ in.

8–19. Work problem 8–17 for a load fluctuating from zero to a maximum.

8–20. Find the permissible static load F for the welded ear, Fig. P8–3, if $l = 10$ in. and $e = 2$ in. Use (a) bare electrodes; (b) coated electrodes.

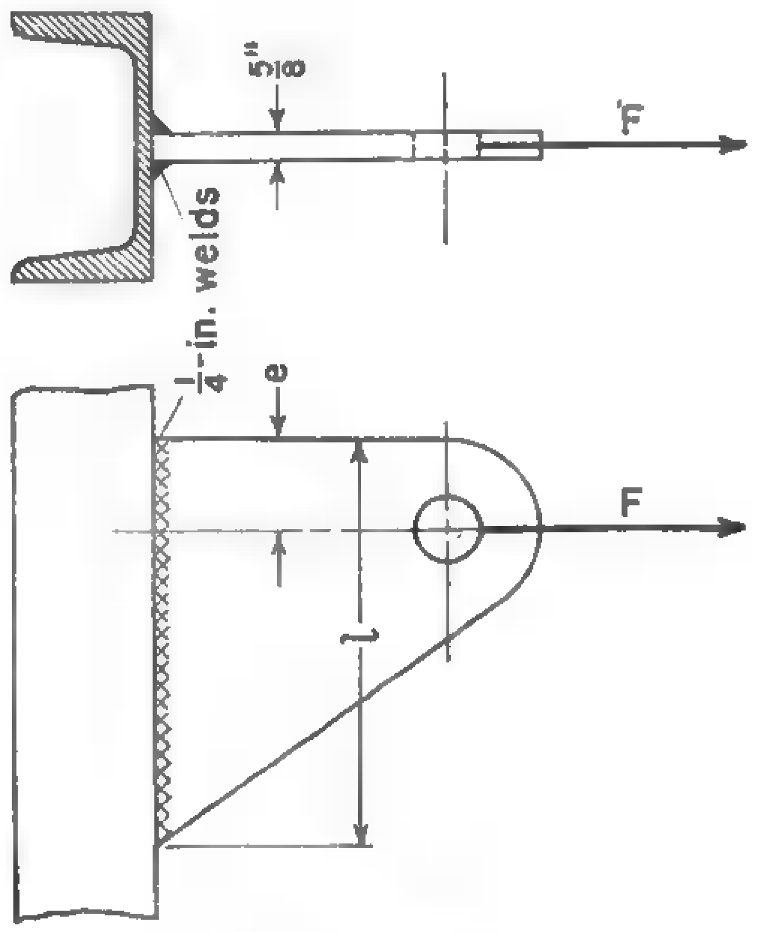


Fig. P8–3.

8–21. Work problem 8–20 for $l = 8$ in. and $e = 2$ in.

8–22. Work problem 8–20 for a load that fluctuates from zero to a maximum.

8–23. Determine the size of shielded-arc welds to transmit the full torque capacity of a $2\frac{15}{16}$-in. mild-steel shaft by a flange coupling (a) of the type in Fig. 8–26a, and (b) of the type in Fig. 8–26b. The torque is steady.

8–24. Work problem 8–23 for a $3\frac{15}{16}$-in. shaft.

8–25. Determine the size of shielded-arc welds to transmit the full torque capacity of a $2\frac{15}{16}$-in. mild-steel shaft by a flange like that in Fig. 8–27 in the text. The torque is steady.

8–26. Work problem 8–25 for a $3\frac{15}{16}$-in. shaft.

8–27. Work problem 8–25, assuming that the torque fluctuates from the maximum value to one-half of the maximum value.

8–28. Work problem 8–26, assuming that the torque fluctuates from the maximum value to one-half of it.

8–29. Design a welded turnbuckle to replace the drop-forged one in Fig. 11–19a, for a pull of 20,000 lb.

8–30. Design a welded clevis to replace the drop-forged one in Fig. 11–19b, for a pull of 12,000 lb.

8–31. Work problem 8–30 for a pull of 25,000 lb.

CHAPTER 9: Design of Riveted Constructions

9–1. The girth seam of a horizontal cylindrical boiler is of the single-riveted lap-joint type. The thickness of the shell is $\frac{3}{8}$ in., the diameter of the rivets is $\frac{7}{8}$ in., the pitch is 2 in., and the margin is $1\frac{7}{16}$ in. Using the allowable stresses recommended by the ASME Boiler Code, determine the manner in which this joint can fail, and its efficiency.

9–2. The longitudinal seam of an air-pressure tank is of the double-riveted lap-joint type. The shell is $\frac{1}{2}$ in. thick, the nominal diameter of the rivets is 1 in., the pitch is $3\frac{3}{8}$ in., the transverse pitch is $2\frac{1}{8}$ in., and the margin is $1\frac{5}{8}$ in. Determine the manner in which this joint can fail, and its efficiency, using the allowable stresses recommended by the ASME Boiler Code.

9–3. The longitudinal seam of a cylindrical boiler drum is of the double-riveted butt-joint type. The shell is $\frac{1}{2}$ in. thick, the nominal rivet diameter is $\frac{7}{8}$ in., the pitch is 5 in., the short pitch is $2\frac{1}{2}$ in., the transverse pitch is $2\frac{3}{4}$ in., and all margins are $1\frac{7}{16}$ in. The thickness of the inner, wide cover plate is $\frac{3}{8}$ in., and that of the narrow cover plate is $\frac{1}{2}$ in. Determine the manner of possible failure and the efficiency of this joint, using the ASME Boiler Code allowable stresses.

9–4. Determine the manner of the possible failure and the efficiency of a triple-riveted butt joint, Fig. 9–10 in the text, having the following dimensions: $h = \frac{3}{4}$ in., the thickness of both cover plates is $h_1 = h_2 = \frac{1}{2}$ in., $d = 1\frac{1}{8}$ in., $p = 8\frac{1}{4}$ in., the inner transverse pitch is $p_{t1} = 2\frac{3}{8}$ in., and the outer pitch is $p_{t2} = 3\frac{1}{4}$ in., and each margin is $m = 1\frac{13}{16}$ in. Use allowable stresses as given by the ASME Boiler Code.

9–5. Determine the manner of possible failure and the efficiency of a quadruple-riveted butt joint having the following dimensions: $h = 1$ in., the thickness of the cover plates is $h_2 = \frac{3}{4}$ in., $d = 1\frac{3}{8}$ in., $p = 19$ in., the middle pitch is $p_2 = 9\frac{1}{2}$ in., and the pitch of the two inner rows is $4\frac{3}{4}$ in.; also, the inner transverse pitch is $2\frac{7}{8}$ in., the next one is $3\frac{3}{4}$ in., and the outer one is $4\frac{1}{4}$ in.; all margins are $2\frac{3}{16}$ in., and the width of the inner cover plate is $30\frac{1}{2}$ in., and that of the outer one is 22 in. Use ASME Boiler Code design stresses.

9–6. An air tank 42 in. in diameter has a double-riveted, lap-joint longitudinal seam. The thickness of the shell is $\frac{9}{16}$ in., the rivet-hole diameter is $1\frac{1}{16}$ in., $p = 3\frac{1}{4}$ in., $p_t = 2\frac{1}{8}$ in., and $m = 1\frac{5}{8}$ in. Determine the safe air pressure and the efficiency of the joint, so that the working stresses in tension, shear, and crushing shall not exceed the ASME Boiler Code values of 11,000, 8,800, and 19,000 psi, respectively.

9–7. A compressed-gas tank 36 in. in diameter has a single-riveted, lap-joint girth seam whose main dimensions are as follows: $h = \frac{5}{16}$ in., $d = \frac{3}{4}$ in., $p = 2$ in., and $m = 1\frac{1}{4}$ in. Determine the safe gas pressure and the efficiency of the joint, using SAE 1010 steel for rivets, SAE 1020 steel for plates, and a safety factor of 3.

9–8. Determine the safe gas pressure and the efficiency of the joint if the shell of the tank in problem 9–7 is made of Alcoa 24S aluminum alloy and the rivets are of Alcoa 17S aluminum and are hot-driven. Determine what changes should be made in the pitch and margin to obtain a maximum safe pressure. Also find the new joint efficiency.

9–9. A pressure tank 60 in. in diameter for hot salt water has a double-riveted, butt-joint longitudinal seam. The thickness of the shell is $h = \frac{3}{8}$ in., that of the straps is $h_1 = h_2 = \frac{5}{16}$ in., the rivet-hole diameter is $\frac{13}{16}$ in., the long pitch is $p = 4\frac{1}{2}$ in., the short pitch is $p_2 = 2\frac{1}{4}$ in., $p_t = 2\frac{7}{16}$ in., and $m = 1\frac{1}{4}$ in. Determine the safe working pressure and the efficiency of the joint, using hot-rolled Monel metal for both the shell and the rivets. Also find the widths of the wide and narrow straps.

9–10. Design a single-riveted lap joint for the girth seam of a pressure tank 36 in. in diameter and subjected to an internal pressure of 150 psi. Use design stresses given in the ASME Boiler Code. Also find the efficiency of the joint.

9–11. Design a double-riveted lap joint for the longitudinal seam of a pressure vessel 24 in. in diameter and with an internal pressure of 180 psi. Use SAE 1010 for rivets and SAE 1020 for the shell plate and straps. Take a safety factor of 2.5.

9–12. Design a double-riveted butt joint for the longitudinal seam of a steam-boiler drum 36 in. in diameter and sustaining a steam pressure of 180 psi. Use allowable stresses given in the ASME Boiler Code. Determine the efficiency of the joint.

9–13. Design a triple-riveted butt joint for the longitudinal seam of a steam-boiler drum 40 in. in diameter and sustaining a steam pressure of 300 psi. Use allowable stresses given in the ASME Boiler Code. Also determine the efficiency of the joint.

9–14. The beam or jib supporting the head sheave of a crane consists of two 10 in. × 15.3 lb channels, Fig. P9–1. Determine the thickness h of the square reinforcing steel plates supporting the stationary sheave pin, the number of rivets to fasten them, and the dimension b. Use the maximum diameter of the rivets recommended by the steel manufacturers. The load is $Q = F = 9$ tons, and the sheave-pin diameter is $3\frac{1}{2}$ in. Use a safety factor of 2.5.

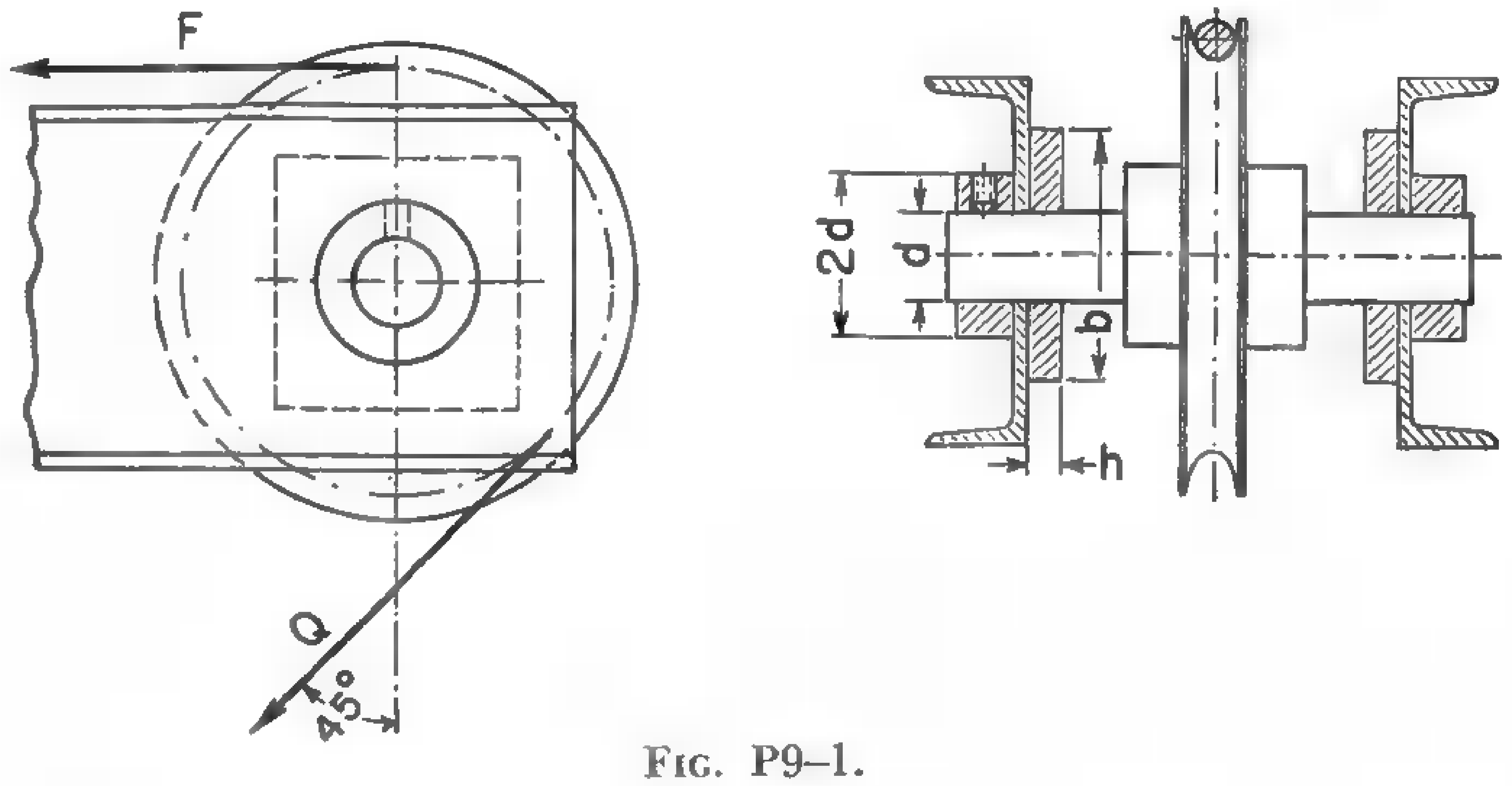


Fig. P9–1.

9–15. An angle a, Fig. P9–2, with equal legs, is riveted to the gusset plate b, which in turn is riveted between two angles c. A load F of 10,000 lb is acting on a free end of the beam formed by the angles c. Assuming a safety factor of 2.5, determine (a) the size of the angle a if the relative positions of the rivets are as shown in Fig. P9–2, $\alpha = 30°$ and $l = 3$ in., and (b) the size, number, and spacing of the rivets in both angle a and angles c. Make the gusset-plate thickness equal to the leg thickness of the angle a.

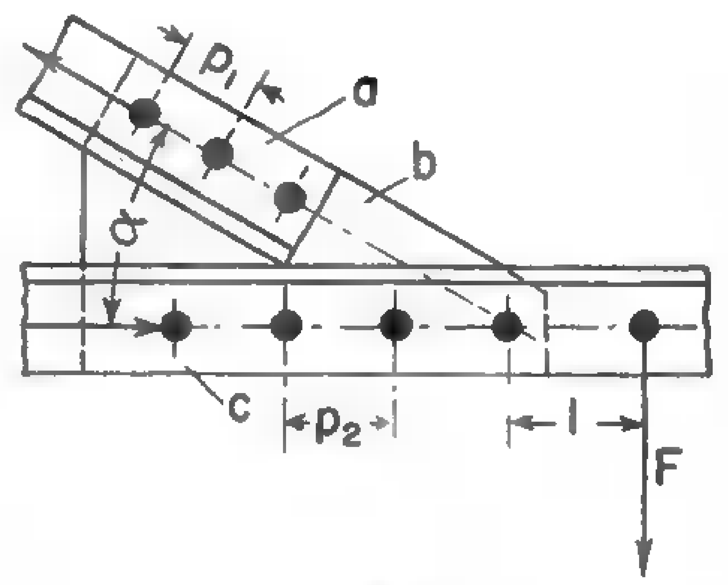


Fig. P9–2.

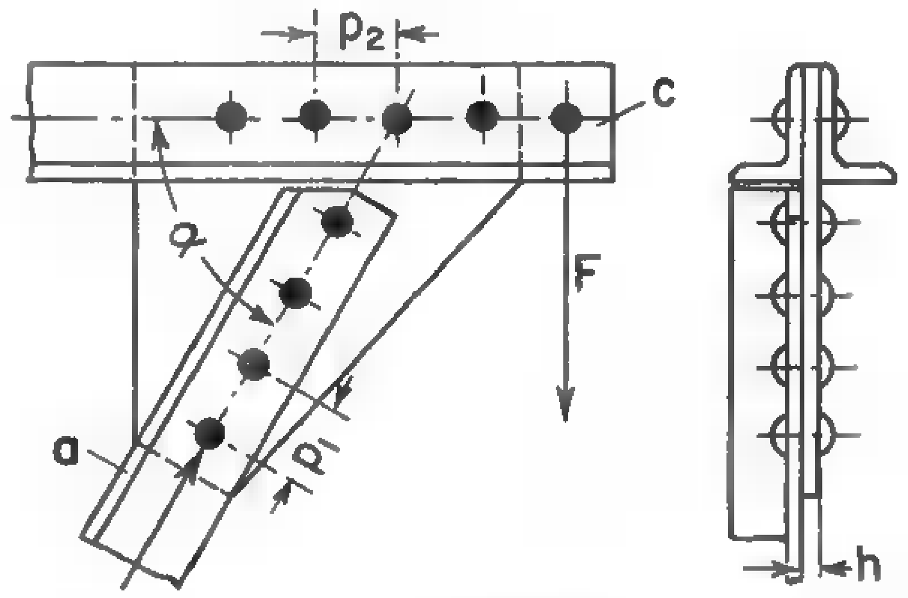


Fig. P9–3.

9–16. Determine the shearing and crushing stresses in each of the rivets in a connection similar to that shown in Fig. P9–2, for the following data: $F = 6{,}000$ lb; the thickness of the gusset plate is $h = \frac{3}{8}$ in.; the rivet diameter is $d = \frac{3}{4}$ in.; the rivet spacing in the horizontal angle is $p_2 = 2\frac{3}{4}$ in.; the leg thickness of the single angle a is $\frac{1}{4}$ in., and that of the angles c is $\frac{5}{16}$ in.; and the line of action of F makes an angle of 60° with the horizontal and passes through the end rivet of the horizontal row, or $l = 0$.

9–17. A small crane has a riveted connection similar to that shown in Fig. P9–3. The strut a is made of a $6 \times 3\frac{1}{2} \times \frac{5}{16}$ in. angle and has a free length of 8 ft. Determine the shear and crushing stresses in each of the eight $\frac{3}{4}$-in. rivets if the load F is 15,000 lb, the upper pitch is $p_2 = 4$ in., the thickness of the gusset plate is $h = \frac{9}{16}$ in., and $\alpha = 60°$.

9–18. Determine the safe load F which can be carried by the crane beam in Fig. P9–3. The beam is supported by a strut a made of a $5 \times 3 \times \frac{1}{2}$ in. angle having a free length

of 7 ft 6 in., and the lower end is riveted to a gusset plate similar to that at the upper end. Assume that there are eight $\frac{3}{4}$-in. rivets, $p_1 = 3$ in., $p_2 = 3\frac{3}{4}$ in., $h = \frac{1}{2}$ in., the safety factor is $n = 2$, and $\alpha = 60°$.

9–19. Determine the safe load F which can be carried by the crane bracket in Fig. P9–4, using the following data: The steel plate a is $\frac{5}{16}$ in. thick; the size of the angle b is $3\frac{1}{2} \times 2\frac{1}{2} \times \frac{1}{4}$ in.; the 8-in. channel weighs 13.75 lb per ft; all rivets are $\frac{5}{8}$ in. in diameter; and $l_1 = 8\frac{1}{2}$ in., $l_2 = 5$ in., and $m = 1\frac{1}{2}$ in. Assume a safety factor n of 2.25.

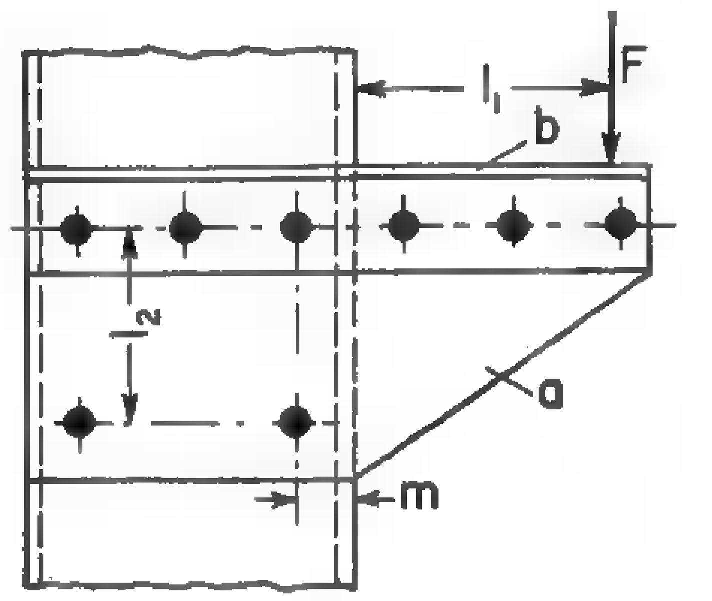


FIG. P9–4.

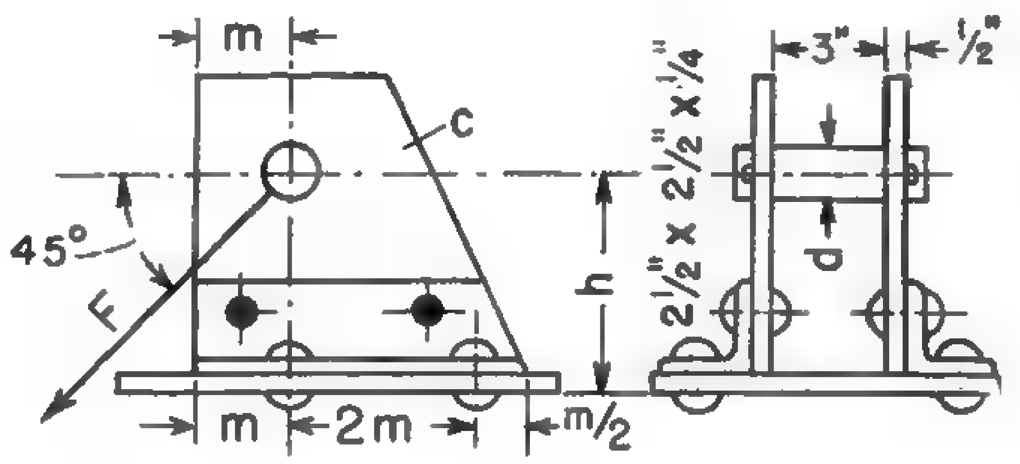


FIG. P9–5.

9–20. Design a bracket, similar to that in Fig. P9–4, to carry a load F of 10,000 lb with a leverage l_1 of 8 in. Use a 10 in. × 20 lb American standard channel and a factor of safety n of 2. Also use a $\frac{3}{8}$-in. plate a.

9–21. Determine the safe load F that the guide-sheave bracket shown in Fig. P9–5 can support with $\frac{5}{8}$-in. rivets if $h = 5$ in., $m = 2\frac{1}{2}$ in., $d = 1\frac{3}{4}$ in., and the factor of safety is $n = 1.8$.

9–22. Design a guide-sheave bracket, similar to that in Fig. P9–5, for a load F of 10,000 lb and a height h of 6 in. Use $\frac{1}{2}$-in. plates c and a safety factor n of 1.75.

9–23. A built-up gear has a forged steel rim and hub connected by a steel plate riveted to flanges on the hub and rim. The pitch diameter of the gear is $D_1 = 28$ in. and the tooth pressure is $F = 4{,}500$ lb. The plate has an outside diameter D_2 of $24\frac{1}{2}$ in. and an inside diameter D_3 of $10\frac{1}{4}$ in. Determine the necessary thickness of the plate, and the size, number, and hole circle of the rivets.

9–24. Work problem 9–23, using the following numerical data: $D_1 = 32$ in., $D_2 = 28$ in., $D_3 = 14$ in., and $F = 6{,}200$ lb.

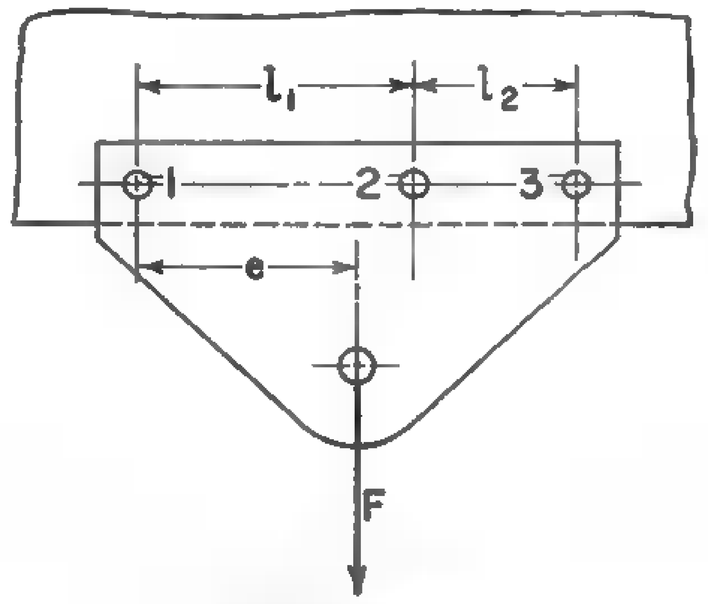


FIG. P9–6.

9–25. Find the size of rivets and the distance e, Fig. P9–6, to be used for fastening a $\frac{1}{2}$-in. plate so that the rivets *1* and *3* are equally stressed when $l_1 = 6$ in., $l_2 = 4$ in., and $F = 12{,}000$ lb.

9–26. Work problem 9–25 for the following data: $l_1 = 8$ in., $l_2 = 5$ in., and $F = 16{,}500$ lb.

CHAPTER 10: Design of Forgings

10–1. Enumerate the primary operations which are used in forging.

10–2. Enumerate the four main groups of forging methods.

10–3. State one of the main advantages of forgings and the requirements to be followed in producing forgings.

10–4. Show by a sketch an example of a hand-forged machine part.

10–5. Explain the process called machine forging.

10–6. What is the difference between a die forging and a drop forging?

10–7. Enumerate some constructive criticisms that may be expected from a practical designer who is working in the field of die forgings.

10–8. Enumerate the requirements that a designer of a drop forging must observe in order that a superior product may be obtained.

10–9. State what requirements a designer of a drop forging must keep in mind in regard to the subsequent machining.

10–10. State in what respect the design of press forgings differs from that of drop forgings.

CHAPTER 11: Screw Fastenings

11–1. Determine the relative strength in tension and torsion of a $1\frac{1}{2}$-in. UNC standard thread, as compared to the UNF standard thread.

11–2. Determine the relative strength in tension and torsion of a 1-in. UNC standard thread, as compared with that of the UNF standard thread.

11–3. (a) Find the pull that must be applied to a 15-in. wrench when tightening a $1\frac{1}{4}$-in. UNC standard bolt to produce a maximum tensile stress of 6,000 psi with no lubrication. (b) Find the maximum true normal stress.

11–4. Determine the efficiency when screwing up a 2-in. UNC standard bolt (a) by using the theoretical expression and assuming oil-graphite lubrication and (b) by using the empirical formula.

11–5. Determine the initial stress set up in a $1\frac{1}{2}$-in. bolt with the UNC standard thread, using for the pull the empirical expression established by Professor Barr.

11–6. Determine the tensile, torsional, and resultant stresses set up in a $\frac{5}{8}$-in.–11 UNC standard bolt when the nut is tightened by a torque wrench with a torque equal to 300 lb-in. Assume the coefficient of friction to be 0.10 for the threads and 0.12 for the nut surface, and take the outside collar diameter as equal to the width across the flats of the hexagonal nut.

11–7. Work problem 11–6 for a 1-in. 12 UNF bolt when the applied torque is 480 lb-in.

11–8. Determine the torque to which a torque wrench must be set when tightening a $\frac{7}{8}$-in.–14 UNF bolt, in order not to exceed a tensile stress of 6,000 psi. Assume that the coefficient of friction is 0.12 in the threads and 0.15 on the bottom of the nut.

11–9. Work problem 11–8 for a 1-in.–8 UNC standard bolt.

11–10. Determine the magnitude of the effort required at the end of a 12-in. wrench to cause failure by tension of a $\frac{5}{8}$-in. UNC standard bolt. Assume that the ultimate strength of steel is 60,000 psi, that there is no lubrication of the thread and only fair lubrication of the bottom of the nut, and that $f_2 = 0.15$.

11–11. Determine the maximum effort which can be exerted at the end of a 16-in. wrench so that the tensile stress in a $1\frac{1}{4}$-in. UNC standard stud bolt does not exceed the safe limit. Assume material of class B, a factor of safety n of 2.5, and fair lubrication of the threads and the bottom of the nut.

11–12. Two 6-in. cast-iron pipes with flanges are bolted together with eight $\frac{3}{4}$-in. bolts located on a $9\frac{1}{2}$-in. bolt circle. The thickness of each flange is 1 in. Determine the bending stress created in the most-stressed bolt if, when a $\frac{1}{16}$-in. gasket between the flanges

is compressed, its thickness becomes $\frac{1}{64}$ in. smaller on one side than on the opposite side. Assume that the surfaces of the back sides of the flanges are parallel to the faces.

11–13. In a swinging 1-in. eye bolt, Fig. 11–14, the center line of the holes drilled in the cast-iron lugs makes an angle of 0.1° with the surface of the flange. The distance from this center line to the bottom of the nut is $2\frac{5}{8}$ in. Determine the bending stress in the bolt shank, assuming that the pin a is a tight fit in the bolt eye, that the eyehole is exactly perpendicular to the center line of the bolt, and that bending starts from the top of the eye.

11–14. Determine the magnitude of the safe load in tension uniformly distributed between four $1\frac{3}{4}$-in. bolts made of open-hearth steel of class A. Assume that there is no shock action, and that the length between the head and the nut is $4\frac{1}{2}$ in.

11–15. Determine the magnitude of the safe load for the data of problem 11–14 if the external load varies suddenly from zero to a maximum and (a) the nuts always remain tight, and (b) the nuts can unscrew and allow a movement of $\frac{1}{64}$ in. when the load is suddenly applied.

11–16. The cylinder head of an air compressor is held in place by steel studs. The cylinder bore is 14 in., and the maximum air pressure is 125 psi. The head-to-cylinder contact surfaces are ground together, no packing being necessary. Determine the number and size of studs to be used, if made of class A rolled steel. Use a factor of safety n of 3.5.

11–17. Find the size of studs which must be used in problem 11–16 if instead of a ground joint a flexible copper-asbestos gasket is used. All other data are the same.

11–18. Data for a booster water pump are the same as for the air compressor in problem 11–16. Compute the size of steel studs (a) for a ground joint and (b) for an elastic gasket.

11–19. Find the size of studs for the cylinder head of problems 11–16 and 11–17 based on empirical expressions for the allowable stresses. Assume that SAE 1120 steel is used.

11–20. Determine whether the size of studs for the pump in problem 11–18 must be changed, and how, if instead of being of open-hearth class A rolled steel the studs are made of (a) hot-rolled phosphor bronze and (b) hot-rolled Monel metal.

11–21. Determine the stresses produced in the $1\frac{1}{4}$-in. UNC bolts used in fastening the crane-runway bracket, Fig. P11–1, by a load F of 5,000 lb.

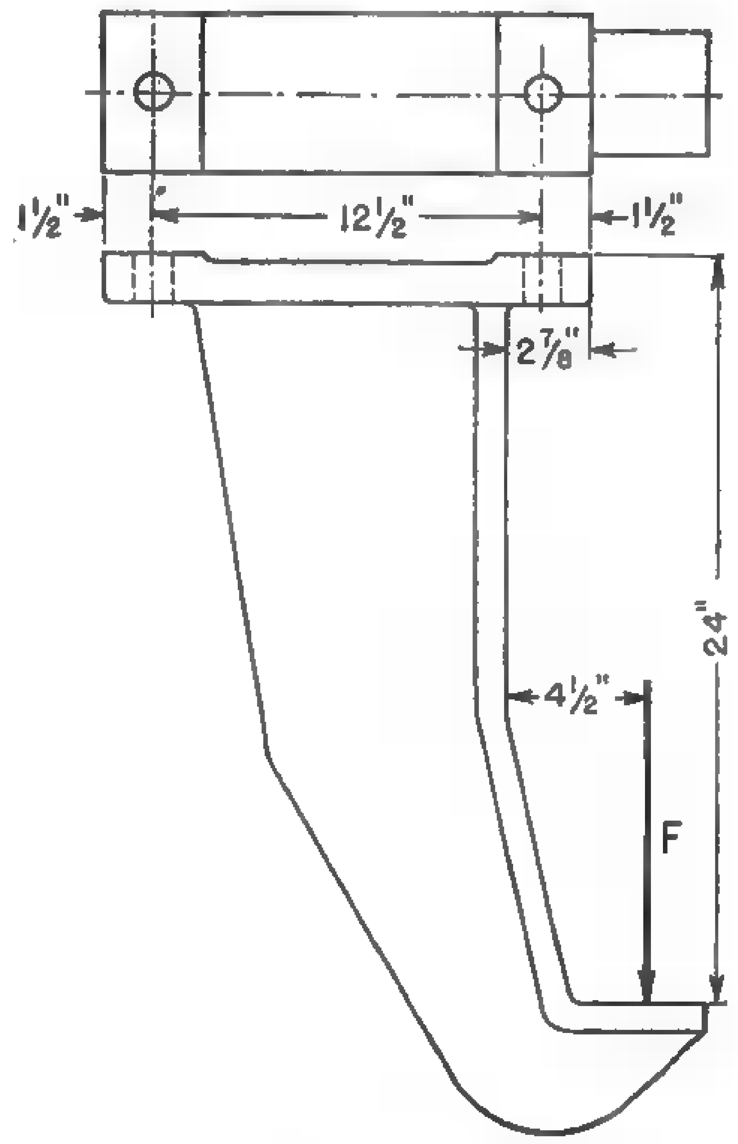


FIG. P11-1.

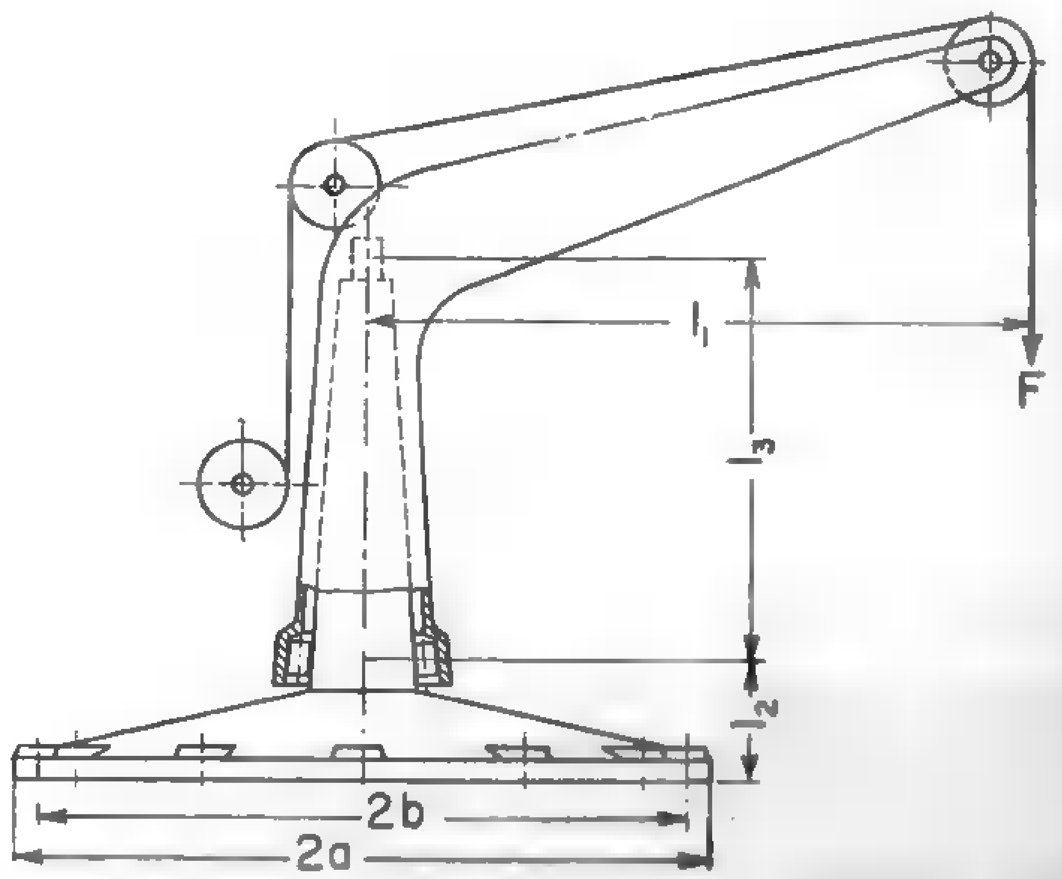


FIG. P11-2.

11–22. A bracket, similar to the one shown in Fig. 11–28, carries a load F of 5,000 lb at a distance l of 18 in. It is fastened by four bolts located so that $l_1 = 22$ in. and $l_3 = 2$ in. Determine (a) the size of the bolts necessary, using class B steel, and (b) the tensile, shear, and resultant stresses, when using $\frac{3}{4}$-in. bolts without the lip a.

11–23. (a) A bearing, similar to the one shown in Fig. 11–29, is fastened to a column by means of four $\frac{7}{8}$-in. cap screws spaced equally on an $8\frac{1}{4}$-in. bolt circle. The bearing-flange diameter is $2a = 10\frac{1}{2}$ in., and the load F of 3,600 lb acts with an arm l of $4\frac{1}{8}$ in. Determine the tensile, shear, and resultant stresses coming upon each of the cap screws. (b) If the screws are to be relieved from shearing stresses by the use of two dowel pins, find the necessary size of these pins.

11–24. Derive the formulas for the loads in tension coming on (a) any bolt and (b) on the highest-stressed one in a bearing similar to the one shown in Fig. 11–29 but equipped with six bolts.

11–25. A bearing similar to the one shown in Fig. 11–29 is fastened to a frame by six bolts spaced equally on a $10\frac{1}{2}$-in. bolt circle. The bearing-flange diameter is $12\frac{1}{2}$ in., and the load of 8,000 lb is applied 15 in. from the frame. Determine the size of bolts and the tensile, shear, and resultant stresses produced on each of the bolts (a) with two bolts located in the vertical plane of symmetry of the bearing and (b) with two bolts located in the horizontal plane of symmetry of the bearing.

11–26. A pillar crane, Fig. P11–2, is fastened to the foundation by 12 bolts spaced equally on a bolt circle. The bolt-circle diameter is $2b$, and the pillar-flange diameter is $2a$. (a) Establish a formula for the maximum load which may come on a bolt. (b) Determine the size of foundation bolts for a load F of 10 tons, if the radius l_1 is 16 ft and the diameters of the pillar flange and bolt circle are 6 ft and 5 ft 4 in., respectively. Use for the bolts SAE 1120 steel and a factor of safety n of 3.

11–27. Determine the type, the size, and the number of setscrews necessary to fasten a belt pulley to a $2\frac{7}{16}$-in. shaft to transmit 25 hp at 300 rpm.

11–28. This problem is identical with problem 11–27, except that the shaft diameter is $2\frac{3}{16}$ in. and the load is 20 hp at 350 rpm.

CHAPTER 12: Keys, Pins, and Cotters

12–1. Determine the dimensions of a standard square key for a gear bored for a $3\frac{7}{16}$-in. shaft and designed to transmit 60 hp at 225 rpm. The outside load is steady.

12–2. Determine the dimensions of a taper key to be used for the same load as given in problem 12–1.

12–3. Find (a) the force necessary to drive home the taper key of problem 12–2, (b) the rate of speed which must be used with a 10-lb sledge hammer to drive this key home, assuming that the last blow will move the key $\frac{1}{64}$ in., and (c) the number of hammer blows of the same intensity required to drive the key home. (Refer to section 3–3.)

12–4. Determine the size and the number, if more than one key is required, to transmit 5 hp at 200 rpm by a pulley to be fastened by Woodruff keys to a $1\frac{7}{16}$-in. shaft. Assume that the torque is steady.

12–5. How many horsepower can be transmitted safely at 100 rpm by a Woodruff key No. 25 in a $2\frac{7}{16}$-in. steel shaft?

12–6. Determine the dimensions of a round taper key, Fig. 12–2a, for a gear on a $2\frac{15}{16}$-in. shaft to transmit 65 hp at 350 rpm. In a gear the maximum tooth load may be two to three times as great as the average one.

12–7. (a) Determine the dimensions of a Barth key, Fig. 12–2b, for a belt pulley on a $3\frac{15}{16}$-in. shaft to transmit 125 hp at 275 rpm. (b) Determine the specific pressure on each of the six sides of the key.

12–8. Determine the dimensions of Kennedy keys for a 10-in. shaft to fasten a heavy gear transmitting 1,200 hp at 100 rpm.

12–9. Determine the dimensions of Lewis keys for the data of problem 12–7.

12–10. Determine all the dimensions of a permanent 4-spline fitting for a $2\frac{3}{16}$-in. shaft to transmit 100 hp at 800 rpm.

12–11. Determine all the dimensions of a 6-spline fitting for a $2\frac{3}{16}$-in. shaft that will permit sliding when not under load, to transmit 100 hp at 800 rpm.

12–12. Determine all the dimensions of a 10-spline fitting for a $2\frac{3}{16}$-in. shaft that will permit sliding when under load, to transmit 100 hp at 800 rpm.

12–13. Find, from tables in the SAE Handbook, the diametral pitch and the number of teeth for an involute spline suitable for a $2\frac{3}{16}$-in. shaft.

12–14. Determine, by using SAE Handbook tables, whether an involute spline or an involute serration will be more suitable for a $2\frac{3}{16}$-in. shaft.

12–15. Determine all dimensions and tolerances necessary for machining the involute spline on a $2\frac{11}{16}$-in. shaft for a sliding fit on the major diameter, and make a detail drawing of the external and internal splines as recommended by the SAE Handbook.

12–16. Work problem 12–13 for a $4\frac{7}{16}$-in. shaft.

12–17. Work problem 12–14 for a $4\frac{1}{2}$-in. shaft.

12–18. Work problem 12–15 for a $4\frac{1}{2}$-in. shaft.

12–19. Work problem 12–13 for a $5\frac{1}{2}$-in. shaft.

12–20. Work problem 12–14 for a $5\frac{1}{2}$-in. shaft.

12–21. Work problem 12–15 for a $5\frac{1}{2}$-in. shaft and for a press fit on the sides of the teeth.

12–22. (a) Determine all the dimensions of a knuckle pin which carries a load of 1,200 lb and has a rocking motion of 20°. (b) Find the stresses in the pin. (c) Give a sketch of the joint.

12–23. Determine all the dimensions of a cotter joint similar to that in Fig. 12–24a, if the external tensile load F is 12,000 lb and it is applied alternatively to the rod ends.

12–24. Determine all the dimensions of a cotter joint similar to that in Fig. 12–24b in the text, for an external load of 20,000 lb which changes continuously from tension to compression.

12–25. Determine the main dimensions of a cotter joint for a flywheel rim similar to that in Fig. 12–24c in the text, assuming that a centrifugal force of 32,000 lb tends to separate the two halves.

CHAPTER 13: Press, Shrink, and Friction Joints

13–1. A hollow steel shaft with an outside diameter of 5.998 in. and a 1-in. inside diameter is to be pressed into a steel disk whose outside diameter is 27 in. Determine the maximum stresses in the disk and the shaft, and also the pressure between the disk and shaft, using a standard class 8 fit and the proper tolerance for the hole in the disk.

13–2. (a) A solid steel shaft with an outside diameter of 4.937 in. is to be pressed into a steel flange that has a $9\frac{1}{2}$-in. outside diameter and is 8 in. long. Determine the proper bore of the hub so that the maximum stress does not exceed 24,000 psi, either in the hub or in the shaft. (b) Find the pressure between the shaft and the hub.

13–3. (a) Find the proper interference for a class 35 cast-iron flange to be pressed on a 4.937-in. shaft. The outside diameter of the flange is $9\frac{1}{2}$ in., and the length is 8 in. (b) Find the maximum stress in the flange. (c) Also find the pressure between the shaft and the flange.

13–4. Determine the proper bores for the flanges in problems 13–2 and 13–3 by using Jenkins' semirational formulas.

13–5. Find the pressure necessary to assemble the shaft and disk in problem 13–1, if the thickness of the disk is 9 in. For the sake of comparison use both the theoretical equations and Jenkins' empirical formulas.

13–6. Using the theoretical and empirical formulas, determine the pressure necessary to assemble the shafts in problems 13–2 and 13–3.

13–7. Several oval links, Fig. 13–3a, with the inside diameter of the semicircles 3 in. and the straight parts 4 in. long, are to be shrunk on bosses to connect two pieces of a heavy cast-iron frame. Determine (a) the interference between the lengths of the links and the bosses to produce a force of 75,000 lb by each link and (b) the cross-sectional dimensions of the link.

13–8. Determine the length, cross-sectional dimensions, all other dimensions, and the necessary interference of four anchor links, Fig. 13–3c, to be shrunk on the rim of a 6-ft cast-iron flywheel and to take up a total centrifugal force of 100 tons. The rim width is $9\frac{1}{2}$ in.

13–9. Find the temperature to which the anchor links in problem 13–8 must be heated when being assembled.

13–10. Find the temperature to which the flange in problem 13–2 must be heated to shrink it on.

13–11. Find the temperature to which an SAE 39 sand-cast aluminum disk must be heated to be shrunk on a $1\frac{1}{2}$-in. steel shaft. Use a class 8 fit.

13–12. Find the temperature to which a 1.500-in. steel pin must be cooled by solid CO_2 to form a shrink fit with an aluminum disk having a 1.498-in. reamed hole.

13–13. Determine the size of the end nut necessary for a Tobin-bronze fan shaft $2\frac{7}{16}$ in. in diameter, the connection being similar to that in Fig. 13–5 in the text. The fan is driven by a 26-hp engine at 780 rpm, and the shaft end is proportioned according to data of section 13–4. For the sake of safety do not take into account the torque which can be transmitted by an additional key.

13–14. Determine the stress set up between the nuts and their seats by a 2-in. threaded rod 48 in. long which, when being assembled, is heated to 260 F on a length of 15 in. The nut is tightened to take up any slack and is then turned one-sixth of a turn more.

CHAPTER 14: Springs

14–1. A simple-beam spring of uniform strength, Fig. 14–2e, made of SAE 1095 steel $\frac{3}{8}$ in. thick, must absorb an impact of 150 ft-lb with a deflection of 2 in. Using a safety factor of 3, find (a) the length of the spring, and the maximum width, and (b) the width and the number of leaves of an equivalent laminated spring.

14–2. A quarter-elliptic (cantilever) laminated spring 27 in. long has, in addition to the master leaf, five graduated leaves, all made of BWG No. 4, SAE 9250 steel, $1\frac{3}{4}$ in. wide. For a safety factor n of 2.25, find the maximum load, the deflection, and the energy absorbed.

14–3. Using data of problem 14–1, work out the answers if the material is SAE 6150 chrome-vanadium steel.

14–4. (a) Design a semielliptic spring to carry a steady load of 900 lb with a deflection of 2 in. The distance between the hinges is $2l = 30$ in. (b) Find the necessary camber of the unloaded spring for the condition that if the load is applied suddenly the camber becomes -1 in.

14–5. A cantilever spring made from SAE 9250 steel has two full-length leaves and six graduated leaves, all $1\frac{7}{8}$ in. wide. The spring length is 34 in., and the static load is 600 lb. Determine (a) the necessary thickness of the leaves to give a deflection of 3 in. and (b) the maximum stress for these conditions.

14–6. A semielliptic automobile spring 50 in. long carries a load of 1,500 lb. The spring has two full-length leaves and eight graduated leaves, all 2 in. wide. Determine the necessary thickness of the leaves to give a deflection of $2\frac{3}{4}$ in. Use SAE 6150 steel and a safety factor of 2.5.

14–7. A semielliptic street-car spring has a length of 42 in. and carries a load of 9,000 lb. It is made up of 18 leaves, 3 in. wide, two of which are full length. Use SAE 6150 steel and a safety factor n of 2.25. Determine (a) the necessary thickness of the leaves and (b) the deflection of the spring.

14–8. The top of a shaking table is supported on four flat spring leaves S, Fig. P14–1, with the ends rigidly fixed both in the floor and to the table. The free length l is 26 in., and the spring strips are of SAE 1050 BWG No. 6 steel $1\frac{1}{2}$ in. wide. Find (a) the force F necessary to move the top $1\frac{1}{2}$ in. from its middle position and (b) the maximum bending stress in the strips.

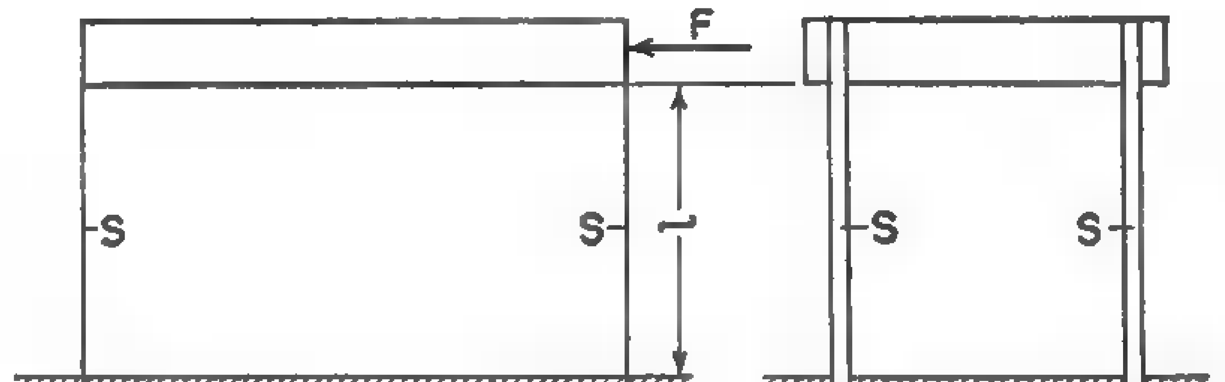


FIG. P14–1.

14–9. Assume that in problem 14–8 the upper ends of one pair of the spring strips are hinged to one table end and those of the other pair are fixed to the other table end. Find (a) the force F necessary to move the table top $1\frac{1}{2}$ in. and (b) the maximum bending stress in each of the two pairs of strips.

14–10. A round-wire cylindrical compression spring has an outside diameter of 3 in. and is made of $\frac{1}{2}$-in. steel wire. The spring supports an axial load of 1,000 lb. Determine (a) the maximum shear stress and (b) the total deflection, if it has 8 coils with closed and ground square ends and is made of SAE 9250 steel.

14–11. Determine the continuously repeated load, varying from the maximum value down to 25 per cent of it, which the spring of problem 14–10 can carry if made of SAE 9250 steel (a) with a safety factor of 3, when using the simplified method of calculation; (b) with the same safety factor of 3, when using the endurance-limit method; and (c) with a safety factor of 1.5, when using the endurance-limit method.

14–12. Determine the amount of energy absorbed by the spring of problems 14–10 and 14–11 at each compression for the cases a, b, and c.

14–13. (a) Find the general relation for the resilience of a cylindrical helical spring with a rectangular-wire cross section, and compare it with the relation for the resilience of a round-wire spring of the same weight and material. (b) Show by a diagram the influence of the ratio b/h on the resilience of the spring.

14–14. Design a helical spring for the front-wheel suspension of an automobile, using silicon-manganese steel. The spring must have a scale of 200 lb per in., an inside diameter of $4\frac{1}{8}$ in., and a free length of 14 in. with closed and ground ends. The spring must be able to carry a static load of 800 lb, and when compressed solid it should not receive a permanent deformation.

14–15. Design a cylindrical, round-wire spring for an overhead valve for a vertical four-stroke oil engine. When the valve is closed the spring force must be 130 lb. The valve lift is $\frac{3}{4}$ in., the valve weighs 7 lb, the weight of other parts which must be accelerated is 8 lb, and the engine runs at 300 rpm. The valve must close during 95 deg of the crank travel. The outside diameter of the spring must be not over $3\frac{1}{8}$ in. Use the simplified method first, and then check the design by the endurance diagram.

14–16. For the data of problem 14–15, design a valve spring with a square-wire section by using the endurance diagram of Fig. 14–14 in the text.

14–17. A helical spring of round wire must support a load of 200 lb. The inside diameter must be not smaller than 2 in. When the load is applied, the spring must compress 2 in. Use SAE 1095 steel wire and a safety factor n of 2. Find (a) the wire size, (b) the number of active coils, and (c) the maximum shear stress, taking into account also the direct stress.

14–18. Work problem 14–17, using a square wire.

14–19. Using data of problem 14–15, design a set of two concentric springs, as in Fig. 14–15 in the text. Each spring is to carry half of the load.

14–20. An airplane-engine valve spring is made of two concentric helical coils wrapped together. The pitch diameter of the springs is $1\frac{1}{8}$ in., and the springs must have a length of $1\frac{11}{16}$ in. when the valve is closed and must exert a pull of 70 lb. The valve lift is $\frac{9}{64}$ in. When the valve is fully open, the combined spring force should be increased 35 per cent. (a) Determine the diameter of wire of SAE 6150 steel for continuous operation and with the condition that should the springs be compressed solid, while assembled in the engine, there will be no permanent deformation. (b) Indicate the number of coils in each spring. (c) Find the length of stock to make each spring. (d) Determine the free length of the springs.

14–21. Design two concentric springs, Fig. 14–15, to take the place of the springs in problem 14–20.

14–22. Design a set of three concentric helical springs with gradually decreasing pitch diameter for a centrifugal variable-speed governor. At 100 rpm the centrifugal force of the governor weights is 100 lb. The highest speed is 200 rpm. The travel of the sleeve compressing the spring from low speed to high speed is $1\frac{1}{2}$ in. Assume that the leverage of the centrifugal force does not change.

14–23. Design a conical spring, for the governor of problem 14–22, whose stiffness will be gradually increased by bottoming of the coils with the larger diameters.

14–24. A torsion spring is made of No. 3 Monel-metal wire, wound in a coil with an inside diameter of $1\frac{3}{4}$ in. Assuming a safety factor n of 2, find (a) the maximum force F which can be applied with a 2-in. leverage, Fig. 14–20, and (b) the corresponding deflection, in degrees, if the spring has $4\frac{1}{2}$ coils.

14–25. Design a torsion spring, Fig. 14–20, for a maximum torque of 20 lb-in. The lever arm is $l = 1\frac{3}{4}$ in. Make the spring of round wire, and assume a sudden torque application.

14–26. Using data of problem 14–25, design a spring made of square wire.

14–27. A window-shade spring is made of steel wire 0.0475 in. square. The outside diameter of the coil is 0.75 in. The spring must exert a pull on the shade of $2\frac{1}{2}$ lb after it is wound 12 revolutions. The wooden roller is $1\frac{1}{4}$ in. in diameter. Determine (a) the number of coils to be used and (b) the maximum flexural stress.

14–28. A flexible coupling transmits 20 hp at 1,000 rpm through a torsional helical spring made of $\frac{5}{8}$-in. square SAE 1050 steel. Find (a) the pitch diameter of the coil, using a safety factor n of 2, and (b) the number of active coils if the torsional deflection should not exceed 5°.

14–29. Work problem 14–28, using $\frac{5}{8}$-in. round wire.

CHAPTER 15: Cylinders, Heads, and Cover Plates

15–1. (a) Find the thickness of the shell of a 60-in. boiler drum for a steam pressure of 175 psi. The longitudinal seam is a triple-riveted butt joint having an efficiency of 85 per cent. Use an allowable stress of 11,000 psi, as prescribed by the ASME Code. (b) Determine the true hoop stress in this shell induced during a hydrostatic test when the pressure is raised to $1\frac{1}{2}$ times the working pressure.

15–2. A standard lap-welded 6-in. pipe has an inside diameter of 6.065 in. and an outside diameter of 6.625 in. Determine the stress induced in the pipe when it is tested with a pressure of 600 psi.

15–3. Determine the stress induced in a 6-in. extra-strong seamless open-hearth steel pipe having a 0.385-in. wall thickness and the same outside diameter as a standard 6-in. pipe (a) under the recommended maximum steam pressure of 600 psi and (b) under a hydrostatic test pressure of 1,100 psi at the mill.

15–4. Determine the limits of safe use of the thin-wall formula (equation 15–6), as compared with equations 15–8 and 15–12, expressed in terms of h/d and p for low-carbon open-hearth steel. Assume an elastic limit in tension of 30,000 psi.

15–5. Determine the wall thickness of a removable cast-iron oil-engine liner if the engine bore is $14\frac{1}{2}$ in. Assume a maximum gas pressure of 600 psi.

15–6. A cast-steel cylinder 10 in. in diameter has closed ends and is subjected to an internal pressure of 2,000 psi. Determine the minimum permissible thickness of the cylinder walls, assuming a safety factor n of 2.25 and a soft steel in the casting.

15–7. A cast-iron drum with integral heads must be designed for an internal pressure of 500 psi. The inside diameter must be 14 in. Assume a better grade of cast iron and a safety factor n of 2. After the thickness h_1 of the cylinder walls is determined, find the radius r of the inner curvature necessary to make the thickness h_2 of the head equal to that of the cylinder.

15–8. Determine the minimum thickness of a dished head, Fig. 15–1, for the boiler drum of problem 15–1, assuming (a) that there is no manhole and $r = 0.8d$, and (b) that $r = 0.8d$ but a manhole is cut in the head.

15–9. A cast-iron steam chest on an engine cylinder has a rectangular opening 14 in. long and 10 in. wide, and the steam pressure is 170 psi. (a) Determine the thickness of a flat cover for this opening made of a better grade of cast iron. (b) Determine the number and size of studs to fasten the cover, using an allowable stress recommended by Unwin (see section 11–13).

15–10. A flat cast-iron cylinder head $1\frac{1}{4}$ in. thick is fastened to the flange of a 9-in. cast-iron cylinder by means of through bolts. Assuming for cast iron an allowable stress of 5,000 psi, determine (a) the pressure that can be carried in the cylinder, (b) the thickness of the cylinder walls and of the cylinder flange, (c) the number and size of bolts to fasten the head, and (d) the outside diameter of the head.

15–11. Assuming that the bolt-circle diameter of the cover in problem 15–10 is $11\frac{1}{2}$ in., find (a) the pressure which the cover can carry and (b) the maximum deflection of the cover under this pressure, assuming a modulus of elasticity of 12,000,000 psi.

15–12. (a) Determine the minimum thickness of a cast aluminum cover of SAE 35 alloy for a vacuum apparatus in which the maximum vacuum reaches 20 in. mercury. The opening to be covered is oval, 8 in. by 12 in. (b) Determine the number of studs, their size, and their pitch, or spacing, to hold the cover tight.

15–13. Determine the thickness of the head in Fig. 15–2e in the text, using all other data from problem 15–6.

CHAPTER 16: Packings and Seals

16–1. (a) Determine the main dimensions for a simple stuffing box for a water-pump piston rod. The rod diameter is 2 in., and the pump delivers water against a head of 320 ft. (b) Give a sketch with dimensions of the box assembly.

16–2. (a) Determine the main dimensions for a stuffing box with a soft packing to be used on a steam engine. The piston-rod diameter is $2\frac{1}{2}$ in., and the steam pressure is 125 psi, gage. (b) Give a sketch with dimensions of the box assembly.

16–3. Determine the main dimensions of a stuffing box with a threaded cap, Fig. 16–3, for a $\frac{7}{8}$-in. rod. Make both threads of the same size. Assume that the box is made of cast bronze and that a steam pressure of 240 psi is used.

16–4. Determine the main dimensions of a stuffing box with a threaded cap for a valve spindle $\frac{3}{4}$ in. in diameter. Make all parts of Monel metal. The valve is used for air under a pressure of 750 psi.

16–5. Determine the main dimensions of a self-sealing leather packing, Fig. 16–6a, for the piston rod of problem 16–1.

16–6. Determine the friction forces which must be overcome by the piston rod of problem 16–1, using the packing box of that problem, and compare them with those of problem 16–5. Assume that the friction force without fluid pressure is 4 lb.

16–7. (a) Determine the main dimensions of a leather cup packing for the plunger of a hydraulic elevator, similar to that of Fig. 16–6b. The plunger diameter is 4 in., and the pressure is 1,500 psi. (b) Find the friction force and compare it with that of a soft packing gland. Without pressure the gland friction is 20 lb.

16–8. A hydraulic-press plunger 12 in. in diameter is fitted with a U-shaped leather collar. The water pressure is 1,000 psi. Determine (a) the total friction of the collar, assuming normal conditions; (b) the total friction of the collar, assuming lack of lubrication; (c) approximately what percentage of the useful power of the press is absorbed by the collar in each of the previous cases.

16–9. Determine the leakage of oil per hour past the packingless plunger of a two-stroke-cycle oil-engine injection pump. The fuel oil is 30° Bé, and its temperature is 80 F. The injection pressure is 1,650 psi and is maintained during 30 deg of crank travel. The plunger diameter is $\frac{1}{2}$ in., the radial clearance is 0.0001 in., and the effective joint length is $2\frac{1}{2}$ in.

16–10. Determine the maximum radial clearance which can be allowed if the leakage in the pump of problem 16–9 should not exceed 0.25 cu in. per hr. The conditions are the same as in problem 16–9, but the injection pressure is raised to 3,500 psi.

CHAPTER 17: Chains and Wire Ropes

17–1. (a) Determine the size, and find from a handbook all other dimensions and weight per foot of length, of an open-link crane chain to withstand a pull of 10 tons for continuous machine operation but without shock action. (b) Find the number of links in a 100-ft length.

17–2. Determine the size and all other dimensions of a wrought-iron open-link chain for a working load of 10 tons for intermittent machine operation with heavy shocks. Also find the number of links and the weight of a 100-ft length.

17–3. Determine the diameter, length, and wall thicknesses h and h_o of a cast-iron drum, Fig. 17–3a in the text, for a $\frac{3}{4}$-in. open-link chain 200 ft long.

17–4. Determine the diameter, length, and wall thickness h and h_o of a cast-iron drum, Fig. 17–3b in the text, for a 1-in. crane chain 300 ft long.

17–5. (a) Determine the size of a 6×19 standard plow-steel rope to be used with a drum hoist to lift 2.5 tons from a depth of 200 ft. Assume a rope speed of 1,000 fpm and an acceleration of 6 fpsps when starting, with no slack. (b) Find the influence of a slack of 2 ft when starting. (c) Find the necessary drum diameter and length. (d) Check the rope size and drum diameter by the bearing-pressure ratio.

17–6. (a) Determine the size of a 6×19 wrought-iron rope used with a drum hoist to lift 1 ton from a depth of 500 ft. Assume a rope speed of 500 fpm and an acceleration of 5 fpsps when starting, with a slack of 2 ft. (b) Determine the desirable diameter of the drum.

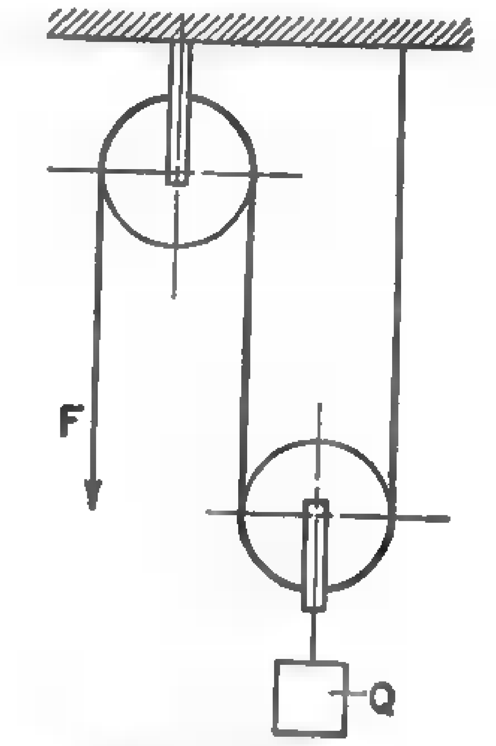


Fig. P17–1.

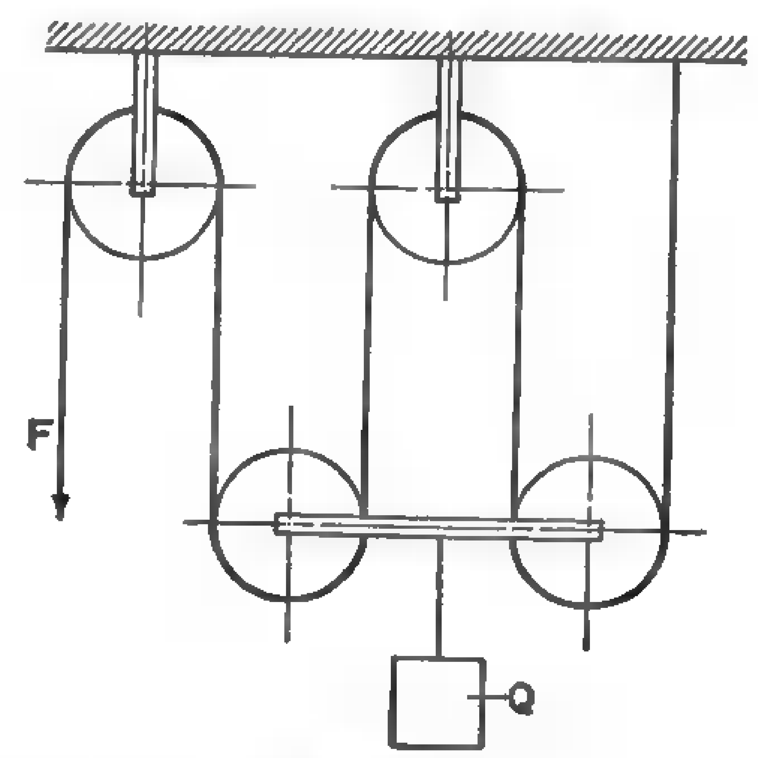


Fig. P17–2.

17–7. The pulley system shown in Fig. P17–1 is used to raise a load Q to 6,000 lb with a maximum acceleration of 8 fpsps. Assume a 6×19 standard-plow-steel rope, sheaves 42 rope diameters in diameter, and a loss of 5 per cent with each pulley. (a) Determine the diameter d of the rope required for a working factor n_o of 5. (b) Determine the effort F. (c) Check the rope size by the bearing-pressure ratio.

17–8. Work problem 17–7, assuming the following sheave diameters: (a) 24d, (b) 36d, and (c) 48d.

17–9. The block and tackle in Fig. P17–2 is used to raise a load Q by an effort F. Assume that $T_1/T_2 = T_2/T_3 = T_3/T_4 = T_4/F = e_r$, where T_1, T_2, T_3, and T_4 are the rope tensions, and derive an expression for F in terms of Q.

17–10. Find the effort F necessary to raise a load Q of 4,000 lb with a block and tackle, Fig. P17–2. Use a sheave diameter D of $30d$.

17–11. (a) Determine the size of mild-plow-steel rope necessary for the data of problem 17–10, using a working factor n_o of 6 and an acceleration of 7.5 fpsps. (b) Check the rope size by the bearing-pressure ratio.

17–12. Determine the diameter, length, and minimum wall thickness of a cast-iron mine-hoist drum for a 1-in. improved-plow-steel 6 × 19 hoisting rope. The depth of the mine is 1,500 ft.

17–13. Determine the diameter, length, and minimum wall thickness of a cast-iron hoist drum for a $1\frac{1}{2}$-in. extra-flexible 8 × 19 plow-steel rope. The length of rope to be wound on the drum is 1,750 ft.

17–14. (a) Determine the size of special flexible 6 × 37 wire ropes and the drum diameter required for an elevator in a building 450 ft tall for a load of 2 tons. Assume a maximum rope speed of 1,000 fpm and an acceleration of 6 fpsps when starting, with no slack. According to the state law, not less than four ropes must be used. (b) Check the ropes and drums by the bearing-pressure ratio.

17–15. (a) Determine the size of 6 × 19 plow-steel rope for the elevator in a building 600 ft high for a total load of 2.5 tons. The desired speed is 1,000 fpm, and the full speed must be reached in 40 ft. Use four ropes, according to the state law, and use large sheaves, selecting twice the minimum diameter recommended in Table 17–1. (b) Check the rope size and the sheave diameter by the bearing-pressure ratio.

17–16. An oil well is drilled to a depth of 5,000 ft with the use of $4\frac{1}{2}$-in. drill pipe which has the same dimensions and weight as 4-in. standard pipe. Assume a weight of 50 lb for pipe joints in every 40 ft of pipe length. The pipe must be raised by using 1-in. special improved plow-steel 6 × 19 rope with 36-in. sheaves and an acceleration of 10 fpsps. (a) Determine the number of ropes required, using a working factor n_o of 3. (b) Check the rope size by the bearing-pressure ratio.

CHAPTER 18: Brakes

18–1. (a) Determine the total energy which must be absorbed to slow down to 450 fpm a mine-hoist cage descending at a rate of 1,500 fpm. The weight of the cage with the load is 2 tons, the hoist-drum diameter is 6 ft, and the brake-sheave diameter is 4 ft 8 in. The rotative speed of the brake sheave is one-third that of the driving engine, and the speed of the hoist drum is one-tenth that of the engine. The weight of the hoist drum is 7.5 tons, and the weight of other rotating parts may be neglected. (b) Determine the normal and tangential forces which must be applied to the brake sheave to slow down the cage from 1,500 fpm to 450 fpm in 15 sec.

18–2. (a) A street car weighing 5 tons and moving on level ground at 30 mph must be stopped in the shortest distance possible. The diameter of the steel wheels is 24 in. Assuming a coefficient of friction of 0.35 between the wheels and the rails, and a friction area of the brake shoes of 30 sq in. at each of the four wheels, determine the distance and the time needed to stop the car. (b) Assuming a friction coefficient of 0.33 between the cast-iron brake shoes and wheel rims, determine the normal force which must be applied to stop the car in the distance computed in (a).

18–3. Determine the tangential force at the brake-shoe surface necessary to bring to a complete stop the hoist cage of problem 18–1 (a) in 30 sec and (b) in 5 sec.

18–4. Determine the normal braking force which must be applied at the brake surface for the conditions of problem 18–1 if the load is lowered by the force of gravity at a speed of 450 fpm.

18–5. Assume that operation is intermittent; that cast-iron shoes run dry on a steel brake sheave which has a diameter of 20 in. and runs at 87 rpm; and that the area of the heat-dissipating surface is 50 per cent larger than the braking surface. Determine the braking area for absorbing 70,000 ft-lb in 30 sec.

18–6. For conditions as given in problem 18–1, assume that the brake has wooden blocks on a cast-iron drum and that the area of the radiating surfaces consists of the inner surface of the brake-drum rim and 80 per cent of the outer rim surface. Find the minimum brake-drum width required.

18–7. A block brake similar to that in Fig. 18–2 is used for lowering a load of 1,200 lb. The main dimensions are: $D = 36$ in., $a = 6$ ft, and $b = 15$ in. Assume that the rotation is clockwise and that $r = 90$ rpm, and that the coefficient of friction is $f = 0.25$. The hoist drum has a diameter of 20 in., and it and the brake drum are on the same shaft. Find the effort F for each of the following positions of the fulcrum: (a) $c = 8$ in., (b) $c = 1$ in., and (c) $c' = 8$ in.

18–8. On a block brake similar to that in Fig. 18–2, $D = 18$ in., $a = 32$ in., $b = 10$ in., and $c = 6$ in. The surface of contact between the wooden block and the cast-iron wheel subtends an angle of 84°, the face of the wheel is 9 in., the wooden block is 7 in. wide, and the wheel rotates at 200 rpm. Determine (a) the approximate braking torque that may be continuously applied without overheating the brake, (b) the force F necessary to produce this torque, and (c) the reactions at the bearings, if the wheel is supported by two bearings located at equal distances from the wheel.

18–9. Work problem 18–8 with the following data: $D = 16$ in., $a = 30$ in., $b = 9$ in., and $c = 5\frac{1}{2}$ in.; the face of the wheel is 8 in.; the steel block is $6\frac{1}{4}$ in. wide; and the wheel rotates at 180 rpm. Compute the coefficient of friction as a function of the rim velocity, and answer all parts of problem 18–8.

18–10. A double-block brake built according to the scheme given in Fig. 18–3 is used to lower a load of 1 ton at a speed of 800 fpm. Assuming for wood on cast iron a friction coefficient of 0.33, determine the necessary effort F and the tension F' of the spring s, if the bell-crank leverage is 2 in. to 18 in., the diameter of the brake is $D = 30$ in., and the diameter of the hoist drum is $D_2 = 18$ in.

18–11. A double-block brake, Fig. 18–3, is used for lowering loads. The brake diameter is 32 in., and other dimensions may be scaled from Fig. 18–3. (a) Determine the horsepower which this brake can absorb at 300 rpm of the sheave with a pull F of 90 lb. (b) Determine the necessary width of the sheave drum from consideration of heat dissipation. (c) Check whether the specific pressure on the wood blocks with an angle of contact of 90° will not require an increase of the drum width.

18–12. A brake similar to that in Fig. 18–3 in the text must absorb 5 hp at 225 rpm. The brake diameter is 18 in., and all other necessary dimensions may be scaled. The drum and the blocks are of cast iron, and no lubrication is used. Determine (a) the tension of the spring s and (b) the pull F necessary to lower the load.

18–13. Work problem 18–12, assuming that the sheave has a groove as shown in Fig. 18–4, with an angle 2α of 48°. The blocks are made of asbestos, and the mean diameter of the brake sheave is 18 in.

18–14. (a) Determine the capacity in horsepower, at 175 rpm of the brake sheave, of a differential band brake, Fig. 18–6. The principal dimensions are $a = 42$ in., $b_1 = 2$ in., $b_2 = 5$ in., $D = 18$ in., the distance from the fulcrum *1* to the sheave center is 12 in., and the line from the fulcrum to the sheave center forms a right angle with the center line *1–4* of the operating lever. The band is asbestos-lined and can stand a tensile load of 4,000 lb. (b) State the direction of the force F, upward or downward, for a clockwise rotation of the sheave. (c) Find the magnitude of the force F.

18–15. Work problem 18–14, assuming that the direction of rotation of the sheave is counterclockwise.

18–16. (a) For a simple band brake as shown in Fig. 18–8, determine the magnitude of the effort F necessary to hold the load on the drum for clockwise rotation. The principal data for the brake are: $a = 54$ in., $b = 5\frac{1}{2}$ in., the sheave diameter is 28 in., and the

angle of contact is $\theta = 255°$. The coefficient of friction is 0.25, the diameter of the hoist drum is 20 in., and the load is 1,500 lb and is suspended directly from a $\frac{3}{8}$-in. steel-wire rope. (b) Determine the thickness and width of the steel brake band, considering stiffness and strength and assuming that the end lugs are welded to the band. (c) Determine the width of the band and the size and number of rivets if the end lugs are riveted to the band. (d) Show a sketch of the rivet spacing.

18–17. In a simple band brake, Fig. 18–8, the band is wound $1\frac{3}{4}$ times around the sheave. The principal dimensions are: $a = 20$ in., $b = 3$ in., $D = 24$ in., and the steel band is 2 in. wide and $\frac{1}{8}$ in. thick. The asbestos lining is fastened with $\frac{5}{32}$-in. copper rivets. The magnitude of the effort is $F = 15$ lb. Determine (a) the horsepower which the brake can absorb at 250 rpm rotating in one direction, (b) the horsepower for rotation at 250 rpm in the other direction, and (c) the maximum stresses in the band for both cases.

18–18. A band brake, Fig. 18–8, is geared to the hoisting motor of a traveling crane. The gear ratio between the motor and the brake shaft is 6 to 1, and the motor can develop 8 hp at 900 rpm. The principal dimensions are $a = 21$ in., $b = 3$ in., $D = 12$ in., and $\theta = 270°$. Assume that the friction coefficient is $f = 0.27$. Determine (a) the magnitude of the effort F to hold the maximum load which the motor can lift, (b) the width of the brake sheave, (c) the width and thickness of the brake band made of SAE 1010 steel, and (d) the size, number, and spacing of rivets to fasten the band to the end lugs (see Fig. 18–9).

18–19. A band brake, Fig. 18–9, is built to absorb a torque of 450 lb-ft. The principal dimensions are $a = 36$ in., $b = 2$ in., $D = 21$ in., and $\theta = 270°$. Determine the coefficient of friction which was used in calculating the brake capacity if the effort is $F = 50$ lb.

18–20. A cone brake, Fig. 18–11, is mounted on a shaft which transmits 6 hp at 225 rpm. The small diameter of the cone is 9 in., and the cone face is 2 in. wide; $\alpha = 15°$; the friction coefficient is 0.33; and the lever dimensions are $a = 24$ in. and $b = 5$ in. Find (a) the effort F necessary to stop the shaft and (b) the specific normal pressures on the cone surfaces.

18–21. Find the torque which can be absorbed by the brake of problem 18–20, using cork lining and such an effort F that the normal unit pressure on the cone surfaces will reach the permissible high limit.

18–22. A hoist drum is bolted to a multiple-disk brake, Fig. 18–12. The drum diameter is 18 in.; the tangential load is 1,200 lb; a $\frac{1}{2}$-in. wire rope is used; the outside diameter of the small steel friction disks is 10 in.; the inside diameter of the larger cast-iron disks is 5 in.; and the lever dimensions are $a = 30$ in. and $b = 4$ in. Determine (a) the force F required on the lever when applying the brake, (b) the number of friction disks required, assuming that the permissible unit pressure is as recommended in the text and that the disks run in oil, and (c) the number of friction disks required if F must not exceed 50 lb.

CHAPTER 19: Screws for Power Transmission

19–1. Determine the pitch, width, and depth of the thread, and the root diameter, of a $1\frac{1}{2}$-in. screw with Sellers' square threads.

19–2. Make a sketch showing all dimensions which determine a 2-in. Acme thread.

19–3. Make a sketch showing all dimensions of a $2\frac{1}{2}$-in. trapezoidal thread.

19–4. Determine the efficiency of a $1\frac{3}{4}$-in. steel screw with Sellers' square threads. The screw is well-lubricated and has a bronze nut.

19–5. Determine the efficiency of a $1\frac{3}{4}$-in. steel screw with Acme thread and a bronze nut (a) if well-lubricated, and (b) when operated dry.

19–6. Determine the load which can be lifted by a 2-in. jackscrew with Sellers' square threads, if a force of 45 lb is applied with an 18-in. lever arm. The friction collar has a $3\frac{3}{4}$-in. outside diameter and a 2-in. inside diameter. The friction coefficients are 0.14 in the threads and 0.11 at the collar.

19–7. A sluice gate weighing 60 tons is raised and lowered by means of two $2\frac{3}{4}$-in. square-thread screws. The screws are operated by an electric motor running at 600 rpm. A ball thrust bearing is used, reducing the apparent friction coefficient to 0.003 on a 2-in. radius. Bronze nuts and fair lubrication are used. If the gate must be raised at the rate of 2 fpm, determine (a) the number of revolutions per minute of the screws, (b) the power of the motor required to raise the gate, assuming a mechanical efficiency of 0.85 for the speed reduction mechanism, and (c) the power requirement to lower the gate.

19–8. Answer the questions of problem 19–7, using the same data but assuming that a double-thread screw is used.

19–9. A $2\frac{3}{4}$-in. screw with Sellers' square threads is used in a press. It has a maximum unsupported length of 18 in. Using SAE 1035 steel for the screw and phosphor bronze for the nut, determine (a) the safe capacity of the press, (b) the proper length of the nut, (c) the necessary torque, assuming 0.13 as the friction coefficient in the threads and 0.15 at the thrust collar, which has an outside diameter of $3\frac{1}{2}$ in. and an inside diameter of 1 in.

19–10. (a) Determine the dimensions d_1, d_2, d_3, d_4, and l, Fig. P19–1, for a steel screw used in a hand punch to make $\frac{7}{16}$-in. holes in a $\frac{1}{16}$-in. 0.10 per cent carbon steel plate. Use a trapezoidal thread. The punch body is a steel casting. (b) Determine the length L of the lever to be used if the force at its end must not exceed 60 lb. Use equation 9–1.

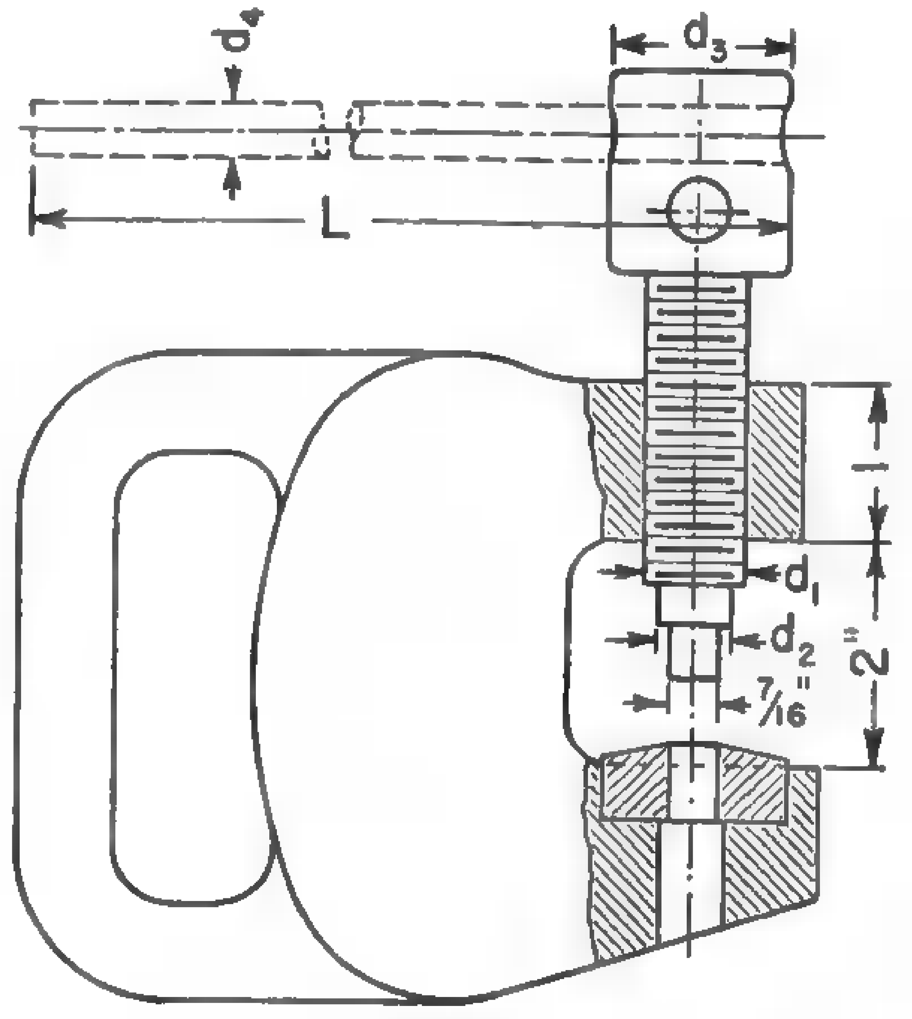


FIG. P19–1.

19–11. (a) Find the capacity of a screw jack, Fig. 19–11, made with a $1\frac{3}{4}$-in. Acme thread and having a lift of 14 in. (b) Assuming that the friction coefficient f_1 in the threads is 0.15, the coefficient f_2 in the upper thrust support is 0.2, and the outside diameter of the thrust collar is $1\frac{3}{4}$ in., determine the efficiency of the screw jack. (c) Determine the length of the lever L required to raise the maximum load that can be lifted by applying a force of 2×60 lb.

19–12. Determine the main dimensions of a screw jack for a load of 5 tons and a lift of 10 in. Make the screw of SAE 1030 steel, and use a cast-iron nut.

19–13. The screw of a toggle press is driven by a gear g, Fig. P19–2, and turns at 75 rpm. The crossheads c move along the screw in opposite directions, since one has a right-hand thread and the other has a left-hand thread. The crossheads move against the axial forces F when the press is operated. The screw has Sellers' square threads with $2\frac{1}{4}$-in. major diameters. Determine the horsepower required to drive the gear g if $F = 2$ tons. Assume that the coefficient of friction in the lubricated threads is $f = 0.11$, and neglect the friction in the bearings b.

19–14. An automatic machine has a power screw with right-hand and left-hand Acme threads and nonrotating nuts, an arrangement similar to that in Fig. P19–2. The

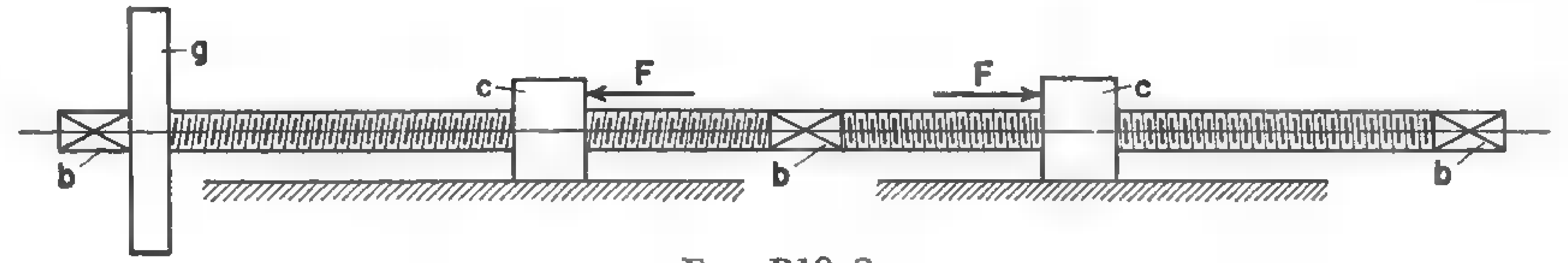


FIG. P19-2.

nuts must be moved against forces F of 1,750 lb. The screw has an outside diameter of 2 in. Assume a friction coefficient f_1 of 0.12, and neglect the friction in the ball bearings b. Determine (a) the efficiency of the screw, (b) the horsepower required to turn the driving gear g to move the nuts with a speed of 10 fpm, and (c) the necessary length of the bronze nuts.

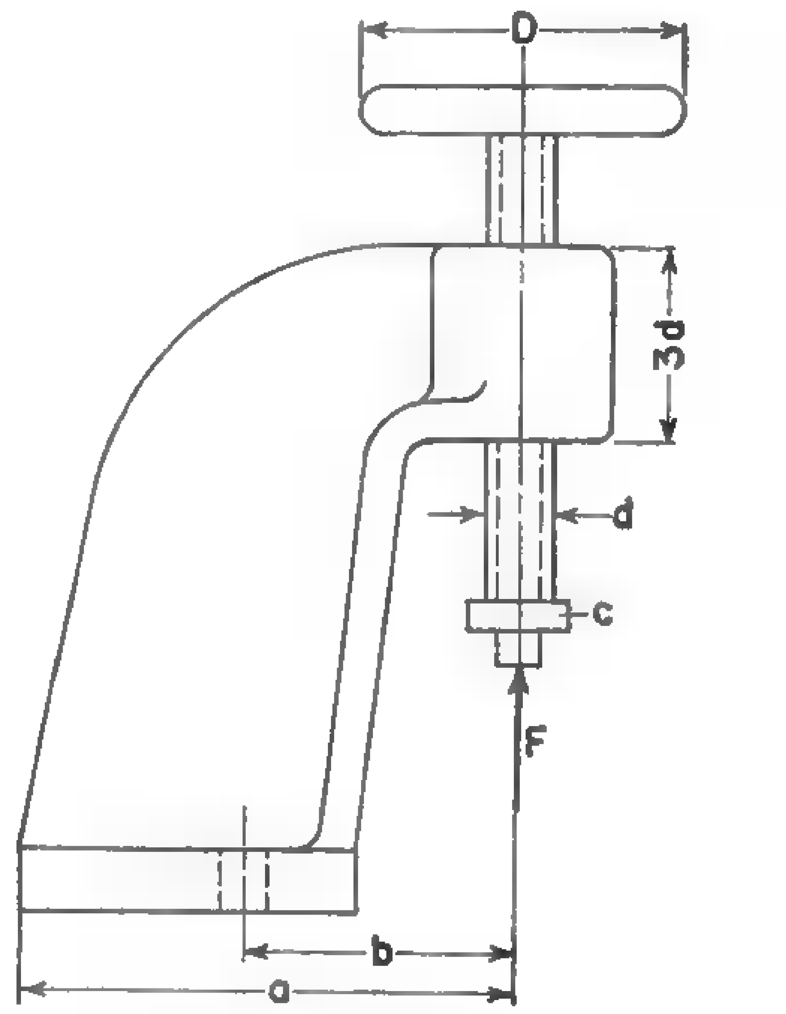


FIG. P19-3.

19–15. The 3-in. screw of a 3-ton shop press, Fig. P19–3, has a Sellers'-type thread. There are two parallel threads, to obtain a greater mechanical advantage. The handwheel has a diameter D of 65 in., and the mean diameter of the thrust collar c is $2\frac{1}{2}$ in. Determine (a) the force that must be applied to the handwheel, assuming that the coefficient of friction in the threads is $f_1 = 0.12$ and that at the thrust collar is $f_2 = 0.125$, (b) the efficiency of the press, (c) the maximum compressive stress in the screw, and (d) the maximum shear stress and the maximum bearing pressure in the threads.

CHAPTER 20: Shafts

20–1. A line shaft rotating at 200 rpm must transmit 60 hp. Assuming that the shaft is of T&G steel shafting, determine (a) the necessary diameter, (b) the commercial size which should be used, and (c) the angle of twist of both shafts in a length of 10 ft.

20–2. Determine the diameter of the shaft, using data of problem 20–1, for the condition that the angle of twist is (a) 1° in a length l of $20D$, (b) 0.75° per foot of length, and (c) 1° per foot of length. (d) Taking into consideration that the total length of keyways cut in the shaft is equal to $0.1l$, find D for an angle of twist of 1° per foot of length.

20–3. (a) Determine the commercial size for a stationary shaft made of SAE 1020 steel supported by two bearings 3 ft apart when a load of 8,000 lb is applied halfway between the bearings. (b) Find the deflection of the shaft. (c) Determine the diameter of the shaft under the condition that the maximum deflection of the shaft should not exceed 0.010 in.

20–4. Determine the shaft diameter for load conditions similar to those shown in Fig. 20–1 in the text, with the following data: The pulley B receives 75 hp at 250 rpm, $D_1 = 42$ in.; $W = 320$ lb; the gear G delivers this power to a shaft located in the same vertical plane but under the shaft to be designed; $D_3 = 14$ in.; $l_1 = 14$ in., $l_2 = 30$ in., and $l_3 = 7$ in.; there is no outside gear H. Find the moments by analytical methods. Assume that the shaft is made of SAE 1020 steel.

20–5. (a) Find the moments and bearing reactions for the data of problem 20–4 by the direct method, and (b) also find the approximate maximum transverse deflection of the shaft.

20–6. Sketch shear diagrams for the load conditions of problem 20–4 in the vertical and horizontal planes.

20–7. Find the diameter of a ship-propeller shaft to transmit 1,600 hp at 95 rpm. The material of the shaft is equivalent to SAE 1025. The thrust from the propeller is 36,000 lb, and the torque is uniform.

20–8. (a) Find the outside diameter of a hollow shaft for the data of problem 20–7, assuming that the inner diameter is about 0.4 of the outside diameter. (b) Find the saving in weight as compared with the solid shaft.

20–9. Find the diameter of the shaft in problem 20–7, using SAE 2340 nickel steel.

20–10. A revolving shaft is subjected to a maximum torque of 15,000 lb-in., and to a maximum bending moment of 32,000 lb-in. Determine the theoretical and commercial-size diameters for the shaft for the following cases, using analytical formulas and assuming SAE 1020 steel: (a) The load is steady. (b) The load is applied gradually. (c) The load is applied suddenly, but with a small shock only. (d) The load is applied suddenly and with heavy shocks.

20–11. Work problem 20–10, using the procedure recommended by the Standard Transmission Shaft Code.

20–12. Work problem 20–10, using the endurance diagram for SAE 1020 steel.

20–13. Determine the shaft diameter in problem 20–4, using the formulas of the Standard Transmission Shaft Code.

20–14. An electric motor transmits 15 hp to a centrifugal pump through a train of gears with a speed ratio of 2.5:1. The motor speed is 1,800 rpm. Determine (a) the diameter of the motor shaft, made of commercial steel shafting, neglecting bending but taking into account the keyway, and (b) the diameter of the pump shaft, made of Tobin bronze, also neglecting bending but not keyseating.

20–15. A piece of $2\frac{7}{16}$-in shafting is supported on bearings 10 ft apart. Determine the maximum permissible speed for this shaft.

20–16. A line shaft rotating at 150 rpm must transmit 60 hp with a torsional deflection not to exceed 1° in a length l of $20D$. For the shafting material the elastic limit in shear is $S_{es} = 22{,}000$ psi and $G = 11.6 \times 10^6$ psi. Determine (a) the commercial diameter of the shaft and (b) the actual safety factor of the suitable commercial size.

20–17. A long line shaft is driven in a machine shop at 210 rpm by a belt drive from an electric motor and must transmit 80 hp. Find the commercial size of shafting to be used.

CHAPTER 21: Couplings and Positive Clutches

21–1. Determine the main dimensions of a flange coupling, Fig. 21–1, for a $4\frac{7}{16}$-in. shaft to transmit the full torque capacity of the shaft.

21–2. (a) Determine the main dimensions of a clamp coupling, Fig. 21–2, for a $4\frac{15}{16}$-in. shaft to transmit the full torque capacity of the shaft. (b) Make a freehand sketch of the assembled coupling, more or less to scale.

21–3. Determine the main dimensions of a compression coupling, Fig. 21–3, for a $3\frac{15}{16}$-in. shaft to transmit the full torque capacity of the shaft.

21–4. (a) Determine the main dimensions of a compression coupling, Fig. 21–4, for a $3\frac{7}{16}$-in. shaft to transmit the full torque capacity of the shaft. (b) Make freehand sketches, more or less to scale, of all details.

21–5. Work problem 21–4 for the compression coupling of the type in Fig. 21–5.

21–6. Work problem 21–4 for the Oldham coupling, Fig. 21–6.

21–7. Work problem 21–4 for the American flexible coupling, Fig. 21–8.

21–8. Determine the main dimensions and give a freehand sketch to scale of an Ajax flexible coupling, Fig. 21–12, for a $3\frac{15}{16}$-in. shaft for heavy-duty service.

21–9. Determine the main dimensions and give freehand sketches of all details for a Westinghouse-Nuttall coupling, Fig. 21–14, for a $4\frac{15}{16}$-in. shaft of an electric generator with fluctuating load, driven by an oil engine.

21–10. Determine the main dimensions and give freehand sketches to scale of all details of a universal joint, Fig. 21–16, for two $2\frac{15}{16}$-in. shafts.

21–11. Determine the main dimensions and give a freehand sketch, to scale, of a flexible coupling, Fig. 21–18, to transmit 50 hp at 600 rpm.

21–12. A slip coupling, Fig. 21–19, is assembled to slip at a torque corresponding to 20 per cent overload on a 300-hp motor running at 360 rpm. The diameters are $D_2 = 31$ in. and $D_1 = 19$ in., the coefficient of friction may be taken as 0.125, the number of bolts and springs is 12, and the spring scale is 2,500 lb per in. Determine the deflection with which each spring must be set up. Assume uniform pressure distribution.

21–13. Work problem 21–12, assuming that the normal wear of the friction surfaces is proportional to the work of friction.

21–14. (a) Determine the main dimensions of a slip coupling, Fig. 21–19, to connect two $3\frac{15}{16}$-in. shafts. Assume a maximum shear stress in the shafts of 6,400 psi and an overload of 20 per cent when the coupling will slip. State reasons for all necessary assumptions and selections. (b) Design the springs and bolts to hold the torque.

21–15. Determine the main dimensions of a jaw clutch coupling for two $3\frac{15}{16}$-in. shafts to transmit the full torque capacity of the shafts. Give freehand sketches of all details and make an assembly view.

CHAPTER 22 Friction Clutches

22–1. Determine the main dimensions and the necessary force of the spring for a cone clutch, Fig. 22–1a, to transmit 45 hp at 2,000 rpm, (a) with a leather facing, (b) with an asbestos-fabric facing, and (c) with cast-iron contact surfaces and cork inserts. Use $\alpha = 12\frac{1}{2}°$.

22–2. (a) Determine the main dimensions, the necessary axial force at the cone surfaces, and the effort which must be applied to the shifting collar *s*, Fig. 22–2, for this clutch to transmit 75 hp at 300 rpm, using cast-iron contact surfaces. (b) Work the problem, using contact surfaces lined with wood blocks. (c) Make a sketch, to scale, of the clutch and the engaging mechanism.

22–3. Using the data of problem 22–2, design a double-cone clutch, Fig. 22–3, applying asbestos-fabric lining.

22–4. Using the data of problem 22–2, design a disk clutch, Fig. 22–4, applying wood blocks on the friction surfaces.

22–5. Determine the main dimensions, the necessary axial force at the friction surfaces, the proper lever dimensions of the engaging mechanism, and the effort which must be applied to the shifting collar for a clutch similar to Fig. 22–4 to transmit 350 hp at 275 rpm from a diesel engine to a boat propeller.

22–6. Using the data of problem 22–1, design a disk clutch similar to that in Fig. 22–5 of the text.

22–7. Using the data of problem 22–2, design a disk clutch, Fig. 22–6, with asbestos-fabric lining.

22–8. Using the data of problem 22–5, design a disk clutch, Fig. 22–7.

22–9. Determine the main dimensions, the necessary axial force at the friction surfaces, the dimensions of the toggle mechanism, and the effort that must be applied to shift the collar when engaging a clutch similar to Fig. 22–8 to transmit 125 hp from an electric motor running at 600 rpm to a line shaft driving metalworking machinery.

22–10. Using the data of problem 22–2, design a cone-disk clutch, Fig. 22–9, applying (a) wood blocks and (b) asbestos-molded blocks.

22–11. Using the data of problem 22–5, design a rim clutch, Fig. 22–10.

22–12. Using the data of problem 22–5, design a rim clutch, Fig. 22–11.

22–13. Determine the main dimensions and the necessary axial force to be applied to the shifting collar of an expansion clutch, Fig. 22–13, to transmit a torque of 2,000 lb-in. and to go on a $2\frac{7}{16}$-in. shaft.

22–14. Determine the necessary dimensions of a roller clutch, Fig. 22–15, to transmit 45 hp at 2,000 rpm.

CHAPTER 23: Bearings with Sliding Contact

23–1. Calculate the viscosity Z at 145 F of an oil for which SSU = 620 sec at 145 F and $\gamma_{60} = 0.9254$ at 60 F.

23–2. Calculate the viscosity Z, at 150 F, of oil D, Table 23–1.

23–3. Using equation 23–10, calculate the safe load on a bearing with a 4-in. diameter and a 6-in. length, if the viscosity Z at the operating temperature is 30 centipoises and the journal rotates at 780 rpm.

23–4. The radial load on a 3-in. oil-ring bearing 4 in. long is 1,100 lb. The bearing clearance is 0.004 in., and the journal turns at 300 rpm. Oil D, Table 23–1, is used, and the operating temperature is 125 F. Determine the load the bearing can carry when running at 480 rpm, taking into account (a) only the influence of speed, equation 23–9, and (b) the heat generated by friction at an oil temperature of 155 F.

23–5. Work problem 23–4, assuming that the loss of power due to friction is twice as great at 480 rpm as at 300 rpm, but that because of better cooling the oil temperature goes up only to 150 F.

23–6. Work problem 23–5, but assume that the bearing clearance was increased to 0.005 in.

23–7. The main bearings of a steam engine are 7 in. in diameter, the load coming upon each bearing is 15,000 lb, and the engine speed is 135 rpm. Determine (a) the length of the bearing, (b) the necessary running clearance, using SAE 60 oil, and (c) the horsepower lost in friction at each bearing.

23–8. Determine the necessary diameter, length, running clearance, and loss of power due to friction of the crankpin bearing of a stationary steam engine for the following conditions: The cylinder bore is 20 in., the admission pressure is 145 psig, the back pressure is 20 psig, the mean indicated pressure is 31 psi, and the speed is 200 rpm. Select a suitable lubricant.

23–9. (a) Determine the necessary length of the bearing, running clearance, side relieves, and loss of power due to friction for a journal which has a $2\frac{7}{16}$-in. diameter and rotates at 175 rpm. The total load on the bearing due to belt pull and weight of pulleys is 1,400 lb, and the load is steady. Use a lubricating oil which has a Saybolt viscosity of 270 sec at 100 F and for which $\gamma = 0.905$ at 60 F. (b) Determine the temperature of the oil film and the necessary area to dissipate the heat of friction.

23–10. A $2\frac{11}{16}$-in. transmission shaft running at 260 rpm is supported on five 7-in.-long Sellers' oil-ring bearings, Fig. 23–14. Determine the expected oil temperature and the horsepower loss in the bearings if oil D, Table 23–1, is used and the average unit pressure on the bearings is 100 psi.

23–11. A locomotive weighs 60 tons, and 15 per cent of this weight is carried by four truck wheels 3 ft in diameter having journals $4\frac{1}{2}$ in. in diameter and 6 in. long. The rest of the weight is distributed uniformly over eight 6-ft driving wheels having journals 8 in. in diameter and $9\frac{1}{2}$ in. long. (a) Find the loss of power due to friction in the journals of the truck wheels with SAE 110 lubricating oil at a speed of 20 mph. (b) Check the mini-

mum oil-film thickness. The heat-dissipating area of each bearing is 430 sq in., and the air temperature is 80 F.

23–12. A journal bearing $2\frac{1}{2}$ in. in diameter and $3\frac{3}{4}$ in. long with a relief of 15° on each side of the bearing shells carries a load of 1,800 lb at 1,200 rpm. Assume a cooling area of 450 sq in., a cooling rate of 2 Btu per hr per sq ft per deg F, and a room temperature of 72 F. Use oil B, Table 23–1. Compute the oil-film temperatures and the minimum film thicknesses for relative clearances c/d of 0.001, 0.00133, 0.00167, and 0.002. Tabulate the calculations and plot the results.

23–13. Work problem 23–12, using oil D, Table 23–1.

23–14. Work problem 23–12 for a load of 1,200 lb at a speed of 1,000 rpm, using oil B, Table 23–1.

23–15. Work problem 23–12 for a load of 1,500 lb, using oil D, Table 23–1.

23–16. Find the loss of power due to friction, and check the minimum oil-film thickness, for the journals of the driving wheels of problem 23–11. The heat-dissipating area of each bearing is 1,100 sq in., and the locomotive speed is 20 mph.

23–17. Work problem 23–11 for a speed of 50 mph.

23–18. Work problem 23–16 for a speed of 50 mph.

23–19. The maximum load which is taken by the bearings of a punch press as the punch enters a steel plate is 75 tons. Assuming that this load is distributed equally between the two bearings, one on each side of the eccentric, determine the minimum diameter and length of the bearings.

23–20. (a) Determine the main dimensions of an oil-ring bearing for a steam turbine that runs at 3,600 rpm. The load on the bearing is 8,000 lb. Use oil A, Table 23–1. (b) Find the expected oil temperature if the room temperature is 80 F.

23–21. Work problem 23–20, assuming that an ordinary bearing with force-feed lubrication is used. Determine the minimum amount of oil that must be supplied, assuming that the temperature rise of the oil when passing through the bearing is (a) 50 deg F and (b) 70 deg F.

23–22. Determine the amount of water that must be circulated to cool the bearing of problem 23–21 if it is desired to maintain the oil temperature at 150 F. Assume that the temperature of the water entering the heat exchanger is 70 F and that a temperature rise of 40 deg F is allowed.

23–23. A walking beam is 12 ft long, and the bearing fulcrum is 4 ft from one end and 8 ft from the other. The long end pulls a vertical load of 16,000 lb. The short end is connected by a 6-ft rod to a crankpin with a 12-in. radius of rotation. The diameter of the fulcrum pin is 6 in. Assuming that oil H, Table 23–1, is used, and that thin-film lubrication exists, determine (a) the length of the fulcrum pin necessary to support the load and (b) the work lost in friction, in ft-lb per min, when the crankpin turns at 30 rpm.

23–24. Determine the necessary width and length of a crosshead shoe for a double-acting air compressor. The intake pressure is 14.3 psia, and the discharge pressure is 75 psia. The bore is 10 in., the stroke is 15 in., the length of the connecting rod is $33\frac{3}{4}$ in., and the shoe length should be 1.25 times its width.

23–25. Determine the minimum height of the piston required to take the side thrust on the piston of a $3 \times 3\frac{3}{4}$ in. automobile engine. The firing pressure is 360 psig, the maximum side thrust occurs when the crank angle is 35°, the amount of this thrust is about 0.1 of the firing pressure, and the length of the connecting rod is $8\frac{1}{2}$ in.

23–26. Design a foot-step bearing similar to that in Fig. 23–29, for a column crane. The crane weighs 10 tons, and the maximum useful load is 25 tons.

23–27. A multicollar bearing similar to that shown in Fig. 23–30 has six collars which have an inside diameter of $2\frac{1}{2}$ in. and an outside diameter of $3\frac{1}{2}$ in. Determine the thrust load which the bearing can carry and the horsepower lost through friction if the shaft speed is (a) 200 rpm, (b) 600 rpm, (c) 1,200 rpm, and (d) 20 rpm (for slow and intermittent service).

23–28. Determine the amount of oil and water that must be circulated through the bearing of problem 23–27, for cases a, b, and c if the rise of the oil temperature is 50 deg F and the water temperature rise is 20 deg F.

23–29. The oil engine of a motorboat develops 1,000 hp at 200 rpm. The speed of the boat is 14 knots, and the propeller slip is 27 per cent. (a) Determine the main dimensions of a multicollar thrust bearing similar to that in Fig. 23–30. (b) Find the loss of horsepower caused by friction. Select the oil to be used for lubrication.

23–30. Determine the amount of oil and water that must be circulated through the bearing of problem 23–29. The rise of the oil temperature is 52 deg F, and the rise of the water temperature is 20 deg F.

CHAPTER 24: Bearings With Rolling Contact

24–1. State the conditions for which a ball bearing should be used in preference to a sliding bearing (a) for a radial load and (b) for an axial thrust load.

24–2. Determine the type and size of a ball bearing for a $2\frac{15}{16}$-in. shaft. The shaft speed is 325 rpm, the radial load is 2,000 lb, with very light shocks, and the axial load is 750 lb. The installation is a temporary one, to serve not over 1 year with 8-hr service per day. The bearing is to be placed 3 ft from one end of the shaft.

24–3. Sketch the method of mounting the bearing of problem 24–2, showing the methods of lubricating it and preventing dust from entering the housing.

24–4. Determine the radial capacity of an SKF 6211-Z bearing running at 500 rpm and carrying an axial load of 500 lb, for 2 years of continuous service.

24–5. A ball bearing is to be used on a drill press operating at 3,000 rpm with a 250-lb maximum thrust and a 500-lb radial load. The press will be operated five 8-hr days a week but will be idle 20 per cent of the time. Determine the type and size of the bearing (a) if it should last 1 year and (b) if it should last 2 years.

24–6. Select suitable ball bearings for the spindle of a woodworking machine revolving at 1,200 rpm. One bearing is subjected to a radial load of 600 lb and a thrust load of 450 lb; the other carries only a radial load of 650 lb. The machine is to be used 8 hr per day and 5 days a week, and a service life of 10 years is desired. The diameter of the spindle is 2 in., and it can be turned down slightly.

24–7. Work problem 24–6, assuming that the speed is 2,500 rpm and the desired life is 5 years.

24–8. Select a ball bearing to support a radial load of 2,500 lb on a $1\frac{11}{16}$-in. shaft turning at 640 rpm. The shaft is to be operated 10 hr per day for 8 years.

24–9. State the conditions for which a roller bearing should be used in preference to a ball bearing.

24–10. Determine the capacity of a Hyatt bearing with a solid outer race and no inner race, for a 4-in. shaft at 650 rpm. The shaft is used in a hoist, its surface has a hardness of 250 Bhn, and the temperature of the shaft is 175 F. Use Kent's Handbook.

24–11. Determine the capacity of a medium series Norma-Hoffman roller bearing for a 100-mm shaft running at 650 rpm. The temperature of the shaft and bearing housing is 175 F, the load is steady, and the life of the bearing should be 3 years with a daily service of 10 hr.

24–12. Design the wristpin, and a needle-roller bearing for it, for a 10-in. oil-engine piston. The maximum pressure is 650 psi, and the engine speed is 310 rpm. Give all dimensions with the necessary tolerances.

24–13. Determine the radial and axial capacities of a Timken taper-roller bearing for a 4-in. shaft running at 850 rpm. The roller temperature is 125 F.

24–14. A shaft is supported by two bearings 16 in. apart and carries a bevel gear of 7.750-in. pitch diameter 6 in. from one end. The gear produces a radial load of 2,150 lb and a thrust load of 625 lb when rotating at 525 rpm. Determine (a) the shaft diameter if the shaft is made of SAE 1045 steel and (b) the proper type and size of ball bearings to be used on each end of the shaft. The desired life is [illegible] years, at 50 hr per week.

CHAPTER 25: Crankshafts

25–1. Determine the main dimensions of a crankshaft with center cranks for a single-acting 7×7 in. air compressor running at 360 rpm. Maximum air pressure: 100 psig.

25–2. Work problem 25–1 for an 8×8 in. compressor running at 300 rpm.

25–3. Work problem 25–1 for a shaft with an overhung side crank, Fig. 25–6. The distance from bearing center to bearing center is 12 in.

25–4. Work problem 25–2 for a shaft with an overhung side crank, Fig. 25–6. The distance between bearing centers is 16 in.

25–5. Determine the main dimensions of a crankshaft for a $3\frac{7}{16} \times 3\frac{3}{4} \times 6$ in. autobile engine. The maximum pressure is 360 psi at speeds of 2,000 to 3,600 rpm. The center lines of the cylinders are $4\frac{3}{4}$ in. apart.

25–6. Determine the main dimensions of a crankshaft for a $8 \times 9\frac{1}{2} \times 4$ in. two-stroke gas engine running at 480 rpm. The maximum pressure is 375 psi. The distance between cylinder center lines is $11\frac{1}{2}$ in.

25–7. Find the equivalent length of a shaft system consisting of the following: an oil-engine crankshaft 8 in. in diameter and 10 ft 11 in. long; and intermediate shaft $5\frac{3}{4}$ in. in diameter and 32 ft long; and a propeller shaft $6\frac{1}{4}$ in. in diameter and 20 ft long.

25–8. Find the equivalent length of the shaft system of problem 25–10, taking into account the rigidity of the crank webs. Their width must conform to the American Bureau of Shipping requirements. The crankpin diameter is 8 in., the length of the crankshaft from one end to the first cylinder center line is $28\frac{1}{2}$ in., the length of the main journals is $7\frac{1}{2}$ in., the distance between cylinder center lines is $20\frac{1}{2}$ in., the crank webs are $3\frac{5}{8}$ in. thick, and the piston stroke is 17 in.

25–9. Find the natural frequency of torsional vibration of the combined shaft of problems 25–1 and 25–2, assuming that there is between the crankshaft and the intermediate shaft a flywheel that weighs 2,830 lb and has a radius of gyration of 2.13 ft. The propeller weighs 675 lb and has a radius of gyration of 19 in. The weight assumed as concentrated at each crankpin is 655 lb.

CHAPTER 26: Flywheels

26–1. In a certain engine the mean flywheel diameter is 72 in. The weight of the rim is 1,500 lb. The maximum and minimum instantaneous wheel speeds are 195 and 185 rpm, respectively. Determine (a) the excess energy stored and given up by the flywheel, (b) the coefficient of uniformity of rotation, and (c) the kind of driven machinery that can be connected to this engine, and the type of drive which must be used in each case.

26–2. Find the flywheel effect necessary to obtain a coefficient of steadiness m of 50 with a four-stroke, four-cylinder 12×15 in. gas engine running at 300 rpm. The 6-in. length of the torque diagram covers 180 deg of the crank travel, its scale of ordinates is 64 psi per inch, and the excess area is $e = 2.71$ sq in.

26–3. Find the main dimensions of the flywheel in problem 26–2. Assume that the maximum cylinder pressure is $p_{\max} = 280$ psig and that, at the crank angle of maximum torque, $p = 0.4p_{\max}$.

26–4. Find the stresses in the rim of the flywheel of problem 26–3.

26–5. Compute the stress in the arms of the flywheel of problem 26–3 when the engine load is increased so suddenly that the engine slows down to 240 rpm after three revolutions.

26–6. Determine the weight of the flywheel required for a punch press for the following conditions: The press requires 15 hp; the complete cycle consists of 7 revolutions of the flywheel, only two and one-half of which take place during the working portion of the cycle; there are 30 complete cycles per minute; and the desirable mean diameter is 52 in.

26–7. Design a flywheel for a punch press which must be brought to rest by one punching operation if the power has been shut off. The maximum work required consists in punching a $1\frac{3}{16}$-in. hole in a $\frac{3}{4}$-in. mild-steel plate. The punch capacity is 24 holes per minute, and the speed ratio of the driving shaft to the eccentric shaft operating the punch is 9:1. In order to clear the floor the wheel diameter cannot be larger than 42 in. The wheel is keyed to the driving shaft, and the mechanical efficiency of the press and drive is 72 per cent. Determine (a) the weight of the flywheel rim, (b) the cross section of the rim, (c) the maximum stress in the rim, assuming six arms, and (d) the coefficient of steadiness of the driving shaft, if the duration of the working stroke is one-half that of the idle stroke.

26–8. Design the flywheel for the punch press of problem 26–7. The requirement is that the flywheel must give a coefficient of steadiness m of 5.

26–9. A cast-iron flywheel running at an average speed of 92 rpm must supply 7,200 ft–lb of energy to a punch press during 0.2 revolution with a 15 per cent change of speed. The maximum velocity at the mean radius must be 1,000 fpm. Determine (a) the dimensions of the rim and (b) the horsepower of the motor required to drive the press if the over-all efficiency of the machine is 78 per cent.

26–10. A one-piece cast-iron flywheel has an outside diameter of 54 in. and a rectangular rim section with a 7-in. face and 4-in. depth. Determine (a) the safe speed in rpm of the wheel, (b) the theoretical bursting speed, and (c) the safe speed of the flywheel if it were made of cast steel.

26–11. A flywheel 12 ft in diameter rotates at 160 rpm. The rim has a rectangular section with a face of 16 in. and a depth of 6 in. and is cast in two parts held together by two shrink links, Fig. 26–5, at each joint. Determine the main dimensions, the cross section, the length of the links, and the temperature to which the links must be heated before being put in place.

26–12. (a) Find the probable maximum stress which would be induced in the rim of a flywheel with an outside diameter of 12 ft and a rim 15 in. wide and 9 in. deep, operating at 175 rpm. Assume that the wheel is cast in one piece and has eight elliptical arms. (b) State whether this stress and speed are safe in a cast-iron wheel. (c) Indicate methods of increasing the safety of a flywheel at the given speed.

26–13. The wheel diameter in problem 26–12 was increased to 13 ft, and the wheel had to be cast in two parts. The rim section was retained unchanged. Assuming that the rim joints are located at the center lines of the arms, determine the safe speeds for (a) joints bolted at the hub and at the rim, (b) joints using shrink links, Fig. 26–5, (c) joints using shrink anchors, Fig. 26–6, and (d) joints of the type shown in Fig. 26–7.

26–14. Design a rim section and a cotter joint, Fig. 26–8, for the flywheel of problem 26–13.

CHAPTER 27: Belt Drives

27–1. A $\frac{3}{8}$-in. chrome-tanned leather belt 14 in. wide is running over a 72-in. cast-iron driving pulley and an $18\frac{1}{2}$-in. paper driven pulley. The larger pulley is keyed to the shaft of a gas engine and turns at 235 rpm. Assuming an endless belt and a center distance of 22 ft, determine the horsepower which this belt will transmit and the probable speed of the driven compressor pulley.

27–2. Using the data of problem 27–1, find the horsepower and speed of the driven pulley if the center distance is reduced to 5 ft and an idler pulley is used to increase the arc of contact.

27–3. A line shaft must be connected to the driving motor by a medium double leather belt. The cast-iron motor pulley is 17 in. in diameter and runs at 900 rpm. The actual speed of the line shaft should be reduced in the ratio 4.75:1. If all the machines driven from the line shaft are operating simultaneously, the total maximum load is 112 hp. However, the actual load is never higher than 72 percent of the maximum load. Determine the necessary width of the belt, and the diameter and width of the line-shaft pulley. The center distance is 18 ft and makes a 40° angle with the horizontal line.

27–4. Using data of problem 27–3, select the proper number of plies of a rubber belt to replace the leather belt, and determine its width and the diameter and width of the line-shaft pulley.

27–5. A seven-ply 20-in. balata belt is to be replaced by an oak-tanned leather belt. The diameter of the driving pulley is 68 in. and its speed is 215 rpm; the diameter of the driven pulley is 40 in.; and the center distance is 22 ft. Assume all other necessary data and determine the width and thickness of the leather belt.

27–6. A heavy triple leather belt that is 36 in. wide and travels at the rate of 5,500 fpm over pulleys 42 in. and 78 in. in diameter must be replaced by a rubber belt. Determine the number of plies and the width of the rubber belt. The center distance is 25 ft.

27–7. (a) Using the data of problem 27–6, and assuming that the smaller pulley is the driver, determine the speeds of the pulleys and the power transmitted by the belt. (b) Find the initial tension which must be put in the belt to transmit the load.

27–8. A $7\frac{1}{2}$-hp motor running at 1,175 rpm is installed with a Rockwood mounting, Fig. 27–4a in the text. The diameter of the cast-iron motor pulley is 9 in., the motor weighs 260 lb; the tight belt side is horizontal, and the angle of belt wrap is $\theta = 165°$; $b = 12$ in.; and the starting torque is 200 per cent of the motor rating. Determine (a) the belt tensions F_1 and F_2 at rated load of the motor, (b) the distance a at full load, (c) the tensions F_1 and F_2 at one-half of motor load with unchanged distance a, and (d) the tensions F_1 and F_2 when the motor is running idle.

27–9. (a) Using data of problem 27–8, find the belt tensions F_1 and F_2 for starting the drive under load. (b) Assuming that during starting the distance a is $9\frac{1}{2}$ in., explain the operation of the Rockwood mounting during starting.

27–10. A 5-hp electric motor running at 1,180 rpm is installed with a Rockwood mounting, Fig. 27–4a. The diameter of the paper motor pulley is 9 in., the motor weighs 200 lb, the tight side of the belt is horizontal, the driven pulley must run at 480 rpm, and the distance between the pulley centers is 20 in. (a) Determine the distance a for full-load operation of the drive if $b = 10$ in. (b) Compare the belt tensions F_1 and F_2 at full, three-quarter, and half-load operation if distance a is adjusted for full-load conditions and kept unchanged.

27–11. (a) Design a leather-belt drive from a 160-hp gas engine running at 360 rpm to a vertical deep-well centrifugal pump to run at 1,150 rpm. (b) After all necessary data are obtained, make a layout for a horizontal quarter-turn drive with a long center distance. (c) Make a layout for a drive using a center distance of 18 ft and a double-pulley idler, as in Fig. 27–7.

27–12. Design and make a sketch of the main pulley for data of problem 27–11.

27–13. Design and make a sketch of a pulley 40 in. in diameter to transmit 45 hp at 175 rpm by means of a leather belt. The pulley shaft is $3\frac{7}{16}$ in. in diameter.

27–14. Design a V-belt drive for a 125-hp centrifugal water pump running at 1,200 rpm and driven by an oil engine running at 350 rpm. Make the center distance as short as possible.

27–15. Design a V-belt drive, using data of problem 27–14, for a V-flat drive, and make sketches of both pulleys.

27–16. Design a V-belt drive for a 5-hp motor running at 1,180 rpm and driving an air compressor at 500 rpm. Select the center distance and give reasons for the selection.

27–17. (a) Design a V-belt drive for the data of problem 27–16, using a V-flat belt. Select the center distance and give reasons for the selection. (b) Give a sketch of the small pulley.

CHAPTER 28: Chain Drives

28–1. Design a roller-chain drive for a small fan. The motor speed is 850 rpm, the desired fan speed is 550 rpm, and the power requirement is 5 hp.

28–2. Design a roller-chain drive to transmit 45 hp from a gas engine running at 240 rpm to a pump with a speed of 75 rpm. Determine the length of the chain, using a center distance equal to the sum of the two sprocket diameters.

28–3. Determine the horsepower which can be transmitted by a four-strand, [illegible]-in.-pitch roller chain driven by a motor at 1,750 rpm. The motor sprocket has 17 teeth, and the driven sprocket has 42 teeth.

28–4. A roller chain operates under a steady load and transmits 7.5 hp from a motor shaft rotating at 720 rpm to a shaft running at 950 rpm. Determine (a) the chain and sprockets required, (b) the pitch diameters of the sprockets, (c) the shortest suitable center distance, and (d) the length of the chain in number of links and in feet.

28–5. A 10-hp motor running at 1,170 rpm drives a line shaft at 240 rpm through a roller chain. The motor-shaft diameter is $1\frac{1}{2}$ in. The starting torque is two times the running torque, and the load produces moderate shocks. Determine (a) the type and size of a suitable roller chain, (b) the pitch diameters of the sprockets, (c) the closest advisable center distance and the corresponding length of chain, and (d) the longest permissible center distance.

28–6. A roller chain must be used to drive the camshaft of a four-stroke gasoline engine running at 720 rpm. The center distance is approximately 24 in. The crankshaft diameter is $5\frac{1}{4}$ in. The camshaft drive requires 3 hp. Determine all necessary dimensions for the chain and sprockets, and check the chain for impact and centrifugal force.

28–7. An oil engine developing 250 hp at 1,200 rpm drives a wire-rope reel on an oil-well rig through a roller chain. The speed of the reel varies from 10 to 50 rpm. The engine can be slowed down to 240 rpm, and the torque remains approximately constant. The load is applied with heavy shocks. (a) Select a suitable roller chain. (b) Determine the numbers of teeth, the pitch diameters, and the outside diameters of the sprockets. (c) Determine a suitable center distance and length of chain.

28–8. A 60-hp truck engine uses a roller chain as the final drive to the rear axle. The driving sprocket runs at 225 rpm, the driven sprocket runs at 100 rpm, and the center distance is approximately 36 in. The efficiency of the transmission between the engine and the driving sprocket is 85 per cent. (a) Select a suitable chain, using a low velocity, not over 700 fpm. (b) Determine the number of teeth in the sprockets, their pitch diameters, and their outside diameters, and (c) find the length of the chain.

28–9. Design a drive, using a Morse silent chain, to transmit 150 hp with a motor speed of 439 rpm and a speed of 88 rpm of the driven compressor. The drive must operate 15 hr per day.

28–10. Using data of problem 28–9, design a drive using a Whitney silent chain.

28–11. Design a drive, using a Morse silent chain, to transmit 500 hp at a motor speed of 320 rpm and a driven speed of 55 rpm. Assume a uniform power requirement and also operation for 12 hr per day.

28–12. Design a drive, using a Morse silent chain, to transmit 50 hp at a motor speed of 700 rpm to a centrifugal pump running at 1,100 rpm.

28–13. With data of problem 28–12, design a drive using a Whitney silent chain.

28–14. Determine the horsepower which can be transmitted by a Morse silent chain 30 in. wide with a pitch of 2 in. The motor speed is 253 rpm, the driven-shaft speed is 56 rpm, and the number of teeth in the motor sprocket is 21. Also find the minimum desirable center distance and the length of chain. Assume continuous operation (a) for 9 hr per day and (b) for 18 hr per day. The load is fairly steady in both cases.

28–15. Determine the horsepower which can be transmitted by a Morse silent chain drive. The chain pitch is 1 in., its width is 7 in., the motor speed is 580 rpm, its sprocket has 29 teeth, and the driven centrifugal pump has a speed of 800 rpm. The pump is operated 22 hr per day for four months each summer.

28–16. Check whether the drive of problem 28–15 complies with the recommendations of the chain manufacturer.

CHAPTER 29: Friction Gearing

29–1. Two shafts 24 in. between centers are connected by a pair of plain spur friction wheels. The driver makes 380 rpm, and the follower makes 140 rpm. Select the materials for both wheels, and find the thrust and width of face necessary to transmit 12.5 hp.

29–2. Using data of problem 29–1, design wheels with grooved faces (a) if both wheels are of cast iron and (b) if the driver face is made of maple. (c) Also find the bearing pressures for both cases.

29–3. A small printing press is driven by a 3.5-hp motor running at 1,175 rpm. On the motor shaft is a rubber spur friction wheel $5\frac{1}{2}$ in. in diameter. Find (a) the necessary width of the wheel and (b) the required bearing pressure.

29–4. A pair of grooved spur friction wheels is used to connect two shafts. The driver makes 260 rpm, and the follower makes 200 rpm. The power to be transmitted is 45 hp. Using cast iron for both wheels, find the diameters of the wheels. Select the angle of grooves and their depth, and find their number and the least pressure on the bearings.

29–5. If the driver in problem 29–4 has three grooves $\frac{1}{4}$ in. deep, with $\alpha = 16°$, find (a) the least pressure per inch of contact line and (b) the bearing load.

29–6. Find the horsepower that can be transmitted by a grooved-face spur friction driver with a 20-in. diameter running at 340 rpm. Both wheels are of cast iron, the groove angle is 14.5°, the wheels are pressed together with a force of 950 lb, and the driver has two grooves with an effective depth of $\frac{7}{32}$ in.

29–7. A hoist driven by spur friction gears has two driving wheels 8 in. in diameter with tarred fiber faces, the two followers are 48 in. in diameter and are keyed to the drum shaft, and the faces are 6 in. wide. Determine (a) the load which can be hoisted if the drum diameter is 24 in. and a $\frac{3}{4}$-in. cable of mild plow steel is used and (b) the horsepower required if the drivers make 215 rpm.

29–8. A small fan requiring 1.5 hp is to be driven from a 1,200-rpm motor by means of bevel friction gears. (a) Using a speed ratio of 1.9 :1, select the materials and determine the main dimensions of the wheels. (b) Determine the bearing pressures when starting and when running.

29–9. A grinding machine requires 1.2 hp and must operate at speeds varying from 300 to 2,400 rpm. The motor runs at 1,175 rpm. Select all materials for a suitable disk friction drive, and determine the main dimensions and all forces involved in the operation of the drive.

CHAPTER 30: Straight and Helical Spur Gearing

30–1. A gear having a pitch diameter of 6 in. must transmit 4 hp at 100 rpm. The service is intermittent. Determine the pitch, the number of teeth, and the width of the gear face, using cast iron for (a) cast teeth and (b) cut teeth.

30–2. A cast-tooth cast-iron gear has 28 teeth of $1\frac{1}{4}$-in. pitch and a 3-in. face. Determine the horsepower which it can transmit (a) at 80 rpm and (b) at 160 rpm.

30–3. An SAE 1030 steel pinion has 18 teeth of $14\frac{1}{2}°$ full-depth type with a $2\frac{1}{4}$ pitch and a $4\frac{1}{2}$-in. face. At 600 rpm it transmits a torque of 6,500 lb-in. to a cast-iron gear with 60 teeth. Determine the stresses in the teeth of the pinion and of the gear, the margin of safety for dynamic loads, and whether the pinion and the gear are suitable for continuous service if both are ordinary commercial products. State the changes which may be necessary to obtain a sufficient margin of safety for dynamic loads and to resist wear in continuous service.

30–4. A 40-tooth phosphor-bronze gear runs with a 20-tooth steel pinion of 175 Bhn. The gears are cut on the 20° full-depth involute system and are of 4 diametral pitch with a $2\frac{1}{2}$-in face. The pinion speed is 1,000 rpm. Determine the horsepower that the gears can transmit, based on (a) static strength, (b) dynamic load, and (c) wear resistance.

30–5. A pair of 20° involute stub-tooth gears must transmit 120 hp at 720 rpm of the pinion, with a 3 :1 speed reduction. The pinion has 24 teeth, and the gears have a 5 diametral pitch and $2\frac{1}{2}$-in. faces. Find the minimum necessary hardness of the teeth of the pinion and of the gear, based on wear resistance.

30–6. Two shafts running at 100 and 135 rpm and transmitting 75 hp are connected by cut spur gears of high-grade cast iron. Determine the necessary pitch, number of teeth, face of the gears, and exact center distance. The service is 18 hr per day.

30–7. Using data of problem 30–6, design the gears if the desired center distance is approximately 25 in.

30–8. A 25-hp electric motor runs at 1,175 rpm and drives through a train of spur gears a shaft whose velocity is about 225 rpm. Use a silent-material pinion. Compute the main dimensions of the gears, using $14\frac{1}{2}°$ full-depth involute teeth.

30–9. With the data of problem 30–8, design the drive if the 20° full-depth involute-tooth system is used.

30–10. Using the data of problem 30–8, determine all main dimensions of a drive with herringbone gears. The pinion is of hardened steel, and the gear is of cast iron.

30–11. A pair of spur gears must transmit 50 hp from a shaft running at 300 rpm to another shaft with a speed reduction of 3.5 :1. The center distance of the shafts is 15.750 in. Determine (a) the diametral pitch and the number of teeth of the gears, (b) the face of the gears, taking into account dynamic load and wear, and (c) the materials that must be used for the pinion and the gear.

30–12. A reciprocating compressor must be driven by an 870-rpm electric motor through a pair of straight spur gears. The compressor should run at about 200 rpm and requires a torque of 2,500 lb-in. Assume a starting overload of 25 per cent and determine (a) the necessary horsepower of the motor, (b) the diametral pitch and face of the gears, using 20° stub teeth, and (c) the number of teeth and the pitch diameter of each gear. (d) Specify the materials for the pinion and the gear.

30–13. A 15-hp motor running at 1,170 rpm drives a fan through a pair of spur gears with a reduction ratio of about 3.9 :1. A micarta pinion and a cast-iron gear are specified. Determine (a) the diametral pitch and the number of teeth in the gears, (b) the gear face, and (c) the pitch diameters and the center distance.

30–14. The gate of a sluice valve weighing 6 tons is raised by means of a cast-iron rack and pinion. Design a train of gears, including the rack, so that the gate may be raised by two men working on 15-in. crank handles and exerting a pressure of 35 lb each. Give also the linear speed of the rack motion, assuming that the hand crank makes 25 rpm.

30–15. A punch press running at 42 rpm is driven by a 12.5-hp motor running at 1,200 rpm. Using a double-gear reduction, select all materials and design the gear train, stating all other assumptions. Give a sketch of the gear train.

30–16. The drum diameter of a hoist is 15 in., and the drum is to be bolted by a flange to a gear approximately 26 in. in diameter. The capacity of the hoist is 2 tons, and the motor speed is 700 rpm. A hoisting speed of about 125 fpm is desired. Select the number of gear pairs in this train, and materials for the gears. Determine the pitches, using Fellows stub teeth; the faces; the number of teeth; and the center distances of the whole mechanism. Illustrate the results by sketches.

30–17. Design a train of gears to transmit 800 hp from a shaft running at 3,600 rpm to one running at 200 rpm. Use herringbone gears, and select the materials, the helix angle, and the pressure angle.

30–18. Make a sketch of the large gear of problem 30–3, showing all dimensions which can be determined by the data given in section 30–10.

30–19. Make a sketch of the large gear of problem 30–17, assuming that a welded construction similar to that in Fig. 30–14 in the text is used.

CHAPTER 31: Bevel Gearing

31–1. Determine the pitch, the width of face, the number of teeth, the outside diameter, and all angles—pitch, face, and cutting—for a pair of cast-iron straight bevel gears with tooth proportions according to the AGMA standard, to transmit 20 hp. The speed of the driving shaft is 600 rpm, and that of the driven shaft is 190 rpm. Check for dynamic load and wear. Assume continuous operation and very light shock.

31–2. A pair of straight bevel gears must transmit 20 hp at 1,250 rpm of the 18-tooth pinion. The speed-reduction ratio is 3.5 :1. Use $14\frac{1}{2}°$ full-depth teeth. Select the materials to obtain a compact design. Determine the diametral pitch, the gear face, the pitch diameters, and the pitch-cone angles for both gears.

31–3. A pair of straight bevel gears, with a 5 diametral pitch and $14\frac{1}{2}°$ machine-cut teeth, are made of SAE 3245 steel and have a 2:1 reduction. The pitch diameter of the driver is 5 in., and the gear face is 1.80 in. Determine (a) the pitch angles of the pinion and gear, (b) the face angles of the pinion and gear, (c) the cutting angles of the pinion and gear, (d) the maximum diameters of both gears, (e) the formative numbers of teeth, (f) the equivalent tangential load at the large end of the teeth at 300 rpm of the driver, (g) the pitch diameter of the effective tooth load, (h) the effective tooth load, and (i) the horsepower transmitted at 300 rpm of the driver.

31–4. Design a pair of straight cast-iron bevel gears to transmit 120 hp from a shaft running at 235 rpm to another running at 75 rpm. Make a sketch of the gear with all dimensions pertaining to the rim, arms, and hub. Check for continuous operation.

31–5. Design ■ pair of Gleason straight bevel gears to transmit 65 hp from a gas-engine-operated driving shaft making 450 rpm to a vertical deep-well pump making 1,100 rpm. The operation is to be considered continuous.

31–6. Design a pair of Gleason straight bevel gears to transmit 65 hp from ■ gasoline engine making 1,500 rpm to a jackshaft making 400 rpm. Assume continuous operation.

31–7. A pair of Gleason straight bevel gears consists of a pinion with 16 teeth and a gear with 48 teeth. The diametral pitch is 2.5, the face is $2\frac{1}{2}$ in., the material is nickel cast iron, and the service is continuous with light shocks. Find what horsepower can be transmitted if the pinion speed is (a) 600 rpm and (b) 1,200 rpm.

31–8. Design a pair of Gleason gears to transmit 20 hp at 900 rpm with ■ speed ratio of 2.1:1 and straight teeth. Assume intermittent service with light shocks. Use steel for the pinion and class 35 cast iron for the gear.

31–9. The ring gear of a truck differential has 50 Gleason straight teeth of 4 pitch and is made of SAE 2345 steel hardened to 240 Bhn. The pinion has 13 teeth with a $1\frac{3}{8}$-in. face and is made of SAE 2345 steel hardened to 300 Bhn. Determine (a) the horsepower that can be transmitted at 1,100 rpm of the pinion, (b) the effective tooth load and the radius of its application, and (c) the magnitude of the axial thrust.

31–10. The Gleason straight-tooth bevel pinion driving the differential of an automobile has 15 teeth with 5 diametral pitch and a $1\frac{1}{4}$-in. face. The pinion transmits 42 hp at 2,500 rpm. The pinion is supported on two bearings placed $1\frac{1}{2}$ and $5\frac{1}{4}$ in. behind the large pitch circle. The gear has 60 teeth. Determine (a) the beam strength of the teeth, (b) the effective tooth load and the radius of its application, (c) the axial thrust, and (d) the radial load on each bearing.

31–11. Using data from problem 31–10 but substituting spiral bevel teeth, a $17\frac{1}{2}°$ pressure angle, and a 35° right-hand spiral (on the gear), determine: (a) the effective tooth load and the radius of its application; (b) the axial thrusts on the pinion and gear; (c) the radial load on the bearings.

31–12. (a) Determine the axial thrusts for direct and reverse rotation of a pair of Gleason spiral gears. The number of teeth in the pinion is 14, the number of teeth in the gear is 42, the spiral angle is 35°, angle $\beta = 14\frac{1}{2}°$, the pinion speed is 1,000 rpm, and it transmits 18 hp. (b) Compare these thrusts with those produced in the pinion and gear shaft of a straight bevel gear. The pinion diameter is $3\frac{7}{8}$ in., and the faces are $1\frac{5}{8}$ in.

CHAPTER 32: Worm Gearing

32–1. A traction-type elevator is operated by a 50-hp, 1,170-rpm motor through a worm drive. The worm has four threads with a 20° pressure angle and a pitch diameter of 4.250 in. The worm gear has 52 teeth of $1\frac{3}{4}$-in. pitch and has a 4-in. face. The worm bearings are 18 in. center to center, and the gear bearings are 9 in. center to center. Determine (a) the center distance of the shafts, (b) the loads on each bearing, (c) the coefficient of friction between the worm thread and the gear teeth, (d) the efficiency of the drive, and (e) the friction heat that must be dissipated, in Btu per min.

32–2. A cast-iron worm running at 240 rpm receives 2 hp from its shaft. The speed reduction is 10:1, and the distance between the shafts is 8 in. The bearings of the worm are on 6-in. centers, and those of the gear are on $5\frac{3}{4}$-in centers. Assume that the over-all efficiency of the worm and the bearings is 86 per cent. Determine (a) the circular pitch, the number of threads, and the number of teeth so that the helix angle is greater than 15°; (b) the diameter of the worm shaft, with a combined torsional stress not to exceed 8,000 psi; (c) the radial, thrust, and tangential loads on the worm and gear; (d) the loads on each of the four bearings; and (e) the friction heat that must be dissipated, in Btu per min.

32–3. Design a worm drive for a speed reducer, to transmit 40 hp at a worm speed of 600 rpm. The desired velocity ratio is 25:1, and an efficiency of at least 87 per cent is desired. Assume that the worm is made of hardened steel, and select the material of the gear.

32–4. The motor of a truck develops its maximum power at 1,200 rpm, and 91 per cent of this power is transmitted to the worm of the worm-gear drive on the rear axle. The speed reduction in the transmission is 3.33:1, and that in the worm and gear is 12.5:1. The worm has ■ quadruple thread of 3.75-in. lead, a pitch diameter of 2.635 in., and a pressure angle of 30°. Determine the maximum power of the motor based on the load which the worm-gear drive can stand, considering (a) strength alone, (b) permissible wear, and (c) heat-dissipating capacity. The worm is made of hardened steel, and the gear is of phosphor bronze and has a face $2\frac{1}{8}$ in. wide.

32–5. If the motor of problem 32–4 develops 35 hp at 1,200 rpm and all other data are the same, determine (a) the magnitude of the forces F, N, Q, and R; (b) the efficiency of the worm and gear; (c) the heat generated, in Btu per hr, at the surfaces in contact; and (d) the necessary area of the housing surface to dissipate that heat.

32–6. Design ■ worm and worm gear to transmit 60 hp at a worm speed of 480 rpm. The desired velocity ratio is 14:1. The efficiency must be not less than 92 per cent. Use a worm with three threads.

32–7. For the data of problem 32–6, design a drive using a worm with four threads.

32–8. An elevator cage is lifted at the rate of 280 fpm. The elevator-drum diameter is 24 in., and the load to be lifted is 4,000 lb. A worm gear is keyed to the drum shaft. Assuming a speed of the driving motor of 900 rpm, determine (a) the worm and gear proportions, (b) the efficiency of the worm drive, and (c) the required horsepower of the motor if the efficiency of the hoist itself is 94 per cent.

32–9. In a worm gear of $\frac{3}{4}$-in. pitch the double-threaded worm is proportioned according to Table 32–2. Assuming a worm speed of 1,075 rpm and 30 teeth in the gear, determine the efficiency of the drive.

32–10. Determine the safe horsepower which the drive of problem 32–9 can transmit if the worm is of 0.10 C alloy steel (Table 32–4) and the gear is of chill-cast SAE 43 bronze. Consider (a) the strength of the gear teeth; (b) the limiting load for wear; (c) the heat-radiating capacity; (d) the method of changing the materials which would increase the allowable horsepower without changing the diameters or speed, and the obtainable gain by this method for continuous operation.

32–11. A triple-thread worm and its gear have a pitch of $\frac{5}{8}$ in. and a velocity ratio of 20:1. The worm and gear are proportioned as recommended by the AGMA. The material of the worm is steel, and that of the gear is Bakelite. The worm speed is 1,750 rpm. (a) Determine the strength of the worm-gear teeth. (b) Find the limiting load for wear. (c) Check the limiting load based on heat-radiating capacity, using equation 32–23. (d) Determine the horsepower which the gear can transmit safely. (e) Describe the method of changing the specifications or proportions which would increase the allowable horsepower without changing the diameters or speed. (f) Compute the possible horsepower increase for continuous operation by this method.

32–12. Using data of problem 32–5, find the bearing loads for the worm shaft and gear shaft if the center distance of the worm-shaft bearings is 12 in. and that of the worm-gear bearings is 9 in. and they are symmetrically located.

CHAPTER 33: Screw Gearing

33–1. Design a pair of screw gears. The speed of the driver is 520 rpm, and that of the follower is 200 rpm; the shaft angle is 72° and the center distance must be 4.573 in.; and the teeth are to be $14\frac{1}{2}°$ involute with American standard full-depth proportions.

33–2. Assuming that the coefficient of friction is $f = 0.07$, determine the axial thrusts for the gears of problem 33–1. The driving torque is 26 lb-in., and the helix angle of the driver is approximately 32°.

33–3. Design a pair of screw gears to transmit $\frac{1}{4}$ hp. The speed of the driver is 600 rpm, and a speed reduction of 4:1 must be obtained. The shafts are at right angles to each other. Select the materials and probable coefficients of friction, and compute the efficiency of the drive.

33–4. Using data from problem 33–3, determine the bearing pressures on all four bearings. Assume that the distances from center to center of each pair of bearings is 5 in. and that the gears are located centrally.

33–5. Design a pair of screw gears to drive a governor of an oil engine from the crankshaft, which makes 180 rpm and has a diameter of 7 in. The governor speed increase should be about 1:4. The torsional resistance which the governor must overcome is 80 lb-in. The shafts are at right angles.

33–6. Indicate what factors should be taken into account if it is desired to develop a formula for the design of screw gears based on values of allowable pressures as used in other machine parts and rubbing speeds between the teeth. Indicate how each factor should influence the design.

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